Fair Value of Life Liabilities with Embedded Options: an Application to a Portfolio of Italian Insurance Policies*

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Abstract

In this paper we present a pricing application analyzing, in a contingent-claims framework, the two most common types of life policies sold in Italy during the last two decades. These policies, characterized by different premium payment styles (single and constant periodical), are endowments including both a bonus option and a surrender option. The bonus option's benefit is annually adjusted according to the performance of a reference fund and a minimum return is guaranteed to the policyholder. The surrender option is an American-style put option that enables the policyholder to give up the contract receiving the surrender value. We propose to price this American-style put option by Montecarlo simulation according to the Longstaff and Schwartz Least-Squares approach [17] giving a comparative analysis with the results obtained by Grosen and Jørgensen [15] according to a Recursive Tree Binomial approach. We then proceed to present an application to a relevant portion of a major Italian life policies' portfolio. We make use of a Black&Scholes-CIR economy to simulate the reference fund, composed by equities and bonds, according to De Felice and Moriconi [14] and Pacati [21], and we estimate the fair value of portfolio's liabilities extending the framework in order to price also the embedded surrender options.

Keywords: Surrender Option; Longstaff-Schwartz Least-Squares Approach; Minimum Interest Rate Guaranteed; Black&Scholes-CIR Framework.

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1 Introduction

The most common types of life policies issued by italian companies present two intimately linked faces: one actuarial and the other financial. From an actuarial point of view, these products provide a financial service to individuals that wish to insure themselves against financial losses which could be the consequence of death, sickness or disability. At the same time these products often include interest rate guarantees, bonus distribution schemes and surrender options that represent liabilities to the insurer. In the past, for example in the 1970's and 1980's when long term interest rates were high, some of these options have been viewed by insurers as far out of the money and were ignored in setting up reserves, but the value of these guarantees rose as long as term interest rates began to fall in the 1990's. If the rates provided under the guarantee are more beneficial to the policyholder than the prevailing rates in the market, the insurer has to make up the difference.

The problem of accurately identifying, separating and estimating all the components characterizing the guarantees and the participation mechanism has attracted an increasing interest both of researchers and practitioners from a risk management and option pricing point of view. In their seminal contributions, Brennan and Schwartz [9] and Boyle and Schwartz [7] have employed the techniques of contingent claims analysis to provide a valuation framework in order to estimate the fair value of a guaranteed equity-linked contract.

According to the recent literature (Grosen and Jørgersen [15], Bacinello [4]), a life policy contract can be viewed as a participating american contract that can be splitted into various components:

1. the basic contract, a risk free bond;

2. the bonus option, a participating European-style option where the benefit is annually adjusted according to the performance of a reference fund and a minimum return is guaranteed to the policyholder; the literature concerning the bonus option is rich and we recall Norberg [18] and [19], Bacinello [3], De Felice and Moriconi [14], Pacati [21], Consiglio, Cocco and Zenios [10] and [11] and Consiglio, Saunders and Zenios [12];

3. the surrender option, an American-style put option that enables the policyholder to give up the contract receiving the surrender value; The literature concerning the surrender option is more recent and we recall Albizzati and Geman [1], Bacinello [4], Grosen and Jørgersen [15].

In addition, traditional italian policies enable the policyholder to give up the contract either receiving the surrender value, a cash payment, or converting the surrender value into a guaranteed annuity, payable through the remaining lifetime and calculated at a guaranteed rate, which can be greater than market interest rate as outlined recently by Boyle and Hardy [8] and Ballotta and Haberman [5]. Another factor added to the cost of these guarantees, according to Ballotta and Haberman [6], is the following: the mortality assumption implicit in the guarantee did not take into account the improvement in mortality which took place in the last years.

In this paper we present a pricing application analyzing, in a contingent-claims framework, the two most common types of life policies sold in Italy during the last two decades. These policies, characterized by different premium payment styles (single and constant periodical), are endowments including both a bonus option and a surrender option. We propose to price the surrender option by Montecarlo simulation according to the Longstaff and Schwartz Least-Squares approach [17] giving a comparative analysis with the results obtained by Grosen and Jørgensen [15] according to a Recursive Tree Binomial approach. We then proceed to present an application to a relevant portion of RAS SpA life policies' portfolio. The results are purely indicative and the comments do not represent the views and/or opinion of RAS management. We make use of a Black&Scholes-CIR economy to simulate the reference fund, composed by equities and bonds, according to De Felice and Moriconi [14] and Pacati [21], and we estimate the fair value of portfolio's liabilities extending the framework in order to price also the embedded surrender options.

Section 2 discusses the Longstaff and Schwartz Least-Squares approach [17] to price an American-style option by Montecarlo simulation and presents a comparative analysis with the results obtained by Grosen and Jørgensen [15]. Section 3 describes the significant portion of life policies' portfolio analyzing two types of life insurance contracts. The approach followed in the simulation of the reference fund is exposed and the estimates of the fair value of liabilities and the results are discussed. Finally, Section 4 presents conclusions and possible future extensions.

2 A Longstaff Schwartz Approach to Price the Surrender Option

Our purpose is to value the surrender option embedded in the endowment policies considered in this work. The surrender option is an American-style put option that enables the policyholder to give up the contract and receive the surrender value. We implement a method that uses Monte Carlo simulation, adapting it, so that it can work also with products that present American-exercise features. In particular, we follow the least squares Monte Carlo approach presented by Longstaff and Schwartz [17].

We first adapt the above method trying to replicate the results in the article by Grosen and Jørgensen [15], where only financial risks are treated, the effect of mortality is not considered and the riskless rate of interest is assumed to be constant. In subsection 3.1 we then apply the method to our model considering both financial and actuarial uncertainty, and where the term structure of interest rates is assumed to be stochastic.

We briefly summarize the problem analyzed by Grosen and Jørgensen [15]: at time zero (the beginning of year one), the policyholder pays a single premium P_0 to the insurance company and thus acquires a contract of nominal value P_0. The policy matures after T years, when the insurance company makes a single payment to the policyholder. However, the contract can also be terminated depending on the policyholder's discretion before time T. The insurance company invests the trusted funds in an asset portfolio whose market value $A(t)$ is assumed to evolve according to a geometric Brownian motion,

$$dA(t) = \mu A(t)dt + \sigma A(t)dW(t), \quad A(0) = A_0, \tag{1}$$

where μ, σ and $A(0)$ are constants and $W(\cdot)$ is a standard Brownian motion with respect to the real-world measure. Under the risk neutral probability measure Q the evolution is given by

$$dA(t) = rA(t)dt + \sigma A(t)dW^Q(t), \quad A(0) = A_0, \tag{2}$$

where $W^Q(\cdot)$ is a standard Brownian motion under Q and r is the instantaneous spot rate. The interest credited to the policyholder from time $t-1$ to time t, $t \in \{1, \dots, T\}$, is denoted $r_P(t)$ and is guaranteed never to fall below r_G, the contractually specified guaranteed annual interest rate. In their paper, Grosen and Jørgensen [15] define $r_P(t)$ as

$$r_P(t) = \max\left\{r_G, \alpha\left(\frac{B(t-1)}{P(t-1)} - \gamma\right)\right\}, \tag{3}$$

where α is called by the authors the distribution ratio, γ is the target buffer ratio and $B(t) = A(t) - P(t)$. P_0 grows according to the following mechanism:

$$P(t) = (1 + r_P(t)) \cdot P(t-1), \quad t \in \{1, 2, \dots, T\}, \quad P(0) = P_0. \tag{4}$$

Grosen and Jørgensen [15] define two contract types: the European contract and the American contract. The former is simply the contract that pays $P(T)$ at the maturity date T, whereas the latter can be exercised depending on the policyholder's discretion at any time t in the set $\{0, 1, 2, \dots, T\}$. If the policyholder decides to exercise at time t, he receives $P(t)$. The surrender option value is given by the difference between the American contract value and the European contract value.

The time zero value of the European contract $V_E(0)$ is calculated by the authors via Monte Carlo simulation under the risk neutral measure: they simulate the value of the asset portfolio until time T with an annual step, determine $r_P^i(t)$ and $P^i(t)$ at each step t and for each path i and then make the average and discount to find the value of the contract:

$$V_E(0) = \frac{e^{-rT}}{M} \sum_{i=1}^{M} P^i(T), \tag{5}$$

M being the number of simulated paths. In order to price the American contract, Grosen and Jørgensen [15] implement a binomial tree model á la Cox, Ross and Rubinstein [13]. We note that, because of the dependence of the contract values on both $A(\cdot)$ and $P(\cdot)$, the size of the trees which keep track of these variates grows exponentially with T.

We now briefly describe the method suggested in the paper by Longstaff and Schwartz [17] in order to price American options by Monte Carlo simulation (LSM: least squares Monte Carlo approach), and then compare the numerical results we have obtained with those presented by Grosen and Jørgensen [15]. The mechanism underlying an option with american exercise features is the following: at any exercise time, the holder of an American option compares the payoff from immediate exercise, which we refer to as intrinsic value, with the expected payoff from continuation, and exercises if the immediate payoff is higher. In other words, at each simulated time instant, the value of the contract is the maximum between the intrinsic value and the continuation value. Thus, the optimal exercise strategy is determined by the discounted conditional expectation under the risk-neutral probability measure of the future cash flows, assuming an optimal exercise policy is adopted in the future. For example, in the case of an American put option written on a single non-dividend paying asset, the value (cashflow) $V_i(S_i)$ of the option at time i, conditional on the current asset price S_i, is given recursively by

$$V_i(S_i) = \max\left\{I_i(S_i), E_i^Q\left[e^{-r\Delta t}V_{i+1}(S_{i+1})|S_i\right]\right\}, \tag{6}$$

where $I_i(S_i)$ is the intrinsic value and Δt is the discretization step. The difficulty in using Monte Carlo derives from the fact that we should know the conditional expected value of the future option value, but this depends on the next exercise decisions.

The approach developed by Longstaff and Schwartz [17] is that this conditional expection can be estimated from the cross-sectional information in the simulation by using least squares, that is by regressing the discounted realized payoffs from continuation on functions of the values of the state variables (the current underlying asset price in this example). For example, the use of a quadratic polynomial would give

$$E_i^Q\left[e^{-r\Delta t}V_{i+1}(S_{i+1})|S_i\right] \approx a_1 + a_2 S_i + a_3 S_i^2. \tag{7}$$

While the generation of sampled paths goes forward in time, the regressions go backwards, starting from time T. At this time, the exercise strategy is trivial: the option is exercised if and only if it is in-the-money. If the strike is X, the cashflows for each path j are $\max\{X - S_T^j, 0\}$, provided that the option has not been exercised yet. Going backwards in time to time step $T-1$, if on a path the option is in-the-money, one may consider exercising it. The continuation value is approximated regressing the discounted cashflows only on the paths where the option is in-the-money. The important point is that the early exercise decision is based on the regressed polynomial, in which the coefficients are common on each path, and not on the knowledge of the future price along the same path. In subsection 2.1 we compare the results obtained by Grosen and Jørgensen [15] with the binomial model with those we have produced through LSM.

2.1 Numerical Results

In the section devoted to numerical results, Grosen and Jørgensen [15] analyse contracts for which $P_0 = 100$, $B_0 = 0$, $T = 20$ years and $r_G = 4.5\%$. The volatility σ of the reference portfolio is set to 15% and 30%, whereas the riskless interest rate assumes values in the set {8%,6%,4%}.

In Tables 1-6 we present the results we obtain through LSM, simulating 50,000 paths, for different values of α and γ and compare them with Grosen and Jørgensen's [15] results (last column). In the column denoted by E.C. we report the values of the European contract. The other columns contain the values of the American contract we have produced using different combinations of A, P and r_P as state variables. The use, for example, of the two state variables A and P together, means that we base the regression on the quadratic polynomial given by a linear combination of the powers and cross products of A and P up to the second order. The reason for using more than one state variable is that, as pointed out by Longstaff and Schwartz [17], if the regression involves all paths, more than two or three times as many functions may be needed to obtain the same level of accuracy as obtained by the estimator based on in-the-money paths. This is our case, since the intrinsic value is not, for example, the standard payoff of a put, but is given by the surrender value, so we cannot limit the number of values used in the regression to those where the option is in-the-money, but we have to consider all of them.

We now look at the values in the tables. In the cases σ=15%, r=8% and σ=15%, r=6%, our results are close to the values by Grosen and Jørgensen [15] when we use as state variable the value A of the reference fund, but we obtain a better approximation when we use two state variables A and P. Moreover, it seems that adding a third state variable doesn't improve the results. We observe that the choice of P (the benefit) as state variable would significantly under-estimate the value of the American contract. When we assume σ=15%, r=4% the situation is different: the use of the state variable A produces an American contract value lower than the corresponding European value. When we regress with respect to P only, either to A and P, the results are more accurate. As in Grosen and Jørgensen [15], the American contract has the same value as its European counterpart, meaning that early exercise is never optimal when the riskless rate of interest r is lower than the minimum guarantee.

The case σ=30% reflects the behaviour of the corresponding cases with σ=15%, but the results are worse. In particular, in the cases r=8% and r=6% it is more evident that the use of two state variables does better than using only A, even if the values obtained are still lower than those given by the tree. This could be due to the greater dispersion of Monte Carlo paths when the volatility is high. When we assume r=4%, the use of the state variable P seems to work well, but the of A and P together gives values for the American contract nearer to the European contract values than those obtained using only A, but still lower.

In general, our results produce a not significant error, since the difference between the values obtained with the two approaches is less than 2.5%; except for the cases with high volatility or interest rates close to the minimum guarantee, in which cases the difference is higher. In addition, the differences are both positive and negative. These results suggest that the LSM algorithm is able to approximate closely the binomial tree values.

3 An Application to an Italian Life Policies' Portfolio

In this section we explain the assumptions, the model and the method we follow in our analysis, according to De Felice and Moriconi [14] and Pacati [21].

We consider two types of participating endowment policies, those payed by a single premium and those payed by constant annual premiums, part of a significant portion of a major Italian life portfolio. The premiums earned are invested into a reference fund. The benefit is annually adjusted according to the performance of the fund and a minimum return is guaranteed to the policyholder.

We define x as the age of the policyholder at the inception of the contract and n as the term of the policy. We define a as the number of years between the inception date of the contract and the 31 December 2002, which is our valuation date. We assume that a is integer and therefore the policy starts exactly a years before the valuation date, which we denote by t. $m = n - a$ is the time to maturity. We suppose that all the contractually relevant future events (premium payments, death and life to maturity) take place at the integer payment dates $a+1, \ldots, a+m$. In particular, benefit payments occur at the end of the year of death, if the policyholder dies within the remaining m years, or at the end of the m-th year, if he is alive after m years. Premium payments occur at the beginning of the year. Finally, we do not consider possible future transformations of the policies, such as reduction and guaranteed annuity conversion options.

Let C_a be the sum insured at the payment date a. We define C_{a+k} and F_k respectively as the benefit eventually paid at time $a+k$ and the market value of the reference fund at time $t+k$, $k = 1, \ldots, m$. We define i as the technical interest rate, i_{min} as the minimum guaranteed and i_{tr} as the minimum rate retained by the company. $\beta \in [0,1]$ is the participation coefficient. The annual rate of return of the reference fund at time $t+k$, I_k, is defined as:

$$I_k = \frac{F_k}{F_{k-1}} - 1, \tag{8}$$

and the readjustment measure is

$$\rho_k = \max\left(\frac{\min(\beta I_k, I_k - i_{tr}) - i}{1+i}, s_{min}\right), \qquad s_{min} = \frac{i_{min} - i}{1+i}. \tag{9}$$

For C_{a+k}, in the case of single premiums, we have

$$C_{a+k} = C_{a+k-1}(1+\rho_k) = C_a \Phi(t,k), \tag{10}$$

$$\Phi(t,k) = \prod_{h=1}^{k} (1+\rho_h), \tag{11}$$

(full readjustment rule), whereas in the case of constant annual premiums, we have, according to Pacati [21],

$$C_{a+k} = C_{a+k-1}(1+\rho_k) - \frac{m-k}{n} C_0 \rho_k = C_a \Phi(t,k) - \frac{1}{n} C_0 \Psi(t,m,k), \tag{12}$$

$$\Psi(t,m,k) = \sum_{l=0}^{k-1} (m-k+l)\rho_{k-l} \cdot \prod_{j=k-l+1}^{k} (1+\rho_j). \tag{13}$$

Our valuation consists in the calculation of American contract value and the European contract value, defined also as stochastic reserve according to De Felice and Moriconi [14] and Pacati [21]. We derive the surrender option as the difference between American contract value and European contract value. The European put option, the minimum guarantee, is a component of the European contract value. We value the stochastic reserve on first order bases using conservative probabilities excluding surrenders and considering net premiums. We define P_k as the net premium due at time $a+k$: since we suppose the valuation takes place soon after the premium payment, $P_k = 0$ for single premiums and $P_k = P$ constant for annual premiums, $k = 1, \ldots, m-1$. We define ${}_kp_{x+a}$ as the probability that the policyholder, which age is $x+a$, is alive at time $a+k$, and ${}_{k-1|1}q_{x+a}$ as the probability that the policyholder, alive at time $a+k-1$, dies between $a+k-1$ and $a+k$. The policies payed by constant annual premiums that we consider are also characterized by the presence of a terminal bonus. We define b_M as the bonus payed in case of death and b_V as the bonus payed in case of life at maturity; both expressed as percentages of the benefit. The quantities $C_{a+k}b_M$ and $C_{a+m}b_V$ are also readjusted according to equation (12), but are probabilized by modifying the survival probabilities with the percistency frequencies in the contract. We will not explicitly derive the terminal bonus and the relative probabilities, but the extension is immediate.

We assume that

$$F(t) = \alpha S(t) + (1-\alpha)W(t), \quad 0 \leq \alpha \leq 1, \tag{14}$$

where $S(t)$ is a stock index and $W(t)$ is a bond index. According to De Felice and Moriconi [14] and Pacati [21], we model $W(t)$ as the cumulated results of a buy-and-sell strategy, with a fixed trading horizon δ, of zero coupon bonds with a fixed duration $D \geq \delta$.

Given the continuous-time perfect-market assumptions, we use a 2-factor arbitrage model, modelling interest rate uncertainty with the CIR model and stock price uncertainty with the Black-Scholes model. Defining $r(t)$ as the instantaneous interest rate (spot rate) and $S(t)$ as the stock index, we thus have the following stochastic equations under the natural probability measure:

$$dr(t) = \kappa(\vartheta - r(t))dt + \rho\sqrt{r(t)}dZ^r(t), \quad r(0) = r_0, \tag{15}$$

where κ is the mean reversion coefficient, ϑ is the long term rate, ρ is the volatility parameter, r_0 is the initial spot rate, and

$$dS(t) = \mu S(t)dt + \sigma S(t)dZ^S(t), \tag{16}$$

where μ is the istantaneous expected return and σ is the volatility parameter. The two sources of uncertainty are correlated:

$$Corr(dZ^r(t), dZ^S(t)) = \rho^{rS}dt. \tag{17}$$

In order to estimate the American contract value and European contract value, we consider the risk neutral measure, substituting the original drift coefficients with $\kappa(\vartheta - r(t)) + \pi r(t)$ and $r(t)S(t)$. π identifies the market price of interest rate risk $q(r(t),t)$: $q(r(t),t) = \pi\sqrt{r(t)}/\rho$. In this model, the price at time t of a deterministic contract that pays at time $s > t$ the amount X_s, is given by

$$V(t, X_s) = E_t\left[X_s e^{-\int_t^s r(u)du}\right], \tag{18}$$

where E_t is the conditional risk-neutral expectation.

The stochastic reserve is the difference between the fair market value of the future benefits payable by the insurance company and the fair market value of the eventual future premiums payable by the policyholder, both multiplied for the corresponding survival probabilities:

$$\begin{aligned} V_t &= {}_mp_{x+a}C_aV(t,\Phi(t,m)) + \sum_{k=1}^{m} {}_{k-1|1}q_{x+a}C_aV(t,\Phi(t,k)) \\ &- \sum_{k=1}^{m-1} {}_kp_{x+a}P_kv(t,t+k) \end{aligned} \tag{19}$$

for single premiums, and

$$\begin{aligned} V_t &= {}_m p_{x+a}(C_a V(t,\Phi(t,m) - \frac{C_0}{n} V(t,\Psi(t,m,m))) \\ &+ \sum_{k=1}^{m} {}_{k-1|1}q_{x+a}(C_a V(t,\Phi(t,k) - \frac{C_0}{n} V(t,\Psi(t,m,k))) \\ &- \sum_{k=1}^{m-1} {}_k p_{x+a} P_k v(t,t+k) \end{aligned} \tag{20}$$

for constant annual premiums. We can apply the survival probabilities after calculating the non-probabilized values, since we suppose that actuarial and financial uncertainties are independent.

We observe (see Pacati [21]) that we can decompose the expression for C_{a+k} into the sum of two components. If we assume that it is always $\min(\beta I_k, I_k - i_{tr}) \geq i_{min}$, we obtain the base component:

$$\mathcal{B}_{a+k} = C_a \prod_{l=1}^{k} \left(1 + \frac{\min(\beta I_l, I_l - i_{tr}) - i}{1+i}\right) \tag{21}$$

in the case of a single premium payment, and

$$\begin{aligned} \mathcal{B}_{a+k} &= C_a \prod_{l=1}^{k} \left(1 + \frac{\min(\beta I_l, I_l - i_{tr}) - i}{1+i}\right) \\ &- \frac{C_0}{n} \sum_{l=0}^{k-1} \left[(m-k+l)\frac{\min(\beta I_{k-l}, I_{k-l} - i_{tr}) - i}{1+i} \cdot \right. \\ &\cdot \left. \prod_{j=k-l+1}^{k} \left(1 + \frac{\min(\beta I_j, I_j - i_{tr}) - i}{1+i}\right)\right] \end{aligned} \tag{22}$$

for constant annual premiums. Observing that $\mathcal{B}_{a+k} \leq C_{a+k}$, we define the put component as

$$\mathcal{P}_{a+k} = C_{a+k} - B_{a+k}. \tag{23}$$

The equation (23) is the payoff of an european put option of annual cliquet type, that guarantees a consolidation of the results obtained year by year, with annual strike rate i_{min}, and where the underlying is the minimum between the return of the reference fund, multiplied by the participation coefficient, and the minimum retained by the company.

3.1 An Extension of Longstaff Schwartz Approach

We now describe how we apply LSM to our model. In considering actuarial uncertainty, we follow the approach adopted by Bacinello [4]. As Grosen and Jørgensen [15], Bacinello [4] implements a binomial tree model, where the riskless interest rate is constant, taking into account the survival probabilities. From a LSM point of view, the approach is the following (to ease the notation, for a given policy we denote by T the time it expires): at step $T-1$ and for path j (if the insured is alive), the continuation value is given by

$$W_{T-1}^j = e^{-\int_{T-1}^{T} r(u)du} C_T^j - P_{T-1}, \tag{24}$$

since the benefit C^j_T is due with certainty at time T. P_{T-1} (zero in the case of single premiums) is the premium due at time $T-1$. The value of the contract F^j_{T-1} is therefore the maximum between the continuation value and the surrender value R^j_{T-1} (that is the benefit due in $T-1$ eventually reduced):

$$F^j_{T-1} = \max\{W^j_{T-1}, R^j_{T-1}\}. \tag{25}$$

Assume now to be at time $t < T-1$: to continue means to immediately pay the premium P_t and to receive, at time $t+1$, the benefit C^j_{t+1}, if the insured dies within one year, or to be entitled of a contract whose total random value (including the option of surrendering it in the future), equals F^j_{t+1}, if the insured is alive. We suppose that the benefit received in $t+1$, in case of death between t and $t+1$, is C^j_t: we thus have a quantity known in t and we can avoid taking the conditional discounted expectation of C^j_{t+1}. The continuation value at time t is then given by the following difference:

$$W^j_t = \left\{ {}_{1|1}q_{x+t} e^{-\int_t^{t+1} r(u)du} C^j_t + (1 - {}_{1|1}q_{x+t}) E(e^{-\int_t^{t+1} r(u)du} F^j_{t+1}) \right\} - P_t. \tag{26}$$

We use the regression to estimate the value of $E(e^{-\int_t^{t+1} r(u)du} F^j_{t+1})$. At each step and for each path we compare the intrinsic value with the continuation value and take the optimal decision. For those paths where the optimal decision is to surrender, we memorize the time and the corresponding cashflow. When we arrive at time 1 (or the time the insured can start to surrend from), we have a vector with the exercise times for each path and a vector with the cashflows (benefits) corresponding to that time. At this point, taking into account that the policyholder can survive until the exercise time or die before, we multiply the benefits due until exercise time by the corresponding survival or death probabilities, sum them all and finally calculate the average on the number of paths.

3.2 Numerical Results

In Tables 7-14, we present the results we obtain analyzing 944 policies payed by single premium and 1,000 policies payed by constant annual premiums. The values are expressed in euro. We make the calculations on every policy without aggregating and group the results by age layers. The two types of policies we consider are characterized by: i=3%, i_{min}=4%, β=80%. We suppose that the policyholder can surrender after the first year from the inception of the contract and we assume that i_{tr}=1%. We calibrate the parameters of the CIR model at 31 December 2002, our valuation date, over a cross section of LIBOR interest rate swaps at different maturities. We simulate 5,000 paths for the reference fund, discretizing the equations for the spot rate and the stock index under the neutral probability measure according to the stochastic Euler scheme with a monthly step. We assume ρ^{rS}=-0.1 and δ=4 months and analyze two different combinations of α, σ and D.

We choose to base the regression for the valuation of the continuation value at time $a+k$ on two state variables, the value of the reference fund F_k and the value of the benefit C_{a+k}. The surrender value at time $a+k$ is $C_{a+k}(1+j_{risc})^{(k-m)}$, where j_{risc} represents an annual compounded discount rate. In the tables we compare the results we obtain making different assumptions about the composition of the reference fund and the values of j_{risc}. In particular, we analyze the combinations of j_{risc}=0 (no penalty in case of surrender) and j_{risc}=0.5%, with α=10%, σ=15%, D=5 years or α=30%, σ=30%, D=10 years. We remark that when the value of j_{risc} is greater than i, this penalty reduces the value of surrender options.

Looking at the values of the stochastic reserve, we observe that it increases as α, σ and D increase. This is due to the fact that the insurance company's liabilities grow. The values of put options increase too, meaning that the weight of the minimum guarantee becomes greater. Put option values too, become more valuable with increasing uncertainty, due to increasing volatility of the underlying portfolio. In the case of single premiums, the value of the surrender option decreases as j_{risc} increases. We also note that the value of the surrender option decreases with a more "aggressive" composition of the reference fund. This is pointed out also in Grosen and Jørgensen [15] and is due to the fact that a more aggressive policy determines an advantage for the policyholder only, and so his incentive to prematurely exercise may be partly or fully reduced. In the case of constant annual premiums instead, the effect of increasing values for α, σ, D and j_{risc} doesn't seem to produce a lessening of surrender option values. The future premiums to be paid could play a significant role in leading the policyholder's decision.

4 Conclusions

In this paper we have presented a pricing application analyzing, in a contingent-claims framework, the two most common types of life policies sold in Italy. These policies, characterized by different premium payment styles (single and constant periodical), are endowments including both a bonus option and a surrender option. We have proposed to price the surrender option by Montecarlo simulation according to the Longstaff and Schwartz Least-Squares approach [17] giving a comparative analysis with the results obtained by Grosen and Jørgensen [15] according to a Recursive Tree Binomial approach. Our results are preliminary but encouraging; the differences between the binomial tree and LSM algorithm showed to be not significant. An extensive work has to be developed in order to make the Longstaff and Schwartz Least-Squares approach [17] more accurate as outlined in Tables 1-6.

We have then proceeded to present the application to a significant portion of a major Italian life policies' portfolio. We have adopted a Black&Scholes-CIR economy to simulate the reference fund and we have estimated the fair value of portfolio's liabilities viewed as American contracts pricing the single components.

In our project we want to consider other types of policies and to extend the Black&Scholes-CIR framework using a two-factor interest rate model. We also aim to aggregate the single policies in "macro-policies" without a significant loss of precision and reducing the calculation time.

Software: The results in this paper were carried out in Matlab using functions implemented by the authors. We are grateful to John Brunello for providing the dataset of life policies.

Tables 1 and 2

$\sigma = 15\%$, $r = 8\%$							
α	γ	E.C. [a]	L.-S. [b] s.v.: A	L.-S. [c] s.v.: P	L.-S. [d] s.v.: A, P	L.-S. [e] s.v.: A, P, $(1 + r_P)$	G-J [f]
0.5	0.05	94.29	109.04	99.09	109.82	109.91	109.49
0.5	0.15	88.28	104.18	96.47	104.96	105.04	104.61
0.5	0.25	83.25	101.02	96.47	101.64	101.70	101.15
0.75	0.05	101.00	115.07	104.45	115.89	115.94	115.69
0.75	0.15	94.09	108.88	98.13	109.68	109.71	109.23
0.75	0.25	88.32	104.24	96.47	105.03	105.08	104.50
1	0.05	105.46	119.48	108.39	120.25	120.26	119.79
1	0.15	98.01	112.35	101.08	113.17	113.16	112.05
1	0.25	91.81	106.89	96.51	107.73	107.78	106.65

[a] European Contract
[b] Longstaff-Schwartz approach, state variable: A
[c] Longstaff-Schwartz approach, state variable: P
[d] Longstaff-Schwartz approach, state variables: A and P
[e] Longstaff-Schwartz approach, state variables: A, P and $(1 + r_P)$
[f] Grosen-Jorgensen

$\sigma = 15\%$, $r = 6\%$							
α	γ	E.C. [a]	L.-S. [b] s.v.: A	L.-S. [c] s.v.: P	L.-S. [d] s.v.: A, P	L.-S. [e] s.v.: A, P, $(1 + r_P)$	G-J [f]
0.25	0.05	97.62	105.79	101.34	107.23	107.29	106.66
0.25	0.15	93.60	103.03	98.74	104.24	104.29	103.85
0.25	0.25	90.34	101.18	98.41	102.15	102.18	101.84
0.5	0.05	109.64	116.19	111.43	117.63	117.70	117.79
0.5	0.15	103.54	110.75	105.90	112.15	112.20	112.09
0.5	0.25	98.63	106.69	101.34	108.09	108.12	107.81
0.75	0.05	116.18	122.32	117.54	123.89	123.91	124.31
0.75	0.15	109.01	115.58	110.47	117.08	117.10	117.18
0.75	0.25	103.24	110.53	104.94	111.92	111.95	111.71
1	0.05	120.66	126.67	121.75	128.36	128.36	128.48
1	0.15	112.82	119.18	114.02	120.70	120.70	120.34
1	0.25	106.52	113.44	107.72	114.88	114.89	114.11

[a] European Contract
[b] Longstaff-Schwartz approach, state variable: A
[c] Longstaff-Schwartz approach, state variable: P
[d] Longstaff-Schwartz approach, state variables: A and P
[e] Longstaff-Schwartz approach, state variables: A, P and $(1 + r_P)$
[f] Grosen-Jorgensen

Tables 3 and 4

$\sigma = 15\%$, $r = 4\%$							
α	γ	E.C. [a]	L.-S. [b] s.v.: A	L.-S. [c] s.v.: P	L.-S. [d] s.v.: A, P	L.-S. [e] s.v.: A, P, $(1 + r_P)$	G-J [f]
0.25	0.05	125.81	121.94	125.81	125.43	125.74	125.88
0.25	0.15	122.25	118.80	122.25	121.77	122.09	122.31
0.25	0.25	119.52	116.48	119.52	118.92	119.30	119.58
0.5	0.05	136.36	131.51	136.36	135.61	135.72	136.46
0.5	0.15	130.48	126.15	130.48	129.64	129.76	130.57
0.5	0.25	126.01	122.14	126.01	125.10	125.25	126.10
0.75	0.05	142.51	137.16	142.50	141.58	141.68	142.64
0.75	0.15	135.32	130.54	135.31	134.37	134.47	135.44
0.75	0.25	129.86	125.56	129.86	128.88	128.94	129.98
1	0.05	146.88	141.05	146.87	145.85	145.89	147.04
1	0.15	138.82	133.68	138.82	137.80	137.83	138.96
1	0.25	132.70	128.09	132.70	131.66	131.68	132.84

[a] European Contract
[b] Longstaff-Schwartz approach, state variable: A
[c] Longstaff-Schwartz approach, state variable: P
[d] Longstaff-Schwartz approach, state variables: A and P
[e] Longstaff-Schwartz approach, state variables: A, P and $(1 + r_P)$
[f] Grosen-Jorgensen

$\sigma = 30\%$, $r = 8\%$							
α	γ	E.C. [a]	L.-S. [b] s.v.: A	L.-S. [c] s.v.: P	L.-S. [d] s.v.: A, P	L.-S. [e] s.v.: A, P, $(1 + r_P)$	G-J [f]
0.25	0.05	101.47	121.39	110.55	122.66	122.75	124.32
0.25	0.15	97.07	117.49	106.04	118.63	118.73	119.99
0.25	0.25	93.25	114.26	102.50	115.28	115.36	116.60
0.5	0.05	123.10	142.19	129.27	144.08	144.14	148.48
0.5	0.15	116.31	135.40	122.14	137.05	137.14	141.02
0.5	0.25	110.48	129.72	117.33	131.15	131.18	134.91
0.75	0.05	135.76	154.74	140.42	157.23	157.26	163.63
0.75	0.15	127.63	146.58	132.46	148.68	148.72	154.39
0.75	0.25	120.71	139.62	125.76	141.57	141.56	146.50
1	0.05	144.95	164.36	148.97	167.31	167.21	174.43
1	0.15	135.94	155.04	140.02	157.52	157.48	163.63
1	0.25	128.29	147.17	132.58	149.46	149.49	154.35

[a] European Contract
[b] Longstaff-Schwartz approach, state variable: A
[c] Longstaff-Schwartz approach, state variable: P
[d] Longstaff-Schwartz approach, state variables: A and P
[e] Longstaff-Schwartz approach, state variables: A, P and $(1 + r_P)$
[f] Grosen-Jorgensen

Tables 5 and 6

$\sigma = 30\%$, $r = 6\%$							
α	γ	E.C. [a]	L.-S. [b] s.v.: A	L.-S. [c] s.v.: P	L.-S. [d] s.v.: A, P	L.-S. [e] s.v.: A, P, $(1 + r_P)$	G-J [f]
0.25	0.05	124.54	132.11	127.84	134.38	134.26	136.34
0.25	0.15	119.70	127.72	123.06	129.68	129.58	131.46
0.25	0.25	115.54	124.01	118.83	125.68	125.62	127.50
0.5	0.05	147.50	154.10	149.40	157.26	157.09	161.91
0.5	0.15	139.91	146.80	141.98	149.62	149.50	153.79
0.5	0.25	133.47	140.61	135.64	143.21	143.11	147.02
0.75	0.05	161.21	167.35	162.31	171.14	170.94	177.73
0.75	0.15	152.03	158.26	153.36	161.81	161.68	167.74
0.75	0.25	144.30	150.77	145.86	154.07	153.95	159.20
1	0.05	171.23	176.75	171.76	181.44	181.21	189.04
1	0.15	160.98	166.82	161.83	171.03	170.81	177.38
1	0.25	152.38	158.47	153.44	162.33	162.18	167.41

[a]European Contract
[b]Longstaff-Schwartz approach, state variable: A
[c]Longstaff-Schwartz approach, state variable: P
[d]Longstaff-Schwartz approach, state variables: A and P
[e]Longstaff-Schwartz approach, state variables: A, P and $(1 + r_P)$
[f]Grosen-Jorgensen

$\sigma = 30\%$, $r = 4\%$							
α	γ	E.C. [a]	L.-S. [b] s.v.: A	L.-S. [c] s.v.: P	L.-S. [d] s.v.: A, P	L.-S. [e] s.v.: A, P, $(1 + r_P)$	G-J [f]
0.25	0.05	159.70	150.81	159.70	155.78	156.07	159.93
0.25	0.15	154.33	146.02	154.33	150.57	150.77	154.37
0.25	0.25	149.78	141.88	149.78	146.17	146.28	149.83
0.5	0.05	184.36	173.22	184.36	179.85	179.70	187.01
0.5	0.15	175.79	165.32	175.78	171.53	171.36	177.87
0.5	0.25	168.61	158.76	168.60	164.52	164.43	170.30
0.75	0.05	199.55	186.94	199.55	194.64	194.39	203.75
0.75	0.15	189.05	177.30	189.05	184.44	184.22	192.59
0.75	0.25	180.31	169.34	180.30	175.93	175.77	183.09
1	0.05	210.85	197.40	210.85	205.65	205.51	215.88
1	0.15	199.05	186.55	199.04	194.13	193.88	202.91
1	0.25	189.25	177.62	189.24	184.56	184.55	191.81

[a]European Contract
[b]Longstaff-Schwartz approach, state variable: A
[c]Longstaff-Schwartz approach, state variable: P
[d]Longstaff-Schwartz approach, state variables: A and P
[e]Longstaff-Schwartz approach, state variables: A, P and $(1 + r_P)$
[f]Grosen-Jorgensen

Table 7

Single Premiums, α=10%, σ=15%, D=5, j_{risc}=0					
Age	Time to Term (Average)	Stochastic Reserve	American Contract	Surrender Option	Minimum Guarantee (Put Option)
15-20	4.11	44,959	48,411	3,452	3,287
21-25	5.02	187,170	229,776	42,607	20,321
26-30	5.55	647,722	729,453	81,731	53,399
31-35	5.38	983,618	1,082,313	98,695	73,739
36-40	5.25	765,117	856,630	91,513	64,966
41-45	4.46	1,025,194	1,114,189	88,996	74,687
46-50	3.66	1,316,850	1,394,116	77,266	82,882
51-55	3.37	415,667	437,568	21,901	25,140
56-60	3.34	520,591	543,038	22,447	29,258
61-65	2.75	105,344	107,752	2,408	5,097

Table 8

Single Premiums, α=30%, σ=30%, D=10, j_{risc}=0					
Age	Time to Term (Average)	Stochastic Reserve	American Contract	Surrender Option	Minimum Guarantee (Put Option)
15-20	4.11	48,734	49,342	608	7,942
21-25	5.02	220,370	234,169	13,799	59,962
26-30	5.55	720,739	743,413	22,673	141,833
31-35	5.38	1,078,181	1,103,083	24,902	188,719
36-40	5.25	850,599	873,078	22,480	168,886
41-45	4.46	1,115,899	1,135,525	19,627	185,621
46-50	3.66	1,406,978	1,421,155	14,177	194,039
51-55	3.37	442,328	446,175	3,847	58,053
56-60	3.34	550,122	553,631	3,509	65,925
61-65	2.75	109,743	109,843	100	10,552

Table 9

Single Premiums, α=10%, σ=15%, D=5, j_{risc}=0.5%					
Age	Time to Term (Average)	Stochastic Reserve	American Contract	Surrender Option	Minimum Guarantee (Put Option)
15-20	4.11	44,961	47,689	2,728	3,273
21-25	5.02	187,249	221,220	33,971	20,199
26-30	5.55	647,884	712,831	64,947	53,057
31-35	5.38	983,764	1,062,177	78,413	73,263
36-40	5.25	765,213	837,992	72,779	64,609
41-45	4.46	1,025,301	1,095,670	70,369	74,249
46-50	3.66	1,316,928	1,377,539	60,611	82,218
51-55	3.37	415,691	432,794	17,103	24,937
56-60	3.34	520,636	538,057	17,421	29,006
61-65	2.75	105,371	107,178	1,807	5,021

Table 10

Single Premiums, α=30%, σ=30%, D=10, j_{risc}=0.5%					
Age	Time to Term (Average)	Stochastic Reserve	American Contract	Surrender Option	Minimum Guarantee (Put Option)
15-20	4.11	48,840	48,972	132	8,010
21-25	5.02	221,358	227,163	5,804	60,341
26-30	5.55	723,048	731,705	8,656	142,980
31-35	5.38	1,081,016	1,089,739	8,723	190,436
36-40	5.25	853,258	860,767	7,509	170,545
41-45	4.46	1,118,585	1,124,442	5,858	187,339
46-50	3.66	1,409,579	1,413,275	3,697	195,923
51-55	3.37	443,051	443,947	896	58,617
56-60	3.34	550,986	551,643	657	66,541
61-65	2.75	109,849	109,859	10	10,702

Table 11

Constant Annual Premiums, α=10%, σ=15%, D=5, j_{risc}=0					
Age	Time to Term (Average)	Stochastic Reserve	American Contract	Surrender Option	Minimum Guarantee (Put Option)
16-20	5.39	405,416	408,610	3,194	36,236
21-25	5.95	1,743,016	1,759,786	16,770	176,223
26-30	6.00	2,278,048	2,300,023	21,975	226,444
31-35	5.77	2,698,273	2,722,775	24,503	257,279
36-40	5.28	2,649,655	2,672,797	23,142	240,556
41-45	4.33	2,321,054	2,338,789	17,735	180,840
46-50	3.49	979,013	984,818	5,805	64,026
51-55	3.81	341,743	344,208	2,465	24,568
56-60	2.75	107,630	108,140	510	5,812

Table 12

Constant Annual Premiums, α=30%, σ=30%, D=10, j_{risc}=0					
Age	Time to Term (Average)	Stochastic Reserve	American Contract	Surrender Option	Minimum Guarantee (Put Option)
16-20	5.39	455,296	458,936	3,640	97,079
21-25	5.95	1,993,997	2,013,188	19,190	483,945
26-30	6.00	2,599,625	2,624,752	25,127	620,370
31-35	5.77	3,058,312	3,086,228	27,916	697,841
36-40	5.28	2,975,296	3,001,463	26,168	640,031
41-45	4.33	2,540,934	2,560,484	19,551	451,370
46-50	3.49	1,050,982	1,057,294	6,313	152,320
51-55	3.81	370,342	373,026	2,684	59,679
56-60	2.75	113,583	114,130	547	13,033

Table 13

Constant Annual Premiums, α=10%, σ=15%, D=5, j_{risc}=0.5%					
Age	Time to Term (Average)	Stochastic Reserve	American Contract	Surrender Option	Minimum Guarantee (Put Option)
16-20	5.39	405,462	408,644	3,183	36,376
21-25	5.95	1,743,324	1,759,992	16,667	176,963
26-30	6.00	2,278,480	2,300,326	21,846	227,264
31-35	5.77	2,698,729	2,723,103	24,374	258,064
36-40	5.28	2,650,384	2,673,391	23,007	241,301
41-45	4.33	2,321,791	2,339,437	17,646	181,418
46-50	3.49	979,290	985,068	5,778	64,044
51-55	3.81	341,840	344,292	2,453	24,609
56-60	2.75	107,663	108,171	508	5,798

Table 14

Constant Annual Premiums, α=30%, σ=30%, D=10, j_{risc}=0.5%					
Age	Time to Term (Average)	Stochastic Reserve	American Contract	Surrender Option	Minimum Guarantee (Put Option)
16-20	5.39	454,654	458,340	3,686	97,741
21-25	5.95	1,991,242	2,010,717	19,475	486,475
26-30	6.00	2,596,068	2,621,564	25,496	623,491
31-35	5.77	3,054,413	3,082,721	28,308	701,675
36-40	5.28	2,971,254	2,997,768	26,513	643,658
41-45	4.33	2,536,913	2,556,617	19,704	454,664
46-50	3.49	1,049,676	1,056,029	6,353	153,604
51-55	3.81	369,787	372,489	2,702	60,144
56-60	2.75	113,412	113,961	549	13,157

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