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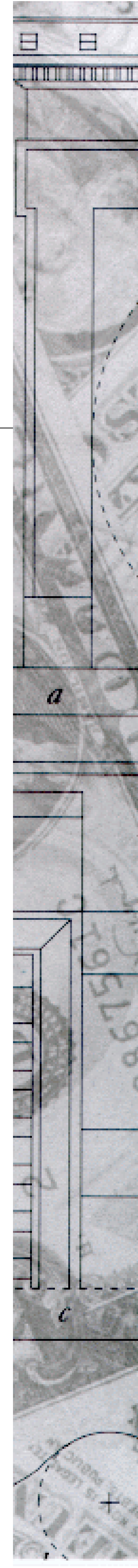

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Ratings Migration and the Business Cycle, With Applications to Credit Portfolio Stress Testing

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Abstract: The turmoil in the capital markets in 1997 and 1998 has highlighted the need for systematic stress testing of banks' portfolios, including both their trading and lending books. We propose that underlying macroeconomic volatility is a key part of a useful conceptual framework for stress testing credit portfolios, and that credit migration matrices provide the specific linkages between underlying macroeconomic conditions and asset quality. Credit migration matrices, which characterize the expected changes in credit quality of obligors, are cardinal inputs to many applications, including portfolio risk assessment, modeling the term structure of credit risk premia, and pricing of credit derivatives. They are also an integral part of many of the credit portfolio models used by financial institutions. By separating the economy into two states or regimes, expansion and contraction, and conditioning the migration matrix on these states, we show that the loss distribution of credit portfolios can differ greatly, as can the concomitant level of economic capital to be assigned to a bank.

Keywords: Credit risk, stress testing, ratings migration, credit portfolio management
JEL Classification Code: G11, G21, G28

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1. Introduction

The evolution of modern risk management can be traced back to Markowitz and his portfolio theory for investments. His fundamental concept of diversification, of considering the joint distribution of portfolio returns, has gradually migrated to risk management. Market risk measurement techniques were the first to mature, mainly due to the richness of available data. Risk managers now routinely measure portfolio change-in-value distributions and compute statistics such as Value-at-Risk (VaR), which are used to determine trading limits and assess risk capital. Moreover, the regulatory community has broadly accepted such a model-based approach to assessing market risk capital. On the credit risk side, however, even with the new BIS accords of June 1999 (BIS publication no. 50), formal credit portfolio models (CPMs) are not permitted for use in the determination of bank credit risk capital. Nevertheless CPMs are becoming more widespread in their use among financial institutions for economic capital attribution, and as the use of new risk transfer instruments such as credit derivatives increases, so by necessity will the use of CPMs.[4]

Recent turmoil in the capital markets has highlighted the need for systematic stress testing of banks' portfolios, including both their trading and lending books. This is clearly easier said than done. Although we have a wealth of data at our disposal in market risk, even there it is not obvious how best to implement stress testing. A short decision horizon, one on the order of hours or days, forces the thinking towards specific

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[4] For insightful comparisons of various CPMs, see Koyluoglu and Hickman (1998), Saunders (1999) and Gordy (2000).

scenarios or the “tweaking” of volatilities and correlations between dominant risk factors. Particularly the latter is important when designing a strategy for stress testing, as the LTCM debacle so poignantly demonstrated (Jorion, 1999). It appears, for example, that correlations increase during times of high volatility (Andersen et al. 2000).

Much less, however, is known about stress testing credit portfolios, and both practitioners and regulators are clamoring for guidance. We propose that underlying macroeconomic activity should be a central part of a useful conceptual framework for credit portfolio stress testing, and that credit migration matrices provide the specific linkage between underlying macroeconomic conditions and asset quality. Credit migration matrices, which characterize the expected changes in credit quality of obligors, are cardinal inputs to many applications, including portfolio risk assessment, modeling the term structure of credit risk premia, and pricing credit derivatives.[5] They are also an integral part of many of the credit portfolio models used by financial institutions. By separating the economy into two states or regimes, expansion and contraction, and conditioning the migration matrix on these states, we show that the loss distribution of credit portfolios can differ greatly, as can the concomitant level of economic capital to be assigned. We believe, therefore, that our analysis provides a useful framework for stress testing a credit portfolio using any of the credit portfolio models currently available.

The paper proceeds as follows. In Section 2 we review the ways in which asset values are tied to credit migration. In Section 3 we describe the ratings data on which our

[5] There is a rich literature on bond rating drift and migration, which we do not seek to address here. See

subsequent migration analysis depends and proceed in Section 4 to discuss estimation issues and the properties of the migration matrices. In Section 5 we integrate business cycle considerations into a migration analysis, and in Section 6 we apply our methods to stress testing a credit portfolio using CreditMetrics™. We conclude and offer suggestions for future research in Section 7.

2. Asset Values and Credit Migration

Let us consider a simple structural approach to modeling changes in the credit quality of a firm. The basic premise is that the underlying asset value evolves over time (e.g. through a simple diffusion process), and that default is triggered by a drop in firm's asset value below the value of its callable liabilities. Following Merton (1974), the shareholders effectively hold a put option on the firm, while the debtholders hold a call option. If the value of the firm falls below a certain threshold, the shareholders will put the firm to the debtholders. The concept is shown schematically Figure 2.1.

Altman and Kao (1991, 1992) and Nickell, Perraudin and Varotto (2000).

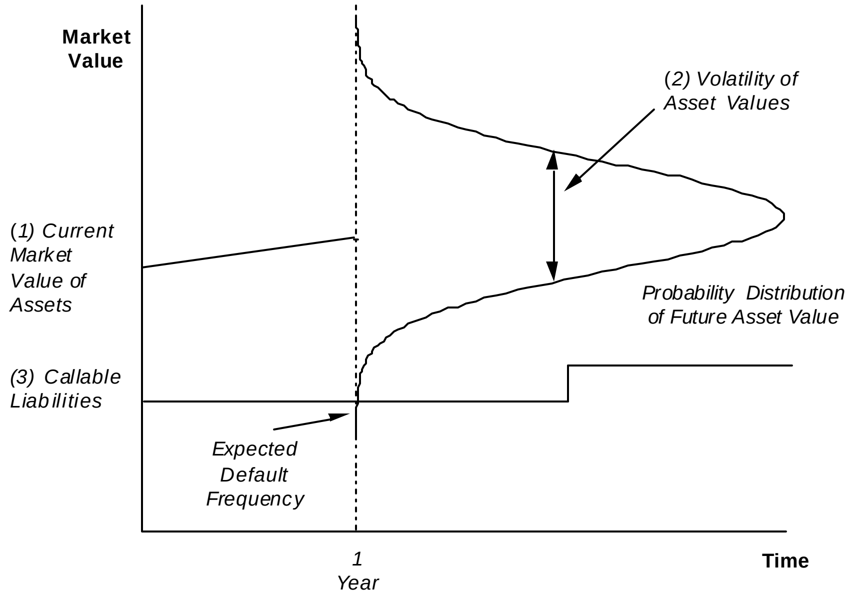

Figure 2.1: Distribution of future asset values, and expected default frequency

Assuming that changes in a firm's asset value can be related to a single systemic factor (for instance, some measure of the state of the economy) via a factor model, conceptually not unlike the CAPM, results in the common specification

$$DA_j = a_j x + s_j \ e_j,$$

where DA_i is the change in standardized asset value (i.e. with a mean of zero and a standard deviation of one) for firm *j*, *x* is the systematic risk factor that denotes the state of economy, *e* is a random shock with zero mean and unit variance, a_j (also known as the factor weight) is the correlation of changes in firms asset value with changes in economic factor *x*, and s_j is the magnitude of the residual volatility in asset value not explained by the systemic risk factor. In the special case of a homogeneous portfolio, when all firms have the same factor weight ($a_j = a_k = a$), the above one-factor model can be re-written as

$$\Delta A_j = \sqrt{r} \cdot x + \sqrt{(1-r)} \cdot e_j,$$

where (r = correl(DA_k, DA_j) = a^2) is the asset correlation between any two firms in the portfolio. The coefficient of the second term comes from the simplifying assumption that $DA_{,}$ x, and e_j all have unit variances. The first term in the equation drives systematic credit risk, while the second term drives idiosyncratic credit risk. Conceptually the systematic (economic) factor will become important later when we consider rating dynamics conditional on the economy. So-called default correlation enters through x.

Assuming that changes in asset value are normally distributed, the default probability can be expressed as the probability of a standard normal variable falling below some critical value. Similarly, thresholds can be established for transitions to other rating states. This is graphically represented for a BBB rated issuer in Figure 2.2

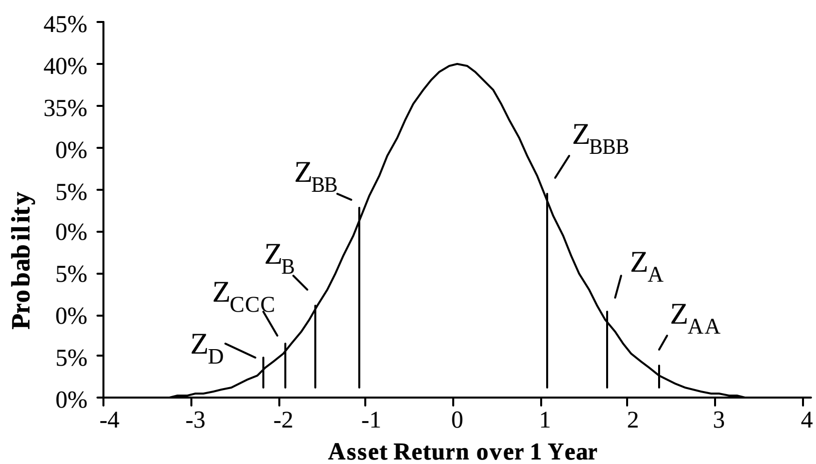

Figure 2.2: Asset Return Distribution with

Rating Thresholds for a BBB Issuer

As Figure 2.2 shows, these asset value thresholds can also be modeled as return thresholds Z_i, with i being the credit class. If the actual asset return DA_j now falls between two asset return thresholds Z_i and Z_{i+1} with $Z_{i+1} < Z_i$, whereby i+1 is the credit class below i, the company's rating next period will be i. In standard implementations of Merton's approach, the percentage changes in asset value are normally distributed with mean m and standard deviation s. Credit portfolio management models such as CreditMetrics™ moreover adjusts the asset returns to be standard normally distributed. Given these assumptions and the probabilities from the actual transition matrix, every asset return threshold corresponding to a specific credit rating can be computed. Employing the empirically estimated expected default frequency (EDF) for a specific initial rating class, the asset return threshold for the default state can be derived by the following relationship:

$$EDF = \Phi(Z_D)$$
$$Z_D = \Phi^{-1}(EDF),$$

where $\Phi(\cdot)$ denotes the cumulative standard normal distribution.

The remaining thresholds can be computed similarly, because for all non-default rating categories with the exception of the highest rating class the following equation holds:

$$q_{ij} = \Phi(Z_j) - \Phi(Z_{j+1}),$$

with q_{ij} indicating the actual probability of migrating from credit class i to category j. To calculate, for example, the upper asset return threshold for migrating to CCC, Z_{j+1} would be the above-computed asset return threshold for the default state Z_D. Finally, for the transition probability to the highest rating class, the following identity must hold:

$$q_{i1} = 1 - \Phi(Z_2).$$

With the asset value process for each issuer and the correlation between the asset value processes of two issuers given, the joint migration behavior can be computed from a joint bivariate normal distribution of the asset returns under their asset correlation:

$$q(q_{aj}^{A} \wedge q_{bk}^{B}) = q\left\{Z_{j+1} < R_A < Z_j, Z_{k+1} < R_B < Z_k\right\} = \int_{Z_{j+1}}^{Z_j} \int_{Z_{k+1}}^{Z_k} f(r_A, r_B, \Sigma)\, dr_B\, dr_A,$$

where $f(r_A, r_B, \Sigma)$ denotes the density function of the bivariate normal distribution for the asset returns r_A and r_B and Σ the covariance matrix between *A* and *B* derived from their equity correlation. Converting the computed joint asset returns then into credit ratings according to the before-derived asset return thresholds, the formula directly provides the joint rating behavior between two obligors for all possible rating combinations.

The elements of the transition matrix therefore provide an easy recipe for simulating credit migrations for credit risk applications. Given a set of transition probabilities for each credit rating, the critical distances can be calculated for each rating category. For a portfolio of credits, the changes in underlying asset values can be simulated easily. The asset value changes can be compared to critical distances to determine ratings transitions for each asset. The ratings transitions (of which default is a specific case) are correlated due to the correlation in asset value changes between firms through the systematic risk factor.

Hence the problem of joint rating migrations has now been reduced to the problem of estimating the asset return correlation between two issuers. Asset return correlations, however, are also not directly observable and thus must be approximated. CreditMetricsTM, for example, approximates asset return correlations via equity correlations, using a market model to link equity correlation to index correlation.

3. The Ratings Data

Here we describe some basic aspects of debt ratings obtained from the Standard & Poor's CreditPro™ 3.0 database, as relevant for our subsequent estimation of ratings transition matrices and their application to stress testing credit portfolios.

3.1. Basic Properties

Our analysis takes advantage of the Standard & Poor's CreditPro™ 3.0 database, which contains issuer credit ratings history for 7,328 obligors over the period from January 1, 1981 to December 31, 1998 (see Figure 3.1). The universe of obligors is mainly large corporate institutions around the world. Ratings for sovereigns and municipals are not included. The share of the most dominant region in the data set, North America, has steadily decreased from 98% to 75%, as a result of increased coverage of companies domiciled outside US (see Figure 3.2). The obligors include both US and non-US industrials, utilities, insurance companies, banks & other financial institutions and real estate companies. The representation of financial services, insurance and real estate has increased, while that of manufacturing, energy and utilities has decreased (see Figure 3.3). The database has a total of 38,588 obligor years of data excluding withdrawn ratings, of which 469 ended in default yielding an average default rate of 1.22% for the entire sample. On average, investment grade rated obligors were 72% of the dataset (see Figure 3.4).

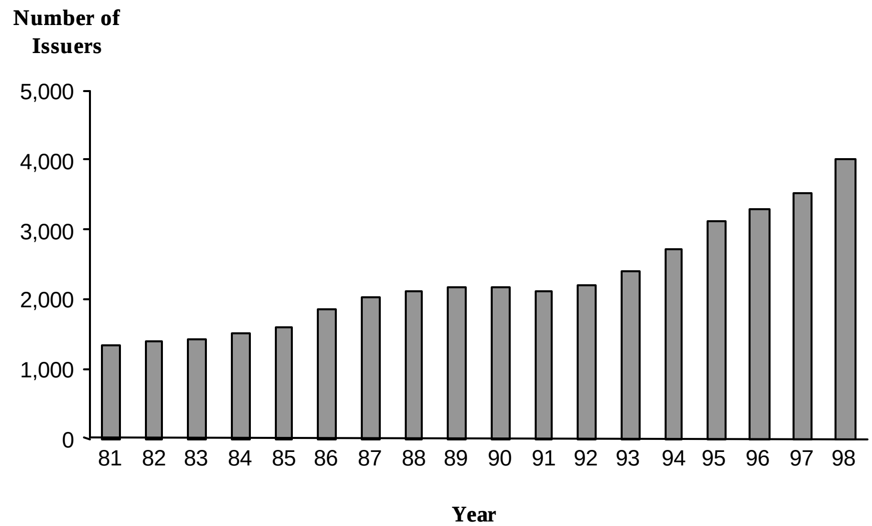

Figure 3.1: Evolution of S&P Rating Universe Over Time

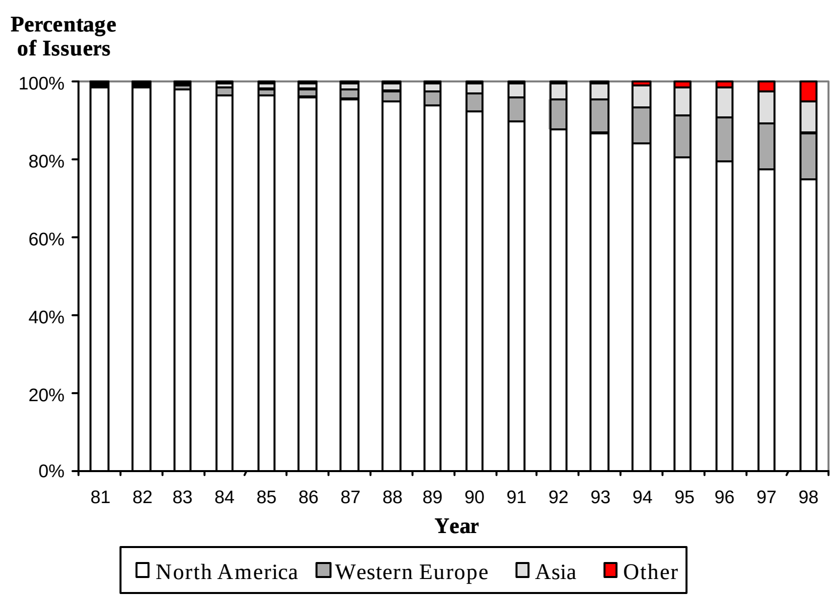

Figure 3.2: Rating Distribution by Geography

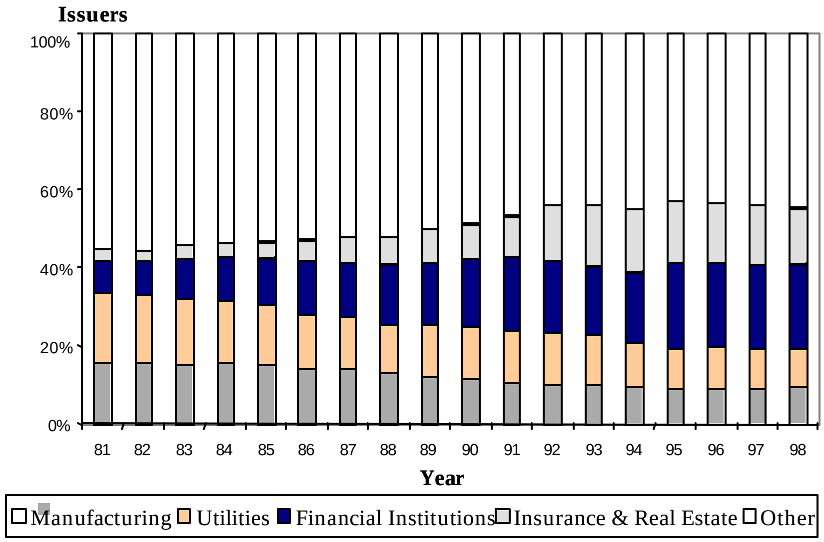

Figure 3.3: Rating Distribution by Industry

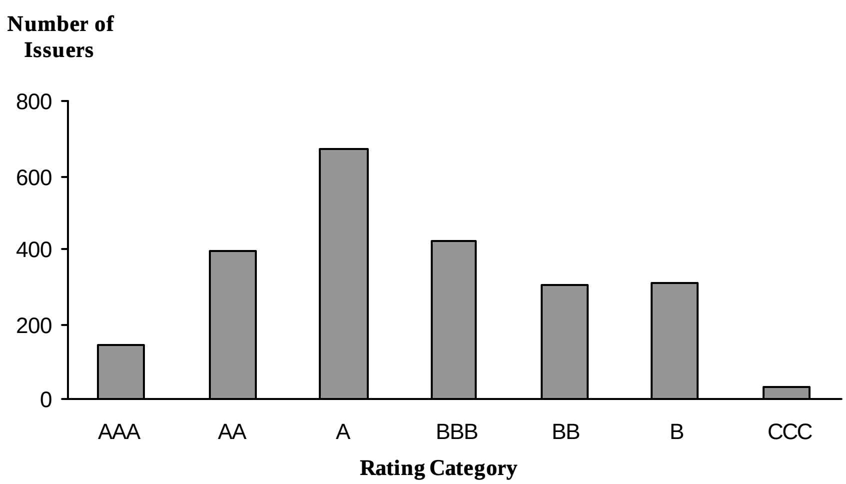

Figure 3.4: Average Ratings Distribution

To capture credit quality dynamics, the creditworthiness of obligors must be assessed, as credit events typically concern a firm as a whole. Unfortunately, published ratings focus on individual bond issues. Therefore, S&P implements a number of transformations:

i. Bond ratings are converted to issuer ratings. By convention, all bond ratings are made comparable by considering the implied long-term senior unsecured rating, i.e. the rating a bond would hold if it were senior unsecured.[6] This rating is then considered the issuer rating.

ii. Issuers are clustered into economic entities. This promotes correct representation of credit quality dynamics by accounting for parent-subsidiary links, mergers, acquisitions, and contractual agreements about recourse.

The transformations significantly improve the applicability of S&P's ratings. Nevertheless, the provided data needs additional adjustment to account for sample size problems relevant for the estimation of transition matrices. Specifically, the CreditPro™ database uses S&P's letter rating scale including the rating modifiers +/-. Hence in total the database comprises 17 different rating categories as well as the default (D) and the "not rated" (NR) state. Although the rating modifiers provide a finer differentiation between issuers within one letter rating category, they pose two problems: the sample size of issuers per rating class including rating modifiers is not sufficient for low rating categories, causing small sample size concerns that affect statistical inference. Moreover, transition matrices are generally published and applied without rating modifiers, as this format has emerged as an industry standard. Therefore, we exclude the rating modifiers in the course of this paper. Consequently, for example, we consider BBB+ and BBB- ratings as BBB ratings. This methodology reduces the database from 17 to 7 rating categories, which ensures sufficient sample sizes for all rating categories.[7] After these

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[6] For a detailed discussion of conversion criteria, see Standard & Poor's (1999).

[7] For a discussion of ratings dynamics for 17 states, see Bahar and Nagpal (2000).

transformations, the universe provided by S&P encompasses the rating histories of 7328 holders of S&P ratings from 1981 to 1998. This translates into nearly 166,000 obligor-quarters of data.

3.2. Selection of Samples for Estimation of Transition Matrices

Subsequently we will estimate and use both *conditional* and *unconditional* ratings transition matrices. By conditional, we refer to conditioning on the stage of the business cycle, expansion or contraction. By unconditional, we refer to averaging across stages of the cycle.

We enforce three criteria in the estimation of transition matrices. First, because our interest is partly in unconditional transition probabilities, the sample period should encompass both expansions and contractions.[8] Second, a minimal sample size threshold has to be met at all times to ensure statistical reliability of the estimates. Third, the time-*t* transition matrix should reflect the time-*t* rating universe, as opposed to some outdated sample that is no longer representative.

It might be desirable to estimate transition matrices based on a constant sample over a given period of time. We could, for example, track the 1981 universe from 1981 through 1998, leaving aside new firms that received their first rating after the beginning of 1981. This approach suffers from several problems. First, any given cohort quickly becomes outdated and hence less interesting as new issuers emerge, mergers and acquisitions

[8] We will use the term “contraction” and “recession” interchangeably.

transpire, and some industries decline while others flourish. Second, the fundamental characteristics of the underlying firms would evolve over time, producing results of dubious interpretation.[9] Third, the statistical size of a cohort would fall below the threshold level as issuers perish, default or retire their rating over time, for example by calling their outstanding debt.

We could attempt to mitigate the problems associated with tracking a fixed cohort simply by using a recently formed cohort. The resulting estimates, however, would reflect only the current economic situation, which might be purely expansion or purely contraction.

Alternatively, we could allow the sample composition to vary over time, incorporating new issuers and discarding those who default. We could, for example, use all issuers outstanding as of January 1, 1981 to estimate the 1981 transition matrix, all issuers outstanding as of January 1, 1982 to estimate the 1982 transition matrix, and so on. This procedure helps ensure that the sample size is always large enough to facilitate sharp statistical inference, that new firms are included in the sample, and that average transition matrices incorporate all states of the economy. We shall use this method.

[9] Altman and Kao (1991), for example, show that the migration drift is dissimilar between issuers from manufacturing vs. financial institutions and public utilities. In addition they find that there are non-negligible differences between the transition probabilities of the 1970's and 1980's.

3.3. Treatment of Transitions to "Not Rated" Status

Before proceeding to estimate transition matrices, we must deal with transitions to "not rated" (NR) status. A total of 2,447 companies were classified as NR from Jan-81 to Dec-98. Transitions to NR may be due to any of several reasons, including expiration of the debt, calling of the debt, failure to pay the requisite fee to S&P, etc. Unfortunately, however, the details of individual transitions to NR are not known. In particular, it is not known whether any given transition to NR is "benign" or "bad." Bad transitions to NR occur, for example, when a deterioration of credit quality known only to the bond issuer (debtor) leads the issuer to decide to bypass an agency rating.

There are at least three methods for removing NR's from the dataset. The first method is conservative and proceeds by treating transitions to NR as negative information regarding the change in credit quality of the borrower. Here the probability of transiting to NR is distributed amongst downgraded and defaulted states in proportion to their values by allocating NR values to all cells to the right of diagonal. The second method is liberal and treats transitions to NR status as benign. The probability transitions to NR are distributed among all states, except default, in proportion to their values. This is achieved by allocating the probability of transiting to NR to all but the default column. The third method, which has emerged as an industry standard, treats transitions to NR status as non-information. The probability of transitions to NR is distributed among all states in proportion to their values. This is achieved by gradually eliminating companies whose ratings are withdrawn. We use this method, which appears sensible and allows for easy comparisons to other studies.[10]

[10] See, for example, the discussion in Carty (1997).

Having reviewed the basic properties of the ratings data, we now proceed to the primary object of interest, migration matrices, which are constructed from the ratings data.

4. Estimation and Properties of Migration Matrices

4.1. The migration matrix

Conditional upon a given grade at time *T*, the transition, or migration, matrix **M** is a description of the probabilities of being in any of the various grades at *T+1*. It thus fully describes the probability distribution of grades at *T+1* given the grade at *T*. We seek to estimate the 7x7 = 49 unique elements of **M**, a conceptual rendition of which appears in Figure 4.1.

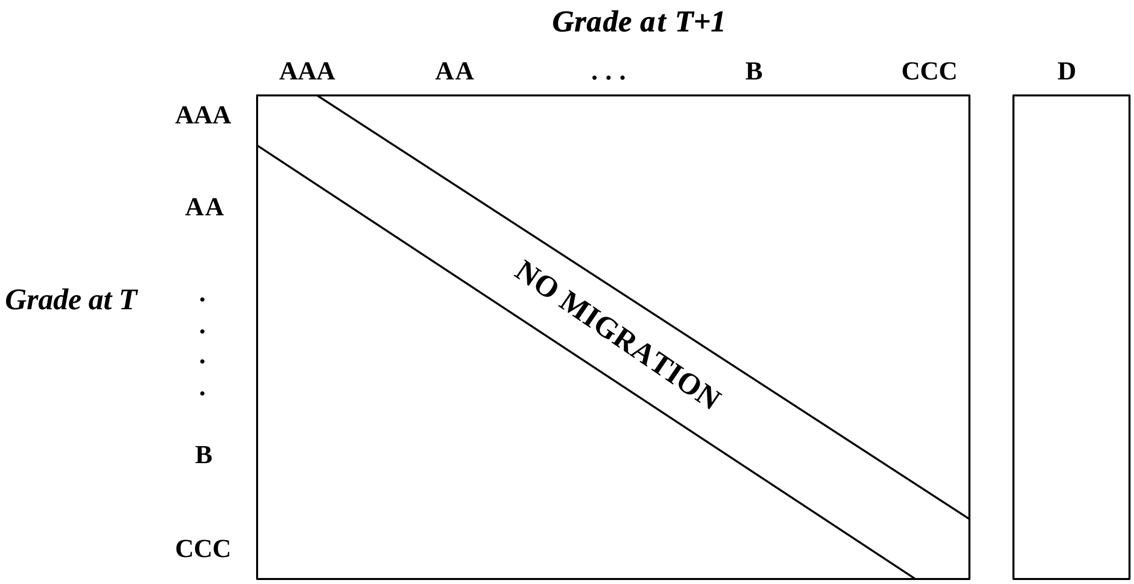

Figure 4.1: Structure of the transition matrix

4.2. Transition Horizon

Theoretically, transition matrices can be estimated for any desired transition horizon. As the ongoing coverage follows at least a quarterly review pattern, transition matrices estimated over short time periods best reflect the rating process. The shorter the measurement interval, the fewer rating changes are omitted. However, shorter duration also results in less extreme movements, as large movements are often achieved via some intermediary steps.

The other factor determining the transition horizon is the application purpose. For the calculation of credit risk exposures by portfolio models, a 1-year transition horizon is standard. Other applications such as the pricing of credit derivatives require shorter horizons; however in practice only annual transition matrices are typically used as shorter transition horizons have yet to be published by the rating agencies.[11]

Although the first two arguments clearly vote in favor of short-term transition matrices, matrices estimated over longer time periods offer the advantage of less noise inherent in the data, as short-term noise cancels itself out for longer horizons. It is noteworthy that matrices for longer transition horizons inhibit a trade-off between overlap and sample size. For instance S&P calculates 2-year transition matrices for every year; for 18 years of data this results in 17 biannual matrices. However, overlap induces the problem of double counting. In fact by applying S&P's method, rating events in the middle of the data series receive a higher weight than the rating behavior in the first and final period. Thus in the course of the paper longer horizon matrices are calculated using non-overlapping periods although this results in fewer data points, e.g. only 9 in contrast to 17 in case of the 2-year matrices.[12]

4.3. Unconditional estimates of **M**

Before proceeding to any conditional estimates of the migration matrix **M**, we consider first their unconditional estimates. Specifically we present the unconditional estimates

[11] Current credit derivatives pricing models, as e.g. the JLT framework, solve the problem of relatively long transition horizons by calculating probability intensities, i.e. continuous time probabilities, from the 1-year matrix.

[12] In order to implement the non-overlapping method, the first two years of data, i.e. 1981 and 1982, are omitted in the derivation of the 4-year transition matrix.

of the global quarterly and annual matrices, shown below in Table 4.3 and Table 4.3. As expected, the transition matrices exhibit higher default risk and higher migration volatility for lower quality grades. Specifically we see that default likelihood increases exponentially with decreasing grade.

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	97.92%	1.95%	0.10%	0.02%	0.01%	-	-	**-**
AA	0.16%	97.95%	1.75%	0.10%	0.01%	0.02%	0.00%	**-**
A	0.02%	0.57%	97.91%	1.34%	0.10%	0.06%	0.00%	**0.00%**
BBB	0.01%	0.07%	1.37%	96.90%	1.38%	0.23%	0.02%	**0.03%**
BB	0.01%	0.03%	0.17%	1.87%	95.35%	2.26%	0.18%	**0.13%**
B	-	0.02%	0.07%	0.11%	1.66%	95.72%	1.46%	**0.96%**
CCC	0.04%	-	0.16%	0.20%	0.41%	3.28%	87.18%	**8.72%**
							Average Default Rate	**0.284%**

Table 4.1: Unconditional Quarterly Migration Matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	91.93%	7.46%	0.48%	0.08%	0.04%	-	-	**-**
AA	0.64%	91.81%	6.75%	0.60%	0.06%	0.12%	0.03%	**-**
A	0.07%	2.27%	91.69%	5.11%	0.56%	0.25%	0.01%	**0.04%**
BBB	0.04%	0.27%	5.56%	87.88%	4.83%	1.02%	0.17%	**0.24%**
BB	0.04%	0.10%	0.61%	7.75%	81.48%	7.89%	1.11%	**1.01%**
B	-	0.10%	0.28%	0.46%	6.95%	82.80%	3.96%	**5.45%**
CCC	0.19%	-	0.37%	0.75%	2.43%	12.13%	60.45%	**23.69%**
							Average Default Rate	**1.215%**

Table 4.2: Unconditional Annual Migration Matrix

A characteristic of all matrices is the high probability load on the diagonal: obligors are most likely to maintain their current rating. Considering the rating transition probability distribution of an obligor given its initial rating, the second largest probabilities are usually in direct neighborhood to the diagonal. In general, the further away a cell is from the diagonal, the smaller is the likelihood of such an occurrence. This rule has frequently been addressed as monotonicity.[13] However, the annual transition

[13] J.P. Morgan (1997), p. 73

matrix depicts some exceptions. For A, BBB, and B rated issuers, the likelihood of defaulting is larger than the likelihood of ending with a CCC rating. Conversely, the probability for a CCC issuer to attain an AAA next year is higher than the chance of being upgraded to AA. There is no single explanation for these violations of monotonicity. For medium quality categories the violations are weaker or even non-existent for smaller transition horizons, suggesting the violations to be a result of intra-interval rating activity omissions inherent in longer transition horizons. The inconsistency for the CCC category might be attributable to noise in the underlying data. However, it is noteworthy that recent articles have challenged the assumption of strict monotonicity as partially unreasonable as "... certain CCC-rated firms are 'do-or-die' type firms. Their very risky nature makes them highly default prone, but if successful they have a significant chance of skipping a few categories on their way to higher ratings."[14]. Moreover, with increasing transition horizon the violation of (row) monotonicity for the default rates becomes more prominent. This is expected since default is an absorbing state.

We have noted that the sample is concentrated along the diagonal; the observation density diminishes rapidly as we move away. We can display the level of parameter uncertainty by computing the coefficient of variation of each cell. The reliability of the cell values decreases as we move farther from the diagonal as we see in Figure 4.2 below.

[14] Lando (1998), p. 151

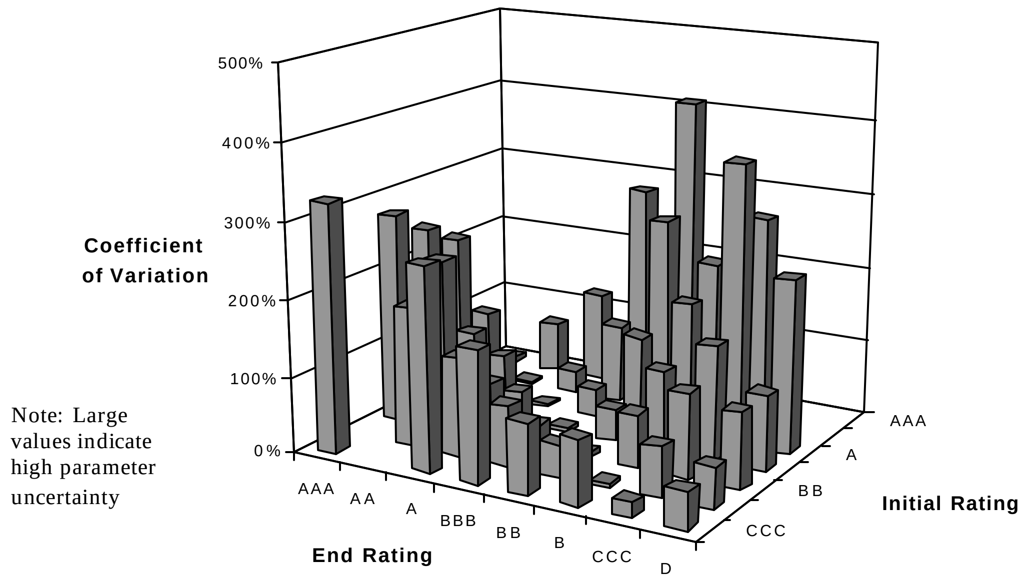

Figure 4.2 : Coefficient of Variation for 1 Year Transition Horizon

Many of the recently developed credit risk models assume the credit migration process to be Markovian; more precisely, the distribution of default time is modeled via a "... discrete time, time-homogeneous finite state space Markov chain."[15] Here we assess whether ratings dynamics are Markovian in two ways: analysis of eigenvectors and eigenvalues, and analysis of path dependence.

4.4.1. Analysis of eigenvalues and eigenvectors

One of the most common ways of testing the Markovian property of a matrix is through eigenvalue and eigenvector analysis. All transition matrices have a trivial eigenvalue of unity; this eigenvalue also has the highest magnitude and stems from the symmetry in the matrix (all rows add up to unity since all transition probabilities sum to one). The remaining set of eigenvalues of the transition matrix have magnitudes smaller than

[15] Jarrow, Lando, Turnbull (1997), p. 487.

unity. A transition matrix can be taken to the k^{th} power by simply splitting the matrix into its eigenvalues and eigenvectors and taking the eigenvalues to the k^{th} power while leaving the eigenvectors unchanged. Thus for transition matrices to follow a Markov chain process, two conditions have to be met. First, the eigenvalues of transition matrices for increasing time horizons need to decay exponentially. In other words, if all the eigenvalues, e_i, of (empirical) transition matrices (of varying time horizon) were ranked in order of their magnitude, one should observe a linear relationship between Log(e_i) and the transition horizon T for each i. Second, the set of eigenvectors for each transition matrix need to be identical for all transition horizons.

Using such an eigenanalysis, we found it very difficult to reject first-order Markov structure.[16] Figure 4.3 displays the second, third, and fourth largest eigenvalues of transition matrices with transition horizons varying from one quarter to four years. The calculated eigenvalues do in fact show a strong log-linear relationship with the transition horizon, thus providing strong evidence for the assumption of Markov properties.

[16] A battery of other tests is conducted by Kronimus & Schagen, 1999, with the same conclusion as we report here.

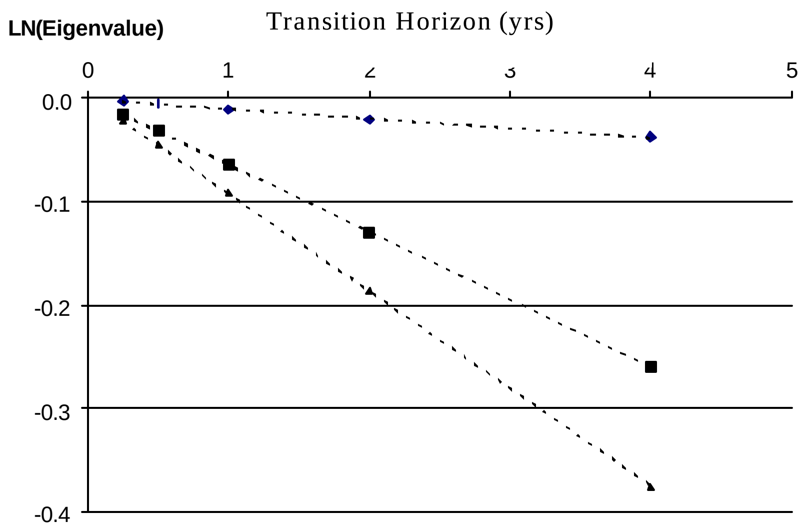

Figure 4.3: Decay of Eigenvalues with Transition Horizon

The structure of the eigenvector corresponding to the second largest eigenvalue provides key insight on the dynamics of ratings migration. Specifically, it indicates the long-term (asymptotic) distribution of companies not ending in default and thus the direction of rating convergence of the surviving population. As Figure 4.4 demonstrates, all 2nd eigenvectors peak at 'A' rating and share a fairly similar shape. Hence, independent from the transition horizon, the long-term survivor distribution trends towards the middle rating classes. The similarity of the 2nd eigenvectors fails to reject the Markov assumption.

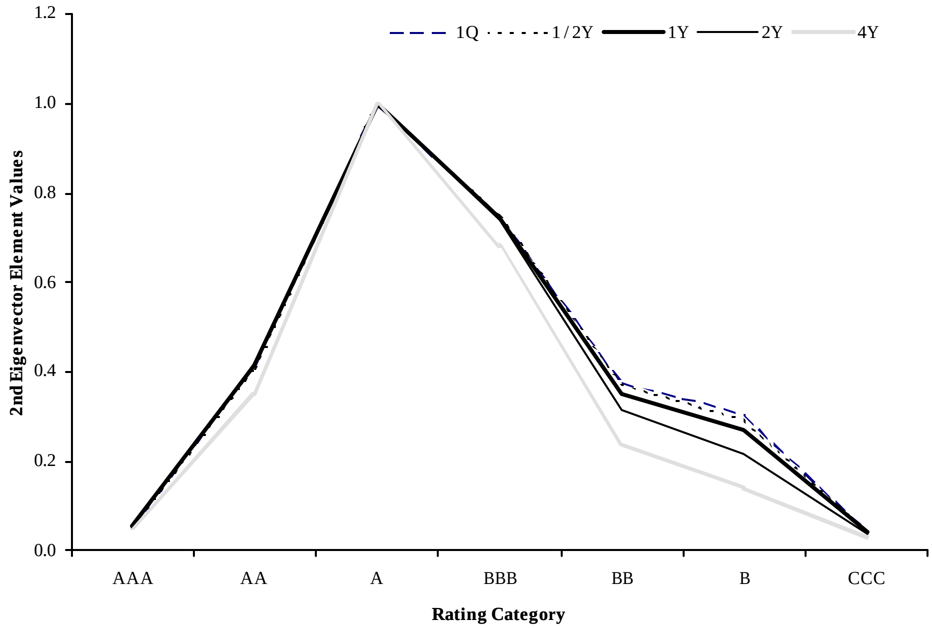

Figure 4.4: 2nd Eigenvector of Matrices with Different Transition Horizons

4.4.2. Path dependence

Although the standard tests reported above fail to reject the Markov property of the transition matrices, a key assumption in many applications in credit risk, a more careful analysis reveals path dependence, which is a clear violation of first-order Markov behavior. In a first-order Markov chain process next period's distribution is only dependent on the present state and not on any developments in the past. In other words, transitions have only a one-period memory.[17]

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[17] Previous research has also provided some indication of a rating memory, though only for Moody's data. See Carty and Fons (1993).

It is useful to define the notion of memory. Path dependence "... presupposes that prior rating changes carry predictive power for the direction of future rating changes."[18] The initial hypothesis is that issuers that have experienced prior downgrading are prone to further downgrading, while issuers that have been upgraded before are less frequently downgraded. The momentum is captured through the directional movements in the path period, defined for simplicity to be one year, i.e. specific transition matrices are computed for issuers that have been upgraded, remained unchanged, or have been downgraded in the previous period.[19] Specifically, the cohort of companies each year was separated into three subgroups according to their rating experience in the previous year: upward trending, downward trending and no trend. Each sub-cohort was followed for a year to observe rating transition probabilities for upgrade, downgrade and no ratings change. The results are summarized in Table 4.3.

This momentum hypothesis is supported by the data as most downgrade probabilities for the down-momentum matrix are larger than the corresponding values in the unconditional matrix. The exact opposite is true for the up-momentum matrix, which exhibits smaller downgrade probabilities than the unconditional matrix. The upgrade probabilities of the up-momentum matrix for below investment grade classes are higher than for the unconditional matrix, while for investment grade categories they are lower. Overall, reduced upgrading and downgrade probabilities for the up-momentum matrix lead to an increased probability mass on the diagonal, while the down-momentum matrix displays the exact opposite. Only the maintain-momentum matrix does not display a systematic trend nor differs to a large extent from the unconditional matrix.

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[18] See Carty and Fons (1993), p. 20

This is however fairly intuitive, as the universe used for the calculation of the unconditional matrix consists to nearly 90% of issuers whose ratings did not change in the previous period, resulting in significant data overlap.

Up Momentum Transition Matrix

Initial Rating	Terminal Rating								No. of Issuer Periods
	AAA	AA	A	BBB	BB	B	CCC	D	
AAA	92,45%	5,66%	1,89%	-	-	-	-	-	53
AA	0,39%	95,33%	4,28%	-	-	-	-	-	257
A	0,26%	1,29%	93,54%	4,91%	-	-	-	-	387
BBB	-	-	3,78%	90,12%	4,94%	0,87%	-	0,29%	344
BB	-	-	1,38%	8,30%	85,81%	3,46%	0,69%	0,35%	289
B	-	-	1,96%	5,88%	5,88%	82,35%	1,96%	1,96%	51
CCC	-	-	-	-	-	-	-	-	-

Maintain Momentum Transition Matrix

Initial Rating	Terminal Rating								No. of Issuer Periods
	AAA	AA	A	BBB	BB	B	CCC	D	
AAA	91,84%	7,58%	0,48%	0,05%	0,05%	-	-	-	2070
AA	0,58%	91,60%	7,02%	0,60%	0,07%	0,12%	-	-	5642
A	0,06%	2,29%	91,56%	5,20%	0,60%	0,26%	-	0,03%	9308
BBB	0,06%	0,33%	5,96%	87,57%	4,53%	1,12%	0,23%	0,19%	5183
BB	0,06%	0,13%	0,50%	9,11%	80,43%	7,60%	1,07%	1,10%	3173
B	-	0,16%	0,29%	0,42%	8,38%	80,82%	3,76%	6,17%	3113
CCC	-	-	-	0,77%	1,54%	12,31%	70,00%	15,38%	260

Down Momentum Transition Matrix

Initial Rating	Terminal Rating								No. of Issuer Periods
	AAA	AA	A	BBB	BB	B	CCC	D	
AAA	-	-	-	-	-	-	-	-	-
AA	0,60%	90,48%	7,74%	0,60%	-	0,60%	-	-	168
A	-	0,93%	93,02%	5,12%	0,47%	0,23%	-	0,23%	430
BBB	-	0,18%	5,42%	85,90%	6,69%	1,27%	-	0,54%	553
BB	-	-	1,12%	7,30%	79,21%	8,71%	1,69%	1,97%	356
B	-	-	0,24%	0,73%	6,05%	75,79%	7,99%	9,20%	413
CCC	0,52%	-	1,05%	0,52%	4,19%	9,95%	42,93%	40,84%	191

Note: Highlighted Cells denote Significance on a 95% Confidence Level (One

Table 4.3: Path Dependent Transition Matrices

The most striking finding, however, is the extreme difference in average default rates. The down momentum average default rate is nearly five times as large as the unconditional one, whereas the up momentum average default rate is less than one fifth

[19] A detailed analysis is given in Kronimus & Schagen 1999.

of the unconditional expectation. Thus the default probability is most sensitive to a prior downgrading history.[20]

Why does this matter? One particular example is the pricing of credit sensitive securities. Such instruments typically employ individual obligations as underlying so that the momentum of an issue might have a significant effect on value.[21] Beyond this example, any instrument with an asymmetric payoff structure, such as options on the yield spread, should be affected by both the increased negative drift for issuers with a down momentum and the increased volatility of migration in general.

5. Expansions and Contractions: Conditioning Transition Matrices on the Stage of the Business Cycle

For more than a century, scholars have productively divided the cycles in real economic activity into separate phases or regimes, in particular treating expansions separately from contractions. Burns and Mitchell (1946) is a seminal work, and Hamilton (1989) is a rigorous modern econometric characterization. Recent evidence, moreover, suggests that parallel regime-switching structures may exist in financial markets. Regime switching has been found, for example, in the conditional mean dynamics of interest

[20] Extreme default rates might be even amplified by more than a factor of five because of the higher volatility of the average down momentum default rate and the highly right-skewed and platykurtic character of EDF probability distributions, as discussed in Koyluoglu and Hickman (1998).

[21] The effect on a portfolio of risky debt would be much smaller as the different momentums of several issuers might cancel one another.

rates (Hamilton, 1988; Cecchetti, Lam and Mark, 1990) and exchange rates (Engel and Hamilton, 1990), and in the conditional variance dynamics of stock returns (Hamilton and Susmel, 1994). Following this tradition, we now assess whether credit rating transition matrices differ across expansions and contractions, and we investigate the credit risk management implications of any such shifting.

5.1. Expansion and Contraction Transition Matrices

In seeking to explain ratings volatility generally and default volatility specifically, we view asset return volatility through a CAPM lens as described in more detail in Section 2. Total volatility (risk) is composed of a systematic and an idiosyncratic component. Because ratings are a reflection of a firm's asset quality and distance to default, a reasonable definition of "systematic" is the state of the economy. The simplest way of characterizing this state is one of expansion and contraction. Considering that U.S. issuers account on average for 88% of the total rating universe between 1981 and 1998, it appears appropriate to focus the analysis exclusively on U.S. issuers and U.S. economic indices.

The National Bureau of Economic Research (NBER) has categorized each month since 1959 into either an expansion or contraction state.[22] Because our highest frequency is quarterly, we in turn label each quarter in our sample period of 1981Q1 to 1998Q4 as expansion or contraction following NBER's own definitions. We proceed to re-estimate

[22] The NBER business cycle chronology is constructed from a subjective examination of the concordance of a large number of business indicators—a much greater variety of series than those included, for example, in the components of real aggregate output.

the migration matrix **M**: we obtain $\mathbf{M}_E$ and $\mathbf{M}_C$, the expansion and contraction migration matrices.

The first question to ask is whether the expansion and contraction matrices are different. The results appear in

Table 5.1. The numbers in bold are significantly different at the 5% level from the unconditional matrix.

The most striking difference between the expansion and contraction matrices are the downgrading and especially the default probabilities that increase significantly in contractions. For the credit grade B the default frequency doubles from an average of 0.9% in expansions to 1.8% in contractions.

1/4-Year US Expansion Matrix

Initial Rating	Terminal Rating								No. of Issuer Periods
	AAA	AA	A	BBB	BB	B	CCC	D	
AAA	98,21%	1,66%	0,11%	0,02%	0,02%	-	-	-	6581
AA	0,15%	98,08%	1,61%	0,12%	0,01%	0,03%	0,01%	-	19458
A	0,02%	0,53%	98,06%	1,21%	0,11%	0,06%	0,00%	0,00%	36404
BBB	0,01%	0,07%	1,47%	96,94%	1,25%	0,22%	0,02%	0,02%	24529
BB	0,01%	0,03%	0,19%	1,93%	95,31%	2,25%	0,16%	0,12%	18161
B	-	0,02%	0,07%	0,10%	1,70%	95,91%	1,31%	0,88%	20002
CCC	0,05%	-	0,19%	0,23%	0,47%	3,57%	87,32%	8,17%	2129

1/4-Year US Recession Matrix

Initial Rating	Terminal Rating								No. of Issuer Periods
	AAA	AA	A	BBB	BB	B	CCC	D	
AAA	97,99%	1,76%	0,25%	-	-	-	-	-	795
AA	0,18%	96,89%	2,79%	0,05%	0,09%	-	-	-	2186
A	0,02%	0,88%	96,44%	2,59%	0,07%	-	-	-	4330
BBB	0,04%	0,04%	1,11%	96,31%	2,33%	0,07%	-	0,11%	2708
BB	-	0,06%	0,06%	1,39%	94,98%	2,72%	0,42%	0,36%	1655
B	-	0,06%	0,06%	0,11%	0,72%	95,02%	2,27%	1,77%	1806
CCC	-	-	-	-	-	1,20%	85,60%	13,20%	250

Note: Highlighted Cells denote Significance on a 95% Confidence Level (One Tailed)

Table 5.1: U.S. Expansion and Contraction Transition Matrices

However, not only extreme migrations become more probable in economic contractions but also the downgrading to neighboring classes is much more likely. On the other hand most upgrade probabilities remain constant or even decrease during contractions. Finally, it is noteworthy that comparatively small deviations on a quarterly basis result in significant differences over a 1-year horizon.

Next we examine parameter volatility by computing the coefficient of variation of each cell for $\mathbf{M}_E$ and $\mathbf{M}_C$. We see the clear emergence of two regimes as evidenced by the dramatic reduction in this parameter uncertainty.

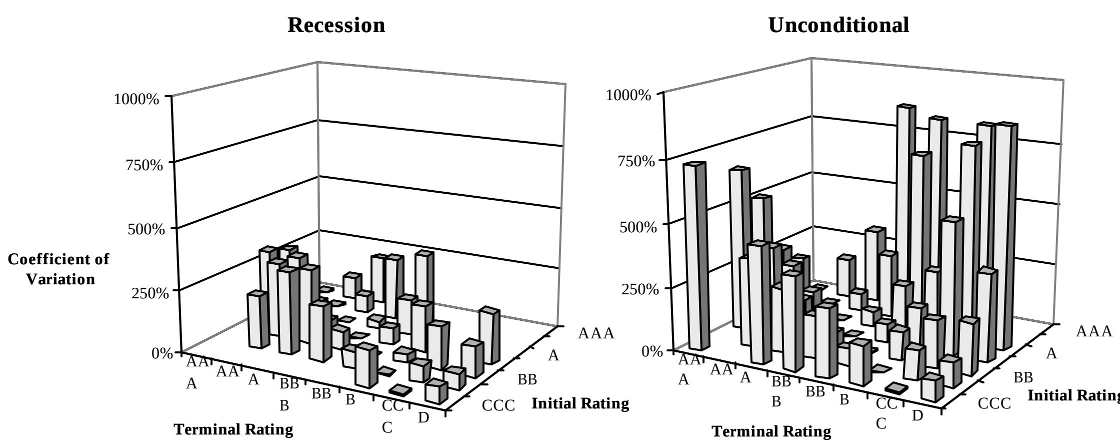

Figure 5.1: Coefficients of Variation of U.S. Contraction Transition

As Figure 5.1 demonstrates, most of the coefficients of variation of the contraction matrix are much lower than those of the unconditional one. While for the expansion matrix the coefficients of variation are on average reduced by only 2% compared to the unconditional matrix, the contraction matrix exhibits about 14% less volatility. Furthermore it is striking that many of the largest reductions in variation coefficients for the contraction matrix actually stem from elements on or close to the diagonal supporting the reliability of the results. It is also noteworthy that the coefficients of variation of the default probabilities decrease even further (by 40% and more), indicating that these time series are particularly distinct between the economic states. Overall, these results reveal that migration probabilities are more stable in contractions than they are on average, supporting the existence of two distinct economic regimes.

The findings so far indicate that the rating universe should develop differently in contraction periods compared to expansion times. This is further corroborated by considering the long-term behavior in form of 2nd eigenvalues and eigenvectors. A lower 2nd eigenvalue in contractions, for example, would imply more rapid credit quality deterioration. Different 2nd eigenvectors between expansions and contractions would indicate that the survivor population trends towards a different distribution.

The 2nd eigenvalues in fact depict a clear difference between expansions and contractions. The contraction eigenvalue is substantially smaller than the expansion one. On a yearly basis, the contraction eigenvalue is only 0.97 vs. 0.99 for the expansion matrix. When recalling that the 2nd eigenvalue is taken to the k^{th} power for the k^{th} period population distribution, the difference indeed results in a much faster decay for the contraction matrix.

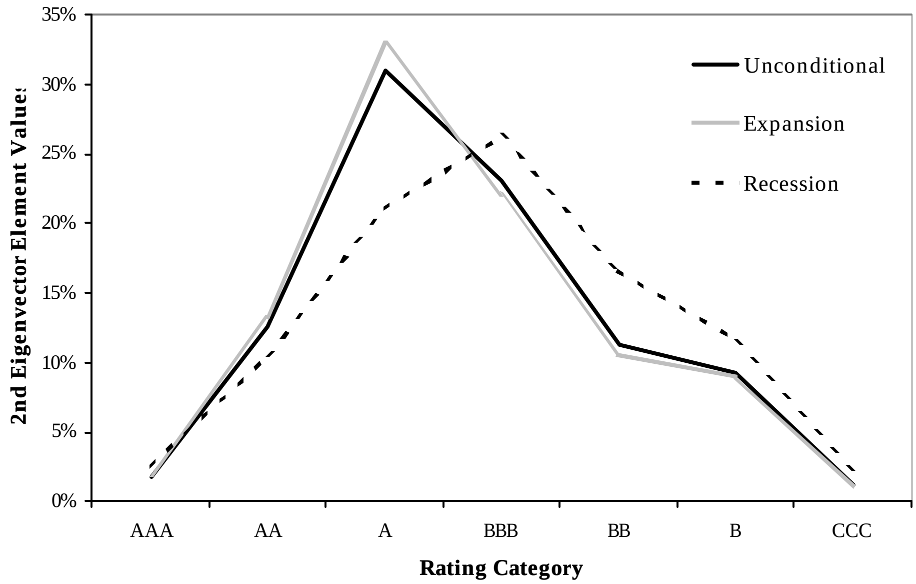

Figure 5.2: 2nd Eigenvector of U.S. Expansion and Contraction Matrices

The 2nd eigenvector of the quarterly U.S. contraction matrix is also very different from the 2nd eigenvector of the U.S. unconditional and expansion matrices (see Figure 5.2). The system no longer tends towards the "A", but towards the "BBB" category. Moreover the population evolution in contractions displays significantly fewer issuers in the investment grade categories and more issuers in the below investment grade classes than in expansion.

5.2. Regime switching

We have seen distinct differences between the U.S. expansion and contraction transition matrices. The straightforward application of these matrices however would normally be restricted to situations where the future state of the economy over the transition horizon under consideration is assumed to be known. Since in reality this is never the case, it might be useful to view this problem through the lens of regime switching matrices.[23] It would be straightforward to link the regime switching and different transition matrices. Moreover, general changes in business cycles, such as longer cycle durations or changing transition probabilities, might be easily incorporated by simply adapting the regime-switching matrix.

Regime switching matrices in their simplest form are 2x2 matrices that depict the probability of being in an expansion or contraction next period conditional upon the current regime. The monthly expansion and contraction definitions of the NBER can be aggregated into quarterly and yearly expansion and contraction classifications. Given the small sample size of only three yearly contraction matrices, we focus on quarterly data. This 2X2 regime switching matrix is estimated twice: first, employing the entire NBER data from 1959 to 1999 and second using only the NBER data from 1981 to 1998.

Regime Switching: 1959 - 1998			**Regime Switching: 1981 - 1998**		
	Expansion	Recession		Expansion	Recession
Expansion	84.8%	15.2%	Expansion	85.0%	15.0%
Recession	57.5%	42.4%	Recession	69.2%	30.8%

Table 5.2: Quarterly Regime Switching Matrices

[23] For an introduction, see Diebold and Rudebusch (1999, 117-143).

Table 5.2 highlights that the last two decades have seen few contractions in relation to the entire economic history over the last 40 years. This becomes even clearer when comparing the steady states of the two regime switching matrices. While the 1981-1998 regime switching matrix implies that on average 17.8% of all quarters are contraction quarters, the 1959-1998 regime switching matrix predicts 20.9% of all quarters to be contractions, indicating that the economic development over the last 20 years has been relatively favorable. Moreover, recessions seem to be getting shorter as evidenced by the maintain probability declining from 42.4% ('59 – '98) to 30.8% ('81 – '98).

5.2.1. Regime Switching Simulation

A combination of the regime switching matrix estimated over the same sample period as our S&P data, namely 1981-98, and the expansion and contraction matrices should yield the same credit quality distribution as the unconditional migration matrix. Given a structurally different history of business cycles over the last 20 years, the unconditional transition matrices, which are estimated based on data from 1981 onwards, also imply the recent regime characteristics with short contractions and long expansions. When forecasting credit quality developments using an unconditional matrix, one implicitly assumes the favorable business cycle pattern to be persistent going forward. If however one deems the structural change in business cycles to be a temporary effect and the long-term estimate to be the best predictor of future regime switching, the unconditional matrix would overstate the overall credit quality evolution.

The following Monte Carlo simulation investigates the effect using the (more pessimistic) *long-term* regime switching behavior on a given rating distribution and compares the results to the unconditional distribution outcome. Specifically, the 1959-1998 regime-switching matrix is combined with the expansion and contraction matrices to get an estimator of credit quality dynamics for a period from 1959 onwards. Figure 5.3 summarizes the Monte Carlo exercise after 100 runs.

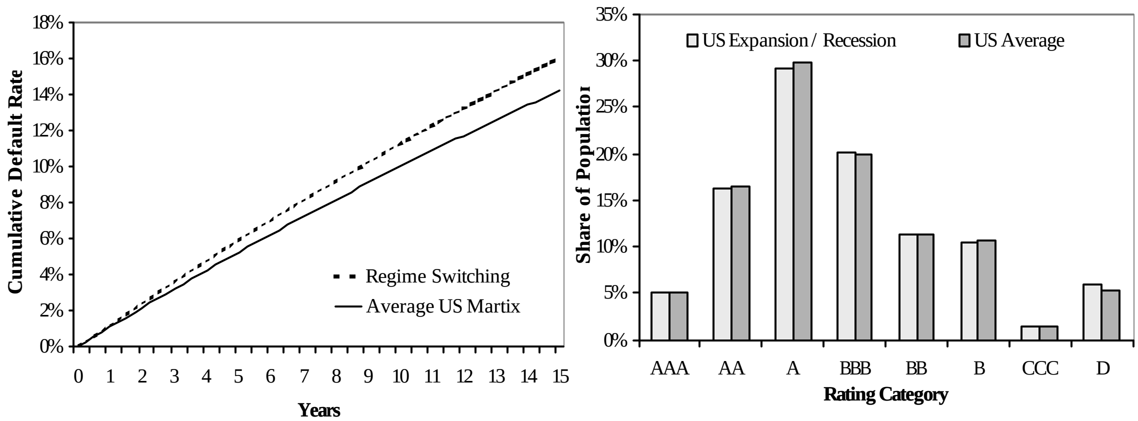

Figure 5.3: Trajectory of Defaults and Population after 5 Years

As expected, the trajectory of defaults for the unconditional transition matrix understates the cumulative percentage of defaulted companies. The deviations between the regime switching and the unconditional simulation however are relatively small over short-term horizons, e.g. with a difference of only 40 basis points after three years. Over longer time horizons however, the differences become more prominent, amounting to 180 basis points after 15 years. In order to infer statements about the overall trend in credit quality evolution, the rating populations are compared.

Figure 5.3 also displays the rating distribution for both simulations after five years. The fraction of issuers in default is understated by the unconditional transition matrix. The difference in probability mass mainly stems from an overstated fraction of A-rated companies. Thus, it seems fair to state that the unconditional transition matrix, estimated, after all, during the recent more robust economic experience, indeed overestimates the credit quality development, though only to a rather limited extent in the short-run.

In summary, the assumption of business cycle structures reverting to the historical long-term estimate does not introduce significantly different results for the evolution of credit quality over the short-term. For longer horizons however, the regime switching approach might supplement the unconditional expectation.

6. Incorporating business cycle views into credit portfolio stress tests

The results presented so far could enhance numerous credit risk operations. First, the design of stress test scenarios can be guided by the observed behavior of default and transition probabilities. This could be implemented by not only simulating the term structure of default but by also employing varying transition matrices in order to incorporate the uncertainty in credit risk models.

Second, the state of the economy clearly is one of the major drivers of systematic credit risk, especially as lower credit classes are much more sensitive to macro-economic factors. Consequently it should be integrated into credit risk modeling whenever possible, otherwise the downward potential of high-yield portfolios in contractions might be severely underestimated. Moreover, internal credit grading systems calibrated on EDFs measured only in expansion times, such as the 1990's, might result in mispriced loans or even suboptimal capital allocation in the lending business. Consequently, these measures need to be calibrated in order to reflect cycle-neutral parameters.

Third, the currently dominant *un*conditional view of credit risk can be extended to a conditional perspective. Current applications rarely use more than an unconditional matrix, often times misrepresenting the underlying portfolio of issuers. Modern credit risk models such as CreditMetrics™ account for different industries only through different term structures, but not through industry dependent transition matrices. The same holds true for different regions. Adding the additional information can yield substantially different credit exposures as we will demonstrate below.

Fourth, a forecasting model for transition matrices can be constructed on the basis of the revealed dependencies on macro-economic indices and interest rates. Such a model could also incorporate additional information such as the momentum path or regime switching probabilities.

6.1. Bank Capitalization

The purpose of capital is to provide a cushion against losses for a financial institution. There are two distinct approaches to calculating capital: a regulatory one which uses simple rules (e.g. for a loan to a corporate entity, 8% capital must be assigned) to arrive at *regulatory* capital, and an economic one which relies more on first principles and formal models to calculate *economic* capital. While both approaches are trying to achieve the same end, inevitably the regulatory approach is a cruder approximation. Moreover, modern financial instruments such as credit derivatives allow for substantial regulatory arbitrage, all of which has heated the debate of migrating towards a models-based approach to computing credit risk capital, much as has already happened for market risk.[24]

Much as in market risk, the economic capital in credit risk is commensurate with the risk appetite of the financial institution. This boils down to choosing a confidence level in the loss (or value change) distribution of the institution with which senior management is comfortable. For instance, if the bank wishes to have an annual survival probability of 99%, this will require less capital than a survival probability of 99.9%, the latter being typical for a highly rated bank. The loss (or value change) distribution is arrived at through internal credit portfolio models. Even if regulators do not assign capital according to such models, they still serve as a valuable tool for managers.

The relation between actual[25] and regulatory capital is obvious. If actual capital *exceeds* regulatory capital, there is room to expand the bank's activities or return capital to the shareholders. If the reverse is true, then the bank needs to contract risk-taking activities or raise more capital. Thus financial institutions treat regulatory capital largely as a constraint. However, the more likely situation is when actual and regulatory capital broadly match. Then if economic capital *exceeds* both regulatory and actual capital, the regulatory rules likely *under*estimate the risk inherent in the bank's activities and the institution may nevertheless want to scale back its risk-taking activities. If, on the other hand, economic capital is *less* than both regulatory and actual capital, the risk of institution's activities are actually *over*estimated by the regulatory rules and there is

[24] See for instance Mingo (2000) and the references cited therein.

probably scope for the bank to take on more risk. Alternately, in the latter case financial institutions have often used regulatory capital arbitrage techniques to reduce both the book and regulatory capital without any significant affect in the economic capital.

6.2. A credit portfolio management example

Having motivated the importance of using formal models for determining economic capital, consider now an example with one of the popular credit portfolio models, CreditMetrics™, that follows an approach that was preciously described in Section 2. It utilizes an unconditional transition matrix to determine asset return thresholds (or critical distances), and simulates the joint distribution of underlying asset values. A cardinal input to the model is the grade migration matrix as it describes the evolution of the portfolio's credit quality. As an illustration of stress testing, we analyze the impact on the portfolio value distribution of replacing the unconditional transition matrix with two conditional matrices: the expansion or contraction matrix.

We constructed a bond portfolio with 148 exposures with a current exposure of $154.6 MM which mimics the ratings distribution of the S&P U.S. universe as of January 1, 1998. The exposures range in maturities from one month to sixteen years. Some have fixed and others have floating interest rates. Interest is paid quarterly, semi-annually, or annually. Several exposures are denominated in foreign currencies. As CreditManager™ does not allow to input recovery rates, we use the preset mean recovery rates and standard deviations from Altman and Kishore (1996). We take the yield curves and credit spreads as of February 17, 1998. We then ask the question: what is the portfolio

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[25] Also referred to as book capital, although actual capital may also include some additional things

value distribution one year hence if 1998 had been an expansion or a contraction year? We generate this distribution with 100,000 simulations using CreditMetrics™. The value distribution is shown below in figure 6.1.

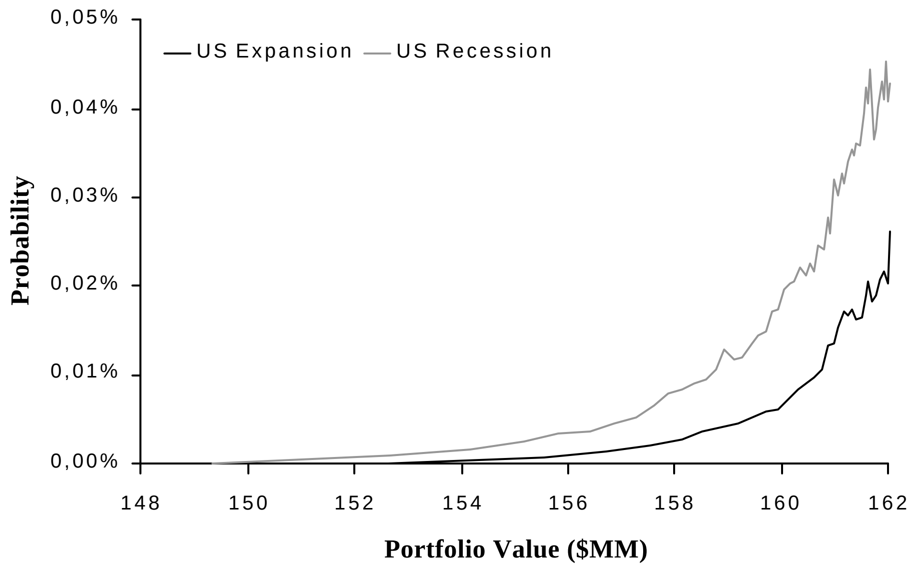

Figure 6.1: Portfolio value distributions

Financial institutions are more interested in the behavior of the portfolio value distribution at the extreme lower end. Specifically for the calculation of VaR, the possible losses on a 99% or 99.9% confidence level are the key figures. Therefore Figure 6.1 depicts only this region of the portfolio value distributions. It clearly shows the much longer downward tail of the contraction distribution and the overall higher probability to obtain values below the mean. The differences are nontrivial, and this becomes even more poignant once we consider the economic capital attribution as seen in Table 6.1. At the 99% confidence level, the amount of economic capital needed for a contraction year is nearly 30% higher than for an expansion year. For the 99.9% level, roughly appropriate for an A- rating, the difference is about 25%!

Portfolio Parameters (all values in $ MM)	***Transition Matrix***			
	Expansion	Recession	Unconditional	% Δ recession / expansion
Mean Value	163.8	163.3	163.7	-0.3%
Econ. Capital (99%)	6.5	8.4	6.6	29.9%
Econ. Capital (99.9%)	12.3	15.4	12.6	25.2%

Table 6.1 Economic Capital Attribution

Finally, the overall deviations can also be described by the overall losses, i.e. expected appreciation minus unexpected losses, that might occur on a given percentage level. While during an expansion the maximum loss in value at a 99.9% confidence level is $3.2 MM, it is $6.8 MM during a contraction -- more than double! This result clearly demonstrates the importance of accounting for business cycle effects when assessing the credit risk of a portfolio. Taking into account that an AA-rated financial is expected to be able to withstand losses even at a 99.97% confidence level, one can easily imagine how this difference becomes even more dramatic in these extreme regions of the value distribution.

7. Summary, conclusions and directions for future research

Utilizing an extensive database of S&P issuer ratings, we have presented a systematic study of rating migration behavior and its linkages to the macroeconomic conditions and asset quality. Our analysis suggests that first-order Markovian ratings dynamics, while not strictly correct, provide a reasonable practical approximation, so long as we allow for different transition matrices in expansions and contractions.

We are not the first to note that ratings transition probabilities may vary with the business cycle, and surely we are not the last. In an interesting contribution done independently of this paper, for example, Nickell, Perraudin and Varotto (2000) use Moody's data from 1970 to 1997 to examine the dependence of ratings transition probabilities on industry, country and stage of the business cycle using an ordered probit approach, and they find that the business cycle dimension is the most important in explaining variation of these transition probabilities. Our work complements, enriches and extends theirs, pointing in particular to the potential usefulness of regime-switching models in the context of credit portfolio stress testing.

As for future research, at least two promising directions are evident. First, for estimating actual credit risk as opposed to stress testing, one may weight the simulations obtained under assumed takeoffs from expansion and from contraction by p and (1-p), where p is the probability that the economy is currently in expansion, obtained for example using the methods of Hamilton (1989). Second, it is of obvious interest to extend out methods to countries other than the U.S., whose typically less well developed business cycle chronologies present a challenge for our approach.

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