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Ph.D thesis in Economics and Applied Statistics :

Longevity and pension plan sustainability in Algeria: Taking the retirees mortality experience into account

Farid FLICI

Jury :

Pr. Rabah BELAID - ENSSEA (Algiers, Algeria)............................ .... President
Pr. Azzedine BELKACEM-NACER - ENSSEA (Algiers, Algeria)... .... Supervisor
Pr. Frédéric PLANCHET - ISFA (Lyon, France)............................ .... Co-supervisor
Pr. Pièrre DEVOLDER - UCL (Louvain, Belgium).......................... .... Examiner
Pr. Mohamed SALHI - Oran II Univ. (Oran, Algeria) .... Examiner
Pr. Nadjia EL SAADI - ENSSEA (Algiers, Algeria)......................... .... Examiner

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"*Shoot for the moon, because even if you miss, you'll land in the stars.*"

— Les Brown

Longevity and pension plan sustainability in Algeria: Taking the retirees mortality experience into account

Farid FLICI

March 16, 2017

Abstract

The role of pension plans in poverty reducing among the old population requires the intervention of governments to keep their long run financial sustainability when the insurance mechanism tends to be failing. In this sense, Pay-As-You-Go pension schemes are the most sensitive pension schemes to longevity and aging effects. The worldwide population is now passing throughout an accelerating aging process. Following the recommendations of the World Bank, many reforms have been introduced in many countries over the world. Most of these reforms aimed to adopt a multi pillars system, in which the existing PAYG plans are reinforced by a parallel funded plan. The Algerian population is not excepted from this aging process which may affect the long run stability of the public pension plan still working following a PAYG system. At present, it is necessary to evaluate the long run sustainability of the Algerian pension plan under the aging perspectives. To proceed to such an evaluation, two sets of factors must be addressed : demographic and economic. Population projection allows to expect the evolution of the populations at working and retirement ages. The adoption of simplest scenarios about the future evolution of activity, employment, affiliation to social security and economic growth allows to expect the evolution of the financial balance of the pension plan. Mortality forecasting is much needed to evaluate the future improvement of longevity of the Algerian population. However, the historical data series and methods should be investigated, treated and made in the required shape for forecasting purposes. Also, the mortality experience of the retired population must be taken into account because the global population life tables are not well adapted to the retirees' experience.

Key-words: *Algeria, aging, pension plan, PAYG, longevity, mortality forecasting, population projection, experience mortality.*

Longévité et viabilité des régimes de retraite en Algérie : Prendre l'expérience de mortalité des retraités en considération

Farid FLICI

16 mars 2017

Résumé

Le rôle des régimes de retraite dans la lutte contre la pauvreté des personnes agées requiert l'intervention de l'État pour maintenir leur stabilité financière lorsque le mécanisme assuranciel tend vers la défaillance. Dans ce sens, les systèmes dits par répartition sont les plus sensibles aux effects de la longévité et du vieillissement parmi les systèmes de retraite. La population mondiale vit un processus de vieillissement accéléré. Suite aux recommandations de la Banque Mondiale, de nombreuses réformes ont été introduites dans de nombreux pays du monde. La plupart de ces réformes visent à adopter un système à plusieurs piliers, dans lequel les régimes par répartition existants sont renforcés par des régimes parallèles financés par capitalisation. La population Algérienne n'est pas exclue de ce processus de vieillissement qui est censé affecter la stabilité à long terme du régime public de retraite fonctionnant encore par répartition. À présent, il est nécessaire d'évaluer la viabilité du régime Algérien étant donné les perspectives de vieillissement. Pour procéder à une telle évaluation, deux ensembles de facteurs doivent être considérés : économiques et démographiques. La projection de la population permet d'anticiper l'évolution des populations à l'age de travail et de retraite. L'adoption de scénarios simplistes sur l'évolution future de l'activité, l'emploi, l'affiliation à la sécurité sociale et la croissance économique permet d'anticiper l'évolution de l'équilibre financier du régime de retraite. La projection de la mortalité reste nécessaire pour évaluer la longévité future de la population Algérienne. Cependant, les données historiques de mortalité ainsi que la méthodologie de calcul devraient être étudiées, traitées et mises en une forme adéquate à des fins de projection. En outre, la mortalité d'expérience de la population retraitée doit être prise en compte du fait que les tables prospectives globales ne sont pas adaptées à l'expérience des retraités.

Mots-clés : *Algérie, vieillissement, régimes de retraite, retraite par répartition, longévité, projection, projection de la population, mortalité d'expérience.*

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Chapter 1

Context, motivation and methodology

1.1 General context

1.1.1 Pension schemes as a part of social protection system

According to Norton et al. (2001), social protection system can be defined as *"the public actions taken in response to levels of vulnerability, risk and deprivation which are deemed socially unacceptable within a given polity or society".* We can understand that social protection systems represent a set of actions entertained by governments in order to protect their population and most precisely vulnerable people against social and economic risks. Socioeconomic risks exist under different forms and there are different approaches to define the "Socioeconomic risks" and also the "vulnerable populations". In their paper prepared in the framework of the European report on development 2010, Brunori and O'Reilly (2010) shed light on the three main approaches of social protection. The first approach is that of the International Labor Organization (ILO) which considers social protection as a human right. States are then considered as a "duty-bearers" and citizens as "rights-holders". This point of view is shared by the United Nations Organization (UNO). A research program was launched in 2013 on the theme of "Linking Social Protection and Human Rights". This project is managed by the United Nations Research Institute for Social Development (UNRISD) in collaboration with the ILO and the Office of the High Commissioner for Human Rights (OHCHR) and other Non Governmental Organizations (ONG's)[1]. The second approach considers that social protection must allow the vulnerable populations to escape poverty by accumulating wealth. In World Bank (2004), social protection is defined as "*a set of public measures that support society's poorest and most vulnerable members and help individuals, households and communities to better manage risks*". The third approach supports the hypothesis that social protection is an avoidable condition for economic growth. That means that social protection is seen as a main to protect vulnerable populations against socioeconomic risks and to allow them improving their life conditions. The real meaning of social protection is how to allow the whole population to contribute to the economic growth.

Giving a definition to the social protection concept returns to the enumeration of a set of socioeconomic risks that public policies may focus on. The ILO social security convention n° 102/1952[2] defined a standard minimum for nine branches of social security, these branches are: medical care, sickness benefit, unemployment benefit, old-age benefit, employment injury

[1] For more details see: http://www.socialprotection-humanrights.org/

[2] www.ilo.org

benefit, family benefit, maternity benefit, invalidity benefit and survivors benefit. The ILO 2012 convention is concerned the social protection floors recommendations (n° 202/2012). Two main recommendations were carried out : ensure a social protection floor for the whole population and extend social security coverage. A set of basic needs are then defined as following : access to essential health care, including maternity care, basic income security for children (nutrition, education, care ... etc), basic security income for persons in active age who are unable to earn sufficient incomes, in particular in cases of sickness, unemployment, maternity and disability, basic income security for older persons. Even if, according to ILO definitions, there is no difference between Social Protection and Social Security, but Social Security is an element of Social Protection and the difference between the two is the contributory or noncontributory of the system. Social protection aims to protect people without asking them any contribution to the system. Social security is an advanced form of social protection and it works following the insurance principle. In concerns of older persons for example, social protection aims to give them a basic income without asking them to take part in the contributory system (Universal protection) and social security consists to pay contributions in the ordinary conditions to have access to incomes / recovery when social risks occur.

A pension system aims to guarantee for old persons a basic income allowing them to satisfy their basic needs. States are asked to care about the vulnerable classes of the population. Old people are considered as a vulnerable population because starting from a certain age, they lose, partially or fully, their working capacity. In the other side, aged population has usually a specific needs: Health care, dependency ... etc. So, at old age, we see our income to be more and more restricted and our outcomes to be more and more numerous.

1.1.2 Pension systems designing

The functioning of pension systems combines social protection and insurance principles (OECD, 2005). This combination aims to integrate two main objectives: ensure retirement benefits smoothed to individual saving efforts and ensure equity between individuals within the same population (Merton and al., 1987). Even if usually, retirement benefits are linked to contributions following the insurance techniques, the social assistance part in pension systems aims to provide the low incomes population a minimum income to satisfy their basic needs. The insurance principles try to keep a relative equivalence between the individual contribution during the working career with retirement benefits. In an insurance context, contributions represent the premiums that we pay in ordinary conditions to benefit a recovery when the insured risk occurs. People must pay contributions during their working age to benefit a pension benefits at retirement. For this, pension plans are managed into two phases in order to convene the life cycle of people: Accumulation phase which usually coincides with the working age and the distribution phase which coincides with the retirement age. The insured risk here is related to the fact to survive to an advanced age. With aging, people see their physical and intellectual capacities reducing (Ilmarinen and Tuomi, 1992). That includes a loss of competitiveness in the labor market called loss of earning power (Wang and al., 2014). As a final result, income resources become more and more restricted with aging. The unknown variables in the insurance equation are represented by the additional life duration, and also the future health conditions. Longevity improvement weighs too heavy for insurers if we consider the pure insurance point of view. Also, health care for old persons is expected to require as more incomes as people live longer. The situation becomes more complicated if longevity improvement is not associated to an extension in the healthy life expectancy or more precisely the working age (Bovenberg and Van Ewijk, 2011-b). In addition, retirement must be planed for many decades in advance. Retirement benefits are almost defined during the contributing phase or at least by the age of retirement. Because of inflation, retired people lose consequently their purchasing power. The inflation effect can be reduced by indexing the pension benefits to inflation which implies in the other side a reevaluation of contributions (Bodie and Pesando, 1982; Clark, 1991). Unfortunately, earning of a great part of people does not allow them to support such a contribution level. Here, we must also consider the individual time preferences and its impact on saving for retirement (Goda and al., 2015). At working age, priority is almost given for the actual living needs more than for the future retirement conditions.

On the lights of all the elements presented until now, the pure insurance system (private pensions / Annuities) seems to be far from being an optimal solution in all situations. Such a system can be convenient only for the high-earning people but not for the whole population. For this, the intervention of the government is needed to manage the whole system and to protect the low earning people. The maintaining of the long-run pension system sustainability is more than to keep equivalence (in relative sense) between contributions and benefits. It aims to keep equality between the successive generations. The ideologies in pension systems designing are various and the final results are due to an accumulation of a political, social, and historical contexts and circumstances (Bovenberg and Van Ewijk, 2011-b). The discussion of such a point is not our main objective in the present work. So, we will focus only on the main contributions in this sense. The main idea behind the pension systems designing is that, as well as the difference is, all pension systems all over the world are built around three or four main functions as it was explained by Bovenberg and Van Ewijk (2011-a) :

- Ensure an optimal life cycle planning for individuals ;
- Ensure an intra-generational risk sharing ;
- Ensure an inter-generational risk sharing ;

These are the three main functions of a pension system, for which we can add poverty reducing as a fourth function which is also embedded in the three first cited ones.

Bovenberg and Van Ewijk (2011-b) gave a detailed explanation of these three functions and the role of the government in maintaining the global system adequacy. The first function makes evidence on the necessity for saving during the working age to better manage its retirement life. Equilibrium must be taken between the two parts of the equation. The more we expect to leave longer, the more we need to improve our saving efforts during the working age. This function can not be kept in an individual way because of the heterogeneity of the population. This heterogeneity can be due to many factors. Thus, inequalities start to rise at birth (Health situation at birth). Others are due to the childhood environment (early life conditions). Individuals within the same population do not have the same chances in labor market, wages and professional careers. Also, preferences and utility functions are different. The risk included by this heterogeneity must be pooled within the same group in order to make lucky people helping unlucky people. This last sentence returns to the definition of the second function of pension systems : ensure an intra-generational risk sharing. The more the number of the insured population is large the better the global risk is shared and managed. When pension systems are voluntary, there will be only the “bad-risks” to be interested by retirement insurance (Adverse selection). Imposing a mandatory system can be perceived by the government as a solution to force working people to contribute to the pension system. By increasing the number of the ensured population, government ensures a good risk sharing within the same generation and saves the low earning people. Maintaining the long run stability of a pension system consists to ensure a risk sharing between the different generations. The government is asked to force individuals to contribute for the social security system. Inequalities between generations are resulted from the demographic and economic changes. The government in the framework of its redistribution function can proceed to a risk transfer intra and inter generations by taxation and public expenditure.

1.1.3 Review of pension systems in the world

The allowance of the contributions to pay the pension benifits can be managed in different ways: Pay-As-You-Go System (PAYG), Fully Funded system or any combination of these two main systems. PAYG system is the most classical way for pension schemes financing. This system was first founded in Germany starting from 1889 (Henke, 2002; Lui, 1998; Wong, 2015). England followed up starting from 1892 (Wang, 2014). Following Wong (2015), a PAYG pension system can be defined as "*a plan in which the current pension welfare of the elderly is financed by contributions from the current working population*". That lets understand that the incomes/ outcomes of pension systems are managed in year-to-year or period to period point of view. The contribution of the insured population at working age are directly used to pay the pension benifits of the retired population. In other words, the contributing generations pay for the retired generations leading to a kind of inter generational solidarity.

As mentioned earlier, the design of pension systems combines the social assistance and life insurance principles. PAYG system is more oriented to be a form of social security than a form of life insurance. The insurance component in the actual pension schemes design were inspired from the life insurance principles which existed long before. The origin of life insurance dates back to the invention of "Tontines" in the early 1650's. Tontines refers to the Italian banker "Lorenzo Tonti" who proposed a scheme to raise money for the french king Louis XIV (Mc Keever, 2009). The principle of the Tontines involves constituting an investment fund shared between a group of persons. The future investment incomes will be distributed only to the surviving ones. Later, many improvement occurred on the primitive form of Tontines before giving the modern notion of life insurance. In 1654, Pascal invented the mathematical theory of insurance with the "random geometry", and he referred to the law of large numbers. In 1657, Huygens brings up the first life tables (female - male). Between 1671 and 1673, Johan De Witt; Dutch Prime Minister of that time, with the help of Johannes Hudde established the first formula for calculating the price of a life annuity based on survival probabilities along age with a placement rate and has thus developed a method for pricing annuities, in order to manage the crisis of public finances of his country during the war waged by France (Hebrard, 2004). The formula proposed by Witt has been developed in 1693 by the astronomer Edmund Halley (Pitacco, 2003), and is still used nowadays in annuities pricing. In 1762, we witness the creation in London of the first company practicing a variable rate with age. Previously, prices were uniformed. Despite these considerable progresses, life insurance was still in a primitive form. It was only in 1762 that life insurance was born in the true sense of the term after an enterprise practiced it successfully in England. In France, it was until 1789 that the Royal Insurance Company was authorized to practice this type of insurance. In

1818, the French State Council had to pronounce on the status of a life-insurance company and emphasized the positive aspects of the insurance contracts on life as an operation for foresight individuals to preserve the randomness of those dear to them. The opinion of the French State Council was issued on March 23, 1818. In his paper, Mc Keever (2009) gives a huge description of the evolution of Tontines from its primary form to the current life insurance and life annuities formulas. Readers interested by the actuarial calculations for life annuities can refer to Hess (2000) or Denuit et Robert (2007).

Note that the current pension schemes try to combine the social function of the government which aims to protect the poor people and maintaining a relative equivalence between the individual contributions and retirement benefits following the life insurance principle. Following a set of historical and political contexts, pension schemes were designed in various combinations of the two following extremes : Social assistance system and the life insurance one. Baldwin (1910) achieved to divide the pension systems into 6 major categories :

- Universal non contributory schemes;
- Partial non contributory schemes;
- Mandatory contributory insurance with State subsidy;
- Voluntary contributory insurance with State subsidy;
- Annuity schemes under public administration;
- Voluntary insurance under private management;

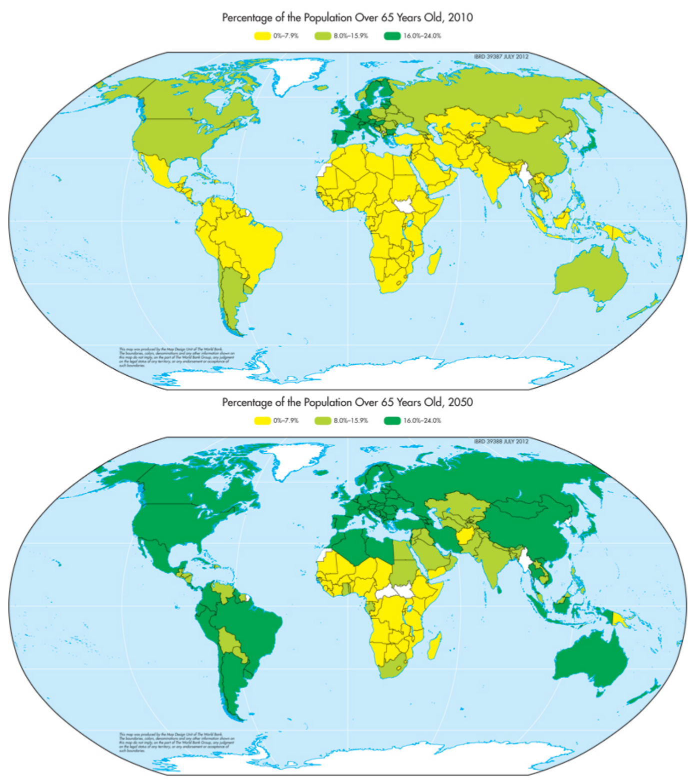
The first scheme is universal and non contributory. This involves a minimal welfare incomes which is guaranteed for old persons among the whole population without paying any contributions. This scheme is becoming an avoidable solution against the social security coverage failure (McKinnon and Sigg, 2003). A great part of the old population in developing countries are excluded from the contributory social security system. In many countries, in Africa in particular, it is difficult to extend social security among older people because of the weakness of the administrative capacities (Johnson and Wiliamson, 2008). Then, policymakers try to recover the social security coverage failure by imposing a non contributory pension schemes accessible by the whole old population. The partial non contributory schemes is based on some illegibility criteria rather than to be accessible by all aged people. This system appeared in Great Britain and Australia. But, the third system was founded in Germany. It is based on the obligation of working people to contribute to the system in function of their wage (/earning). This contribution is augmented by the employer's contribution and the State Subsidy. The fourth system is based on an individual saving for retirement for which the State add a bonus. This scheme is issued from the Belgian system. The fifth schemes is somehow different from the Belgian scheme. Here, the subsidy of the State covers only the administration fees. The Insurance fund is also managed and guaranteed by the Government. The Canadian public annuities are the most former example in this sense. The last scheme represents simply the private life annuities, where the insurance fund is managed without a direct intervention of the State.

These types of schemes have led later to the well-known multi-pillars pension system that we intend to present in the following elements.

1.1.4 A long way of reforms

Demographic and economic circumstances have been changed during these last decades. When environment is changing, tools must be changed in order to maintain the initial objectives on target. With population aging and micro / macro economic changes, it turned out that the pension schemes financed by PAYG systems are not guaranteed in terms of long run stability. In 2015, the world's population is estimated at around 7.35 billions. The part of the population aged 60 and over is estimated to 12.3%. According to the projections of the United Nations Organization (UNO, 2015), this part is expected to increase to 21.5% in 2050 and to 28.3% in 2100. The aging of the world population is then so evident even if its degree is various from a country to another. Figure 1.1 shows the evolution of the part of the population aged more than 65 years compared to the whole population in the world's countries between 2010 and 2050.

Figure 1.1: World population aging process 2010 - 2050



Source : World Bank / Pensions Data (downloaded in June 2016)

According to Figure 1.1, we can see that old people represent a part of less than 8% among the populations of Africa, central and south eastern Asia and also Latin America. In United States, Canada, Eastern Europe, East Asia, this proportion is comprised between 8 and 16%. Only in western Europe, more than 16% of the population is aged 65 years and over. By a comparison with the figure down; which simulates the situation in 2050, most of the world's countries will have older populations by 2050. This process of aging is threatening the stability of pension schemes financed by PAYG system. The number of people at working age which are theoretically supposed to contribute for one individual at retirement is falling down. The effects of aging on the stability of the pension systems can not be temporary but permanent. The design of pension schemes all over the world must undergo a deep reforms. The World Bank, being conscious of the effects of aging on pension schemes stability and also the old people's life conditions, proposed in 1994 a general framework of pension systems reform in the world (World Bank, 1994). According to the World Bank point of view, the classical way to finance pension schemes based on PAYG system will not support the pressure imposed by the population aging for the long run (Holzmann and al., 2008). What could be the optimal solution? A set of key elements were discussed in the world Bank report in concern of the design of pension systems in order to satisfy the basic functions of a pension system under the new economic and demographic constraints: Poverty alleviation and saving-incomes smoothing. The government has to decide about the targeted emphasis between the two objectives, and has to classify the population to sub-categories in term of the needs to be satisfied. Then, the secondary elements need to be discussed about how to satisfy the first fixed objectives:

- Mandatory or voluntary?
- Fully funded system or financed by PAYG system (unfunded)?
- Public or private management?

If the government aims to protect all the older population against the risk to remain in poverty for a long term, a mandatory plan is needed. A voluntary plan is not always sufficient to attract enough people to contribute for the pension plan. By contributing to the pension fund during the working age, people compare the actual contribution with the purchasing power of their promised pension incomes. People can be encouraged by the economic stability of the country : stable return rate, stable inflation ... etc. Also, the time preferences are different by agent : some people prefer to spend more and save less, others prefer to spend less and save more for retirement. The benefit guaranteed by the pension plan (Defined Benefit) must attract the heterogeneous population. If the voluntary plan is not sufficient, the regulation power of the government must play its role to extend social security coverage among the working population and impose a mandatory plan. But, in most of developing countries, public management is not enough competent to ensure this function.

A funded plan serves more the second function of a pension scheme: saving-incomes smoothing. In a funded scheme, the lifetime value of the total contributions is equal to the lifetime value of the future incomes. In such a system, otherwise called Defined Contribution system (DC), the future benefits are defined by the actual contribution combined with their future investment. Differences in life duration between individuals are compensated by pooling a maximum number of individuals inside the same group. The most formal examples of DC schemes are : Occupational plans which are proposed by employers to attract workers, and personal annuities plans. The first is partially funded system while the second is fully funded. In unfunded plan, called also Public PAYG system or Defined Benefits plan (DB), the government promises the contributing people a guaranteed replacement rate of their actual wages at retirement. A lower limit is guaranteed for the low wage category. The guaranteed value is almost expressed in terms of purchasing power. The loss of value because inflation is supported by the government. So, at retirement, individuals do not receive the exact lifetime value of their contributions. Usually, in order to reduce social inequalities, low wages receive more and high wages receive less subcidies. A funded plan encourages individual saving efforts and discourages evasion, but in the other side it does not serve the poverty alleviation. In addition, the development of such a system requires a developed capital and insurance markets.

The recommendation of the world bank was to pass from a one-pillar system to a multi-pillars system. The one-pillar systems have the disadvantage to do not allow a good perfection in terms of the different functions of a pension system which are not well correlated. In the beginning, a three pillars system was proposed :

1. First pillar : Mandatory, wages-linked, Defined Benefit system, Publicly managed;
2. Second pillar : Mandatory, funded schemes, Defined Contributions system, privately managed;
3. Third pillar : Voluntary, fully funded and privately managed;

Each pillar is conceived to serve a certain function of the pension systems. The 1*st* pillar has the objective of poverty alleviation, it can be an universal system offering a minimum guaranteed income for the whole older population or a limited benefit system based on some eligibility criteria : needs-linked or employment-related incomes.

The second pillar may endorse the saving- incomes smoothing. Then, it encourages individual saving and economic capital accumulation. This second pillar can take two forms : individual saving plan or occupational plan.

The third pillar allows to persons with high wages to augment their saving effort in the intention to receive more incomes at retirement.

All these three pillars are contributory. Later, the World Bank added two additional non-contributory pillars (Holzmann and Hinze, 2005) :

- Zero-pillar: It is a contributory universal system which deals directly with the vulnerable old population by offering them a minimum protection level;
- Fourth Pillar : Informal old age protection system is based on the intra family solidarity, not necessary based on financial supports : social supports, housing, health care;

According to the suggestions of the World Bank, a Multi-pillars system is very suitable with a multi-objectives pension system. In such a combination, each pillar is supposed to face a certain kind of problems and circumstances (Holzman and Hinze, 2005). The formal one-pillar system is less flexible in this sense. Also, the World Bank does not propose a standard solution to be used in all circumstances but just a general framework to be adapted with the specificity of each county (Holzmann and al., 2008) and taking into account the moving reform needs (Holzman, 2012).

Many countries were motivated by the World Bank recommendation (World Bank, 1994) to move from the unfunded to the fully funded pension scheme. Following the experience of Chile which adopted a DB schemes as a main pension scheme starting from 1980, 30 other countries has experienced a similar experience by adopting the DB scheme either as a complementary, optional or main pension scheme. Australia followed in 1992 by implementing a DC main pension scheme, Mexico in 1997, Kazakhistan in 1998, Kosovo in 2002 and Nigeria in 2005.

Parallel to the reforms occurred in the framework of one-pillar pension schemes, in developed countries where the financial and the insurance markets were enough developed, pension systems started to take a numerous forms combining the first defined pension schemes in order to make more coherence in the functioning of multi-pillars system and to convine all parts of the population (Holzmann, 2012). Many forms appeared under the zero-pillar (social-pension) and we can distinguish : 1) Targeted programs destined to protect the older low income population, 2) Basic pensions functioning by flat rate pensions or employment-linked pensions 3) Universal schemes which are completely non contributory. Under the first-pillar, we can find : 1) Defined-benefit schemes (DB) making actuarial relationship between retirement incomes and individual contributions. 2) Notional defined-contribution schemes (NDC), in which benefits are defined by the individual contributions and returns augmented by a non financial interest; 3) Provident Funds (PF), where benefits depend on the individual contributions and their investment returns under public management. Pillar-2 contains two secondary schemes : fully-funded Defined Contributions (DC) or Defined benefit (DB), mandatory and privately managed.

Table 1.1: DC system implementation - Some worldwide exemples

Country	Type of scheme	Year of Implementation
Chile	MS	1980
Australia	MS	1992
Peru	OP	1993
Colombia	OP	1994
Uruguay	CO	1996
Mexico	MS	1997
Kazakhstan	MS	1998
El Salvador		
Poland, Swetzerland	CO	1999
Latvia, Costa Rica	CO	2001
Bulgaria, Estonia, Croatia	CO	2002
Kosovo	MS	
Norway	-	
Russian Federation	CO	2003
Dominican Republic	-	2003
Lithuania	CO	2004
Slovak Republic	OP	2005
Nigeria	MS	
Macedonia, FYR	CO	2006
Romania, Panama	CO	2008
Kyrgyz Republic	CO	2010
Egypt, Arab Rep.	CO	2012

Type of scheme: MS (Main Scheme), CO (Complementary Scheme), OP (Optional Scheme). – (Missing)

Source: World Bank : Pensions Data (Downloaded in June 2016)

Also, some developing countries and other medium incomes developed countries adopted similar reforms. For example, according to table 1.1, The Eastern European and central Asian countries tend to implement a DC funded system to complete the existing PAYG system. Many reforms occurred between 2002 and 2008 : Bulgaria, Estonia and Croatia in 2002, Russian Federation in 2003, Lithuania in 2004, and Macedonia, Romania, Panama during the period [2006-2010]. Egypt followed a similar way of reform by implementing a complementary funded pension scheme starting from 2012.

1.1.5 Pension system risks

In order to better manage pension system for the long run, the underlying risks must be well known and well evaluated. According to the main principles which allow the running of the pension systems already presented, we can conclude that the nature of the risks to manage inside a pension plan depend firstly on the structure of the system itself. Mainly, there are two approaches to finance pension systems : a PAYG system with Defined Benefit Scheme and a Defined contribution scheme. The first system works following the inter-generational risk sharing while the second is managed in a similar way as an individual saving principle with an intra-generational risk sharing. The PAYG system is very sensitive to the demographic and economic balances between the contributing and the retired generations. In a Defined

Contribution scheme, the risk function is defined on the basis of the characteristics of each generation. Here, we will shortly present the main risks under each systems.

Pay-As-You-Go System

The long term stability of PAYG systems is defined by the stability of the population structure. A reduction in fertility rates accompanied with longevity improvement will lead to an unbalance between the working population and the population at retirement in the future. The risk of aging can heavily affect the stability of pension plans under PAYG system, especially when it is associated with a set of other economic factors. The management of such a system requires to have a continuous look on the future changes in population structure, aging phenomenon and longevity improvement.

Demographic risks:

- Aging : The aging process is caused by a significant reduction in fertility rates. When it is accompanied with an improvement in longevity, that leads to an acceleration in the aging process. A decrease of the ratio (population at working age / population at retirement age) must be recovered by either a relative reduction in retirement pension or an augmentation of the contributing rate accompanied by an extension of the number of contributing years required to access to retirement pension. Such a demographic change can be managed in many ways. The government can augment taxes for the future working population to ensure the payment of the future retired generations (redistribution), but what about ensuring equity between generations?

Economic risks:

The economic risk are related to a change in the economic growth between the working period and the period of retirement.

- Wages: An increase / decrease in the general level of wages in a country affects directly the volume of the future contributions to the pension system. Also, when wages are relatively low, people care about their current life conditions rather than saving for retirement;

- Unemployment : an economic recession leads automatically to an increase in employment and by the way to an increase of the informal employment. In economic crisis, it becomes more difficult to keep people inside the social security system;

- Affiliation to social security : The affiliation rate to social security can be affected by a change in the micro-economic individual behaviors. Also when the social security affiliation is mandatory, the government shall keep encouraging people to contribute for social security and retirement. People must be motivated by the stability of inflation, economic stability ... etc. because contributing people are almost carrying about the conditions of their own retirement;

- Inflation : retirement pension are usually defined at the year of departure for retirement on the basis of the number of contributed years and also the final wage. The delay between the age of 60/65 and the surviving age limit can be too long and can reach

20-30 years in average. The effect of inflation can be very important on the whole period and can push-down the purchasing power. Usually, the public fund support certain categories with low retirement incomes. In a Defined Benefits schemes, a relative wages replacement rates must be guaranteed by the government in the framework of its redistribution function ;

In addition, there are many ultra economic factors which may affect the long run stability of PAYG pension schemes : Political risks, wars, economic crisis, ...etc.

Defined Contribution system

Also known under Defined Contribution system, risks can be grouped into two main categories : demographic and economic. The difference consists in the way that these risks affect the stability of the pension system. In DC systems, the future retirement incomes are defined in function of the accumulated contributions, the future return rate and also by the survival function of the considered generation of retirees. In such a system, individuals support more risks as in PAYG schemes. This risk is related to the fact that the actual contributions are defined on the basis of the future expected evolution of the elements in the equivalence formula, and these underlying elements are expected to vary over time.

Demographic risks:

- Specific Mortality : In DC pension schemes, the future outcomes are mainly defined by the residual life expectancy of the retirees portfolio. According to this, the number of payments which are to be payed to survival retirees are related directly to the mortality function of these least. In some cases, the mortality function of retired people is different from the mortality function of the whole population. When a specific life table for the retired is missed, the use of a life table constructed on the whole population data can be used as a model to describe the mortality function of retirees.

- Longevity : In addition to mortality, longevity is affecting the future outcome of pension funds. Longevity is the phenomenon of continual reduction in mortality at all ages. The probability to reach a high age become more and more important. This phenomenon affects directly the death calendar of retirees and makes retirement benifits payment much longer than it was expected;

Economic risks:

- Working capacity (disability): A loss of the working capacity due to an accident results in the reduction of the contributing capacity of the insured. That leads to the reducing of the contributing years and then the future retirement benefits;

- Unemployment : The number of years in employment defines directly the accumulated contribution at time of departure for retirement. The more the employment period is longer the most the contributing effort (contribution / earning) is moderated;

- Future return rate: A great part of uncertainty in pension schemes functioning under DC system is due to the future evolution of the contributions return rate;

- Inflation: In the same case as in PAYG systems, inflation affects the purchasing power of the future retirees. The inflation risk is supported by the insured people themselves. Transferring such a risk to the insurer is traduced by an augmentation of contributions;

1.1.6 Review of the Algerian pension system

The Algerian pension system was unified since 1984. Before this date, the pension planing was separately managed by 7 funds. These 7 funds were offering the same advantages in the same conditions for different professional categories.

- la CAAV (Algerian Fund for Pension Insurance) dedicated for the pensioners of the general regime;
- La CGR (General Retirement Funds) for public servants retirees;
- La CNMA (National Agricultural Mutual Funds);
- La CSSM (Social Security Fund for Minors);
- La CAVNOS (Pension Insurance Fund for non-Salaried);
- L'EPSGM (Establishment for Social Protection of Seafarers);
- La CAPAS : covering the pensioners of the public company "SONELGAZ" (Electricity and Gas of Algeria);

The architecture of the Algerian pension system after the independence in 1962 was inherited from the french system which has been extended to the whole salaried population in 1953. Before this date, only some professional categories were covered by the pension insurance system such : railway workers, transit workers and the employees of the public company "Electricity and Gas of Algeria". The pension insurance was extended to the non-salaried categories starting from 1956. Starting from 1984, all these pension regimes were unified under the tutorship of the "Caisse Nationale des Retraites" (CNR).

The Algerian pension system is mandatory, financed by PAYG system, earning-linked Defined Benefits, Publicly managed. The contribution is directly linked to wages. Benefits are defined in function of the average wages of the last 60 months and the number of years of contribution. A minimum pension income is guaranteed and defined to be no less than 75% of the National Minimal Guaranteed Wage (NMGW). That comprises survivor's benefits for the spouse, parents or orphans. Further details about the characteristics of the Algerian Pension System are given in chapter 7.

To summarize, the Algerian pension plan is still based on single pillar system (Pillar 1), as it is mostely the case in the Middle East and North Africa (MENA) region. Pension systems is MENA region are still young, and they marked lot of weaknesses since they were first implemented. In the first stage of each pension fund history, contributing population is much more important than the population of retirees. Consider the financial equilibrium of the system from the cash-flows point of view is not sufficient to assess the sustainability of the system for the long run. Robalino and al. (2005) focused on the structural problems that include pension systems in MENA region and averted the necessity and the emergency to proceed to a deep reform. According to the same author, even without taking into account the effects of population aging, the actual pension design in MENA region suffers some permanent inefficiencies (Robalino and al., 2005):

1) The first problem is the abusive generosity of the system: while the replacement rate of the working age income is almost 50% in average, MENA's pension system offers a replacement rate of around 75%. The problem is that the promised pension benifits are large and seem to be unsatisfied without a significant intervention of the public treasury or a parametric reforms : augmentation of the contribution rates. Ben Brahem (2009) discussed the pension systems generosity in North African Countries (Tunisia, Algeria, Morocco) and confirmed that the high generosity of the pension systems in the cited countries can not be kept for the long run because of population aging.

2) The second element which threatens the pension systems stability in MENA region consists in how these systems were designed. The way that benefits are defined distorts labor market supply and saving decision. Linking benefits to the final wage rather than to the average earning conduces to systemic generous benefits and then to some manipulation in salary definition along the working career. The configuration of the system encourages evasion and discourages the individual saving efforts and makes working population to prefer early retirement. For the case of Algeria for example, even if the age for retirement is fixed at 60 years by the regulation, an important part of workers get retired long before. As we will see in chapter 7, some retirement formula (retirement without age condition, early retirement, 5 years bonus for women ... etc) allows to reduce significantly the average age for retirement. Also, the current system leads to generate some inequalities in terms of notional benefits distribution in intra and inter generations (Mendil, 2014).

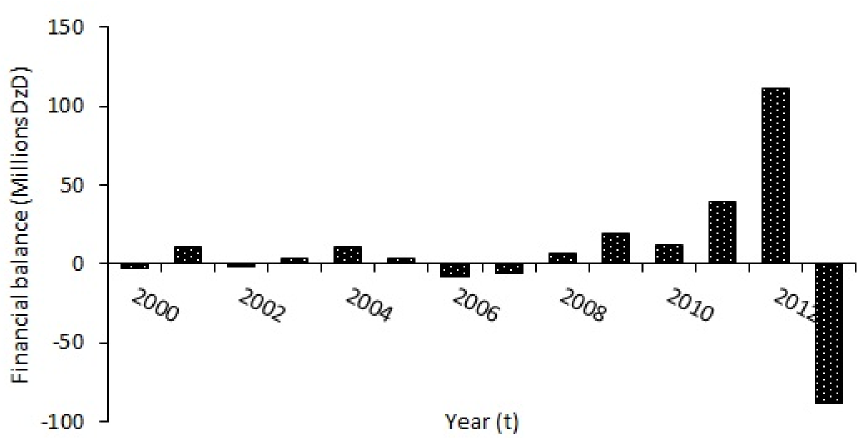
In addition to these two flagrant elements, many other sources of long run pension system unsustainability can be cited:

3) The weakness of the administrative efficiency;

4) The modesty of coverage rates;

The fact that pension systems represent a long term commitment requires an efficient planing at all sides. It will be very difficult for MENA countries, including Algeria, to pay the promised benefits under the new economic and demographic challenge. If we observe the recent evolution of the financial balance of the "Caisse Nationale des Retraites CNR" treasury (Figure 1.2), we can easily observe the difficulties which start to rise.

Figure 1.2: Recent evolution of the financial balance of the CNR



Source : CNR (2014)

From Figure 1.2, we can easily observe the financial instability which starts to emerge in the Algerian pension plan. The situation will be more complicated with the forthcoming population aging. A parametric reforms will not be sufficient to keep the long run stability of the system. It is not easy to extend social security among the working population in a short period. Such an alternative must focus on the determinant of social security demand in the actual circumstances (Merouani et al., 2016). A significant augmentation of the contribution rates for social security will encourage evasion and will affect the existing labor market equilibrium. Reforms must be oriented to pass from a one-pillar to a multi-pillars pension system, by reducing the incidence of the unfounded schemes and to complete it by a funded scheme (Chourouk, 2003). In such a perspective, longevity risk must be considered. For each pillar, the future time distribution of pension benefits is, directly or indirectly, affected by the continuous reduction in mortality.

1.2 Longevity risk in pension plans

As we have already seen, longevity risk is prominent in concern of the pension system long run stability. In a PAYG system, it affects the aging process and provokes an unbalance between the current contributing generation; which is the future retired generation, and the future contributing generation. In DC system, it directly affects the duration of payment of the pension benifits. In the two systems, the longevity risk affects also the health care expenditure. The more people are living longer the more their health care requires more incomes. However, the pension system is designed, future benefits are directly or indirectly affected by improvements in longevity. In PAYG Defined Benefits schemes, longevity risk can not be shared within each generation. The promised benefits can be covered only by an augmentation of the contributions of the working generation (Hassler and Lindbeck, 1997). Such a parametric adjustment must be based on the evaluation of the time distribution of the promised benefits which underlay an estimation of the future improvement in longevity.

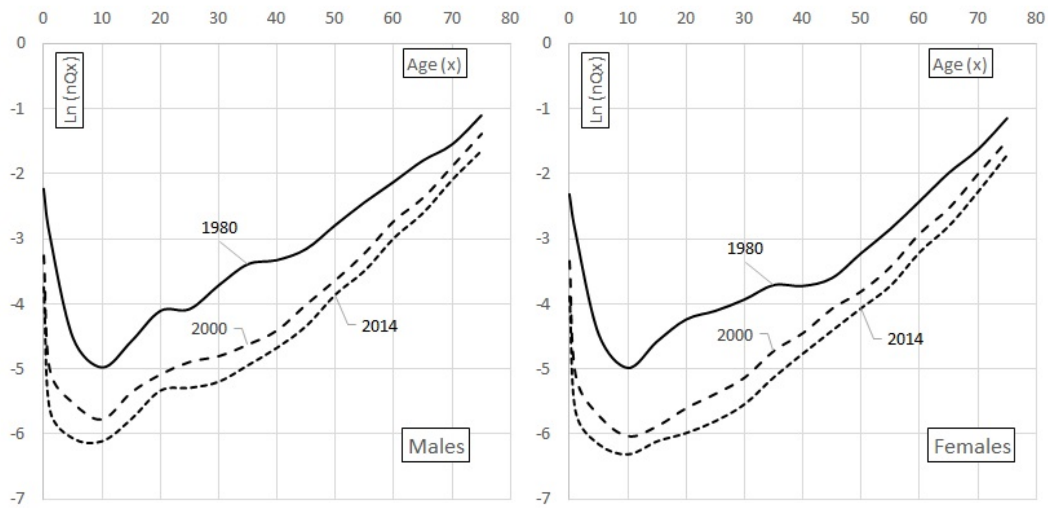
The Notional Defined Contributions (NDCs) Pension Schemes become a suitable reformulation of the classical PAYG system. The advantage of such a scheme is that the future benefits are defined on the basis of contributions, notional return rate and the future improvement in longevity. The more longevity improves the more future benefits decrease (Cipriani, 2013). That is why it is important to study the effects of longevity improvement on the stability of NDCs pension schemes (Knell, 2013; Devolder and Melis, 2015). In their paper, Arnold and al. (2016) focused on the problem of longevity improvement among a NDCs pension plan. Their main idea was that the future improvement in longevity can be covered by keeping the accumulated contributions by persons dying before getting retired. Note that usually, these Survivors Dividend (SD) are distributed to survivors among the insured cohort. The fully funded pension contracts ensure a suitable framework for insurance against longevity risk. The annuities market proposes many solutions for risk transfer (Blommestein and al., 2009). The reader can refer to Richards and Currie (2009) for further details about life annuities pricing and reserving when longevity is improving.

We can conclude that the longevity risk importance depends on the pension system design. The effects of such a risk and the reasonable solutions are related to the way that the pension system is managed. But, in all case, longevity risk evaluation allows to approach the future promised benefits and to adjust contributions in their function.

1.2.1 Recent improvement of longevity in Algeria

Since 1977, life expectancy has improved in Algeria and mortality rates have decreased at all ages. Life expectancy at birth has improved from about 56 years for females and 54.1 for males in 1977 to respectively 77.8 and 76.6 in 2014. This evolution is presented with further details in section 2.3. Figure 1.3 shows the evolution of the mortality age pattern between 3 time points : 1980, 2000 and 2014.

Figure 1.3: Mortality reduction in Algeria 1980, 2000 and 2014



Source : Annual publications of ONS (1981, 2001 and 2015)

This improvement has been resulted from several factors: improvement in education, in medicine, hygiene, economic ..... etc. This improvement is supposed to keep its trend in the future, but we need to know with more accuracy how mortality schemes will be during the coming decades. Several models have been proposed to model longevity. Most of the proposed models are based on the extrapolation of the historical trend in the future.

1.2.2 Some notes on longevity modeling

If mortality models have appeared and been used for actuarial calculations since the $17th$ century (Halley, 1693), longevity modeling has been developed only by the end of the $20th$ century. In the beginning, researchers (actuaries and demographers) tried to forecast independently age specific mortality rates by a direct use of the time series models. In a second stage, the mortality forecasting methodology has marked some improvement. Rather than forecasting each series of the observed age specific mortality rates independently, the new methodology consisted to : fit the annual life table by some parametric mortality model (Gompertz, 1825; Helligman and Pollard, 1980), then the historical series of the models parameters are projected as time series. A rich review of these two first methods are presented in Tabeau (2001). The prospective mortality models as they are known today are principally issued from the contribution of Roland Lee and Lawrence Carter (Lee and Carter, 1992). The main idea of the LC model is to reduce the matrix of the historical mortality surface (ages in lines and years in columns) into 3 vectors: 2 are related to age and 1 to time. The problem of forecasting age specific mortality rates is then reduced to forecast the time components. A detailed review of the prospective mortality models is presented in section 5.2.

1.2.3 Mortality Data in Algeria

The first Algerian life table based on the whole population data has been constructed in 1977 by the Office of National Statistics (ONS). Until 1998, it was not easy to publish such a data with an annual frequency. Some calendar years life tables were missing (see Chapter 3). Also, for some years, the published life tables were closed out before the open age group [80 and +[. Beyond the closure age, the results of the forecast and their quality are widely related to the quality of the data used as basis for forecasting. A deep analysis of the historical evolution trend is supposed to provide a clearer idea about the evolution of mortality level, the methodological changes and data quality improvement occurred during the observation period. Therefor, we dedicated all the 3 first chapters to study and to complete the Algerian mortality surface. The implementation of the mortality forecasting models requires the disposal of a complete historical mortality surface containing ages (x) on lines and years (t) on columns and the corresponding age specific mortality / death rates in cells. Before to proceed to the forecasting of mortality rates for the Algerian population, we start by making the data at our disposal in the required format. Then, the specific mortality of retirees will be projected by a positioning on the global population mortality forecast.

1.3 Retired population life tables

1.3.1 General context

The mortality of retirees is supposed to be different from the mortality of the whole population. The use of the global population life tables for actuarial calculations related to a portfolio of retirees can lead to a king of mis-estimation of the mortality function of the insured population. To improve the calculation accuracy, the risk function must be adapted to the experience of the concerned population. The differences in life expectancy between sub-groups within the same population can be explained by ethnic differences, life conditions, health care, professional category... etc. When data is enough available, the experience of mortality of the pensioners can be studied by taking into account the socio-professional category, but when it is not, the construction of a life table summarizing the mortality experience of all the pensioners in a country can be more suitable for actuarial calculation than the use of the global population life table.

1.3.2 Experiences of other countries

The experience of the USA in the use of life tables specific to the annuitants population is very huge. The first American life table specific for annuitants has been constructed in 1951. These actuarial life tables were known under "Group Annuity Mortality : GAM-51" and have been based on the mortality data of the public sector and used to calculate both private and public life annuities in USA. An updated version has been published in 1971 (GAM-71). Starting from 1983, and in order to specify the used toolss for actuarial calculation to the real mortality function of the insured population, the Actuarial life tables has been separately constructed for private and public annuitants populations: GAM-83 and UP-84 "Uninsured

Pensioners". The separation between males and females has been first introduced in 1994. The new published actuarial life tables (UP-94 and "The Group Annuity Reserving GAR-94") which separate between males and females and also allow to do projections on the basis of an improvement scale « AA ». This scale is constructed on the basis of the observed trend in mortality reduction for 'the Civil Service Retirement System – CSRS' between 1977 and 1993 (SOA, 1995). Since 2000, UP and GAR Life tables were grouped under a common nomination « Retirement Plan 2000 life table: RP-2000 » including 3 types: Public sector, private sector and a global life table. In 2014, 11 tables have been proposed for males and females (RP-2014) including a projection scale (2014 Mortality Projection: MP-2014).

In Canada, the absence of specific life tables for the annuitant population pushed the authorities to use the US life table rather than to use the Canadian global population life tables. For a long time, the actuarial calculations of the Canadian pension schemes have been based on the UP-94 and RP-2000. Starting from January 2015, prospective life tables specific to the Canadian pensioners have been established CPM-2014 (2014 Canadian Pensioners Mortality).

In the United Kingdom, « Continuous Mortality Investigation : CMI » publishes regularly dynamic life tables adapted to the retirees mortality experience : (Series « 92 », « S1 » series, « 00 »). These periodic life tables include a projection scale allowing to forecast the age specific mortality rates (IAFA, 1990). In 2003, this projection scale was replaced by the use of the stochastic mortality models (CMI, 2004). Starting from 2009, new series of life tables were published on the basis of the mortality data of the period 1961 – 2008. These data are annually revised.

In France, Life annuities are priced starting from 1994 on the basis of a female generational life tables (TPG 1993). This table has been revised in 2005 separating males and females (TGH 05 and TGF 05). In all circumstances, in developed countries, we almost tend to use an Actuarial life tables adapted to the insured population than the whole population life tables. Also, it was noticed that the use of prospective life tables is recommended for life annuities pricing and reserving and also for pension systems actuarial evaluation.

Developing countries are doing some efforts in order to build actuarial life tables adapted to the mortality experience of their whole population or of the retirees / annuitants populations. For a long time, some African countries have been using a former French life tables for life annuities calculations. But recently, many attempts have achieved to the construction of their own actuarial life tables. In Tunisia, starting from January 2009, a generational life tables constructed on the whole population data (TGEN99) has been officially imposed for life annuities calculations. In Sub-Saharan Africa, the works of Aymeric Kamega (Kamega, 2011; Kamega et Planchet, 2011) allowed to constructed a dynamic life tables specific to life annuities market in the sub-Saharan region.

1.3.3 Some perspectives for Algeria

The general constant that can be turned out through the presentation of all these exposed experiences is that there is an evident tendency to adapt the actuarial life tables to the retirees' experience rather than the use of the global population life tables. Equally, the importance of the use of the dynamic life tables rather than the periodic life tables appears clearly. Algeria is still lagging behind that vision. Although, the actuarial life tables (used for both life insurance products and life annuities reserving and pricing) are based on the global population mortality data. These tables have been constructed in 2004 by the National

Council for Insurance NCI (CNA, 2004). These life tables were constructed on the mortality data of the global population observed during the period [1997-1999]. The table TV 97-99 (TV as Table Vie which means in french Table for Life) is used for pricing the Life products (Life annuities) and the table TD 97-99 (Table of Deaths in french) is used for the insurance products related to death. Tables TV 97-99 and TD 97-99 are still used nowadays for life insurance pricing. In concern of the retired population, there is no specific life table published yet. The global population is almost heterogeneous, and the use of the whole population life table for actuarial calculation is not supposed to give the best estimates of the annuitants or the pensioners mortality. For more accurate actuarial calculations, the actuarial life tables must be adapted to the experience of the directly concerned population.

1.3.4 Dynamic experience life tables with limited data

When the insured population data is relativelly important and available for a long period, the construction of dynamic life tables for the pensioner's population can be done following the same methodology used to construct a dynamic life table for the global population. To cite some examples is this sens, we can refer to those cited in section (1.3.2). The generational french life tables TGH 05 and TGF 05, tables RP-2014 in the USA and also the life tables published by the Continuous Mortality Investigation (CMI) in UK are directly based on the pensioner's data. In the case of new insurance company, or a new insurance market, the use of an external reference is needed to project the mortality experienced by the insured population. The interested reader can refer to Thomas and Planchet (2014) or Planchet (2005 ; 2006) for more detailed presentation of the most used methods in this sense.

1.4 Objective and Methodology

The main objective of the present work is to address the effects of longevity on the stability of the Algerian pension plan. The evaluation of the future engagement of the pension fund under PAYG system is directly related to the estimation of the mortality function of the pensioner's population and to expect its future reduction. The future balance incomes - outcomes of the Algerian retirement funds will be determined by a set of parameters : Aging, inflation, wages, employment, labor market participation ... etc. Here, we will suppose that, the economic factors will stabilize at their recent observed level or continue to evolove according to the recent observed trend. Hence, we will focus only on the effects of aging and longevity improvement. So, it will be required to construct dynamic life tables based on the retired population mortality experience. Unfortunately, this kind of data is not available for a long period (only 10 years). That implies that we can not do a robust forecast. The practice in such circumstances aims to position the experience mortality future trend on an external reference : the global population projected mortality. The global population mortality data of Algeria is available starting from 1977. The length of data allows to use and compare a set of dynamic mortality models in order to project the age specific mortality rates. However, the data quality must be addressed before to construct dynamic life tables.

1.5 Outline of the thesis

In summary, our work is arranged into 7 chapters including the current introductive one. In first (Chapter 2), we would give a pertinent analysis about mortality evolution in Algeria during the past half century. The evolution of life expectancy from the independence of Algeria in 1962 till today has marked some changes in terms of evolution trends and level. Except the terrorism events that happened in Algeria during 90's, there is no other event which is supposed to provoke a change point on the life expectancy evolution series. Methodological changes have had a significant effect on the observed change points.

In Chapter 3, we estimate the missing data in the historical mortality surface of Algeria. We remind that there are some years of the period [1977 - 1997] which are still with no data, and there are also some life tables which were closed out before the age of 80. Mortality forecasting models must be based on a complete and continuous mortality surface. Since that, we propose in this chapter to complete the missing data in the Algerian mortality surface before to do a forecast.

Chapter 4 focuses on the old age mortality. Algerian life tables were almost closed out at 75, 80 or 85 years old. Mortality beyond the closure age is summarized by the residual life expectancy which is indirectly estimated on the basis of the Coale-Demeny model life tables (Coale and al., 1983). This estimation process can lead to a missestimation of the old age mortality especially in the case of African populations (Enakem and Som, 1984). Actuarial calculations need detailed mortality rates extended until the surviving age limit or near to. Several models have been proposed to extend old age mortality rates starting from the mortality trend observed at adult ages. The usual models proposed in this issue are presented in (Section 4.3). In this chapter, we propose to extend the old age mortality rates in Algeria by using and comparing some extrapolation models.

Mortality forecasting is treated in Chapter 5. After presenting the main contributions on mortality forecasting models, we compare these models to fit the historical mortality surface. For models comparison, we use mainly the goodness-of-fit and the predictive capacity. In some cases, it turned out that the use of these only two criteria does not allow to obtain coherent results. for this, some complementary criteria are used : regularity of the projected mortality surface, evolution of the Mortality Sex Ratio ... etc. The obtained mortality surface will be extended until the age of 120 on the basis of the results issued from Chapter 4. The age of 120 is supposed here to represent an estimation of the surviving age limit for the Algerian Population. This assumption is based on the maximal surviving age observed throughout the MICS IV survey (Flici et Hammouda, 2016).

Chapter 6 will be dedicated to construct a dynamic life table adapted to the retired population specific mortality. In first, crude mortality rates will be estimated from the distribution of the observed deaths and the population at risk. We note that the retired population mortality data are available for the period [2004-2015] by sex and by five-age groups going from [50-55[to [95-100[. The obtained mortality surface will be fitted to reduce sampling fluctuations due to the reduced size of the population at risk. Then, we will try to define a regression model allowing to position the retired population mortality (experience mortality) on the external reference which is supposed to be the global population mortality. Once the regression model is defined and estimated, the experience mortality surface will be projected to the future.

In Chapter 7, we will present the elements which may affect the financial stability of the Algerian pension system in the long terms. The projection of the Incomes/ outcomes of the pension fund must be based on a population forecast. Unfortunately, Algeria does not have a population projection for the period beyond 2030. In a first part, we do a population forecast by the component method. For mortality, we do a coherent mortality forecast by using the Lee Carter model under the Ratio-Product-Method proposed by Hyndman and al. (2013). For fertility, we adopt as a central scenario that the Global Fertility Rate will full down from 3 births per woman in 2015 to 2.5 in 2070. On the basis of the population forecast, we will define the expected evolution of the contributing population and the retired population numbers. Then, we fix a scenario about salaries and pension amounts evolution. The mortality schemes of retired population is supposed to be defined by the dynamic life tables calculated in Chapter 6. In final, the evolution of total expenses and total incomes are projected.

In final, we will conclude with a general conclusion summarizing the main findings.

Chapter 2

Review of a half-century of mortality changes in Algeria : 1962 -2012

Joint Work with Dr. Nacer-Eddine Hammouda, Research Director, CREAD Algiers, Algeria.

2.1 Introduction

Following the independence of Algeria in 1962, the Algerian population knew a period of recovery in marriage and birth rates accompanied by a progressive decrease of mortality. During this period, the Algerian administrative and statistical services were still unable to ensure an exhaustive coverage of the changes that were going through the population after the withdrawal of the colonial administration. Therefore, it took time to achieve the first studies about the Algerian population. In 1966, a first general census of population and housing was executed (GCPH 1966), followed by a national statistical population study (NSPS 1969-70). Both operations were carried out mainly through the French technical cooperation.

Efforts were intensified by the early seventy to achieve in 1977 the first Algerian life tables based on civil registration data. The civil registration coverage remained insufficient and not well known especially for deaths. This situation harms a good knowledge of demographic phenomenon; hence an estimation of the missing information by specific investigations was needed. The reliability of the life-tables and the parameters which they provide are mainly based on a better knowledge of the phenomenon of mortality. In this sense, the estimations of life expectancy at birth in particular still raise many reservations on the part of international institutions. The calculation of the human development indicator (HDI) and the human poverty indicator (HPI) is respectively based on life expectancy at birth and the probability of dying before age 40.

Several studies have attempted to analyze the evolution of mortality in Algeria after the independence. For a long time, the unavailability of a long and homogeneous data series has disallowed any attempt in this sense. However, the works of Salhi (1984) allowed to reconstruct the evolution of mortality in Algeria from 1965 to 1981.

For international analysis and comparison, the estimates of the United Nations Organization (UN) or those of the World Health Organization (WHO) are often used (see for example Tabutin et Schoumaker, 2005). The purpose is that, the national estimates made by the Office of National Statistics (ONS) and those established by international organizations tend to diverge over time. In 2012, life expectancy at birth in Algeria is estimated by the WHO at about 72 years (WTO, 2013), those of United Nations are around 70.9 years on average for the period 2010-2015 (UN, 2013), while the ONS gave a value of 76.4 years (ONS, 2013). This difference, which is resulted from the difference in terms of the methodology and the used data, makes researchers and academicians confused about data that could lead to a coherent analysis of mortality changes in Algeria. It is still early to comment on such an issue that could be the subject of other research papers.

The objective of this Chapter is to assess the evolution of mortality in Algeria from independence to today. This analysis will be based on data published by the ONS and take into account the methodological changes.

2.2 Overview of mortality data in Algeria

The calculation of mortality indicators in Algeria is mainly based on data continuously recorded by civil registration services. However, this registration is incomplete and needs some corrections. This correction requires specific investigations and comparison with data from other sources such as hospital data and census data.

The data recorded in the civil status (number of live-births, stillbirths and deaths by age-groups) are transmitted to the ONS in monthly digital slips (MDS). While these bulletins ensured full information of the different demographic events, the transmission and the exploitation delays were very long. In some cases, some bulletins were not exploited. Also for the events recorded by judgments, the transmission delays are so long.

The vital statistics service in Algeria was first founded by the French colonists in 1882 (Biraben, 1969). During the colonial era, the population data collecting lacked rigor, particularly when it was concerned with the indigenous population. Thus, the bulletins were filled irregularly and only in the 58 largest municipalities (Negadi, 1974). After the effective resumption of the vital statistics services in 1963, the challenge was how to improve the quality of population statistics. The result was the widespread of the recording over the entire population starting from January 1964. This led to the publication in 1968 of the first demographics data of Algeria after its independence. It gathered the vital statistics for the years 1964 and 1965 (SEP, 1968).

The GCPH 1966 provided the age structure of the Algerian population and the needed denominators for mortality indicators calculation. In contrast, some weaknesses were alleged to the census data like a slight underestimation of the female population (Maison, 1973; De Lamaze, 1977). This underestimation may distort the calculation of the different demographic indicators.

Certainly, the vital registration coverage of the different demographic events remains far to be perfect and quantified. Moreover, the statistics recorded between 1964 and 1970 have marked a significant irregularities (Mouffok, 1984). The correction of such ledge requires essentially an estimation of the coverage rate. Then a correction factor is applied to approach the real occurrence from the recorded numbers. In addition, it is necessary to correct the recorded numbers of live-births and infant deaths by false stillbirths[1]. The proportion of stillbirths among infant deaths was estimated at 40% during the period 1964-1970 (Tabutin, 1975). But, this is expected to decrease over time. Unfortunately, the ONS publications do not address this point and do not show how it is used to correct the crude data.

[1] A birth gives rise to either a live birth or a stillborn. Children who die before being registered in the vital registration services are sometimes recorded as a stillbirths. Those least are called false stillbirths.

In the early 70's, a multi-round demographic survey (MRDS) has been concluded in the framework of the NSPS 1969-70 and has consolidated the information about mortality in Algeria. The NSPS, in its section dedicated for fertility, permitted the estimation of mortality rates at very young ages of the 1960-69 generations (Vallin, 1975). When analyzing results, Tabutin (1974) pointed out the possibility that the observed low level in infant mortality, compared to the African estimates, could be resulted from an eventual deaths under-coverage.

The MRDS which allowed the establishment of the first Algerian life table for its northern population, provided the vital registration coverage rate for births and deaths (Daoudi, 2001). A confrontation of the results of this survey to the vital registration statistics demonstrated that over a third of deaths were not recorded. The survey also revealed that female deaths and those occurred in sparse area were relatively less covered (Negadi, 1974). On the basis of the obtained coverage rates, the ONS has corrected the recorded numbers of deaths and births. However, coverage rates are not expected to keep constant over time, hence it is needed to carry out a periodic surveys to allow updating the correction factors. In such a context, keep using the same correction factor over a long period may lead to an exaggerated overestimation of mortality rates, particularly for infanthood (MSPRH, 2002).

The quantitative improvement of civil registration data allowed the ONS to ensure, starting from 1977, publishing national life tables for based on civil registration data, with almost an annual frequency. The second attempt to estimate the coverage rate was concertized on the occasion of the workforce and demography survey conducted in 1981. Compared to the situation given by the 1969-70 survey, it is turned out that the coverage rate has improved in 1981. The coverage rate for deaths increased from 60.60% to 81.06%. This survey supervised by the ONS showed that only 77.5% of infant deaths were covered. The under-coverage of births was relatively less important. This differential under coverage between births and deaths leads to an underestimation of the IMR. According to the mortality rates observed during the period 1976-1981, Mouffok (1984) referred the impact that could have an over-recording of births on the underestimation of the IMR for the cited period. The coverage rate of births was estimated at 86.8% in 1970, at 92.8% in 1977 and at 98.72% in 1981. Again, the ONS publications did not include any technical note about the correction process.

These coverage rates estimated in 1981 continued to serve for the correction of the crude recorded data until 2002 (Bedrouni, 2009), where they have been revised on the basis of the results of the Algerian family health Survey (AFHS 2002) featured with those of the GCPH 1998. Correction factors were applied with retroactive effect from 1998 and continue to serve, at least until now, to correct vital statistics (ONS, 2012a) without being published. Furthermore, the adoption of the same correction factor for the different ages is problematic because it assumes implicitly the uniqueness of deaths coverage at all ages (Mouffok, 1974; Daoudi, 2001). This hypothesis seems far from being verified and no study has been conducted in this sense. It is assumed that deaths in low and high ages are less well covered (Iles, 1990).

In all cases, births are better covered than deaths. The gap in this area is expected to decrease with the improvement of the vital registration coverage. Therefore, the correction of the recorded numbers by an old correction factor should lead to an overestimation of births and deaths with a slight favor for deaths. That may lead to an overestimation of mortality indicators with an increasing effect as further away from the reference date of correction factor estimation. The update of correction factors will bring the series of the corrected data in its natural course, and leads to an apparent variation due to the removal of the over-estimator effect of the old correction factors.

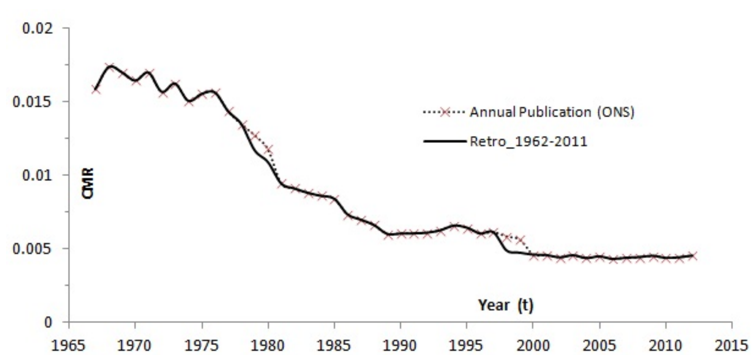
Moreover, the IMR sensitivity for changes in calculation parameters is very important especially if we consider the confusion that continues to be made between early infant deaths and stillbirths. The stillborn concept includes any death of a fetus inside the uterus after six months of pregnancy, before or during childbirth. Children who showed a sign of life at birth and died before to be recorded in the vital services as stillbirths are in reality false stillbirths. This last should be firstly recorded as a live births and then as an infant deaths, which is not always the case in Algeria (MSPRH, 2006). The biggest problem is that, when recorded at the vital services, some false stillbirths are confused with stillbirths, which is probably caused by the absence of a clear definition of the concept. The MDS in their section dedicated for stillbirths included a question about the life span and the life signs exhibited by the stillborn. To correct this kind of errors consisting on a systematical confusion in recording, the number of live births and infant deaths must be corrected according to the false stillbirths.

The effect of change in the correction factor is more or less apparent following the nature of each mortality indicator. The IMR seems to be the most sensitive indicator for such changes compared to the crude mortality rate (CMR). The evolution of CMR can also be affected by changes in the age structure of the population. The effect on life expectancy at birth is even lower. The study of the IMR seems to be well suited to view the variation in mortality due to changes in correction methods. An analysis based on the life expectancy at birth is supposed to give an approached analysis about the natural evolution of mortality.

The Crude Mortality Rate (CMR)

Figure 2.1 shows a comparative evolution between the initial and the revised estimation made by ONS about CMR during the period 1967-2012. The effects of the correction methods revision are obvious. We recall that the data published in 1979 and 1980 have been corrected on the basis of the coverage rate estimated by ONS without using any other source of estimation parallel to the civil registration data (Daoudi, 2001). Later, the new correction factor estimated in 1981 was used to correct the CMR corresponding to 1979 and 1980. The 2002 revision has had a retroactive effect till 1998. This correction procedure does not eliminate sudden drops already seen in the CMR series. We have just observed recoil in the sudden drops from 1981 and 2000 respectively to 1979 and 1998. The ONS could have used any regression to interpolate the correction factor between two estimation dates in order to remove the sharp falls in data series.

Figure 2.1: CMR evolution in Algeria between 1967 and 2012



The data of the period 1967 -1978 were estimated by Salhi (1984). Thereafter, the ONS published periodically the CMR in its annual publications. After re-estimating the new correction factor, the ONS was conducting a review of published data for some previous years. The CMR evolution series has been corrected and included in the retrospective of demographic statistics 1962 -2011 published in 2012 (ONS, 2012-a).

Except these comments, the evolution of the CMR had marked some variation periods which could be easily distinguished. After a slight downward trend during the period 1967-1976, a sharp reduction was observed during the period (1977-1981) as it was carried out in some previous works (Hamza Cherif, 2011; Daoudi, 2001). After a moderate reduction during the 80s, the CMR had stagnated during the black decade (Hamza Cherif, 2011) or rather had slight upward trend, especially in 1994 and 1995. There has been a sharp fall towards the years of the black decade, which coincided with the second revision of the correction factors.

The general remark that can turns-out from the reading of the CMR evolution is that this last was greatly affected principally by the correction factor revision. That prevents any attempt to approach the part of change attributed to the natural reduction in mortality. The brusque falls cannot be explained by a natural variation. Also, the CMR evolution remains impaired by changes in the population age structure. The observation of IMR can provide us with a clearer reading in this issue.

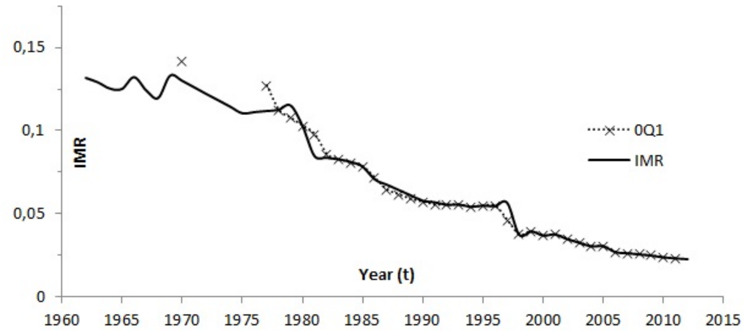
The Infant Mortality Rate (IMR)

The IMR is usually used as a measure of the population health statue. Despite it concerns only the mortality level during the first year after birth, it seems to be well correlated with the health statue at all ages (Reidpath and Allotey, 2003). According to the same authors, the IMR represents a good measure of the population health in the inability to perform a more sophisticated measures. In the other side, the simplicity to calculate the IMR makes it a good indicator to evaluate the improvement over time of the population health statue. However, its evolution in time can be biased by a set of methodological effect as we have pointed it out in the previous section. In Figure 2.2, two significant decline points can be observed in the IMR evolution corresponding respectively to 1981 and 1998. These two points correspond to the years where the crude data correction factors has been revised and coincide partially with those recorded by the CMR (Figure 2.1). The two indicators series have not been corrected exactly at the same time points, but there is a little difference in concern of the first revision. It seems that the correction factors estimated in 1981 were not used similarly to correct IMR and CMR. We remind that the ONS retrospective publication (ONS, 2012a) has no dedicated part for IMR, and we simply analyze the estimates given in the annual publications [2]. That presents another imperfection in the correction process and we wonder about the age specific mortality rates and the life expectancy which results from. In order to have a more clearer idea about this difference, we associated in Figure 2.2, the evolution of the mortality rate between 0 and 1 year ${}_0Q_1$ to the IMR.

Infant mortality can be measured in several ways. If the IMR seems to be the most relevant measure of mortality at this age, mortality rate between 0 and 1 (${}_0Q_1$) can be used for the same purpose. Although these two indicators are calculated on different basis, their evolution over time should correlate, whether in terms of natural evolution or in terms of brusque change-points due to methodological changes.

[2]The data from 1965 to 1986 after revision was published in Yeghni et Abdoun (1988).

Figure 2.2: IMR VS ${}_0Q_1$ evolution in Algeria between 1962 and 2012



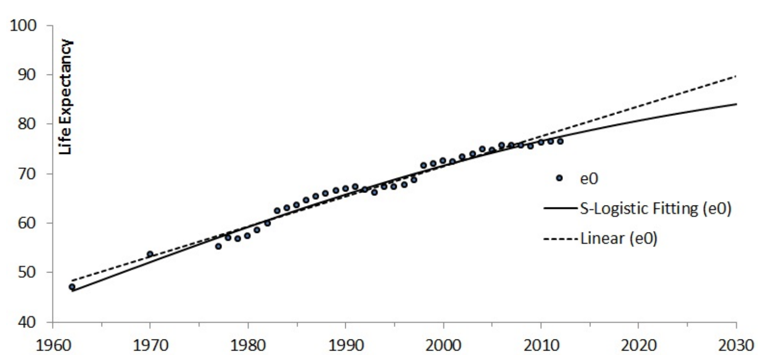
The comparison allows to clarify if life tables have been corrected on the basis of the new correction factor estimated in 1981 as the IMR or not. A change point in mortality rates will be reflected in the evolution of the life expectancy evolution series. As shown in Figure 2.2, the two curves are perfectly correlated starting from 1982. For the period prior to 1982, the evolution of the two indicators was disassociated and that may indicate an inadequacy in the correction methods and that the coverage rates estimated in 1981 were not similarly used to correct the two indicators. Far from extending this finding to all ages, an analysis of changes in the mortality age pattern may provide a clearer reading about this element.

As we noted earlier, the change points due to methodological change which are principally related to the improvement of coverage rates are more important in the IMR than in the CMR evolution series. We expect that the effect will be less important on life expectancy.

2.3 Evolution of life expectancy at birth

The observation of the evolution of life expectancy at birth in Algeria from 1962 to 2012 reveals a general improvement trend. It results in an annual average gain of 7 months, and a total gain of nearly 30 years in half century allowing to pass from 47 years in 1962 (Ministère du Plan, 1979) to 76.4 years in 2012 (ONS, 2013-a). However, this improvement (Figure 2.3) remains unstable over time.

Figure 2.3: Evolution of Life expectancy at birth (1962 – 2012)



Source of data : 1962 estimations of the Ex- Planning Ministry (1979), 1970 : National Statistic Population Study (NSPS, 1970). 1977-2012 : ONS annual publications.

Between 1962 and 1970, the Algerian population earned 6.5 years in life expectancy at birth involving an annual average gain of 2 years per year. The RPDS 1969-70 resulted in a slightly lower value (53.5 years). Giving the value recorded in 1977, the Algerian population has gained nearly 8 years in a range of 15 years. Starting from 1977, the life tables annually published by the ONS ensured a continuous and homogeneous data series.

By analyzing the evolution of life expectancy at birth in Algeria, we aim to detect the time points where the historical evolution series marked a significant change. Later, we will try to seek explanations for this trend change. Our idea is to model this general evolution with an adequate model and then to detect the changes occurred around this general trend.

By the late of the 20^{th} century, many attempt have been recorded to model the time evolution of life expectancy as an S-logistic function (Bulatao et al., 1989; Marchetti, 1997). In a first stage, life expectancy starts to increase slowly before that the improvement rate accelerates. When life expectancy attains a relatively high level, it continues to increase but following a decelerating improvement rates. This thesis is based on the hypothesis of existence of a biological limit of the human lifespan forcing life expectancy deceleration when this limit approaches. But, no proof of the existence of an upper limit of life expectancy has been presented yet. In contrast, Oeppen and Vaupel (2002) demonstrated that the world record life expectancy of females has improved following a steady line at least since the mid-19th century and no deceleration has been observed. For males, the record life expectancy marked a slight decline in terms of evolution rate during the last 50 years. Oeppen and Vaupel suggested that the life expectancy evolution in a given country cannot be treated independently to the rest of the world. Their argument is that life expectancy in any country evolves with an accelerated trend in order to catch-up the world record line, the more the record line approaches the more the improvement rate decelerates. This is the explanation that they gave for the deceleration of life expectancy in single country context. In his paper, Lee (2003) was carrying more about mortality forecasting and the new challenge made by the findings of Oeppen and Vaupel (2002) which contradicts the life expectancy deceleration for the long run and the existence of a human lifespan limit. Life expectancy evolution results from the combined evolution of the age specific mortality rates over time. The Lee-Carter model (Lee and Carter, 1992), which represents a landmark in mortality forecasting, is based on the extrapolation of the historical reduction trends in ASDRs to the future. When ASDRs are expected to fall exponentially, it results to a decelerating life expectancy. Few years after the Lee and Carter contribution, many experiences on life expectancy evolution have shown that there is no deceleration (Boongarts, 2004; Debonneuil and al., 2015) and others have shown that in many industrialized countries the trend have been linear for a long periods (White, 2003). In his paper, Lee (2003) seemed to be not enough convinced that the linear evolution of life expectancy can be maintained for a long time. If so, he expects that the female life expectancy record will be at around 108 years by 2100. His second argument was that, for males, the record life expectancy has shown a deceleration during the last half century. To go further with his idea, Lee (2003) tried to fit the female record life expectancy evolution series with a quadratic function and compared it to the results obtained with a linear fitting. It turned out that the evolution of the record life expectancy has a slight S-shape. Recently, in a paper that he presented in Longevity 11 Conference, Lee (2015) insisted on the fact that life expectancy increases following an S-shape curve for the long run. Also, he address the possibility that the linear trend that we can observe for some countries can be incorporated as the medium phase of the S-shape transition cycle.

In this paper, we are not addressing the forecasting issue of mortality analysis but just analyzing an evolution already occurred. So, modeling life expectancy evolution in our case will not take the expected future evolution into consideration. More consideration will be given to the fitting quality of the historical series. Projection issues can be separately discussed in other papers. If we look at Figure 3.3, we observe that the S-logistic function fit the historical life expectancy series in Algeria much better than the linear function. The difference is obvious, and starting from 2007, the deceleration takes evidence. So, the S-logistic function will be used to fit the historical life expectancy evolution for the Algerian population in the present work.

The S-logistic function which is supposed to represent the evolution of life expectancy at birth in the year (t) according to its reference value (k_0) is given by the following equation (see Bulatao and al., 1989):

$$e_t = k_0 + \frac{k}{1+exp[logit(e_0)+rt]} \quad \text{..................................} \quad \text{Equation [1]}$$

with $logit(e_0) = log[\frac{k_0+k-e_0}{e_0-k_0}]$, k represents the potential total gain in life expectancy at birth in the time interval, and r is a parameter to be estimated.

Certainly, when we try to fit the evolution of life expectancy by an S-logistic function, we suppose that life expectancy is converging to a known upper limit represented here by (k_0+k). In our case, even if we believe about the human lifespan limit, the Algerian level still far under the international record. In second, this international record itself is still improving. That makes difficult to find a convincing interpretation of ($k_0 + k$) in our model. The only interpretation that we can advance, is that ($k_0 + k$) represent the maximal level that life expectancy could reach in Algeria under the current life conditions of the population. Also, since we are only interested by fitting an historical series without addressing any projection to the future, the needed interpretation becomes useless. Under any conditions, a ceiling ($k_0 + k$) can be fixed as a minimum of the values that never have been reached by the Algerian population during the considered period.

To go on our initial idea, Marchetti (1997) used a similar formula as equation [1] to model the potential gain in life expectancy at birth by replacing $logit(e_t)$ by a constant (b):

$$e_t - k_0 = \frac{k}{1+exp-[at+b]}$$

It is turned out that this second model fits well the evolution of life expectancy in Algeria for both sexes population. As reported by Vargas (2004) and in order to improve the fitting quality, k and k_0 can be estimated by the same process as a and b in order to minimize the sum of squared errors. Then, Marchetti (1997) estimated the model parameters by a linearization of the logistic curve using the Fisher-Pry transformation (Fisher and Pry, 1971). Letting $F_t = \frac{(e_t-k_0)}{k}$ be the saturation level of the function, the Fisher-Pry transformer results in :

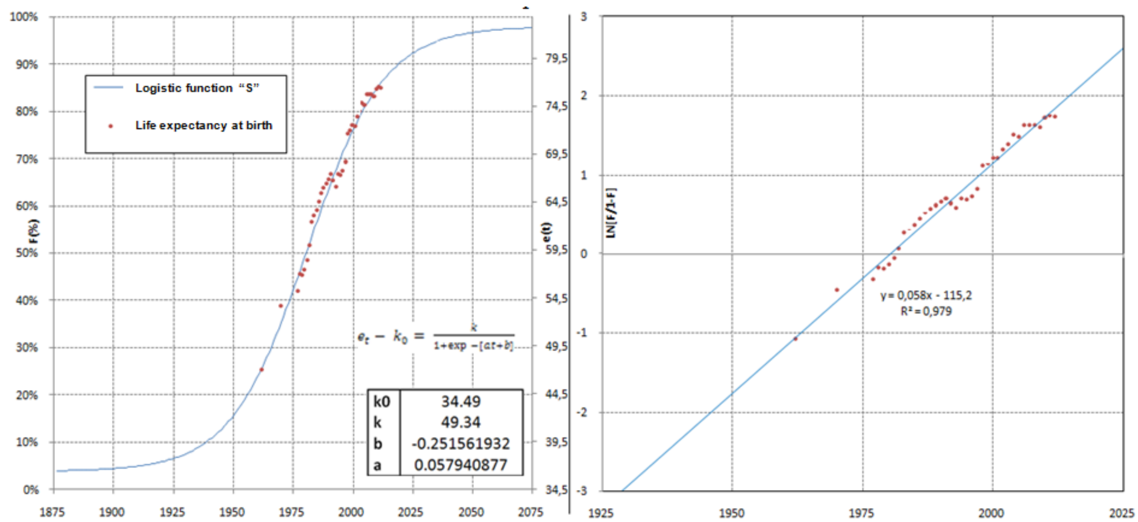
$$ln[\frac{F_t}{1-F_t}] \approx -at - b$$

The linear transformation of the logistic curve can also be made by a simple manipulation of equation (1). That leads to:

$$ln[\frac{k}{e_t-k_0} - 1] = -at - b$$

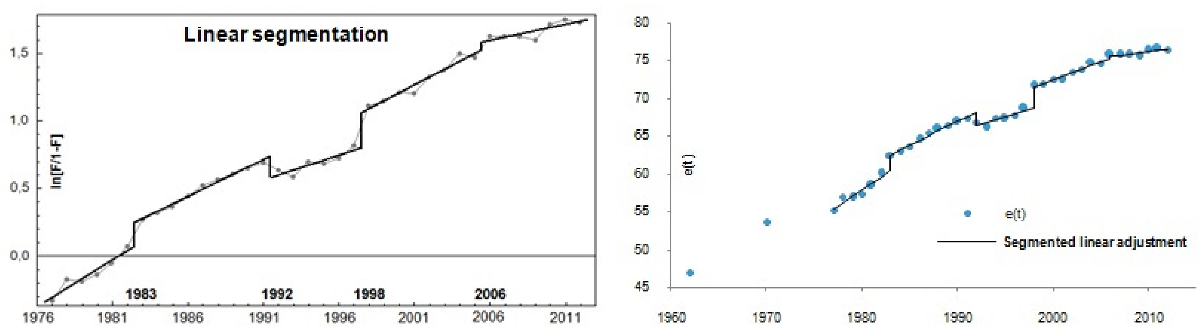
Figure 2.4 shows the fitting of the series of the observed life expectancy at birth with an S-Logistic function and also the linearized series.

Figure 2.4: logistic adjustment and linear transformation of the series of e_0



The new expression of the series of life expectancy allows a linear segmentation which provides the possibility to determine the trend change points occurred during the analysis period. The segmentation has been performed using the tool SEGMENTER V 2.1 following the methodology proposed by Askoy and al. (2007). Results are shown in Figure 2.5.

Figure 2.5: Segmentation of the life expectancy linear transformed series



Le figure left shows the linear segmentation results of $Ln[F/1-F]$obtained with SEGMENTER 2.1. The figure right shows the segmentation results reported on the real scale.

Except for the period of 1962-1977 which was characterized by an important weakness in term of data availability, the segmentation gave 4 significant change points of life expectancy at birth, therefore 5 distinct periods can be distinguished : [1977; 1982], [1983; 1991], [1992; 1997], [1998; 2005] and [2006; 2012].

According to the observation of the linear segmentation results, we notice that the change points are considered in our application according to two dimensions: change in the general level and change of slope. These are respectively translated by a punctual gain /loss in the global level of life expectancy and improvement /deterioration of the average annual gain. At each change point, the curve of life expectancy changes its general level and continues to evolve according to the new slope. The gain /loss in the general level of life expectancy is estimated by the difference between distinct values obtained with two successive curve-pieces for the same change point. The consideration of these changes levels certainly modifies the reading of the life expectancy evolution compared to a reading based solely on change in the annual average gain. The characteristics of the main periods of life expectancy evolution in Algeria are shown in Table 2.1.

Table 2.1: Characteristics of life expectancy evolution by period

Period [t, t+a]	Initial value (e(t)) in years	Ponctual Gain/Lose at (t) (in years)	Average Annual Gain/Lose (in days/year)
1962 -1976	47.0	-	203
1977 -1982	55.1	-	302
1983 -1991	62.4	+2.08 (+3.44%)	228
1992 - 1997	66.7	-1.89 (-2.77%)	149
1998 - 2005	71.9	2.63 (+3.83%)	173
2006 - 2012	75.7	0.29 (0.39%)	56

As shown in Table 2.1, life expectancy at birth has overall growing trend. In 1977, it was at 55.1 years. With an annual average gain of 302 days/year, life expectancy at birth could have stagnated at a level of 60.4 in 1983, except that the change point level observed in 1983 brought it to 1.8 years below (62.4 years). This droop can signify a permanent transition from one situation to another. The most probable interpretation for such a change may consist of an overestimation of mortality during the period [1977-1982], which may be due to a change in correction factors since it cannot be explained by a natural reduction. Algeria during the considered period did not experience any event that could even affect mortality level. Whereas, the quality of vital statistics data, even after correction, can provide a convincing explanation. The death coverage rate was estimated at 81.1% in 1981 after it was at 60.1% in 1970 (Daoudi, 2001). An under estimation of deaths coverage may lead to an overestimation of deaths during the cited period.

Considering the jump-up of 1983, the annual average gain in life expectancy recorded between 1983 and 1991 a value of 228 days/year, which has brought life expectancy at birth from 62.4 to 67.3 years in 1991.

During the black decade, mortality rates have marked a meaningful rise. In 1992, the curve of life expectancy recorded another level change point that drove nearly 2 years down maintained until 1998. The annual average gain decreased to 149 days/year. Again, the death coverage problem rises concerning the civil war lost individuals. The non-recording of deaths among this category have certainly involved an underestimation of mortality. Publications of the ONS have not addressed any details about this point.

In 1998, Life expectancy evolution recorded a level change point (+2.6 years). After, it resumed its overall improvement trend after recording with an average annual gain of 173 days/year, which is higher than the value recorded during the previous period. Admitting that life expectancy evolves according to an S-logistic function, we should expect a deceleration in the annual gain even more life expectancy increases. This significant improvement can be seen as a catching-up effect after the black decade. However, it is difficult to explain the change point observed in 1998 and to separate the natural improvement from the method change effect because the correction rates revision coincided with the end of the black decade.

The year 2006 saw a slight jump-up in the general level by 3.5 months accompanied by a decline in the average annual gain to 2 months/year. Life expectancy cannot keep a constant improvement rate for a long period, especially if this improvement is not accompanied by a gradual improvement of life and health conditions. Improving health care and life environment are the main factors which can have a significant effect in life expectancy improvement. The more life expectancy improves the more the marginal effect of public policies decreases. The same effort makes increasingly less gain in life expectancy at birth.

In practice, improvement of life expectancy results from a reduction of mortality rates at different ages. However, and as previously reported, the natural evolution of life expectancy at birth, as it was described by the publications of the ONS, may be altered by a set of methodological changes effects such the average age at death for the different age groups, the closing-out methodology, the correcting of the crude numbers of deaths and population number and the process of mortality curves adjustment.

In the next part, we will try to isolate the natural evolution of life expectancy at birth from the methodology changes effects. Of course, in the absence of the crude data which have been used to calculate the Algerian life tables throughout the analysis period, it would be difficult to perform this task in its entirety. Except, it would be appropriate to reduce the effect of the methodology change and approach to natural mortality evolution.

2.4 Analysis of the methodological effect on the life expectancy evolution

The low sensitivity of life expectancy at birth for changes in correction methods is supposed to allow more approximate analysis of the natural evolution of mortality. However, some elements related to the life table construction methodology: average age at death under 5 years, graduation methods and the closing-out methods may partially lead to irregularities in estimating life expectancy at birth. The analysis of the overall mortality trend should be based on synthetic indicators summarizing the mortality experienced by a population at different ages. In this sense, life expectancy at birth, taking into account the age at death without being affected by the population structure changes, seems to provide a good indicator for the analysis of the mortality global evolution. Moreover, this indicator, because of its structure, appears to be less sensitive to change in correction methods. This quality makes it to be the most suitable among the mortality indicators to analyze the natural evolution of the studied phenomenon.

A relevant analysis of the evolution of life expectancy at birth can only be done by a detailed analysis of the technical elements used for its calculation. Life expectancy at birth is the average age at death of a fictive population exposed to the living conditions of the moment. In other words, it is the total number of years lived by a population (T_0) divided by the initial population (l_0). We can write :

$$e_0 = \frac{T_0}{l_0}$$

for the age interval [x, x+n], if we consider l_x to be the initial population at age x, the surviving population l_{x+n} live n year each surviving individual. For the dying people, noted $d(x, x+n)$, the number of years lived in the age interval [x, n[is noted by ${}_na_x$. The total number of years lived by l_x in the age interval [x, x+n[is simply the sum of the years lived by both the surviving and dying people which can be expressed by $d(x, x+n).{}_na_x + l_{x+n}.n$. T_0 represents the sum of the years lived in between age 0 and the surviving age limit w. According to this, life expectancy at birth can be expressed by the following formula:

$$e_0 = \frac{\sum_{x=0}^{w-n}[d(x, x+n).{}_na_x] + l_{x+n}.n}{l_0}$$

For a closure age of 80, e_0 can be expressed as (see Hinde, 2014) :

$$e_0 = \frac{\sum_{x=0}^{75}[d(x, x+n).{}_na_x] + l_{80}.e_{80}}{l_0}$$

To better understand the evolution of e_0, it would be necessary to examine the evolution of the various parameters used for its calculation. In this sense, the age structure of mortality, life expectancy at the closure age and the average number of years lived by deaths in each age interval noted ${}_na_x$ are to be analyzed. In general, the deaths occurred in the age interval $(x, x+n)$ are supposed being lived the half of the age interval length $(n/2)$. Infant and child deaths are generally excepted from this general rule.

Average number of years lived by infant and child deaths ${}_1a_0$ and ${}_4a_1$

The exception made by infant and child deaths in terms of average number of years lived is related to the fact that deaths at such ages are usually concentrated in the early age intervals (see Bouregeois (1946) for the case of infant mortality and Coale and Demeny (1966) for child mortality). Therefore, this indicator can be estimated from a detailed calendar of deaths: the weekly or monthly schedule of infant deaths and the annual calendar of child deaths resulted from specific investigations or even vital records. In the absence of this information, ${}_1a_0$ and ${}_4a_1$ can be deducted from ${}_1q_0$ following the formula proposed by Coale and Demeny (1966).

Generally, ${}_na_x$ does not refer to items published in a life table, but can be deducted from the other parameters. The observation of the evolution of this parameters issued from the ONS publications reveals some anomalies. In addition to the unexplained variations, some aberrant values are observed.

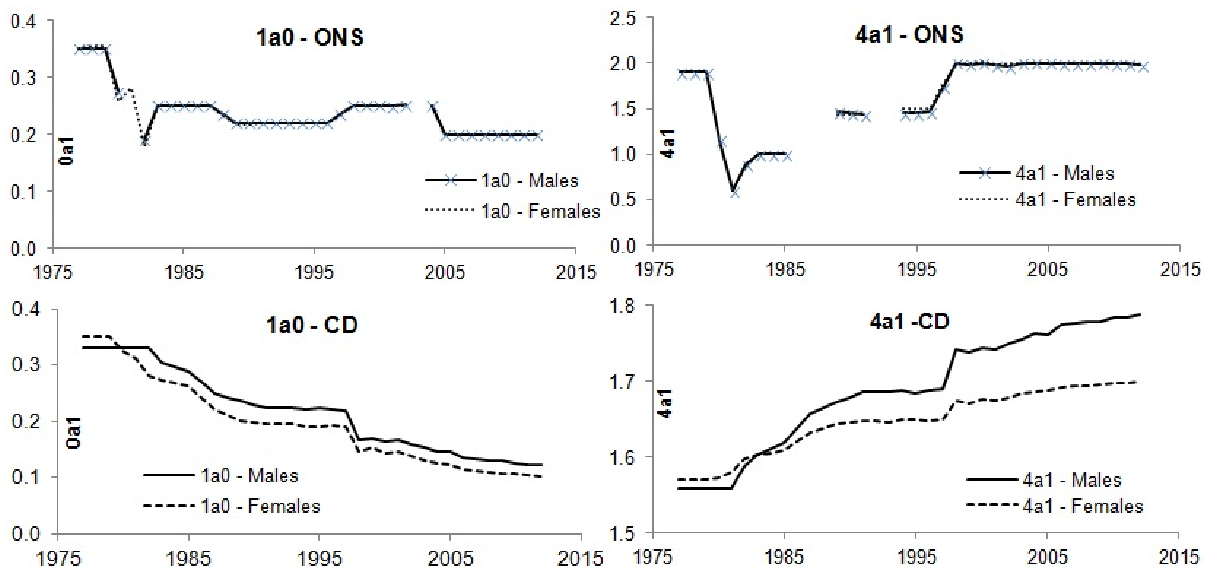
On the basis of a regression based on ${}_1q_0$, Coale and Demeny (1966) achieved a function to estimate ${}_1a_0$ and ${}_4a_1$ for the four families of model life tables. In the model "North", ${}_1a_0$ and ${}_4a_1$ can be defined as shown in Table 2.2.

Table 2.2: estimations of ${}_1a_0$ and ${}_4a_1$ according to Coale-Demeny method

		Males	Females
${}_1a_0$	${}_1q_0 \geq 0.1$	0.33	0.35
	${}_1q_0 < 0.1$	$= 0.05 + 3\ {}_1q_0$	$= 0.045 + 2.875\ {}_1q_0$
${}_4a_1$	${}_1q_0 \geq 0.1$	1.558	1.57
	${}_1q_0 < 0.1$	$= 1.859 - 3.013\ {}_1q_0$	$= 1.733 - 1.627\ {}_1q_0$

Figure 2.6 shows ${}_1a_0$ and ${}_4a_1$ issued from the Algerian life tables compared to those obtained with the Coale-Demeny formula.

Figure 2.6: ${}_1a_0$ and ${}_4a_1$ estimated by the ONS compared to those obtained with CD method



The upper figures show the series of ${}_1a_0$ and ${}_4a_1$ deducted from the ONS life tables. Missing values in the series correspond to aberrant values and even negative sign in some cases. The figures below represent the re-estimated series by the CD formula.

Sudden changes were recorded on the obtained series with the CD formula because of the sudden changes in the underlying ${}_0q_1$ series. As we have already shown, the change levels are mainly driven by the adoption of the new correction rates to correct the recorded deaths and live births respectively in 1981 and 1998. For this, the evolution of ${}_1a_0$ and ${}_4a_1$ redrawn by the CD formula are still indirectly affected by the correction methodology change.

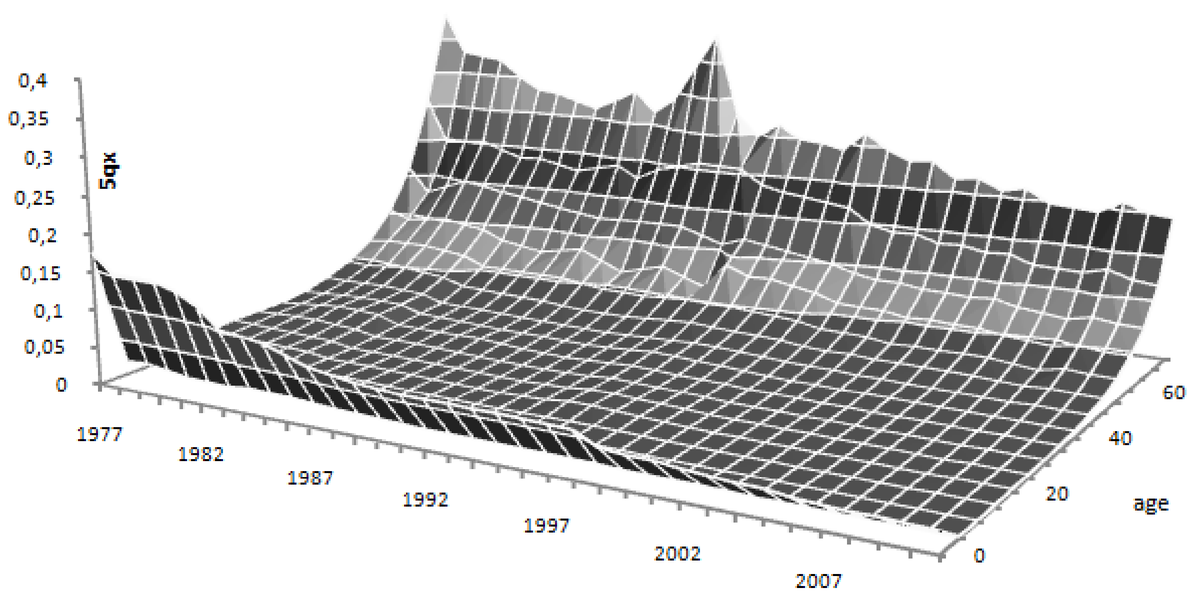
The effect of the life-table closing-out method

Because of information's lack about mortality at advanced ages and irregularity, if available, the use of model life tables as a substitute is highly recommended. Once the standard mortality pattern corresponding to the national mortality data is determined among the model life tables, the residual life expectancy at the closure age is directly deducted. Life expectancy at lower age is determined by recurrence. The estimates obtained by this way for the case of Algeria mark a certain irregularity in terms of their evolution in time. This last depends greatly on the method used to select the compatible model life table with the national data. Algeria as other African countries suffer a kind of wrong use of the model life tables to close-out national life tables (Ekanem and Som, 1984). This can have a meaningful effect on the changes observed in the evolution of the life expectancy at birth.

In addition, the change in the closure age from one period to other remains unexplained. Until 1982, the last age group in the Algerian life table was [80 and +[. From 1983 to 1988, it became [70 and +[. Since 1989, we return to [80 and + [and to [75 and + [since 1993. In 1998, we went back again to [80 and +[to move to [85 and +[starting from 2010.

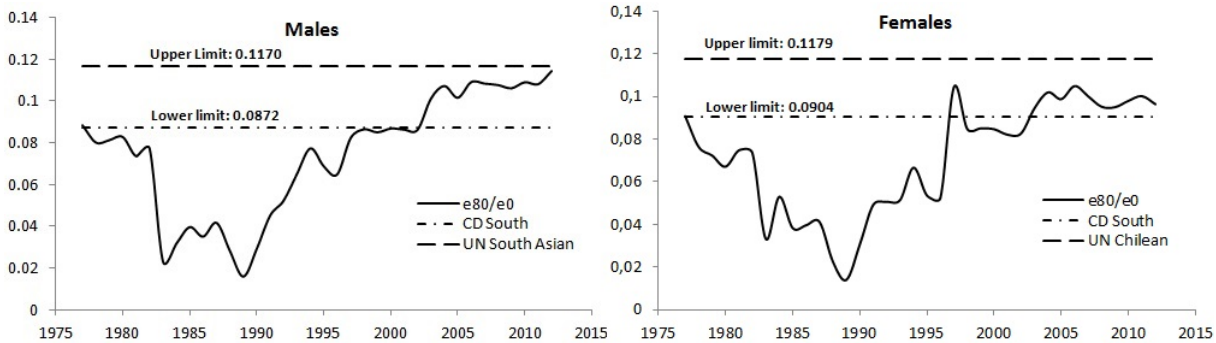
Our objective at this stage is to identify the part of the variations in life expectancy at birth which are related to the closing-out methodology imperfection. In order to conduct such analysis, it is necessary to constitute a complete mortality surface closed out at a common age. Recall that the ONS has not established a life tables for some years of the period 1977-1997. As a result, the mortality surface we have at present has some missing tables. Similarly, some life tables were closed out before the age of 80. To complete this surface and to adapt it to the needs of our analysis, we proceeded to estimate the missing data. Missing life-tables are estimated by the average of the two neighboring tables. The life tables which were closed-out at 70 or 75 years are completed; up to 80 years using the Gompertz formula (Gompertz, 1825). The obtained mortality surface is shown in Figure 2.7.

Figure 2.7: Algerian mortality surface 0 - 80 years (1977 -2012)



To show the imperfection in estimating life expectancy at the closure age, we compared (Figure 2.8) the report $\frac{e_{80}}{e_0}$ as reported in the ONS publication with the upper and lowest bounds issued from the nine families of the model life tables (UN and CD).

Figure 2.8: Evolution of $\frac{e_{80}}{e_0}$ issued from the Algerian life tables compared to the extremes values given in the UN and CD model life tables



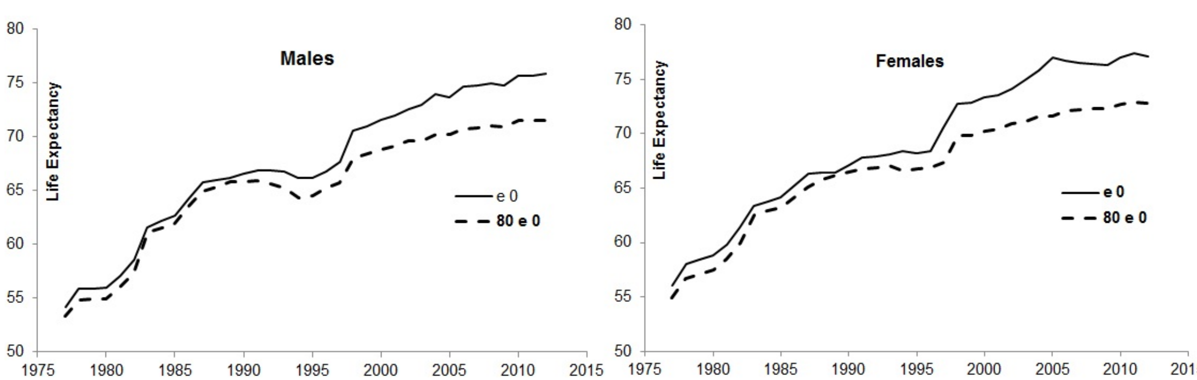
The observation of the evolution of the ratio $\frac{e_{80}}{e_0}$ as described by the ONS publications, shows a kind of under estimation of life expectancy at the age of e_{80} compared to e_0 especially during the period prior to 1998. The nine families of United Nations and Coale-Demeny life tables give a constant minimal and maximal ratio for all mortality levels among the same family. In the figure 2.8 and according to the model life tables, the ratio $\frac{e_{80}}{e_0}$ must be comprised between 0.087 and 0.117 for males and between 0.09 and 0.118 for females.

Starting from 2003, this ratio has a relatively an acceptable level. This is likely to alter the reading we seek for the change points previously identified. To avoid such imperfection, mainly due to the closing out assumptions, Arriaga (1984) suggested the use of temporary life expectancy to approach the natural evolution of mortality. In this sense, the use of the temporary life expectancy between 0 and 80 years (noted ${}_{80}e_0$) will eliminate disturbances caused by indirect methods of old ages mortality estimation. Temporary life expectancy between age x and $x+i$ can be estimated by:

$${}_ie_x = \frac{T_x - T_{x+i}}{l_x} = \frac{\sum_{x=X}^{X+i} \cdot {}_nL_x}{l_x}$$

The series of temporary life expectancy (0-80 years) obtained for male and female populations are shown in the Figure 2.9. By the same, ${}_1a_0$ and ${}_4a_1$ have been corrected by using the Coale-Demeny method without having a significant effect on ${}_{80}e_0$.

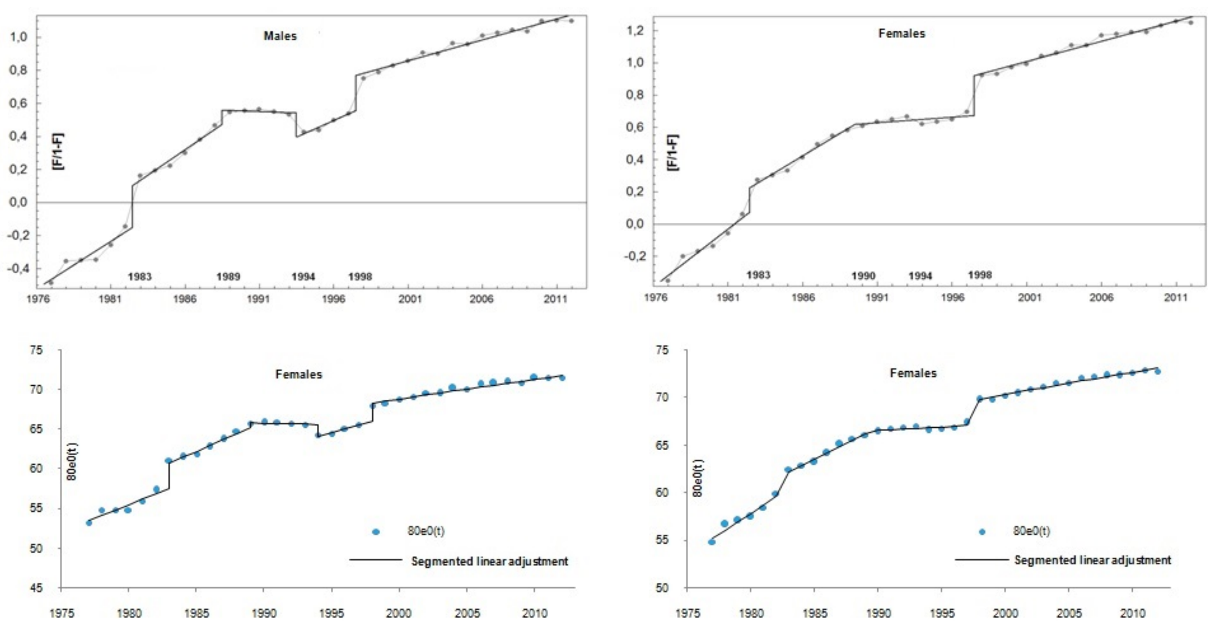
Figure 2.9: life expectancy and temporary life expectancy ${}_{80}e_0$



The comparison of e_0 and ${}_{80}e_0$ reveals some difference in terms of evolution trends. e_0 shows a stagnation during the black decade while ${}_{80}e_0$ shows a decline. This allows to suggest that the decline in female life expectancy at birth during the black decade was partially mitigated by the closing-out methods. Similarly, the drop observed on e_0 in 2005 disappears on the curve of ${}_{80}e_0$. This finding reflects a relative underestimation of old age mortality for females in 2005. Probably, the change of the reference model life table used for old age mortality estimation starting from 2006 may explain such finding. Unfortunately, the ONS again did not publish any detail about this point.

In order to show the effect of the imperfection of the closing-out methodology on the life expectancy evolution more clearly, we proceed to a change points analysis of ${}_{80}e_0$ and we compare results with those which were already shown in Figure 2.5. Close out a life table mains to estimate the mortality age pattern beyond the closure age or simply estimate the residual life expectancy at this age. In any case, the resulted estimates must be coherent with the mortality at younger ages and is supposed to do not have any particularity compared to it. In this sense, the changes occurred on the mortality trend at younger ages will be automatically reproduced at ages beyond 80. In other words, the segmentation of ${}_{80}e_0$ may lead to the same results obtained with e_0. Any difference in this sense can be imputed to an imperfection in the closing out methodology. Figure 2.10 shows the change points analysis results obtained on ${}_{80}e_0$ for males and females life expectancy.

Figure 2.10: Segmentation of the temporary life expectancy series



Figures up show the change points in the linear transformed temporary life expectancy. Figures down report the segmentation results on the real scale.

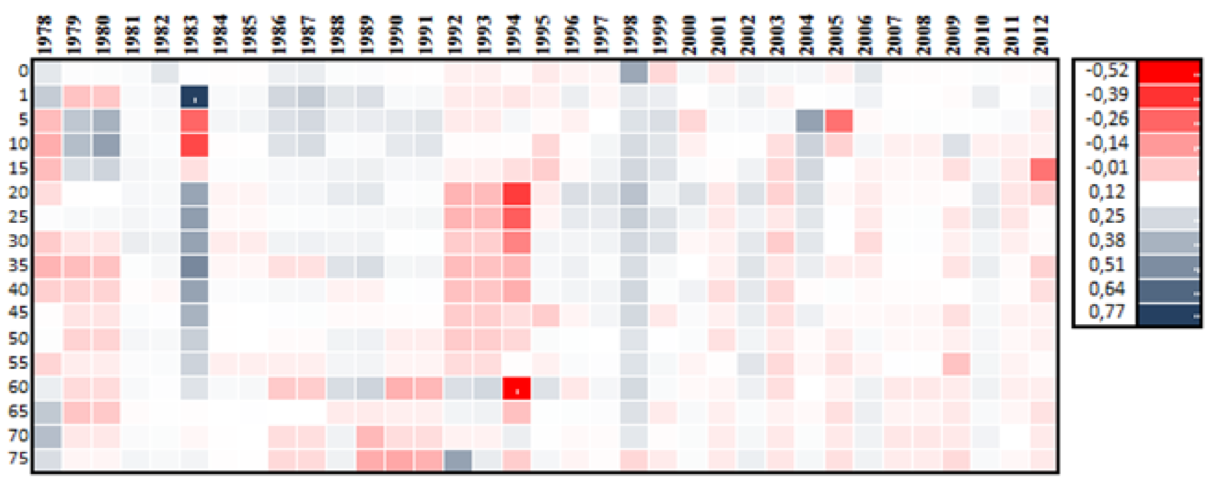
The results of the temporary life expectancy segmentation accentuate the evidence of the change points that were previously detected. The change points related to correction factor change in 1983 and 1998 are complying with. Unlike, the 2006 change point is mitigated by the removal of the closing out imperfection effect. The effect of the black decade is obvious with some differences by sex. For men, a slight tendency to decline was observed in 1989 followed by a sharp deterioration in 1994 which continued until 1998. For women, a near stagnation is observed from 1990 until 1998.

Although terrorism events have started only towards the end of 1991, the trend change observed in 1989 and 1990 respectively for men and women are still without any convincing explanation. We recall that the ONS has not published a life table for 1990. As we previously noticed, for the needs of the present work, this missing life table was estimated by the average of the neighboring tables. Therefore, 1990 may correspond to an artificial change point. The real change point should be attributed to the previous or the next year.

2.5 Age specific mortality evolution

After presenting the evolution of the various elements affecting the life expectancy at birth, it is appropriate to seek explanations based on changes in the specific age mortality. Variation in mortality at age x over time can be measured by the age-specific mortality reduction rates (Andreec and Vaupel, 2005). For age x, mortality reduction rate from a year t to $t+1$ can be expressed by the following formula: : $r_{xt} = -ln[\frac{m_{x,t+1}}{m_{x,t}}]$. with $m_{x,t}$ represents the death rate specific to age x at year t. This formula can be adapted to five ages mortality rates and be written as: $r^*_{xt} = -ln[\frac{{}_nq_{x,t+1}}{{}_nq_{x,t}}]$. The obtained reduction surface is shown in Figure 2.11.

Figure 2.11: Age specific mortality reduction surface



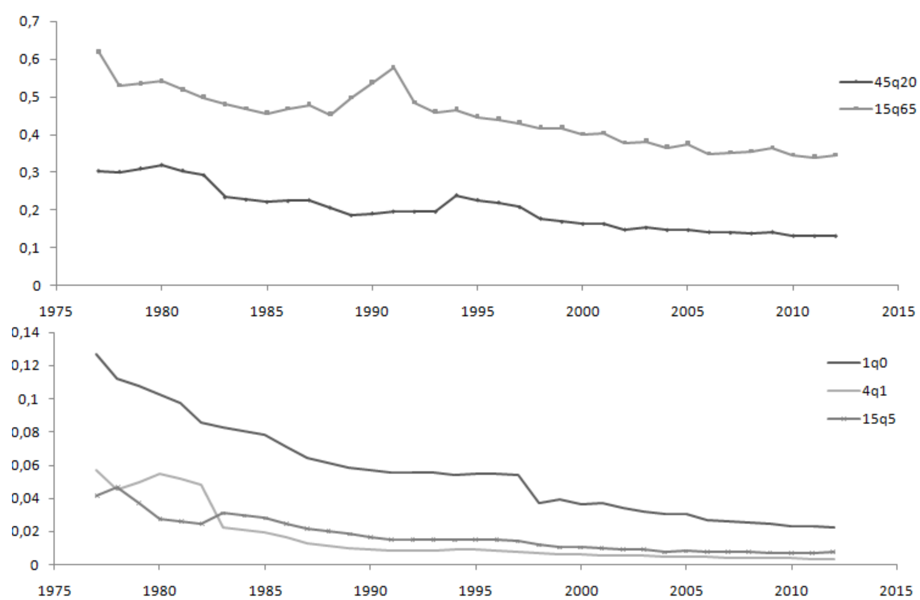
The visualization of the mortality reduction surface allows to better detect the occurred mortality changes by age and time. According to the year of observation, changes in 1983 and 1998 can easily be observed. In 1998 we observe that there is a significant reduction at all ages. In 1983 however, we can see a kind of deformation of the mortality curve due to a sharp reduction in mortality at ages between 20 and 65 years accompanied by an increase in mortality between 5 and 20 years. The effects of the black decade are observable particularly for the age range 20-60 years and with less importance for ages under 20. Moreover, we observe an increase in mortality between 1992 and 1994. This let suppose that the mortality reduction has not been uniform for all age categories. However, we can detect some correlation when we consider this reduction by large age groups.

To make this last finding more evident, it is necessary to proceed of an analysis by large age groups. For this, we do a partitioning of the age range (0 to 80 years) to sub age ranges having a similar mortality reduction profile. For this, we apply an ascending hierarchical classification (AHC) under XLSTAT while infant and child age groups will be kept to be analyzed separately to the other age groups. Results are shown in Figure 2.12.

Our objective in this step is to study the sensitivity of the different ages to the general mortality reduction trend. The analysis of the evolution rates of the age specific mortality rates reveals that the age group [1-4] marked the most significant declines in mortality. Between 1977 and 2012, the probability of dying between 1and 5 years declined by over 94%. Then, the decline becomes less important when age increases. Infant mortality recorded a significant decline of 82%.

Compared to other age categories, the absence of strong disturbances on the curve of ${}_1q_0$ (see Figure 2.12) during the period prior to 1983 is remarkable. This will suppose that the rates and methods used for the data correction of the infant population were more appropriate compared to other categories. The use of a correction rate specific to this age may explain this result. The effects of the black decade are apparent especially in 1991. We note, moreover, that instead of maintaining its downward trend, ${}_1q_0$ has almost stagnated between 1991 and 1998. The break point that was observed starting from 1998 is most likely due to the change of correction methods as mentioned above. The relatively high level of child mortality recorded during the period before 1983 can not only be due to a natural effect but probably to correction method effects. Methodology change effects are more significant and apparent on this age group than on the other age categories. The effect of terrorism events during 90^{th} is also observed but with less gravity than for infant mortality.

Figure 2.12: Evolution of mortality rates by large age groups



In adverse to other age groups, the curve of ${}_5q_{15}$ is the only curve that knew relatively an important mortality level during the period 1977 - 1983. This can reflect a kind of under recording of deaths or an overestimation the population of this age. From there, the curve follows the same trend as ${}_4q_1$ with a slight bump between 1991 and 1999, and a more moderate decrease starting from the end of the black decade.

After its overall decline observed during the first period, ${}_{45}q_{20}$ starts to increase from 1990 to record in 1994 its highest level during the black decade. Thereafter, it decreases gradually before resuming its initial trend starting from 1999.

The evolution of the probability of dying between 65 and 80 years followed approximately a straight trend. This quotient has taken an upward trend from 1988 to 1991 and maintained it until reaching a value that is displayed since 1977.

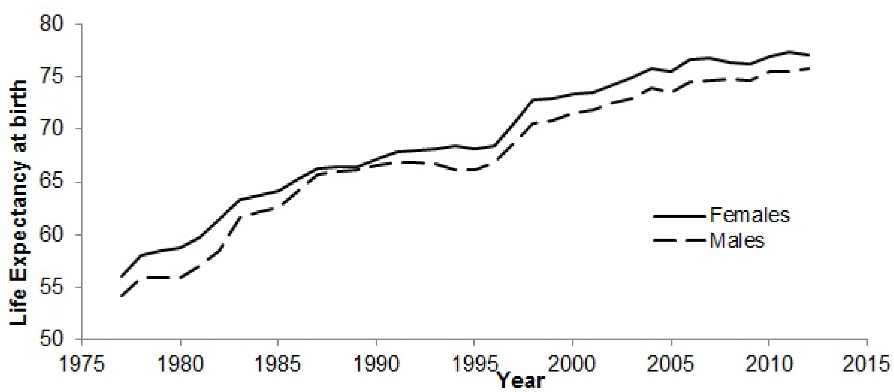
2.6 Sex differential mortality evolution

The differential life expectancy between males and females and the mortality sex ratio (MSR) represent a complementary tool to measure mortality underestimation by the civil statue (Tabutin, 1991). Now, we will try to present the evolution of such indicators according to data availability.

2.6.1 Male and female life expectancy

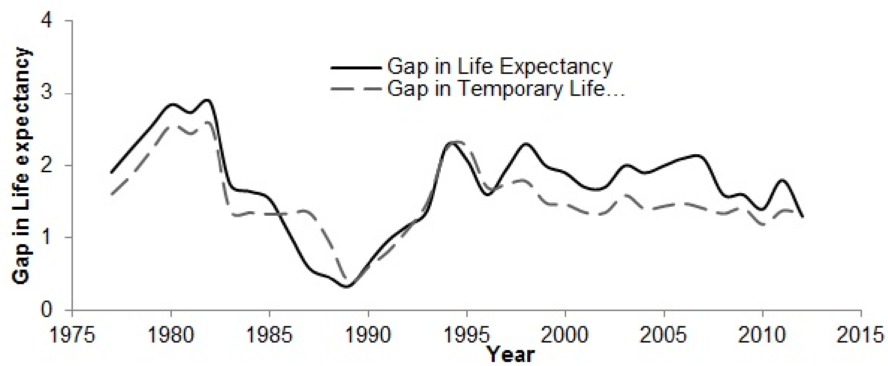
Observing the comparative evolution of life expectancy of males and females (Figure 2.13) reveals that the two curves are changing following the same trend with an average deviation of 1.73 years for the whole observed period.

Figure 2.13: Life expectancy evolution for males and females



Between 1977 and 2012, men earned 21.6 years in life expectancy going from 54.2 to 75.8 years with an average annual gain of 225 days/year. However, women won a little less, with a total gain of 21 years, going from 56 to 77.1 years and an average annual gain of 219 days/year. Even if in overall, men earned more than women in life expectancy between 1977 and 2012, the gap between the two sexes, as we can see in Figure 14, had not have a constant level over the whole period. This gap was up to 2.5 years in the first period. Such a gap should not be natural because it inexplicably down 60% in the next period. In the absence of any other explanation, it is likely that this gap can be explained by the difference in coverage of deaths between males and females and even highlights once again that deaths under recording was more important for women than for men.

Figure 2.14: Evolution of the gap in life expectancy between males and females



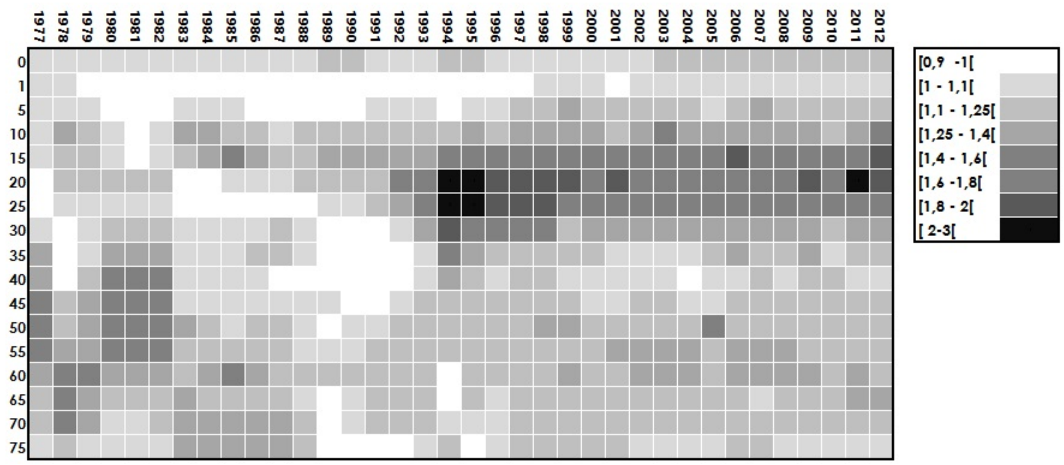
Starting from 1989 (Figure 2.14), the gap in life expectancy between males and females takes up an hawser which allowed to bring it to 2.28 in 1994. This coincides with the year that marked the highest level of mortality during the black decade. The effect of the black decade on sex differential mortality is therefore very obvious. After 1994, except the peak of 2005 which corresponds to a value of 3.4, this gap has taken a slight downward trend to reach the value of 1.3 in 2012. The visualization of the evolution of the gap in the temporary life expectancy allowed to conclude that the importance of the life expectancy of women compared to men in 2005 is simply due to an imperfection in old age mortality estimation methods.

2.6.2 Age specific mortality rates evolution

In this part, we focus on the mortality sex ratio. A particular attention will be attributed to some age groups: [0-1[, [1-4], [15-50[and [50-80]. A such analysis is supposed to inform us on the mortality of certain categories: infant, child and maternal mortality.

From reading the evolution of the MSR as it is reported in Figure 2.15, many pertinent remarks can be highlighted. The MSR for the infant age category has evolved irregularly from 1.01 in 1977 to 1.13 in 2003 and kept at that level until 2012. In concern of the child age category, an over female mortality was observed and persisted until the late 1990's and we return to a slight over male mortality resulted in a sex ratio of 1.03. This over female mortality at the age category [1-4] is not new in all; Tabutin (1991) noted a similar finding during the period 1965- 1977. This phenomenon is probably due to a sociological factor (SEP, 1975), translated by a kind of discrimination in treatment and nutrition in favour of boys. If we consider that the over female mortality observed at some years for the age category [5-10[can be considered as an extension for what it is observed at the previous age category, many other similar cases can be detected at different ages during the period prior to 1998 without having any explanation for the moment. The age category [20-50[has almost marked during the period from 1983 to 1992 an MSR very closer to 1, translated either by an over female mortality or a very slight over male mortality. The same is observed at high ages between 1989 and 1992 in addition to some other cases.

Figure 2.15: Mortality sex ratio surface



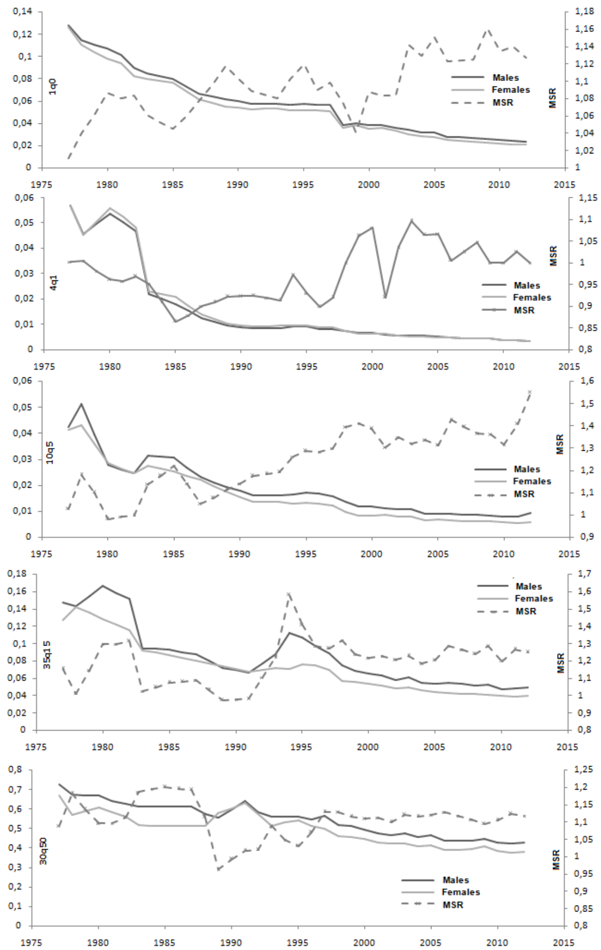
Starting from 1992, mortality rates for males aged between 15 and 35 started to rise away above those of females because of the terrorism events. Men between 20 and 30 years were more touched. During the most terrible years (1994 and 1995), the MSR approached the value of 2.4 before it started to decrease gradually to 1.7 by the end of the black decade. Thereafter, the male mortality rate between 15 and 35 years kept involving around these relatively high level for the rest of the observation period. This relatively high MSR comparatively to the other age categories is due to differences in life style, risk taking behaviour and healthy habits between males and females.

In overall, the period prior to 1998 was marked by a high irregularity in the MSR. We can easily observe the sudden droop fall occurred in 1983 at the age group [35-70[and the evident bump of the SMR for the age category between [70-80[starting from 1983 before to droop suddenly in 1989 to give a female mortality excess. Starting from 1998, the SMR marked more regularity and a general SMR age pattern began to draw a stable shape. That can give an evidence about the improvement in data quality and a diminution in the methodological imperfections.

The analysis of the evolution of the SMR by large age groups [0-1[, [1-5[, [5-15[, [15-50[and [50-80[is supposed to allow a clear reading of the evolution of the differential mortality (Figure 2.16).

Globally, the series of ${}_{35}q_{15}$ for males and females evolved following a similar downward trend except that the gap between sexes is more or less important by period. The mortality sex ratio has basically recorded two peaks (1979-1982 and 1991-1996) explained by the relative increase in male mortality rates. If the peak of 90's can be related to the high male mortality resulted from terrorism events, the peak of the period before 1982 can not find any natural explanation. Contrary to what was observed for other age categories, the male curve was not accompanied by the female curve. Since 1992, the sex ratio began to change in terms of level. After scoring a peak of 1.58 in 1994, it stabilizes near 1.2 starting from 1996.

Figure 2.16: Evolution of mortality sex ratio for large age groups



The mortality sex ratio between 15 and 50 years can serve as an indirect estimator of maternal mortality. Female mortality excess can reiterate a significant maternal mortality (De Forts, 1998). In such a case, the weakness of the mortality sex ratio reported before 1994, may indicate a significant level of maternal mortality. The disability to isolate methodological effects would be inappropriate to confirm such a finding. The analysis of the period 2000-2012 that has marked a relative stability reveals no significant events. The sex ratio has changed slightly between 1.22 and 1.25. This increase may reflect a reduction in maternal mortality comparatively to the previous period.

On the same pace seen on the other age groups, the mortality sex ratio of the age category [50-80[has had an upward trend since 1989 and almost stagnated since 1998.

2.7 Conclusion

From independence to today, mortality in Algeria has experienced significant declines resulted in a gain of 30 years in life expectancy at birth in the range of half century passing from about 47 years in 1962 to 76.4 years in 2012. Parallel to this, many improvements have been recorded in terms of civil statue coverage and many surveys and studies have been dedicated to strengthen knowledge about mortality. Coverage rates has improved from 60.6% in 1970 to 81.1% in 1981 for deaths and from 86.8% to 98.7% for births. These rates were updated in 2002 and used to correct the recorded data with retroactive effect starting from 1998, without being published as the calculation methodology. The few works dedicated for mortality evolution analysis in Algeria concluded until now (Hamza Cherif, 2011; Iles, 1990) are still superficial and more oriented to analyze the evolution of mortality according to official statistics and without addressing the effect of methodological changes and imperfections on this evolution. Our main objective in the present work was to give a deep analysis in concern of mortality in Algeria as it was described by national statistics while addressing the effect of the methodological changes and imperfections on this evolution. Life expectancy at birth was the main indicator used for such issue but other mortality indicators were also used in order to support our results. The crude mortality rate (CMR) and the infant mortality rate (IMR) were used mainly to show the methodological imperfections and their effect on the evolution of mortality indicators. As a result, it turned out that two droop fall points can be observed on the two indicators and were caused by the revision of the crude data correction factor. By the same, it appeared that there was some differences in concerns of the date of effect of the correction factor estimated in 1981 between the CMR and the IMR. The second revision of 2002 had took effect starting from 1998 and has had an apparent effect on the two indicators.

The evolution of life expectancy has had an S-logistic shape during the observation period traduced by a slowdown trend in the annual gain. Even if the level of life expectancy in Algeria remain far below the international record, the evolution of this indicators seems to be constrained by health and life conditions. In contrast of some industrialized countries where life expectancy attained a high level and have been growing with a linear trend since more than a century (White, 2003), the Algerian population could not hope to live longer in the current conditions. Pushing up life expectancy requires a heavy investment in the population health and life conditions. Although, this general trend dissimulates some change points where the series of life expectancy changed suddenly its general level and its annual improvement rate. Certainly, a change points analysis based on both level and slope changing may provide a more appropriate analysis compared to an analysis based only on the level or the slope changes. The first change point occurred in 1983 pushing life expectancy 2 years up compared to its initial course. The only explanation that we advanced was an over estimation of mortality during the period prior to 1983 due to an imperfection in the coverage rates estimation and crude data correction. The terrorism events brought life expectancy 1.9 years down starting from 1992 before a recovery (+2.6 years) in 1998. The year 1998 corresponded conjointly to two events which should affect mortality statistics: the end of the terrorism which was official announced in the late 1999, and the adoption of the new crude data correction factor combined with the 1998's census data. Thus, it was made difficult to separate the natural reduction in mortality from the methodological effect. In final, 2006 marked a last change point where the annual improvement rate reduced to the third compared to its value during the previous period by passing from 6 to 2 months per year. At that stage, it was necessary to seek a convincing explanation for all these recorded change points. For this, we pushed up our investigation to inspect the evolution of the elements involved in life expectancy calculation: number of years lived by deaths in each age category, closing out methodology and the mortality age pattern. It turned out from this analysis that in 1983, mortality rates between 20 and 65 has decreased suddenly compared to 1992. In 1998, we observe a sudden reduction at all ages compared to 1997 and that gives more evidence for the methodological change effect. It turned out, also, that the change point of 2006 was due to an imperfection in the closing out methodology for the female life tables and there is not any sudden change in the mortality age pattern compared to the previous years. In final and in order to confirm all these finding, an analysis of the evolution of the differential mortality between males and females was concluded. That provided us with some additional information in concern of the data quality and mortality evolution. It revealed that data quality has been significantly improved starting from 1998 and that the terrorism events concerned much more males than females aged between 15 and 35 years.

The quality of any analysis remains dependent on the quality of the used data. From independence to today, civil registration coverage has undergone significant improvements. The estimation of civil registration correction factor and the methods used to correct and complete the crude data are still marked by some imperfections and this can biases any attempt to do a meaningful analysis of mortality evolution in Algeria. To avoid such inconvenient, it will be necessary to correct the historical mortality surface while correcting all the imperfections cited above or at least give more coherence and adequacy to the mortality indicators evolution as a time series. Rather than using a fixed correction rates for a long periods, any interpolation based on the coverage rates estimated in 1977, 1991 and 2002 will allows to extenuate the sudden change observed on the different mortality indicators. The other imperfections can be treated on the basis of the view of experts in the population studies domain. However, the fact that the ONS did not publish any details about the crude data and the estimation methodology makes this challenge extremely difficult and pushed the international organizations to use an indirect estimation process to approach mortality level and life expectancy in Algeria. In all cases, indirect estimates cannot replace estimates based on real data and differences in the estimation methodologies can affect significantly the estimation results. From our point of view, that is why UN estimates about life expectancy in Algeria differ from those of the WHO and from those of the ONS. An inter institutional collaboration is much needed in order to bring closer data sources and estimation methodology which will certainly reduce the gap between the estimates of the different institution about mortality in Algeria.

Chapter 3

Estimation of the missing data in the Algerian mortality surface (1977 – 2000) by using an age-time-segmented Lee-Carter Model

This work has been presented in Stochastic Modeling and Data Analysis Conference. SMTDA2014. Lisbon, Portugal, June $13th$, 2014.

3.1 Introduction

After the independence in 1962, the Algerian administration made great efforts to improve the quality of civil registration data. A first General Census of Population and Housing (GCPH) has been achieved in 1966. 4 years later, a largest demographic survey was taken. The National Statistical Population Study (1969-1970) allowed the estimation of the first life table for the Algerian northern population. After the second GCPH in 1977, the first Algerian life table based on civil registration data has been published by the Office of National Statistics (ONS). After that, the ONS has continued publishing these tables by an irregular frequency principally because of the shortage and the unreliability of the data if available. It is only since 1998 that national life table have been annually constructed. In addition, some life tables of the previous period have been closed earlier than 80 years. However, no one has tried to estimate the missing life tables for the previous period. Today, it is very necessary to make this data available, especially for the governmental previsions and actuarial prospective calculations.

In Flici (2016-a), for the needs of dynamic life tables construction which requires a continuous mortality surface, we estimated the missing mortality rates by a polynomial fitting of the specific age survivals series. The obtained results suffered some weaknesses regarding the consistence between the estimated values and the existing ones. Here we aim to achieve a more consistent estimation.

The main objective of the estimation of the missing data in the historical mortality surface is to allow forecasting the mortality age pattern in the future. In fact, the implementation of the mortality forecasting models requires the disposal of a complete and a continuous historical mortality surface (Lee and Carter, 1992; Renshaw and Haberman, 2006; Cairns and al., 2008 ... etc). Li, Lee and Tuljapurkar (2004) proposed an adapted version of the Lee-Carter model (Lee and Carter, 1992) for the case of incomplete mortality surfaces. Following their idea, the mortality variation index is projected with a Random Walk with drift model calibrated only on years with available data. However, the mortality time variation index in the Lee-Carter model is much more complicated to be projected with other time series models. Even if the random walk with drift ensures an acceptable predictive capacity in some contexts, it can lead to inconsistent forecasts when the historical trend of mortality is not linear. In such cases, other time series models must be compared and applied to extrapolate the time component in the future. Also, the quality of the forecast is mainly related to comparing a set of candidate mortality models. Hence, the method proposed by Li and al. (2004) is only adapted to the Lee-Carter model. The implementation of the models including the cohort effect (Renshaw and Haberman, 2006; Currie, 2006; Cairns and al., 2008) is much more complicated. That is why it is so important to estimate the missing data in the Algerian mortality surface which represents the main objective of this work.

Several methods can be used to estimate the missing life tables. Because of particular structure of the missing data among the existing one in our case, the set of methods that can be used is restricted. In first, we note that the jump on data is no more than one year. In other words, there are no two consecutive years with no data. The second element that we must take into account is that some life tables were closed out before the age of 80. Our idea is based on the original model of Lee and Carter (1992), which was initially developed to project the age mortality pattern in the future. Here, we use it just to fit the historical mortality surface. The ages parameters are estimated on the basis of the complete existing annual life tables. The time component is estimated for years with partially / fully available data. The values corresponding to the missing years are estimated by using a polynomial function fitted on the existing data. The more the fitting quality is better, the more the estimated values are consistent with the existent ones.

3.2 Data

As mentioned bellow, the first Algerian life table based on civil registration data has been established in 1977. Between 1997 until 1998, the mortality surface stilled discontinuous and there are some years with no life tables: 1979, 1984, 1986, 1988, 1990, 1992 and 1997. By the same, the closure age varied inexplicably from a year to year as showed in table 3.1.

Table 3.1: Closure age for the Algerian life tables (1977-2012)

Years	Closure Age
1983, 1985, 1987	70
1993 - 1996	75
1977, 1978, 1980, 1981, 1982, 1989, 1991 and [1998-2009]	80
2010-2012	85

As shown in Table 3.1, the closure age varied permanently during the observation period. It was at 80 years during the period 1977-1982, 70 between 1983 and 1987, 80 years from

1989 to 1991, 75 years during 1993- 1996, 80 during 1998- 2009 and 85 years starting from 2010. In addition to the estimation of the missing calendar years life tables, we aim, in the present work to complete life tables that were closed-out before the age 80.

Table 3.2: Missing data in the Algerian life tables (1977-1997)

	Female_dath_quotient_qx																						
qx	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
0	0,1263	0,1102		0,0963	0,0956	0,0639	0,0603		0,0766		0,0620		0,0554		0,0550		0,0533	0,0518	0,0517	0,0522		0,0506	0,0556
1	0,0570	0,0455		0,0558	0,0527	0,0461	0,0251		0,0209		0,0159		0,0105		0,0092		0,0094	0,0094	0,0097	0,0090		0,0054	0,0064
5	0,0173	0,0166		0,0113	0,0104	0,0099	0,0151		0,0113		0,0066		0,0070		0,0050		0,0053	0,0051	0,0048	0,0047		0,0042	0,0051
10	0,0112	0,0114		0,0068	0,0065	0,0061	0,0075		0,0074		0,0067		0,0050		0,0040		0,0056	0,0056	0,0056	0,0057		0,0051	0,0023
15	0,0151	0,0155		0,0102	0,0096	0,0059	0,0071		0,0067		0,0066		0,0055		0,0048		0,0046	0,0045	0,0049	0,0047		0,0040	0,0051
20	0,0162	0,0154		0,0144	0,0156	0,0126	0,0096		0,0092		0,0079		0,0060		0,0053		0,0056	0,0056	0,0067	0,0063		0,0046	0,0041
25	0,0213	0,0180		0,0164	0,0154	0,0145	0,0100		0,0096		0,0087		0,0075		0,0063		0,0066	0,0066	0,0074	0,0066		0,0059	0,0049
30	0,0157	0,0216		0,0196	0,0179	0,0165	0,0120		0,0121		0,0099		0,0095		0,0089		0,0064	0,0064	0,0063	0,0066		0,0077	0,0063
35	0,0181	0,0251		0,0244	0,0255	0,0220	0,0154		0,0152		0,0150		0,0129		0,0113		0,0124	0,0119	0,0121	0,0120		0,0109	0,0090
40	0,0211	0,0269		0,0241	0,0254	0,0226	0,0157		0,0171		0,0159		0,0164		0,0148		0,0168	0,0170	0,0179	0,0172		0,0141	0,0123
45	0,0266	0,0307		0,0272	0,0256	0,0245	0,0251		0,0204		0,0189		0,0166		0,0152		0,0192	0,0194	0,0214	0,0225		0,0191	0,0172
50	0,0364	0,0412		0,0399	0,0372	0,0346	0,0314		0,0319		0,0297		0,0261		0,0266		0,0301	0,0306	0,0292	0,0288		0,0257	0,0223
55	0,0554	0,0621		0,0576	0,0546	0,0519	0,0459		0,0506		0,0509		0,0459		0,0450		0,0470	0,0452	0,0480	0,0435		0,0391	0,0325
60	0,0907	0,0756		0,0864	0,0854	0,0794	0,0759		0,0693		0,0764		0,0556		0,0711		0,0495	0,0975	0,0751	0,0776		0,0655	0,0555
65	0,1445	0,1020		0,1367	0,1346	0,1268	0,1228		0,1196		0,1066		0,1406		0,1167		0,0967	0,1214	0,1062	0,1069		0,0950	0,0645
70	0,2477	0,1604		0,1960	0,1663	0,1766							0,1946		0,1952		0,1955	0,1676	0,1759	0,1750		0,1600	0,1564
75	0,5772	0,5141		0,5169	0,2956	0,2766							0,5049		0,4057							0,2575	0,2355

Table 3.2 shows the shape of the missing data among the existing data. Blank cells in the table represent the missing five-age mortality rates that we propose to estimate in the present chapter.

3.3 Objective and Methodology

The main objective of the present chapter is to estimate the missing life tables as it should be done by the ONS. It means that we suppose the same conditions and the same data quality in the considering years. In other words, our objective is not to approach the real mortality level at the considered period, but just to give a coherent estimate with the existing life table while respecting the specificity of each period. In Chapter 2, we have shown that mortality evolution in Algeria; according to official statistics, was heavily affected by a set of methodological change effects. The mortality scheme has changed many times principally because of the revision of the crude data correction factor in 1983 and 1998. The 1990's knew a relatively high mortality level due to terrorism events that have known Algeria during the period 1992 – 1999.

The second objective is to complete the life tables closed-out before the age of 80, in order to ensure a common closure age for all the tables. The choice of the age of 80 is explained by the fact that a big part of life tables was closed at the age of 80.

To summarize, the main idea that we propose to estimate the missing data is to proceed by a parametric fitting of the historical mortality surface by using the Lee-Carter model. The parameters of the estimated model will then be used to estimate missing data.

The choice of the period 1977-1999 to be a basis of fitting is principally justified by the mortality trend changing of 1998 (Chapter 2). During the period 1991-1999, Algeria has lived a terrorism decade. The mortality caused by terrorism was very important. The adoption of the national reconciliation pact had significantly limited the terrorism acts after 1999. Also, the revision of the crude data correction factor starting from 1998 has contributed to the cited trend change. To be more clear, the evolution of the life expectancy at birth in Algeria marked a changed level point in 1998 (Chapter 2). In reality, this change-point should occured in 1999 if we consider only the data annually published by the ONS. On the basis of the crude data correction factor estimated through the Family Health Survey (2002) and data issued from the 1998's Population Census, the ONS revised the 1998's life table. The corrected series has been published in 2012 (ONS, 2012-a) and contained a retrospective review of the ONS publications about the Algerian demographics main indicators. 1997 is a missing calendar year life tables. Its estimation is much related to the neighboring years. But the question that we should ask in such conditions is how the 1997's life table could looks like being constructed by the ONS? Certainly, it should be closest to the original 1998's life table rather than the revised one especially if we consider that the ONS had revised the 1998's life table because the Population Census has been done in the same year.

In the other side, we notice that 1999 will be included in the mortality surface to be fitted even if an apparent change trend occurred in 1998. Similarly, the life tables of the period 1993-96 were closed out at the age of 75. In addition, the life table of 1997 is missing, considering an additional year in the fitting process has to improve the weight of the cited age group (75-80) compared to the others. Enlarge the reference calculation period beyond 1999 may have to influence the fitting quality since it will lead to an apparent change in the time index trend.

To conclude about the time range used in the fitting process, we prefer using the period 1977-1999 for the arguments cited above while considering the original 1998's life table rather than the revised one.

In concern of the methods which can be used to estimate the missing data in our case, several approaches can be envisioned, but the results are less sufficient in the Algerian context. The structure of the missing data imposes to propose an adapted estimation method. The most important point is that there is not more than one missed year between two published life tables. So, we can simply extend all the annual life tables until the age of 80 by using the usual mortality models and then use the average to estimate the missing table between two tables as it has been done in Chapter 2. In Flici (2016-a), we used a polynomial function to fit the time series on the surviving number series for each age, and then calculated the resulted mortality rates. The difference is not important, but the results lack some regularity and coherence when the estimated data is compared to the existing one.

When we are dealing with mortality surfaces, specific age mortality rates can either be considered as an age function or as a time series. In each case, mortality evolution must follow a well-defined function. If we take only one dimension into account, this can lead to some incoherence when the resulted estimates are taken by the point of view of the second dimension. The idea is to combine these two dimensions: age and time. The Lee-Carter model was initially developed for mortality forecasting based on the decomposition of the historical mortality surface on three components: two are related to age and one to time. The extrapolation of the time components to the future allows to construct the prospective mortality surface. By the present work, we aim to use the same principal to complete the mortality surface. More details will be exposed bellow.

3.4 Lee - Carter Model (1992)

The Lee-Carter formula was developed for mortality forecasting. The idea is to decompose the logarithm of the central death rate into three components: the average age mortality pattern (α), the mortality time variation index (k) and the age specific sensibility (β). The calculation formula is given by:

$$ln(\mu_{xt}) = \alpha_x + \beta_x * \kappa_t + \varepsilon_{xt}$$

$\alpha_x$represents the average, for each age, of the death rate logarithm:

$$\alpha_x = \frac{1}{n}\sum_{t=t_1}^{t=t_n} ln(\mu_{xt})$$

The residual matrix is then decomposed into two vectors β_x and k_t:

$$ln(\mu_x) - \alpha_x = \beta_x * \kappa_t$$

while respecting the identifiability constraints : $\sum \beta_x = 1$ and $\sum \kappa_t = 0$.

Several methods can be used to estimate β_x and k_t. In the original paper (Lee and Carter, 1992), the authors used the Singular Values Decomposition method based on the Principal Components Analysis. Nevertheless, other techniques were developed later.

In this chapter , we propose to use a decomposition techniques based on the optimization process. The idea is to estimate the two components by an optimization problem aiming to reduce the gap between the initial matrix $ln(u_x) - \alpha_x$ and the matrix obtained by the product of the estimated vectors β_x and k_t. For that , we can use any optimization criterion : Ordinary Least Squared Errors (OLS), Absolute Main Error (AME). This point is discussed in further details in section 3.3. The use of the XL solver facilitates the computation process. A detailed calculation example is shown in Flici (2016-a).

Once the time index estimated, the k_t values corresponding to the missing years in the series are estimated by a polynomial fitting of k_t series. We note that for our application we use the global, males and females life-tables with five age description.

3.5 Fitting criterion

In this section we will discuss different fitting criteria in order to select the adapted one for mortality surface fitting issues. A mortality surface describes the evolution of mortality rates in two dimensions: age and time. To simplify the discussion, we will firstly focus only on the age dimension.

The adjustment of mortality curves passes necessarily by choosing the model that best describes the evolution of mortality rates over age. The estimation of the model parameters must be based on a convenient fitting criterion. That can be understood in different ways and different criteria can be used. Historically, various criteria were developed for this issue. For the simple linear regression, the criteria the most commonly used are maybe the Ordinary Least Squared Errors (OLS) and the Least Absolute Deviation (LAD) (Chen and al., 2016). Mortality curves have approximately an exponential form. The estimation of the parameters of the fitted curve passes by making the curve to fit on a linear form by introducing the log function. Then, the evolution of the death rates over age is treated like a line. Both OLS and LAD have to eliminate the offsetting between opposite sign errors. The difference is that with the OLS method, making errors on square allows to enlarge the relative value of the biggest errors. Then, the regression process is more oriented to reduce the biggest errors compared to the smallest ones. This is the advantage of OLS method compared to LAD, which consider all errors with a similar importance.

Here, we illustrate the result obtained with the two methods applied to fit the Algerian male life table (1999) by using Gompert'z formula (Gompertz, 1825) and for the age range: 30–65 years. The linear form of Gompertz formula is given by passing to the ln of the central death rate μ_x.

$$\mu_x = -ln(1 - q_x)$$

Passing to the logarithmic allows getting the linear form:

$$ln(\mu_{xt}) = ln(-ln(1 - q_x)$$

With the OLS regression method, the parameters are defined in order to minimize the following objective function :

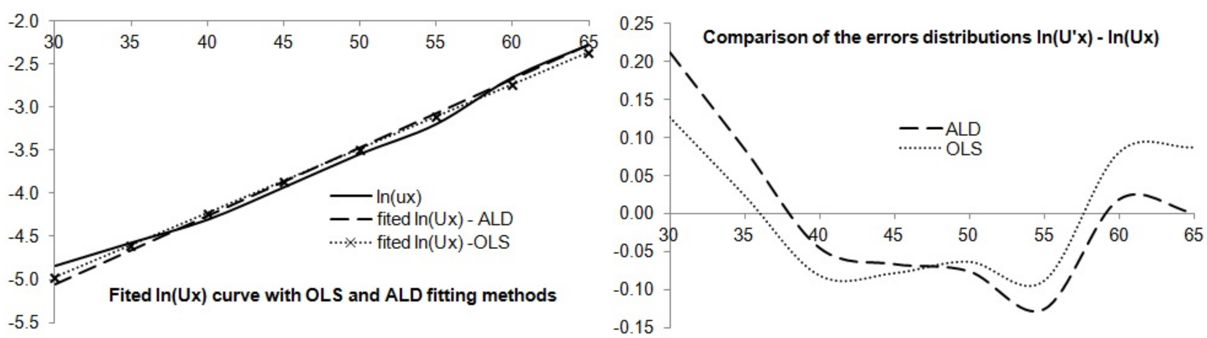
$$MinA = \sum_{x=x_1}^{x_n} [ln(\hat{u}_{xt}) - ln(u_{xt})]^2$$

With the LAD, we have to minimize:

$$MinB = \sum_{x=x_1}^{x_n} |ln(\hat{u}_{xt}) - ln(u_{xt})|$$

The results are given by Figure 3.1.

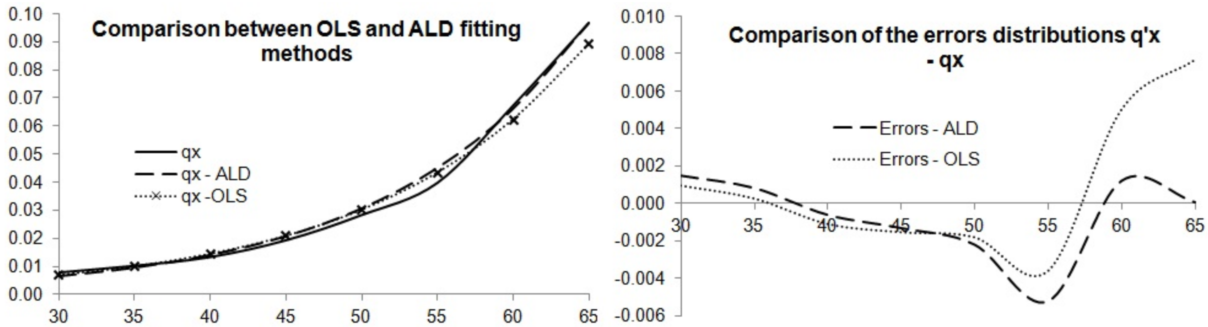
Figure 3.1: Comparison of OLS and ALD errors distribution- Log scale



The figure left shows the fitted curve compared to the original one. The figure right illustrates the errors distribution obtained under the two regression techniques.

It is difficult to observe the difference between the results obtained by the two methods comparing only the fitting results. The observation of the errors distribution has to clarify this difference. We mentioned earlier that the OLS method focuses more on the biggest errors. Here, we observe the opposite. Compared to ALD, OLS gives more accurate results at lower ages corresponding to lower death rates (μ_x). The result is logical if we consider the effect of the function (ln) on the death rates pattern. The linear form was obtained by introducing the (ln) function. Death rates are for the considered ages comprised between zero and one. The (ln) function has to reverse the death rates importance with negative sign. Considering in absolute, the smallest values μ_x give the biggest value $ln(\mu_x)$ and vice versa. The difference between OLS and ALD in term of fitting quality rises from this point. The corresponding results obtained on mortality rates (q_x) are given by Figure 3.2.

Figure 3.2: Comparison of OLS and ALD errors distribution- Natural scale



The observation of the results obtained on the mortality rates (q_x) confirms that the effect expected by the use of the OLS method is reversed following the introduction of the logarithm. In other words, when the mortality rates curve is fitted by the procedure previously explained, the obtained adjustment is more adequate for the linear form (ln) but note for the original curve (q_x). To support this argument, we show in Table 3.3 a comparison based on the Standard Deviation (SD) and the Coefficient of Determination (CD) obtained on the mortality rates curve and the corresponding linear transformation.

Table 3.3: Adjustment quality on transformed and original curves

	Standard Deviation			*Coefficient of Determination*	
	Errors on ln(ux)	**Errors on q(x)**		**ln(ux)**	**q(x)**
OLS	0.0888	0.0038	**OLS**	99.06%	99.29%
ALD	0.1074	0.0022	**ALD**	99.06%	99.51%

We observe that, whatever the used regression method is : OLS or ALD, the quality of the fitting is better on the transformed curve compared to the initial one. Since the result depends largely on the selected criterion, we must fix the ultimate use of the obtained model parameters. We remind that we are interesting by the fitting quality on the mortality rates curves and not on the quality of the linear adjustment itself. For that , we must adapt the objective function on the objective that we have:

$$MinA = \sum_{x=x_1}^{x_n} (\hat{q}_x - q_x)^2$$

or

$$MinA = \sum_{x=x_1}^{x_n} |\hat{q}_x - q_x|$$

Here we show the example of a curve fitted with the two formulas with a comparison between $ln(u_x)$ and q_x (Figure 3.3 and Table 3.4).

Figure 3.3: Fitting quality comparison q_x Vs. $ln(q_x)$ / OLS vs ALD

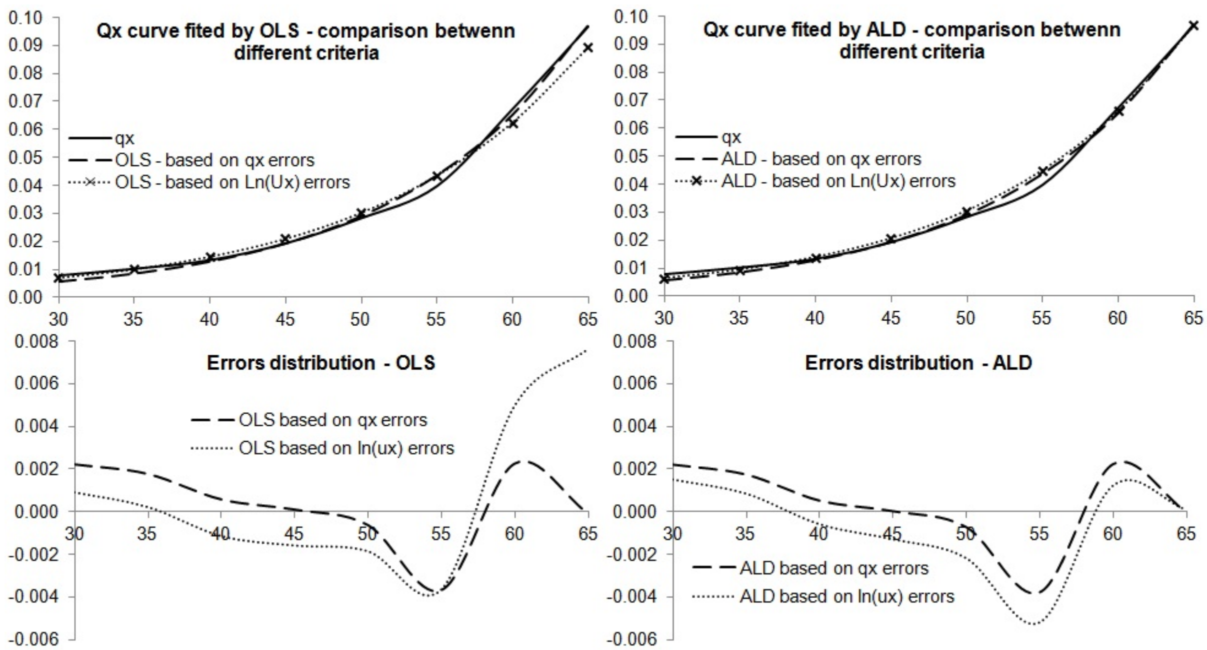


Table 3.4: Adjustment quality with Mean Absolute Deviation as optimization criteria

	Errors Standard Deviation			*Coefficient of Determination*	
	Based on ln(ux)	**Based on q(x)**		**Based on ln(ux)**	**Based on q(x)**
OLS	0.0038	0.00196	**OLS**	99.29%	99.63%
ALD	0.0022	0.00197	**ALD**	99.51%	99.63%

We confirm that whatever the used regression method is, the results are better when the model parameters are calculated with an objective function based on the mortality rates errors reducing. And we can easily observe that ALD regression method allows obtaining better adjustment quality compared with the OLS.

We remind once again the interest to consider the final objective of the work when we are choosing the optimization criterion. Our objective is to estimate the parameters of the adequate mortality model, i.e. Lee Carter model. The estimation method that we need is the one which allows reducing the relative importance of the errors at each age. The importance must be the same for each age, whatever the level of the considered mortality rates is. To do this, we must consider the relative error. In this sense, the Mean Percentage Absolute Deviation (MPAD) can be a very suitable criterion for our objective. MPAD was already used in many applications (Thomas and Planchet, 2014; Felipe et al., 2002).

3.6 Implementation of the Lee Carter Model

Since the objective of the present work is not approach the real level of the Age Specific Mortality Rates during the corresponding years but approach the life tables like that could be done by the ONS. According to that , the Lee-Carter model will be used for a different purpose than for whom it was first proposed. The Lee-Carter model was developed to forecast the age specific mortality rates in the future, by extending the historical observed trend. Here, we use the same model to estimate the missing data in a mortality surface. The model will be fitted on the available data, and then, the estimated parameters will be used to estimate the missing values. Better the fitting quality is, better the coherence between the existing and the estimated data will be. Our idea consists simply to estimate the parameters of the Lee-Carter model by using only the available complete life tables (extended till 80 years). Then, missing life tables will be reconstructed by using the first estimated age parameters α_x and β_x. The value κ_t corresponding to the missing years are interpolated from the first estimated κ_t by using any polynomial function.

Usually, Lee-Carter model is used to model the age specific death rate. The shape of the age specific death rates is more easily adjustable than the curve of mortality rates (Gavrilova and Gavrilov, 2014). In our case, mortality is principally described by five-age mortality rates. The death rates have been published only for some years in the ONS annual publications. Completing the existing mortality surface which is the main objective of the present work returns to estimate the missing five age mortality rates ${}_nQ_x$. For that , we reformulate the Lee Carter model to suit the case of ${}_nQ_x$. There is another alternative consisting to convert ${}_nQ_x$ into ${}_nM_x$. The estimated missing ${}_nm_x$ is then reconverted into ${}_nQ_x$ by using the approximation of Kimball (1960): ${}_nQ_x = (2 * n *_n M_x)/(2 + n *_n M_x)$. However, this double stage of conversion process may lead to an important loss in fitting quality. Also, this approximation relationship is based on the hypothesis of uniform distribution of deaths in each square [x.t]. Deaths occurred at the age categories 0 and [1-4] are concentrated by the beginning of the age intervals (Bourgeois, 1946; Coale and Demeny, 1966). In such a case the Kimball's approximation may lead to inaccurate values. Some adapted formulas are needed in this case. In order to simplify the fitting process and the estimation of the missing data, we propose simply to use the Lee-Carter model to fit the five age mortality rates rather than death rates. The final formula becomes:

$$ln({}_nQ_{xt}) = \alpha_x + \beta_x * \kappa_t + \xi_{xt}$$

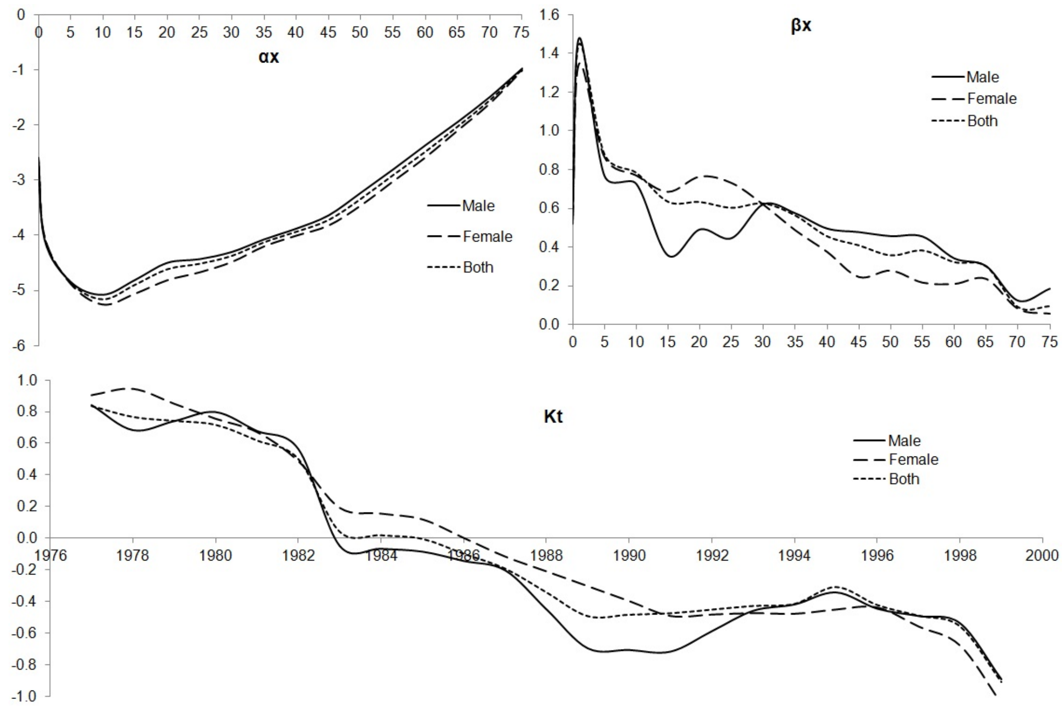
With ξ_{xt} representing the error term.

The first parameter to be estimated is:

$$\alpha_x = \frac{1}{T}.\sum_{t=1}^{T} ln({}_nQ_{xt})$$

Then, the residual matrix is decomposed into two vectors: $ln({}_nQ_{xt}) - \alpha_x = \beta_x * \kappa_t$ while verifying the identifiability constraints: $\sum \beta_x = 1$ and $\sum \kappa_t = 0$. The parameter α is defined to be the average of the log of the five-age mortality rates by age. The matrix $Z = [ln({}_nQ_{xt}) - \alpha_x]$ was decomposed by an optimization process on XL-Solver, the objective function was the Mean Percentage Absolute Deviation (MPAD) on${}_nQ_x$. This estimation process was achieved by introducing two constraints ensuring the uniqueness of the solution: $\sum \kappa_t = 0$ and $\sum \beta_x = 1$. The first constraint is verified only on the available data. When the missing data will be estimated, this constraint will not be verified and it loses all its mathematical significance. Note that, this constraint allows adapting the parameter to the extrapolation process but it is not essential. In addition, the respect of this constraint decreases lightly the adjustment accuracy. In the other side, the β_x parameter can be negative for some ages (Lee and Carter, 1992). The obtained results are shown in Figure 3.4.

Figure 3.4: Lee Carter parameters estimation



The obtained results regarding the goodness-of-fit given by Table 3.5, gives the MPAD for the male, female, and both sex populations.

Table 3.5: MPAD - first estimation

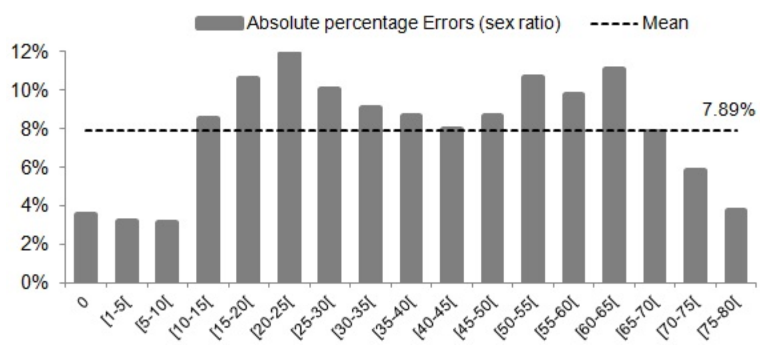
	Males	Females	Both sexes
0	6.97%	4.80%	5.70%
[1-15[	18.99%	13.63%	16.50%
[15-60[	11.96%	4.86%	8.07%
[60-80[	7.93%	10.05%	8.14%
Mean	9.90%	7.65%	8.28%

The model gives a best fitting quality on the female mortality surface, especially for the age range [15, 60[. The variation on the mortality trend during the considered period is important because of the methodological changes and the terrorism events that has known Algeria between 1991 and 2000 (Chapter 2). By the same, we observe that the adjustment quality is lower for ages between 1 and 15 years. The use of a common fitting model for all ages may lead to such effects. Proceed by an adjustment by age categories having a similar mortality profile may improve the quality of the adjustment.

Equally, it is necessary to take into account the coherence between the results obtained on the three populations. For that we have introduced two control variables on the estimation process: the Males Females Mortality Ratio MFMR, and the coherence between the global mortality rates (both sex) and mortality rates for males and females.

The MFMR follows generally a specific trend (De Jastrzbski, 1919). The value obtained on the estimated tables (males and females) must be coherent with the observed age pattern. To measure this quality, we have to compare, for the available years data, the original and the estimated curves of the MFMR by age, and use the Relative Percentage Deviation as optimization criterion. The results are shown in Figure 3.5.

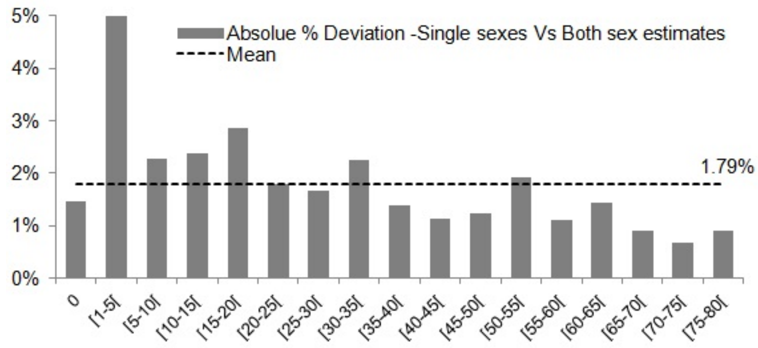
Figure 3.5: Deviation between original and estimated sex-age ratio curves



On the basis of Figure 3.5, we can deduce that the used method gives the best quality at the extreme ages [0, 10] and [65, 75]. That confirms the necessity to proceed by an independent fitting by large age groups.

The second control criterion allows to compare the estimated single sex life tables to the estimated both sex life tables. The estimated mortality rate at age x for the both sex population is supposed to be equal to the average of the mortality rates for males and females weighted by the sex structure of the population at age x. More the two values are near, more the estimation results are coherent. The deviation between the global population mortality rates and the weighted average of males and females mortality rates for age x going from 0 to 80 is shown in Figure 3.6.

Figure 3.6: Deviation between the both sexes mortality rates and the weighted average of male and female death rates



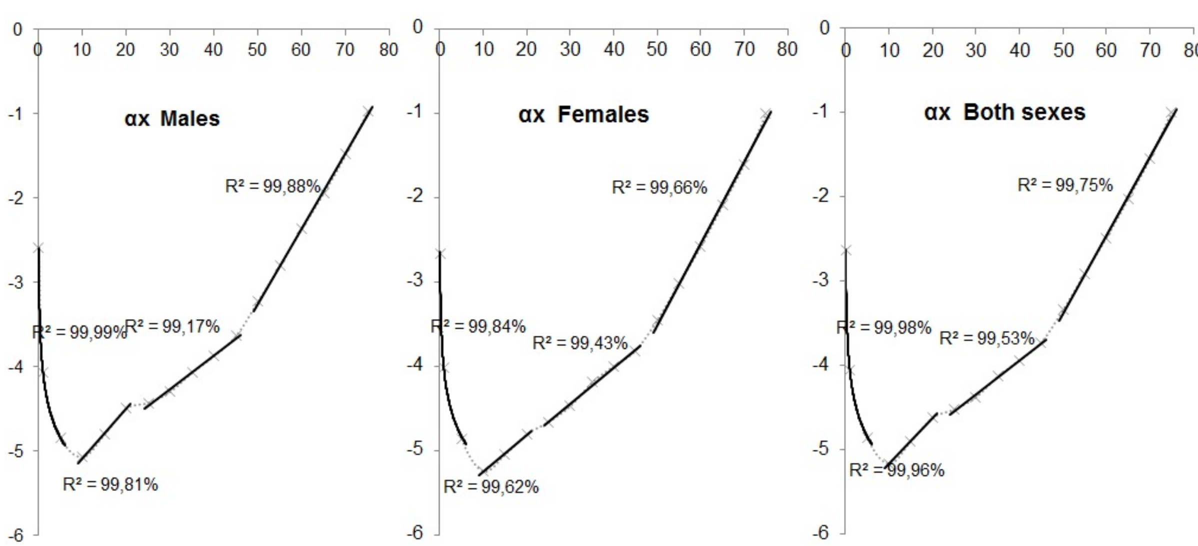
In terms of the adequacy of the estimated data between single sex's tables and both sexes table, we can say that the age group [1-4 years] has marked the most important error. For the ages beyond, the error decreases continuously.

Globally, the mean value of error which is equal to 2% is acceptable. Here, we aim to optimize the goodness-of-fit within improving the quality of the control variables. To improve the fitting quality, we prefer to proceed by a partial adjustment. In other words, we will segment the age range [0-80[into sub-ranges having a similar mortality profile.

3.7 Lee Carter adjustment on sub-ranges ages

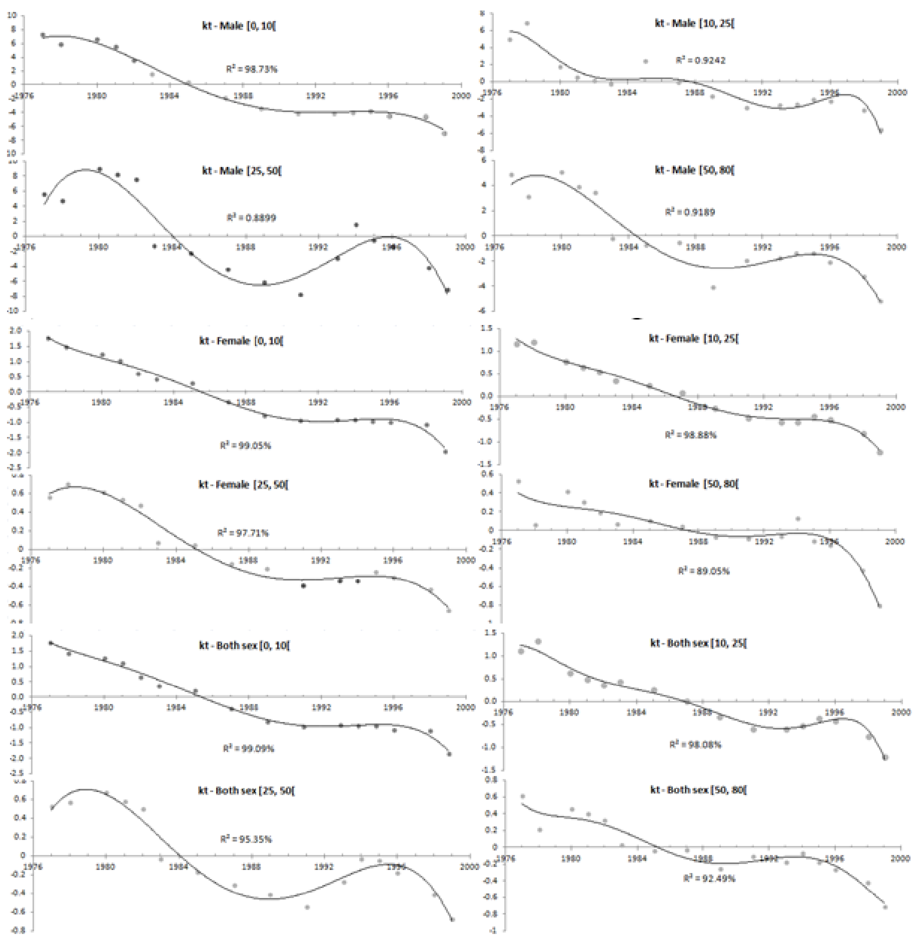
The segmentation will be done on the basis of the trend of the parameter α. This least represents the average mortality rate by age. The idea is to construct a sub-ranges where the evolution trend is relatively regular. The segmentation results are shown in Figure 3.7.

Figure 3.7: Age segmentation of the mortality surface



By this segmentation, we meant to constitute homogeneous age groups that have a similar mortality profiles. We aimed to constitute a series as long as possible with as a fewer number of parameters as possible while keeping the coefficient of determination as high as possible (>99%) and as similar as possible for all the constituted age groups. For the three tables, we got 4 age groups: [0, 10[, [10, 25[, [25, 50[and [50, 80[. For the first group, the $ln(q_{xt})$ evolves following a quadratic function. Thereafter, the mortality evolution can be represented by a linear function. Except for the third age group, the male table gave better fitting quality than females and both sexes curves. In the rest of this application, we keep the original values of α. The fitting serves only as segmentation method. The obtained k_t curves are represented by Figure 3.8.

Figure 3.8: k_t – by age ranges



We observe that the fitting function passes perfectly through the κ_t points. The fitting quality is much better at lower age categories than at higher ages. We recall again that we aim to ensure a certain coherence between the existing data and the estimated (missing) data from the point of view of time evolution. In order to improve this coherence, we have to improve the goodness-of-fit of the existing κ_t since the same fitting function will be used to estimate the missing values of κ_t. For this, we propose to improve again the fitting quality by introducing a time segmentation. The number of parameters is not important here. The main objective is to reduce errors. If we observe the fitting results obtained until now on ${}_nQ_x$, we note an improvement of the goodness-of-fit for the three surfaces (Table 3.6).

Table 3.6: MPAD - Age segmentation

Males	Females	Both sexes
7.60%	5.52%	6.48%

Without age segmentation, the MPAD corresponding for males, females and both sex mortality surfaces were respectively 11%, 7% and 9%. We observe that the age segmentation led to significant improvements in fitting quality especially for the male surface. Furthermore, the fitting quality has not been improved similarly for all age categories as shown in Table 3.7.

Table 3.7: Detailed MPAD - Age segmentation

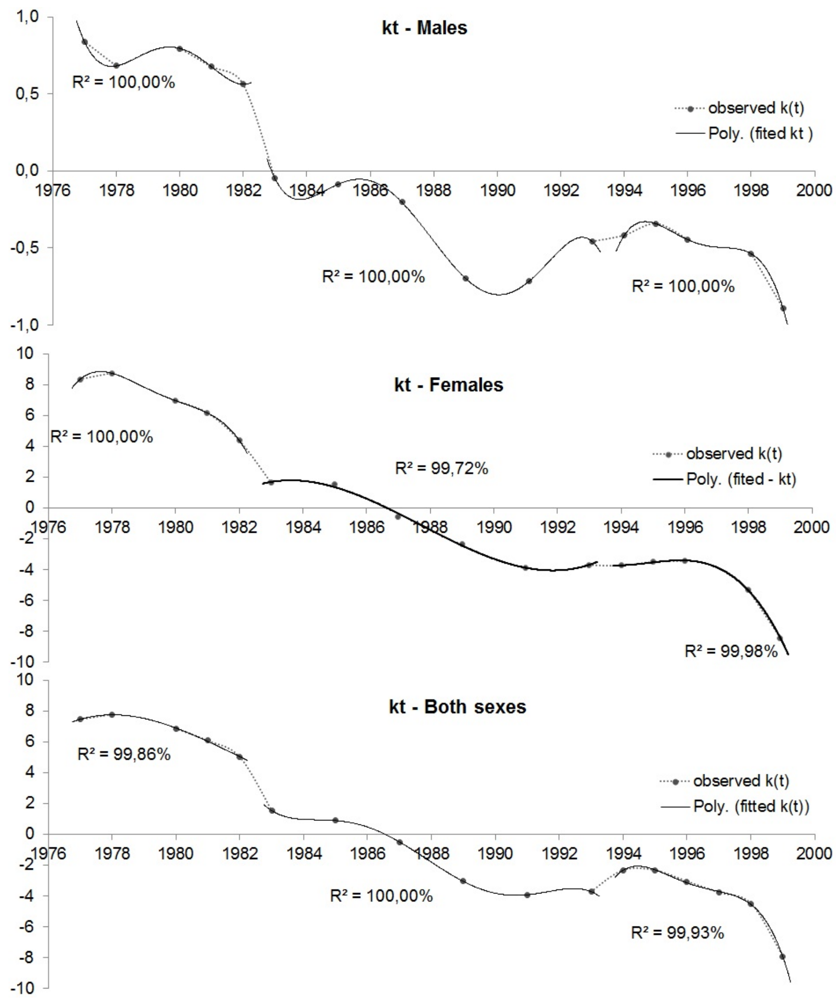
	MPAD - Global Surface Fitting			*MPAD - Fitting by Age Ranges Surface*			
Age	**Male**	**Female**	**Both Sexes**	**Age**	**Male**	**Female**	**Both Sexes**
[0, 10[	14.45%	8.77%	11.49%	**[0, 10[**	8.04%	7.27%	8.08%
[10, 25[	17.39%	7.54%	12.08%	**[10, 25[**	11.07%	5.82%	8.70%
[25, 50[	20.59%	5.26%	8.40%	**[25, 50[**	6.69%	3.68%	4.65%
[50, 80[	9.12%	7.56%	12.28%	**[50, 80[**	6.41%	6.01%	6.10%

The age range that has benefited the more from the age segmentation is the class [25, 50[. Here, we do not aim only to improve the average fitting quality but by the same way to reduce the variability of the observed MPAD at different ages. The mortality trend has changed over time and marked some change-level points. Fitting the mortality age-range surfaces that lies on long periods with changing mortality trend may have negative effects on the fitting quality. To escape this disadvantage, we will try to proceed to a time segmentation based on significant change points.

3.8 Time segmentation

The estimation of the calendar years missing data is based on the estimation of the κ_t values for the considered years. For that, we must estimate a function that better describes the evolution of κ_t over time. We have tried to estimate a unique function capturing the mortality trend during the whole period for each age category. The fitting quality is still not much good and does not allow to give the attained estimation quality. The idea is to segment the analyzed period on sub-periods where the local trend can be represented by one function. Since we have already segmented the age range [0-80[into 4 age sub categories, we will try in the following step to define a common time segmentation for all age categories and for the three populations. The idea is to fit the maximum number of years by the fewer number of parameters as possible. We reconsider the κ_t series for males, females and both sex populations already shown in Figure 3.4. The results of the time segmentation are shown in Figure 3.9.

Figure 3.9: Segmentation and fitting of the sub-period mortality trends



Time segmentation is supposed to improve the fitting quality. Each sub-period has a different mortality trend. Considering these three sub-periods like one period may conduct to reduce the fitting quality in this sense that we try to resume different trends by only one function. This segmentation allows separating the effect of each sub-periods trend from the others. The coefficient of determination is higher than 99% for the three sub-periods and for the three surfaces. In final, we applied the Lee carter formula by small areas and we have in all 12 small surfaces to fit for each population (male, female and both sex). Figure 3.10 shows the new estimated k_t after age and time segmentation. The figure shows also the fitting of the different series by a polynomial function.

The parameters α_x and β_x re-estimated after age and time segmentation are respectively represented in Figure 3.11 and 3.12.

Figure 3.10: κ_t segmented curves

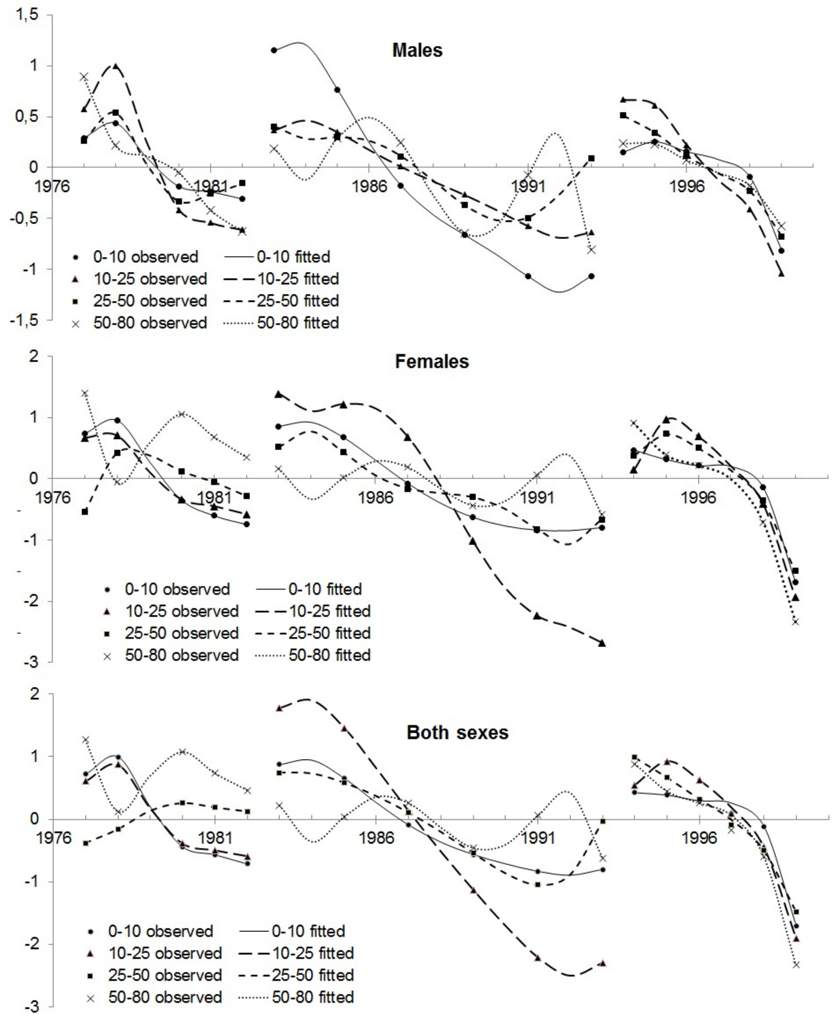


Figure 3.11: Time segmentation - α_x parameters

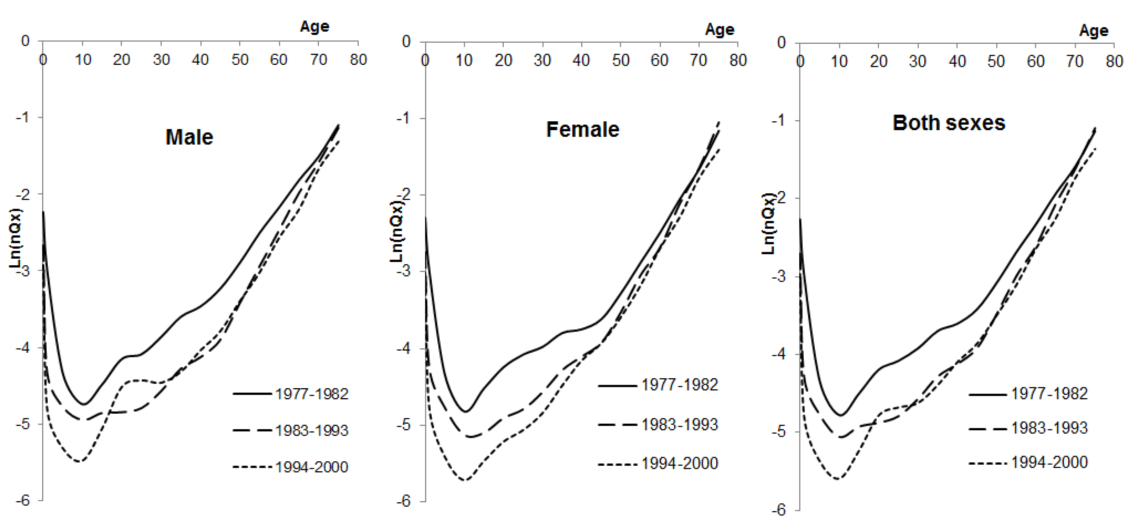
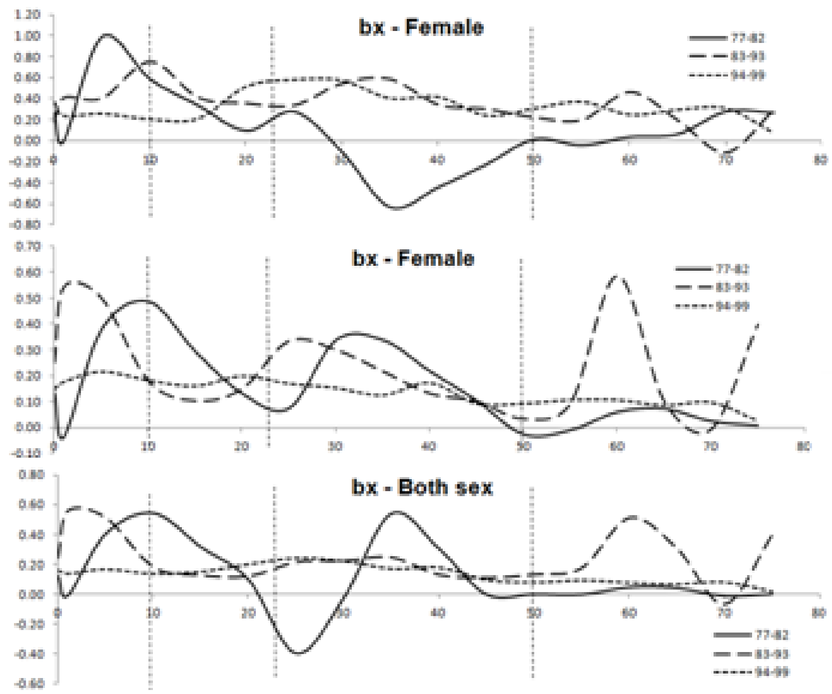


Figure 3.12: Time segmentation - β_x curves



The obtained functions are used to estimate the missing years κ_t values. To improve again the quality of these estimated κ_t values in the sense to better associate with the existing κ_t values, we prefer to replace the existing κ_t values by those expected by the fitting function. This process allows to ensure the coherence between the existing data and the estimated (missing) values of κ_t. Thereafter, we readjust the β_x vector to better fit changes occurred after replacing the original values of κ_t by those expected by the fitting function. The quality of the fitting obtained is shown in Table 3.8.

Table 3.8: MPAD - age and time segmentation

Males	Females	Both sexes
3.16%	3.50%	3.18%

The female population has benefited the least from the fitting quality improvement. However, the fitting quality of the three populations are closer. We noticed earlier that we do not aim only to improve the global quality adjustment but by the same way to reduce the gap between the three populations estimates quality.

3.9 Estimation of the κ_t missing values

As we mentioned earlier, once we got a function that represents the evolution of the mortality trend with a coefficient of determination higher than 99%. This function is used to estimate the missing κ_t values. The first and the third time interval are fitted by a 5-degree polynomial function; the second is fitted by 6-degree function. The estimated parameters of these functions are represented in Table 3.9.

Table 3.9: κ_t functions withinage and time segmentation

	1977- 1982					1983- 1993						1994-2000				
	a	b	c	d	e	a	b	c	d	e	f	a	b	c	d	e
0-5	-1.3002	2.7321	-1.3835	0.2591	-0.0166	-0.1570	2.2514	-1.1466	0.2162	-0.0183	0.0006	-0.7004	1.4943	-0.8042	0.1755	-0.0139
10--20	-3.2355	6.4422	-3.1841	0.5846	-0.0368	-0.0142	0.6288	-0.2915	0.0523	-0.0045	0.0002	-0.4189	2.0087	-1.1414	0.2301	-0.0164
25-45	-2.3617	4.5068	-2.2795	0.4300	-0.0275	0.2356	0.1313	-0.0354	-0.0003	0.0000	0.0000	0.3152	0.5523	-0.4365	0.1009	-0.0080
50-80	2.9850	-3.2713	1.4138	-0.2661	0.0174	3.0045	-4.7747	2.5405	-0.5625	0.0540	-0.0019	-0.3298	1.0462	-0.5983	0.1266	-0.0095
						Male										

	1977- 1982					1983- 1993						1994-2000				
	a	b	c	d	e	a	b	c	d	e	f	a	b	c	d	e
0-5	-1.9097	4.5324	-2.2637	0.4076	-0.0251	0.2110	1.0487	-0.4647	0.0679	-0.0044	0.0001	0.1851	0.7847	-0.6772	0.1969	-0.0191
10--20	-1.3629	3.6174	-1.9322	0.3684	-0.0239	3.1629	-3.0252	1.5643	-0.3400	0.0305	-0.0010	-3.5402	5.9054	-2.6863	0.5011	-0.0350
25-45	-3.7125	4.7879	-1.9158	0.3145	-0.0187	-1.3458	3.1502	-1.5747	0.3190	-0.0290	0.0010	-1.4474	2.9989	-1.4139	0.2688	-0.0194
50-80	8.8083	-12.0504	5.6152	-1.0348	0.0658	3.3055	-5.3192	2.7060	-0.5800	0.0547	-0.0019	2.1900	-1.7854	0.5703	-0.0629	-0.0006
						Female										

	1977- 1982					1983- 1993						1994-2000				
	a	b	c	d	e	a	b	c	d	e	f	a	b	c	d	e
0-5	-2.7632	6.0152	-3.0703	0.5756	-0.0369	0.0329	1.4085	-0.6743	0.1163	-0.0091	0.0003	-0.1829	1.2563	-0.8502	0.2233	-0.0206
10--20	-2.4953	5.3311	-2.6998	0.5027	-0.0320	0.5306	2.0657	-0.9712	0.1654	-0.0136	0.0004	-1.6495	3.6303	-1.7460	0.3367	-0.0246
25-45	-0.0432	-0.8329	0.6224	-0.1368	0.0095	0.4858	0.4637	-0.2504	0.0495	-0.0054	0.0002	1.0678	0.1578	-0.3139	0.0829	-0.0078
50-80	7.4106	-10.0620	4.7489	-0.8835	0.0566	3.8508	-6.1320	3.1075	-0.6633	0.0623	-0.0021	1.6704	-0.9494	0.1410	0.0279	-0.0073
						Both sex										

The quality of the estimated κ_t values compared to the original ones measured by the MPAD is given on Table 3.10. which considers the time-age segmentation.

Table 3.10: MPAD based on the k_t polynomial function

	1977-1982	1983-1993	1994-2000	1977-1982	1983-1993	1994-2000	1977-1982	1983-1993	1994-2000
0-5	1.58E-14	2.18E-14	2.18E-14	1.09E-14	9.74E-15	5.72E-15	8.43E-15	1.87E-14	1.36E-14
10--20	1.59E-14	1.68E-14	5.92E-15	8.84E-15	1.80E-14	3.93E-14	1.41E-14	1.16E-14	9.14E-15
25--45	2.58E-14	1.25E-15	3.27E-15	2.06E-14	7.05E-14	8.94E-15	4.72E-15	1.38E-13	8.07E-16
50--50	1.49E-14	2.04E-13	1.05E-14	4.48E-14	4.58E-13	3.77E-15	1.24E-14	3.06E-13	1.32E-15
	MPAD - Male		3.17E-14	MPAD - Female		6.33E-14	MPAD - Both sex		4.95E-14

We observe that the fitting functions capture almost 100% of the variance of the κ_t distribution for all the 36 mortality sub-surfaces. The coefficient of determination is equal to 100% for all these areas. As we mentioned earlier, these functions will serve to estimate the missing κ_t values. To minimize the lack on the fitting quality, and ensure a better coherence between the existing κ_t series and the missing values to be estimated, we preferred to replace all the original κ_t values by those expected by the polynomial functions. Then, we estimate again the β_x vector. The fitting quality measured by the MPAD for the three populations is shown in Table 3.11.

Table 3.11: Death rates MPAD obtained after introducing the k_t polynomial functions

Male	Female	Both sexes
3.16%	3.48%	3.17%

If we compare the results shown in Table 3.11 to those of Table 3.8, we can observe that replacing the existing κ_t by the expected ones while estimating the missing values in the series have not led to any loss in fitting quality. This result is due to the goodness of the fitting and also to the re-adjustment β_x to fit changes in κ_t.

The MPAD shown in Table 3.11 summarizes the average fitting quality obtained on the whole surface. The main objective of the segmentation is to improve the quality of the fitting while reducing the variance in fitting quality by age and time. Table 3.12 shows the MPAD obtained on all the 36 sub surfaces.

Table 3.12: MPAD by small areas

	Male			Female			Both sex		
	1977-1982	1983-1993	1994-2000	1977-1982	1983-1993	1994-2000	1977-1982	1983-1993	1994-2000
[0-5[	4.38%	0.90%	1.85%	4.47%	2.37%	1.38%	4.40%	1.61%	1.50%
[10-20[	2.19%	3.47%	3.97%	1.65%	2.24%	2.15%	1.48%	2.39%	3.10%
[25-45[	2.79%	3.57%	2.04%	4.71%	2.76%	1.84%	2.71%	2.68%	1.53%
[50-80[	2.42%	5.45%	1.73%	5.93%	4.03%	2.50%	4.91%	4.66%	2.00%

As we observe in Table 3.12, there is not an important difference in terms of MPAD between the different sub mortality surfaces. It varies between 0.9% to 5.45% for males, between 1.38% to 5.93% for females, and between 1.5% to 4.91% for both sex population.

The re-estimated alpha parameter

In the classical Lee-Carter formula, α_x is calculated as the average over time of the log of the age specific mortality rates at age x. In order to improve the fitting quality of the model, Renshaw and Haberman (2006) proposed to use this first estimated α_x only as a starting value. Then it should be re-estimated by the same computation process as β_x and κ_t. Here, we propose to proceed similarly and to re-estimate α_x to improve again the quality of our estimates. In other words, we use α_x, β_x and κ_t obtained until now as a starting values and we decompose the log of the five-age mortality rates into three components by the same decomposition process. Effectively, this process led to improve the fitting quality as shown in Table 3.13.

Table 3.13: MPAD obtained after replacing calculated alpha by the estimated alpha

Male	Female	Both sexes
3.04%	3.28%	3.03%

The re-estimation of the alpha parameter led to an insignificant variation that we cannot observe on the graphical representation.

3.10 Completed surfaces adequacy

Until now, we had completed separately the three mortality surfaces. The goodness-of-fit of each of the three completed surfaces cannot necessarily imply the adequacy of males Vs. females and single sexes Vs. both sexes estimates. To verify this quality, several methods can be used. The Male female mortality ratio and its time / age evolution can give an idea about the coherence of the estimated data. Here, we propose another criterion that can be measured and can also be introduced as a constraint in the optimization problem. In reality, the both sexes surface depends on the male and female surfaces. The estimated mortality rates for both sexes population must be equal or near to the weighted average of the males and females mortality rates at the same age x and the year t.

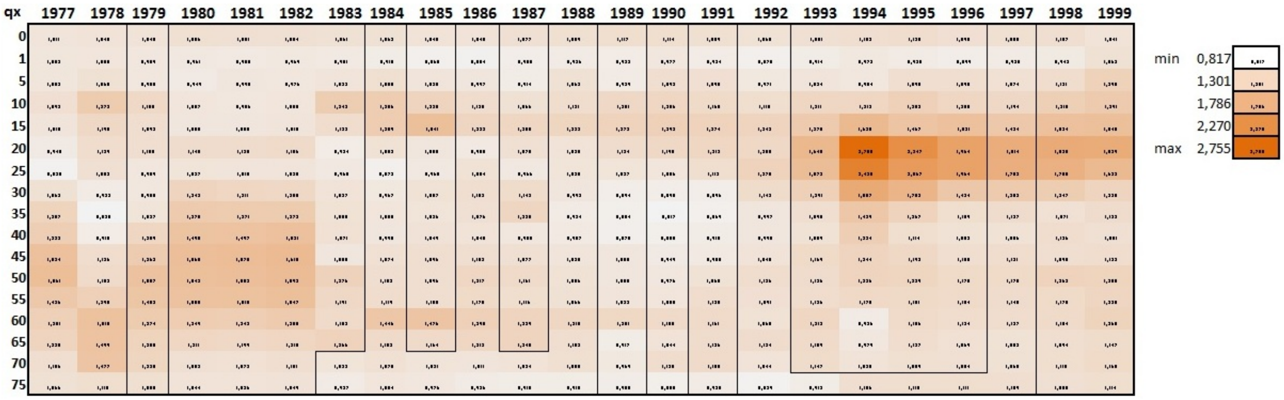
3.10.1 Male Female Mortality Ratio MFMR

The male female mortality ratio represents the report of the male mortality rate with the female mortality rate. The evolution of this ratio by age during each year follows usually a well-defined trend (De Jastrzbski, 1919). Usually, the risk of death is more important for males than for females, especially at younger ages. This difference is common all over the world wide countries and it has many explanations: risk taking, unhealthy behavior, cultural, environmental ... etc. (Kruger and Nesse, 2004). A such indicator can be used to assess the quality of the estimated data. The estimated data can be qualified to be adequate if it presents some adequacy with the existing data in terms of MFM. In Figure 3.13, we show the surface of the MFMR including the original data and the estimated one. The two data categories are separated by a bolt lines.

The original values of the MFMR represent some apparent variation by age and by time. We can easily observe some differences between the two first tables (1977 and 1978) in terms of MFMR. For the first table, the MFMR is higher for the age range [30 – 60[. The reverse is observed for the following year which have higher MFMR for [15- 30[and [60 – 75[. The years 1980, 1981 and 1982 have almost the same MFMR profile as 1977. The period 1983 – 1992 have different characteristics, with an MFMR higher at the age range [10 – 20[and [60 – 70[and lower for the other ages. The period [1993 – 1999[is characterized by an MFMR sometimes superior then 2 for the young ages [20 – 30[essentially due to the terrorism events during 90's. The estimated values that are framed by bold lines do not represent any anomaly.

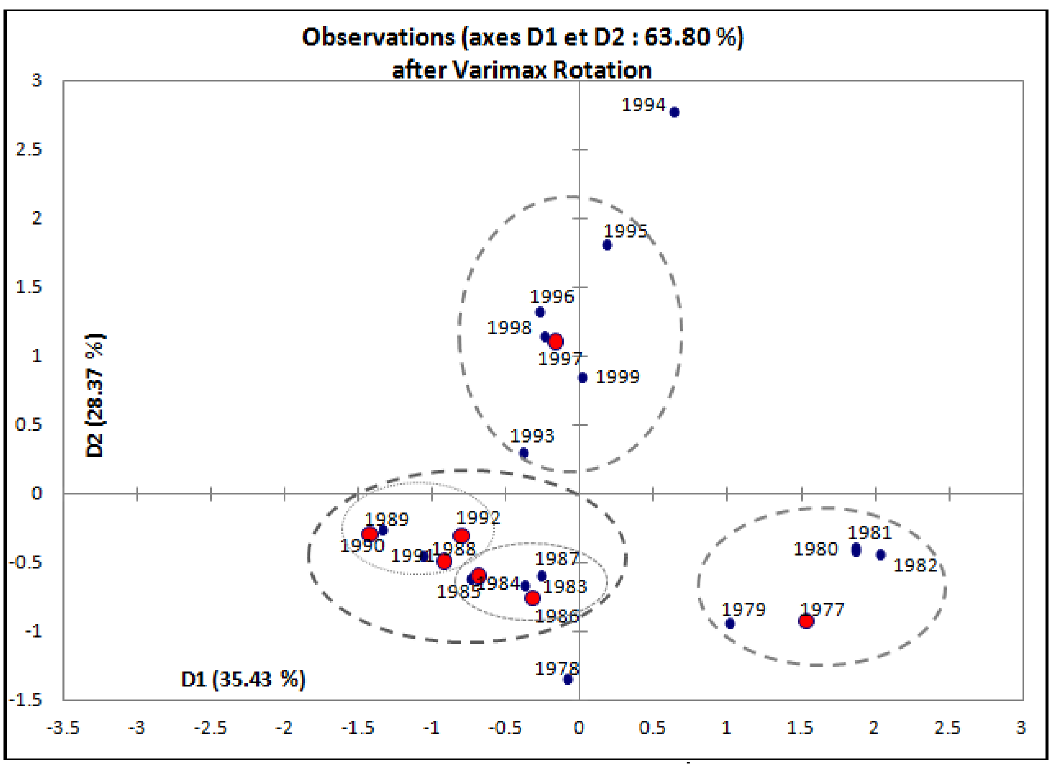
They follow the trend of the original values ether by age or by time. It is exactly the objective that we meant to reach during the present work.

Figure 3.13: The MFMR Surface



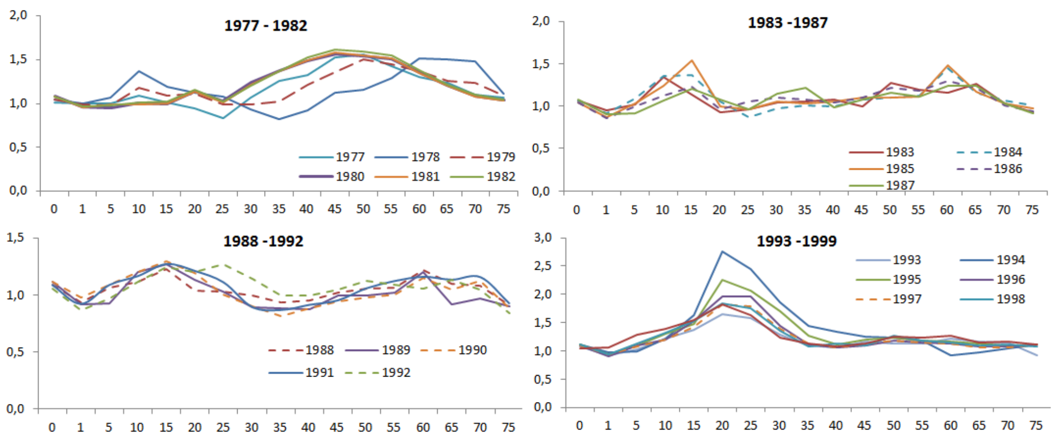
To better visualize the quality of the MFMR based on estimated values, we prefer to use a criterion that allows a clear lecture and allows by the same way to distinguish the years that have a similar characteristics. For that we use the Principal Component Analysis (PCA). Results are shown in Figure 3.14.

Figure 3.14: Time Segmentation by the Principal Components Analysis



The results obtained with 63% of captured variance, confirms the assessment about the coherence of our estimates already shown in Figure 3.13. We confirm that the estimated years data, represented by red points, correlate with the neighboring years. The years 1978 and 1994 have singular trends. The period 1983 – 1992 can be segmented to two sub-periods. To know more about the trend that had characterized each period, we show here the curves representing the age pattern of the MFMR for each year of the period of analysis. Each sub period having a similar profile are plotted in the same figure including the original and the estimated data (Figure 3.15).

Figure 3.15: MFMR by period

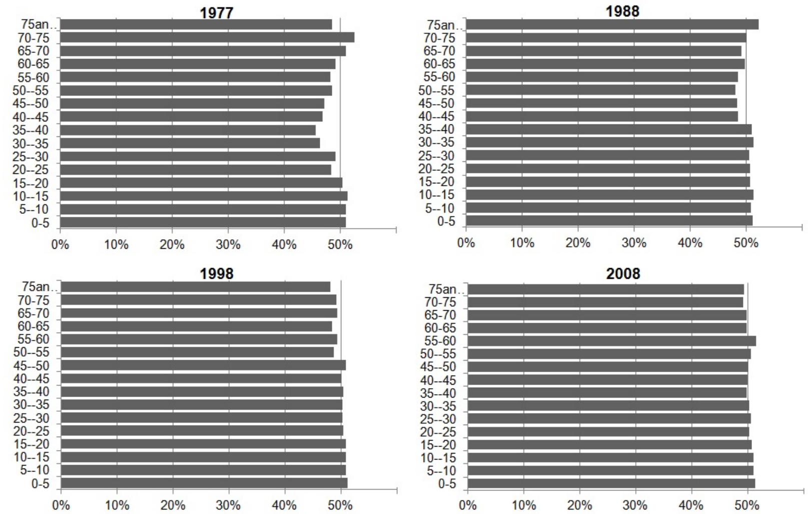


From Figure 3.15, we observe that during the considered period, the evolution of the MFMR has had four different trends. In the item representing the 1977 -1982 period, the MFMR has marked two bumps: initially between 5 and 15 and the second, more larger, between 25 and 70 years. The MFRM reaches its highest value of 1.6 around the age of 45 years in 1982. During the second period, the trend is characterized by a huge bump between 5 and 20 years, followed by a linear trend until the age of 60 where the MFRM reach the mean of 1.4 and follows by a decrease. The third period has a trend similar to the second. This similarity was already shown in Figure 3.14. The only difference is the width of the bumps. For the last period, the bump is around 15 and 35 years. To conclude, we can say that regarding the MFMR, the estimated data does not present any anomaly compared to the original data.

3.10.2 Both sex surface VS male and female surfaces

To perform our assessment about the goodness of the estimated life tables, we propose to evaluate the coherence between the existing life tables and those estimated in the present work by evaluating the coherence between the both sex and the single sexes estimated life tables. The mortality rate at age x and year t for the both sex population must be equal to the weighted average male and female mortality rate at the same age. The weight can be the population age structure by sex. According to the Algerian population censuses data (1977, 1987, 1998 and 2008), we observe that the proportion of males is almost equal to the proportion of females at all ages as shown in Figure 3.16.

Figure 3.16: Males proportion according to the Algerian population censuses



The averaged mortality rates for males and females must be calculated by the following formula:

$$_nQ_x^b = \pi_x . _nQ_x^m + (1 - \pi_x) . _nQ_x^f$$

With π_x to be the proportion of males in the age group $[x, x+n[$. $_nQ_x^m$: mortality rate at the age group $[x, x+n[$ for males, $_nQ_x^f$ corresponds to females and $_nQ_x^b$ for both sex population.

By considering the almost equality of the males and females proportions in the Algerian population structure over the different censuses, the weighted average of the male and female mortality rates can be approximated by a simple arithmetic average. The gap between the estimated and the calculated (average of the male and female mortality rate) can be used as a measure of the incoherence of the estimated life tables. To do this, we propose to re-estimate the parameters of the Lee-Carter model for males, females and both sex population by the same estimation process including a constraint about the coherence between these three populations. The new objective function of the optimization problem can be written as:

$$MinB = \frac{(MPAD_{males} + MPAD_{Females})}{2}$$

With the constraint:

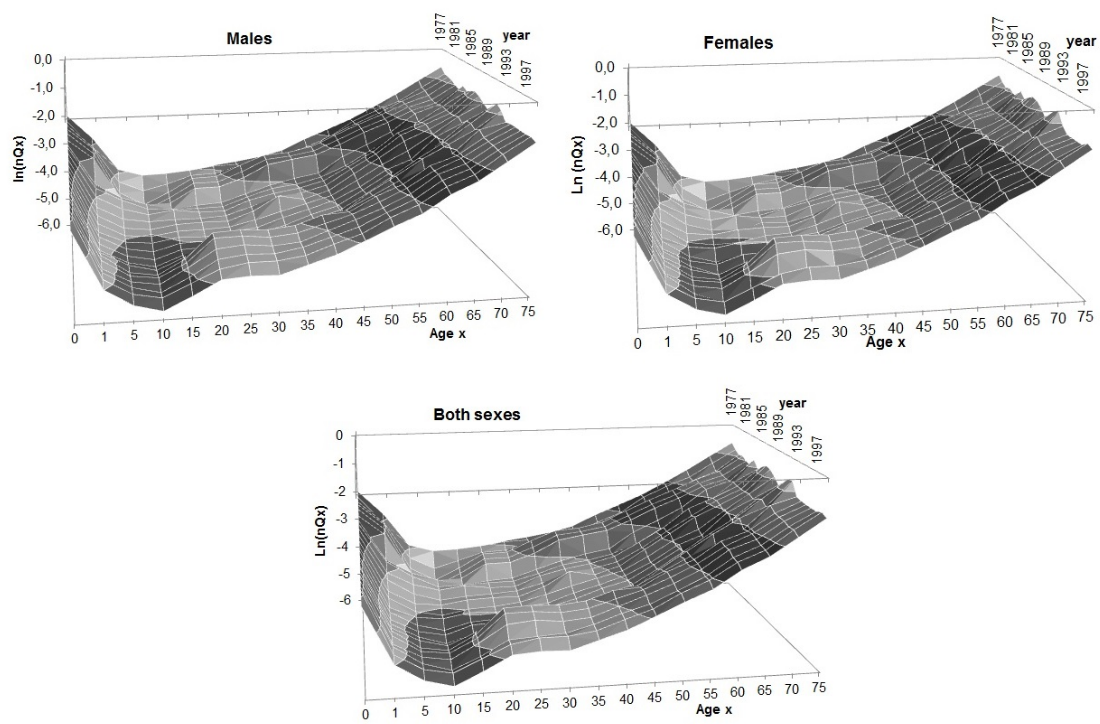
$$\frac{1}{XT} \sum | \frac{\widehat{q}_{xt}^b - \frac{(\widehat{q}_{x_t}^m + \widehat{q}_{x_t}^f)}{2}}{\widehat{q}_{xt}^b} | \approx 0$$

So, we have to estimate, in all, 36 Models by the same optimization problem using XL-Solver.

3.11 Completed Algerian mortality surface

The completed mortality surfaces (males, females and both sex) are presented in Figure 3.17.

Figure 3.17: Completed mortality surfaces



The observed turbulences in the mortality surfaces presented in Figure 3.17 is due to the original data rather than to the estimation results. The obtained new results do not present any significant variation compared to those obtained in the last section in terms of goodness-of-fit and coherence. A comparison between the single sexes and the coherent male and female estimates in terms of fitting quality are presented in Table 3.14.

Table 3.14: Comparison between Both and singles surfaces estimation processes

	Male	Female	Both sex	Mean	MPAD EC
Both sexes estimation	3,21%	3,35%	3,18%	3,25%	0,87%
Singles sexes estimation	3,04%	3,28%	3,03%	3,12%	2,14%

When analyzing Table 3.14, we observe that the introduction of the coherence constraint has not significantly affected the fitting quality. The MPAD calculated under the three surfaces has slightly increased by +0.17%, +0.07% and +0.13% respectively for males, females and both sex population. In the other side, the gap between the calculated and the estimated mortality rates for both sex population (MPAD EC) has been greatly decreased by passing from 2.14% to only 0.87%. We can conclude that in overall, the lack in fitting quality was recovered by the decrease in the MPAD EC. The obtained results are satisfactory.

3.12 Conclusion

Demographic calculations and all the economic and actuarial accounting that it implies, need a complete and continuous mortality surfaces. In this sense, mortality projections and construction of prospective life tables which are largely used to calculate the equilibrium of retirement and life annuities systems need to make available a complete mortality surfaces. The pure demographic analysis must be based on continuous data series, if we want to improve the quality and the usefulness of all study in this field. Several methods can be used to estimate mortality missing information, however, what we need is an estimation that approaches and correlates the existing data. The estimation method that we have to use must be adapted to the final objective and the ultimate use of the estimated data. By the way, the characteristics of the available data must be taken into account. The problem with the Algerian mortality data is that, in the beginning, life tables were not annually constructed and there are some calendar years without life table. Also, some life tables were closed earlier than 80 years. However the accuracy and the goodness of the estimation method is, it could not give as good estimates as real data. Our main objective of the present work was to approach the mortality profile in the considered years. The adequacy of the estimated data compared to the existing data is the most important point. In other words, we aimed to approach the estimated life tables to how they should be estimated by the ONS in the corresponding years by assuming the same methodology and the same data quality.

The idea is to use the Lee Carter Model that was initially conceived to project the age specific death rates. It is based on the decomposition of the log of the death rates surface on three components. Then, the extrapolation of the time component allows projecting the age mortality pattern in the future. For our work, the idea was to estimate the parameters of Lee-Carter model on the existing data and then the missing values in the mortality time index are estimated by a local polynomial fit. This estimated values in the time trend index allow to estimate the missing mortality rates. The estimation of a fitting function of the mortality time index passes by the improvement of the quality of the fitting itself. To improve it, we proposed an age / time segmentation and applied the Lee Carter model by 12 reduced mortality surfaces for each of the three populations: male, female and both sex population. To simplify the calculation process, we proposed to base the decomposition of the residual mortality matrix by optimization process in XL-Solver. The optimization criterion has a great effect on the fitting result and its quality. The MPAD is the criteria that we used to better orient the estimation process to suit the characteristics of our data and also our final objective. In final, the adequacy of the obtained information was verified using the MFMR and the gap between the estimated and the calculated both sex mortality rates. In order to control this gap, we introduced it as an additive constraint in the new optimization problem. This new optimization process allowed to estimate, by the same process, the Lee Carter parameters for the three surfaces while controlling coherence and fitting quality.

In Final, we got complete mortality surfaces for the three populations. The use of the control variables allows to compare these results with other works that may have the same objective.

Chapter 4

Closing-out the Algerian life tables : for more accuracy and adequacy at old-ages

This work has been presented in the International Association of Actuaries / ASTIN Colloquium 2016. Lisbon, Portugal, June 1st, 2016.

4.1 Introduction

The improvement of life expectancy in Algeria during the recent decades has resulted from a mortality reduction at all ages (Chapter 2). As a result of this reduction, the probability to reach the age of 80 passed from 0.74 in 1977 to 0.95 in 2014, achieving 22% of reduction in the space of 37 years. In addition, The chance to reach the age of 60 marked an improvement of 15% passing from 0.82 in 1977 to 0.96 in 2014. Parallel to this, the size of the population aged 60 and over increased from 790 000 according to the General Census of Population and Housing (GCPH 1966) to more than 2.5 millions in 2008. The population aged 75 and over was estimated according to the results of 2008's Census of about 683 500 individuals. The size and the part of the old-age population should continue to increase during the forthcoming century. According to such a perspective, the old age mortality should be attentively considered as well as the mortality at younger ages.

A life table is supposed to describe the mortality level lived by a population at all ages, from birth till total extinction. For each age x, that life tables might give the correspondent risk of death. When mortality data are available and reliable, this kind of calculation can be made easily. But In practice, the quality and the availability of this data is varying with age and adult ages that are usually well covered compared with lower and higher ages. The lack of the mortality data and the exposure to the death risk beyond a certain age leads to some irregularities in mortality curves. For a long time, some conventional mortality models, used in principal to graduate adult mortality curves, have been used to extend mortality to the older ages. The most practical example in this sense is the fact that the United Nations Population Division have used a Makeham-type function to extend the model life tables until the age of 85 years. For the Coale-Demeny model life tables revised in 1983, a Gompertzian function was used to extend mortality rates until 100 years old (Coale and al., 1983). All these methods were used under the assumption that mortality rates keep growing at old ages according to the same acceleration rate observed on the adults ages. As mentioned in Heligman and Pollard (1980), when childhood and young ages are surpassed , the mortality rates can be represented by an exponential function (Gompertz, 1825). Parallel to this, the deceleration of mortality rates at older ages was noticed in many works starting by Gompetrz and then Pearks (1932) (see Gavrilova and Gavrrilov, 2014). Unfortunately,the assessment of such a preliminary finding needs various comparisons with a real observed data. The improvement of the population life conditions in developed countries resulted the improvement of life expectancy at birth and the growth of centenarian population. By the end of 20^{th} century, the good data quality reached by some developed countries provided a consistent old age population data bases allowing more flexibility in studying and modeling old age mortality. Coale and Guo (1989), in the third version of model life tables, confirmed mortality deceleration on the observation of mortality rates until the age of 100 for some countries (Netherlands, Japan, France, West Germany, Austria, Sweden, and Norway). Their idea was that even if mortality rates keep growing with age, the growth of rate starts to back down starting from a certain age (80 or 85). This deceleration takes a linear trend. The same approach with some adaptation was developed one year later by Coale and Kisker (1990). This approach is still used nowadays for closing out the model life tables.

There are some observed differences between demographers and actuaries in terms of old-ages mortality modelling practices. Other methods were developed separately by actuaries, but there is some convergence regarding the final objective which is to set more accuracy to the mortality curve span at old age. Demographers focused generally on the life expectancy at birth. The effect of the Closing-out method is not very important in calculating life expectancy at birth (Buettner, 2002). The effect is more important in terms of life expectancy at 50 or 60 which coincides with the age range that actuaries are interested in.

For Algeria, the national life-tables published recently (2010-2014) by the Office of National Statistics (ONS) are closed-out at the open age group [85 and +[. For the years before 2010, the closure age was bellow. On the basis of the selected model life table, the corresponding life expectancy at the age of closure is estimated. Combining it with the other calculated parameters for the youngest ages, the life expectancy at birth is concluded. The mortality rates corresponding to the oldest ages are not published in any way. Although the chosen life expectancy is estimated and used, the adequacy of the corresponding mortality rates at old-age do not fit necessarily the existing mortality curves. A change in the reference model life tables or a wrong use of this last can have an effect on the estimated life expectancy at birth. In Chapter 2, we have shown how this practice can affect the evolution of the life expectancy at birth and how it can distort all analysis about its evolution.

To estimate old age mortality summarized by the life expectancy at the closure age, we need to select an adequate model life tables among Coale-Demeny (CD) or the United Nations (UN) model life tables. Between the two, African countries prefer to use the first model life tables to complete their mortality data (Ekanem ans Som, 1984). Mortality of the northern African countries is generally represented the south type of CD life tables while the North type is used to describe the mortality pattern of sub-Saharan countries. However, this procedure implies many inadequacy. In practice, the method of stable population is used to select the adequate model life table. As shown in (Ekanem and Som, 1984), this process leads generally to an inadequate selection compared with the use of the ordinary least squared deviations method. In addition to these two types, CD model life tables includes two other types : West and East types, and data of any African country has been considered to construct these model life tables. Here, the use of CD model life table can, in any way, guarantee a good estimation and is the best among the existent method to estimate old age mortality.

Model life tables were principally conceived to allow estimating life tables from unreliable or incomplete national mortality data. Estimating mortality at older ages for developing countries is considered from this point of view. A such estimation process can be very useful when the shape of the mortality curve at adult ages is not that regular to allow a consistent extrapolation to the older ages. However, another problem can rise if we consider this estimation process by the view of the trend of the estimated old age mortality rates and its compatibility with the adult age observed trend. Each type is based on the observation of a set of life tables having a similar mortality schemes ordered in equidistant scale in function of the corresponding life expectancy at birth. The selection process is based on the comparison of the national mortality data, at adult ages, to the mortality pattern given by the different types of model life tables. The residual life expectancy at the age of closure is directly deduced. Then, the life expectancy at birth is deduced by recurrence. In some cases, a moving average adjustment around the connection point is needed, because the two curves have basically a different evolution trends.

Our first objective here, is to reduce the fluctuations due to closing out method changes and to give more homogeneous old age mortality surface for the Algerian population. The second objective is to provide readers with a general methodology to extend mortality to the old age for the Algerian population starting from adults mortality curves. The dynamic life tables or some specific life tables which can be estimated on the basis of the Algerian mortality data, should be closed out following a methodology taking into account the specificities related to Algeria. Here, we aim to make evidence of these specificity.

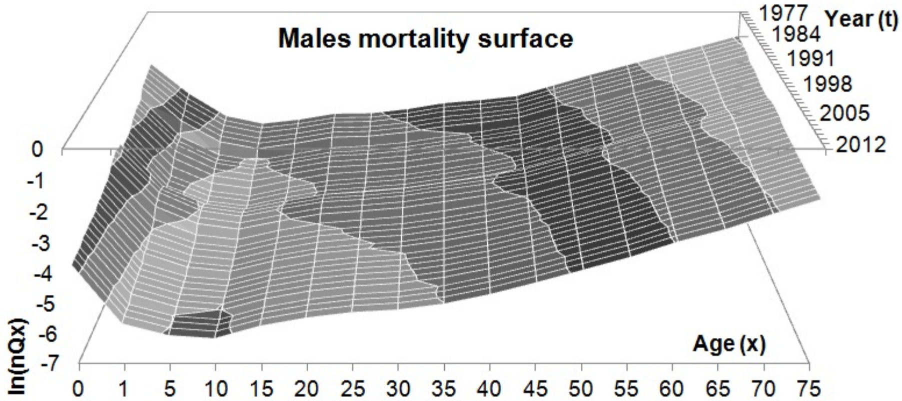
For this, some old age mortality models will be compared to extend mortality beyond 80 years on the basis of the observed mortality curves shape at youngest ages. The extrapolation results are generally related to the quality of the existing data and also to the shape of the mortality curve at the age range used as a basis of the extrapolation. This last can vary from a population to another and from a period to another in the same population according to the regularity of the data. Generally, starting from a certain ages (35, 40 or 45) the mortality rates grow following an exponential function. This regular trend allows to extend mortality to the older ages easily. But the final result is very related to the method and the age range chosen to calibrate the model, in the absence of observed data to be used for comparison. Some other criteria can be used to orient the model calibration. Usually, an assumption about the ultimate surviving age is defined and the mortality rate at that age might converge to 1. The mortality curve for the Algerian population has changed many times over the observed period and the challenge is that it is so complicated to find a unique model which allows a good fitting quality all over the period, ensuring some adequacy between males and females and an adequacy regarding to the year-to-year mortality rates varying. Here, we will try a set of proposed methods.

4.2 Data

4.2.1 Presentation

For the needs of the present work, we will use the Algerian life tables published by the ONS during the period 1977-2014. The missing data has been estimated in Chapter 3. So, our database is composed by a five age mortality rates from 0 to 75 years during 1977-2014 (Figure 4.1).

Figure 4.1: Five-age mortality surface (Algeria: 1977-2014)



Considering the closure age, the Algerian life tables have been stopped at the open age group [85 and +[since 2010 only. For the previous years, the closure age varied between [70 and +[and [80 and +[. (more details in chapters 2 and 3). For the present application, we use a completed mortality surface with the open age group [80 and +[as a common closure age for all the Algerian life table from 1977 until 2014.

Some old age mortality models are more suitable for single ages data. For this issue, we propose to interpolate the single age mortality surface from the data that we have until now and which is structured following a five ages structure.

4.2.2 Single-ages mortality surface

In the literature, several methods were proposed to interpolate single-age values (Exposure, Observed deaths, Deaths rates) from five-age description (king, 1908; Sprague, 1880; Beers, 1944 and 1945). Usually, some complications can be found because of the structure of the abridged life-table. The length of the age groups is 5 years except the two first groups [0-1[and [1-5[. Some interpolation methods were proposed to deal with unequal age-intervals principally based on the Karup-King and Lagrange interpolation methods (Crofts, 1998). For the present work, we preferred to use both Lagrangian and Karup-King methods (Shrock and al., 1993). The first method is more adequate with the lower age groups while the second is more adapted with the higher age groups. The connection point is defined in the way to minimize the gap between the two curves around this points. The connection point is different by period and sex.

The obtained single age mortality surfaces for males and females are shown in figure 4.3.

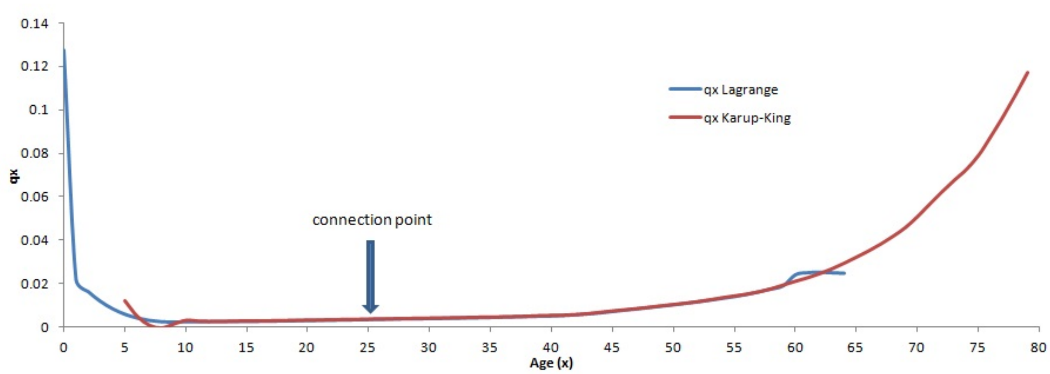
A significant part of extrapolation models of mortality at older ages are based on a single ages mortality description. For this aim, and to avoid missing the opportunities that may present a model or another, we saw that it is necessary to interpolate death rates before extrapolation to the older ages. To reach this purpose, we have used and compared a set of interpolation methods: Lagrange, Karup-King and Sprague. For the interpolation of the extreme class (75-80 years), the first method requires the disposal of mortality rates up to 95 years. In the absence of such data, we saw that an extrapolation based on models unsuitable for old ages mortality does not guarantee a good extrapolation quality. The Lagrange method can be used until 65 only, and, the other two methods have the disadvantage of interpolating the quotients for only five ages groups. The age group [1- 4 years] in this case is our problematic. The breaking-out of the whole ${}_5q_0$ without distinction of infant and child mortality will leads to an overestimation of death quotients at 1, 2, 3 and 4 years at the expense of infant mortality rate. This last is relatively high by nature and remains a key indicator in the abridged and detailed life tables. In order to mitigate these drawbacks, the idea is to combine two formulas Lagrange (0 - x years) and karup-king (x - 80 years). X being the connection age which is fixed to optimize the quality and the regularity of the interpolated series. However, it remains preferable that the connection point corresponds to a five-year age, in order to avoid the possibility of gaps if we aim to restore the five-age mortality rates.

If we consider q^*_x to be the mortality rate at age (x) obtained with the Karup-king Method and $q^+_x$is the rate obtained with Lagrange for the same age (x), the connection point (k) is defined in order to minimize the Mean Absolute Errors (MAE) between the two curves around this point. This implies :

$$MinA = \frac{1}{5}\sum_{x=k-2}^{k+2} \left\|q^*_k - q^+_k\right\|, ...k = 20......50,$$

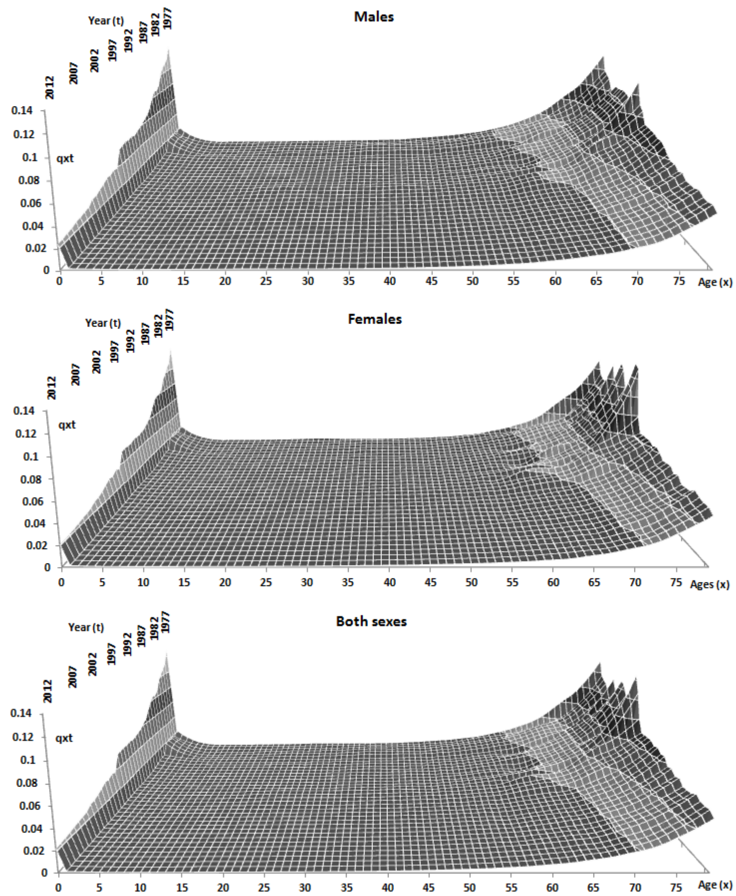
The connection point corresponds to the point where the gap is minimal. This coupling point varies from a table to another (Both sexes, males, females) and from a year to another. Figure 4.2 traces the coupling of two sets of quotients: quotients obtained by the method of Lagrange and those obtained with Karup King for male life table of 1977.

Figure 4.2: Curves junction : Karup-King VS Lagrange



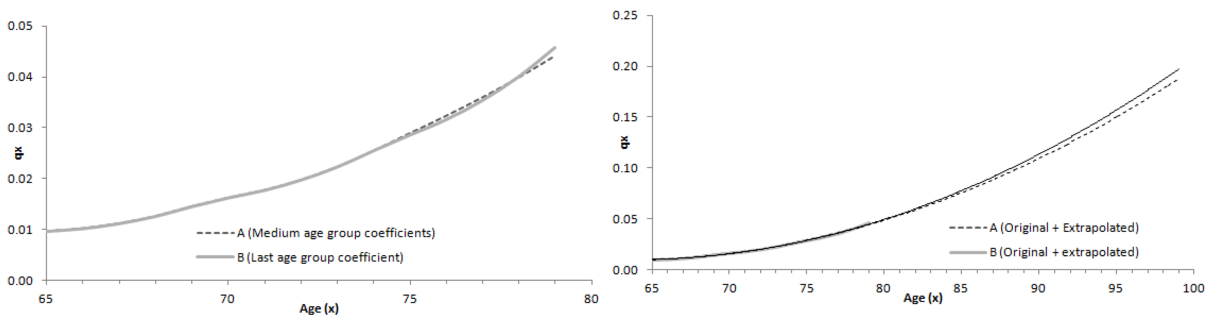
As shown in Figure 4.2, the connection point is 25 years. We can easily notice that the method of Karup-King gives unsatisfactory results for ages below 10 years. The same is observed with the Lagrange method for the age group of 60 and over, for all tables. Automatically, the connection age is chosen in the age range [10, 55]. For the 1977's males life table, the mean squared deviation recorded its lowest value (3.79E-11) in the neighbourhood of 25 years. Thus, from 0 to 25 years we used Lagrange interpolated rates while Karup-King's interpolated rates are adopted for ages of 25 years and over. The obtained single ages mortality rates are shown in Figure 4.3.

Figure 4.3: Single ages mortality surfaces



Although these two formulas are used to interpolate the single ages mortality rates from the five ages rates by interpolating the observed deaths, the interpolation quality can not be of the same degree in the extremes ages as in medium ages. The reason is that the interpolation formula is based on a combination of the trends before and after the targeted age group. Therefore, these interpolation methods assign coefficients to reconstruct the single age deaths number which is a fraction of five age deaths. For extreme classes, the interpolation is based solely on the basis of previous classes and the target class itself, using other coefficients that are different from those used to break out the medium classes. The adaptation of the interpolation coefficients is supposed to improve the quality of interpolation at the extreme age groups. Therefore, we would like to compare the quality that can be achieved by the two formulas in this matter. The importance of such an element is increasing if we take into account the purpose of this work that requires more accuracy regarding to mortality data for the oldest ages. in this work, we are interested in an extrapolation based on single ages data rather than five ages data. The quality of the extrapolated data depends largely on the database as well as the control data. Again, We remind that most of the available data, as published by the ONS and completed in Chapter 3 , is extended until the five age group [75 - 79] , and only the five most recent tables are up to [80 - 84] years. Here we would like to highlight the definition of extreme class arguing that it is not common for all of the series (1977 - 2014) and is varying from [74-79] to [80-84]. Treating a class as extreme or medium is not supposed to give the same results. Interpolate ${}_5q_{75}$ as a common extreme age group for the whole period (1977-2014) might ensure the homogeneity of the single ages interpolated mortality rates between 75 and 79 years old. The concern is how to break out ${}_5q_{80}$ given that the interpolated rates will be used to orient the the extrapolation process at advanced ages. The use of the coefficients of the extreme ages groups might lead to irregularities since ${}_5q_{75}$ was broken-out by using the same coefficients too. Figure 4.4 shows this irregularity with more evidence.

Figure 4.4: Interpolation of the death rates of the age group [70-75[by using medium and last age group coefficients and its impact on the old ages extrapolated rates



The figure on the left shows the difference which can be created when "Medium" or "Last" age group karup-king coefficient are used to interpolate the mortality rates of the age group [70-75[. The figure on the right shows the effect of the considered coefficients on the extrapolated mortality rates beyond the age of 80.

The curve (B) shows the death rates extrapolated with considering 75-79 as a medium class and 80-84 as extreme class. The curve (A) recounts the situation ignoring the existence of the ${}_5q_{80}$, and by considering ${}_5q_{75}$ as a common extreme class with the intention of approaching the case of the post-2010 period. Lateral, the ${}_5q_{80}$ quotient is interpolated basing on the coefficient of extreme classes of the Karup-King formula. The figure in the left shows the fluctuations in the 75-79 age caused by the procedure (A).

Similarly, it would be impossible to interpolate the ${}_5q_{75}$ on the basis of medium coefficients for the entire area (1977-2014). The figure in the right shows the effect of the adoption of the curve (A) compared to the results obtained with the curve (B). The effect is more important when the extrapolation is done on a more distant horizon. What is certain is that; this is the solid line (poly. B) that can reconstitute the initial five-year quotient ${}_5q_{80}$. Dashed line (poly. A) generates an overestimated quotient.

Recall that our mortality surface is extended from 1977 to 2014 and life tables are almost extended until the age group [70 - 79]. To better orient the mortality extrapolation to older ages, we must take profit of ${}_5q_{80}$ contained in the five recent life tables. Considering this additional data have help us to perform model selection and calibration in this sense. The concern is to ensure the homogeneity of the time series of the interpolated mortality rates of the last age category regarding to the resulted homogeneity on the extrapolated old ages mortality rates. For this, 75 - 79 mortality rates should be interpolated; for the whole period 1977-2014, using the same formula and the same coefficients. So how do we proceed? on one hand, using extreme class coefficients for all years will result to a kind of irregularity of the 75-79 interpolated mortality rates because this process implies to interpolate the 80-84 mortality rates for the years 2010-2014 with the same extreme class coefficients too. Regularity will be missed for years 2010-2014 since both ${}_5q_{75}$ and ${}_5q_{80}$ were broken out using the same extreme class coefficients. On the other hand, it is impossible to consider the class [75-79[as a medium class for the whole period [1977 - 2014] as in the absence of ${}_5q_{80}$ for the period before 2010. The idea is to define a new interpolation coefficients which combine the characteristics of the medium and extreme class coefficients to be used for the whole period 1977-2014. On the basis of the observation of the interpolated rates between 75 and 79 obtained by using the medium coefficients on the data of the period 2010-2014, we define a new extreme class coefficients which lead to similar results when ${}_5q_{80}$ are ignored. These coefficients will be applied to interpolate 75-79 mortality rates for the period 1977-2009. By this way, we ensure homogeneity of the interpolated 75-79 mortality rates for the whole period 1977-2014 while ensuring some regularity when passing from [75-79] to [80-85] interpolated mortality rates for the years 2010-2014. The regularity of this series is so important since we will use it to orient the old age mortality trend.

Interpolation of ${}_5q_{75}$ by karup-king adjusted coefficients

In order to make the procedure of the ${}_5q_{75}$ interpolation more clear and understood, we expose, in first, the Karup-King method based on 3 interpolation coefficients before spreading how to adjust the original formula to suit our case.

karup-King formula :

The Karup-King formula is designed to interpolate the number of deaths occurring between age x and age $x + 5$. The evolution of the survivors population at the exact age x will be subsequently interpolated. The interpolation tries to break out the five-ages mortality rate into a single ages ones which will allow to reconstruct the original five ages rates with an acceptable fidelity degree. This relationship can be written as follows:

$${}_5q_x = 1 - \prod_{s=0}^{4}(1 - q_{x+s})$$

Although this characteristic can be satisfied by different combinations (q_x, q_{x+1}, ..., q_{x+4}), the solutions which can ensure continuity and consistency of the evolution trend of the single ages mortality rates are limited. The interpolation methods assume that the interpolated rates grow following the same trend already observed on the five-ages rates. The structure of the single ages rates must join the trend footprint by five-ages rates of the age groups before and after the target age group.

For the first class, the number of deaths can be calculated by the formula is:

$$d_{x,x+i} = A_{1i}.D_{x,x+5} + A_{2i}.D_{x+5,x+10} + A_{3i}.D_{x+10,x+15} \ with \ i = 1, 2, .., 5$$

For medium classes:

$$d_{x,x+i} = B_{1i}.D_{x-5,x} + B_{2i}.D_{x,x+5} + B_{3i}.D_{x+5,x+10} \ with \ i = 1, 2, .., 5$$

For the ultimate age groups, the interpolation formula is as following:

$$d_{x,x+i} = C_{1i}.D_{x-10,x-5} + C_{2i}.D_{x-5,x} + C_{3i}.D_{x,x+5} \ with \ i = 1, 2, .., 5$$

The Coefficients A, B and C are presented in Table 4.1.

Table 4.1: Karup-King interpolation coefficients

	first group N0			*Middle groupe Ni*			*Last groupe Nk*		
	N0	**N1**	**N2**	**Ni-1**	**Ni**	**Ni+1**	**Nk-2**	**Nk-1**	**Nk**
first fifth	0.344	-0.208	0.064	0.064	0.152	-0.016	-0.016	0.112	0.104
second fifth	0.248	-0.056	0.008	0.008	0.224	-0.032	-0.032	0.104	0.128
third fifth	0.176	0.048	-0.024	-0.024	0.248	-0.024	-0.024	0.048	0.176
fourth fifth	0.128	0.104	-0.032	-0.032	0.224	0.008	0.008	-0.056	0.248
last fifth	0.104	0.112	-0.016	-0.016	0.152	0.064	0.064	-0.208	0.344

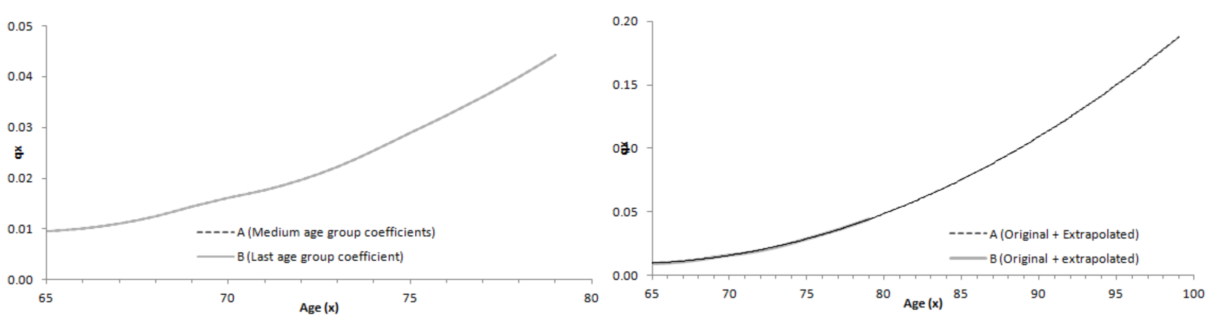
Our idea is to re-edit the coefficients used to interpolate ${}_5q_{75}$ based on the formula of extreme classes giving the same results as those obtained by considering ${}_5q_{75}$ as medium class (as of 2010-2014). For this, the redefinition of these coefficients will move towards the goal of achieving similar results on a set of 15 mortality tables: for males, females and both sexes for the period 2010-2014. The new redefined coefficients are shown in Table 4.2.

Table 4.2: Modified Karup-king last group interpolation coefficients

	Original coeficients				*Modified coeficients*			
	Nk-2	**Nk-1**	**Nk**	***Sum***	**Nk-2**	**Nk-1**	**Nk**	***Sum***
first fifth	-0.016	0.112	0.104	0.2	0.006	0.078	0.119	0.203
second fifth	-0.032	0.104	0.128	0.2	0.012	0.037	0.158	0.207
third fifth	-0.024	0.048	0.176	0.2	0.009	-0.003	0.199	0.205
fourth fifth	0.008	-0.056	0.248	0.2	-0.003	-0.039	0.240	0.198
last fifth	0.064	-0.208	0.344	0.2	-0.023	-0.074	0.284	0.187
Sum	0.000000	0.000000	1.000000	1	0.000317	-0.00057	1.000249	1

The obtained results are shown in Figure 4.5.

Figure 4.5: The effect of karup-king interpolation coefficients correction



4.3 Old ages mortality extrapolation methods

Numerous studies have shown that mortality rates changes its evolution trend beyond a certain age; usually the age of 80 or 85 years (Coale & Guo, 1989; Coale & Kikser, 1990) . According to this, the use of conventional mortality models may lead to results that do not reflect the old ages mortality shape. In such a context, we use appropriate models adapted to the old ages specificity. Quashie et Denuit (2005) provide us with a fairly broad overview of the various methods designed to extrapolate mortality to the older ages. We expose in this part the different extrapolation methods that appear to be applicable on our data.

Gompertz-Makeham model (GPZ, MKM)

For a long time, the classical mortality models have been used to extend mortality to the older ages. Gompetrz (1825) discovered that the force of mortality can be expressed as an exponential function with age :

$$\hat{\mu}_x = \alpha.e^{\beta x}$$

In 1860, Makeham (Makeham, 1860) developed this model by adding a constant term representing the risk of death by accident which is not related to age:

$$\hat{\mu}_x = c + \alpha.e^{\beta x}$$

Despite that the Gompertz model became a special case of Makeham model when $c = 0$, Makeham model has almost a better fitting quality compared to Gompertz model. Both these models were used to extend mortality beyond the age of 80. A Makeham-type function was used to extend the United Nation's model life tables beyond the age of 80 (UN, 1981). The used function was expressed by the following formula:

$$\frac{{}_n\hat{q}_x}{1 -_n \hat{q}_x} = A + B^x$$

The sixth last estimated${}_nq_x(x = 50 - 75)$ were used to estimate the function parameters and to extend${}_nq_x$until the total extinction of survivals $(l_x = 0)$.

The exposure of the accident death risk become negligible, because old people are generally exempt from any risky activity (Gavrilov and Gavrilova, 2011). In such a case, Makeham model does not provide any added value compared to Gompertz model. Simply, both models lead to approximate goodness-of-fit and Gompertz model becomes preferable. For this, Gompertz model was used in 1983 to extend the Coale-Demeny model life table until the age of 100 (Coale and al, 1983).

When we extend mortality just until the age of 100, the choice of the extrapolation method is of a little importance. The shortage of old people mortality data prevented any comparison in this sense and the majority of attempts have been based on simple assumptions. Since the 80's, life expectancy has improved as well as the number of centenarians (Robine and Vaupel, 2001; Buettner, 2002). The need for more sophisticated tools for old age mortality extrapolation became required. The improvement of the old people mortality data quality permitted actuaries and demographers to evaluate the goodness-of-fit of their models and to propose adapted ones.

The availability of reliable old population mortality data allowed researchers to study the shape of the mortality curve at the older ages. Its main finding was that the old age mortality does not follow a Gompertzian function, but slows down slightly. According to this, Gompertz model over-estimate mortality at older ages. This phenomenon was well known as old age mortality deceleration, was observed on several populations and many of studies have tried to find a convincing explanation. A huge number of researchers assumed to relate this deceleration to a selection process concluding that when people reach a certain age, their death risk continue to grow-up but with decreasing rate (Kannisto and al., 1994; Kannisto, 1992; Coale and Guo, 1989). In their paper, Gavrilov and Gavrilova (2011) have shown that there is no old age mortality deceleration, thus the weakness of the quality of the old age mortality data and the exposure to the death risk might lead to such a consequence. In the absence of data allowing to verify these two hypothesis on the Algerian population, we keep supposing that the first assumption is more near to be verifiable.

In the same sense, Coale and Guo (1989), then Coale and Kisker (1990) achieved to formulate a model describing the old age mortality evolution. This method is still used nowadays to close-out the Coale-Demeny model life tables.

Many other models which were initially conceived to fit adult ages mortality have been tested on the old age mortality data as Perk's, Weibull and Heligman-Pollard models.

Weibull's model (WBL)

Weibull (1951) proposed the following formula:

$$\hat{\mu}_x = \alpha . x^{\beta}$$

Helligman and Pollard model (HP)

Helligman-Pollard model (1980) is the only mortality model which allows to fit mortality at all ages including infant, child and young ages.

$$\frac{q_x}{1-q_x} = A^{(x+B)^c} + D.exp(-E(ln(x) - ln(F))^2 + G.H^x$$

Each one of the three terms of the model fit a mortality at certain ages (Helligmand and Pollard, 1980). At the age of 40 or 50, the two first terms become useless and only the third term can be kept. then it can be written as :

$$Logit[\hat{q}_x] = \pi + \varphi . x \;\; with \; \pi = ln(G) \; and \; \varphi = ln[H]$$

This last formula is similar to the Gompertz mortality law. The only difference consists on the used mortality measure q_x and μ_x.

This method was examined by the United Nations Population Division for old age mortality extrapolation issue (Buettner, 2002).

Coale-Guo and Coale-Kisker methods (CK)

The main idea of the Coale-Guo model (1989) is that, at older ages, mortality rates keep growing-up but with an increasing growth rate. This deceleration is supposed to follow a linear trend. Firstly, the model was expressed in 5 years age groups, and adapted after to the single ages description. The growth rate of the mortality from an age to another was defined by:

$$k_x = ln[\frac{_5m_x}{_5m_{x-5}}]$$

k_x is supposed to increase between two consecutive ages by a constant R:

$$k_{x+5} = k_x - R$$

For two ages x and $x + 5.i$, we can write:

$$k_{x+5.i} = k_x - i.R$$

The Coale-Guo model was first applied to extend central death rate until the age of 110 starting from the age of 75. When k_{80} is known, for $x \geq 80$ and according to the previous formula, the estimation of the death rate at certain age starting from a known death rate at the previous age can be done by the following formula:

$$_5\widehat{m}_{x+5} =_5 \widehat{m}_x.exp(k_{80} - \frac{(x-80)}{5}.R)$$

and as a general formula, we can write:

$$_5\widehat{m}_{80+5.i} =_5 \widehat{m}_{75}.exp(k_{80} + k_{80} - R + k_{80} - 2R + k_{80} - 3R + ... + k_{80} - i.R)$$

Which implies:

$$_5\widehat{m}_{80+5.i} =_5 \widehat{m}_{75}exp(ik_{80} - \tfrac{i(i+1)}{2}R) \;\; i = 1, 2, 3, ...$$

The objective of the method was to derivate the mortality rate at the age 80, 85, 90, 95, 100 and 105 from $_5\widehat{m}_{75}$.That leads to an age closure equal to 110 years. The choice of this value was justified by the fact that the observed highest age of death is around 110 (Coale and Kisker, 1987). The last mortality rate can be found by using:

$$_5\widehat{m}_{105} =_5 \widehat{m}_{75}exp(6k_{80} - 15R) \;\; i$$

Authors have arbitrarily imposed as a constraint:

$$_5\widehat{m}_{105} -_5 \widehat{m}_{75} = 0.66$$

This formula is more adapted to populations with high life expectation (Coale and Guo, 1989), that means 70 years or higher.

Coale and Kisker (1990) reformulated Coale-Guo formula and adapted it for the case of single ages mortality data. Starting from the same principal of mortality deceleration rate with constant decreasing, they achieved the following formula based on the central death rate :

$$\hat{u}_x = \hat{u}_{x-1}exp(k_{80} + s.(x-80)) \; x = 80, 81, ..., 109.$$

k_{80} represents the growth rate of mortality at 80 years. It is defined to be the average growth rate of the age range [65-80] and can be calculated by the following relationship:

$$k_{80} = \frac{ln(\frac{\hat{u}_{80}}{\hat{u}_{65}})}{15}$$

In some cases, the use of the Coale-Kisker formula can lead to some incoherence or to a crossover of the males and females extrapolated mortality curves. To fix this element, the authors have arbitrarily fixed the rates for a relatively high age (110 years):

$$\hat{u}_{110} = \begin{cases} 1 \; for \; males \\ 0.8 \; for \; females \end{cases}$$

This leads to define S which is equal:

$$s = -\frac{ln(\frac{\hat{u}_{79}}{\hat{u}_{110}}) + 31k_{80}}{465}$$

s in Coale-Kikser model is the same R used in Coale-Guo formula. They are calculated from the same formula.

Finally, both of these methods are well known as Coale-Kisker model which is also called Quadratic model (Roli, 2008. Thatcher, 1999). According to this definition, we can understand that Coale-Kisker model consists simply to express the logarithm of death rate as a quadratic function. However, this formulation is not used or explained in the literature:

$$ln(\widehat{\mu}_x) = a + bx + cx^2$$

The mortality growth rate can be expressed as:

$$k_x = ln[\frac{\widehat{\mu}_x}{\widehat{\mu}_{x-1}}] = a + bx + cx^2 - a - b(x-1) - c(x-1)^2 = c(2x-1)$$

Considering a starting age of 80, we can write:

$$k_x - k_{80} = 2.c.(x - 80)$$

and that leads to the same formula seen earlier :

$$\hat{u}_x = \hat{u}_{x-1} exp(k_{80} + 2.c.(x-80)) \; x = 80, 81, ..., 109.$$

Other methods have been developed later for the same issue.

Himes, Preston and Condran (HPC)

Himes, Preston and Cardan (1994) proposed an extrapolation method based on the observation of the single ages death rates at the age range 45-99. Their idea is to smooth the observed mortality curve by an 2-steps 3rd order moving average on the form:

$$g_x = \frac{\mu_{x-1} + \mu_x + \mu_{x+1}}{3}$$

Then, the smoothed death rate g_x is smoothed again by using the formula:

$$h_x = \frac{g_{x-1} + g_x + g_{x+1}}{3}$$

that leads to the following final formulation of the smoothed death rate:

$$h_x = \frac{\mu_{x-2} + 2.\mu_{x-1} + 3.\mu_x + 2.\mu_{x+1} + \mu_{x+2}}{3}$$

For the considered age range, the death rates follow an exponential function which can be approximated to a Gompetz's model or to the third terms of Helligman and Pollard model. The authors proposed to extend mortality until the age of 115 years old by using the following formula:

$$Logit(\hat{\mu}_x) = ln(\frac{\hat{\mu}_x}{1-\hat{\mu}_x}) = \theta + \Pi.\text{x}$$

The parameters θ and Π are estimated on the age ranges [45-79]. In the original application, many age ranges were compared : [45-79], [50-79], [55-79], [60-79] and [65-79]. The comparison of the obtained results has shown that the estimated parameters vary significantly in function of the age range used as a basis for extrapolation.

Perks, Logistic, Kannisto, Thatcher models (PRK, LOG, KST, THT)

The use of logistic functions for mortality curves graduation was first introduced by Perks (1932). He proposed the following formula to fit the mortality hazard rate with age:

$$\widehat{\mu}_x = \frac{\alpha.e^{\beta x}}{1+\gamma.e^{\beta x}}$$

The general form of logistic model can be written like :

$$\widehat{\mu}_x = \theta + \frac{\lambda.\alpha.e^{\beta x}}{1 + \alpha.e^{\beta x}}$$

When Thatcher et al. (1998) tried to use this model which was used to fit old age mortality of 13 developed countries, they found that λ is very close to 1 (Thatcher, 1999). They concluded that the model can be simply written with 3 independent parameters:

$$\hat{\mu}_x = \theta + \frac{\alpha.e^{\beta x}}{1 + \alpha.e^{\beta x}}$$

at older ages, θ become negligible as the constant accident risk in Makeham's model. Since that, the previous formula can be simplified to be (Kannisto, 1992):

$$\hat{\mu}_x \approx \frac{\alpha.e^{\beta x}}{1 + \alpha.e^{\beta x}}$$

In the Logit form, we found:

$$Logit(\hat{\mu}_x) = ln(\alpha) + \beta.x$$

That is the same formula used independently in 1994 by Himes, Preston and Cardan. When kannisto proposed such a simplification, he was just reporting some findings without any intention to give a specific mortality model. This model was reported in (Thatcher et al., 1998) as a Kannisto model.

Denuit et Goderniaux method (DG)

This method relies on a polynomial formula of order 3 of log mortality rate:

$$ln(\hat{q}_x) = a + bx + cx^2$$

In the original article, the authors have set an assumptions about the ultimate survival age which might be equal to 130 years old. In the order to respect this constraint, they imposed $q_{130} = 1$ and $P_{130} = 0$.

Comparison

According to the old age mortality models presented above, there are different ways to extend mortality to the older ages. Globally, 2 mortality measures can be used: the death rate μ_x and the mortality rate q_x. Gavrilov and Gavrilova (2011) explained the difference between the use of these parameters, and detailed how the choice of the indicator that can affect the final result of the extrapolation process. At the older age, the mortality rate reach a high level and keep growing slowly until the value of 1 which represents the limit, while death rates can keep growing without a such a constraint. For this, $\mu_x$is more suitable for the fitting and the extrapolation use especially at the older ages. When we report to the semi-log scale, this last can easily be approximated by a straight line more than q_x. Another way to give more regular linear trend to the mortality age is the introduction of *Logit* which allows passing from [0-1] to]-∞, +∞[. The third element, is that there are principally two ways to extend mortality to the older ages : a transformed linear trend and transformed quadratic trend.

Among the presented models, there are only two that propose an extrapolation based on q_x: HP and DG models. The first propose an extrapolation with a linear trend of $ln(q_x)$, while the second aims to impose a quadratic form to the $Logit(q_x)$. The other models are all based on μ_x. GPZ, MKM, WBL and HPC models express the death rate in log linear form. CK model is oriented to extend $ln(\mu_x)$ with a quadratic function, while logistic models (PRK, LOG, KST, THR models) try to approximate $Logit(\mu_x)$ to a linear function. In addition, two families of models can be distinguished in function of either any closure constraint is imposed or not. GPZ, MKM, WBL, LOG, KST, THT, PRK, HP and HPC are supposed to be a behavioural extrapolative models since the extrapolated rates at older ages are just a result of the model calibration at younger ages. With such models, we can not impose any closure constraint. The life table is closed-out when q_x is equal to 1. For the other models (CK, DG), which are based on a quadratic functions, although the old age mortality are extrapolated with a parametric models, the final result is significantly affected by the imposed closure constraint. In our application, we will try to compare different variants of models. for this, the models to be compared are : HP, DG, GPZ, WBL, CK and KST.

4.4 Old age mortality extrapolation results

4.4.1 Some methodological orientations

To compare mortality models, we use conventionally a set of quantitative and qualitative criteria. The principal ones are the goodness-of-fit and the predictive capacity. Goodness-of-fit aims to evaluate the fidelity of the model of the data used for its calibration. A comparison between observed (or original) and fitted values is necessary in such a case. The selection process must lead us to select a model which have more capacity to predict nearly the real trend of the extrapolated variable beyond the last age for which data is available. It is well known among statisticians that a good fitting quality does not guarantee necessarily a good predictive capacity. In order to make a sense for this evaluation, we need some data to confront the values extrapolated by the model against the observed ones. we remind that according to the data which we have until now, the single age death probabilities are extended in almost case until the age of 79. Only for the 5 last years, these mortality rates were extended until the age of 84. Here, we have to arrange this data in order to allow a good evaluation of the goodness-of-fit and the predictive capacity of the selected model. Using a lower ages to calibrate the models can lead to a good fitting results in spite of its being useless for old age mortality extrapolation issue. In other words, the more the upper limit of the age range used to calibrate the model is highest (near to the extreme available specific-age-mortality rate), the more the calibrated model is suitable for extrapolation issue. Moreover, to evaluate the predictive capacity of a model, some data (ages) must be used for comparison. The More the used age range is longer,the more the evaluation takes a sense. However, given the shortage of the data for age beyond 75 and 80 (for the 5 last years) we must better manage and combine the available data for both issues : goodness-of-fit and and predictive capacity. According to all these elements, we judged the most suitable to use is the age of 74 as a maximum age for model calibration and the age range [75-79] to evaluate the predictive capacity of all models. Since the mortality rates for the age range [80-84] are available but only for 5 years, it will be advantageous to consider it in the predictive capacity evaluation. To avoid the problem of the weight generated by the number of years with available data for each age range, we can use a weighted evaluation criterion which takes the errors observed at [75-79] (38 years) in addition to those observed at [80-84] (5 years) by an equivalent weight. It is clear that the use of data until 84 years does not allow to predict the exact old age mortality trend, but it allows at least to predict the starting-up of this trend. For this, it is necessary to exploit all the available data.

As a fitting and evaluating criteria, the Mean Squared Error (MSE) seems to be suitable for our case. We remind that the six selected models are based on different significant quantities: $ln(\mu_x)$, $logit(\mu_x)$ and $logit(q_x)$. To ensure the comparability of the models for the fitting quality and the predictive capacity evaluations, the MSE is used for all models. It must be expressed in a unified mortality measure that may be $ln(q_x)$ which is used only for comparison issue. The goodness-of-fit can be evaluated on the age range [x-74] by:

$$MSE_{[x-74]} = \frac{1}{t * x} \sum_{x=X}^{74} \sum_{t=1977}^{2014} [ln(\hat{q}_{xt}) - ln(q_{xt})]^2$$

For the predictive capacity evaluation, this criteria is estimated in two steps: In first, we calculate the MSE corresponding for each age range [75-79] and [80-84] by the respective corresponding formula:

$$MSE_{[75-79]} = \frac{1}{5 * 38} \sum_{x=75}^{79} \sum_{t=1977}^{2014} [ln(\hat{q}_{xt}) - ln(q_{xt})]^2$$

and

$$MSE_{[80-84]} = \frac{1}{5 * 5} \sum_{x=80}^{84} \sum_{t=2010}^{2014} [ln(\hat{q}_{xt}) - ln(q_{xt})]^2$$

Then, the weighted MSE corresponding to all the age range [75-84] is simply calculated by :

$$WMSE_{[75-84]} = \frac{1}{2}(MSE_{[75-79]} + MSE_{[80-84]})$$

In first, the six models will be extrapolated to the ages beyond 75 without imposing any closure constraint. We remind that when some extrapolation models were first proposed, they were based on the age of 80 years as the age to start extrapolation. Here, all the used models are adapted to give an extrapolation starting from the age of 75. The extrapolation process will be applied independently for males, females and both sexes population.

In addition to the evaluation of goodness-of-fit and the predictive capacity, some other qualities are required to qualify any model to be a good one. The extrapolation's results must be coherent regarding to some classical finding in the mortality modelling field that we will present as following:

1. Mortality rates keep increasing with age. That allows to write :$\widehat{q}_{x+1} > \widehat{q}_x;\ \forall x \geq 75$. The mortality deceleration which can be observed starting from a certain age (Coale and Guo, 1989; Coale and Kisker, 1990) does not imply a decrease in mortality rates with age but a decrease of the growth rate. We introduce this element, which appears to be evident, because of polynomial functions (DG and CK methods for example) can lead under certain conditions to a reversal in the mortality rates beyond a certain age. This propriety will be only checked-out for DG and CK models.

2. Since extrapolation is made independently for males, females and both sexes populations, the extrapolated mortality rates for both sexes population should be closer to the weighted average of male and female rates. According to the observation of the structure of the Algerian population aged 70 and +, we can observe that males and females have approximately the same weight. That allows to write : for a fixed year (t) : $\widehat{q}_x^{both} \approx \frac{(\widehat{q}_x^m + \widehat{q}_x^f)}{2}; \forall x \geq 75$. In the case of a change in these sex distribution at very advanced ages in any way, the mortality rate for both sexes population q_x^{both} must be situated between the male and female rates, and can not be outside in any case . That implies : $\widehat{q}_x^b \in [\widehat{q}_x^f - \widehat{q}_x^m]$;

3. Male mortality rates are higher than female mortality rates:$\widehat{q}_x^m \geq \widehat{q}_x^f, \forall x \geq 75$. This constraint is checked only for the years which do not have a female over mortality for ages below 75 years. Since the extrapolated old age mortality is widely determined by the trends observed at the prior ages, an observed female over mortality at adult ages will lead to a similar effect at old ages.

4. The last element which can give a clear judgement about the goodness of the extrapolation is the age limit predicted by the model. Theoretically, the age limit (w) is reached when the mortality rate attains the value 1: $\widehat{q}_w = 1$. An old age mortality model can be described as a good extrapolative model when it leads to an age-limit coherent compared to the maximum surviving age observed in reality. For the Algerian population, we remind that, according to the MICS IV (Multi Indicators Clustery Survey) results, a maximal surviving age of 112 years old was observed for females and 110 for males (Flici et Hammouda, 2016). That does not represent an estimation of the maximal surviving age for the Algerian population, but just a minimal of the interval where the ultimate age can be situated.

4.4.2 Model selection

Goodness-of-fit and predictive capacity

In almost cases, goodness-of-fit is widely related to the age range used to calibrate a model. Usually, the quality is higher as the length is shorter, but at the expenses of the robustness of the model. When a model is estimated on long length data, the variation bands of the predicted series are narrow. So, it is so important to define a fitting criteria which combine goodness of fit and robustness. The use of the Bayesian Information Criterion (BIC) is more suitable for such purpose since it considers, in addition to the distance between fitted and original values, the number of parameters and the number of observations. Although the BIC was firstly proposed for Likelihood Estimations issues (Schwarz, 1978), the formula was adapted later to suit the Least Squares Errors (LSE) estimation method (Burnham and Anderson, 1998; Hansen, 2007). The adapted formula can be written as:

$$BIC = n.ln(\frac{1}{n}SSE) + k.ln(n)$$

with n representing the number of observations, and k the number of parameters in the model.

BIC is used to evaluate both the goodness-of-fit and the predictive capacity of the models. For each model, these qualities are related to the age range used to calibration, and it is not evident to define a common age range which ensure the best quality for all models. Also, there is practically no "best model" in all situations. It is very important to reduce the initial set of candidate models to a sub-set leading to approximate qualities. According to this, each model is calibrated on various age ranges : [40-74], [45-74], [50-74], [55-74] and [60-74] then extrapolated beyond the age 75. The models are ranked according the two criteria. Results are presented in Table 4.3.

When we try to compare a set of models with different age ranges, it is not evident to find a model which is the best with any age range. Similarly, we can not find a specific age range which guarantee the best result for every model. That became more complex when we try to combine goodness-of-fit and predictive capacity. But, from the observation of the results given in the tables behind, we can conclude some evidence about the best models and the best age ranges according to the goodness of fit and the predictive capacity criteria.

According to the ranking of the six models and to the two selected criteria obtained on the three populations, we observe some differences in ranking which changes in function of the criteria and of the age range. But, some evidences appear clearly: The three best models for fitting and predicting are DG and CK in the first and the second positions followed by GPZ in the third position. HP, GPZ and WBL came in 4th, 5th and 6th positions. we observe that, every model leads to its best quality in a specific age range. Nevertheless, it is necessary to select a common age range for all models to reduce irregularities which can appear at very old ages. Here, we propose to use the age range [50-74] to fit all models. This age range is the only range which allows to keep the final ranking (Goodness-of-fit and predictive capacity) obtained on the three populations.

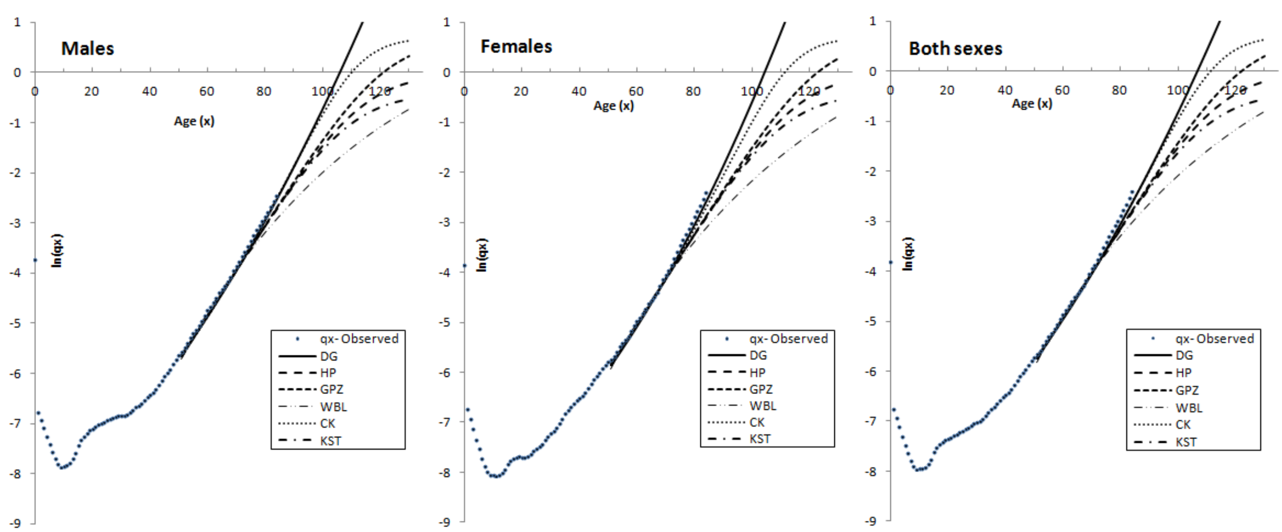
Figure 4.6 shows the mortality rates extrapolated to old ages with the six models.

Table 4.3: Old age mortality models evaluation and ranking

Age range for calibration	40-74		45-74		50-74		55-74		60-74		Models Ranking	
Age range for evaluation	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	Goodness of fit	Predictive capacity
Model												
DG	-173.44	-50.68	-163.82	-50.80	-137.50	-53.83	-107.75	-45.01	-81.20	-46.14	2	1
HP	-173.02	-29.85	-165.40	-33.75	-138.12	-35.92	-109.86	-37.36	-89.85	-36.63	4	4
GPZ	-174.04	-31.39	-165.92	-35.51	-138.49	-37.75	-110.06	-39.06	-89.98	-37.80	3	3
WBL	-166.11	-16.02	-141.67	-16.02	-125.87	-20.25	-105.19	-23.93	-86.76	-26.98	6	6
CK	-178.28	-46.95	-157.36	-45.61	-131.87	-45.91	-102.75	-46.90	-75.63	-48.42	1	2
KST	-171.95	-28.73	-164.81	-32.11	-137.72	-34.23	-109.63	-35.77	-89.70	-35.47	5	5
Females												
Age range for calibration	40-74		45-74		50-74		55-74		60-74		Models Ranking	
Age range for evaluation	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	Goodness of fit	Predictive capacity
Model												
DG	-152.12	-24.75	-139.67	-23.93	-114.48	-32.64	-95.36	-29.48	-82.05	-31.07	2	2
HP	-144.06	-13.61	-140.44	-16.52	-121.59	-20.07	-97.99	-23.05	-84.36	-26.29	4	4
GPZ	-144.81	-14.15	-141.07	-17.13	-121.98	-20.76	-98.20	-23.83	-84.44	-27.01	3	3
WBL	-115.92	-3.48	-119.29	-7.02	-110.25	-11.04	-93.31	-15.09	-81.85	-19.41	6	6
CK	-152.28	-22.83	-139.06	-22.13	-115.29	-33.39	-94.99	-27.90	-69.77	-27.62	1	1
KST	-143.31	-13.05	-139.78	-15.97	-121.19	-19.34	-97.77	-22.35	-84.28	-25.58	5	5
Both sexes												
Age range for calibration	40-74		45-74		50-74		55-74		60-74		Models Ranking	
Age range for evaluation	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	Goodness of fit	Predictive capacity
Model												
DG	-169.38	-36.08	-150.69	-35.17	-124.04	-35.67	-96.69	-36.69	-71.77	-37.53	1	2
HP	-160.24	-20.57	-155.85	-23.87	-132.04	-26.93	-105.61	-29.41	-90.70	-31.24	4	4
GPZ	-161.23	-21.51	-156.64	-24.90	-132.54	-28.05	-105.89	-30.53	-90.85	-32.21	3	3
WBL	-126.52	-7.25	-130.85	-11.24	-118.59	-15.25	-99.95	-19.18	-87.13	-23.00	6	6
CK	-165.80	-30.63	-161.33	-38.73	-133.65	-40.35	-103.78	-40.48	-76.63	-39.99	2	1
KST	-159.24	-19.71	-155.02	-22.96	-131.50	-25.92	-105.31	-28.33	-90.53	-30.30	5	5

Each model is calibrated on 5 different age ranges. BIC is calculated for [x-74] and [75-84] as an indicators of goodness of fit and the predictive capacity. The best age range which leads to the best models regarding to the GOF is marked in grey. The best age range for each model regarding to PC is marked in blue.

Figure 4.6: Extrapolation models comparison

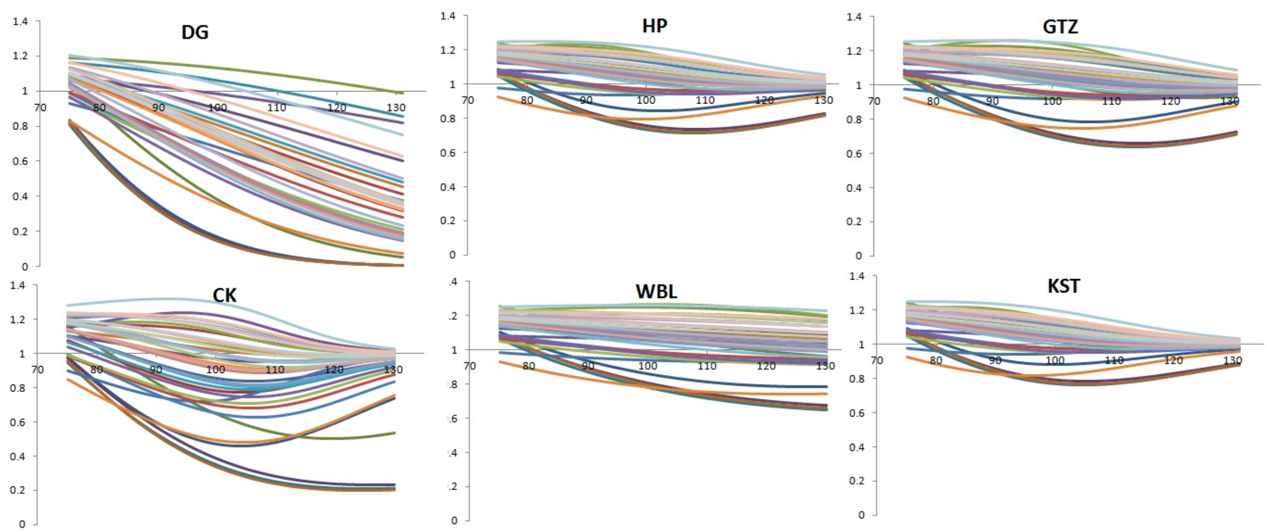


We observe that quadratic models (DG and CK) provide a near qualities regarding to the goodness-of-fit and the predictive capacity compared with the linear transformed model. An evaluation based only on the goodness of fit and the predictive capacity criteria is not sufficient to select the best models. The complementary criteria that we will use for the evaluation and comparison issue is the coherence between male and female expected mortality and the coherence between single sexes and both sexes extrapolations. Also, the expected surviving age limit is used to evaluate the extrapolation results.

Expected males females sex ratio

The observation of the male female sex ratio calculated on the extrapolated mortality rates beyond the age of 75 for the period 1977-2014 are shown separately for each of the six compared models. Results are presented in figure 4.7.

Figure 4.7: Extrapolated Mortality Sex Ratio with the 6 models

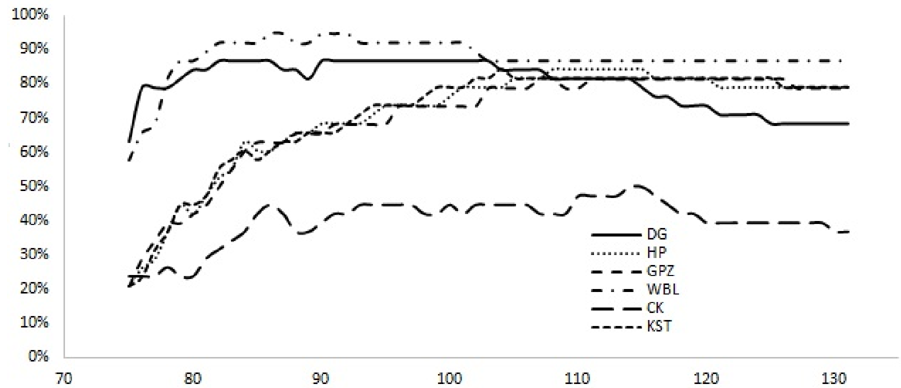


At old ages and very old ages, when mortality rates grow to the value 1, male and female mortality rates tend to converge to each other. In such a case, the male female sex ratio must converge in the 1. KST, HP and GPZ have led to more formal results compared to the three other models.

Coherence between unisex and both-sexes expected mortality rates

To evaluate the quality of each model in terms of coherence between single sexes and both-sexes mortality estimations, we calculate the part of cases where both-sexes expected mortality rate that is situated out of the interval between males and females mortality rates.The more this failure rate is lower, the more the extrapolation is coherent. results are presented in Figure 4.8.

Figure 4.8: Failure rate between single sex and both-sexes expected mortality rates

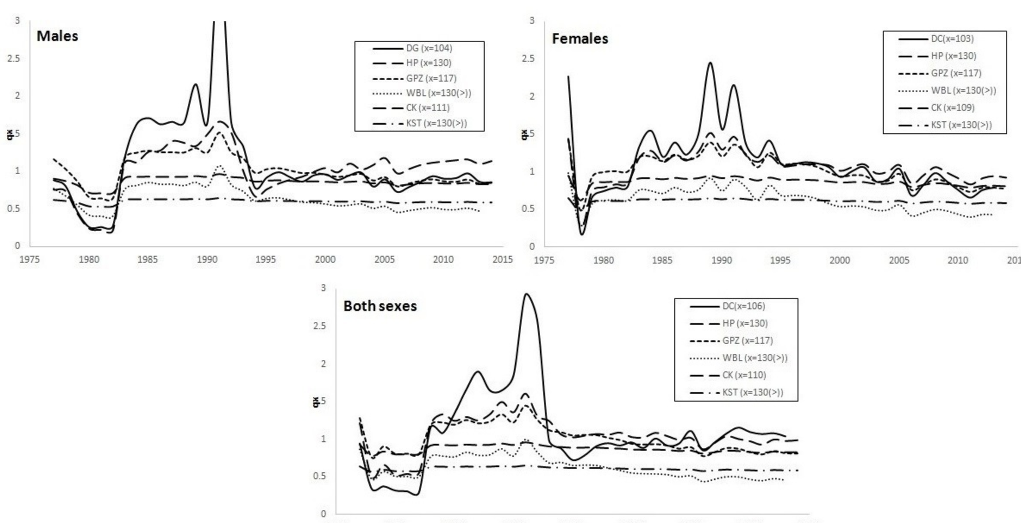


The DG and WBL models led to the most failed result when we compare single sexes and both-sexes expected mortality rates. The other models give an acceptable failure rate under 20% increasing beyond the age of 100 to around 70%. CK model give the less important failure ratio.

Expected age limit

The surviving age limit is attained when the death's probability is equal to 1. One way to evaluate the coherence of the old age extrapolated mortality rates is to compare the different models according to their expected age limit. Surely, the expected age limit obtained with a given model can not be constant over time, but slightly increasing with time. To compare the six models presented above, we will try to present the expected mortality rate at a high age which should be near to the age limit predicted by each model. The obtained result on the period 1977-2014 is shown in Figure 4.9.

Figure 4.9: Comparison of some high ages q_x predicted by the models



Other than the turbulence that mark the evolution of the mortality rates (nearly) at the age limit, some models lead to incoherent age limit. By the age of 130, HP, WBL and KST models give a mortality rate under 1. The other models lead to more coherent results. The age limit expected with DG model is equal to 104, 103 and 106 in average respectively for males, females and both sexes populations. 106 is the age limit expected for the year 2014 for the three populations. For CK model, the expected age limit is situated between 109 and 111 years old. In adverse, GPZ model leads to higher age limit equal 117 in average and between 121 and 122 years for 2014.

According to the maximum surviving age observed for the Algerian population until now, GPZ model seems to be the most coherent model according to the expected age limit. Resulting from the MICS survey, the maximum surviving age was equal to 112 for females and 110 for males (Flici et Hammouda, 2016). This survey is far to provide an accurate estimate of the age limit for Algerians, since it is not exhaustive compared with the civil stats registration data, but it allows to fix the lowest bound of the age limit estimation. Also, we must consider the future evolution of the age limit since longevity keeps increasing. The age limit is defined to be the age which can not be surpassed by any human being in a certain geographic area. Denuit and Goderniaux (2005) supposed this age to be equal to 130 for the developed countries. In the same chain of ideas, we can suppose that this age limit can be lower in developing countries and we can consider 120 as a reasonable limit for the Algerian population.

Result's discussion

We have seen through this comparison, that quadratic models gave a good quality regarding to goodness-of-fit and predictive capacity compared with the linear transformed models. GPZ model makes exception of this rule. The study of the expected age limit predicted by the different models confirmed this finding. The comparison based on other criteria has shown a different judgement. The models ranking overturned in almost case, when we consider the coherence between single sexes and both-sexes estimation and the sex differential expected mortality. GTZ model keep a good score in all the situations.

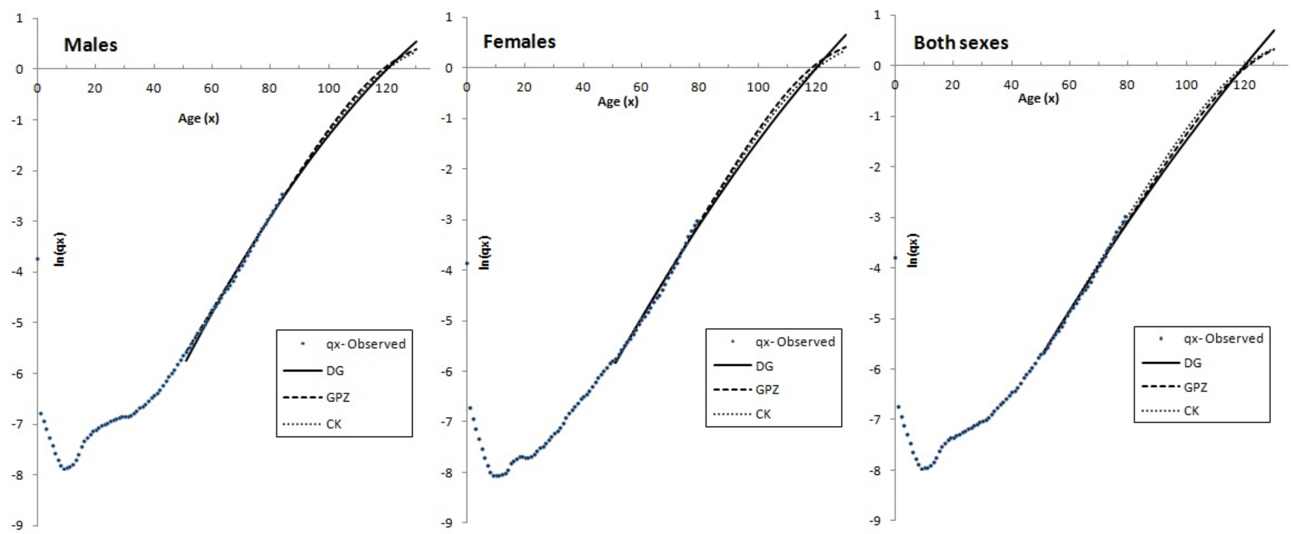
The less performance resulted with the quadratic models (CK and DG) can be explained by the nature of the polynomial functions which can lead to some plausible values when it is used to extrapolate or to extend mortality rates for a huge age ranges. Although they guarantee a better goodness of fit and predictive capacity compared with the linear transformed models, they generate some incoherence regarding the sex differential or single sexes vs both-sexes extrapolated mortality rates. To avoid this disadvantage, these models were usually used under some constraints about the age limit (Denuit and Goderniaux, 2005) or about a fixed mortality level at certain high age (Coale and Kisker, 1990). Imposing such a construct may avoid a crossover of the extrapolated mortality curves considered in the time dimension as in the sex differential one (Buettener, 2002).

In the rest of the present work, we keep working with 3 models: DG, CK model with 120 as an assumed age limit, and GPZ model as the best linear transformed model.

4.4.3 Model calibration under age limit assumption

Until now, we have seen that the quadratic models (DG, CK) and GPZ model are near to describe the old age mortality of the Algerian population than the other models. However, some weaknesses were observed. The trend of mortality rates beyond 80 obtained with the models are almost incoherent especially those obtained with the quadratic models. The introduction of an age limit assumption should reduce the observed incoherences (Buettner, 2002). For this, DG and CK models are re-estimated by imposing 120 years as an age limit. This additive condition have to impair the fitting quality of these models. To recover such a lack in fitting quality and to keep the 2 quadratic models in the same fitting performance as GPZ model, we reduce the length of the age range used to calibrate the two models. we observe that when DG and CK are calibrated on the age range 60-79, they give approximately the same fitting quality as GPZ model calibrated on the age range [50-79]. The evaluation of the three models after imposing the new constraints are given in Figure 4.10.

Figure 4.10: Old age mortality extrapolation with imposing an age limit



We observe that the three models lead approximately to the same results regarding to the extrapolated trend of mortality rates beyond 80 until 120 years. To confirm the goodness of these results, we must check out their coherence, principally regarding the male vs female (Figure 4.11) and single sexes vs both sexes differential mortality (Figure 4.12).

Figure 4.11: Male Female Mortality ratio under the age limit constraint

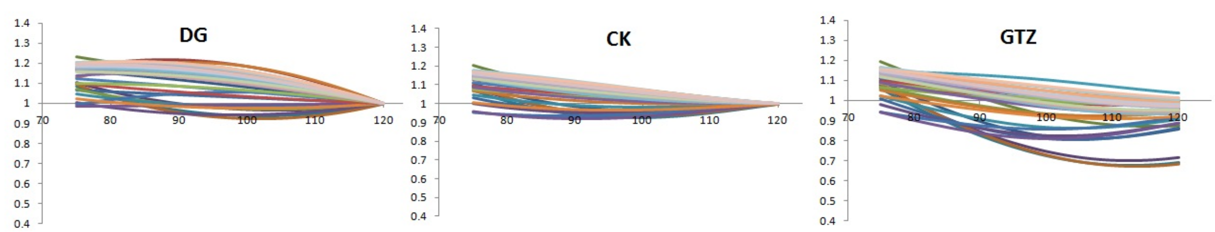
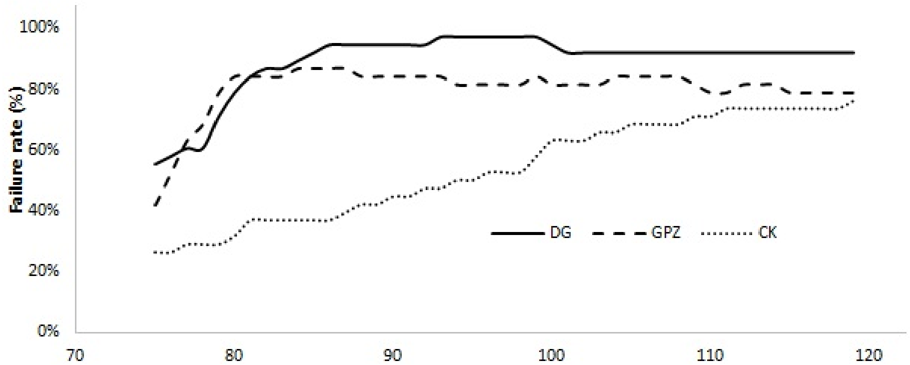


Figure 4.12: Single sex vs Both sexes extrapolation under the age limit constraint



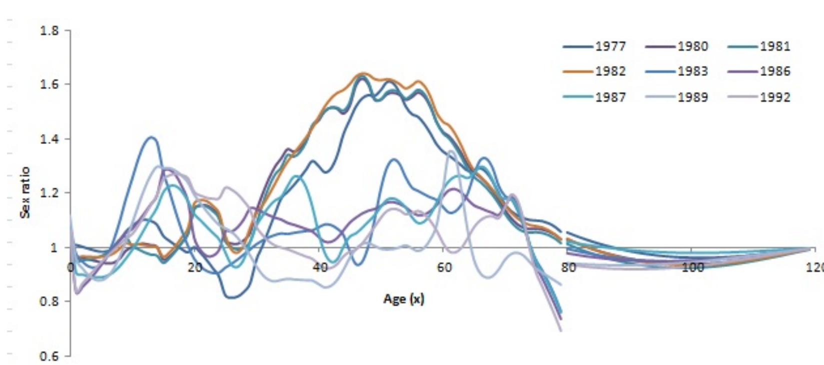
According to these two last criteria, CK seems to be better than DG and GPZ. The expected male female mortality ratio converge to the value 1 more rapidly in CK model than with DG model. For the coherence between singles sexes and both sexes projections, the increasing trend of the failure ratio can be explained by the fact that at near age limit, all mortality rates converge to the unit value. Furthermore, the difference between males, females and both sexes mortality rates become as smaller as the age limit approaches. However, it is necessary to improve the quality of the extrapolation by imposing some additive constraints about the comparative evolution of male, female and both sexes extrapolated rates.

4.4.4 Coherence constraints

Here, we impose two additional constraints; The first one aims to keep the females mortality under the males mortality while the second one will try to keep the both sexes expected mortality in between males and females mortality.

For some years of the period before 1994, we could not obtain a sex ratio greater than 1 for the extrapolation age range [80-120]. These years have marked a slight female overmortality at the age range used for the extrapolation (Figure 4.13). Also, a heavy decrease in sex ratio is observed by the late of the age range used for extrapolation. For the period post 1994, male mortality keeps increasing over the female mortality until the age limit.

Figure 4.13: Years with extrapolated Female mortality excees



For the second constraint about the coherence between both sexes and single sexes extrapolations, we imposed that the both sexes extrapolated mortality rates vary in between male and female extrapolated mortality rates which were estimated in the previous part. This constraint was fully respected without affecting significantly the goodness of fit.

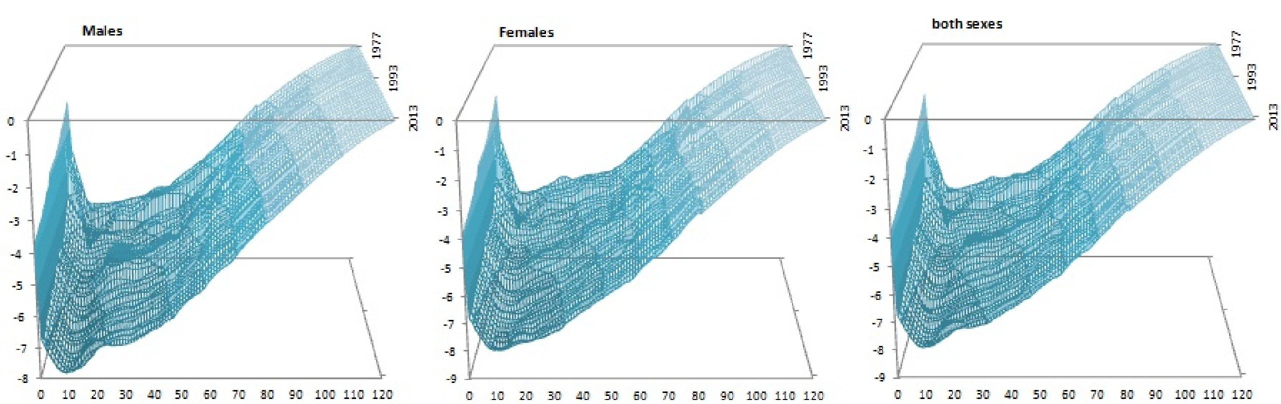
4.5 Final results

After imposing all the necessary constraints on the extrapolation process, we can conclude that the three selected models: GPZ, CK and CG give approximately the same quality regarding the considered evaluation criteria. Therefore, it is very difficult to decide whether one model is more appropriate than the others. The quadratic models allow more flexibility in old age mortality extrapolation. Besides, they ensure a good fitting quality on the age range used as a basis for the extrapolation. When the age limit constraint is imposed, the extended mortality surface shows a high regularity. GPZ model have shown a good performance regarding all selection criteria. However, the use of unfitted adult age mortality surface for its calibration, have led to unstable expected age limit series. This disadvantage does not appear in the case of quadratic models by imposing a common age limit constraints for all years. Consequently, GPZ model can be a perfect one to extrapolate old age mortality on a well fitted mortality surface. As a result and giving that CK model provide better quality compared to DG model on the basis of the other selection criteria, the decision is to adopt the CK model to extrapolate old age mortality for the Algerian mortality surface.

4.5.1 Complete mortality surface

The completed mortality surfaces for male, female and both sexes population are shown in Figure 4.14.

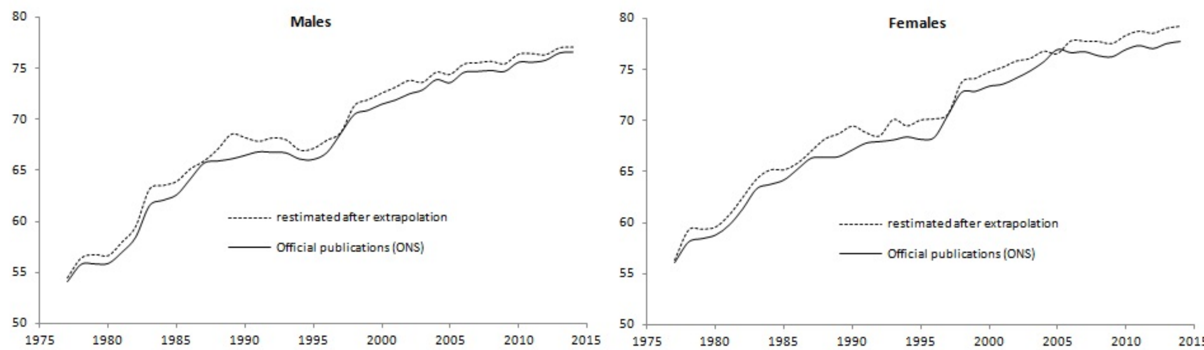
Figure 4.14: Completed mortality surfaces ($ln(q_x)$) 1977-2014



4.5.2 Corrected life expectancy at birth

As we have already pointed out, the life expectancy at birth is largely related to how the life tables are closed out. We will illustrate the impact of the closure of the life tables on the evolution of life expectancy at different ages. Life expectancy following the conventional notation is calculated from a five age mortality patterns. In actuarial calculations, a single ages pattern can be used and lead to a slight difference. To avoid any methodological effect when comparing the official life expectancy and thus re-estimated according to the extrapolation results obtained in the present work, we use five ages mortality rates to re-estimate life expectancy. Results are shown in Figure 4.15.

Figure 4.15: Re-estimated life expectancy after extrapolation compared to official statistics



The first finding we can carry out from figure 4.15 is that the life expectancy as we estimated it by the present work is slightly greater than the values included in ONS official publications. The average gap in this sense is around 0.9 year for males and 1.2 year for females. As mentioned in Chapter 2, some change points observed on the series of life expectancy at birth find an explanation (or confirmation) in the results obtained here. In 2005 for example, the change point observed on the female curve is significantly due to the closing-out procedure.

4.6 Conclusion

We have seen along this paper why it is important to close-out life tables on the basis of the extrapolative approach rather than the use of model life tables as an external reference. The first advantage of the first approach is to allow extending the mortality pattern beyond the usual closure age until the surviving age limit. These details are generally needed in actuarial calculations and population forecasts. Because of the information's lack and unreliability, such detail is usually not included in official life tables which are closed out earlier. Only the residual life expectancy at the closure age is published to summarize the mortality pattern for the ages beyond. In adverse, the quality of the estimates resulted from the use of the model life tables to close-out national life tables is very related to the adequate model life table selection process. Since that, a wrong use of model life tables may lead to unrealistic estimated life expectancy at the closure age. Also, an eventual change in the selection methodology may lead to an apparent level change in the series of the life expectancy at birth. We have seen in the introduction of this work some evidences concerning the wrong use of model life tables in the case of some African countries (Ekanem and Som, 1984).

We have shown also in Chapter 2 that some changes in the life expectancy evolution series in Algeria during the past half century are purely due to methodological effects. In this sense, the closing-out methodology led to some irregularities regarding the resulted life expectancy at birth. For this, we proposed in the present work to try another approach to close-out the Algerian official life tables ensuring more accuracy, adequacy and regularity. Our approach was to estimate the old age mortality by the extrapolation of the observed mortality trend at adult ages by using appropriate models. A set of old age mortality models were presented and compared for this issue : Gompertz (1825), Weibull (1952), Kannisto (1992), Helligman and Pollard (1980), Coale and Kisker (1990) and Denuit and Goderniaux (2005). The model evaluation and selection were based on a set of criteria : goodness of fit, predictive capacity, predicted age limit, coherence between male and female mortality and also the coherence between single sexes and both sexes mortality estimates. After a first comparison, three models were evaluated as suitable to our data : CK, DG and GPZ. To improve the quality of the resulted extrapolation under the three models, we modified the age range used for model calibration and imposed the age of 120 years old as an age limit constraint for the quadratic models (CK and DG). In addition to this, some other constraints were imposed to ensure coherence in males vs females, single vs both sexes extrapolations. In final, we concluded that the three models lead approximately to a similar results. Because of the irregularities in the resulted series of the expected age limit, the GPZ model was excluded from the comparison. Among the two quadratic models, CK model marked better quality than DG model regarding the male vs female and single vs both sexes extrapolations. Once the old age mortality rates were extended until 120 with the CK model, life expectancy at birth was re-estimated. The comparison of the re-estimated series of life expectancy at birth with the national statistics showed that our method leads globally to a gain of about 0.5 year in average on the whole period 1977-2014. Also, the obtained series shows more regularity in terms of the time evolution trends. Finally, we would like to highlight the importance of closing out the Algerian life table by using the extrapolative approach compared to the use of the model life tables. This last approach allows to reduce irregularities in the mortality indicators time series which is supposed to suit perfectly a pertinent analysis of mortality natural evolution. Also, the presented methodology provides lecturers a way to extrapolate mortality at the older ages in all situations especially in dynamic life table's construction.

Chapter 5

Future improvement of longevity in Algeria: Comparison of mortality models

A first version of this paper has been presented in Longevity 11 Conference, Lyon, France, September 8, 2016. A second version has been presented in IAA-Life Section Colloquium, Hong Kong, April 25 th, 2016.

5.1 Introduction

During the past half-century, the Algerian population has earned about 30 years in life expectancy at birth and more than 6 years in life expectancy at 50 (Chapter 2); but the tools used for calculation in the life-insurance market still less developed. Actuarial calculations are still based on a static life table constructed on old mortality data (CNA, 2004). That represents the only attempt where the Algerian authorities have tried to make available an actuarial life table to be used by the life insurance companies for pricing and reserving.

Now, The need is more than updating this life table with recent mortality data but to construct a dynamic life table which allows to take into account the future improvement in life expectancy well-know as the longevity phenomenon. In a previous work (Flici, 2016-a), we have shown that the use of a prospective life table shows an under pricing/ reserving of life annuities by about 23% compared to the use of the Algerian actuarial life table TV97-99. We have shown also that the use of an updated life table based on data of the year 2009 may push-up prices 11% over. According to this finding, the updating of the actuarial life tables can not itself resolve the problem of under-pricing of life annuities products in Algeria. Since they take into account the expected improvement in the life expectancy in the future, the use of dynamic life tables seems to be more convenient as a pricing and reserving tool for life annuities products in Algeria.

The construction of a dynamic actuarial life tables for Algeria have been treated in some previous works. In Flici (2016-a), we have tried to construct a dynamic life table for the Algerian population aged 60 and over by using Lee-Carter model (Lee and Carter, 1992). For this issue, we used the male and female life tables published by the Office of National Statistics (ONS) from 1977 to 2008. The data of the period [2009-2013] was used to evaluate the predictive capacity of the forecasts. In another work (Flici, 2014), we have tried to reach the same objective by using a multi-components Lee-Carter model (LC) as it was proposed by Renshaw and Haberman (2003). The use of LC model with considering more than one component resulted in improving the fitting quality of the model but with increasing the number of the parameters to estimate. Also, the resulted time indexes have irregular trends and are much complicated to be projected with a simple time series models. On the basis of all these experiences with the Algerian data, we achieved to observe that the regularity of the historical mortality data in Algeria was greatly affected by the terrorism events during 90's (See Chapter 2). In most of cases, this observed irregularity increases the difficulties about doing a robust forecast of mortality in the future. According to that , it was necessary to find a model which fit well the historical mortality surface of the Algerian population while ensuring more regularity, robustness and reducing the number of parameters to estimate. In this sense, we have compared in another work (Flici, 2015) between some Lee-Carter generalized models : Age-Period-Cohort model APC (Renshaw and Haberman, 2006) and the simpler APC model (Currie, 2006). The models were implemented on the age range [0-79] to better take into account the cohort effect.

Our main objective in the present chapter is to propose a dynamic life table for the population aged 50 year and over. For this purpose, we compare a set of models, principally Lee-Carter generalized models (Lee and Carter, 1992; Renshaw and Haberman, 2006; Currie, 2006) and Cairns-Blake-Dowd generalized models (Cairns and al., 2006; Cairns and al., 2007). The different models will be calibrated on the mortality surface 1977-2010, the 3 recent years (2011-2013) will be used to evaluate the predictive capacity of the models. In addition to the predictive capacity, goodness-of-fit, the expected life expectancy, the sex differential mortality and also the regularity of the projected mortality surfaces will be used to complete the evaluation and the comparison process. Once the best model is selected, we re-calibrate it on the data series from 1977 to 2013, and we do a forecast.

5.2 Background in mortality modeling

The main idea of the prospective mortality modeling is to reduce the mortality surface into a limited number of parameters. Then, the projection of the mortality age pattern to the future is made easy by just forecasting the components related to time by using the time series techniques.

Lee and Carter (1992) proposed a mortality model based on three parameters. Two of them are related to age, and the third is related to time. The log of the central death rate can be modeled by the following formula (noted M1) :

$$ln(\mu_{xt}) = \alpha_x^{(1)} + \beta_x^{(1)} * \kappa_t^{(1)} + \varepsilon_{xt}..............(M1)$$

with
$\alpha_x^{(1)}$: The average over time of $ln(\mu_{xt})$ for age x ;
$\kappa_t^{(1)}$: The general mortality trend index ;
$\beta_x^{(1)}$: The sensitivity of age x to the general trend of mortality evolution.
$\varepsilon_{x;t}$: Error term.

To estimate the parameters of the model, the authors proposed to estimate in first the parameter α_x by the mean over time of the logarithm of the central death rate at age x noted $\alpha_x^{(1)} = \frac{1}{n}\sum_{t=t_1}^{t_n} ln(\mu_{xt})$. Then, the residual matrix $ln(\mu_{xt}) - \alpha_x^{(1)}$ is decomposed into two components $\beta_x^{(1)}$ and $k_t^{(1)}$. The first parameter represents the sensitivity of the age x to the mortality time variation which is represented by the other component. The decomposition process can be done by using the Singular Values Decomposition technique (SVD) in order to minimize the Sum of Squared Errors (SSE) between the two parts of the equation : $ln(\mu_{xt}) - \alpha_x^{(1)} = \beta_x^{(1)} * \kappa_t^{(1)}$. Then, the estimated parameters are re-estimated to fit the observed number of deaths in each year along the observation period. To ensure the uniqueness of the solution, some identifiability constraints must be imposed to the estimated parameters :

$$\sum_{x=x_1}^{x_n} \beta_x^{(1)} = 1$$

$$\sum_{t=t_1}^{t_p} \kappa_t^{(1)} = 0$$

Wilmoth (1993) proposed a one stage decomposition process based on the Weight Sum Squared Errors minimization (WSSE). The quantity to minimize is : $min(WSSE) = W_{xt}.[ln(\mu_{xt}) - \alpha_x^{(1)} = \beta_x^{(1)} * \kappa_t^{(1)}]^2$. The weight W_{xt} is supposed to be the number of deaths observed at age x during the year t noted D_{xt}.

Several variants for the original Lee-Carter model were proposed to improve the goodness of fit. Renshaw and Haberman (2006) added a cohort effect to M1:

$$ln(\mu_{xt}) = \alpha_x^{(1)} + \beta_x^{(1)} * \kappa_t^{(1)} + \beta_x^{(2)} * \gamma^{(1)}{}_{t-x} + \varepsilon_{xt}........(M2)$$

Compared to M1, one more constraint is added to M2:

$$\sum_{t-x=t_1-x_n}^{t_p-x_0} \gamma_{t-x}^{(1)} = 0$$

Renshaw and Haberman (2006) proposed also to use the mean over time of $ln(\mu_{xt})$ just as a starting value to estimate $\alpha_x$which is re-estimated, in a second step, by the same estimation process as $\beta_x$and κ_t.

Currie (2006) simplified M2 by introducing a constraint on the sensitivity of the mortality at age x for time and cohort variation $\beta_t^{(1)}$ and $\beta_t^{(2)}$which are supposed to be constant and equal to $\frac{1}{n}$. That leads to the model M3 represented bellow:

$$ln(\mu_{xt}) = \alpha_x^{(1)} + \frac{1}{n}\kappa_t^{(1)} + \frac{1}{n}\gamma_{t-x}^{(1)} + \varepsilon_{xt}..........(M3)$$

The same constraints in M1 and M2 are imposed to M3.

Cairns-Blake-Dowd (2006) proposed a mortality forecasting model based on the simplicity of the mortality curve beyond a certain age. Excluding the mortality bumps in childhood and motor and maternal mortality bumps at young ages, mortality curves take an exponential form. The introduction of the *logit* leads approximately to a linear form. The CBD model proposes the following formula:

$$logit(q_{xt}) = ln(\frac{q_{xt}}{1-q_{xt}}) = \kappa_t^{(1)} + \kappa_t^{(2)} * (x - \bar{x}) + \varepsilon_{xt}...........(M5)$$

For each year t, M5 model can be treated as a linear equation having the form $Logit(q_{xt}) = a_t + b_t x$ with a_t as a starting value and b_t as a slope. The only difference is that in M5, the starting point is supposed to be the midpoint of the age axis $\bar{x}$. According to that, $\kappa_t^{(1)}$ can be an approximation to the mid-age corresponding $logit(q_{xt})$ value at time t in the mortality logit axis and $\kappa_t^{(2)}$ as a slope of the mortality Logit line.

M6 is the first generalized M5 with a cohort effect (Cairns et al, 2007):

$$ln(\frac{q_{xt}}{1-q_{xt}}) = \kappa_t^{(1)} + \kappa_t^{(2)} * (x - \bar{x}) + \gamma^{(1)}{}_{t-x} + \varepsilon_{xt}........(M6)$$

Others generalized CBD models were proposed. The most important versions are M7 including a quadratic age terms (M7):

$$ln(\frac{q_{xt}}{1-q_{xt}}) = \kappa_t^{(1)} + \kappa_t^{(2)} * (x - \bar{x}) + \kappa_t^{(3)} * ((x-\bar{x})^2 - \sigma_x^2) + \varepsilon_{xt}.........(M7)$$

and M7 including a quadratic age terms and a cohort effect (M7*)

$$ln(\frac{q_{xt}}{1-q_{xt}}) = \kappa_t^{(1)} + \kappa_t^{(2)} * (x - \bar{x}) + \kappa_t^{(3)} * ((x-\bar{x})^2 - \sigma_x^2) + \gamma_{t-x}^{(1)} + \varepsilon_{xt}........(M7*)$$

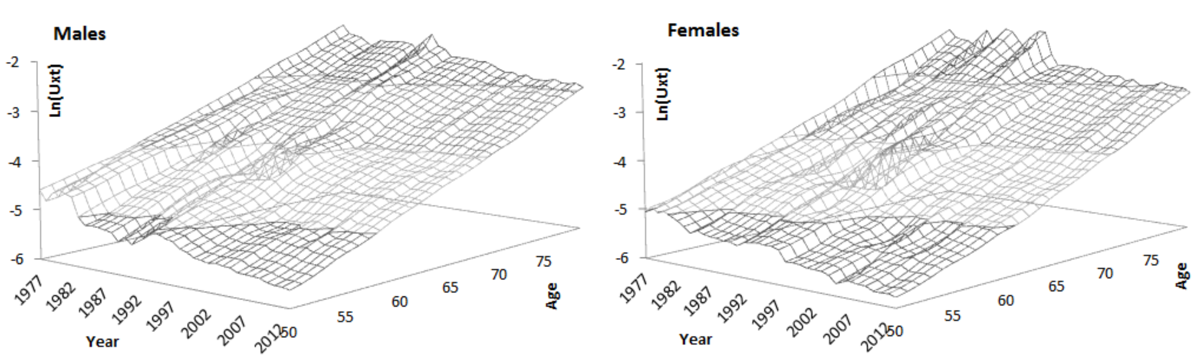
For M6 and M7*, a constraint is imposed to the cohort effect:

$$\sum_{t-x=t_1-x_n}^{t_p-x_0} \Upsilon_{t-x}^{(1)} = 0$$

5.3 Data

The first Algerian life-table based on the civil registration data has been constructed in 1977 by the Algerian Office for National Statistics (ONS). Since that, the ONS has tried to make available this information with an annual frequency, that came possible starting from 1998. So, for some calendar years, there are no published life tables : 1979, 1984, 1986, 1988, 1990, 1992 and 1997. Also, some life tables were closed-out before the age of 80. For the period 1983-1987, the closing age was [70 and +[. For the period [1993-1996], the published life-tables were closed-out at the age group [75 and +[. For the rest, it was [80 and +[or higher. In Chapter 3, we proposed to complete the missing data using a modified LC model with age-time segmentation. The completed mortality surface is shown in Figure 5.1.

Figure 5.1: Crude mortality surfaces (1977-2013)



The implementation of some mortality models, particularly those considering cohort effect, needs that death rates should be expressed in the same unit both for age and time. Since the length of the historical data is limited, the use of five-age description seems to be inappropriate. So, we need to make our data on a single ages formulation. To do this, we simply used the Karup king formula. In final, we have a single ages mortality surface for ages : 50 -79 and for the period 1977 - 2013.

For the present work, we use as a quantitative criterion to evaluate the goodness of fit of the different models the weighted Sum of Squared Errors (WSSE). The weight in a such case can be the population at risk or the observed number of deaths; but in most of cases, the second definition is adopted (Wimoth, 1993; Koissi and Shapiro, 2008). This information is unfortunately unavailable and unpublished in the annual publication of ONS as the age specific death rate (m_{xt}). What wehave do is simply the five age mortality rates (${}_5Q_x^t$,). Once the single ages mortality rates (q_{xt}) are interpolated, the central death rates (μ_{xt}) can be approximated by the following relationship : $\mu_{xt} = -ln(1-q_{xt})$. If we assume a piece-wise constant mortality function for each x and t, we can write : $\mu_{xt} = m_{xt}$. With a known age specific death rates, we will need to the detailed age population at risk to deduce the number of deaths at age x and year t that we will use as a weight to calibrate mortality models.

According to the available official statistics, the population at risk is not well known, or at least, it is not well adapted to the needs of the present work. We try to use the data provided by the different Algerian censuses to approximate the detailed age population numbers for the whole period [1977-2013]. Since the independence in 1962, the Algerian was subject to censuses in 5 times: 1966, 1977, 1987, 1998 and 2008. Census gave up the five age structure of the global population for males and females from the age 0 until the age of 70 or over.

Figure 5.2: Population Pyramid of Algeria according to censuses data

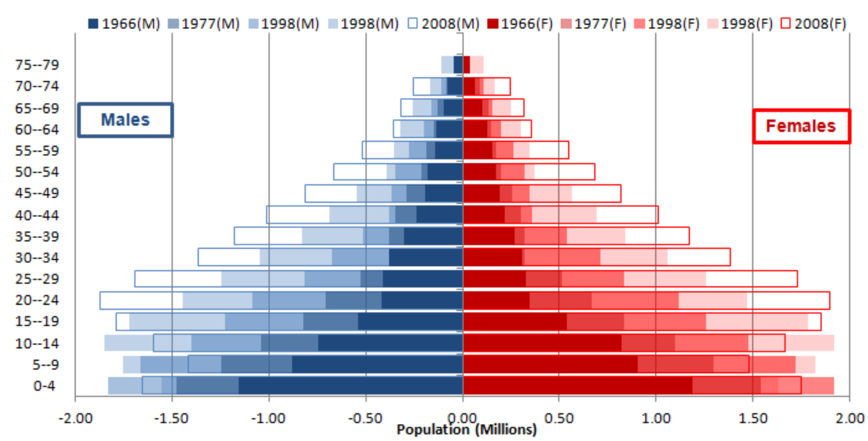


Figure 5.2 allows to visualize the population growth by five ages from a census to another. It is well known that the census gives a more complete data compared to the civil registration data. By using this information, we can interpolate the evolution of the population at each age group $[x, x+5[$ simply by using a linear interpolation between each two consecutive census points. For the period after 2008, which corresponds to the date of the last census, we

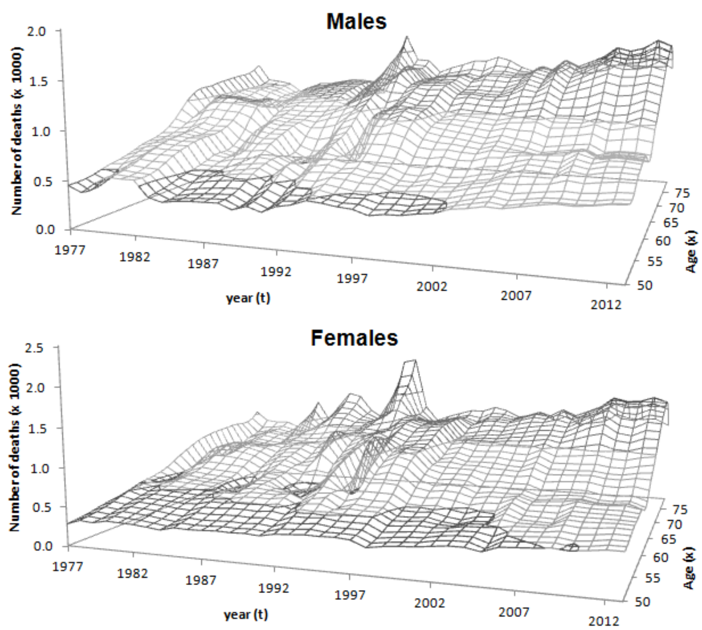
keep applying the growth rates observed between 1998 and 2008 to estimate the population numbers distribution by five age groups and sex.

After that, we achieve to estimate the single ages exposure to the death risk by using the karup-king formula. Since we are working on the retired population, the use of the age range [50-80[seems to be suitable. Once the detailed age population at risk is defined (L_{xt}), it becomes possible to estimate the deaths' distribution by using the simple formula:

$$D_{xt} = L_{xt} * m_{xt}$$

The obtained results are shown in Figure 5.3.

Figure 5.3: Detailed ages Deaths surfaces



5.4 Best model selection

Our objective in the present step is to evaluate and compare a set of mortality models (Lee-Carter, CBD) : M1, M2, M3, M5, M6, M7, M7* that we presented above. The goodness-of-fit is conventionally the most used criterion in order to compare mortality models. It ensures in most of cases a good forecasting capacity. But in other cases, this criterion, either if it gave a good index, the obtained projections are not necessarily good regarding the predictive capacity, especially if we look for more accuracy and we compare several models. This criterion is completed by a set of other quantitative and qualitative selection criteria: predictive capacity, expected life expectancy, coherent between males and females predicted mortality and the regularity of the projected mortality surfaces.

5.4.1 Goodness of fit

In Lee Carter(1992) , the parameter α_x is defined to be the mean over time of the ln of the central death rate (Lee and Carter, 1992) :$\alpha_x = ln(\prod_{t=T_1}^{T_n}(\mu_{xt}))^{\frac{1}{(Tn-T_1)}}$

Then, we decompose the residual matrix into two vectors: $ln(\mu_{xt}) - \alpha_x \approx \beta_x * \kappa_t$ with respecting the constraints $\sum_{x=X_1}^{X_n} \beta_x = 1$ and $\sum_{t=T_1}^{T_p} \kappa_t = 0$ to ensure the uniqueness of the solution.

To decompose this residual matrix, a two stages decomposition process was proposed. In the first stage we decompose the residual matrix by Singular Values Decomposition SVD techniques:

$$minS(1) = \sum_{x=0}^{n-1} \sum_{,t=1}^{p} [ln(\mu_{xt}) - \alpha_x\text{-}\beta_x * \kappa_t]^2$$

In the second estimation stage, β_x and κ_t are adjusted to fit the observed number of deaths at each year t.

$$minS(2) = \sum_{t=1}^{p} \sum_{x=0}^{n-1} [exp(\alpha_x + \beta_x * \kappa_t)L_{xt} - D_{xt}]$$

D_{xt}: observed number of deaths at age x and time t,

L_{xt}: the exposure to the death risque at age x and time t (population at risk).

Wilmoth (1993) proposed a one stage decomposition process based on the Weighed Least Squared Errors. The quantity to minimize becomes the Weighted Sum of Squared Errors (WSSE):

$$WSSE = \sum_{x=0}^{n-1} \sum_{,t=1}^{p} W_{xt}[ln(\mu_{xt}) - \alpha_x\text{-}\beta_x * \kappa_t]^2$$

The weight W_{xt} can be the observed by number of deaths at each point x and t noted D_{xt} (Wilmoth, 1993). Statistically, a weight is defined in order to lead to $\sum W_{xt} = 1$. For this, some authors proposed to divide the number of deaths observed at time t and age x by the total number of deaths occurred during the whole observation period and for all ages (Koissi and Shapiro, 2008). That matches with the definition of the Weighted Average of Squared Errors WASE:

$$WASE = \frac{1}{\sum D_{xt}} \sum_{x=0}^{n-1} \sum_{t=1}^{p} D_{xt}[ln(\mu_{xt}) - \alpha_x\text{-}\beta_x * \kappa_t]^2$$

Renshaw and Haberman (2006) used the original values of α_x as a starting values which was re-estimated by the same optimization process with all the parameters in RH model. We proceed similarly in calibrating M1, M2 and M3. Note that XL-Solver is used for all applications included in the present work.

Results

Figure 5.4: M1 - Parameters estimation (1977 -2013)

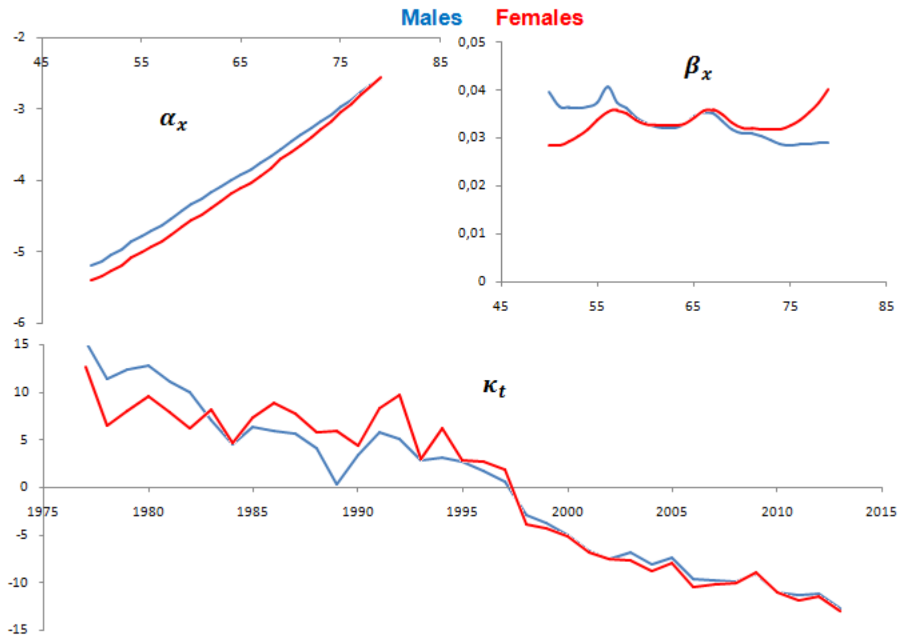
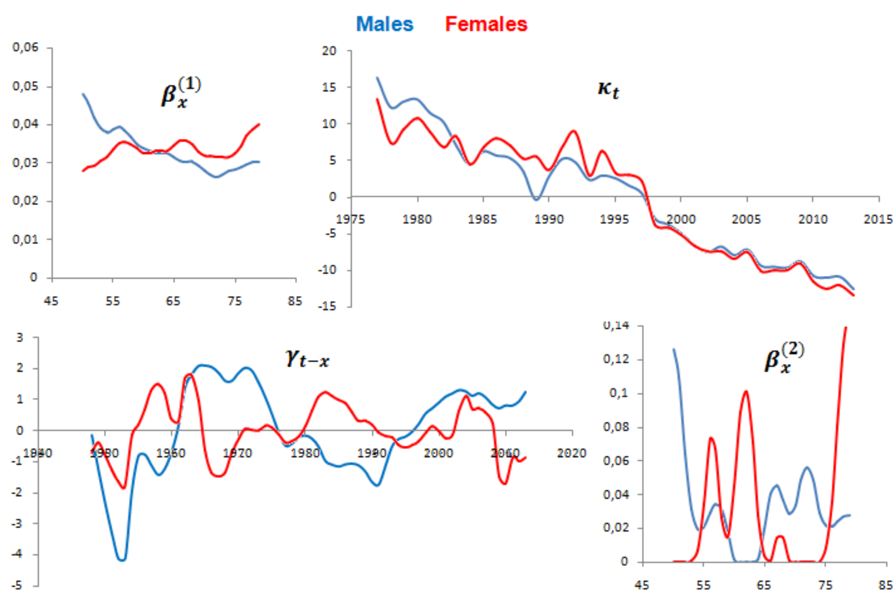
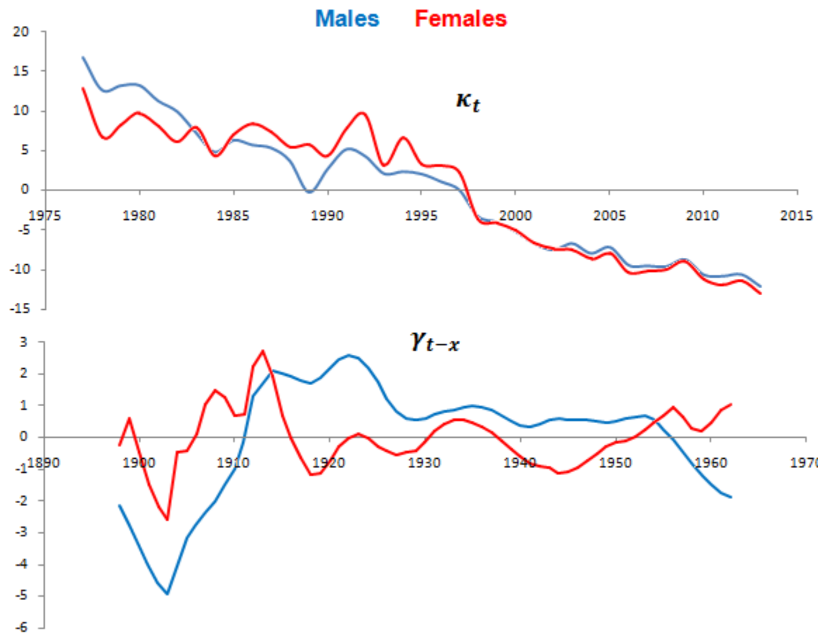


Figure 5.5: M2 - Parameters estimation (1977-2013)



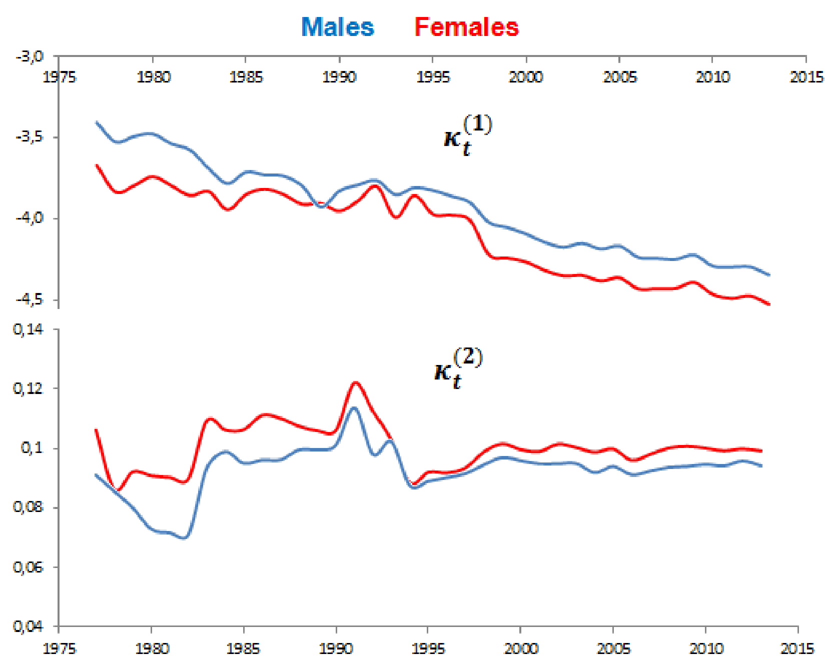
To estimate M2, we set as a starting values $\beta_x^{(1)}{=}\beta_x^{(2)}{=}\frac{1}{n}$. We do not introduce any starting values for time and cohort effect.

Figure 5.6: M3 - Parameters estimation (1977-2013)



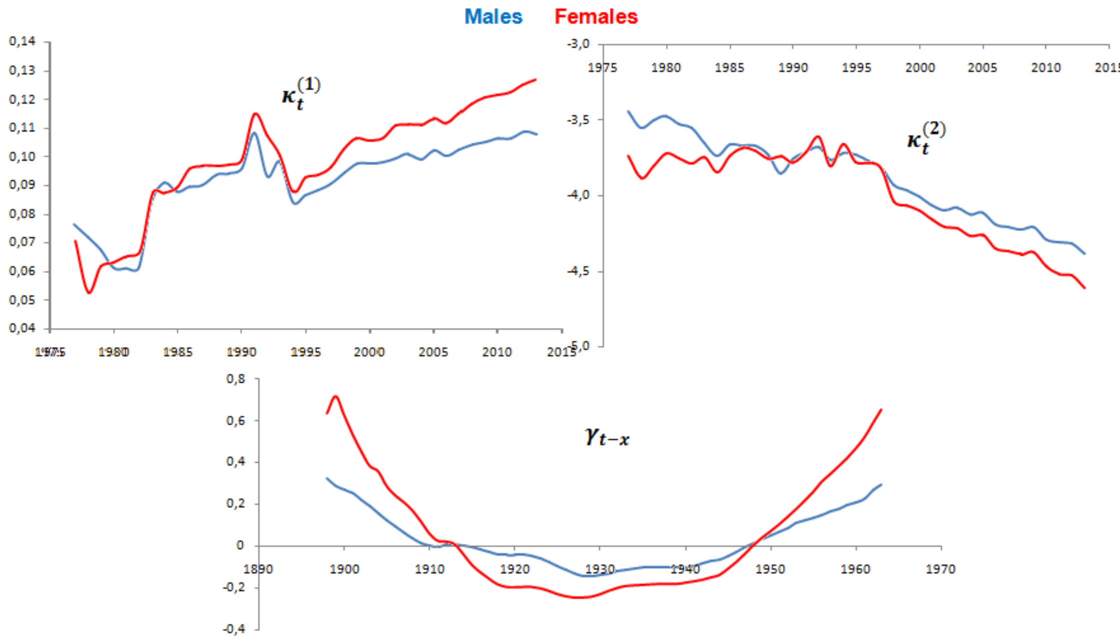
The transition from M2 to M3 was by introducing $\beta_x^{(1)} = \beta_x^{(2)} = \frac{1}{n}$. No starting values fore time and cohort components.

Figure 5.7: M5 - Parameters estimation (1977-2013)



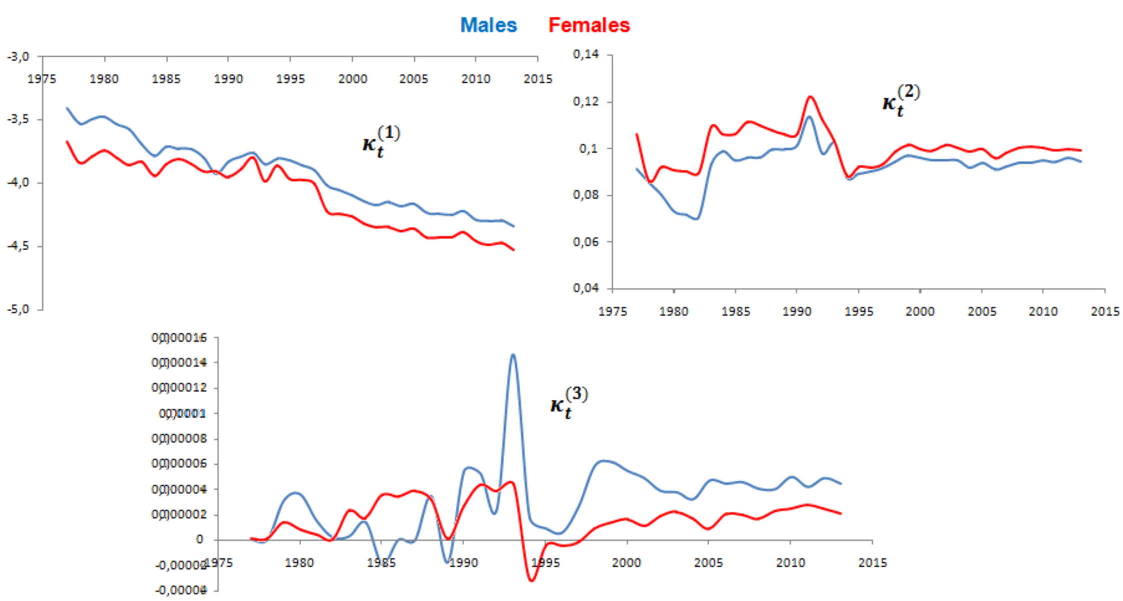
Starting values : $k_t^{(1)} = ln(\frac{q_{65,t}}{1-q_{65,t}})$. $k_t^{(2)} = \frac{Ln(\frac{q_{79,t}}{1-q_{79,t}})-Ln(\frac{q_{50,t}}{1-q_{50,t}})}{30}$.

Figure 5.8: M6 - Parameters estimation (1977-2013)



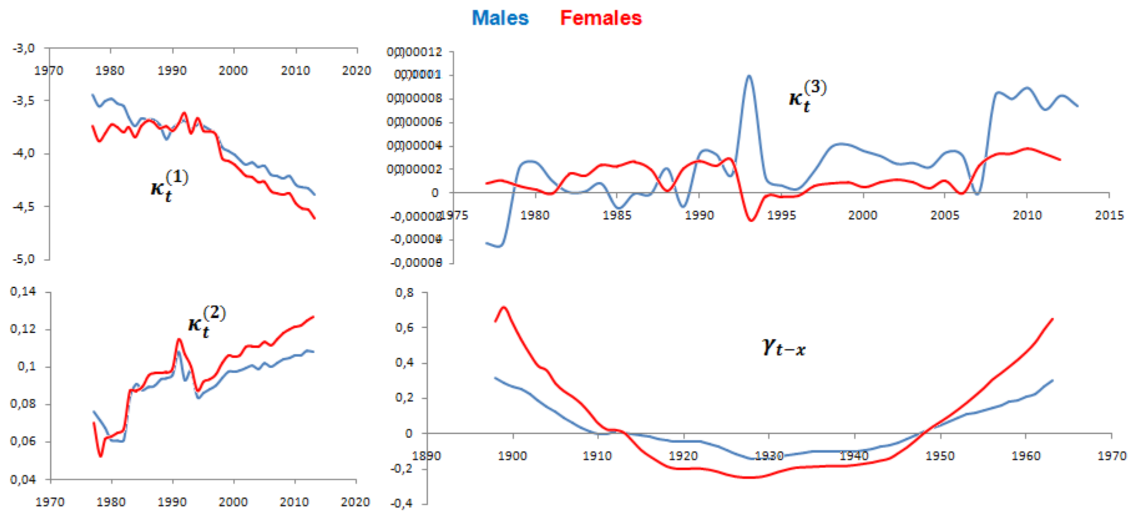
Starting Values : $k_t^{(1)}$and $k_t^{(2)}$ estimated in M5. No starting value for γ_{t-x}.

Figure 5.9: M7 - Parameters estimation (1977-2013)



Starting Values : $k_t^{(1)}$and $k_t^{(2)}$ estimated in M5. No starting value for $k_t^{(3)}$

Figure 5.10: M7* - Parameters estimation (1977 -2013)



Starting Values : $k_t^{(1)}$, $k_t^{(2)}$ $k_t^{(3)}$ estimated in M7 and γ_{t-x} estimated in M6.

Summary

The cohort effect in M6 and M7* has a strange shape showing a high level by the two extremes of the series and a relatively low level in the middle. This can not be only imputed to the real evolution of mortality by generations. This kind of problems can be found white CBD generalized models and it is usually explained by a kind of a mixture between cohort effect and residual errors which is well know as cosmetic effect. Mendes and Pochet (2012) for example concluded to a similar observation on the Belgian mortality data. The problem is about the no evidence to separate perfectly time and cohort effects. Many works showed a similar cohort effect shape obtained under the CBD generalized models (Cairns and al., 2007).

To evaluate the goodness-of-fit of the different models, we firstly use the Weighted Sum Squared Errors WSSE. In our case, the different models are not all based on the same specific age mortality indicators. The models M1-M3 are based on $ln(\mu_{xt})$ while M5-M7* are based on $Logit(q_{xt})$. The comparison between these models should be based on a common indicator and $ln(q_{xt})$ seems to be suitable for all models. This indicator is only used for comparison issues; each of the seven models is calibrated on its appropriate mortality indicator. The comparison criteria can be written as:

$$WSSE = \sum_{x=50}^{79} \sum_{t=1977}^{2013} W_{xt}.[ln(\hat{q}_{xt}) - ln(q_{xt})]^2$$

$\hat{q}_{xt}$ and q_{xt} are respectively the fitted and the observed age specific mortality rates for age x and time t and W_{xt} represents the weight.

The WSSE can perfectly be a good evaluation and calibration criterion but it is not suitable to compare models based on different number of parameters. The goodness-of-fit improves usually in function of the number of parameters of the underlying model. Since that, it is not very convenient to compare models including different number of parameters only on the basis of the WSSE. The comparison criterion should take into account the difference in the number of parameters between the compared models. The use of the Akaike Information Criterion AIC (Akaike, 1973) or the Bayesian Information Criterion BIC (Schwartz, 1978) allows to take into account the number of parameters and the size of the observation used for models calibration. Here, we use the formula which was adapted to suit the Least Squares estimation process (Burnham and Anderson, 1998; Hansen, 2007). AIC and BIC can be expressed in function of the errors variance σ_m^2, the number of the observations m which is equal to $n.p$ and the number of parameters included in the model noted k:

$$AIC = 2.k + m.ln(\sigma_m^2)$$

$$BIC = m.ln(\sigma_m^2) + k.ln(m)$$

The variance can be approximated by $\sigma_m^2 = \frac{SSE}{m}$ in the case of Least Squares estimations. Here, we remind that the principal used fitting criterion was the WSSE. In a such case, the weighted errors variance can be approximated to be the Weighted Average of Squared Errors WASE : $\sigma_m^2 = WASE$.

Table 5.1. summarizes the WSSE, AIC and BIC calculated under the seven models.

Table 5.1: Goodness of fit - Summary

	WSSE		BIC		AIC	
	Males	**Females**	**Males**	**Females**	**Males**	**Females**
M1	5353	6876	-5129,5	-4732,7	-5615,7	-5218,9
M2	*3840*	*5122*	-4825,0	-4386,4	-5792,3	-5353,7
M3	4460	6612	-5079,6	-4523,7	-5746,3	-5190,4
M5	*2926*	*7513*	*-5961,1*	*-4795,5*	-6332,0	-5166,4
M6	*1932*	*4248*	*-5959,1*	*-4965,7*	-6660,8	-5667,4
M7	2926	7511	-5701,7	-4536,4	-6258,1	-5092,8
M7*	1932	4221	*-5699,8*	*-4713,4*	*-6586,9*	*-5600,5*

M7 and M7* are respectively extensions of M5 and M6 which are supposed to improve the fitting quality of the original models. But, according to the results in Table 5.1, these models do not provide any advantage compared to M5 and M6. So, as a first conclusion from the models comparison, M7 and M7* are drooped out the set of the compared models. For the rest of the present work, we keep comparing between the 5 other models which can be ordered as following according to the goodness-of-fit : M6, M5, M1, M3 and M2. This ordering is similar for male and female data. A comparison based on other selection criteria is supposed to perform the evaluation and the comparison of the 5 models : Predictive capacity and the coherence between males and females in the projected mortality.

5.4.2 Short term predictive capacity

To evaluate the short term predictive capacity of the five models, we project mortality on the basis of the data of the period (1977-2010) to the period (2011-2013) and we compare the projected and the observed mortality rates.

M1:

To forecast the specific age death rates with M1, we need just to forecast the mortality trend index (k_t) to the future. To do this, several time series models can be used. Lee and Carter (1992) used a random walk with drift ARIMA(0,1,0):

$$k_t = k_{t-1} + d_1 + \delta_t$$

The drift d_1 represents the mean annual change in κ_t, and δ_t an error term. This model was also used in several works (Dowd et al., 2011; Zhou et al., 2013; Cairns et al., 2011).

In other works, a first order auto-regressive model AR(1) model for which a constant d_1 is added was used:

$$k_t = d_1 + d_2 k_{t-1} + \delta_t$$

The coefficient d_2 represents the slope of decreasing of k_t.

The use of such models is based on the stationarity over time of the k_t series. If this series is not stationary itself, it can be by differentiation. The first difference (or more) can be projected by using ARIMA (1,1, 0) model:

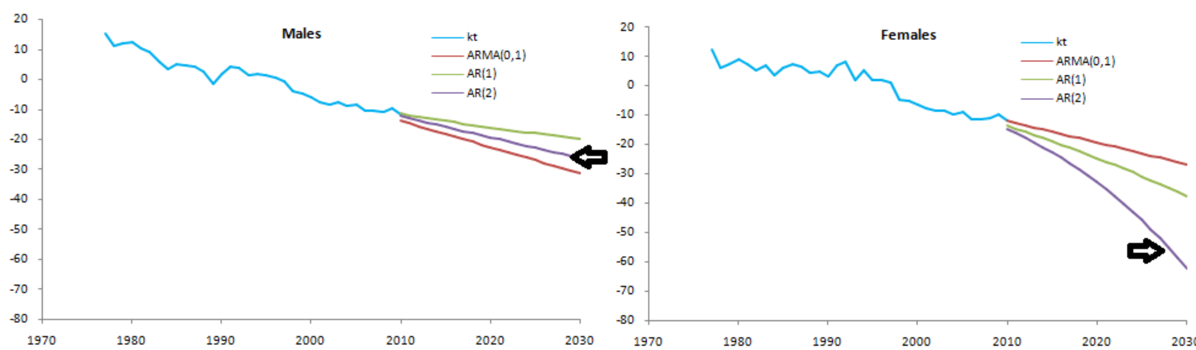
$$k_t - k_{t-1} = d_1 + d_2(k_{t-1} - k_{t-2}) + \delta_t$$

Yang and Wang (2013) used an ARIMA(2,1,0):

$$k_t - k_{t-1} = d_1 + d_2 k_{t-1} + d_3(k_{t-1} - k_{t-2}) + \delta_t$$

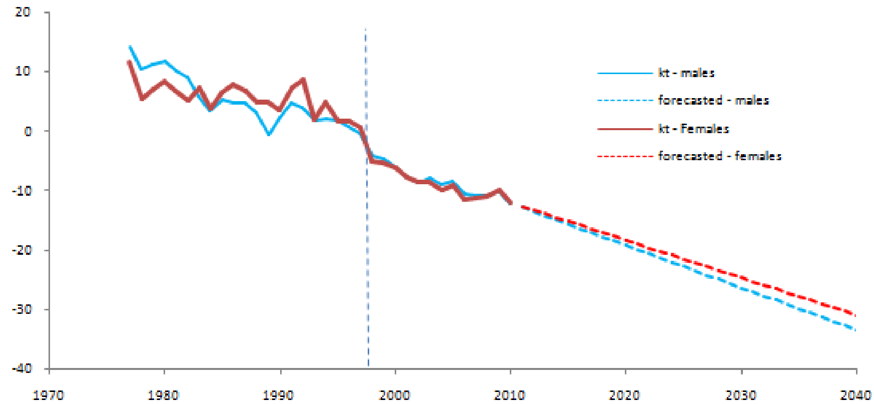
For our case, we compare three models : AR(1), AR(2) and ARIMA(0,1,0). After comparison, AR(2) seems to be the best model to forecast the time mortality index in M1.

Figure 5.11: κ_t forecasting - comparison of some time series models



As we see in Figure 5.11, the three models lead to incoherent forecasting if we consider the male/ female mortality trend evolution. This inconvenient is more apparent with the 2^{nd} order Auto Regressive model which is supposed to be the best model according to the results obtained on the historical trend. The deal can consist on the time range we have chosen for model calibration. As we signaled earlier, the mortality time index has been affected by the high mortality level due to terrorism events during 90's. The idea is to use only the recent period having a regular trend. When the period [1998-2010] is used, we get a good forecasting results with ARIMA(0,1,0) as shown in Figure 5.12.

Figure 5.12: κ_t forecasting based on the recent period data

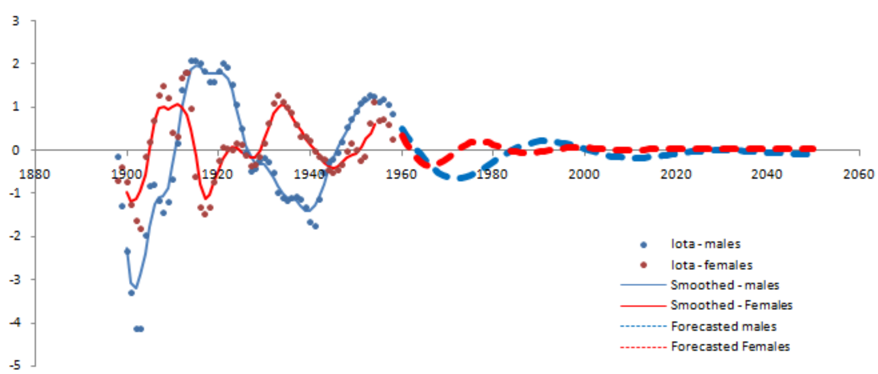


M2

To forecast mortality with M2, we need to forecast both the mortality time index (κ_t) and the cohort effect (γ_{t-x}). To forecast κ_t, we use the same process as with M1. The results are approximately same.

The cohort effect, as the mortality index, must be projected using the time series forecasting techniques with respecting some specificity. To ensure the homogeneity of the series γ_{t-x} and to reduce fluctuations which are due to reducing in the observation number by the two extremes of the series, the observation corresponding to the beginning and the end of the series are neglected (Cairns et al., 2008).

Figure 5.13: M2 : Cohort effect projection



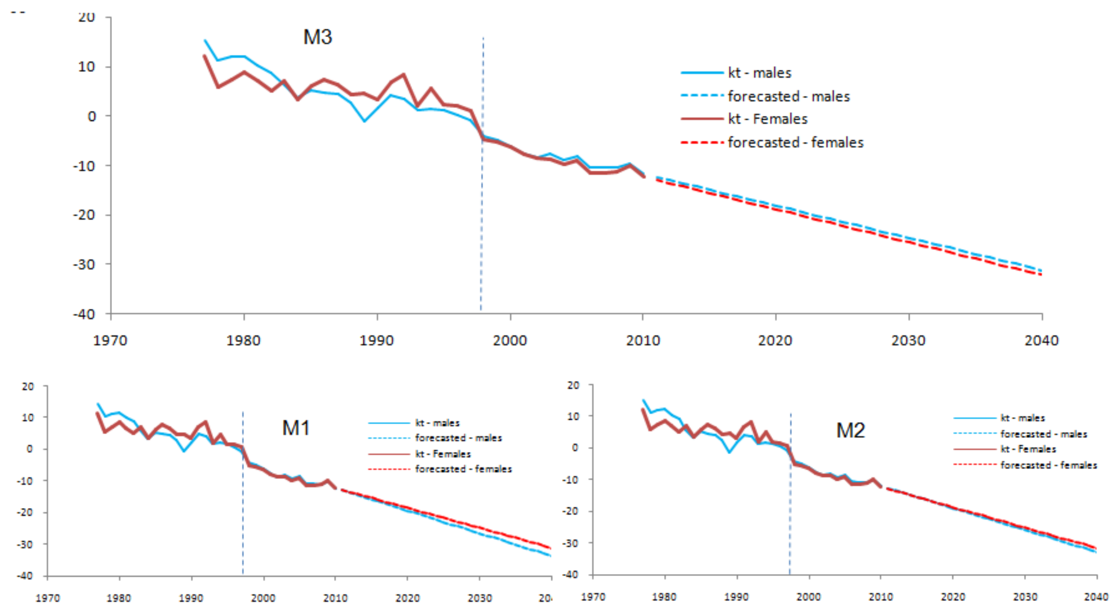
To forecast the cohort effect, Chan et al. (2014) used a first order Auto-regressive model AR(1). If we make $s = t - x$ as a cohort index, the cohort effect corresponding to the s^{th} cohort can be projected by: $\gamma_s = c_0 + c_1\gamma_{s-1} + \delta_s$. Dowd et al. (2011) used an ARIMA(1,1,0) : $\gamma_s - \gamma_{s-1} = c_0 + c_1(\gamma_{s-1} - \gamma_{s-2}) + \delta_s$. This leads to: $\gamma_s = c_0 + \gamma_{s-1} + c_1(\gamma_{s-1} - \gamma_{s-2}) + \delta_s$. Cairns et al. (2011) used an AR(2) model: $\gamma_s = c_0 + c_1\gamma_{s-1} + c_2\gamma_{s-2} + \delta_s$. Here, we compared these three models on the series of γ_{t-x} and it seems that AR(2) is the more suitable model to forecast the cohort effect. The obtained results are shown in Figure 5.13.

We note that just before to be projected, the observed γ_{t-x} has been smoothed with a 5^{th} order moving average to reduce irregularities. We observe that for the long term, the cohort effect tends to turn around zero. That means that the cohort effect does not have a significant impact on the projected mortality rates but has a significant effect on the goodness-of-fit.

M3

For M3 (Figure 5.14), the time index was projected by using an ARIMA(0,1,0) calibrated on the series of the period (1998 - 2010).

Figure 5.14: M3 : Time mortality index forecasting



In contrast to results obtained with M1 and M2, we observe with M3 that the projected male mortality index is somehow higher than the females one: $k_t^{Females} \leq k_t^{Males}$. That may lead to more coherent forecast regarding the sex differential mortality evolution.

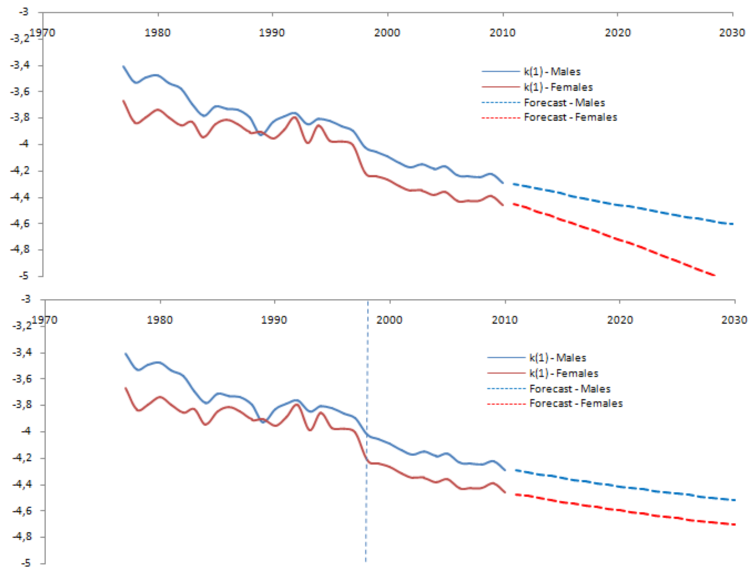
For the cohort effect, we use the same process as with M2.

M5

To extrapolate the age specific mortality rates with M5, $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ should be projected to the future. For $\kappa_t^{(1)}$, an AR(1) model fits well the two series (males and females) but when we do a comparison of males and females projected mortality rates, we observe that the forecast leads to a kind of divergence between the two sexes by the horizon of the projection. The female mortality tends to decrease faster than the male mortality and that may lead to an important gap in life expectancy between males and females. The idea is to use only the recent observed trend for forecasting. When we use the period [1998-2010] to calibrate the time series models, we get a better quality and a coherent differential mortality evolution. Again, the AR(1) model gives a good fitting quality as we can see in Figure 5.15.

A similar process is followed for $\kappa_t^{(2)}$. Results are presented in Figure 5.16.

Figure 5.15: M5 - $k^{(1)}$ projection



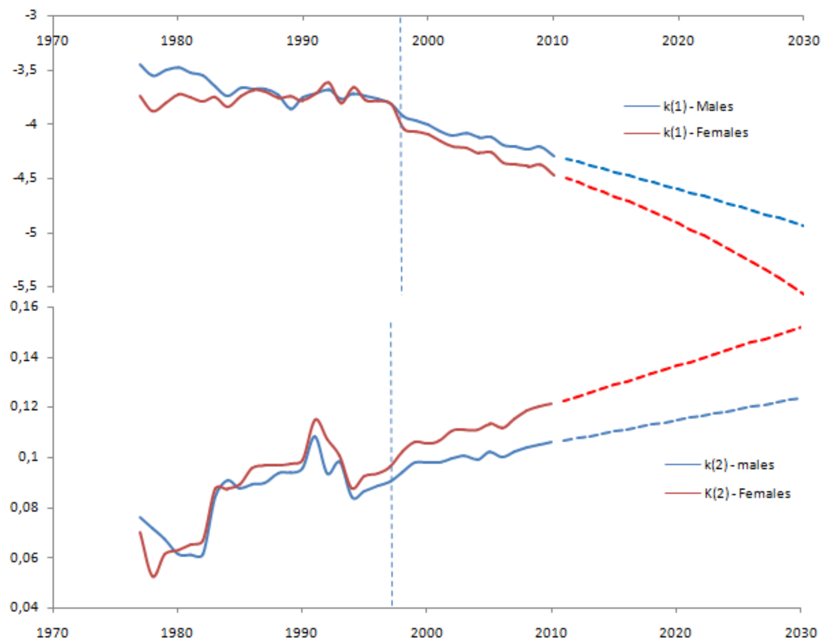
The figure up shows the obtained results when the whole historical data is used. In the figure down, we show the same results when only the recent trend is used (1998 - 2010).

Figure 5.16: M5 - $k^{(2)}$ projection

M6

M6 model is simply M5 model for which a cohort effect is added. So, the components $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ are projected here in the same way as in M5. By using the recent observed trends of these parameters, we noticed that the best model to forecast $\kappa_t^{(1)}$ is ARIMA(0,1,0). For $\kappa_t^{(2)}$, an AR(1) model is needed. The results are shown in figure 5.17.

Figure 5.17: M6 - $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ forecasting

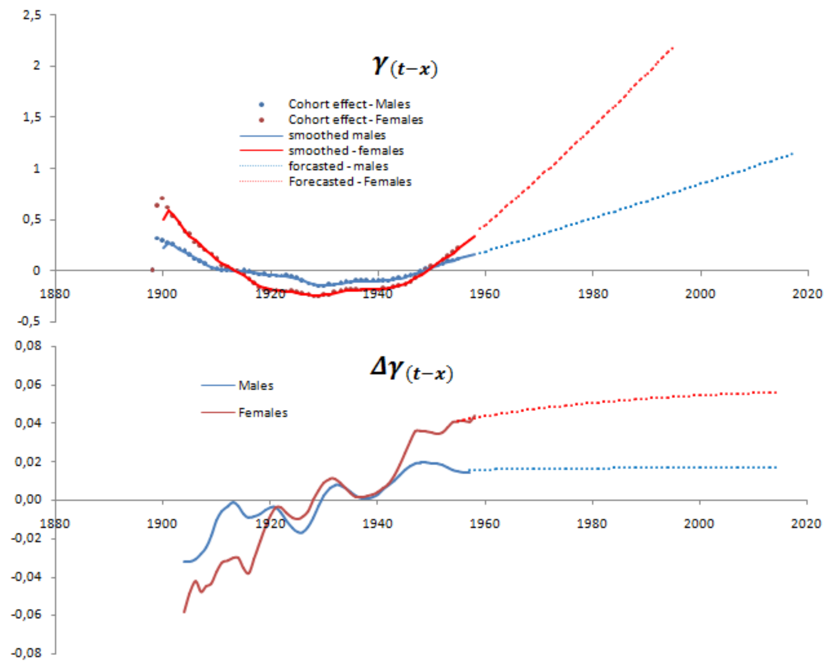


More difficulties were found with cohort effect forecasting. According to the general trend of this parameter, the forecasting can not be done by a simple time series model as is the case of the other time parameters. To do this, we compared a set of time series models which seems suit to the shape. In final, we achieved to use an ARIMA(1,2,0) model which gives good results according to the goodness-of-fit and the sex differential mortality evolution. In first, we estimated the difference function represented by :

$$\Delta_{\ t-x} = \gamma_{t-x} - \gamma_{(t-1)-(x-1)}$$

The new variable Δ_{t-x} is then projected with AR(2) model. The obtained results are shown in figure 5.18.

Figure 5.18: M6 - cohort effect forecasting



Comparison

To compare the five models according to the short term predictive capacity, we calculated the sum of the squared errors (SSE) between the projected and the observed logarithm of mortality rates tor each year $t = 2011, 2012, 2013$:

$$SSE_t = \sum_{x=50}^{79} [ln(\mu_{xt}) - ln(\hat{\mu}_{xt})]^2$$

Then, a sum of the three years is calculated. The five models are ranked according to this criterion as shown in Figure 5.19.

Figure 5.19: Comparison of the short term predictive capacity of the five models

	Males					Females				
	2011	2012	2013	Sum	rank	2011	2012	2013	Sum	rank
M1	0,048	0,094	0,054	0,196	4	0,101	0,069	0,077	0,247	4
M2	0,042	0,101	0,085	0,227	5	0,054	0,044	0,074	0,172	2
M3	0,014	0,063	0,014	0,092	2	0,012	0,050	0,033	0,095	1
M5	0,042	0,034	0,068	0,144	3	0,248	0,175	0,202	0,625	5
M6	0,022	0,041	0,022	0,085	1	0,050	0,030	0,109	0,189	3

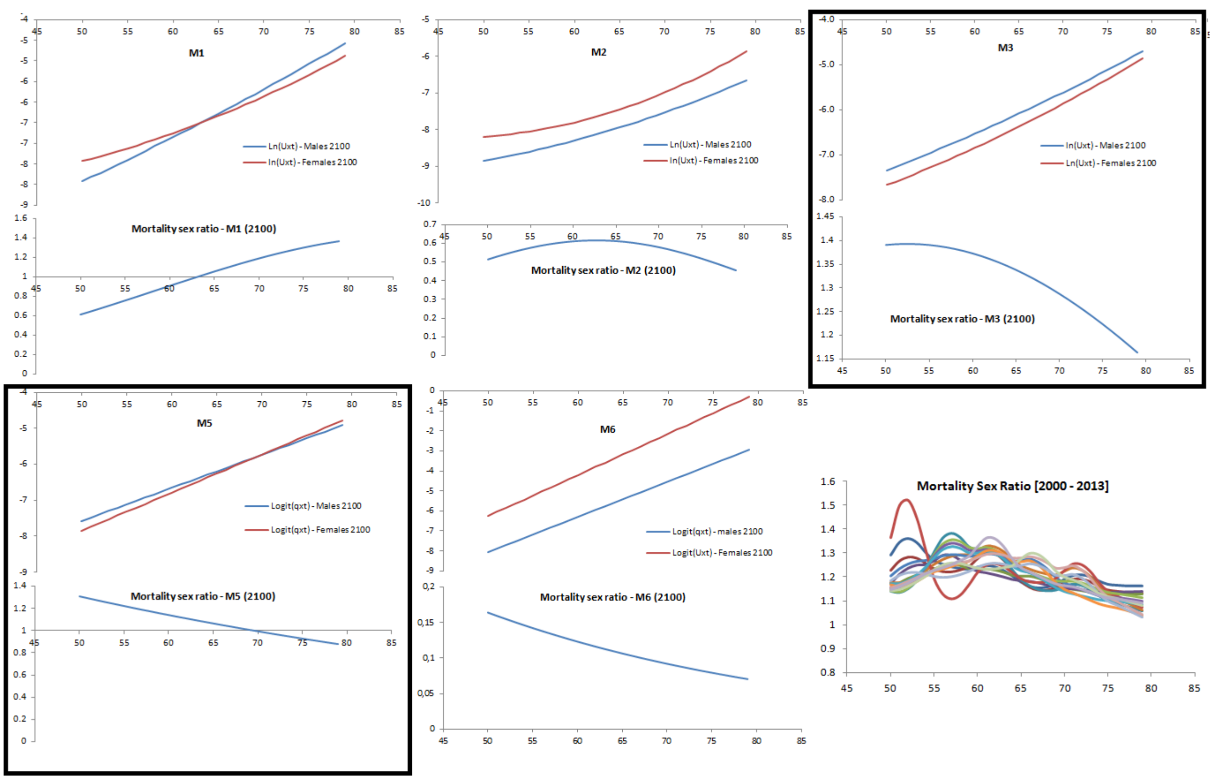
M3 is the best model regarding to the short term predictive capacity, followed by M6.

5.4.3 Sex differential forecasting coherence

We forecast mortality until 2100 and we observe the sex-differential mortality by the horizon of the forecast. To evaluate to forecast coherence, we calculate the mortality sex ratio obtained with each model ($SR_{xt} = \frac{q_{xt}^{Males}}{q_{xt}^{Females}}$) and we compare them to the observed trend of the sex mortality ratio during the period [2000-2013]. The choice of the period [2000-2013] as a basis of the comparison is explained by the relative stability that had known the sex ratio pattern compared to the previous period. Figure 5.20 shows the results obtained with the five models.

As we can see in Figure 5.20, for each model (M1, M2, M3, M5, M6), we have shown the mortality curves by sex corresponding to the year 2100. For each model, we calculate the Mortality Sex ratio. The figure bellow-right shows the observed mortality sex ratio during the period [2000-2013]. This parameter shows a continual decrease between the age of 50 until 79 passing from around 1.25 to nearly 1. This age structure of the mortality sex ratio is supposed to keep fixed by the horizon of the forecast. A coherent mortality forecasting model is supposed to lead to a similar results when male and female mortality are compared. According to this, M3 is the only model providing a coherent forecast. The model M5 led to a less coherent results reflecting a continual increase with age but showing a female mortality excess beyond the age of 70. The other models M1, M2, M6 led to completely incoherent results. M1 leads to a crossover of the male and the female mortality curves. M2 and M6 to a fully female mortality excess.

Figure 5.20: Expected Mortality sex ratio (2100)



According to these results, we keep comparing two models M3 and M5 on the basis of the regularity of the extrapolated mortality surfaces and also the expected evolution of the differential life expectancy between males and females.

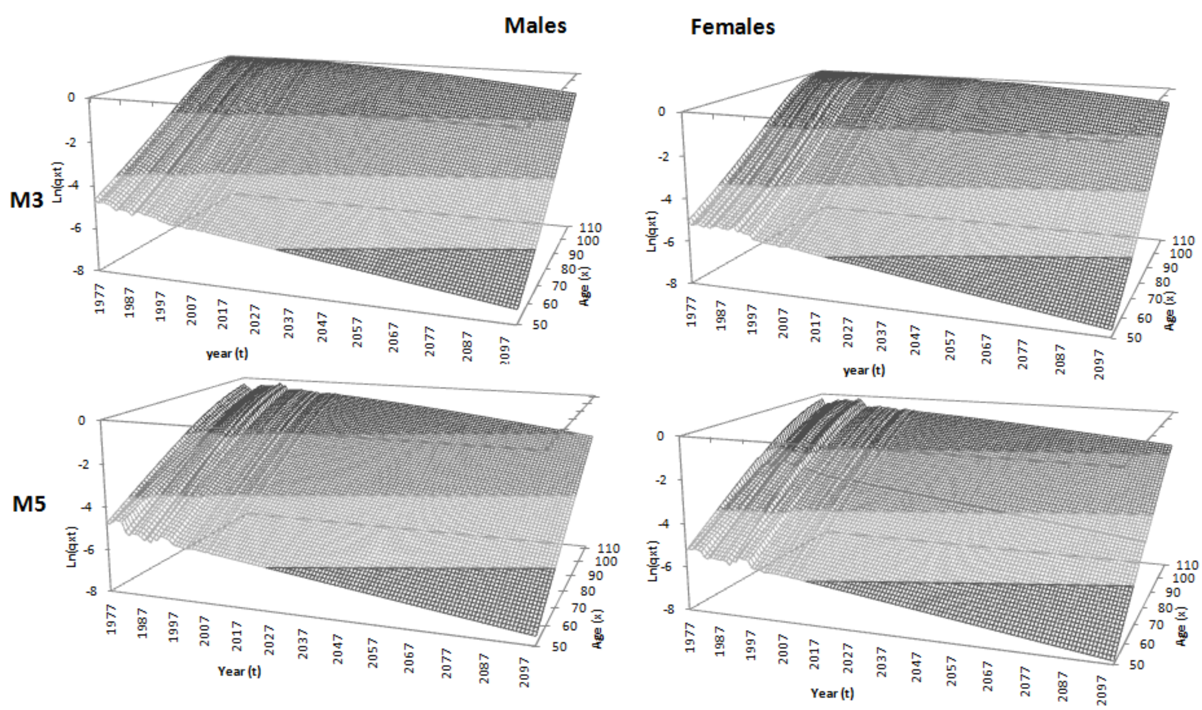
5.5 Construction of dynamic life-tables

In this part, we will construct dynamic life tables with M3, M5. The elements which we have until now allow to project the death rates for ages from 50 to 79. For actuarial uses, life tables must be extended until a high age which is supposed to be nearly the surviving age limit. The projected life tables are closed-out with Denuit et Goderniaux Model (2005):

$$ln(q_x) = a + bx + cx^2 + \xi_x$$

In the original article, authors have set an age limit constraint by imposing the death probability to be equal to 1 by the age of 130: $q_{130} = 1$ and $P_{130} = 0$. Here, we impose a closure constraints at 120 years ($q_{120} = 1$) as it was suggested by Flici et Hammouda (2016). Figure 5.21 shows the projected mortality surfaces extended until the age of 110 and until the year 2100.

Figure 5.21: Projected mortality surfaces : M3 VS M5

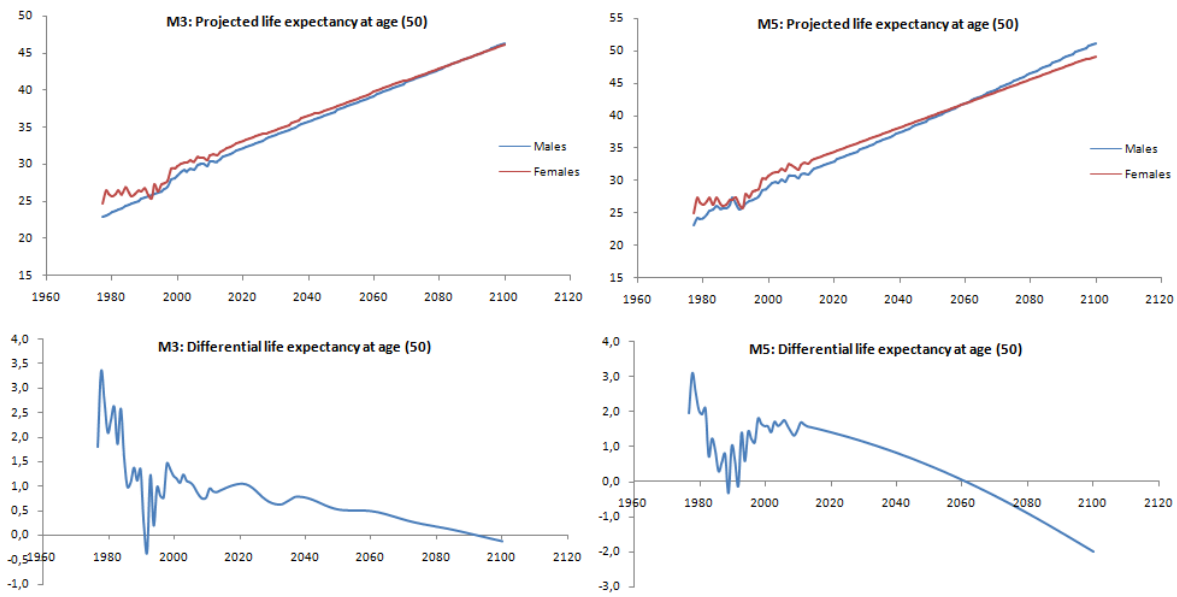


Projected mortality surfaces for males (Left) and Females (Right) obtained by M3 (High) and M5 (Down)

Residual life expectancy: M3 VS M5

A good mortality model may lead to some coherence when male and female expected life expectancy are compared. To evaluate a such coherence, we study the evolution of the life expectancy at 50 for males and females. As demonstrated in other works, the differential life expectancy might decrease when life expectancy is improving (Vallin et Meslé, 1989). Regarding to this logic, M3 leads to a more coherent forecast compared to M5. Figure 5.22 shows a comparison between the results obtained with the two models.

Figure 5.22: Projected life expectancy at age 50 : M3 VS M5



According to M3 model forecast, the gap in life expectancy at 50 years is supposed to decrease from 1 year to 0 between 2015 and 2090. With M5, the males and female life expectancy are supposed to become equal by only 2060.

5.6 Conclusion

Constructing dynamic life tables has a great importance since life expectancy is still improving. Starting from the original contribution of Lee and Carter (1992) who proposed M1 model, many other models have been developed to forecast specific age mortality rates to the future. M2 and M3 were based on an extensions of M1. After, Cairns, Blake and Dowd proposed a new approach for mortality forecasting known under M5 model, which has been modified later to provide a set of generalized models (M5-M7*).

The plurality of the mortality forecasting models allowed performing fitting historical mortality surfaces. We have shown along the present work a comparison of seven mortality models on the Algerian mortality data : M1, M2, M3, M5, M6, M7 and M7*. Since the different models are based on a different numbers of parameters, we saw that it is more suitable to use BIC and AIC rather than WSSE to evaluate the goodness-of-fit. After comparison, we achieved to observe that M7 and M7*, which represent respectively a modified M5 and M6, do not provide any improved fitting quality compared to the original models. For that, these two models were excluded from the models comparison.

When we forecast mortality, we care more about the quality of the forecast than the quality of the fitting. In most of cases, the two elements are somehow correlated but the goodness-of-fit does not imply necessarily a good forecasting quality. For this, other evaluation criteria must be included in the evaluation process : The predictive capacity, the male female coherence and the regularity of the projections. To evaluate the predictive capacity of the five models, we used the data of [1977-2010] to calibrate models and data of the period [2011-2013] to evaluate the predictive capacity. We know that the length of the comparison period is not sufficient to assess the goodness of the long run projection, but that allows to have a summary idea about the launch of the projection trend. We notice that the historical mortality series in Algeria was greatly affected by the events of terrorism during 90's. That made difficult to forecast the time mortality indexes with all the mortality models. The use of the data of the whole period [1977-2013] leads to a kind of incoherence when we compare the male and female projected trends. In our work, without proceeding to a coherent forecast as it was proposed by Li and Lee (2005) or by Hyndman and al. (2013), we have tried to avoid any unrealistic convergence or divergence between the male and female projections. It was evident that the only way to get a coherent projection for males and females is to use the period [1998-2013] to calibrate the time series models. This finding was commonly observed for the five mortality models. The evaluation of the predictive capacity showed an acceptable level for all the five models. We know that the use of only 15 years as a basis of the forecasting is no sufficient to assess the robustness of the results but also the use of a longer data series is not supposed to improve the quality of the forecast.

In addition, the expected mortality sex ratio at the horizon of the projection was used to evaluate the long term coherence of the forecast. Compared to the observed schemes in the mortality sex ratio by age during the period [2000 -2013], only the model M3 gave a coherent forecast. M5 led to a relatively less coherent results. The nonperformance of M1 and M2 compared to M3 might be related to the imposed constraint on the age sensitivity to the mortality time variation which is supposed to be equal for all ages in M3. A difference between male and female in terms of this sensitivity can lead to an incoherence in long term projections. The problem with the CBD models (M5 and M6), is that including the cohort effect into account has to reduce the forecasting capacity. For this, M5 shows more better results than M6 according to the male female coherence. The observed shape of the cohort effect in M6 can not be only imputed to a cohort effect but to a mixture between this last with the residual errors. In addition, is was evident to find a way to separate perfectly the cohort effect from the time effect.

In final, we would conclude that the present work allowed to evaluate the performance of the most commonly used mortality forecasting models and to know the adequacy of each model to the Algerian mortality data. According to the obtained results, M3 can be qualified to be the best model to forecast mortality in Algeria under the conditions and the criteria presented above. However, we gave some tracks to improve the quality of the other models principally in the coherence part. The use of common age sensitivity factor for males and females in M1 and M3 should improve the mortality forecast coherence between males and females and to avoid to obtain incoherent results when we use the whole historical data series as a basis for the forecast. The use of the approaches proposed by Li and Lee (2005) and Hyndman and al. (2013) can be also used for the same issue.

Chapter 6

Construction of a prospective life table based on the Algerian retired population mortality experience

Joint work with Frédéric Planchet, Full Professor, ISFA Lyon (France). This work has been presented in the conference "Statistical and Mathematical Tools for Actuarial Sciences and Finance". MAF2016. University Paris-Dauphine, Paris, April 1 st, 2016.

6.1 Introduction

Life expectancy is still improving in developing countries. This improvement is almost different by sub-populations. Mortality of the retired population is often lower compared to the global population. The use of dynamic life-tables based on the global population data might distort all calculations when used for pension plan reserving. The use of life tables adapted to the retired population mortality experience is more suitable for this issue. Usually, the data of the insured population is not available for a long period allowing to do a robust forecast. Also, this data is issue from reduced samples of population compared to the global population which leads to important irregularities related to the reduced population at risk. In such a case, the direct use of the prospective mortality models such Lee-Carter (Lee and Carter, 1992) or Cairns-Blake-Dowd models (Cairns et al., 2006) to predict the future mortality trend is not practical at all. For that, some methods were proposed to consider the particularities of the insured population mortality while ensuring a good fitting quality and a strong forecasting capacity. These methods aim to position the experience life tables to an external reference (Planchet, 2005; 2006). The main idea was to define a regression relationship between the experience death rates and the reference death rates. This process is principally based on the Brass Logit system (Brass, 1971). The use of the reference life table to estimate mortality schemes starting from incomplete or imperfect mortality data has become a common practice for experience life-tables construction both in developed and developing countries. Kamega (2011) used the same approach to estimate actuarial life tables for some sub-Saharan African countries with taking the French life tables as an external reference (TGH05 and TGF05). The main objective for the present work is to construct a prospective life table based on the mortality data of the Algerian retired population. The data is available for ten years (2004 to 2013) and for the ages [45 - 95[arranged in five-age intervals. This data concerns the observed number of deaths and the numbers of survivals by the end of each year of the observation period. In Chapter 5, we constructed a prospective life-table based on the global population mortality data. The length of the observed data allows doing a strong forecast. However, the forecasting results lack some coherence regarding the male-female coherence. For this, we suggested the use of coherent mortality models. In the present work, the global population mortality will be used as a reference to position and forecast the mortality experienced by the retired population.

6.2 Short presentation of the principal formulas in the Algerian pension system

The Algerian public pension system for the salaried population is managed by the National Pension Fund (NPF) [1]. The pension benefits guaranteed by this fund can be divided into two main categories: Direct Pension Benefits (DPB) and Survivors Pension Benefits (SPB).

The direct pensions feature 5 retirement formulas : Normal Retirement (NR), Early Retirement (ER), Proportional Retirement (PR), Retirement without Age Condition (RAC) and Retirement Allowance (RA).

The classical retirement formula is the Normal Retirement (La pension de retraite). The regulatory age for retirement is 60 for salaried. 32 years of contribution allow to benefit a replacement rate of 80% (full retirement) of the late career wage. Some categories can benefit a retirement age bonus. Salaried women can get retired by the age of 55. Women have 5-years bonus than men regarding the age of retirement. Also, for each birth, they get 1 year more until 3 in maximum. Other categories can also benefit some bonus : Moudjahidhine[2], workers whose have been totally and permanently disabled in case if they can not benefit a disability pension and having accumulated at least 15 years of contribution. Also, employees having been worked in some particular nuisance conditions can, as the cases cited above, get retired at the age of 55.

Retirement Allowance: Retirement allowance is intended to the same categories like in the case of Retirement Pensions but only when some conditions are not satisfied. However, 5 years of contribution are required. Starting from 1999, the legal age for this retirement category has passed from 65 to 60 years old. The calculations are same like in Retirement Pension.

The Early Retirement, the proportional retirement and retirement without age condition were introduced during 1990's.

Early Retirement: To benefit this formula, some conditions are required as being aged 50 and over (45 for women), having at least 20 years (15 for women) of contribution and having been worked for the same employer for 3 years without interruption during the 10 last years. In addition, the employer must pay the contributions related to the anticipation period.

Retirement without age condition: the only condition is having accumulated 32 years of contribution.

Proportional Retirement: two principal conditions are required, being 50 years and having accumulated 20 years of contributions (respectively 45 and 15 for women).

[1] Caisse Nationale des Retraites : www.CNR-dz.com

[2] Moudjahidine are the former combatants of the Algerian liberation war (1954-1962)

6.3 Data

The available data are from the retired population of "Caisse Nationale des Retraites - CNR". For $t = [2004, 2013]$, and x from 45 years to the open age group [95 and +[, by five-ages, we have the observed number of deaths ($D^t_{x,x+5}$) and the survivals by the end of each year ($l_{x,t}$). This data is arranged by sex but the numbers of males are greatly more important than those of females. We will give further details on this point bellow. This data concerns 5 different categories of direct retirement: Normal Retirement (NR), Retirement Allowance (RA), Early Retirement (ER), Proportional Retirement (PR) and Retirement without Age Condition (RAC). There is a sixth direct retirement formula (complementary retirement pension CRP) for which data is not much detailed.

The structure of the data depends on the nature of the retirement formula. For NR, the data are available starting from the age of 50/55. For RA it is 60 years, and for the other formulas, the pension/ allowance starts to be served around the age of 40 and 55 years. Generally, women get retired 5-10 years before men in average. For this, the female data is relatively much available at the lower ages compared with male data. To insure data homogeneity, we prefer to start from the age of 50 for the both sexes.

Table 6.1 presents the structure of the insured population distributed by age, sex and retirement formula during the whole considered period (2004 - 2013).

Table 6.1: Structure of the global retired population by sex and retirement formula (global : 2004 - 2013)

	NR			RA			ER			PR			RAC			
Age	Males	Females	Part. Sum	Males	Females	Part. Sum	Males	Females	Part. Sum	Males	Females	Part. Sum	Males	Females	Part. Sum	
40-44	0	0	0	0	0	0	0	0	0	0	18	18	219	50	269	
45-49	0	0	0	0	0	0	0	157	157	25 756	75 354	101 110	26 349	4 017	30 366	
50-54	84	14 180	14 264	0	0	0	3 453	1 177	4 630	602 621	114 875	717 496	189 338	23 583	212 921	
55-59	16 602	91 999	108 601	0	0	0	25 220	72	25 292	934 391	78 140	1 012 531	463 710	28 680	492 390	
60-64	1 071 454	191 028	1 262 482	241 330	28 199	269 529	588	4	592	607 297	31 785	639 082	397 623	17 659	415 282	
65-69	1 261 454	163 400	1 424 854	234 053	31 582	265 635	48	0	48	204 668	5 932	210 600	199 825	7 880	207 705	
70-74	1 202 953	133 044	1 335 997	169 740	27 891	197 631	22	0	22	36 254	1 079	37 333	55 260	2 393	57 653	
75-79	895 709	96 521	992 230	103 811	22 207	126 018	0	0	0	5	1	6	10	2	12	
80-84	541 684	60 723	602 407	60 303	17 871	78 174	0	0	0	2	0	2	4	0	4	
85-89	253 926	30 702	284 628	34 669	10 704	45 373	0	0	0	2	0	2	3	0	3	
90-94	106 789	16 336	123 125	16 436	5 314	21 750	0	0	0	5	0	5	4	0	4	
95 & +	73 028	15 951	88 979	14 539	3 605	18 144	0	0	0	12	0	12	11	2	13	Global sum
Sum	5 423 683	813 884	6 237 567	874 881	147 373	1 022 254	29 331	1 410	30 741	2 411 013	307 184	2 718 197	1 332 356	84 266	1 416 622	11 425 381

We have in all more than 11 millions person-years along the period 2004 - 2013, which represents an average of 1,4 million per year. The distribution of the corresponding numbers of deaths are presented in Table 6.2.

Table 6.2: Structure of global deaths by sex and retirement formula (global : 2004 - 2013)

	NR			RA			ER			PR			RAC			
Age	Males	Females	Part. Sum	Males	Females	Part. Sum	Males	Females	Part. Sum	Males	Females	Part. Sum	Males	Females	Part. Sum	
40-44	0	0	0	0	0	0	0	0	0	0	0	0	2	0	2	
45-49	0	0	0	0	0	0	0	0	0	1	110	111	66	11	77	
50-54	1	12	13	0	0	0	44	0	44	2 520	320	2 840	814	70	884	
55-59	167	414	581	0	0	0	364	0	364	6 388	289	6 677	2 894	111	3 005	
60-64	12 343	1 265	13 608	2 654	185	2 839	23	0	23	6 042	115	6 157	4 016	97	4 113	
65-69	26 022	1 724	27 746	4 104	303	4 407	0	0	0	2 789	3	2 792	2 914	34	2 948	
70-74	37 541	2 477	40 018	4 873	517	5 390	0	0	0	448	1	449	997	0	997	
75-79	43 634	2 932	46 566	4 708	577	5 285	0	0	0	0	0	0	0	0	0	
80-84	40 211	2 769	42 980	4 637	741	5 378	0	0	0	0	0	0	0	0	0	
85-89	29 559	1 973	31 532	3 447	620	4 067	0	0	0	0	0	0	0	0	0	
90-94	15 233	1 216	16 449	1 914	314	2 228	0	0	0	1	0	1	0	0	0	
95 & +	5 798	673	6 471	834	148	982	0	0	0	0	0	0	0	0	0	Global sum
	210 509	15 455	225 964	27 171	3 405	30 576	431	0	431	18 189	838	19 027	11 703	323	12 026	288 024

For deaths, we observe in Figure 6.2 that the column corresponding for females in PR is empty. It means that any death has been recorded during the observed period among the exposed population (1410) aged 45 - 65. There is two possible explanations for this. In first, it can be explained by the weakness of the observed population, or by a misreporting of deaths due to a confusion of deaths between the different categories.

6.4 Estimation of the Age Specific Death Rates m_{xt}

The objective of the present part is to estimate the mortality surface for males and females on the basis of the available data. Since that the detailed age data is not available, we will estimate the mortality indicators following a five-ages description. The first indicator that we can estimate in a such case, is the observed death rate for the age groups $(x, x+5)$ during the year t noted $M^t_{x,x+5,}$ which represents the number of death observed at age group $(x, x+5)$ and year t $(D^t_{x,x+5})$ divided by population exposed to the death risk during the observation year $L^t_{x,x+5}$. The calculation formula can be written as: $M^t_{x,x+5} = \frac{D^t_{x,x+5}}{L_{x,x+5}}$.

The estimation of the risk exposure can not be currently estimated in the current case. Data is not individualized but aggregated by five-age groups and we can not estimate the current exposure period during a year for each individual.

Until now, the data is arranged by five-ages and by single years in the time dimension.

6.4.1 Estimation of the exposure to the death risk

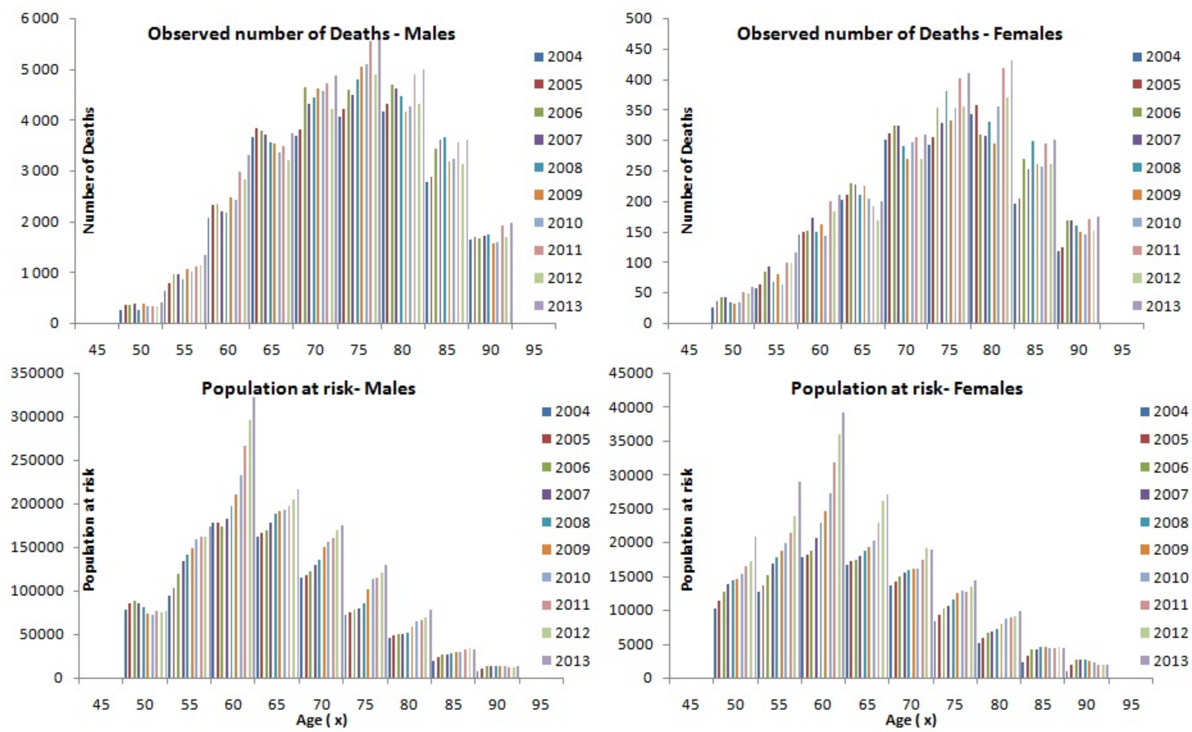
In the absence of a detailed information about the different dates of events occurred in the sample of the retired population: date of entrance/exit in the sample, date of birth/death, death rates can not estimated accurately. The first information allows to estimate the current duration where the individual has been exposed to the death risk and the second allows to estimate the exact age at the different events. The use of aggregated data leads to less accurate results. The estimation of the exposure to the death risk with grouped data must be done by imposing some simplest assumptions.

Since we do not have any information about the flow of individuals into/from the considered portfolio, we prefer to estimate the exposure to the death risk $L^t_{x,x+5}$ simply by the average population by the mid-year t which may be approximated by the average between the surviving population at the beginning and the end of the year by using the following formula :

$$L^t_{x,x+5} = \frac{l^{t-1}_{x,x+5} + l^t_{x,x+5}}{2}$$

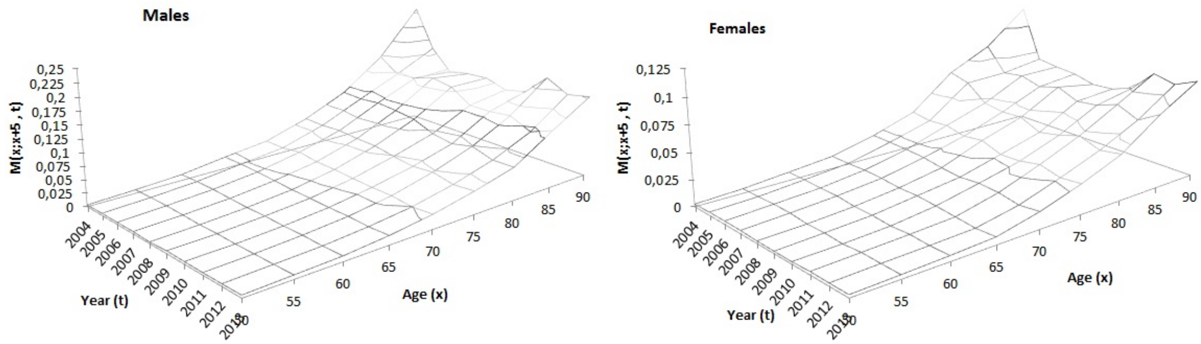
We remind that the available information is the deaths numbers for age $x \in [45-95[$ and time $t = 2004, 2005, ...2013$ and also the survival numbers by age and sex at the end of each year of the considered period. To estimate the population at risk by five age groups in 2004, the population number at the beginning of the year was needed. For simplification issues, we deduced it from the population number at the end of the year by assuming a similarity of the reports of population numbers at the end and the beginning of the year 2005. The observed number of deaths and the population at risk for each age group $[x, x+5[$ and year t are represented in Figure 6.1.

Figure 6.1: Deaths and population at risk by age



The number of death has grown up following a linear trend from 45 to 80 years for men and from 45 to 75 for women. Then it decreases slightly beyond this age. In concern of the distribution of the number of retirees by age, we observe that the age category [60-65[comprises the most important number of retirees men and women. The obtained mortality surface is given by Figure 6.2.

Figure 6.2: Mortality surfaces ($M^t_{x,x+5}$)



We observe that the obtained mortality surfaces show a relative stability until the age 80-85 years. Beyond this age, some apparent irregularities can be observed because of the reduction of the population exposed to death risk.

6.4.2 Detailed ages mortality surfaces

The next step is to construct a detailed age life table. The idea is to suppose the death rate corresponding to the central age of the age group [x, x+5[noted $m_{x+2,5,t}$ to be equal to the observed death rate corresponding to the same five age group that we previously noted $M^t_{x,x+5}$. We can write : $m_{x+2,5,t} = M^t_{x,x+5}$. Then, we interpolate the detailed age death rates $m_{x+0,5;t}$ for $x = 50, 51, 52, ..., 84$ by any smoothing function. The use of mortality models is more suitable to fit the curve of $m_{x+2,5}$ separately for each year t. The quadratic model seems the most suitable for this issue.

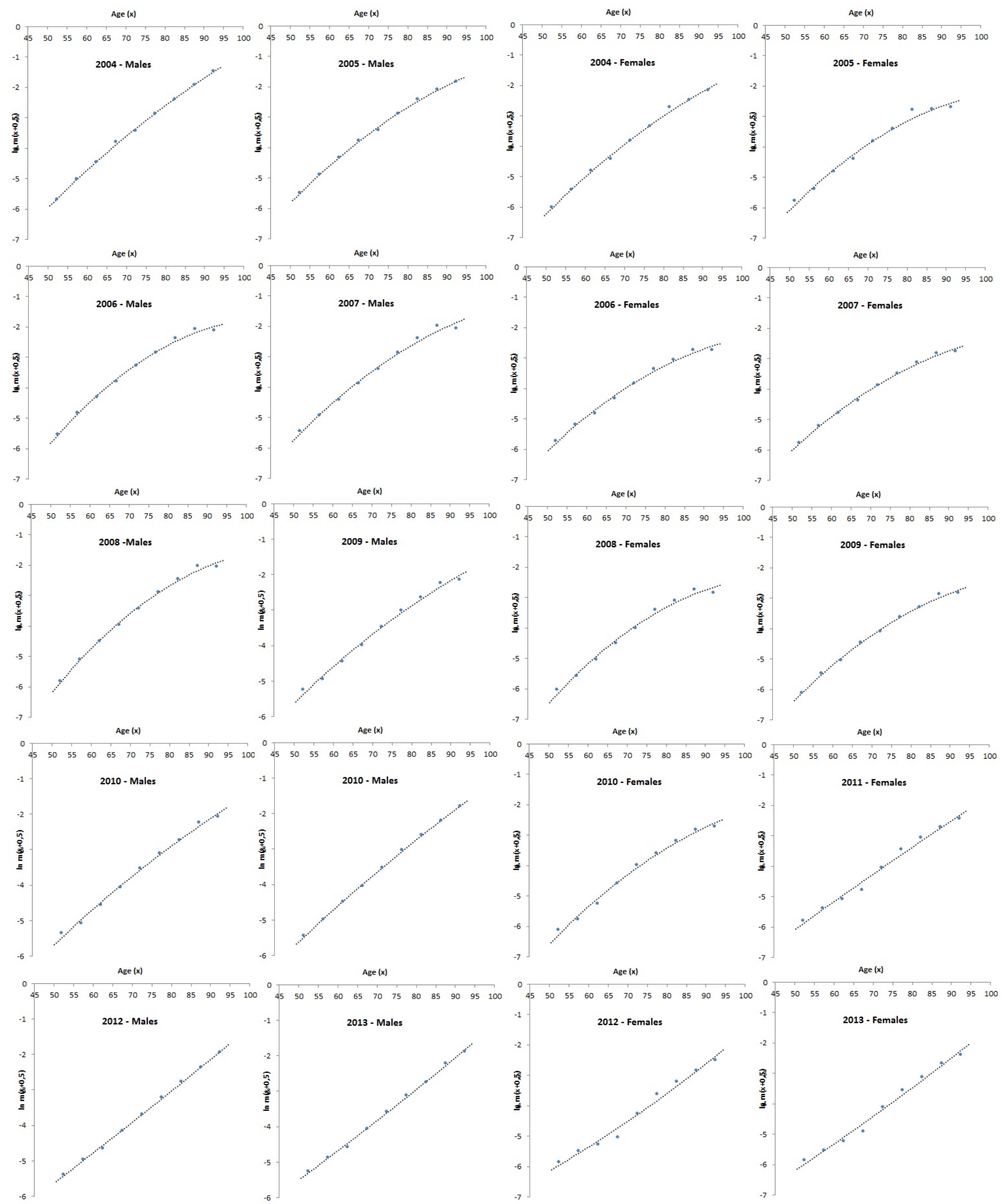
$$ln(\widehat{m}_{x+2,5,t}) = a + b(x + 2,5) + c(x + 2,5)^2$$

The fitting process will be oriented to minimize the weighed squared errors (WSE) at the known points. The observed deaths can be used as a weight. The minimization problem can be written as following:

$$minWSE = \sum_{x=50,55}^{75} D^t_{x,x+5} * (m_{x+2,5,t} - a - b(x + 2,5) - c(x + 2,5)^2)$$

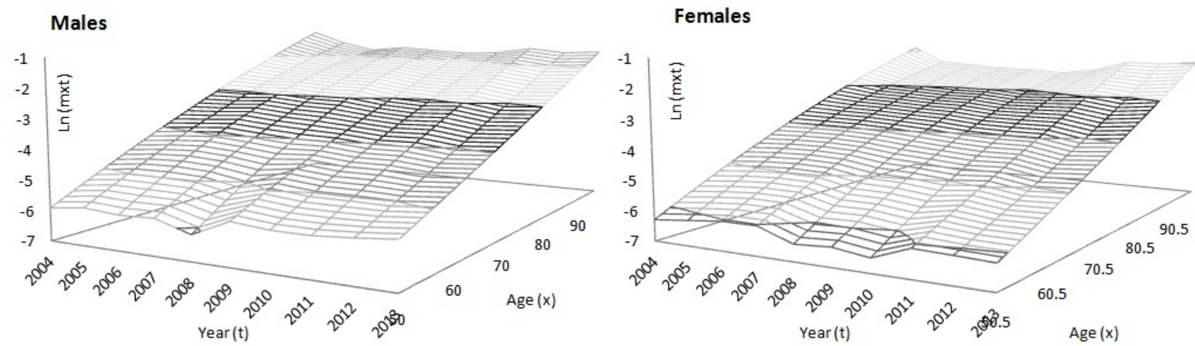
The fitting of the crude annual mortality curves is shown in Figure 6.3.

Figure 6.3: Annual life-tables fitting



The objective of the quadratic fitting already done on the crude death rates aims in first to interpolate the single ages death rates and by the same to reduce the irregularities caused by the reduced sample size for some age categories. As a result of this fitting, the annual life tables were interpolated and smoothed. However, specific death rates at each age x as a time series evolution are still marked by some fluctuations as shown in Figure 6.4.

Figure 6.4: Detailed age mortality surface for males and females $ln(m_{xt})$



In overall, the experience mortality surfaces presented in Figure 6.4 does not present any kind of extreme perturbations compared to the standard shape of mortality curves either in age or in time series evolution.

6.5 External reference mortality

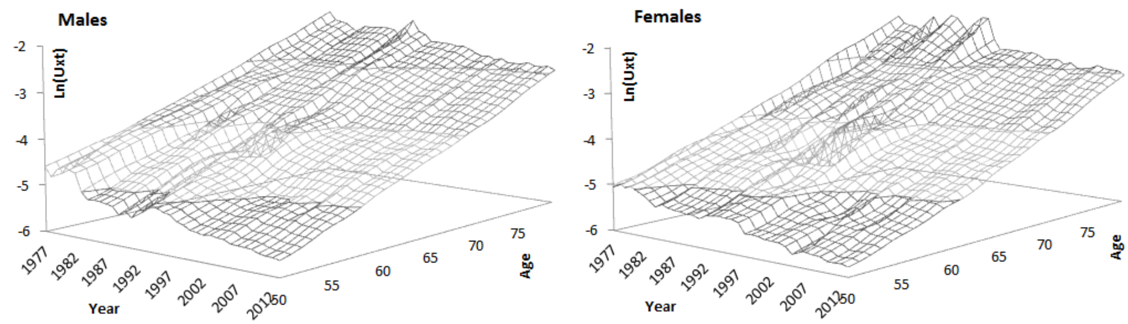
The length of the obtained mortality surfaces is limited and does not allow a robust forecast when the prospective mortality models are directly applied. For this, we need a consistent mortality reference to position the experience mortality. This positioning is traduced by a kind of a regression allowing to pass in any condition from the reference mortality rate to the experience mortality rate corresponding to the same age x at the same years t. Here, we propose to use the Algerian global population mortality as a reference.

6.5.1 Historical mortality surface

The Algerian life table for the global population is published by the Office of National Statistics (ONS) starting from 1977. Mortality rates are given by five age groups starting from 0 to around 75-80 for males, females and both sexes populations. During the period prior to 1998, these life tables were not continuously published. For some years, no life table has been published. An annual publication frequency was ensured only starting from 1998. By the same, the closure age varied between the open age group [70 and +] to [85 and +[in an irregular way. For mortality forecasting issues, the availability of a complete historical mortality surface is required. For this, We dedicated Chapter 3 to estimate the missing five-age mortality rates in the Algerian historical mortality surface.

Actuarial calculations are usually based on single ages description of mortality rates. Because we are dealing with a population of pensioners, we will focus only on the mortality of the population aged 50 years and over. The mortality rates at our disposal until now need to be interpolated in order to suit the requirement of our calculations. The Karup-King method can be used to break-out the five age mortality rates into detailed ages mortality rates. This interpolation method was explained in further details in Chapter 4. The obtained single ages mortality rates for the ages 50 to 80 are represented by Figure 6.5.

Figure 6.5: Single age global population mortality surfaces (1977-2014)



Source : Annual publication of ONS / Missing data : estimated in Chapter 3/ Single age data: Interpolated by Karup King method

The aim of positioning of the experience mortality on a reference mortality surface is to ensure more robustness in forecasting. The main idea is to allow deducing the projected experience mortality rates from the projected reference mortality. So, reference mortality rates must be projected. We have shown in Chapter 5, that the use of the prospective mortality models in an independent way does not guarantee coherent results regarding the male female mortality ratio. For this, the use of the coherent mortality models is highly recommended (Li and Lee, 2005; Hyndman et al., 2013). In Flici (2016-c), we conducted a comparison between these two models in the intention to project mortality for ages going from 0 to 80 years. It turned out from this comparison that the model proposed by Hyndman et al. (2013) leads to better results regarding the goodness-of-fit and the male female coherence. Here, we present shortly the fitting and the forecasting process proposed by Hyndman et al. (2013) well known under The Product-Ratio Method.

6.5.2 The Product Ratio Method

The coherent mortality forecasting method is based on the decomposition of the male female mortality surface into two new components: A joint mortality function and a differential mortality function. The joint mortality function represents a common averaged surface for both males and females. It can be calculated by the geometric average for death rates at each year t and age x by the following formula :

$$m^B_{x,t} = \sqrt{m^m_{x,t} . m^f_{x,t}}$$

with B refers to Both sex population, m : male and f : female.

The differential mortality function is represented by the rooted male female mortality ratio:

$$R_{x,t} = \sqrt{\frac{m^m_{x,t}}{m^f_{x,t}}}$$

The introducing of the root on the male female mortality ratio is supposed to facilitate the reconstruction of the fitted male and female age specific death rates by using the following formulas :$m^m_{x,t} = m^B_{x,t}.R_{x,t}$ and $m^f_{x,t} = \frac{m^B_{x,t}}{R_{x,t}}$.

The joint mortality surface and the differential mortality function issued from the Algerian male and female mortality surfaces are shown in Figure 6.6.

Figure 6.6: $\ln(m^B_{x,t})$ and $R_{x,t}$ - Observed 1977-2014

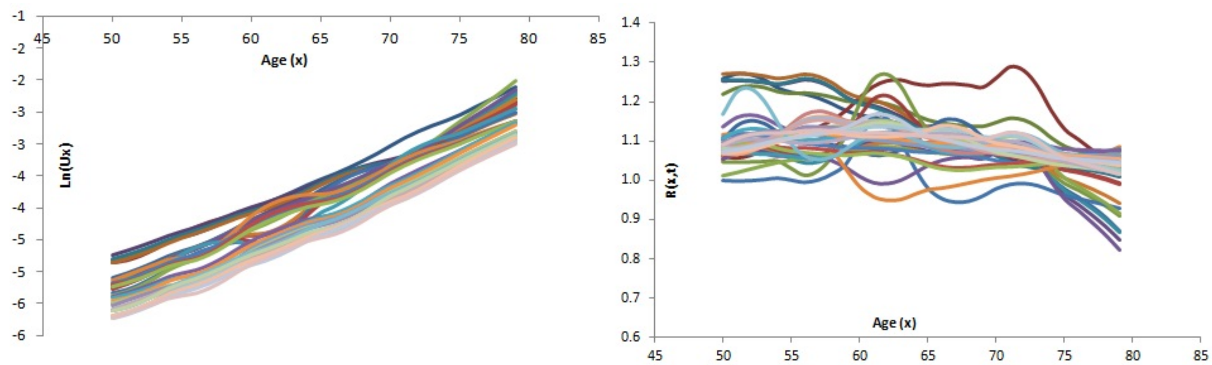


Figure in left shows the the annual mortality curves in log scale. Even if some differences can be easily identified when comparing different years, a relative stability is observed for the recent years [1998-2014] and an overall decrease can be observed regularly at all ages. The figure in right shows the age pattern of the male female mortality ratio for the different years of the period [1977-2014].

6.5.3 Reference mortality coherent forecasting

The next step will be the forecasting of each on these 2 components by using the classical approach proposed by Lee and Carter (1992). For this, two models are estimated: the joint mortality surface and the differential mortality function. The log of the joint death rate at age x and time t is decomposed into three components:

$$ln(m^B_{x,t}) = \alpha_x + \beta_x * \kappa_t + \xi_{x,t}$$

The estimation process will be oriented to minimize the Weighed sum of the squared errors (WSSE):

$$MinWSSE(1) = \sum_{x=50}^{79} \sum_{t=1977}^{2014} W_{x,t}[ln(m_{x,t}) - \alpha_x - \beta_x * \kappa_t]^2$$

The weight $W_{x,t}$ is supposed to be the total deaths occurred at time t and age x for both males and females.

While respecting the identifiability constraints: $\sum \beta_x = 1$ and $\sum \kappa_t = 0$.

for the Sex ratio, we use:

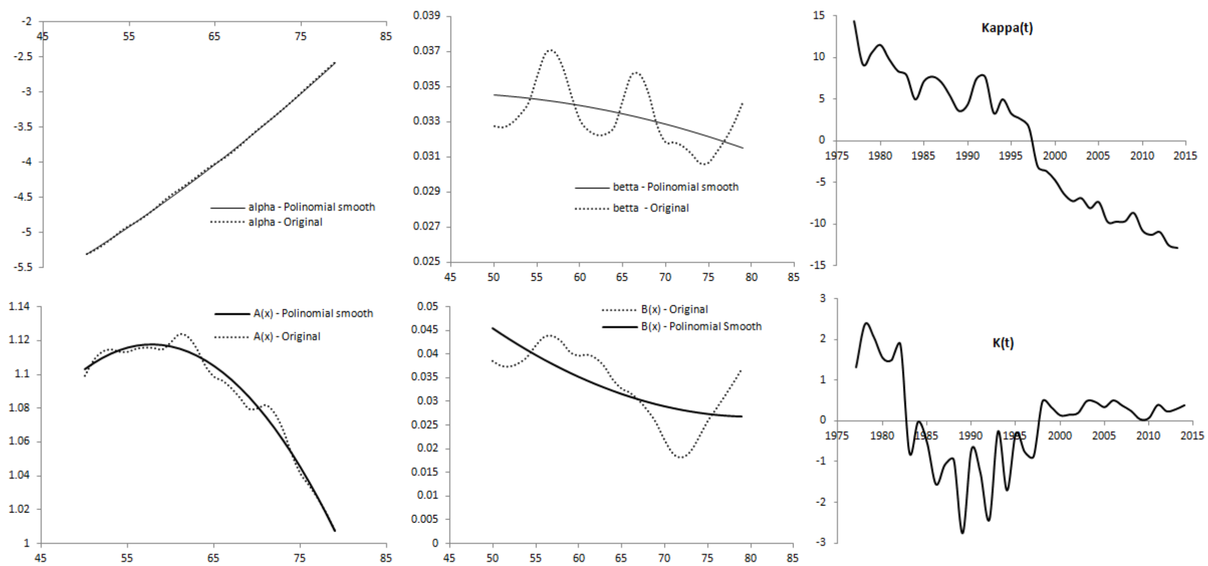
$$R_{x,t} = A_x + B_x * K_t + \varepsilon_{x,t}$$

The estimation process will be oriented to minimize the Weighed sum of the squared errors (WSSE):

$$MinWSSE(2) = \sum_{x=50}^{79} \sum_{t=1977}^{2014} W_{x,t}[R_{x,t} - A_x - B_x * K_t]^2$$

While respecting the identifiability constraints: $\sum B_x = 1$ and $\sum K_t = 0$.
The decomposition process led to the results presented in Figure 6.7.

Figure 6.7: Product Ratio Method - parameters estimation



In order to improve the regularity of the projected mortality surfaces, the age parameters were smoothed by using a 2^{nd} order polynomial function.

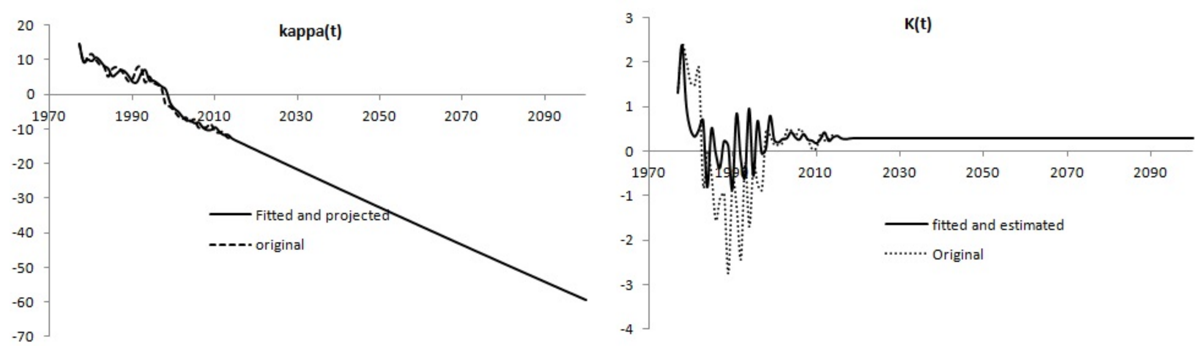
Time indexes forecasting

To forecast κ_t and K_t, many time series models can be used (see Chapter 5). Here, we propose to compare three models: ARIMA(0,1,0), AR(1) and ARIMA(1,1,0). A random walk with drift is the most commonly used model to forecast mortality time index in Lee-Carter model (Lee and Carter, 1992; Li and Lee, 2005). The use of a such model requires that the historical evolution of the modeled parameter has globally a linear trend. This condition is verified in our case, except the bump observed during 90's which is due to the important mortality level caused by terrorism events that have known Algeria during 90's. The model can be written as : $\kappa_t = \kappa_{t-1} + u + \varepsilon_t$ with u as a drift and ε_t as an error term. The second model to be compared is the AR(1) with a constant which can be written as: $\kappa_t = \lambda.\kappa_{t-1} + u + \varepsilon_t$. In final, either to model κ_t itself, we model the differentiated series $\kappa_t - \kappa_{t-1}$ by using an AR(1) with drift. That leads to an ARIMA(1,1,0) for which we add a constant and we can write: $\kappa_t - \kappa_{t-1} = \lambda.(\kappa_{t-1} - \kappa_{t-2}) + u + \varepsilon_t$ which can be simply expressed by: $\kappa_t - \kappa_{t-1} = (1+\lambda).\kappa_{t-1} - \lambda.\kappa_{t-2} + u + \varepsilon_t$. The three models are compared on the basis of the Mean Squared Errors (MSE) and the Bayesian Information Criteria (BIC) which allows to consider the difference of the number of parameters in the three models.

These three models can also be applied to forecast K_t. The historical series of K_t shows high irregularities during the period prior to 1994.

The selection of the best forecasting model is not itself sufficient to guarantee a strong forecasting. The time rang to be used has also a great impact in this sense. We observed in models calibration that the suppression of the years [1992-1998] does not have any effect on the forecasting results under the three models. The use of the recent trend (1998-2014) led however to a significant difference in this sense. We prefer to use the whole period as a basis for the forecasting. The stability of the series K_t allows to obtain a reasonable results. The final results are shown in Figure 6.8.

Figure 6.8: Product Ratio Method - Time components forecast



The extrapolated times indexes κ_t and K_t combined with the age parameters allow to reconstruct the projected mortality rates and sex ratio surfaces. According to that the projected death rates can be obtained by $ln(m^B_{x,t}) = \alpha_x + \beta_x * \kappa_t$ with α_x and β_x the parameters of LC model estimated and fitted in the previous element and κ_t the projected mortality time index. Similarly, the projected age specific sex ratio $R_{x,t}$ can be estimated by the formula: $R_{x,t} = A_x + B_x * K_t$ with A_x, B_x the age parameters estimated and fitted in the previous point and K_t the projected time index.

Then, and using these two extrapolated parameters, male and female mortality surfaces can be deduced by $m^m_{x,t} = m^B_{x,t}.R_{x,t}$ and $m^f_{x,t} = \frac{m^B_{x,t}}{R_{x,t}}$.

Old age mortality extrapolation

The single ages mortality surfaces that we have until now are those projected until 2100 and extended only until the age of 79. To extend death rates at older ages, we use the Coale-Kisker model (Coale and Kisker, 1990). The Coale-Kisker method is based on a quadratic formulation of the growth of death rates beyond the age of 80, which implies a linear deceleration rate. Beyond the age of 80, passing from an age specific death rate to the next can be done by the formula :

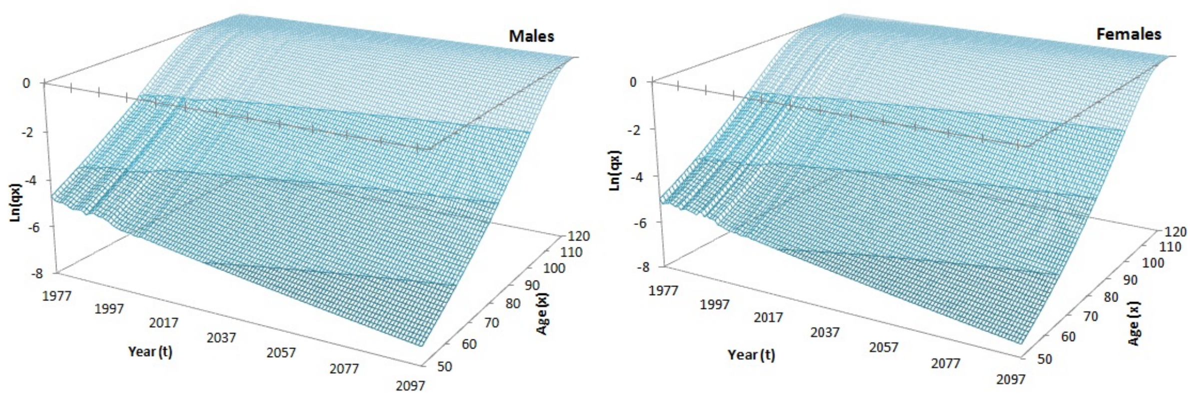
$$\hat{m}_x = \hat{m}_{x-1} exp(P_{80} + s.(x - 80)) \; x = 80, 81, 82, ...$$

P_{80} is the average growth rate of death rate between 65 and 80, and s is the slope of mortality growth rate deceleration which is supposed to decline following a linear trend. To estimate this slope s, authors imposed arbitrary a closure constraint : $m^m_{110} = 1$ and $m^f_{110} = 0.8$. This constraint does not imply that 110 years is defined as an ultimate surviving age as it has been done in Denuit and Goderniaux (2005) where the age of 130 years was defined as an ultimate age. The constraint imposed by Coale and Kisker (1990) meant just to orient the old age mortality trend and escape an eventual crossover of the male and female mortality curves (see Chapter 4).

For our application, we directly use the quadratic form of the Coale-Kisker method : $ln(m_x) = a + b(x-79) + c(x-79)^2 + \xi_x$. The model was calibrated to fit the death rates of the age range [60-79] and with imposing a constraint about the survival age limit which is supposed to be 120. This method is well detailed in Chapter 4.

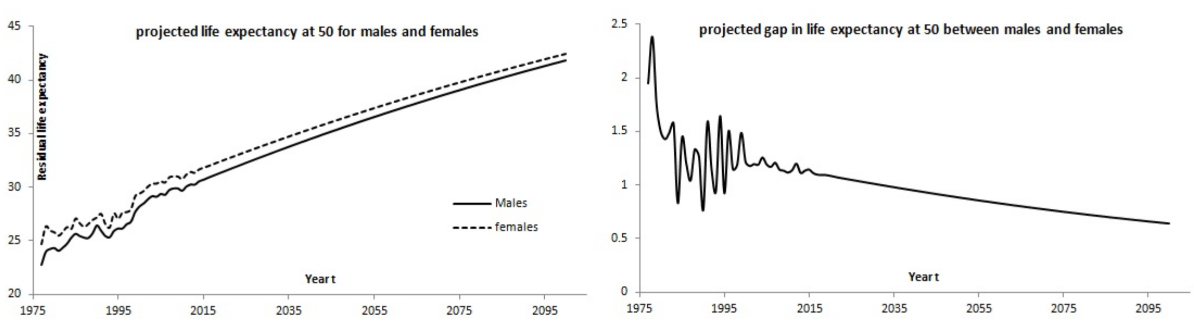
The projected mortality surface for males and females are presented in Figure 6.9.

Figure 6.9: Coherent mortality projected surfaces for males and females



The projected mortality surfaces for male and female populations show a high regularity. On the basis of the extrapolated mortality rates, the evolution of the life expectancy at age 50 can be deduced. Figure 6.10 shows a comparison between males and females and illustrate also the expected evolution of the gap in life expectancy at age 50.

Figure 6.10: Projected life expectancy at 50 for the global population by sex



The residual life expectancy at age 50 corresponding for males is expected to increase from 30.5 in 2014 to 35.9 in 2050 and 41.8 by 2100. For females, it is expected to go from 31.7 in 2014 to 36.8 in 2050 and 42.5 by 2100. The gap in life expectancy between males and females will decrease from 1.14 to 0.65 year between 2014 and 2100.

6.6 Mortality positioning

The construction of experience life tables based on an external reference needs to define a well known relationship between the experience mortality and the reference mortality. Then, the projected experience mortality can be deduced from the projected reference mortality.

Let $m_{x,t}^{ref}$ be the fitted Age Specific Death Rate for age x at time t for the reference population and $m_{x,t}^{exp}$ the fitted Age Specific Death Rate related to the retired population. We remind that, until now, $m_{x,t}^{ref}$ were projected until 2100. $m_{x,t}^{exp}$ are only available for the period 2004-2013. The length of the available historical series is not sufficient at all to do a strong forecast. Positioning the experience mortality on an external reference mortality, consists to define a regression allowing to deduce the projected mortality experience from the projected mortality of the global population. This relationship should be defined by a comparison between the two mortality for a common age range and period. Here, the common surface is defined by the age range [50-79] and the period [2004-2013]. In order to improve the goodness of the estimates obtained by the regression, a comparison will be conducted between the linear and the quadratic regression.

In first, we will test the linear regression proposed by Brass (1971). This method is based on the expression of the Logit of the experience mortality rates in function of the Logit of the reference mortality rates. This can be written as :

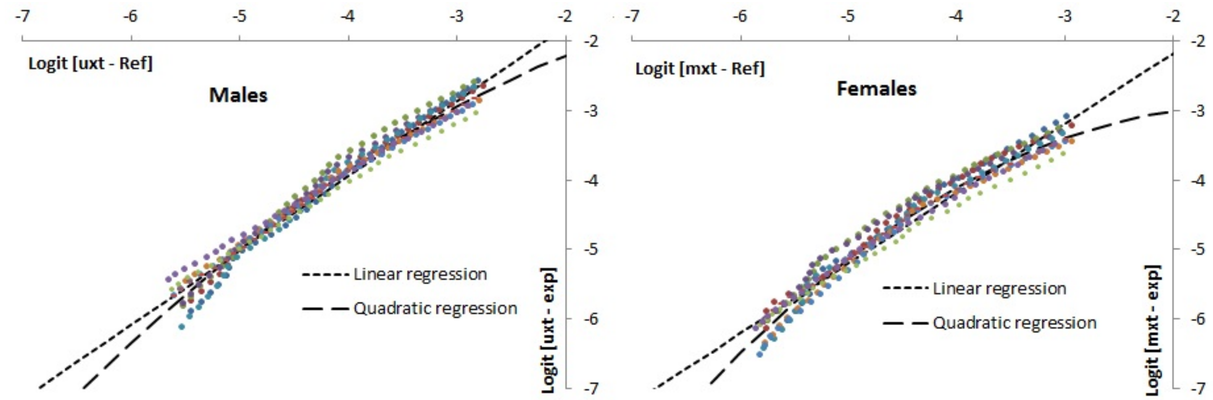
$$Logit(m_{x,t}^{exp}) = \gamma + \delta Logit(m_{x,t}^{ref}) + \zeta_{x,t}$$

With γ and δ to be the parameters of the linear regression, and $\zeta_{x,t}$ is an error term normally distributed. The estimation process will be oriented to minimize the sum squared errors (SSE) between the expected and the observed experience death rates:

$$MinSSE = \sum_{x=50}^{79} \sum_{t=2004}^{2013} [Logit(\widehat{m}_{x,t}^{exp}) - \gamma - \delta Logit(m_{x,t}^{ref})]^2$$

Figure 6.11 shows the obtained results.

Figure 6.11: Linear regression of $Logit[m_{xt}^{exp}]$ on $Logit[m_{xt}^{ref}]$



We observe that the linear equation does not fit well the regression of the experience mortality rates on the reference mortality rates. The regression seems to have a quadratic shape more accentuated in the case of females. To evaluate the quality of the linear regression and its capacity to predict the experience death rates starting from the reference ones, we compare the observed deaths to those expected by the linear model. For that , the expected rates must be considered by admitting an errors threshold. That returns to the estimation of the confidence intervals of the predicted death rates and the number of deaths.

We remind that the crude data that we used in our application concerning the experience mortality was available by 5 age groups from 50 to 95. The regression has been done for a common age range between the global and the retired populations. This age range was [50-80[. For this, the quality of the regression must be evaluated on the basis of the observed and the expected number of deaths for the age groups [50-55[, [55-60[, ... , [75-80[.

If we note $\bar{D}_{x,x+n}$ to be the average number of deaths observed in the age group $[x, x+n[$ during the whole period 2004-2013. $\bar{L}_{x,x+n}$ is the corresponding population at risk. The Crude Death rates averaged on the period [2004-2013] can be calculated as $\bar{M}_{x,x+n} = \frac{\bar{D}_{x,x+n}}{\bar{L}_{x,x+n}}$. The predicted number of deaths can be calculated by : $\widehat{D}_{x,x+n} = \widehat{M}_{x,x+n}.\bar{L}_{x,x+n}$ with $\widehat{M}_{x,x+n}$ is the expected death rates corresponding to the age group $[x, x+n[$ an calculated by the average of the Age specific mortality rates at ages $x, x+1, x+2, ..., x+4$ as following:

$$\widehat{M}_{x,x+n} = \prod_{z=x}^{x+n-1} \hat{m}_z$$

The confidence bound must be calculated for 6 successive five age death rates. Here, the estimation of the confidence bound is different from the case of confidence intervals estimated independently for each age and also from the case of a survival function. For this, we follow simply the methodology proposed by Planchet et Kamega (2013) in confidence bound estimation for single ages death rates. Further details about the estimation process can be found in the same article. Here, we propose to adapt the proposed formula to suit the case of five-age death rates.

Constructing a confidence interval at a confidence level of $1 - \alpha$ for $\widehat{M}_{x,x+n}$ which is supposed to be normally distributed aims that there is a probability $1-\alpha$ that the real death rate is comprised in the confidence interval:

$$P(M_{x,x+n} \in [\widehat{M}_{x,x+n} \pm \mu_{\frac{\alpha}{2}} \sqrt{\frac{\widehat{M}_{x,x+n}(1-\widehat{M}_{x,x+n})}{\widehat{L}_{x,x+n}}}], x = x_1) = 1-\alpha$$

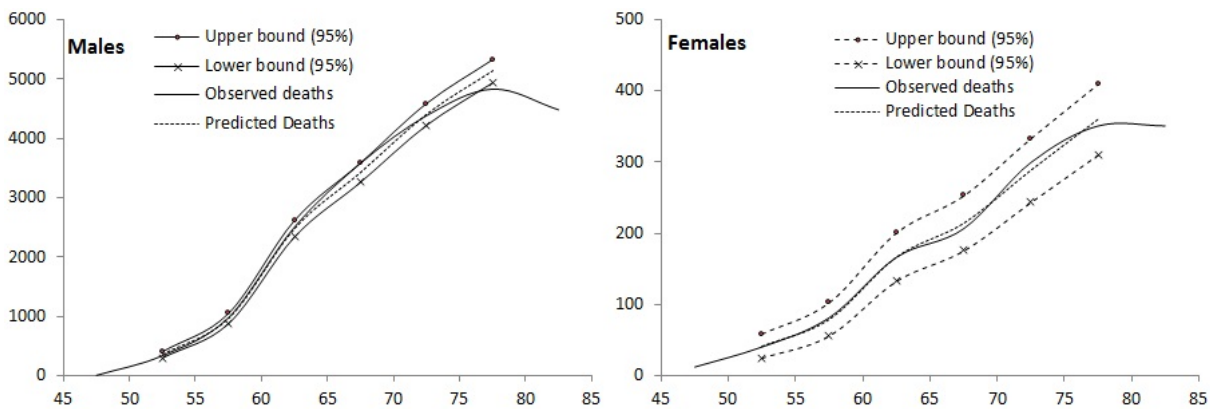
With $\mu_{\frac{\alpha}{2}}$represents the $\frac{\alpha}{2}^{th}$ quantile of the reduced centered normal distribution.

In adverse to the confidence interval which allows to obtain a variation interval of a punctual estimates, the confidence bound tries to take into account the probability that all the considered points will be simultaneously situated in their respective intervals. According to that, if there is a probability $1-\alpha$ that $\widehat{M}_{x,x+n}$ will be situated in a well defined confidence interval, there will be a probability $1-\beta$ that a G number of independent observations will be simultaneously situated in their respective confidence intervals with $1-\beta = (1-\alpha)^G$. The confidence bound can be then calculated by:

$$P(M_{x,x+n} \in [\widehat{M}_{x,x+n} \pm \mu_{\frac{\beta}{2}} \sqrt{\frac{\widehat{M}_{x,x+n}(1-\widehat{M}_{x,x+n})}{\widehat{L}_{x,x+n}}}], x \in [x_1, x_G]) = 1-\alpha$$

The confidence bounds calculated on the number of deaths (averaged on the period 2004-2013) predicted by the linear regression are represented in Figure 6.12 compared to the observed numbers.

Figure 6.12: Confidence bounds of the number of deaths predicted by the linear regression



For females, we notice that the observed number of deaths are situated inside the confidence bound for all the age groups. For males, the predicted number of deaths is situated out of the confidence bound for the age groups [65-70[and [75-80[. This allows to conclude that the linear regression does not capture a sufficient part of the information about the experience mortality scheme starting from the reference.

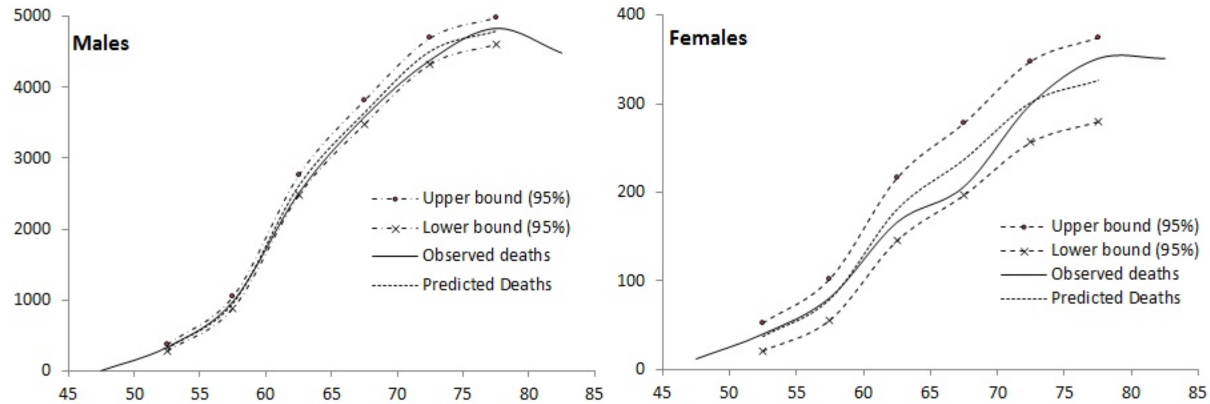
In order to reduce the part of the information loss due to the use of the linear regression, we propose to modify slightly the regression equation in order to capture the curvature of the cloud points. The regression of the experience mortality rates on the reference rates is supposed to be expressed by a 2^{nd} degree polynomial function:

$$Logit(\widehat{m}_{x,t}^{exp}) = \eta + \chi Logit(m_{x,t}^{ref}) + \varphi Logit(m_{x,t}^{ref})^2 + \zeta_{x,t}$$

With η, χ and φ are the regression parameters and ζ an error term.

The re-calculated confidence bounds are presented in Figure 6.13.

Figure 6.13: Confidence bounds of the number of deaths predicted by the quadratic regression



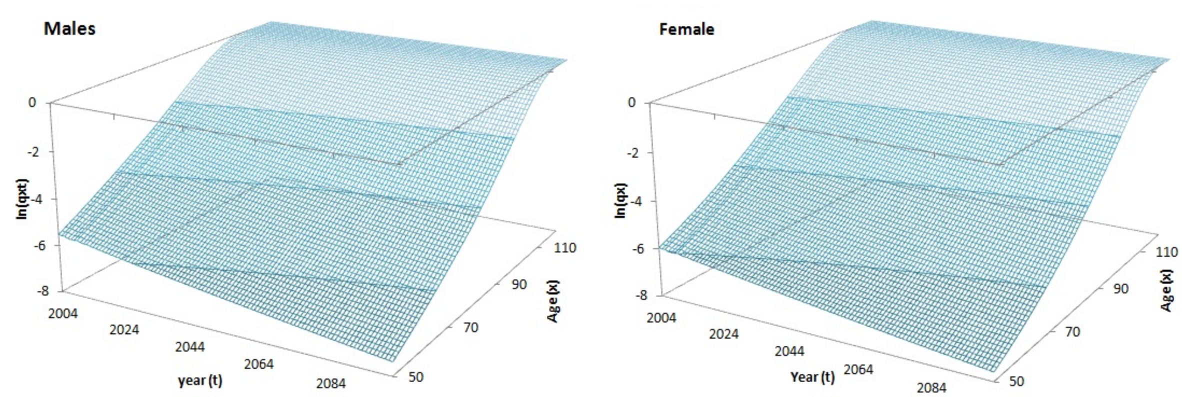
We observe that the quadratic function fits the regression of the experience mortality rates on the reference mortality rates better than the linear regression. The observed number of deaths is situated inside the confidence bound for all ages and for males and females.

The experience mortality rates will then be positioned on the reference mortality by using a quadratic regression rather than a linear regression.

Retired population dynamic life tables

On the basis of the coefficients of the quadratic regression, the Logit of the retired population can be deduced from those of the reference population projected until 2100 for ages between 50 and 79. The age specific death rates for the ages beyond 80 are extrapolated by using the Coale-Kisker model. The extrapolated mortality surface is presented in Figure 6.14.

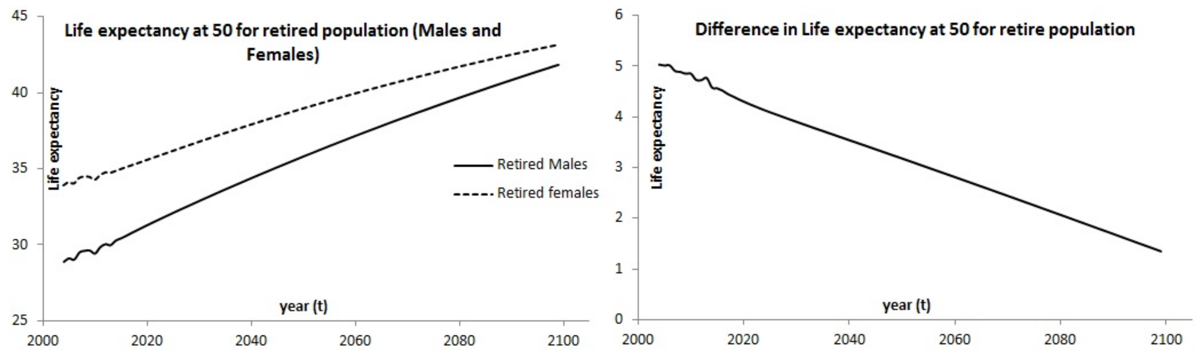
Figure 6.14: Experience mortality surface - projection results



The projected life expectancy at 50 years for the retired population

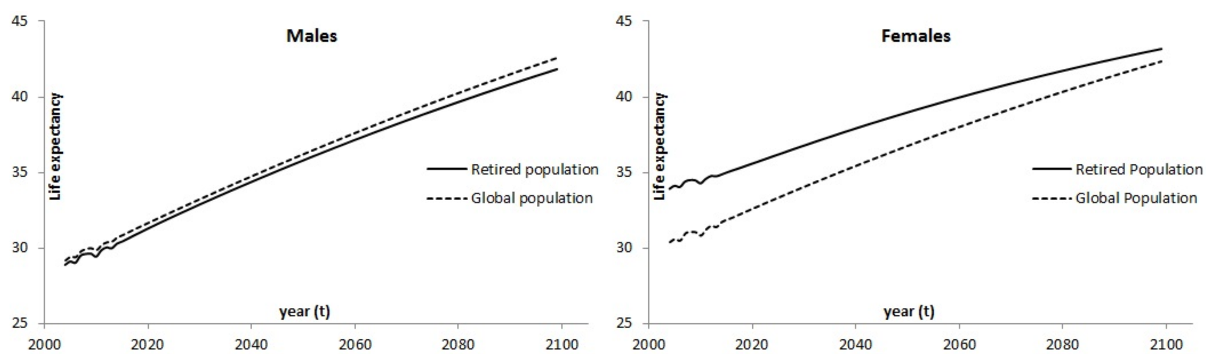
According to the forecasting results, the female life expectancy at age 50 is supposed to increase from 34.8 in 2014 to 39 in 2050 and to 40.9 in 2070. For males, this value is expected to grow from 30.4 in 2014 to 35.8 and 38.5 respectively in 2050 and 2070. The gap in life expectancy at age 50 which is at a level of 4.4 in 2014 is expected to decrease continuously until reaching a value of 2.4 by 2070. Figure 6.15 shows the obtained results in more details.

Figure 6.15: Projected life expectancy at 50 for the retired population



When we compare the results obtained on the retired population to the global population, a significant differences can be observed when comparing males and females. Figure 6.16 gives a comparison in this sense.

Figure 6.16: Life expectancy at 50 - a comparison by sex (Global population Vs. Retirees)



According to Figure 6.16, for males, there is not a significant difference between the global and the retired population in terms of life expectancy at age 50. The average gap is around 5 months during the whole observation period in favor of the global population. For females, the life expectancy of the retired population is 3.16 years higher than the life expectancy of the global population in 2014. This gap is expected to decrease in time and it will passe to 2.2 in 2050 and 1.7 by 2070.

There are two possible explanations for this last finding. The first explanation assumes that working conditions have an effect only on the female population. For men, life conditions are not different for the insured ones compared to the rest of the population. The insured and non-insured, the working and non-working men are approximately exposed to a similar life conditions. In adverse, working women have a better life, healthy and non-risky behavior than women of the rest of the population. Compared to men, working women are more concentrated in non-risky activity sectors (services & administration) (ONS, 2014-c). So, work gives women a particular life style and social appurtenance and better life conditions compared to working men and women of the rest of the population. The second alternative supports the assumption that the ONS underestimates the life expectancy of the female population. This second assumption is supported by the results obtained by Flici et Hammouda (2016). When analyzing the data issued from the health survey MICS-2012 which focused on the period [2008-2012], they found a gap of 3.9 years at age 50 between men and women of the global population. This result is near to the gap obtained on the retired population. Lets suppose that the retired population does not present any particularity in term of residual life expectancy compared to the global population, and the difference mentioned above resulted from a defection of the ONS estimates in concern of the female life expectancy. To affirm a such assumption, we need to compare some additional sources of data (insurance companies, social security, self employed pension regime) and to conduct a deep investigation in the methodology and the crude data used by the ONS to estimate female mortality.

6.7 Conclusion

When the mortality scheme of the insured population is different from the mortality of the rest of the population, it is recommended to adapt the actuarial life tables based to the experience of the concerned population. Usually, the mortality experience data is available for short periods and that reduces the robustness of projections. Also, the fact that the experience life tables are based on a reduced population size compared to the global population leads to important fluctuations regarding the estimated rates. To ensure a good forecasting capacity while taking the retirees mortality experience into account, the experience mortality must be positioned on a reference allowing to do a robust forecast. A regression function must be calculated to allow deducing the experience death rates from the reference rates (Planchet, 2005; 2006). In this sense, the Brass Logit system (Brass, 1971) is used to do such positioning.

Our objective in this chapter was to construct a prospective life tables adapted to the mortality experience of the Algerian retired population. For that, we used the mortality data issued from the portfolio of direct retirement of the Algerian National Retirement Fund (CNR) observed during the period [2004-2013] separately for men and women aged 50 years and +. Firstly, death rates were calculated. Then, the regression equation of the experience rates on the reference rates is calculated. The reference mortality here, was supposed to be the global population mortality already shown in Chapter 5. The global population male and female mortality has been projected by using the coherent mortality model proposed by Hyndman and al. (2013). To improve the quality of the regression which is firstly supposed to be linear, we proposed to use a quadratic regression to represent the experience mortality rates in function of the reference ones. By using this regression, the experience death rates was projected in function of the projected death rates of the global population.

According to the final results issued from Chapter 6, the life expectancy at 50 years of the retired population will evolve from 30.4 and 34.8 years respectively for males and females in 2014 to 35.8 and 39 years in 2050. The gap in life expectancy at 50 years is expected to fall gradually from 4.4 to 3.2 years during the same period. If we observe the difference between the global population and the retired population mortality projections by sex we observe an important similarity for males but a significant difference for females. The life expectancy at 50 for the global male population is 0.27 year higher than the life expectancy of the retired male population in 2014 which is supposed to fall to nearly 0 by 2050. In adverse, the gap is very significant in the case of the female population. The retired women hope survive 3.6 years more than women of the global population beyond the age of 50 years in 2014. This gap is supposed to fall down to a value of 2.2 in 2050.

Chapter 7

Financial sustainability of the Algerian pension system : perspective of the 50 coming years

Joint work with Frédéric Planchet, Full Professor, ISFA Lyon (France). This work has been presented in the conference "Journée d'Actuariat Vie Algérie - JAVA 3". Koléa, Algeria. December 7^{th}, 2016.

7.1 Introduction

The Financial balance of a pension plan is basically defined by the difference between expenditures and total incomes. However, a capital importance must be given for the long run equilibrium. Earnings are constituted by the total contributions payed by the affiliated population during a given period. Expenditures are all the annuities payed for retirees during the same period.

Maintaining pension systems equilibrium became a big challenge for all countries. This equilibrium is widely related to a set of elements supposed to vary over time. Population structure, longevity, economic growth are the main variables considered to keep such equilibrium. The evolution of the demographic parameters can heavily affect pension plan stability for a long term. For the case of Algeria, public pensions work according to the pay-as-you-go principle and equilibrium is maintained by public transfers. But, population is now aging, and longevity is improving. As a result, the number of retirees will grow faster than the population at working age. It will be hard to keep equality between the pension plan incomes and outcomes in such conditions especially if we consider the weakness of affiliation to social security among the working population. The part of this last depends largely of the public employment. The private sector stills much less covered. In the other hand, there are many other socioeconomic factors which can not be ignored when studying the pension plans sustainability. In the present Chapter, we present a general review of the different variables affecting the stability of the Algerian pension plan while focussing on effects of the demographic changes.

7.2 Short presentation of the Algerian Pension system

As it was presented in Chapter 1, the Algerian pension system is a mandatory, publicly managed system financed by a Pay-as-you-go Defined Benefits principle. The affiliated people have to pay a series of contributions along their activity period. The pension regime dedicated for the self-employed population (employers and self-employed) is managed by "Caisse d'Assurance Sociale pour les Non Salariés - CASNOS" and the employee's regime is managed by the "Caisse Nationale des Retraites - CNR" while contributions for retirement are collected by the "Caisse Nationale d'Assurance Sociale - CNAS" as a part of the social security contribution. Here, we will focus on the second pension regime concerned by the employees. The population of self-employed retirees represents only around 10% of salaried retirees.

For the population of the affiliated employees, the amount of the monthly contribution is defined as a part of the monthly salary. Many pension formulas are proposed in function of the number of the years of contribution. These conditions to access for each retirement formulas were fixed by the law 83-12 revised in March 22, 1999 by the law 99-03. The retirement age was fixed at 60 years for employees and at 65 years for self employed. In ordinary conditions, a minimum of 15 years of activity are required. In such conditions, the adequate retirement formula is the "Normal retirement NR". Retirement benefits are defined in function of the average of the last 60 month wage $\bar{W}$ (or, if is more advantageous, the average of the best 60 month salaries) and in order to attribute for each working year a validation rate of 2.5%. The Pension Benefit (PB) is calculated by the following formula:

$$PB = \bar{W} * n * 2.5\%$$

With a number of working years n equal to the maximum required working years (32), the formula above leads to a replacement rate of 80% of the average of the best (or last) 60 mounts wage. This replacement rate is higher than the average replacement rate in the Middle East and North Africa (MENA) region which is evaluated at 70-75% and much more higher than its level in developed countries (Robalino and al., 2005).

In the sense of the law, a working year is validated to give part in the pension benefits if it have led to, at least, 6 months of contribution. This last element with the fact of considering the best (or last) wages rather than the whole career average wage to define the retirement benefits endorses the evidence of the relative high generosity which marques the Algerian pension system already highlighted by Ben Brahem (2009). In addition, some bonuses are provided for some categories :

- A minimal pension equal to 75% of the national minimal guaranteed wage (NMGW);

- A bonus of 5 years is given for women with one additional year for each child reasen with 3 children as a limit without a financial impact;

- A bonus of 5 years is accorded for Moudjahidine (Former combatant of the Algerian liberation war 1954-1962). Years of participation in the war are double counted with a validation rate of 3.5% rather than 2.5%, plus a bonus of 1 years of each 10% disability degree occurred during the liberation war, a replacement rate of 100% and a minimum equal to 2.5 X NMGW;

- A pension complement is provided for one spouse in charge;

- An early retirement for workers under nuisance conditions;

In addition to the Normal retirement formula, many other direct retirement formulas were proposed in order to smooth individual decisions and economic circumstances:

- Proportional Retirement (PR): this formula was introduced starting from 1997 to allow workers whose want to stop working to get retired before fulfilling the basic conditions. Only 20 years of activity and a minimal age of 50 years are required. The pension benefit is then proportional to the contributing effort.

- Early Retirement (ER) : This formula was fixed by the decree 94-10 to suit the case of workers loosing their job because of the particular economic circumstances during 90's. The pensioner must have 50 years (45 for women), 20 years of activity and 10 years of contribution at least. Also, the decision of getting retired must be supported by the employer who must pay the right-opening contribution which is defined in function of the number of anticipated years : 13 months wage for 5 anticipated years; 16 months wages for 5 to 8 years; and 19 months wage for more than 8 years of anticipation;

- Retirement Allowance (RA): The retirement allowance was proposed to allow those who have reached the age of 60 without fulfilling the requirements to access to retirement pension, in order to benefit a minimal earning for a minimum of 5 years of contribution.

- Retirement without Age Condition (RAC): When a worker has accumulated 32 years of contribution, he can enjoy his retirement before reaching the regulatory age of retirement. To compensate for the loss resulted from his early retirement since he is still able to improve his final wage which is serving directly to calculate his retirement pension, his replacement rate is augmented by 2% for each year below age 60 years in the limit of 5 years. This formula was introduced starting from 1997;

The five presented formulas already presented (NR, PR, ER, RAC, RA) can be grouped in one main retirement category : the Direct Retirement DR. The DR concerns the benefits pension formulas dedicated for the future retired employees. A survivors benefits are provided for the family members after the death of the insured worker/ pensioner. We can distinguish two sub-formulas for survivors benefits because ofthe ammount's difference of the initial pension benefit formula: Survivors pension (for NR, PR, ER and RAC) and survivors allowance (for RA). Survivors benefits are provided for:

1. The spouse : In case of remarriage, the pension is transferred to the other rights holders;

2. Orphan: for males until 18 years old or 21 years if they are still studying; for females until occupation;

3. Parents: only if their own pension is under 75% of NGMW;

The initial reversion rate when only the spouse survives without any other right-holders is fixed at 75% of the principal pension benefit. When there is another surviving right-holder (orphan or parent), the reversion rate passes to 80% to be shared between the surviving spouse (50%) and the orphan (or parent) (30%). When the spouse survives with more than one other right-holders, the spouse keeps his 50%, while others share 40%. When there is no surviving spouse, orphans get 45% and parents get 30%.

- Complementary retirement CR: the CR provides two type of benefits : the principal pension and the survivor's pension.

- Other pensions: Some international conventions was signed between Algeria and some countries : Tunisia, France, Morocco, Belgium, Egypt, Spain to allow the payment of pension benefits for Algerians who are living out of Algeria;

The aim of DB systems is to ensure a fixed purchasing power for the pensioners by protecting them against inflation. For that , the retirement pensions and allowances are annually revaluated. Also, the pension benefits are annually adjusted to the augmentation of salaries.

Although these rules which aimed to protect the low benefits pensioners, some additional "Solidarity Subsidy" are provided by the government as public transfers:

- Starting from 2006, an Additional Monthly Indemnity for Retirement Allowance has been introduced to support the beneficiaries of a RA less than 7000 Dzd per month.The indemnity's amount is a proportion varying from 10% to 50% of the principal allowance. The minimum Allowance amount was seated at 3500 dzd.

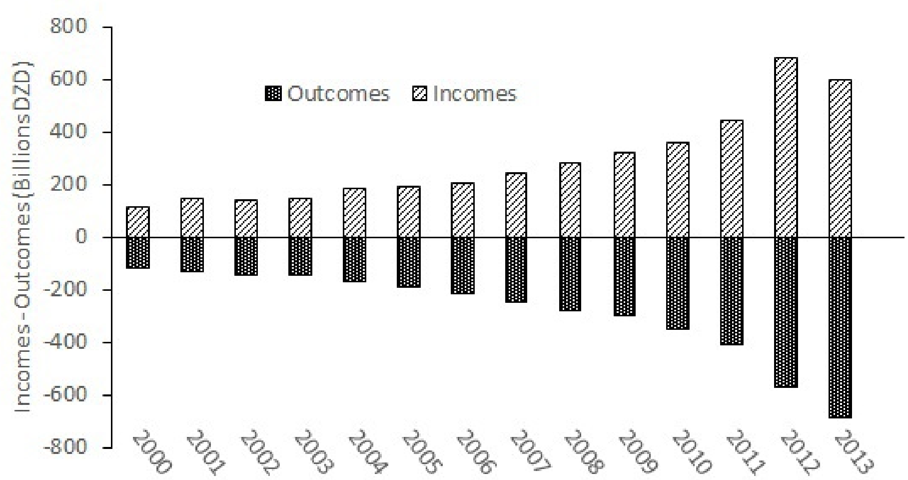
- For pensioners, the minimum was defined to be 13 500 Dzd. An Additional Monthly Indemnity for Retirement Pensions and Disability has been introduced in 2006 (Ordinance n°06-04 of 15 July 2006) in order to keep all pensions beyond this minimum.

- A pension complement of 2500 DZD is provided for the spouse in charge earning less than 75% of the NGMW.

7.3 Pension plan financial balance : evolution review

From 2000 to 2013, the global expenses of the CNR have grown from 117 to 687 billions DZD with an annual exponential growth rate of around 13.7% in average. In the other hand, the global incomes have passed from 114 in 2000 to 599 billions in 2013. Similarly to expenses evolution, total incomes have grown with an annual exponential growth rate of 12.8%. Figure 7.1 draws a comparative evolution of expenses and incomes.

Figure 7.1: Pension Plan Incomes Vs. Outcomes (2000-2013)



Source :CNR (2014)

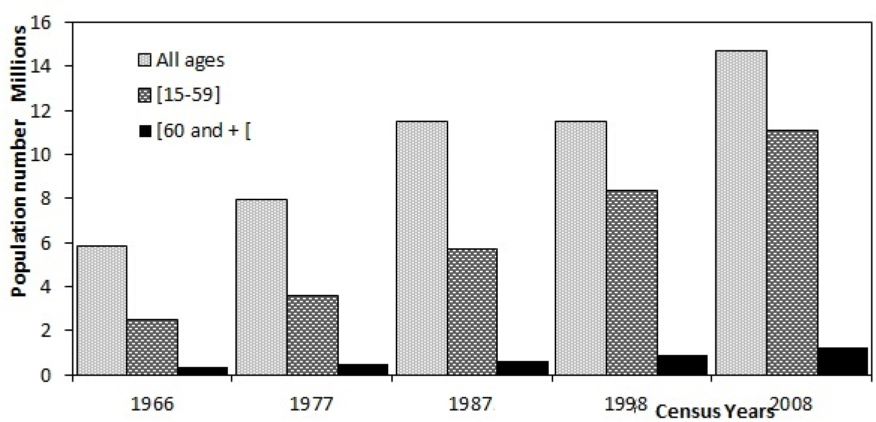
This evolution resulted from a combination of a set of factors, principally the diffencial evolution of the populations of retirees and contributors and also the improvement of salaries and pension benefits amounts. Globally, the financial balance of the CNR has been maintained if we consider the question from the year-to-year point of view, but it is not sufficient to asses about the long run sustainability of whole system. In the following element,we will try to go through the details of this evolution.

In first, we start by giving a short review of the evolution of the Algerian population structure from the independence in 1962 till now. Under PAYG system and particularly under Defined Benefits schemes, changes in the population structure is the main determinant of the financial balance, the economic factors effect comes in the second position.

7.3.1 Population growth and Structure

From the independence to now, the Algerian population has grown from about 11 millions to across 40 millions in January 2016. The evolution in term of numbers does not affect the pension plan stability but the evolution in term of structure does matter. The population at working age (15-59 years) does not represent the labor force in a country. What may affect the retirement plan incomes is the affiliated working population. The comparison of the population at working age to the population at retirement age may give only some indications. In this sense, a comparative evolution for the Algerian population is shown in Figure 7.2 according to population censuses data.

Figure 7.2: Population at working age Vs. population at retirement age

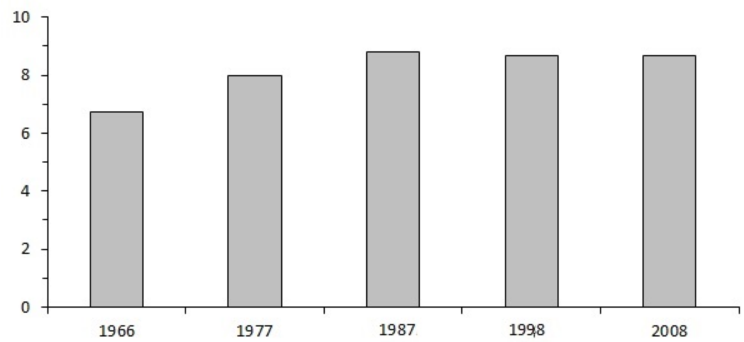


Source: Algerian population censuses: 1966, 1977, 1987, 1998 and 2008.

The report of the population number at the working age divided by the population at retirement age represents, theoretically, the number of individuals whose contribute for 1 individual at retirement age. The Evolution of this report according to the Algerian census data is shown in Figure 7.3.

The population at working age reported to the population at retirement age represents just a theoretical number about the one of individuals who may contribute for 1 theoretical retiree. This report was around 6.7 in 1966, 8 in 1977 and have been varying between 8 and 9 starting from 1987. This report represents a potential which is not exhaustively exploited because of the weakness of the affiliation to social security. By the same, retirement expanses are defined by the retired population and not by the population at retirement age. In the following element, we will show a general view of the evolution of the working population number in Algeria and also the evolution part of the ensured population among working people.

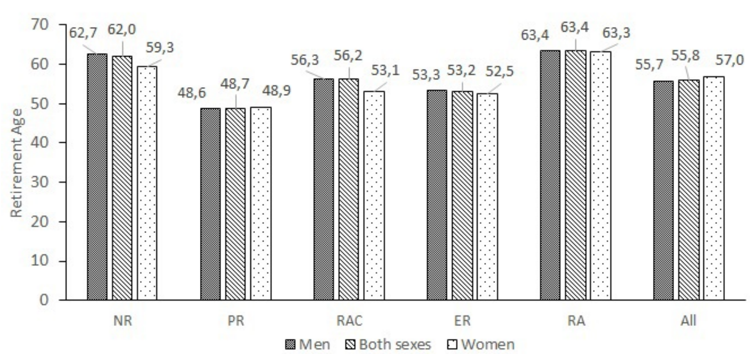
Figure 7.3: People at working age for 1 individual at retirement age



Source : Calculated from the Algerian Population Censuses data.

This finding is based on the hypothesis that the retirement age is 60 years. Even if theoretically, the age of retirement is fixed at 60 years, many options exist to allow people to get retired earlier: 5 years bonus for women. Early retirement, Proportional Retirement, Retirement without age condition. This various advantages push down the real retirement age. Figure 7.4 shows the average age of retirement for the Direct Retirement Formulas (NR, RA, ER, PR and RAC) in 2013. There is no detailed data for other years. Only a new retirees data in 2013 is available by age, sex and Retirement formula (CNR, 2014).

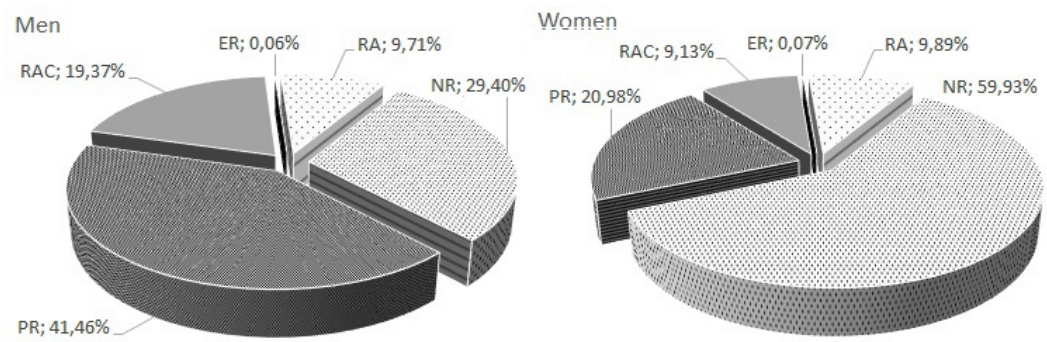
Figure 7.4: Average retirement age for the Direct Retirement formulas (2013)



Source : Calculated from data provided from CNR (2014)

We observe that for NR and RA, for which a minimum age of 60 years is required, the average age of retirement in 2013 was upper to 60 years, respectively 62 and 63.4. For the other retirement formulas, this age was equal to 48.7, 56.5, 53.2 respectively for PR, RAC and ER formulas. A slight difference is observed between men and women. Men retire later then women in all retirement formulas except PR. This last exception makes the average retirement age for women slightly higher than that of men : 55.7 for men and 57 years for women. If we ignore workers whose got retired beyond the minimal age for retirement (50 for men and 45 for women), it turns out the average age for retirement (all direct formulas included), that the average retirement age is 59 years for men and 57 for women. This result is supported by the difference in the distribution of retirees of each sex by retirement formula as we can see in the Figure 7.5. Note that an exception about the minimal age required for getting retired has been introduced in the way to contain the crisis of municipally guard fired starting from 2011. This point is well explained in the rest of the present work.

Figure 7.5: Distribution of new retirees by retirement formulas (men and women 2013)



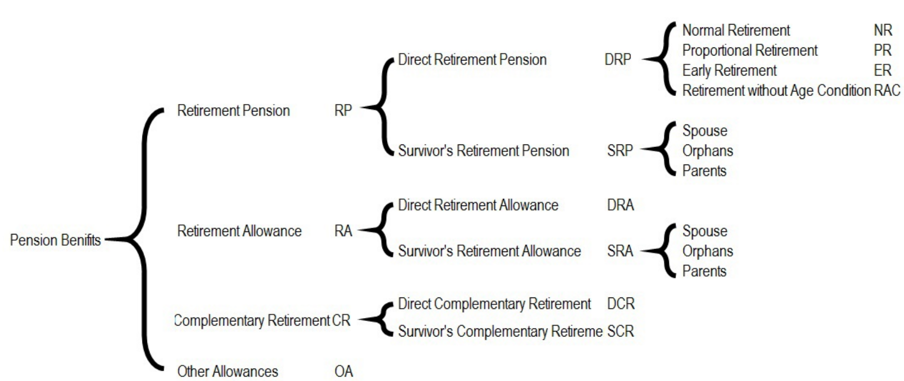
Source: Calculated from data provided in CNR (2014)

As we can see in the figure 7.5, 60% of women retirees are concentrated on NR for which the retirement age is relatively higher. In the other hand, only 29% of men retirees go for NR and more than 40% of them go for PR for which retirement age is relatively lower.

7.3.2 Pension plan outcomes

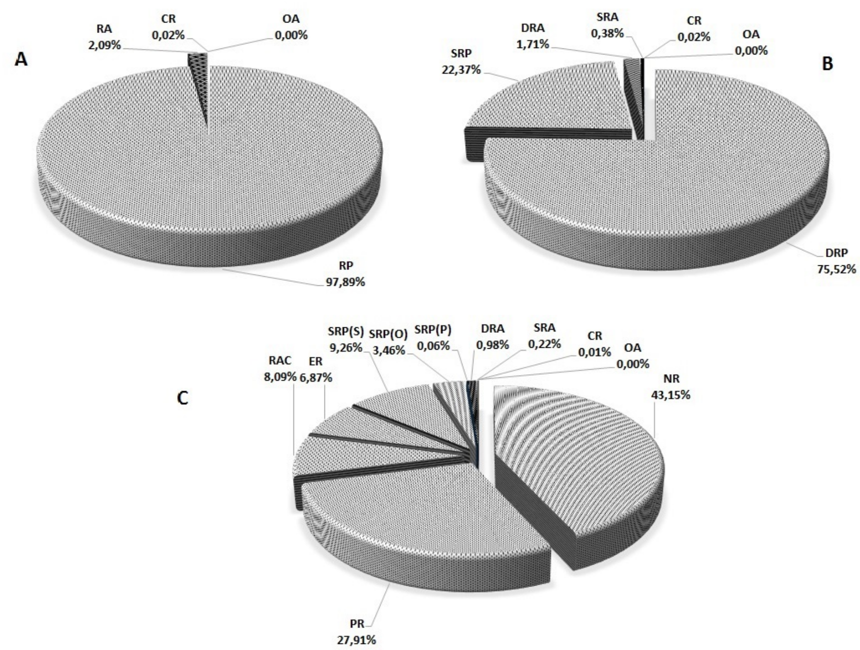
A detailed analysis of the retirement expanses evolution requires analyzing two main elements: the evolution of the population of beneficiaries and the pension benefits amounts. Before all, we take to give a summary about the retirement formulas in the Algerian retirement system as shown in Figure 7.6.

Figure 7.6: Retirement Benefits types provided by the Algerian Pension Plan



According to Figure 7.5, Retirement Benefits in the Algerian System can be grouped in 4 main categories: The Retirement Pensions (RP), the Retirement Allowances (RA), Complementary Retirement (CR) and Other Allowances (OA). Each Category comprises Direct and Survivor's Benefits (spouses, orphans and parents). There is no available details in concern of the "Other Allowances" which represent a part of nearly 0% from the direct retirement expenses as it is shown in Figure 7.7.

Figure 7.7: Retirement expenses average distribution by retirement formula (2000-2013)



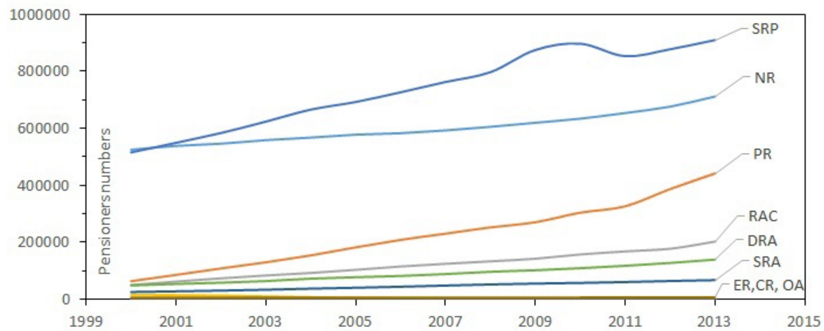
Source : Calculated from data provided in CNR (2014)

Each item (A, B, C) in Figure 7.7 shows the retirement expenses' distribution by a retirement formula according to the hierarchical scheme presented in Figure 7.5. According to the item (A), around 98% of retirement expenses are used to pay Retirement Pensions. A part of 2% is used to cover the Retirement Allowances expenses. The Complementary Retirement and the Other Allowances parts are very near to 0%. Item (B) provides more details about this distribution, 75.5% are served as a Direct Retirement Pensions, 22.4% as a Survivor's Retirement Pensions. Item (C) redraws the parts of NR, PR, RAC and ER which represent respectively 43.1%, 27.9%, 8.1% and 6.9% from retirement expenses.

Evolution of the pensioners numbers

The total number of pensioners passed from 1.25 millions in 2000 to 2.48 in 2013 and to 2.77 millions in 2015 all retirement formulas included. Figure 7.8 shows the evolution of this population arranged by retirement formula from 2000 to 2013.

Figure 7.8: Evolution of the number of pensioners by retirement formula (2000-2013)



Source : CNR (2014)

The numbers of NR and SRP beneficiaries are very important compared to the other retirement formulas. The number of NR beneficiaries passed from 525000 in 2000 to more than 712000 in 2013. For SPR, the evolution was from 515000 to 910000 beneficiaries. The PR beneficiaries number have grown from 64000 in 2000 to 442000 in 2013. In 2013, the RAC, DRA and SRA beneficiaries recorded respectively 203000, 140000 and 67000 beneficiaries. The part of the ER, CR and OA are much smaller with less than 70000 beneficiaries in 2013. In concern of the distribution of pensioners numbers between pensions and allowances, we observe that in 2013, Retirement Pensions takes a part of 91.4% from the total number of pensioners (54.7 for DRP and 36.7 for SRP). The part of Retirement Allowances is 8.35%.

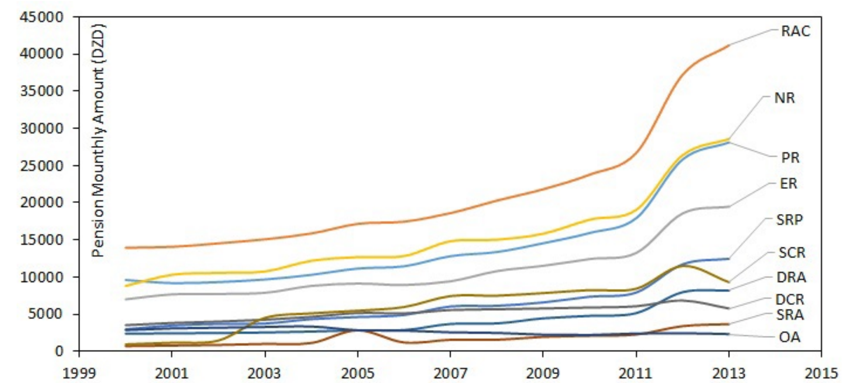
Part of retirees among population at retirement age

Access for retirement requires contributing for social security during a minimum number of years, which can varies from 5 years for the case of RA to 32 years for the case of RAC. To benefit a NR pension, a minimum of 15 years is required against 20 years for PR and ER. If this condition is verified, pension benefits are payed as long as retirees still alive while contributing period can vary in duration and in nature (continuous/ discontinuous) according to the employment conditions, individual preferences and social security design. We can understand that the notion of coverage is different for workers and people at retirement age. Working population social security coverage determines old population retirement coverage but there is no mathematical relationship between them. In other words, it we know only how is the affiliation rate for social security, we can not expect the number of retirees in the future. A part of the actual contributing people will not be acceded to retirement if their whole contribution period falls down the required minimum. Also, when affiliation rates are estimated, some people could not be affiliated, but they can be affiliated after or before the estimation points and then get access for retirement. For that, in the absence of an individual history contribution, it is not evident to determine exactly the number of future retirees among a given generation.

Pension benefit average amount evolution

The monthly pension benefit is defined by a combination of elements : The final wage, the number of years of contribution, the annual revaluation rate and the validation rate of each year of contribution. The average benefit amount by retirement formulas is affected by the age structure of retirees' portfolio. For that, its time evolution is affected by the entrance / exit movement in each portfolio. When wages are growing in time, it results that the new retirees have higher pension benefits than the old ones. In order to ensure some equity and reduce the gap between the succeeding generations in term of purchasing power, an annual revaluation of pension benefits is needed. Figure 7.9 shows the recent evolution of the average pension amount by retirement formulas.

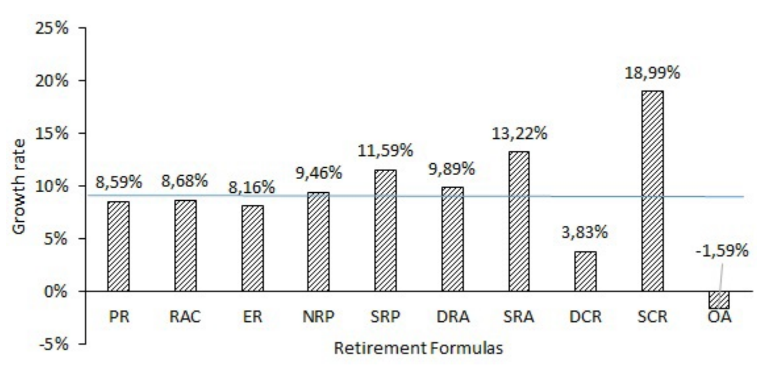
Figure 7.9: Evolution of the monthly pension amount by retirement formula (2000-2013)



Source : CNR (2014)

We observe that RAC provides the highest Benefit amount because it is linked to 32 years of contribution. Within this condition, the replacement rate of the salary before retirement is 80%. The other formulas lead to a lower replacement rate in proportion with the number of years of contribution. PR provides benefits as much as NR. The same minimal number of years of contribution is required for both. The only difference is the retirement age and the salary augmentation that can earn the Normal retirees during the additional working age. The gap that may results from these elements is somehow reduced by the annual revaluation of the old pension benefits which makes them approximately at the level of NR Benefits if we compare two retirees having the same age.

Figure 7.10: Annual growth rates of pension benefits in average (2000-2013)



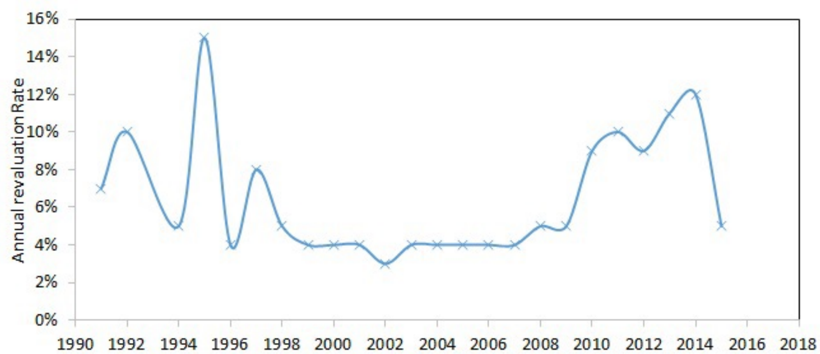
Source : Calculated from data provided in CNR (2014)

In terms of time evolution of the retirement benefits average amount over time some differences can be observed in term of evolution rate within the different retirement formulas. Figure 7.10 shows clearly this difference.

According to Figure 7.10, it turns out that the average benefits in Direct Retirement Pension formulas have grown with an annual growth rate of 8.75%. For the other ones with relatively lower benefits, the annual growth rate was slightly much important because of the additional revaluation provided in the way the guarantee the minimum level for the low earning categories. The average growth rate for all formulas was around 9.28%.

This pension benefits annual growth rate is not only resulted from the annual revaluation which aims to protect pensioners against purchasing power loss due to inflation. Some additional advantages are provided to keep in line the low benefits with a defined minimum. Figure 7.11 gives a review about the annual revaluation rates applied to pension benefits.

Figure 7.11: Annual Retirement Pensions and Allowance Revaluation



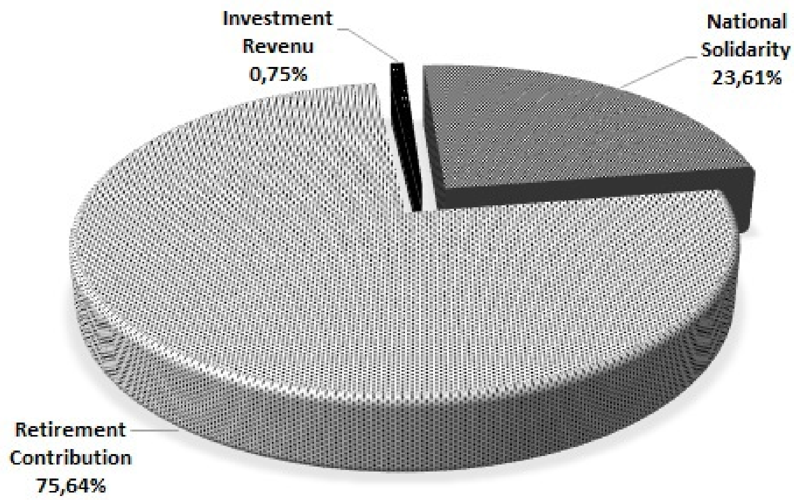
Source : from 1991 to 2013 (CNR, 2014), For 2014, 2015 : Official Journal of Republic of Algeria, n° : 2014/25, 2015/24 (www.joradp.dz)

The annual revaluation rates has stagnated during the period 1998-2009 between 4 and 5%. Starting from 2010, this rate passed beyond 8% and reached an exceptional level of 12% in 2014. Because of the new budgetary constraints imposed by the recent decline of oil prices, the revaluation rate was fixed at 5% in 2015 and at 2.5% in 2016.

7.3.3 Pension plan incomes

The financial sustainability of a pension plan is very sensitive to a set of a demographic and economic factors. Maintaining such stability requires a good planing allowing to manage a long term commitments such retirement contracts. In a Defined Benefits system, as we presented it earlier in the present work, the main objective is to protect old people from poverty. That is why the intervention of the government is needed to adjust the financial situation resulted from the distributed retirement benefits and the collected contributions. Figure 7.12 shows the financial resources of the CNR. Parts in the figure are calculated in average on the period 2000-2013.

Figure 7.12: Pension plan incomes distribution (2000-2013)



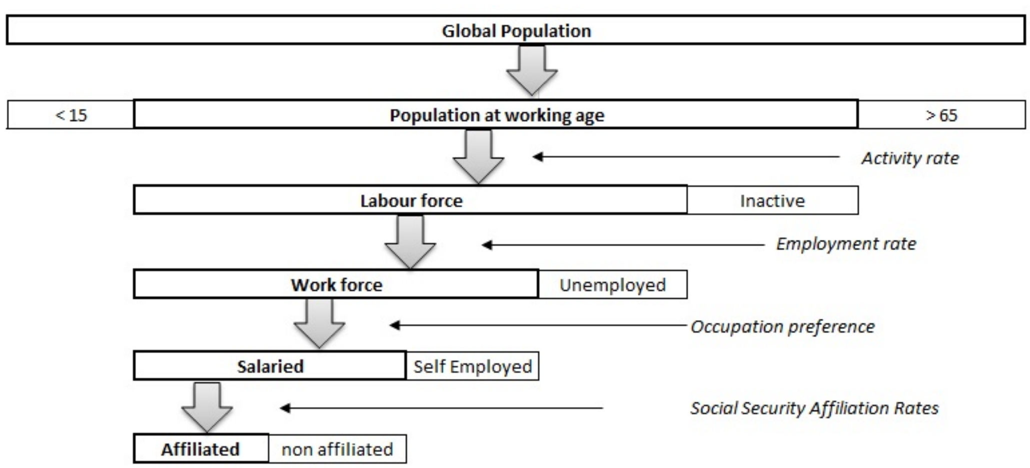
Source: Calculated from data provided in CNR (2014)

As we can see in Figure 7.12, the resources of the Algerian pension plan come mainly from the contributions of the affiliated employees with a part of 75.6%. Public transfers represent around 23.6% while less than 1% only come from the investment of contributions.

The contributing population

Figure 7.13 shows a scheme of the process that goes from the global population to the contributing population passing through the labor market and social security system.

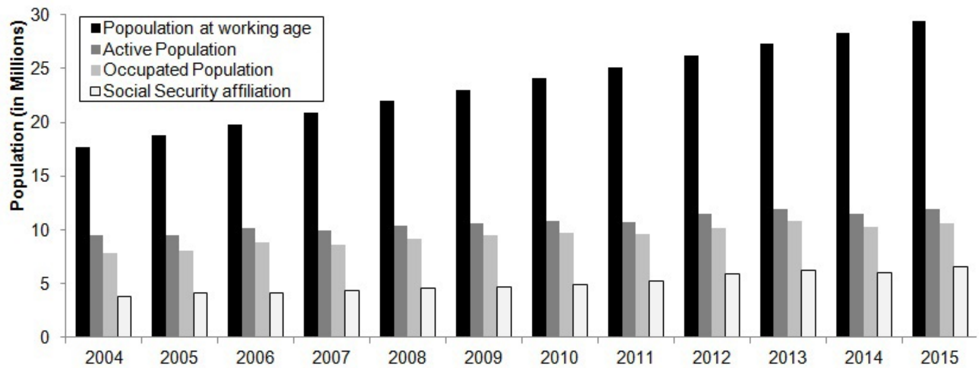
Figure 7.13: Affiliation scheme



It is well known that not the whole population aged [15-60[participates in the labor force. The total labor force records the number of individuals, aged of 15 years and + whose are supplying labor for the production of goods and services. This definition excludes all individuals who are not offering a labor force in the labor market (Students, home keepers... etc) and excludes also disable population. Labor force comprises the "work force" and the "unemployed population". The working population contains employees and employers / self employed. At the end, only the affiliated employees whose contribute for CNR.

Figure 7.14 shows a comparative evolution of the population at working age, the active, the working and the affiliated populations in Algeria from 2004 to 2015.

Figure 7.14: Population at working age, active population and affiliated population in Algeria



Source : Population at working age was calculated by interpolation / extrapolation based on 1998 and 2008 census data. Other indicators are issued from ONS (2015)

During the whole observation period, the part of population at working age which contributes for social security has fluctuated in between 20.15% and 22.75% white with an average of 21.28%. That means that for 5 persons at working age, there is only one who contributes for social security. Even if men and women have equal parts in the population at working age (approximately 50% for each sex), the participation in the labor force and the work force are different and males have the most important parts. In 2015, the participation rate in labor force is evaluated at 65% for men and only 15% for women. The employment rate was at 90% for men and 83.5% for women. Unfortunately, data about affiliation to social security are not available by sex, and we suppose for simplification issues that affiliation rates are same for men and women.

Monthly wage

In a wage-related Defined Benefits pension plan, contributions are not flat-rates but are calculated as a part of the monthly wage. This last varies in function of a set of elements. The National Guaranteed Minimal Wage (NGMW) can maybe have a significant effect on the evolution of the monthly average wage. The NGMW is fixed by authorities in order to protect people earning a very low salaries and it represents a guidance for the general evolution index of the other wages categories. The evolution of the NGMW in Algeria is represented in Table 7.1.

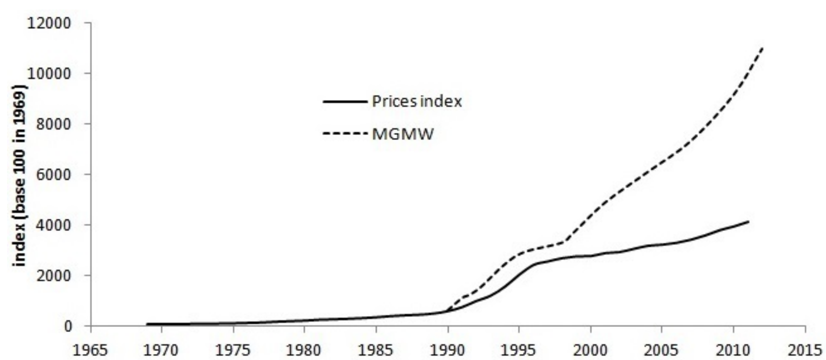
Table 7.1: Evolution of the NGMW in Algeria (1962 until now)

Revision data	NGMW (in Algerien Dinar)
01/01/1990	1000
01/01/1991	1800
01/07/1991	2000
01/04/1992	2500
01/01/1994	4000
01/05/1995	4800
01/01/1998	5400
01/09/0998	6000
01/01/2001	8000
01/01/2004	10000
01/01/2007	12000
01/01/2010	15000
01/01/2012	18000

Source: ONS (2012-b)

As shown in Table 7.1, the NGMW has been revised many times between 1990 and 2012 and has grown 18 times along that period. This augmentation is supposed to allow the low wages earners to save their purchasing power against inflation. To show this relation that we suppose exists between the NGMW and the prices index evolution, Figure 7.15 illustrates two curves representing simultaneously the evolution of the two cited indicators.

Figure 7.15: Comparative evolution of the prices index Vs. NGMW Index in Algeria



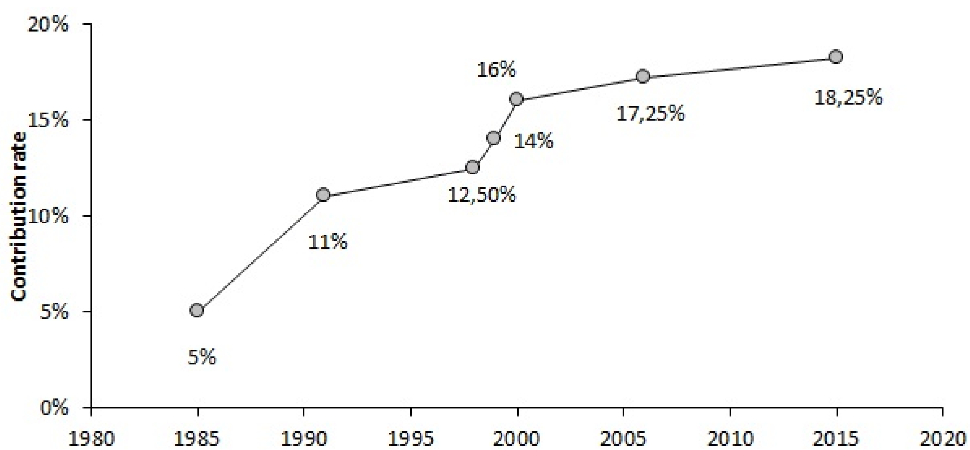
Source : Calculated from : 1) Prices index : ONS (2012-c) 2) NGMW: ONS (2012-b)
To draw the evolution of the NGMW while ensuring its comparability to the price index evolution, we calculated an NGMW index on the base of 611.37 in 1990 corresponding to the value recorded by the prices index at the same year. This process calculation was imposed by the data availability. The MGMW has been introduced only starting from 1990. So, for comparative evolution issue, the two indicators were compared on the same starting index value in 1990.

As we can see in Figure 7.15, the NGMW has grown 4 times faster than prices. When prices grow up, the low wages are usually supposed to be more sensitive to the loss of purchasing power. So, the observed growth rate in the NGMW compared to the prices index evolution is more oriented to protect the low wages rather than the medium or the higher wage classes.

Contribution rates

The contribution rate represents the part of the wage dedicated to pay the monthly contribution for retirement. The contribution for retirement is payed as a part of the contribution for social security. Then, the global contribution is divided between the different social security funds. In Algeria, this contribution rate for social security has marked many revisions and augmentations from 1985 till today. In 2015, the social security contribution rate has been fixed at 34.5% for the monthly wage. 25% are supported by the employer, 9% by the insured itself (employee) and 0.5% represents the contribution of the State. The contribution for retirement represents almost 50% of the contribution for social security. Its evolution over time is shown in Figure 7.16.

Figure 7.16: Evolution of the contribution rate to retirement in Algeria



Source : Official Journal of Algeria, different editions www.joradp.dz

The contribution rate for retirement (CRR), as shown in figure 7.16, has marked important augmentations from 1985 to 2015 by passing from 5% to 18.25%. Two augmentations were highly significant; the revision of 1991 brought up the CRR from 5% to 11%. The three consecutive revisions of 1998, 1999 and 2000 allowed to augment the CRR to 16%. This fast augmentation has had for objectives to keep the pension plan financial balance giving that outcomes tend to grow faster than incomes.

We conclude that the population of contributors is not only constituted by employees but also by employer / self employed. The contribution rate to social security for self employed is fixed at 15%, including 7.5% as a contribution for retirement.

7.4 Population projection

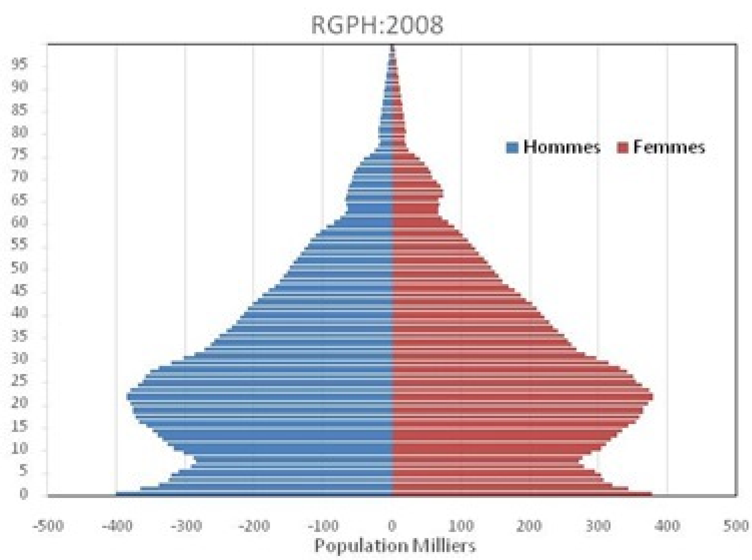
We intend to study the stability of the Algerian pension plan during the 50 upcoming years. In first, a population forecast is needed. Then the populations of contributors and retirees can be deduced on the basis of some simplistic assumptions about activity, working, occupation statue and affiliation to social security.

Population projection is based on a set of hypothesis regarding the future behavior of the demographic phenomena : fertility, mortality and immigration. The recent changes of life expectancy evolution and the Global Fertility Rates (GFR) in Algeria need a longer time length to be assessed and it is so difficult to assess if the recent observed trend will be kept in the upcoming years or it will diverge from the general trend observed since the late 70's. Conscious of all these elements, we do a population forecast until 2070 only in the intention to evaluate the stability of the Algerian pension plan without addressing any critical view of the goodness of the forecast itself. Data availability restricts the quality of the forecast and the projection that we propose can be qualified to be the best possible solution for the moment awaiting data improvement.

7.4.1 Basis of the forecast

Population Structure is mainely given by population censuses. Data recorded by civil statue services allows drawing the year-to-year population evolution. For population forecasts purposes, an initial population structure is needed. In this sense we prefer to use the data issued from the latest Algerian population census of 2008 as a basis of our forecast, rather than using the population structure published by the ONS in 2015 or 2016.

Figure 7.17: Single ages population pyramid - Algeria 2008



Source : General Population census 2008,
single ages numbers were interpolated by karup-king method.

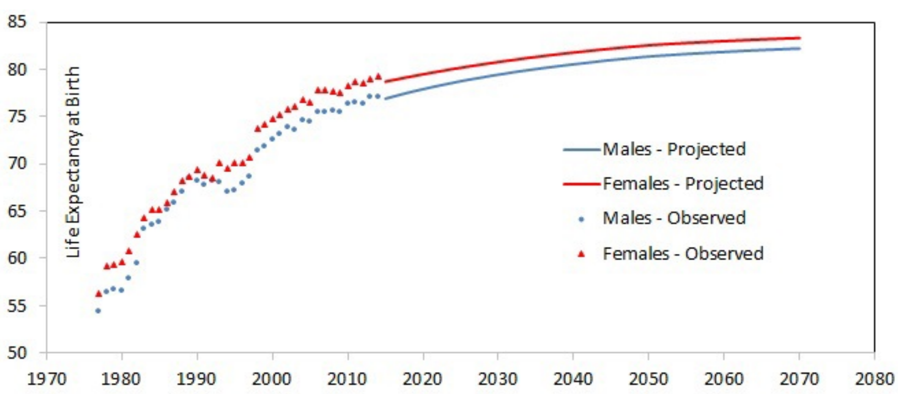
The population pyramid issued from 2008's census is expressed by five age groups from [0, 5[until the open age group [75 and +[for males and females. If we intend to do a single age forecast, it will be necessary to make the population numbers in single age format. The use of the Karup-King formula (see Chapter 4) allows to do such breaking-out. Figure 7.17 shows the single age pyramid of the Algerian population according to the 2008's census. Note that the population aged 75 years and + was broken-out by a linear interpolation between 70 and 100, while the number of population aged 100 years and over is supposed to be equal to 0.

7.4.2 Mortality Evolution scenario

To project the Age Specific Mortality Rates to the future, we proceeded by a coherent mortality forecasting methodology. Throughout the application process, it turned out that the use of the Lee Carter model (Lee and Carter, 1992) to project mortality rates independently for males and females leads to some incoherence regarding the sex mortality ratio by the horizon of the forecast (Flici, 2016-b).

The use of the coherent mortality forecasting approach allows to avoid this kind of incoherence. In another work (Flici, 2016-c), we compared two coherent mortality models: The product Ratio method proposed by Hyndman and al. (2013) and the Lee-Carter model with an additive common component for males and females proposed by Li and Lee (2005). It turned out that the first model leads to better results regarding the Goodness of fit and the coherence. The same paper (Flici, 2016-c) has been used in the official population forecast for Algeria at the horizon of 2030 (forthcoming). Here, we extend the projection until 2070 by using the same parameters used to project mortality until 2030. Figure 7.18 shows the evolution of life expectancy at birth for males and females.

Figure 7.18: Life expectancy forecast for men and women



Source : Flici (2016-c)

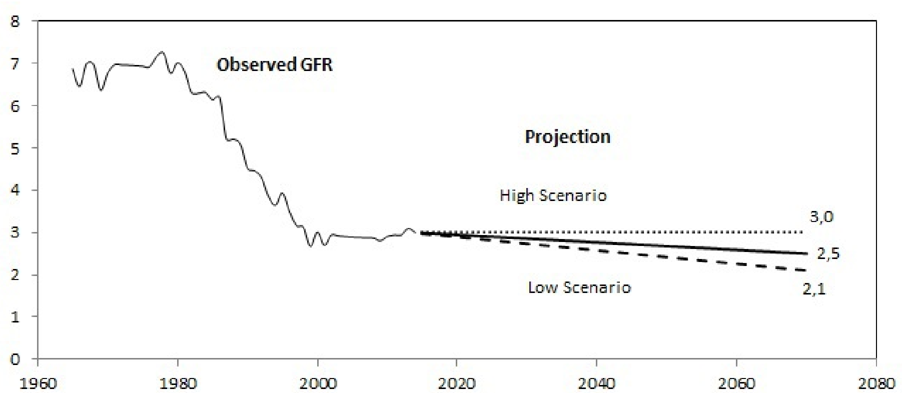
7.4.3 Fertility evolution scenario

To fix the future scenarios of the fertility rates evolution in Algeria during the upcoming half century, we consider the views of experts rather than the a probabilistic forecast. We aim to keep the population forecast results obtained in the present work within the framework of the National official projection methodology. Currently, the final report has not been published yet. But the framework of the projection was discussed and fixed in the meeting of the population forecasts subcommittee of the National Committee for Population (NCP) which depends from the National Health Ministry.

Discussions have mainly focused on a comparison between the probabilistic modeling approach and the view of experts approach. The first approach works well in the case of mortality, but not as well for fertility. The use of the Lee-Carter model as it was adapted to forecast fertility rates (Lee, 1993) led approximately to a constent Global Fertility Rate (GFR) at around 3 births / women by the horizon of the forecast (Flici, 2016-d). Experts of the NCP judged that this scenario can be used as a High Scenario rather than a middle one. Fertility rates are expected to keep slightly decreasing in the coming years. The low scenario was defined in order to respect the minimum GFR required for the population regeneration (2.1). In final, a level of 2.5 was kept as a medium scenario. Since the projection results was greatly depending of these underlying hypothesis which present a kind of weakness over time, the projection's horizon was limited to only 2030 for the National Population forecast.

In order to keep working within these hypothesis while extending the projection's horizon until 2070 to suit the objective of the present work, we keep the same levels (high, medium and low) to be expected at 2070 rather than 2030 as in the national projection. Results are shown in Figure 7.19.

Figure 7.19: Global Fertility Rate Evolution Scenarios



Source : from 1964 - 2014 : Algerian Official publications (Various). for [2015 - 2070] : future evolution scenarios are issued from the working meetings of the National Committee for Population / population projection Sub-committee.

7.4.4 Population forecast results

Until now, we have forecasted two components : Mortality and Fertility. Because immigration's data are missed or not available in the required format, we suppose that the immigration flow is equal to 0. The combination of the age specific mortality and fertility rates applied to the population structure given by GCPH-2008, we obtained the global population evolution and its age and sex structure for the whole period 2015-2075. The population pyramids for the years 2020, 2030, 2040, 2050, 2060 and 2070 are shown in figure 7.20.

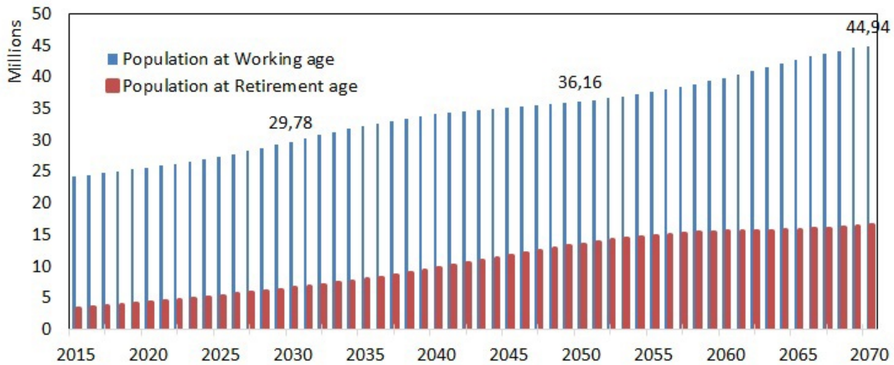
When we observe the transformation of the population pyramid from 2020 to 2070, we can deduce that the top of the pyramid is expected to enlarge, compared to the basis of the pyramid. That reflects the aging process of the Algerian population during the 50 upcoming years. The part of the population aged 60 and over is expected to increase from around 8% in 2015 to keep around 20% starting from 2050.

Figure 7.20: Population Pyramids by 2020, 2030, ... and 2070

7.4.5 The evolution of the populations at working and Retirement ages

The population forecast already done allows to deduce the populations at working age [15-59] and the population at retirement age [60 and +[. Figure 7.21 shows the evolution of this evolution from 2015 until 2070.

Figure 7.21: Population at working age Vs Population at Retirement age expected evolution



These results combine males and females. The segmentation by sex does not lead to any significant difference. The whole population is equally divided between males and females. According to the results of the population forecast, the population aged [15-59] is expected to grow from around 24 millions in 2015 to 30 in 2030, to 36 in 2050 and to 45 millions by the horizon of 2070. In the other hand, the population aged 60 and over is expected to grow from 3.4 millions in 2015, to 13.5 in 2050 and to about 16.5 millions by 2070.

The analysis of this evolution in absolute terms does not highlight the effect which may have the population aging on the pension plan financial balance. Figure 7.22 shows the expected evolution of the report of population ages [15-59] on the population aged [60 and +[which represents the number of individuals at working age corresponding to 1 theoretical retiree.

Figure 7.22: Number of working people for 1 theoretical retiree - forecast results

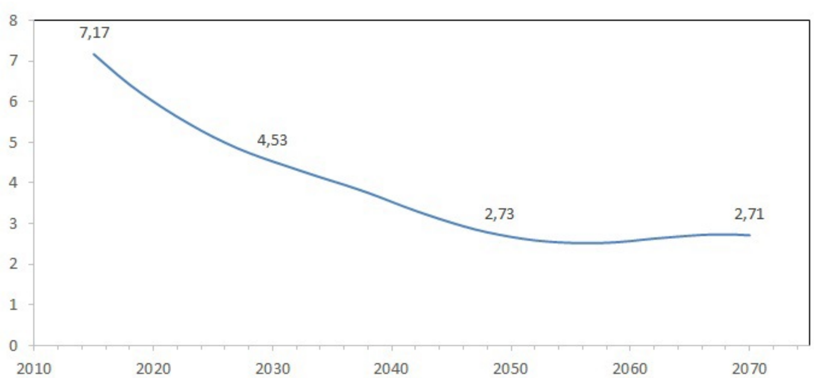


Figure 7.22 lets appear the expected decrease in the report of the population at working age, on the population at retirement age. In 2015, we have more than 7 individuals at working age corresponding to 1 individual at retirement age. This report is expected to fall to 4.5 in 2030 and to stabilize around 2.7 for the period [2050-2070]. This funding means that, if it is difficult to keep equilibrium between incomes and outcomes of the pension plan under a theoretical report of 7, it will be more difficult to keep it under a value of 2.7. The main challenge will be to make a great part of the population at working age in occupation and than within the social security system. In the following parts, we will try to consider these two last elements in order to define the long term sustainability of the pension plan.

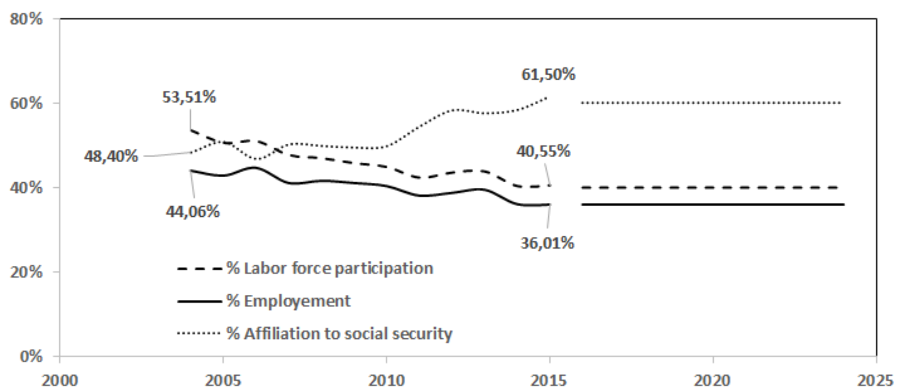
7.5 Economic factors future evolution scenarios

For the economic scenarios, we suppose that all elements will keep constant over the projection period: Activity rates, employment rates, occupation statue preference and also the affiliation social security. Salaries will be supposed to continue their recent observed growth rate.

Labor force, workforce, and affiliation to social security

During these last years in Algeria, there is only 40 % of the population at working age whose are offering their work force on the labor market. This rate was around 53% in 2002. The employment rate has decreased from 44% in 2002 to 36% in 2015. 38.5% of the employed population is still not affiliated to social security. Figure 7.23 summarizes the recent evolution of these three elements:

Figure 7.23: Labor force,work force and affiliated population rates



Source : calculated from data provided by ONS (2015)

For the needs of the present work, we will keep a simplest hypothesis about the expected evolution of the economics elements. According to that, we will suppose that during the 50 coming years, the labor force participation rate will keep constant at the level of 40%. The employment rate will be supposed to be around 36% and the affiliation to social security will be fixed at a part of 60%.

The analysis of the expected incomes - outcomes in a pension plan passes principally by expecting the way how populations' numbers (contributors and retirees) and financial quantities (average retirement expanses, average contribution) will evolve over time. On the basis of the global population forecast, the population at working and retirement age can easily be deduced. By assuming some hypothesis about activity, employment and affiliation to social security, the populations'evolution of contributors and retirees can be deduced. Also, on the basis of the economic growth, the average pension amount and average amount of contribution for retirement can be expected.

Affiliation rate for social security allows to determine the population of contributors among working people while retirement coverage allows determining the population of retirees among the population at retirement age. Usually, for projection issues, we suppose either the recent observed levels in these two parameters will keep constant either we use a simplistic models to forecast the observed trend. However, these two elements can not be supposed to be fully dependent since the access for retirement requires to pay contributions during a minimal number of years. According to that, a retirement coverage among a generation is directly related to the number of years of contribution among the same generation. An active person has to contribute during a minimum of 5 years during his whole working age to get access for retirement when he is 60 years. During the working age length of a generation, especially when social security coverage is low, it is evident to have an important flow into/from social security system. Then in the absence of individual data about contributing history, it is not evident to fix any relationship between contribution rates and retirement coverage in a given generation. The future number of retirees is supposed to be more important than the average number of contributors among the same generation, because contribution rate represents just an average one which does not address entrance and exit into/from the population of contributors. We conclude that even if we can estimate approximately the evolution of the number of contributors from year-to-year among a generation, we can not easily conclude the future number of individuals who will be retired when they reach the retirement age . To avoid this mis-estimation problem, we proceed as following :

- All the population at working / retirement ages will be supposed to be potential contributors / retirees. Hence, the evolution of the population number among the same cohort is only defined by the surviving function.

- Data concerning activity, employment, occupation statue preference (employees or self-employed) and affiliation will be used to estimate, for each five age group and for each sex, the chance to be affiliated as an employee for all individuals aged 15 and over.

- The population forecast combined with the expected evolution of the average wage, the contribution rate, and the chance to be affiliated as an employee is supposed to allow estimating the expected evolution of total contributions.

- The age distribution of the probability to contribute for CNR (affiliated employee) allows to estimate the expected number of contribution's years. Combined with the final wage, this allows to estimate the average pension benefit. Multiplied by the potential population of retirees, that allows to estimate the total expenditure allocated to pay Direct Retirement Pensions at time "t".

- Because survivor's benefits are complicated to be modeled since it does not include only spouses but also orphans and parents without being our main objective in the present work, the survivor's pension total expenditure is calculated as a part from the direct pensions expenditure. The sum of the two gives the total retirement expanses.

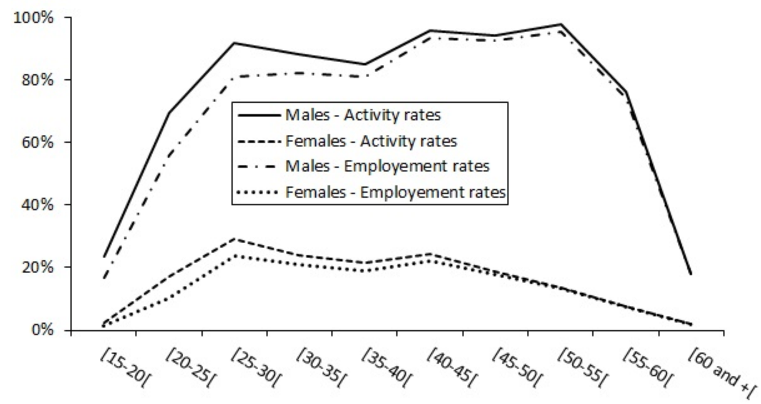
- The administrative fees are calculated as a percentage of the retirement expanses. The total gives the global pension plan incomes.

In the following, the elements cited above will be presented in more details.

7.5.1 The affiliation rate amount the population at working age

By supposing the population at working age to be potential contributors for the CNR. The idea is to calculate, at each single age starting from 15, the rate to be affiliated as an employee among the population at working age. This implies to consider the activity rates, the employment rates, the part of employees among the working population and the affiliation rates among employees. The workforce survey conducted by the ONS in 2013 (ONS, 2014-b) provides the data needed in our calculations. Data about activity and employment are arranged by sex and five age groups starting from [15-20[until [60 and +[. Activity rate represents the report of the labor force on the population at working age. The labor force includes every individual supplying his workforce on the labor marker either is working or seeking for work. The employment rate is calculated by the working population divided by the population at working age. Activity and employment rates in Algeria in 2013 are presented in Figure 7.24.

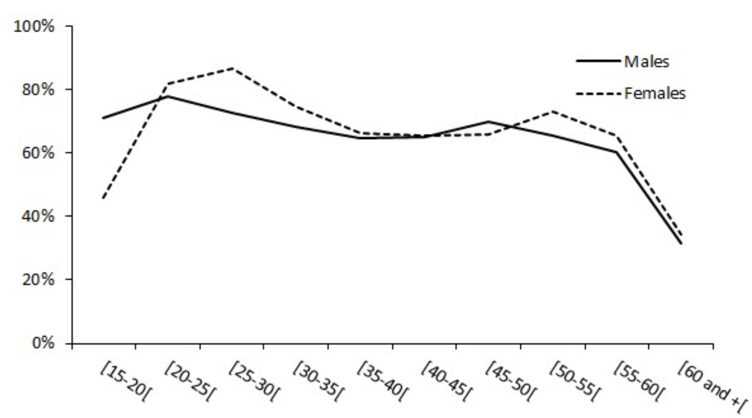
Figure 7.24: Activity and employment rates by 5 age groups - 2013



Source : ONS (2014-b)

The population's activity rate at working age was around 44%, 71% for males and and 16.6% for females. Between 25 and 55, men are the most active during their working age with a rate of 92% while women are more active between 25 and 45 with an average activity rate of 21,5%. The curve representing employment follows nearly the shape of the curve of activity with a slight gap at young ages narrowing gradually with age. The employment rate for men is 65%, for women is about 14%. In concern of workers'distribution by occupation statue (employees and self-employed), the part of salaried is about 69% of workers, 68% for men and 73% for women. The age distribution of the part of salaried among workers is shown in Figure 7.25.

Figure 7.25: Part of salaried among workers by five age groups - 2013

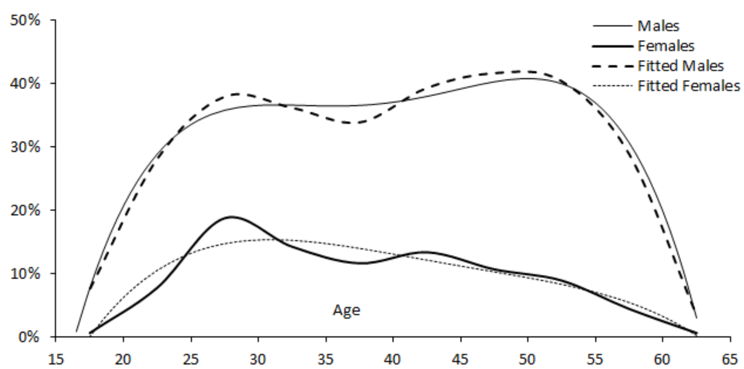


Source : ONS (2014-b)

Data about affiliation to social security is given by type of occupation : employees or self-employed with sex distinction but without age ventilation. Affiliation rate to social security for the whole working population is 57.5%, 54% for men and 72% for women. Employees are well covered than the self-employed. The affiliation rate of employees is 70%, 65% for men and 92% for women. This difference is mainly due to the nature of the employer (private or public) and also the occupation statue (employees or self employed). The salaried women work more in the public sector compared to men with a respective parts of 83% and 54%. Self employment takes a part of 31% among men and 24% among women. The automatic affiliation to social security in the public sector makes the affiliation rate of the public employees equal to 100%. The coverage in the private sector is supposed to be much lower. These informations are unfortunately missed in the ONS report.

By using all these presented informations, the part of the contributors for CNR (affiliated employees) among the population at working age can be deduced. The obtained results are shown in Figure 7.26.

Figure 7.26: Probability to contribute for CNR by age - 2013



Source : Calculated from data provided in ONS (2014-b)

Figure 7.26 represents the part of contributors for CNR among the population at working age by age for men and women. The interpolation at single ages was done by a polynomial fitting : a 5-degrees polynomial for men and a 4-degree polynomial for women. Then, the fitted curves shown in Figure 7.26 show the affiliation rates at single ages from 15 to 65 years, that we note $AS_x^{m,f}$ with x represents the age, m and f denotes males or females. This indicator can be used to estimate the contributors distribution by sex and age among a given population. In the unavailability of data series allowing to forecast this structure in the future, we can suppose simply that it will remain constant during the coming decades. Then, for each cohort, the evolution of the number of contributors over age can be deduced. If we intend to estimate the expected number of years of contribution to CNR during the working career of each individual, that we note $EC^{m,f}$, we can simply do an approximation based on the report of the whole contributed years on the population at working age, which can be simply approximated by the sum of the affiliation rates between 15 and 65 years. This can be written as: $EC^{m,f} = \sum_{x=15}^{65}(AS_x^{m,f})$. It turns out that the expected period of contribution to CNR is equal to 15 and 4.6 years for males and females at working age.

Then, the number of the future retirees among a generation can be deduced from the population at retirement age. If we consider a common retirement age of 59 years for men and 57 for women, then all the population reaching this age will be supposed to be a potential retired population. Hence, the average retirement benefit amount will be calculated on the basis of the expected duration of contribution and the final late career average wage. That will makes possible to estimate the total retirement benefits without fixing an exact number about the population of retirees.

7.5.2 Evolution of the average amount of contribution

The evolution of the average amount of the monthly contribution is directly related to the wages' evolution and the contribution rate. For the needs of this present work, the contribution rates will be supposed to keep constant. Wages are supposed to evolve following the recent observed trend. If we consider the period 2009 - 2013, the average growth rate of the net wages is 9.5%. From 1990 to 2013, the crude average wage has evolved by an annual growth rate of 10.7%. Here, we prefer take the recent observed trend since there was not a significant difference among the whole period.

In 2013, the contribution rate for social security was at 34.5% including 17.25% for retirement, 10% are payed by the employer, 6.75% by the employee itself and 0.5% by the government. Stating from 2015, the part of the employer passed to 11% and the contribution rate for retirement became 18.25%.

The amount of the contribution is calculated by multiplying the average crude monthly wage (50211 dzd) by the contribution rate for retirement. That leads to an annual contribution in 2013 of :

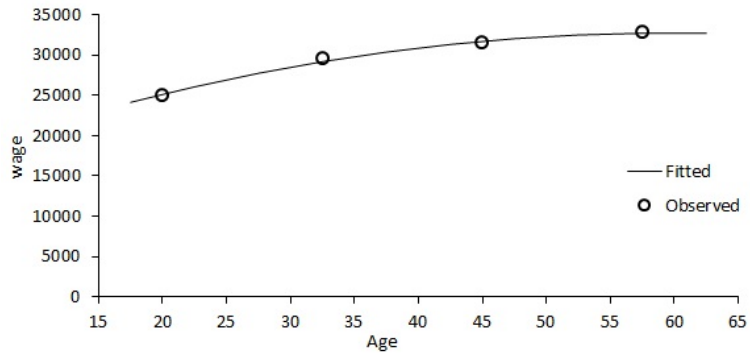
$$50211 \text{ x } 17.25\% \text{ x } 12 = 103957.8 \text{ dzd}$$

For a number of contributors of 5 173 000, which represents the number of the affiliated employees, the total contribution to CNR during the year 2013 is evaluated to 537 773 825 900,68 dzd. The total income of CNR in 2013 is about 599 billions dinars, 430 billions come from the contributions of the affiliated employees, 12 as investment returns and 121 as public transfers. If we compare only the theoretical and the effective total contributions, we can conclude that CNR may suffers a recovery problem evaluated at around 20%. It seems that some employers are not regular in the payment of their employees contributions. Also, the fact to validate a full year of contribution when only 6 months of real contribution are proofed may contribute to the cited gap between theoretical and real incomes.

7.5.3 Evolution of the average retirement benefit

The evolution of the average retirement benefit amount results from the years number evolution of contribution, the annual revaluation rate, and the final wage evolution. The evolution of the net wage with age reveals that in 2011, wage at [55-60[represents 1.11 of the average net wage. To make the relationship between salaries and age, we proceeded to a polynomial fitting based on the data provided in ONS (2014-a) which gives the evolution of the wages by large age groups <25; [25-39[, [40-49[and [50 and +[. The obtained results are shown in Figure 7.27.

Figure 7.27: Wage Vs. Age (2011)



Source : Calculated from ONS (2014-a)

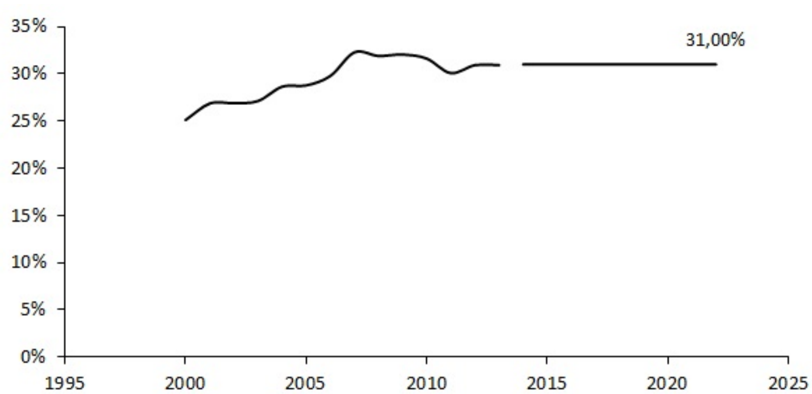
The retirement benefit amount is directly related to the average wage of the last 5 years of the working career. If we admit that the average age for retirement is 59 years for men and 57 for women, the wage to be considered in the calculation of the retirement benefit corresponds to the average wage between 54 and 58 for males and 52-56 females. More details about this point are presented in subsection 7.6.2.

In addition, the evolution of the retirement benefit amount is also affected by the annual revaluation as shown in Figure 7.11. By including all the factors affecting the evolution of the average retirement benefit amount, we have shown in Figure 7.10, that the average growth rate is evaluated at 9.28% during the period 2000 - 2013.

7.5.4 Survivor's retirement expanses and administration fees

In addition to the direct retirement benefits, CNR provides survivors benefits for : spouses, orphans and also parents. Modeling the expenses evolved by survivor's benefits is complicated and can be separatly treated by another work. Our idea is to consider the survivor's benefits expenses as a percentage of the direct benefits ones according to the recent observed trend. The evolution of the report of survivors benefits expenses on the direct benefits expenses during the period 2000-2013 is shown in Figure 7.28.

Figure 7.28: Evolution of the report survivors on direct benefits expenses



Source : Calculated from data provided by CNR (2014)

The part of survivor's benefits expanses represented 25% of the direct benefits expenses in 2000. This part has evolved to stagnate at around 31% starting from 2006. For the needs of the present work, we will suppose that this report will keep turning around this value during the upcoming years.

In addition to retirement expenses (Direct and survivor's benefits), the total outcome of CNR comprises administration fees. These expenses represent in average between 2006 and 2013 a proportion of 1.5% from the retirement expenses. We suppose that this proportion will be kept at this level during the upcoming years.

7.6 Incomes - Outcomes projection

In order to expect the future evolution of the incomes and outcomes of the employee's pension scheme, we preferred to separate in our calculation: the actual portfolio in service and the future generations including that of the current contributors. The actual portfolio of retirees will firstly be supposed to be closed for new entrances. Then, in a second step the late entrances will be addressed. In contrast, the current contributors' portfolio can undergo some exits and entrances depending the affiliation behavior of the working population. For that, calculation concerning this category will be done as part of the future generations.

The age of departure for retirement will be supposed to be equal to the average age observed in 2013 which is 59 for men and 57 for women. Parallel to this, the working age will be extended until 65 years in order to take into account the individuals whose keep working beyond the average age of retirement.

We remind that in the present work, the specificity of the retired population compared to the global population in term of mortality will be considered in the outcomes projection. As shown in Chapter 6, the prospective experience life tables were constructed and projected on the basis of the mortality experience of retirees. It will be more coherent to use these tables to describe the mortality of retirees rather than the use of the global population life tables.

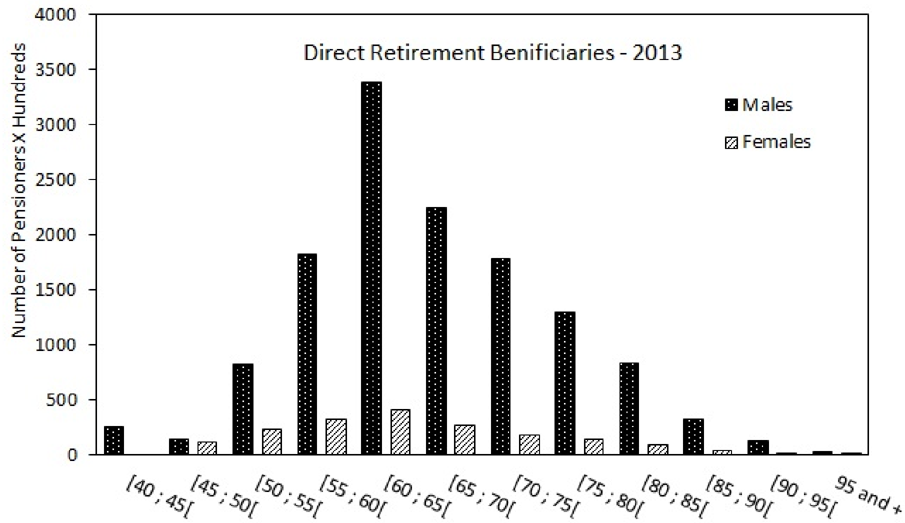
As we mentioned above, the minimal number of years of contribution which allows to accede to retirement is 5 years (retirement allowance), while 32 years of contribution allow to benefit a full retirement. Such a detail is not addressed in our calculation and we suppose simply that workers contributing less than 5 years are compensated by those having contributed for more than 32 years.

7.6.1 Portfolio in payment phase

In first, we will estimate the future distribution of pensions to be payed for the portfolio of retirees in payment phase according to the 2013 closing situation. The distribution by age groups and sex of the direct retirement beneficiaries in 2013 is shown in Figure 7.29.

We observe that the age group [60-65[contains the most important number of retirees. Even if the minimal regulatory age for retirement is fixed at 50 for men and 45 for women, we observe that some men has have retired before this age. This case represents an exception compared comparing to the general rule. During the terrorism decade that Algeria knew during 90's, the government created a security institution to protect the rural areas. After the end of the terrorism events, this institution became useless and a part of the fired agents has benefited from an exceptional early retirement starting from 2011.

Figure 7.29: Distribution by sex, age of direct retirement benificiaries - 2013



Source : CNR (2014)

The objective of the present part is to expect the time evolution of the total retired benefits of the presented portfolio. This evolution will results from the evolution of 2 parameters : 1) The survival function of the concerned retirees and 2) The evolution of the average amount of retirement benefits. The evolution of survivals among the population of retirees is described by the experience prospective life table constructed in Chapter 6 that we baptize EPLT - 2013. In concern of the second element which consists in the evolution of the average amount of the pension benefits, we will suppose that this last will keep growing with a rate of 5% during the upcoming decades.

Formulation:

Let $l^s_{x,t}$ denotes the surviving retirees at age x and year t of sex s = males or females. Until now, the population of retirees is arranged on five age groups. To obtain the single age distribution, we use the Karup-King method to break-out the five age populations into a single age populations. This method is well explained in Chapter 4. The open age group [95 and +[is supposed to be closed at 100 years. The weakness of the number of retirees in this age group permits a such hypothesis without affecting significantly the final results of the present work. Let also ${}_np^s_{x,t}$ denotes the surviving probability between ages x and $x+n$ between the year t and $t+n$ (EPLT 2013), and let R to be the annual average direct pension benefit growing with a rate r=9%. The total pension benefits at year t related to the portfolio in service can be calculated as :

$$TPB(1)^s_t = \sum_{x=x_0}^{w} l_{x,t}.R(1+r)^{t-2013}$$

Note that the number of surviving retirees is evolving from a year to another following the relationship $l^s_{x+1,t+1} = l^s_{x,t}.p^s_{x,t}$. The total retirement expenses covering the global population is calculated by summing the male's and female's REs:

$$TPB(1)_t = TBP(1)_t^{males} + TPB(1)_t^{females}$$

7.6.2 Next generation of contributors - retirees

The projection of the future incomes - outcomes related to the future generations of contributors and retirees must be principally based on the population forecast. As we mentioned it earlier, all the population at working age will be considered as a potential population of contributors. Then, the probability to be affiliated as an employee combined with the expected evolution of wages will allow to estimate the average contribution by a single age between 15 and 64 years. Proceeding similarly, the population at retirement age will be considered as potential retirees. Then, the average duration of contribution combined with the expected final wage will allows to estimate the average pension benefit amount. we remind that the expected duration of contribution is 15 years for men and 4.6 years for women at working age.

Expected contributions

The expected contributions are calculated on the basis of the evolution of the population aged [15-65[issued from the population forecast. The number of the population at age x at time t is denoted $\Lambda^s_{x,t}$ with s =males, females; $t \geqslant 2014$ and $x = 15, 16, \dots 64$. The probability to be affiliated as an employee which represents also the probability to contribute for CNR at age x is noted AS^s_x (see Figure 7.26). From 2009 to 2013, the average crude wage in the public sector has grown with an annual rate of 9.5%. The same annual evolution rate was observed on the average net wage between 2009 and 2013 all sectors included. For that , we suppose that wages will keep growing with the same rate during the future years. We can write: $\bar{W}_t = \bar{W}_{2013}(1 + 9,5\%)^{t-2013}$.

The total contributions that will be payed during the year t (noted TC_t) is calculated by summing the contributions to be payed by different cohorts during the same year noted C_{xt}. The calculation formula can be written as:

$$TC_t = \sum_{x=15}^{65} C_{xt} = l_{xt}.AS_x.\bar{W}_t.18.5\%$$

Expected pension benefits

The calculation of the expected pension benefits for the future generations of retirees returns to expect the average duration of contribution, the final wage evolution, the age of retirement and the annual revaluation rate of pensions benefits. The average age of retirement excluding exceptions is 59 for men and 57 for women. We suppose that the departure for retirement will be fixed around these averages during the upcoming years. Then, the wages corresponding to the last 5 working years serve to calculate the pension benefit amount. Here the wages between [54-58] and [52-56] respectively for men and women will be considered in Pension Benefits calculation. For simplification issues, we replace the age intervals by the central ages. It means that the average wages corresponding to 56 and 54 years ($W_{56,t}$ and $W_{54,t}$) will be considered respectively for men and women. Because these details are not provided on the ONS publication, we will proceed to estimate $W_{56,t}$ and $W_{54,t}$ from $\bar{W}_t$ according to the polynomial function shown in Figure 7.27. We obtain $W_{56,t} = 1.107 * \bar{W}_t$ and $W_{54,t} =$

$1.104 * \bar{W}_t$. The expected duration of contribution is supposed to be 15 years for men and 4.6 for women (see subsection 7.5.1). The annual revaluation of pension benefits amounts will be supposed to keep around 5%. In first, it convenes to calculate the annual pension benefit corresponding to the first year of retirement that we denote $\bar{PB}_{m,t}$ (m denotes the retirement age) and calculated by:

$$\bar{PB}_{m,t} = W_{m,t} * EC * 2.5\%$$

Then, the time and age evolution of the annual pension benefit amount is supposed to keep growing with a constant annual revaluation rate of 5%:

$$\bar{PB}_{m+n,t+n} = \bar{PB}_{m,t}(1 + 5\%)^n$$

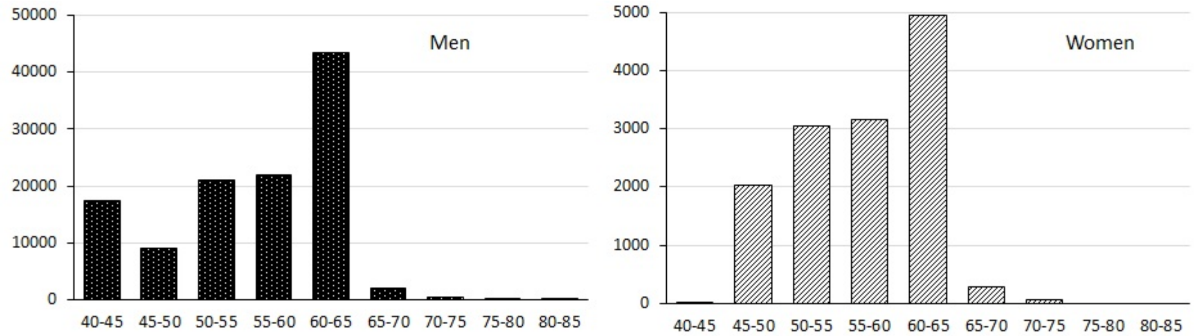
The total pension benefits distributed during a year t is calculated by multiplication on the population number in each generation. If we consider $\Lambda_{m,t}$ as the population number at the age of departure for retirement m during the year t issued from the population forecast, the survival function of this starting population will be estimated in the basis of the life table EPLT-2013 (Chapter 6). The total PB distributed during a year t can be calculated by :

$$TPB(2)_t = \sum_{x=m}^{w} \bar{PB}_{x,t} * \Lambda_{xt}$$

7.6.3 The whole system stability

The evolution of the total pension benefits should be calculated by summing the pension benefit of the current portfolio of retirees and the pension benefits that will be distributed to the future generations. Here, there is a risk of double reserving or an omission in concern of certain categories of retirees. We remind that the first group is observed by the end of 2013 and it is constituted by retirees aged 40 and over. The second group is constituted by the future generations whowill get retired starting from 2014 when reaching the age of 59 for men and 57 for women. The risk of double computation concerns the individuals who are already retired before reaching the average age for retirement. Those retirees will be computed again in the second group when they reach the average age for retirement. To escape such a problem of double computation, we remove the retirees aged below the average retirement age from the first group. In the other hand, the current portfolio of retirees (1^{st} group) is not supposed to be closed for new entrances among the oldest generations not already retired. If the workers aged below the average age of retirement will be computed with the future generations, those aged beyond the average age of retirement in 2013 and not already retired yet, are for now excluded from any consideration. For that, the future entrances in the current portfolio of retirees between the average age of retirement and 65 years must be estimated and added to computation. The population aged between m and 65 years in their evolution over time must be augmented by the future workers to be retired (not retired yet). To do that, a distribution of the probability of getting retired among the working population must be calculated for each age. Then, the retirement schedule will be dressed. CNR (2014) provides the distribution of workers getting retired in 2013 by five age groups. Figure 7.30 shows this distribution for the case of men and women.

Figure 7.30: Distribution of workers getting retired in 2013 by sex and age

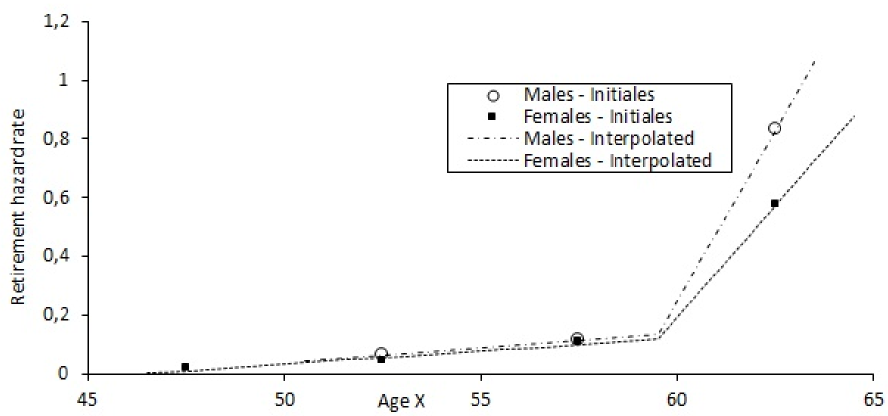


Source : CNR (2014)

The total number of the new retirees in 2013 is evaluated to 129000, 90% are men and only 10% are women. We can observe that an important part of the total number of departure for retirement has occurred at 60-65 for men and women. Beyond the age of 65, the proportions are much less important. The numbers that we can observe between 40 and 50 for men represent a special case in concern of the years 2012 and 2013 (fired municipal guards). If we consider that the minimal regulatory age for retirement is 50 for men and 45 for women and 65 year to be an extension of the normal retirement age which is 60, the probability of getting retired at each age between [45-65[and [50-65[can be calculated.

Costa (1998) tried to approximate the probability of retirement at single age when this late is reached without having retired yet, from studying the labor force participation rates issued from censuses data. Here, we simply estimate this probability (rate) by dividing the number of workers getting retired in 2013 by the number of the affiliated employees during the same year with a distinction by sex and age. In Subsection 7.5.1, we did an approximation about the distribution of the affiliated employees by age and sex. The report of the number of workers getting retired in 2013 at each age group $[x, x+5[$ divided by the structure of the affiliated employees by age gives the hazard rate of retirement for workers not yet retired as it is represented in Figure 7.31.

Figure 7.31: Probability of retirement by age -2013

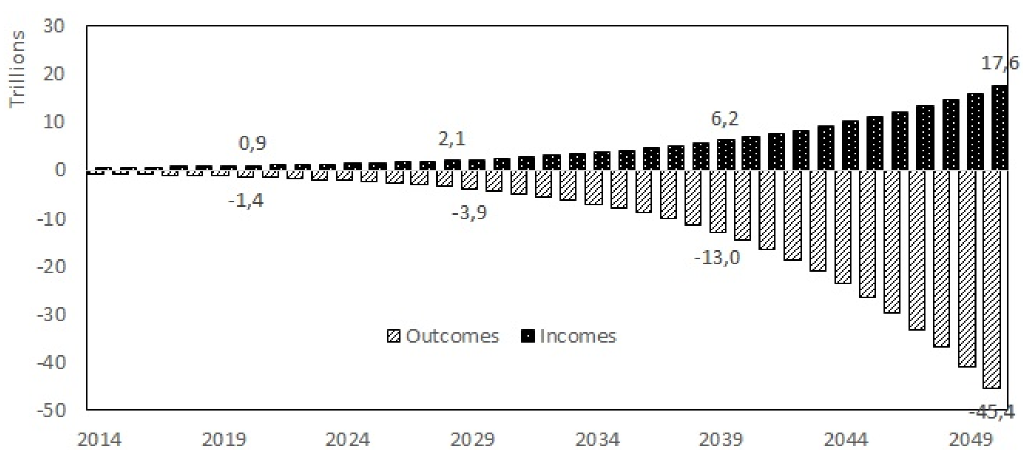


Initial calculations gave the hazard rate for getting retired by five age groups between 50 and 65 for men and 45 - 65 for women. For our application's needs, a description by single ages is needed. For that, we proceeded to an interpolation of the rates at single ages from the five ages rates. A change point can be observed starting from age 60 because the regulatory retirement it fixed at that one, and the use of a unique interpolation function along the whole age range may leads to unsatisfactory fitting quality. The use of a junction of two linear functions allowed to escape this disadvantage. However, and in order to make the interpolation results more adapted to reality, the junction point was extended until 60 years rather than to be the center of the age group [55-60[as it is shown in Figure 7.31. In other words, is more realistic that the probability to retire starts to rise starting from the age of 60 than starting from the age of 57.

In the rest of this application, the retirement probability age pattern is supposed to keep constant during the upcoming years. That allows passing from an annual to a generational description of the retirement probability age pattern. These calculated retirement schedules will be used to estimate the future entrance in the portfolio already retired by the end of 2013. According to that , the evolution of the number of retirees in each generation among the portfolio in service is defined by the death and the decision of retirement.

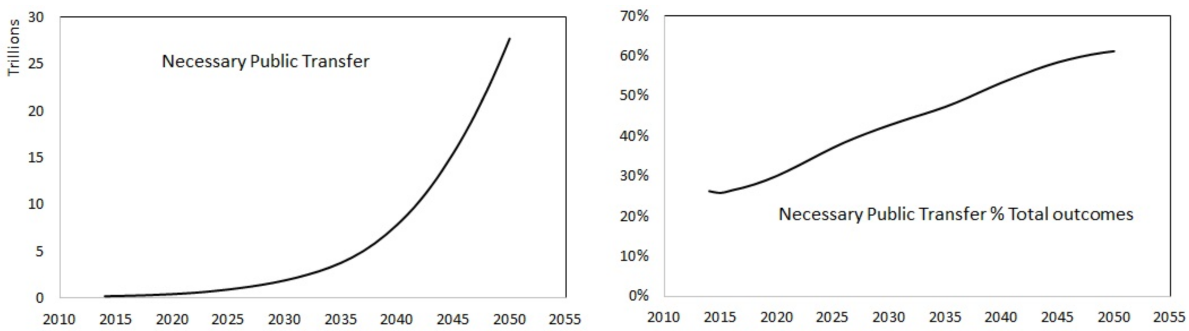
The projection of the financial balance of the Algerian pension plan returns to project separately the total incomes and the total outcomes. The total outcomes are constituted from the total retirement expanses augmented by 1.5% representing the administration fees. Retirement expanses groups the direct retirement benefits plus 31% representing the survivor's benefits. The total incomes are principally constituted from the contributions of the affiliated population augmented by an investment returns of about 2.5%. We remind that there is a difference between the theoretical total contributions and the real contributiosn of about 20%. The difference between the total incomes and the total outcomes are covered by public transfers. Figure 7.32 shows the projected total income and outcomes according to the hypothesis mentioned above.

Figure 7.32: Projected income - outcomes



According to the projection results, total incomes in 2014 are evaluated to 519 trillions dzd against 703 trillions for total outcomes. Public transfers cover 26% of the total outcomes. By 2030, the part of the public transfer needed to keep the financial balance passes to around 43% from total incomes and to 61% by 2050 as shown in Figure 7.33.

Figure 7.33: Projection of public transfer in pension plan incomes



The financial deficit of the CNR in 2013 was at 23% and it is covered by public transfers. This part represents the function of redistribution of the government which aims to maintain the payment of pension benefits by taxation. For now, taxation incomes are greatly based on the gas oil sector. The gap between pension plan incomes and outcomes is supposed to reach a total value of 27.7 trillions dzd in 2050 covering 60% of the total outcomes. In such conditions, we wonder if the government can support this permanent growing gap for the long run and ensure equality between the actual and the future generations.

7.7 Conclusion

The importance of pension plans in reducing poverty and old population protecting highlights the need to study their financial sustainability for the long run. Even if in most of pension schemes pension benefits are somehow related to the payed contributions following the insurance principle, public transfers are necessary to preserve the low earning population in a social protection context. However, there is no evident optimal combination between these two principles and there is not an evident way to design a pension system adapted to the whole population. A preference for the insurance principle will disadvantage the low earning population and the adoption of a pure solidarity system will pucks the high earners out of the system. Maintaining the stability of the system for the long run needs a public managing. In some particular conditions, the private sector can not support the conjunctural losses. The government may support the conjunctural deficit in order to save the well being of the low earning population and to encourage economic development for the long term.

In Algeria, the financial balance of the pension plan is still maintained by public transfers. The actuarial return rates of pension contracts is almost superior to the return rates in financial markets (Mendil, 2014). Even if the public budget is expected to recover the future unbalances, that must be equally distributed between the different generations. Admitting a fixed level of the State contribution in pension plans incomes, this last must be periodically evaluated and updated in order to ensure the viability of the pension system. The generosity of the Algerian pension plan, as in north African countries, is supposed to fall down during the upcoming decades with the increase of the number of retirees and the relative decrease of the public incomes (Ben Brahem, 2009). In such conditions, it will be difficult, but necessary, to ensure equality between the actual and the future generations.

We have shown in this chapter that the effect of longevity and population aging on the pension plan sustainability is permanent and not conjunctural. The financial deficit is expected to rise continually until a level of 61% by 2050. For that, a parametric reforms can not be sufficient to ensure the long run sustainability of the system. Augmenting the affiliation rate or the retirement age can not be sufficient to reduce the financial gap for the long term. A large reform of the whole system is needed. If we intend that it will be possible to keep the public transfers at around 25% of the total outcomes, the constancy of this part will improve equity between the actual and the future generations. For the long run, it will be necessary to revise the actuarial linkage between contributions and pension benefits. Improvement of longevity does increase the duration of payment of pension benefits. If the retirement age is not augmented, that leads to an unbalance between the working career duration and the retirement duration. Population aging provokes a fall in the report of the population at working age on the population at retirement age. To keep total incomes at the level of total outcome in a PAYG system, it will be necessary to ask more contribution efforts to working people while moderating the benefits distributed to retirees.

According to all these finding, it will be necessary to pass from one-pillar to a multi-pillars system as it was suggested by the World Bank (1994). That will allow to reduce the financial deficit of the whole system and restrict public transfers in a way to ensure the long run sustainability. It will be important to keep the actual system with some adjustment and to dress a second pillar, mandatory fully funded and privately managed. The first pillar is to be re-designed in order to manage the public transfers and ensure more equity while the second aims to encourage the individual contribution efforts. However, this second pillar needs to a strong financial market to invest the contributions of the affiliated workers.

General Conclusion

Pension plans as a part of the social protection system aim to protect the vulnerable populations from the late life risks. Losing the working capacity when reaching a high age, and the rise of health care expanses are the main risks conducted by longevity. In pension plans designing, we distinguish two main approaches : a social protection pure approach and a pure insurance approach. The first approach is based on the re-distributive function of the State, non contributory and covers the whole elder population (universal system - Pillar 0). The second approach is fully funded and retirement benefits are linked to the individual contribution efforts (Defined Contribution schemes : Pillars 2 and 3). Between these two extremes, many combinations have been developed. In this sense, the unfunded pension schemes work following the inter generational solidarity principle where the actual working population pays the pension benefits of the actual retired population under a public management. However the pension system is, its designing is oriented to serve two main objectives : poverty reducing and contributions - pensions benefits smoothing. The first objective aims to provide the elder vulnerable population financial sources allowing them to satisfy their basic needs. This objective can be perceived from different points of view. It can consists on a social service made at the disposal of elder people or on a financial subsidy linked or not to the affiliation statue. In a universal scheme, all the elder population get access to a minimal and standard financial subsidy unconditioned by the affiliation statue. In such a case, a parallel fully funded scheme (Pillars 2 and 3) can be implemented in order to allow individuals to adjust their saving efforts to their pension amount preferences. For unfunded schemes, the financial subsidy payed by the government and financed by taxation is added to the pension benefits payed for the retired population (Pillar 1).

Poverty reducing which represents an ultimate objective of the public pension plan, requires a strong management and financial stability for the long run. For the case of Pay-As-You-Go Defined Benefits systems, the financial sustainability is related to the evolution of the population structure and the pension plan cycle. The retirement mechanism implies that a certain number of contributions must be payed before to open right to a pension benefit. According to that, in the first decades following the implementation of a new pension plan, the population of contributors is much more important compared to the population of pensioners. This report has to decrease in function of the maturation process of the pension plan. For that, a positive financial balance during the first decades can not assess about the sustainability of the system for the long run. This last is mainly based on the comparative future evolution of the population of contributors and the population of retirees. In a late maturation stage of a pension plan, we can expect that the second population will grow faster than the first one. In addition, there are many other factors affecting the increase of the pension systems financial unbalance. Longevity improvement accompanied by a reduction in fertility rates leads to population aging and to an apparent and permanent change in the population structure. Affiliation to social security, labor force and workforce participation are also a significant determinants of the financial sustainability of a pension plan.

In its report, the World Bank (1994) averted from the consequences of population aging on pension plans sustainability and insisted on the necessity to proceed to a heavy reform in order to maintain the long term financial sustainability. Since this date, many reforms have been conducted in many countries all over the world. The main recommendation was to pass from one-pillar to multi-pillars system. It was more than evident, that a one pillar system can in any case fling aside the aging consequences.

Algerian pension plan is organized into two regimes : the first one is dedicated for employees (CNR) and the second covers the self-employed population (CASNOS). The whole system works following Pay-As-You-Go defined benefits principle. It is mandatory and publicly managed. When analyzing the recent observed trend in the financial balance of CNR, we observe that some difficulties start to rise in term of incomes -outcomes balance. In 2013, public transfers covered 23% of the total expanses of CNR. The objective of the present work was to provide a prospective analysis of the future evolution of the financial balance of the Algerian pension plan under population aging and longevity improvement.

To give a clearer view of the future sustainability of the Algerian pension system, it was necessary to project the population structure by forecasting mortality and fertility and quantifying the future reduction in the mortality specific to the retired population. Even if fertility rates can be projected by using probabilistic models, the point of view of experts has a capital importance. In a comparative point of view, mortality forecasting tends to be more based on a probabilistic approach. For our objective needs, a relevant part of applications in the present work were dedicated to project the mortality age pattern of the global population and also the mortality specific to the retired population.

Mortality forecasting must be based on a long and strong historical mortality surface. For that, it is necessary to assess the reliability of mortality data at our disposal. In this sens, Chapter 2 was dedicated to give a deep analysis of the mortality evolution in Algeria since 1962. Our analysis focused on the life expectancy at birth which is supposed to summarize the mortality reduction at all ages. Other indicators as the Infant Mortality Rate and the Crude Mortality Rate were used to assess some findings. Our motivation to do such analysis was to separate the natural evolution of mortality in Algeria from the methodological imperfections. It turned out from the analysis that the natural evolution of mortality in Algeria was heavily affected by the methodological changes and imperfections.

The Office of National Statistics (ONS) started from 1977 to publish life tables for the Algerian population. These life tables were based on the data issued from the civil registration and corrected on the basis of specific surveys. Before this date, statistics about mortality in Algeria were somehow poor. A first attempt to estimate the mortality age pattern was conducted in 1969-70 by the occasion of the National Statistic Population Study but it was only based on the northern population of Algeria. By the following, the ONS tried to make the publishing of the national life tables more regular and that could not be possible until 1998. By the same, some life tables were closed out before the age of 80 years. The implementation of the prospective mortality models requires the availability of a complete historical mortality matrix containing ages in columns and years in lines. Chapter 3 was dedicated to estimate the missing data in the Algerian mortality surface. Li et al. (2004) focused on how to project the age specific mortality rates when data is imperfect or incomplete. However, the proposed method allows projecting mortality only when the historical observed trend is linear. In order to reduce model risk when modeling and projecting mortality, a set of candidate models must be evaluated and compared and the hypothesis of linearity must be considered as one among other possible cases. For that, we preferred to estimate the missing data in order to have more choices regarding mortality models selection. Our idea was to use the Lee-Certer model (Lee and Carter, 1992), which was first proposed for projecting mortality surfaces, to estimate the missing data in the existing mortality surface.

The Lee Carter model is based on the decomposition of the historical mortality surface into three components, two are related to age and the third represents the time variation index. For forecasting issues, only the time parameter is projected. In our case, the three parameters are estimated on the basis of the available data. The missing years in the time index series are estimated by a polynomial fit. Then, the missing mortality rates are estimated by combining the initial calculated age parameters with the estimated time parameter. To improve the coherence and the adequacy between the existing data and the estimated one, the Lee-Carter model was implemented by sub-surfaces having a similar local profile. In other words, we proceeded to a time and age segmentation grouping the age categories having a similar mortality profile and considering the break points in the time index series.

Until this point, we have completed the mortality surface for ages between 0 and 80 and for time going from 1977 until now. Usually, the mortality at ages older than 80 is estimated either by using the extrapolative methods or on the basis of the model life tables. The ONS uses the second method and proceeds to the estimation of the residual life expectancy at the closure age to summarize the mortality rates evolution for the ages beyond 80. Ekanem and Som (1984) explained how a wrong use of this estimation method can affect the old age mortality estimates. They also noticed that such practice is very frequent in African countries and we have shown in Chapter 2 that the Algerian life tables present such methodological imperfections. To deal with this and in order to improve the old age mortality estimation methods and results, we proposed in Chapter 4 to complete the old age mortality rates in the historical mortality surface on the basis of an extrapolative approach.

The extrapolative approach is based on the use of specific models to extrapolate mortality rates beyond the age of 80 until (or nearly) the surviving age limit. Several models issued from the literature were evaluated and compared in this sense (Coale and Guo, 1989; Coale and Kisker, 1990; Denuit and Goderniaux, 2005; Kannisto, 1992; Thatcher, 1998;1999; Gompertz, 1825 ...etc). In first, the different models were calibrated and evaluated on different age ranges between 45 and 75 years, the predictive capacity has been tested on the age range [75-85[for the years where mortality rates at these ages were available (2010-2014). Then, some qualitative and quantitative criteria were added to perform the comparison process as the male-female coherence, single sexes and both sex estimates adequacy and also the predicted age limit. It turned out that the quadratic models (Coale-Kisker and Denuit Goderniaux models) give a better quality regarding the combination of all criteria: goodness-of-fit, predictive capacity and the predicted age limit. Gompertz model which is a linear transformed model gave also a sufficient results. However, the quadratic models have a highest capacity to combine fitting quality and expected age limit. Because when quadratic models are calibrated, an age limit constraint can be easily added to the estimation process. To improve the quality and the adequacy of the estimates, some constraints were added to the estimation process concerning the male female coherence and the single-both sex adequacy. After comparison, it turned out that the Coale-Kesker model with an age limit constraint at

120 years is more adapted to the Algerian data. The value of 120 years was fixed according to the finding of Flici et Hammouda (2016) who observed a maximum age of 110 and 112 for males and females already alive when analyzing the results of the Multi Indicators Clustery Survey MICS-2012. According to this, a value of 120 years is very acceptable as a theoretical age limit for the Algerian population.

In a final stage, we completed the existing life tables by the extrapolated mortality rates beyond the age of 80 years. In comparison to the ONS' estimates based on the model life tables, we noticed that life expectancy at birth was augmented by around 0.5 years in average for the period 1977-2014. The obtained results can serve as a methodology for old age mortality estimation in the Algerian case, in the historical context and also when projecting mortality in the future (prospective life tables).

Chapter 5 treated the projection of the age specific mortality rates in the future. A set of a prospective mortality models have been evaluated and compared. We distinguished two families of models : Lee-Carter model and variants (Lee and Carter, 1992; Renshaw and Haberman, 2006 and Currie et al. (2006)) and Cairns-Blake-Dowd model and variants (Cairns et al, 2006; 2008). The Lee-Carter model proposed in 1992 is based on the decomposition of the log of the age specific death rates surfaces into three component. The first component represents the average age mortality pattern on the whole observed period; the second represents the sensitivity of mortality by age to the time variation in mortality which is represented by the third component. Renshaw and Haberman (2006) added a cohort effect, and Currie et al. (2006) simplified the model by applying a common age sensitivity for time and cohort effects. The initial CBD model is based on the linearity of the log mortality curve starting from a certain age. The second variant of the model comprises an additional component representing the cohort effect. In the third variant of CBD model, the age mortality curve is represented by a quadratic term. The forth variant comprises a quadratic age effect and a cohort effect.

In our application, these 7 models respectively baptized M1, M2, M3, M5, M6, M7 and M7* were evaluated and compared regarding the goodness-of-fit. To take into account the difference in the number of parameters between the different models, we used the Bayesian Information Criteria BIC (Schwartz, 1978) and the Akaike Information Criteria AIC (Akaike, 1973) as a fitting criteria. An adaptation of these two criteria was done to suit the case of the least squares estimation process as it was suggested by Burnham and Anderson (1998) and Hansen (2007). A first evaluation revealed that the two models M7 and M7* which represent extensions of M5 and M6 do not have any added value compared to the initial models. So, only the five other models have been kept for the rest of the comparison. To perform the comparison, some additional criteria were used : the short term predictive capacity, the regularity of the forecast, the expected life expectancy and the coherence between the male and female estimates. To evaluate the short term predictive capacity, we calibrated the models on the data of the period [1977-2010], the projected data are compared to the data observed during the three recent years 2011, 2012 and 2013. A graphical analysis of the forecasting results revealed that when the projection of the times parameters (under the 5 models) is based on the whole period (1977-2010), that leads to an apparent incoherence between males and females traduced by an excessive divergence or convergence over time. Better results were obtained when only the trend observed after 1998 is used as a basis of the forecasts. We already signaled in Chapter 2 that mortality data quality has greatly improved starting from 1998 and that can explain the results obtained in Chapter 5. The evaluation of the forecasting results according to the male and female mortality coherence revealed that only the models M3 and M5 provided an acceptable quality. To go ahead in our analysis, the two models were used to project the mortality surfaces in the future. The old age mortality was estimated by using the Coale-Kisker model with 120 years as a theoretical age limit as it was suggested in Chapter 4. Finally, the expected life expectancy were calculated and compared. It turned out from the analysis of the life expectancy evolution that under M5, a crossover of the male and female life expectancy is generated by 2060. Under M3, the female life expectancy keeps evolving beyond the male life expectancy until 2100 when we finish by having a crossover at that date.

In the conclusion of Chapter 5, we highlighted the limits of the independent mortality forecasting models. Even if we try to obtain certain coherence in forecasting when male and female mortality is independently projected, the coherence of the results is not guaranteed. The use of coherent mortality models (Li and Lee, 2005; Hyndman et al., 2013) is highly recommended in this sense.

Among the actuarial best estimates practices, the adaptation of the calculation tool to the covered risk represents a fundamental principle. In this sense, the life tables used for life annuities or pension plans actuarial calculations must be highly adapted to the population directly concerned. The use of the global population life table to dress life annuities pricing or pension plans sustainability can distort all calculations if the mortality of retirees is different from the mortality of the rest of the population. So, the use of the experience life tables is highly recommended. However, the length of the historical experience data series and the limited number of observations reduce the robustness of the forecasting results. In the other hand, global population mortality data is usually available for longer length and calculated on more important number of observations. That makes global population mortality data more suitable for projection issues but less adapted to the experience of the retired (or the insured) population. A way to combine two objectives (forecasting capacity and adaptation to the experience mortality) consists in positioning the experience mortality of the retired population on the global population mortality data allowing a robust forecast. In other words, a regression of the experience mortality rates on the reference mortality rates must be calculated. Then, this regression equation is used to deduce the projected mortality rates for the retired population from the projected mortality of the global population (Planchet, 2005; 2006). This regression can be based on the Brass logit system (Brass, 1971). The same approach was used later by Kamega (2011) to construct a prospective experience life table adapted to the insurance market in subsaharian African countries.

The objective of Chapter 6 was to construct a prospective life table adapted to the mortality experience of the retired population in Algeria. The data of the Algerian National Retirement Fund (Caisse Nationale des Retraites) was used for this issue. The available data covers the period 2004-2013 and concerns the direct retirement beneficiaries issued from the employees' regime (survivor's beneficiaries and self employed are excluded). The retired population as the observed deaths are grouped by sex and five age groups from [45-50[to [95 and +[. Firstly, the five age death rates were calculated. The single age mortality rates were deduced by a polynomial fitting of the five age mortality rates which aimed also to smooth the crude rates and reduce the irregularities due to the reduced sample size. Then, the regression equation of the experience rates on the reference rates is calculated. In order to reduce the information loss generated by the regression, we proposed to use a quadratic rather then a linear regression. The reference mortality here, was supposed to be the global population mortality already shown in Chapter 5. Before to estimate the regression equation, the reference mortality surface was fitted with Lee-Carter model. In order to avoid any incoherent regarding the projected male and female mortality, a coherence mortality forecasting approach was adopted. The model proposed by Hyndman et al. (2013) was used for this issue. Flici (2016-c) conducted a comparison between the two main coherent mortality forecasting models (Hyndman et al (2013) and Li and Lee (2005)) and concluded that the first model leads to better results with fewer parameters. The projected mortality surface of the global population was used to position the experience prospective life table. Old age mortality rates were extrapolated by using the Coale-Kisker model as it was recommended in Chapter 4.

According to the final results issued from Chapter 6, the life expectancy at 50 years of the retired population will evolve from 30.4 and 34.8 years respectively for males and females in 2014 to 35.8 and 39 years in 2050. The gap in life expectancy at 50 years is expected to fall gradually from 4 to 2.8 years during the same period. If we observe the difference between the global population and the retired population mortality projections by sex, we observe an important similarity for males but a significant difference for females. The life expectancy at 50 for the global male population is 0.27 year higher than the life expectancy of the retired male population in 2014 which is supposed to fall to nearly 0 by 2050. We can conclude that there is no significant difference in concern of the male populations. In adverse, the gap is very significant in the case of the female populations. Retired women hope to survive 3.6 years more than women of the global population beyond the age of 50 years in 2014. This gap is supposed to fall down to a value of 2.2 in 2050. There are two possible explanations of this observed difference between the males-female gap in life expectancy between the global and the retired populations : 1) working conditions have an effect on the residual life expectancy. According to this assumption, men, either working or not, affiliated or not, are exposed to similar life and working conditions. For that, no difference is observed between the residual life expectancy between retirees and non-retirees. In adverse, it is observed that working women are more concentrated in non-risky activity sectors (administration & services) more than men (ONS, 2014-b). So, work gives women a particular life style and social statue and better life conditions compared to working men and women of the rest of the population. 2) The ONS underestimates the life expectancy of the global female population. This second assumption is supported by the results obtained by Flici et Hammouda (2016). When analyzing the data issued from the health survey MICS-2012, they found a gap of 3.9 years at 50 years between males and females of the global population. This results is near to the gap obtained between the male and female retired populations evaluated here at 3.95 in 2014. It lets suppose that the retired population (men and women) does not presents any particularity in term of residual life expectancy compared to the global population, and the difference mentioned above is resulted from a defection of the ONS estimates in concern of the female life expectancy. To affirm such assumption, we need to compare some additional sources of data (insurance companies, social security, self employed pension regime) and to conduct a deep investigation on the methodology and the crude data used by the ONS to estimate female mortality.

The prospective life tables based on the mortality experience of the retired population are needed in life annuities reserving, pension plan sustainability analysis and pricing / reserving in funded defined contribution pension schemes. The sustainability of pension systems is constrained by a set of demographic and economic factors. Longevity when combined with a reduction of fertility rates leads to population aging. It is not evident to separate perfectly the effect of longevity from the effect of aging on PAYG defined benefits schemes sustainability analysis. The economic factors consist on the change in the labor force and workforce participation, the affiliation to social security, wages evolution and the linkage earning-pension benefits.

In Chapter 7, we analyzed the long term sustainability of the Algerian pension plan following the expected future improvement in longevity and the population aging process. In first, we gave a short description of the evolution of the financial balance of the public pension regime on the light of the evolution of the demographic and economic factors. The evolution of the retirement incomes is directly related to the evolution of the affiliated employees, wages and contribution rates. The evolution of outcomes is related to the evolution of the population of retirees, the replacement rate, and the average pension benefit amount for which survivor's benefits and administrative expanses are added. The gap between incomes and outcomes of the CNR is estimated at about 23% in 2013 and it is covered by public transfers. Compared to the developed countries, the Algerian pension scheme is highly generous; the replacement rate of the final wage is 80% while this rate is around 70-75% in MENA region and much more less in developed countries (Robalino et al., 2005). Other forms of generosity are represented by the bonuses offered for certain categories of the population : women, Moudjahidine, parents ... etc. and the annual revaluation of the pension amount by about 5%.

In the current stage, the Algerian pension system is still in its first decades of maturation. Even if the total incomes cover the retirement expanses, we can not assess about the long run financial sustainability of the Algerian pension plan. The population is now aging, longevity is improving and the population of retirees is expected to grow faster than the population of contributors. A realistic evaluation of the long run financial sustainability of the pension plan passes necessarily by a projection of the different factors affecting incomes and outcomes. In first, a population forecast has been done in order to deduce the future evolution of the population at working / retirement age. Then, some assumptions about the economic factors evolution were dressed to achieve an analysis of the financial balance evolution for the long term.

The population projection was done by forecasting mortality and fertility rates starting from the population pyramid given by the general population census of 2008. Calculation was done by single ages. Mortality rates were forecasted by using the coherent forecasting model of Hyndman et al.(2013). Fertility rates was forecasted by using the Lee-Carter model (Lee and Carter, 1992) adapted for the case of fertility (Lee, 1993) while considering the views of experts regarding the future general trend of fertility. In concern of immigration, the absence of detailed data let us suppose a null immigration balance. After getting the evolution of the population structure by age and sex until 2070, we deduced the number of the affiliated population from the available rates about participation in labor force, working and affiliation to social security.

According to the results of the population projection, the report of the population at working age on the population at retirement age is supposed to fall down from a value of 7.2 in 2014 to stabilize around 2.7 starting from 2050. It means that the Algerian population is going to exit from an era of a “demographic dividend” where the population at working age is widely larger than the population at retirement age. That will make the maintaining of the financial sustainability of the pension system more harder during the upcoming decades.

The estimation of the expected evolution of total incomes returns to expect the evolution of the population of contributors, their average wage and contribution rates. We remind that in our application, we are interested to the contributors for CNR. That mains the occupied population affiliated to social security as employees. Self-employed are covered by another pension scheme (CASNOS). The ONS workforce survey 2013(ONS, 2014-b) provided us with the activity and employment rates by five age groups and by sex. It provides also the distribution of the male and female working populations by occupation statue (employee or self employed). The affiliation rate to social security is provided in overall for males and females without any detail about the age structure. These information allowed us to approximate the probability that an individual at working age contributes for CNR (as an employee). This probability is in final approximated for each single age between 15 and 65 years for males and females. For projection issues, and because of the unavailability of a long data series allowing to do any kind of forecast, these probabilities are supposed to keep constant over time. The future evolution of salaries was simulated according to trend observed on the ONS data. An annual growth rate of about 10.7% was observed during the period [1990-2013] and 9.5% during the period [2009-2013]. The crude wage is supposed to keep growing with a rate of 9.5% during the future decades. In final, contribution rates are supposed to keep constant at the current level of 18.25%. The evolution of the total contribution is then deduced. In addition to this, the total incomes of CNR comprises an investment return evaluated at around 2.8% from the total contributions and a public transfer evaluated at 23% from the total expanses in 2013. Note that CNR recovers only 80% of the theoretical contributions. The other 20% are supposed to represent the part of the unpaid contributions of the affiliation population voluntary or because of work leaving. We note that in the regulation of the Algerian social security system, a year of contribution is validated when contributions are payed during more than 6 months.

To estimate the evolution of retirement expanses, it was necessary to draw the future evolution of the retired population and the pension amount evolution. Pension benefits are directly linked to the final earning and the duration of contribution and the validation rate of each year of contribution. Access to retirement is constrained by having contributed for a minimum of 5 years during his working age (Retirement allowance) and 32 years of contribution allows benefiting a replacement rate of 80% (full retirement). Because affiliation histories are not or can not be dressed for each generation, it is not possible to deduce the future number of retirees in one generation starting from the expected contributors in the same generation. Our idea was to suppose all individuals at working age to be a potential contributors for CNR with a defined age probabilities. These single age probabilities of contribution for CNR allowed to approximate the expected duration of contribution for males and females. Men and women are expected to contribute respectively during 15 and 4.6 years during their working age. As the population of contributors, all people reaching the retirement age are supposed to be potential retirees. The average pension benefit amount is calculated on the basis of the expected duration of contribution and the final wage. This last is approximated from a regression on the average wage calculated from data provided in the 2011's salary survey (ONS, 2014-a). Note that according to data provided by CNR (2014), the average observed retirement age is 59 for men and 57 for women. Beyond this age, the survival function of retirees is described by the prospective experience life tables constructed in Chapter 6. An annual revaluation rate of 5% was applied to draw the evolution of pension benefits amount over time.

The evolution of total incomes and outcomes was calculated separately for the portfolio already in service by the end of 2013 and for next generations of contributors-retirees. The portfolio already in service is composed from retirees coming from successive old generations. The point was that in some generations, there are some working individuals not retired yet who are expected to get retired in the coming years. So, it was necessary to dress the retirement schedule of the oldest generations. The probability for getting retired at each single age (Costa,1998) was calculated from data about workers retired in 2013 (CNR, 2014) and that allowed us to expect the future new entrances in the current portfolio in service by individuals issued from the oldest generations. Calculation for the next generations was based on a common age forretirement of 59 for men and 57 for women and a working age going from 15 to 65 years.

In addition to the direct retirement expanses, total expanses (outcomes) comprises a survivor's retirement expanses and administrative fees. Because modeling survivor's pensions does not constitute a main objective in the present work, we preferred to project their expanses on reference to the Direct retirement expanses. Survivor's pension expanses represented during the period (2006-2013) a proportion of 31% of the direct retirement expanses and that we suppose to keep constant for simplification issues. Administrative fees represent about 1.5% of the retirement expanses.

According to the final results, the deficit between total expanses and total contributions of CNR is expected to grow up from 23% in 2013 until 61% by 2050. If this gap is currently covered by public transfers, maintaining the financial balance for the long term seems to be very complicated. The CNR should constitute financial reserves from the payed contributions during the first decades following the implementation of the Algerian pension system. In addition to population aging and longevity, the retired population is expected to grow faster than the number of contributors. In such circumstances, maintaining the stability of the pension system requires a deep reform. At least, the linkage of contributions to pensions benefits must be revised in order to ensure a relative equivalence. The part of public transfers can also be fixed at its current level in order to ensure inter generational equality. Ben Brahem (2009) mentioned that the actual generosity can not be kept for long periods. However, the replacement rate which is actually evaluated at 80% provide a reduced utility because of inflation. In the other hand, affiliation rates to social security must be strengthen in order to ensure a better risk sharing between the individuals of the whole population. In order to take into account the affiliation duration and the life expectancy after retirement, it will be necessary to augment the average retirement age.

To summarize, the actual configuration of the Algerian pension plan does not provide a favorable conditions for the long term financial stability. The future generations will be asked to improve their saving efforts and the government will be asked to restrict public transfers. Retirement subsidy must be destined only for the poorest retirees. In this sens, the current system can be kept with some adjustment regarding the linkage of contributions to pension benefits. Public transfers must be moderated and be provided only for the poorest retirees (Pillar 1). Parallel to that, a second scheme must be founded. It must be mandatory, fully funded defined contributions scheme, privately or publicly managed (Pillar 2). In order to convene the consumption - saving preferences of the high earning population who wants to have complementary retirement resources, a voluntary fully funded schemes can also be added as a 3rd pillar to the whole pension system. This third pillar is supposed to attract the high earning population and encourage them to improve their saving efforts. The first pillar will be oriented to satisfy the first objective of pension plans which is poverty reducing among the elder population. Government must have a long view on the evolution and the distribution of public transfers on the successive generations because it is necessary to keep certain equality in this sense. Pillars 2 and 3 will ensure a better linkage of the contributing efforts to pension benefits with some compensation between individuals among the same generation. The success of the 1st pillar is based on the knowledge of the real individual earning which pass necessarily by the regulation of the informal activity. Pillars 2 and 3 will need a strong financial industry and high economic stability. The success of the whole system will require the improvement of the part of formal sector in the economic system and affiliation rates to social security.

Note that the objective of the present work does not consist to the designing of the Algerian pension plan reform, but we take just to give some lights about the possible reforms. Pension plan designing requires a deeper analysis of the different factors (demographic, socioeconomic) affecting its stability. In addition, a pertinent analysis must be conducted in the framework of the economical and social policy of the country. Here, we focused only on the effect of population aging and improvement of longevity and we supposed a simplest scenarios about the future evolution of the economic factors. Participation in labor force and occupation rate can evolve positively or negatively during the next decades. The public sectors is still employing an important proportion of the population and we can imagine the effect that can have a withdrawal of the State from his function of employer. Also, sociological changes may have a heavy effect on the life style. Female activity can rise comparing to male activity and in such case the quantification of its effect on pension plan stability need a more detailed analysis. Affiliation to social security and occupation statue preference have also a great importance when evaluating the pension plans stability.

In the other hand, mortality forecasting either in the context of the global population or that of the retired population must be improved. Forecasting quality is widely related to the data quality and the lenght of the observation period. The historical mortality data in concern of the Algerian population must be revised because, as shown along this work, methodological changes have a significant effect on the historical observed trend of mortality. Also, data about the mortality of the insured population must be collected and arranged more frequently. We note that data and studies in concern of the mortality of the Algerian population still poor. In addition to the official life tables annually published by the ONS, there is any attempt to construct an official prospective life table. On the demand of the Algerian Health Ministry and the local office of the United Nation Fund for Population Studies, Flici et Hammouda (2016) tried to calculate a periodic life table from the MICS-4 crude data. Also, in the academic side, mortality forecasting is not treated enough. In Flici (2015), we have tried to forecast the Algerian mortality surface by comparing the Age-Time and the Age-Time-Cohort approaches. In Flici (2016), we constructed a prospective life table for the population aged 60 years and over for actuarial calculations needs. In concern of the actuarial life tablse, we note that the National Council for Insurance published in 2004 the only Algerian Actuarial life table based on the global population data of the period 1997-1999 (CNA, 2004). These life tables have not been revised or adapted to the insured population. The CNR, even though mortality data of the retired population is available, no attempt has been recorded in the way to construct an official life table, either annual or prospective, adapted to the retired population. At universities, mortality modeling and forecasting starts just to be treated by some students at the “Ecole Nationale Superieure de Statistique et d’Economie Appliquée”, “L’Université des Sciences et Technologies Houari Boumedien” and the “Ecole Supérieure de Sécurité Sociale”. Awaiting that these works give birth to some academic or professional publications, efforts must be intensified in order to improve data quality about mortality in Algeria.

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Appendix

Appendix A - The completed Algerian mortality surface (q_{xt})

Male_ Completed mortality surface qx

qx	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
0	0,128	0,115	0,110	0,107	0,101	0,090	0,085	0,096	0,080	0,073	0,067	0,064	0,062	0,060	0,058	0,056	0,058	0,057	0,058	0,057	0,055	0,056	0,040
1	0,057	0,046	0,050	0,054	0,050	0,047	0,022	0,022	0,018	0,015	0,013	0,011	0,009	0,009	0,008	0,008	0,009	0,009	0,009	0,008	0,008	0,008	0,007
5	0,017	0,020	0,014	0,011	0,010	0,010	0,013	0,014	0,012	0,009	0,008	0,007	0,007	0,006	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,004
10	0,012	0,016	0,010	0,007	0,006	0,006	0,010	0,010	0,009	0,008	0,007	0,007	0,006	0,005	0,005	0,004	0,004	0,004	0,005	0,005	0,004	0,004	0,003
15	0,013	0,016	0,012	0,010	0,010	0,009	0,008	0,009	0,010	0,009	0,008	0,008	0,007	0,007	0,006	0,006	0,006	0,007	0,007	0,007	0,006	0,006	0,005
20	0,015	0,017	0,016	0,016	0,015	0,015	0,009	0,009	0,009	0,008	0,008	0,008	0,007	0,007	0,006	0,006	0,009	0,015	0,015	0,012	0,010	0,009	0,008
25	0,018	0,019	0,017	0,017	0,016	0,015	0,010	0,010	0,009	0,009	0,008	0,008	0,008	0,007	0,007	0,007	0,010	0,016	0,015	0,013	0,012	0,010	0,008
30	0,020	0,020	0,021	0,024	0,022	0,020	0,012	0,013	0,013	0,012	0,011	0,010	0,008	0,008	0,008	0,009	0,011	0,016	0,014	0,012	0,011	0,010	0,008
35	0,023	0,021	0,027	0,034	0,032	0,030	0,016	0,016	0,016	0,015	0,018	0,012	0,011	0,010	0,010	0,011	0,014	0,017	0,015	0,013	0,013	0,012	0,010
40	0,028	0,025	0,031	0,036	0,035	0,035	0,020	0,018	0,018	0,017	0,016	0,015	0,014	0,014	0,014	0,014	0,018	0,023	0,020	0,018	0,017	0,016	0,013
45	0,041	0,035	0,039	0,043	0,041	0,040	0,023	0,023	0,022	0,022	0,020	0,020	0,019	0,018	0,017	0,018	0,022	0,024	0,026	0,024	0,022	0,021	0,019
50	0,057	0,047	0,056	0,061	0,058	0,055	0,040	0,034	0,035	0,035	0,034	0,034	0,028	0,032	0,028	0,034	0,034	0,038	0,036	0,034	0,033	0,032	0,028
55	0,079	0,080	0,081	0,087	0,083	0,080	0,058	0,055	0,056	0,056	0,057	0,055	0,047	0,054	0,048	0,056	0,053	0,053	0,053	0,050	0,048	0,046	0,040
60	0,118	0,114	0,114	0,119	0,112	0,110	0,087	0,079	0,102	0,095	0,095	0,074	0,064	0,064	0,083	0,085	0,080	0,091	0,087	0,087	0,081	0,076	0,067
65	0,179	0,153	0,167	0,165	0,161	0,157	0,155	0,133	0,139	0,136	0,135	0,132	0,129	0,130	0,133	0,134	0,114	0,119	0,122	0,114	0,111	0,104	0,097
70	0,274	0,237	0,228	0,212	0,200	0,194	0,212	0,212	0,212	0,212	0,212	0,212	0,189	0,212	0,224	0,212	0,222	0,194	0,192	0,188	0,182	0,179	0,161
75	0,402	0,350	0,343	0,331	0,306	0,290	0,324	0,321	0,324	0,326	0,324	0,319	0,276	0,315	0,375	0,323	0,314	0,278	0,278	0,275	0,272	0,278	0,260

Female_ Completed mortality surface qx

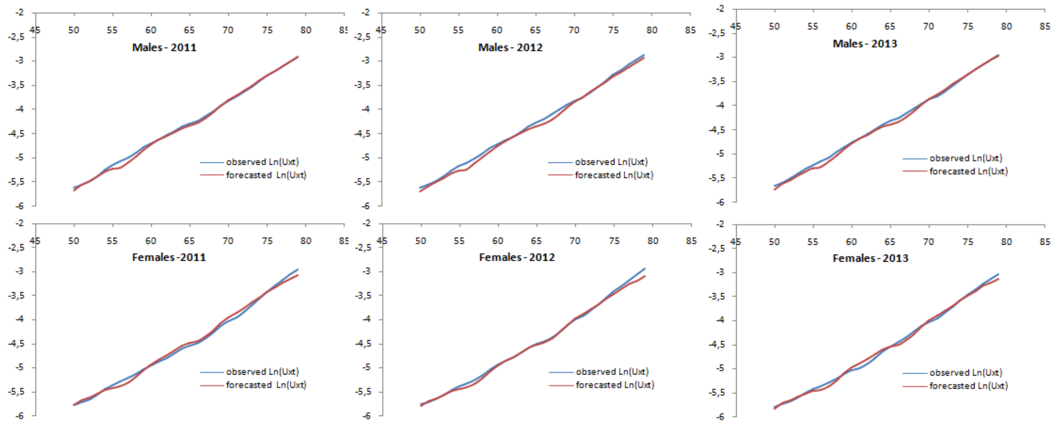
qx	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
0	0,128	0,115	0,105	0,107	0,101	0,090	0,085	0,081	0,080	0,070	0,067	0,059	0,062	0,054	0,058	0,053	0,058	0,057	0,058	0,057	0,051	0,056	0,040
1	0,057	0,046	0,051	0,054	0,050	0,047	0,022	0,024	0,018	0,017	0,013	0,012	0,009	0,010	0,008	0,009	0,009	0,009	0,009	0,008	0,009	0,008	0,007
5	0,017	0,020	0,015	0,011	0,010	0,010	0,013	0,013	0,012	0,009	0,008	0,007	0,007	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,004
10	0,012	0,016	0,009	0,007	0,006	0,006	0,010	0,007	0,009	0,007	0,007	0,006	0,006	0,004	0,005	0,004	0,004	0,004	0,005	0,005	0,003	0,004	0,003
15	0,013	0,016	0,011	0,010	0,010	0,009	0,008	0,007	0,010	0,007	0,008	0,006	0,007	0,005	0,006	0,005	0,006	0,007	0,007	0,007	0,004	0,006	0,005
20	0,015	0,017	0,015	0,016	0,015	0,015	0,009	0,009	0,009	0,009	0,008	0,007	0,007	0,006	0,006	0,005	0,009	0,015	0,015	0,012	0,006	0,009	0,008
25	0,018	0,019	0,016	0,017	0,016	0,015	0,010	0,010	0,009	0,008	0,008	0,008	0,008	0,007	0,007	0,006	0,010	0,016	0,015	0,013	0,006	0,010	0,008
30	0,020	0,020	0,020	0,024	0,022	0,020	0,012	0,012	0,013	0,010	0,011	0,010	0,008	0,009	0,008	0,008	0,011	0,016	0,014	0,012	0,008	0,010	0,008
35	0,023	0,021	0,026	0,034	0,032	0,030	0,016	0,016	0,016	0,014	0,018	0,013	0,011	0,012	0,010	0,011	0,014	0,017	0,015	0,013	0,011	0,012	0,010
40	0,028	0,025	0,026	0,036	0,035	0,035	0,020	0,018	0,018	0,017	0,016	0,016	0,014	0,016	0,014	0,015	0,018	0,023	0,020	0,018	0,016	0,016	0,013
45	0,041	0,035	0,028	0,043	0,041	0,040	0,023	0,021	0,022	0,019	0,020	0,019	0,019	0,019	0,017	0,018	0,022	0,024	0,026	0,024	0,020	0,021	0,019
50	0,057	0,047	0,027	0,061	0,058	0,055	0,040	0,029	0,035	0,030	0,034	0,029	0,028	0,029	0,028	0,030	0,034	0,038	0,036	0,034	0,028	0,032	0,028
55	0,079	0,080	0,056	0,087	0,083	0,080	0,058	0,048	0,056	0,050	0,057	0,048	0,047	0,047	0,048	0,050	0,053	0,053	0,053	0,050	0,042	0,046	0,040
60	0,118	0,114	0,082	0,119	0,112	0,110	0,087	0,057	0,102	0,081	0,095	0,063	0,064	0,056	0,083	0,085	0,080	0,091	0,087	0,087	0,072	0,076	0,067
65	0,179	0,153	0,131	0,165	0,161	0,157	0,155	0,110	0,139	0,121	0,135	0,113	0,129	0,110	0,133	0,122	0,114	0,119	0,122	0,114	0,105	0,104	0,097
70	0,274	0,237	0,181	0,212	0,200	0,194	0,193	0,194	0,193	0,193	0,193	0,194	0,189	0,194	0,224	0,193	0,222	0,194	0,192	0,188	0,170	0,179	0,161
75	0,402	0,350	0,314	0,331	0,306	0,290	0,371	0,315	0,354	0,385	0,374	0,334	0,276	0,311	0,375	0,395	0,288	0,250	0,248	0,247	0,246	0,278	0,260

Both sexes_ Completed mortality surface qx

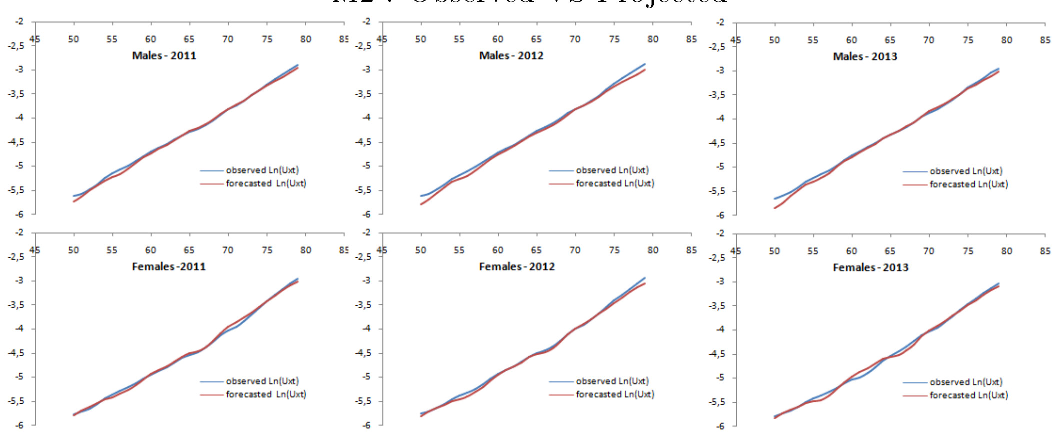
qx	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
0	0,128	0,115	0,107	0,107	0,101	0,090	0,085	0,084	0,080	0,071	0,067	0,062	0,062	0,057	0,058	0,054	0,058	0,057	0,058	0,057	0,053	0,056	0,040
1	0,057	0,046	0,051	0,054	0,050	0,047	0,022	0,023	0,018	0,016	0,013	0,011	0,009	0,009	0,008	0,008	0,009	0,009	0,009	0,008	0,009	0,008	0,007
5	0,017	0,020	0,014	0,011	0,010	0,010	0,013	0,014	0,012	0,010	0,008	0,007	0,007	0,006	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,005	0,004
10	0,012	0,016	0,009	0,007	0,006	0,006	0,010	0,009	0,009	0,007	0,007	0,006	0,006	0,005	0,005	0,004	0,004	0,004	0,005	0,005	0,004	0,004	0,003
15	0,013	0,016	0,012	0,010	0,010	0,009	0,008	0,009	0,010	0,008	0,008	0,007	0,007	0,006	0,006	0,005	0,006	0,007	0,007	0,007	0,005	0,006	0,005
20	0,015	0,017	0,015	0,016	0,015	0,015	0,009	0,009	0,009	0,008	0,008	0,007	0,007	0,006	0,006	0,006	0,009	0,015	0,015	0,012	0,008	0,009	0,008
25	0,018	0,019	0,016	0,017	0,016	0,015	0,010	0,010	0,009	0,009	0,008	0,008	0,008	0,007	0,007	0,007	0,010	0,016	0,015	0,013	0,009	0,010	0,008
30	0,020	0,020	0,021	0,024	0,022	0,020	0,012	0,012	0,013	0,011	0,011	0,010	0,008	0,009	0,008	0,009	0,011	0,016	0,014	0,012	0,010	0,010	0,008
35	0,023	0,021	0,027	0,034	0,032	0,030	0,016	0,016	0,016	0,014	0,018	0,013	0,011	0,011	0,010	0,011	0,014	0,017	0,015	0,013	0,012	0,012	0,010
40	0,028	0,025	0,028	0,036	0,035	0,035	0,020	0,018	0,018	0,017	0,016	0,016	0,014	0,015	0,014	0,015	0,018	0,023	0,020	0,018	0,017	0,016	0,013
45	0,041	0,035	0,033	0,043	0,041	0,040	0,023	0,022	0,022	0,021	0,020	0,020	0,019	0,018	0,017	0,018	0,022	0,024	0,026	0,024	0,021	0,021	0,019
50	0,057	0,047	0,046	0,061	0,058	0,055	0,040	0,031	0,035	0,032	0,034	0,031	0,028	0,031	0,028	0,032	0,034	0,038	0,036	0,034	0,031	0,032	0,028
55	0,079	0,080	0,068	0,087	0,083	0,080	0,058	0,050	0,056	0,053	0,057	0,051	0,047	0,050	0,048	0,053	0,053	0,053	0,053	0,050	0,045	0,046	0,040
60	0,118	0,114	0,098	0,119	0,112	0,110	0,087	0,062	0,102	0,086	0,095	0,068	0,064	0,060	0,083	0,086	0,080	0,091	0,087	0,087	0,076	0,076	0,067
65	0,179	0,153	0,149	0,165	0,161	0,157	0,155	0,119	0,139	0,131	0,135	0,123	0,129	0,118	0,133	0,131	0,114	0,119	0,122	0,114	0,108	0,104	0,097
70	0,274	0,237	0,205	0,212	0,200	0,194	0,202	0,203	0,203	0,202	0,202	0,203	0,189	0,203	0,224	0,202	0,222	0,194	0,192	0,188	0,176	0,179	0,161
75	0,402	0,350	0,329	0,331	0,306	0,290	0,347	0,317	0,338	0,355	0,349	0,328	0,276	0,314	0,375	0,355	0,299	0,263	0,261	0,261	0,260	0,278	0,260

Appendix B - Comparison between the observed and the projected mortality rates obtained with M1, M2, M3, M5 and M6

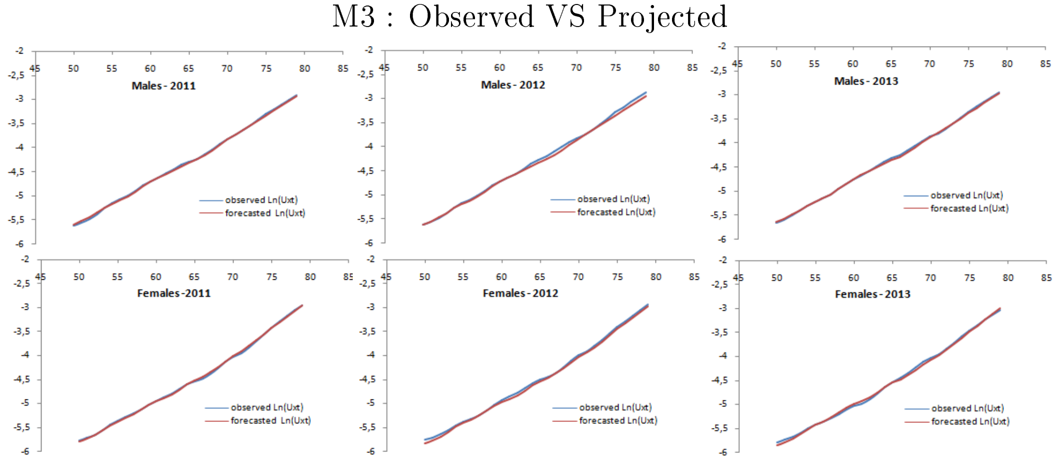
M1: Observed VS Projected



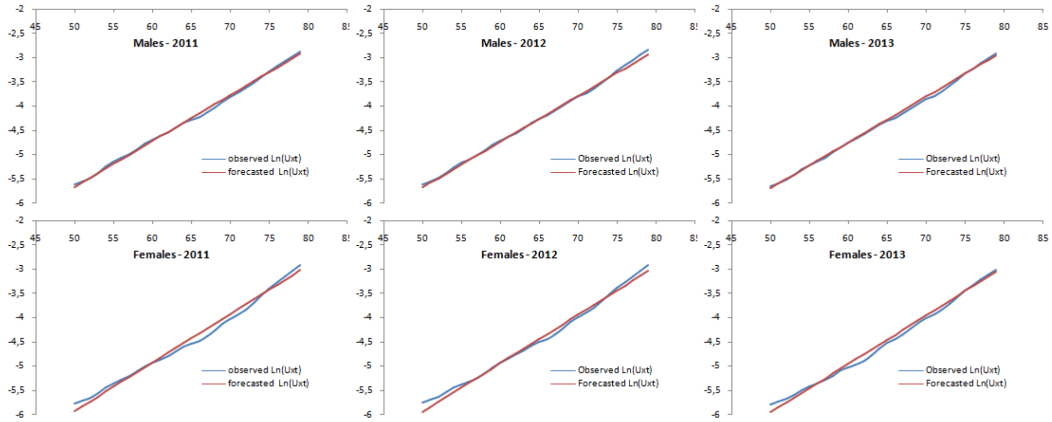
M2 : Observed VS Projected



M3 : Observed VS Projected



M5 : Observed VS projected



M6 : Observed VS Projected