HOW TO DEFINE THE QUALITY OF AN ECONOMIC SCENARIO GENERATOR FOR CALCULATING THE BEST-ESTIMATE OF A FRENCH SAVINGS CONTRACT IN €?

VERSION 0.2

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Summary

French savings contracts in €: principles

General principles

- Capital invested by the policyholder.
- Annual revaluation at least a guaranteed rate.
- In case of surrender or death: payment of the total capitalized investment.
- The insurer transfers the investment to the financial market.

Low risk investment

- Investment management at the discretion of the insurer.
- Nature of assets: bonds, equities and real estate.
- Partial protection of the policyholder's savings in case of insurer default: up to € 70,000 guaranteed by FGAP.

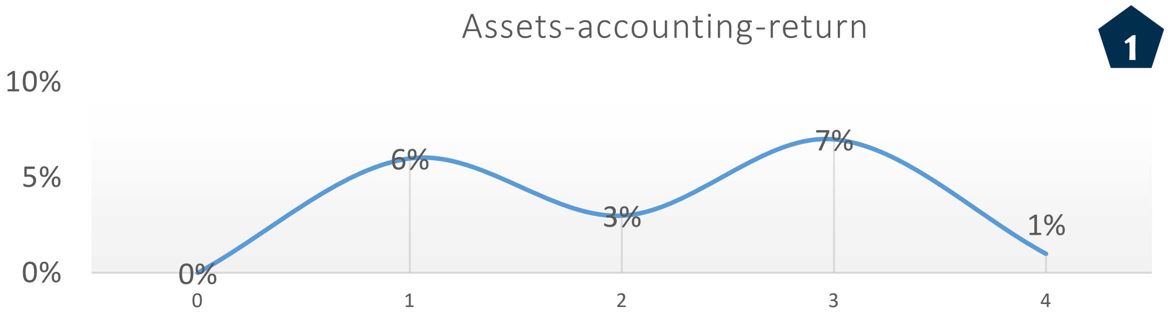
Capitalisation rate

- Guaranteed rate: minimum contractual revaluation rate of the invested capital.
- Profit sharing (PS): the technical and financial balance after payment of guaranteed interest.
- Undistributed PS is provisioned (PS Reserve) for up to 8 years.

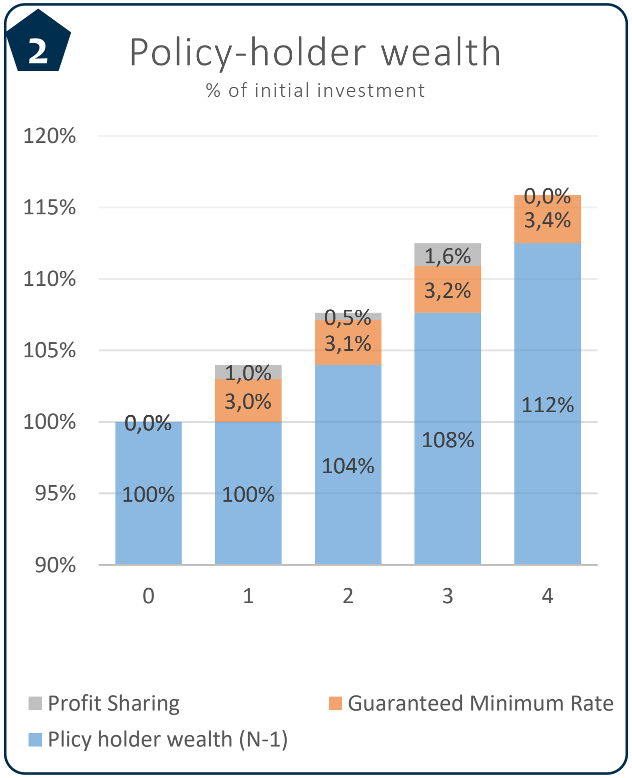
Acquired capital

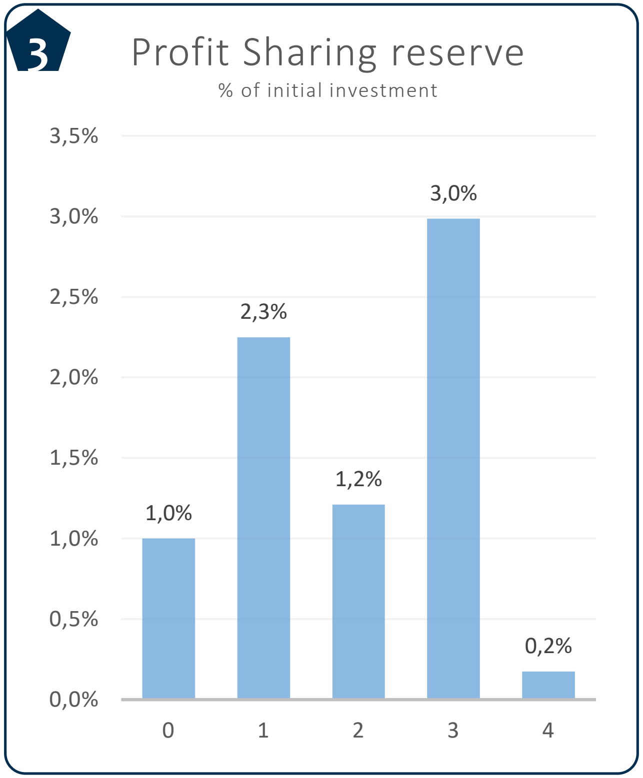
- At each time, the savings acquired by the policyholder corresponds to the investment capitalized whith guaranteed rates and the PS distributed.
- The PSR is acquired by all the policyholders.
- But the distribution of the PSR is at the discretion of the insurer.

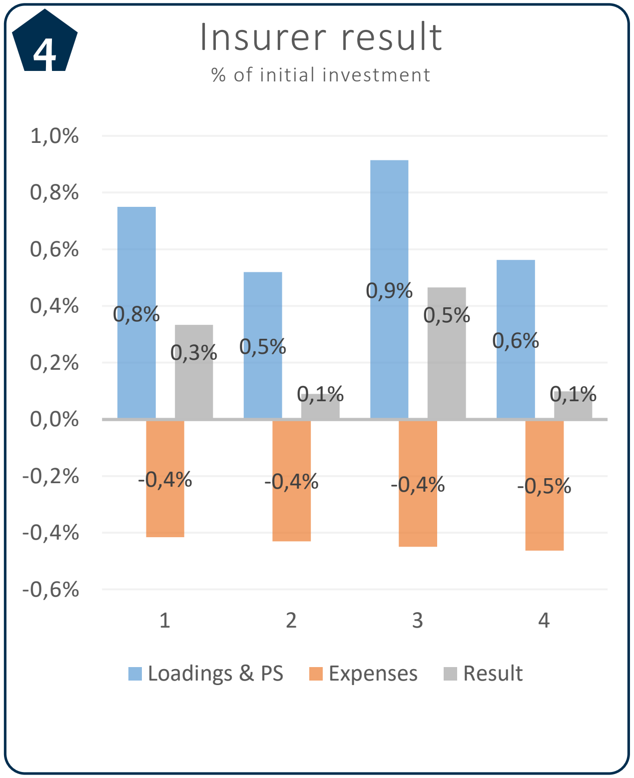
Simplified illustration of cash-flows on a Frensh savings contract without future premiums



Initial investment	***100***
Profit sharing rate	***90%***
Loadings rate	***0,5%***
Expenses rate	***0,4%***







General overview of policyholders options and insurer risks

Policyholder options

- The guaranteed rate or guaranteed PS options can be assimilated to a European vanilla option.
- The surrender option can be assimilated to an American put option.
- The option of guaranteed rate on scheduled payments can be assimilated to a swaption.
- Biometric options: options depending on the risk of mortality (or longevity) as the guarantee of a life table.

Insurer risks: general overview

1 Biometric risks

- Mortality
- Longevity

2 Behaviours of policyholders

- Dynamic lapse: surrender that depends on revaluation rates.
- Structural lapse: all non-dynamic lapses

3 Insurer's policy

- Investment policy.
- Anticipations of the behavior of policyholders.
- Expenses, etc.

4 Financial and economic risks

- Asset return.
- Interest rates.
- Inflation, employment, etc.

Summary

1. General overview of French savings contracts in €
2. ESG models and Liability valuation principles under Solvency 2
3. Sensitivity of the best-estimate to ESG IR models and data
4. Given a risk environment, is the value of the best estimate unique?
5. Conclusions and discussion

Liability valuation principles under Solvency 2 framework (1/2)

- Under Solvency II framework, the economic balance sheet is basically established using at the valuation date:
  - the market value of assets,
  - the market value of liabilities, i.e. the value of technical provisions as the sum of a best-estimate and a risk margin.
- Replicable and non-replicable liabilities are distinguished :
  - Replicable risks are related to the financial markets,
  - Non-replicable risks are related to:
    - Underwriting risks: biometric risks, customer behaviour ...
    - Insurer management actions especially the management of the book yield of the asset.
- The best estimate of the liabilities is calculated as the sum of probability weighted average of future cash-flows, taking account of the time value of money in a runoff environment (see art. 77 directive Solvency II) :

$$BE(0) = E\left(\sum_{i=0}^{+\infty} CF_i . \exp(-i . r_i)\right)$$

Liability valuation principles under Solvency 2 framework (2/2)

Initial investment

Underwriting and expenses

- f_t : is a factor that takes into account
  - The exit probability during the year t due to deaths or lapses.
  - The probability to be under-contract at time t.
  - Expenses or loadings rate.
- f_t is a stochastic factor since it depends on the distributed return

$$BE(0) = PM(0).E\left(\sum_{t=1}^{T} f_t.\psi(t)\right) \quad \Bigg| \quad \psi(t) = \exp\left\{\sum_{i=1}^{t} c_i - i.r_i\right\}$$

$$\psi(t) = exp(Total\_Distributed\_Return_t) * DiscountFactor_t$$

Insurer's policy

- Investment policy and economic environment
- Accounting performance steering:
  - Rate higher than the guaranteed rate
  - Management of a latent wealth
  - Reaction to concurrence

Economic scenarios

- Asset performance.
- Discount factors and therefore risk free interest rates.

ESGs for best-estimate valuation (1/2)

- An economic scenario generator (ESG) is a computer-based model of an economic environment that is used to produce simulations of the joint behavior of financial market values and economic variables.

- Two common applications are driving the increased utilization of ESGs:
  - Market-consistent (MC) risk-neutral valuation for pricing financial derivatives and insurance contracts with embedded options.
  - Risk management : calculating business risk, regulatory capital and rating agency requirements.

- Some constraints appear from the regulatory framework related to the ESG:
  - Risk-free interest rate term structure is given by EIOPA on a monthly basis.
  - The ESG should be market consistent, i.e. it should satisfy some tests guarantying that the results provided by the ESG is consistent with financial market data (see art. 76 directive).
  - The calibration process uses data from financial markets that are deep, liquid and transparent.
  - The insurer should justify that data selected is relevant given the characteristics of the insurance obligations.
  - The random numbers generator is valid and not manipulated.

ESGs for best-estimate valuation (2/2)

- Economic scenarios must be consistent with the financial market (Market Consistent).
- The objective is to produce a fair value of liabilities that is consistent with observable and measurable market prices and risks.
- A Mark to Market approach is not adapted to evaluate the best-estimate in a fair value frame work : insurance policies options and guarantees are not traded in a liquid and deep market.
- The best-estimate is calculated in a Mark-to-Model framework.

The calibration and validation of the economic scenario generator (ESG), used to evaluate the best-estimate, by comparing the simulations to the observed data as part of a statistical approach, cannot be considered.

A conventional process to calibrate ESGs for BE valuation

- In the absence of market observations of the best-estimate, the ESG is calibrated on financial products (caps, floors, swaps ...).
- The quality of the ESG is appreciated by its ability to reproduce the prices of these financial products.
- The assumption underlying this practice is the equivalence between these financial options and the BE.
- This hypothesis can not be verified, however, because the best-estimate is not observable.
- The process of generating economic scenarios can be summarized in three main choices :

1- Modeling environment

- Economic variables
- Risk neutral measure
- Real world measure

2- Mathematical models

- Stochastic univariate models
- Dependence structure (copula)

3- Parameters

- Financial instruments for calibration
- Data (time, periodicity, depth...)
- Optimisation measure/method
- Validation methods

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Market consistant ESGs models studied

1 Modeling environment

Measure	Risk neutral				
Economic variables	Risk free interest rate			Equity	Real-estate

2 Mathematical models

Stochastic models	Hull & White	G2++	LMM	Black-Scholes	Black-Scholes
Dependence structure	Correlation matrix extracted from historical data				

3 Parameters

Instruments	Caps & Swaptions on Euribor		Swaptions on Euribor	Calls on CAC40	FR Historical data
Data periodicity & depth	Yearly data over 10 years. Data observed 02 Jan 2018			3 years	12 years
Optimization method	Euclidean distance (2-norm distance)				Standard deviation

Non market consistent ESGs models studied

1 Modeling environment

Measure	Risk neutral			
Economic variables	Risk free interest rate		Equity	Real-estate

2 Mathematical models

Stochastic models	Hull & White	LMM	Black-Scholes	Black-Scholes
Dependence structure	Correlation matrix extracted from historical data			

3 Parameters

Parameters based on market-consistant calibration	Increase of the volatility parameter	• Increase of the volatility convergence. • Increase of long term limit.	Not changed	Not changed

Calibration results : focus on interest rates models (1/3)

Hull & White

- Model : $dr(t) = [\vartheta(t) - a.r(t)]dt + \sigma.dW(t)$

Hull & White calibration		a	σ	Total relative square error
Market Consistant	Caps	1.00%	0.80%	2.98%
	Swaptions	0.10%	1.62%	8.48%
Non Market Consistant	Increase of σ	0.10%	5.00%	NA

G2++

- Model :
  - $r(t) = x(t) + y(t) + \phi(t)$
  - $dx(t) = -ax(t)dt + \sigma dW_1(t)\ et\ x(0) = 0$
  - $dy(t) = -by(t)dt + \eta dW_2(t)\ et\ y(0) = 0$
  - $dW_1(t)dW_2(t) = \rho dt$ and $-1 \leq \rho \leq 1$

G2++ calibration		a	σ	b	η	ρ	Total relative square error
Market Consistant	Caps	5.14%	0.03%	7.50%	7.07%	0.99	0.005%
	Swaptions	11.49%	8.90%	26.50%	25.67%	0.99	3.47%

LMM

- Model :
  - $dF_k(t) = \sigma_k(t)F_k(t)dZ_k(t)\,;\, t \leq T_{k-1}$
  - $dZ_i(t)dZ_j(t) = \rho_{i,j}dt.$
  - $\sigma_i(t) = (a(T_{i-1} - t) + b)e^{-c(T_{i-1}-t)} + d$
  - $\rho_{ij}(t) = \exp(-\beta|T_i - T_j|)$

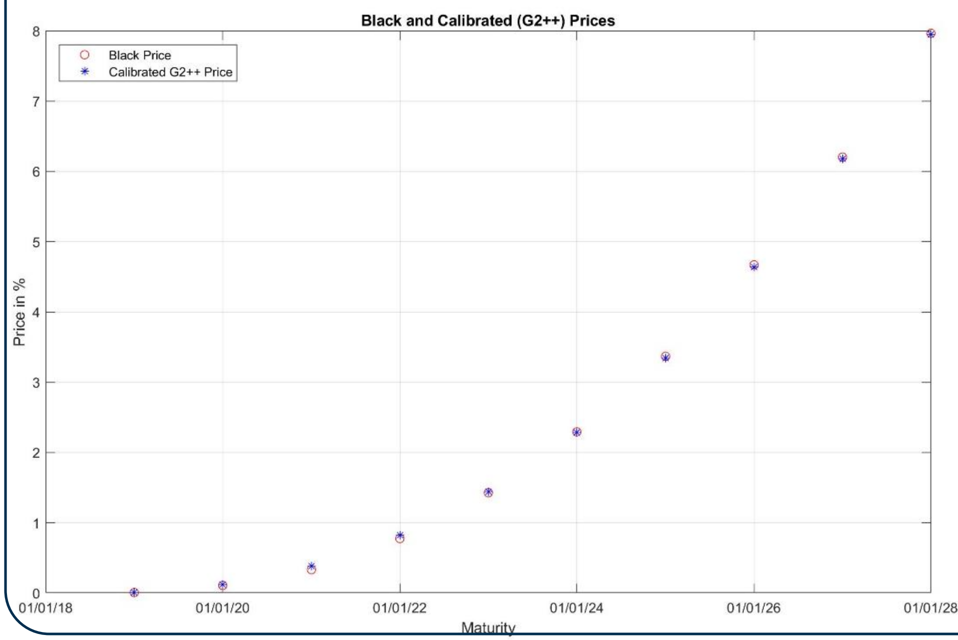
G2++ calibration		a	b	c	d	β	Total relative square error
Market Consistant	Swaptions	18.85%	0.19%	7.45%	0.01%	0.1%	0,73%
Non Market Consistant	Increase of c & d	18.85%	0.19%	20%	1%	0.1%	NA

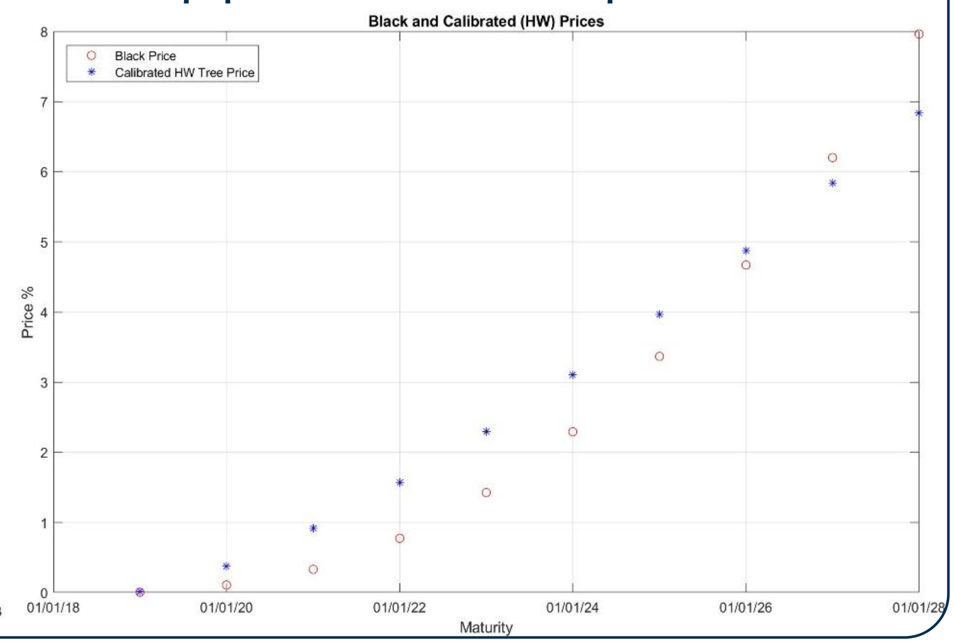
Calibration results : ability to reproduce prices

Cap prices*

Swaption prices*

Illustration of simulated H&W and G2++ cap prices *Vs* Market prices*

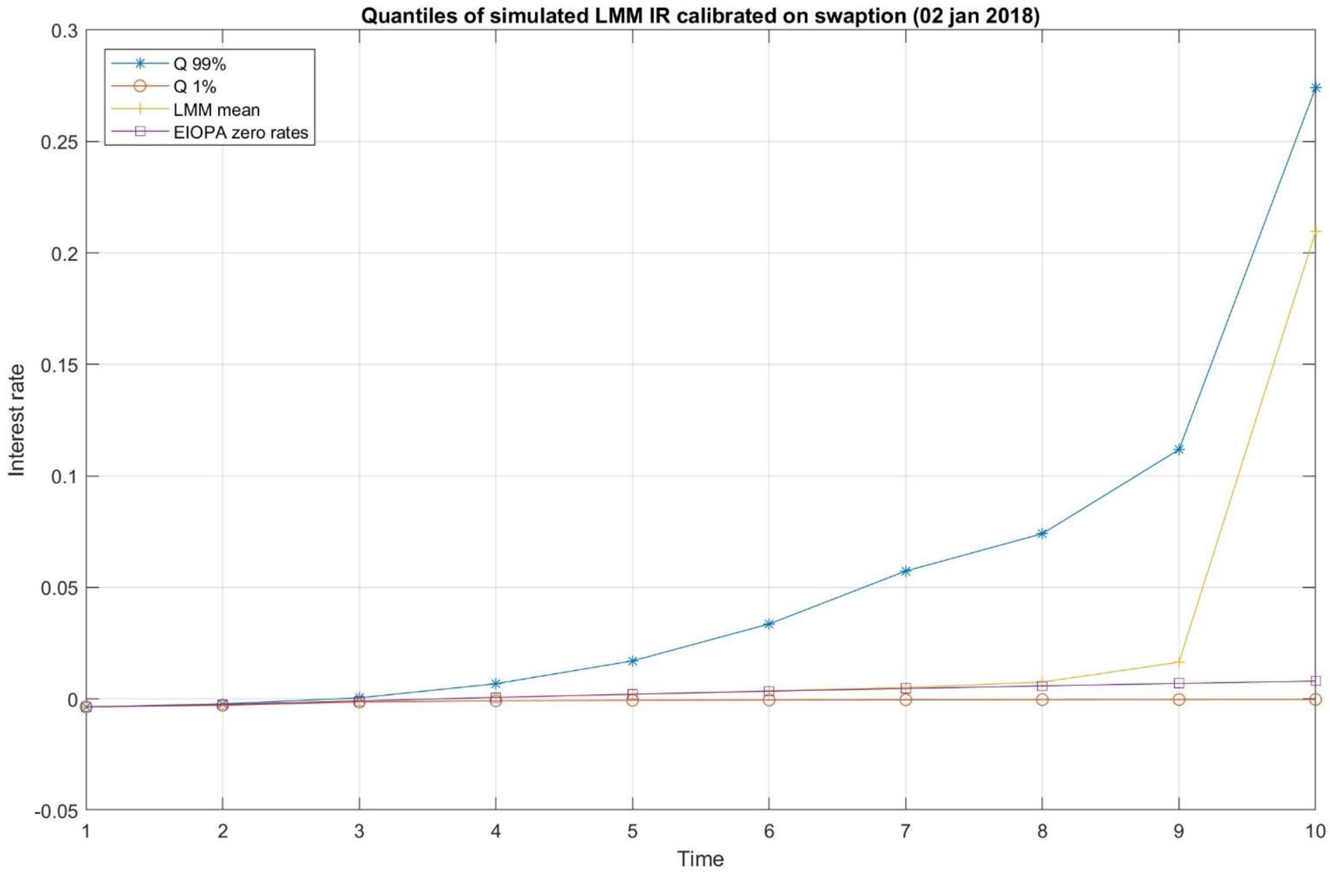




(*) based on market data as of 02/01/2018

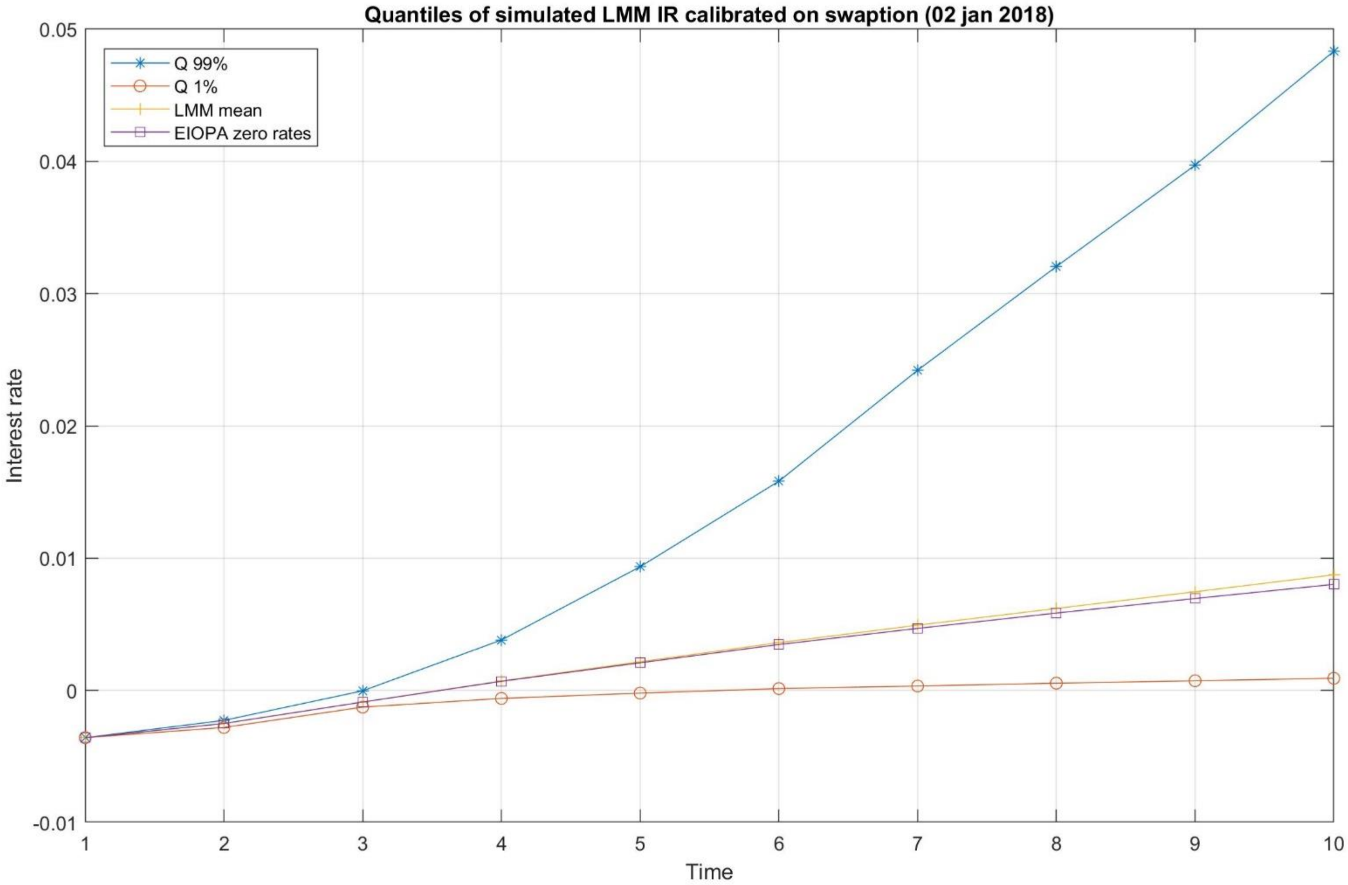
Calibration results : focus on LMM model (1/2)

- Focus on LMM model calibrated on swaptions :



Calibration results : focus on LMM model (2/2)

- Focus on adjusted LMM model (witch is not market consistent) :

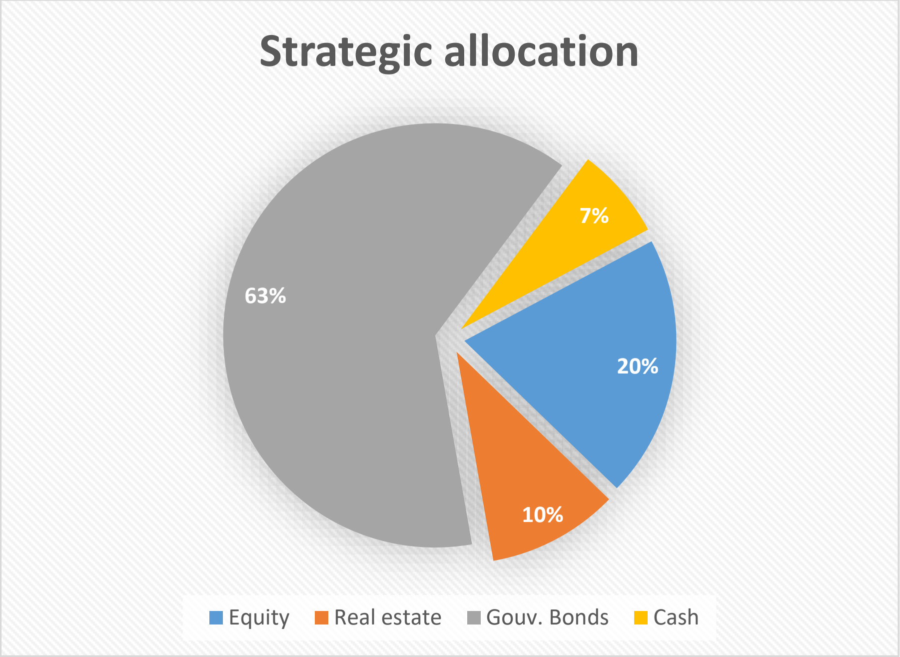


Best-estimate calculation : modeling environment

- The best-estimate is calculated on SimBEL.
- SimBEL is an ALM software allowing a best estimate calculation based on Monte-Carlo simulations developed by Prim'ACT.
- In addition to the economic scenario tables, SimBEL has been set as follows :

Asset (M€)	Market Value	Net book value
Equity	20	19
Real estate	10	10
Gouv. Bonds	63	70
Cash	7	7
Total	100	106

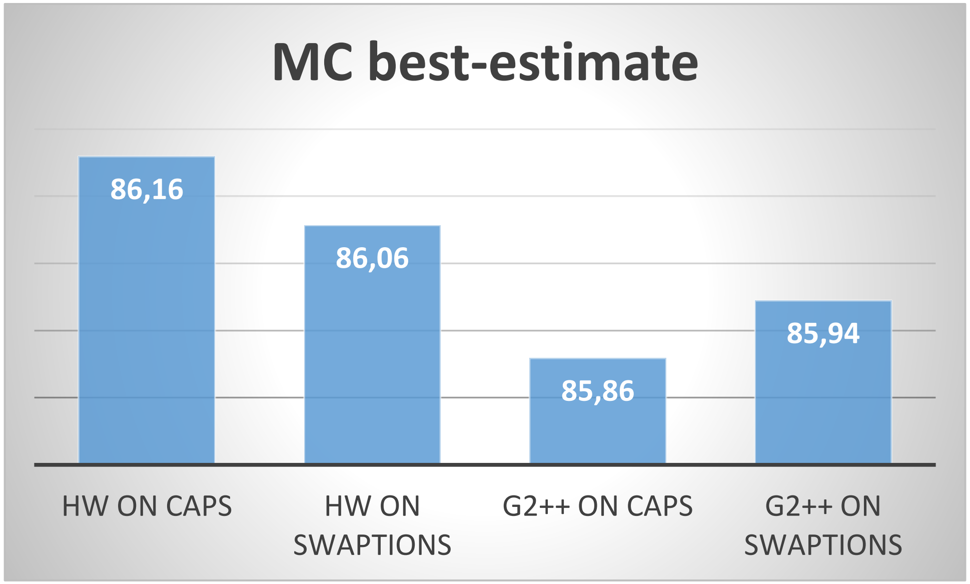
PM (M€)	60
Projection years	25
Target revaluation rate	2.50%
Guaranteed first year rate	0.50%



Best-estimate calculation : results and conclusions

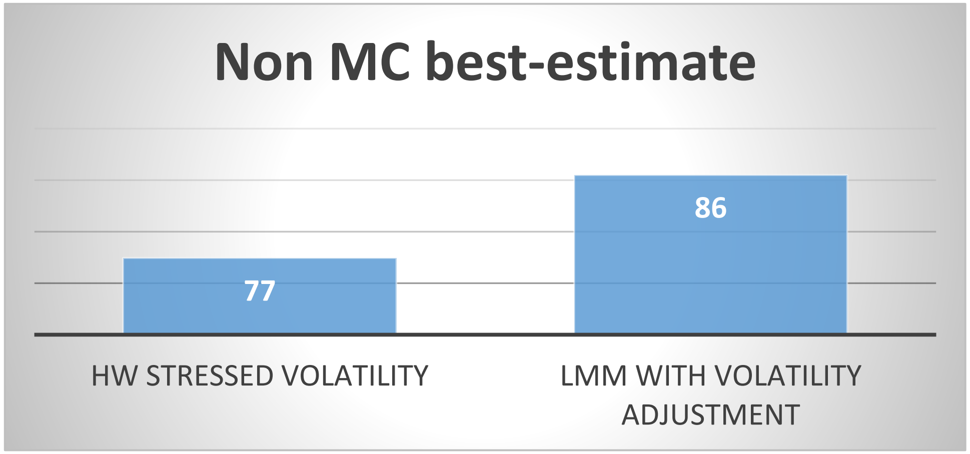
Market Consistant BE

The market consistent H&W and G2++ best-estimates are of the same order of magnitude !



Non Market Consistant BE

Stressing volatility can have an important impact on the best-estimate.



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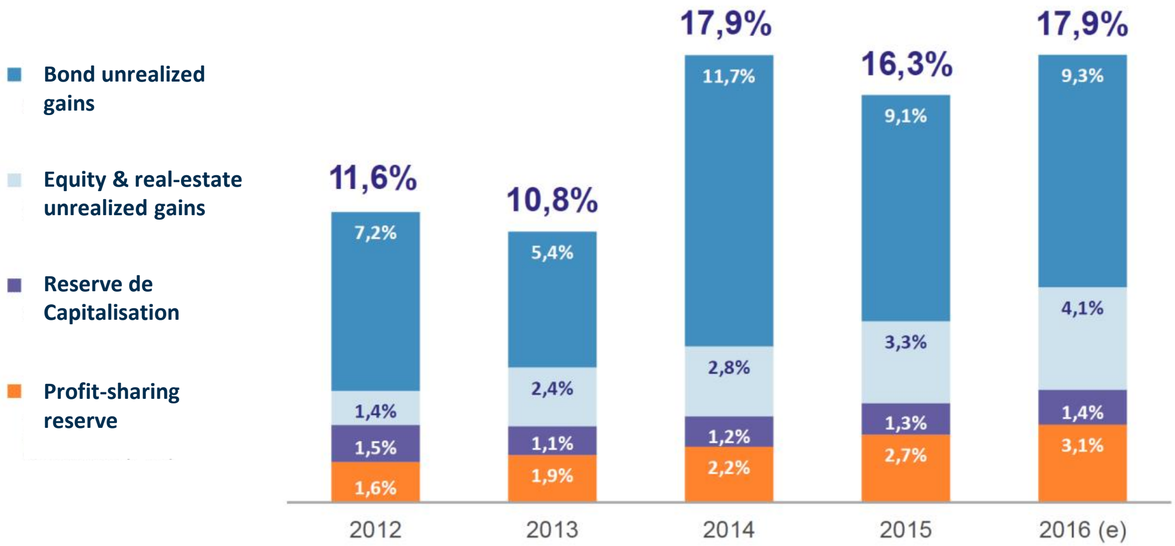
Qualitative analysis (1/2)

- Policyholders, whose savings acquired on valuation date 0 is PM (0), can benefit from an initial wealth cumulated by the insurer. The latter is as follows:
  - **Profit-sharing reserve** which is **totally acquired** to the policyholders. Its distribution is at the discretion of the insurer. The initial value of this provision is denoted PPB (0);
  - **The unrealized gains/losses** or losses generated by the accounting management of revaluation rates already distributed. The initial value is denoted PMVL (0);
  - **Reserves for steering book returns** such as the "Reserve de Capitalisation", the "PAF" and the "PRE". These provisions are at the discretion of the insurer. Their initial value is denoted PR (0).

French market (source ACPR/FFA 2016)

- Acquired savings in 2016: € 1,287 billion.
- Life insurance own funds before appropriation of results: € 68 billion.



Initial wealth cumulated on the French market (source ACPR/FFA 2016)
% of total acquired savings

Qualitative analysis (1/2)

Unrealized gains/losses

- In a **runoff** contract management universe, assets will be sold gradually to serve accounting revaluation rates or to honor outflows.
- The unrealized gain/loss will be gradually distributed in part to policyholders and integrated into future cash flows at the discretion of the insurer.
- The initial and maximum unrealized gains/losses value that can be distributed to the policy holders is: $\boldsymbol{PMVL_{max} = PMVL(0)}$.
- The minimum value depends mainly on the management policy of the asset book return and the profit sharing clauses. In all cases, there exists $PMVL_{min} \leq PMVL(0)$ such that: $\boldsymbol{PMVL_{min} \leq PMVL_{distributed} \leq PMVL_{max}}$.

Steering reserves

- Reserves for the management of revaluation rate will be capitalized or reversed as the asset is sold.
- If Steering reserves are non-zero at the end of the simulation period, the insurer can keep them (example: "reserve de capitalization").
- The current maximum amount that can be distributed to policyholders is $\boldsymbol{PR_{max} = PR(0)}$.
- The current minimum amount that can be distributed depends on the policy of the insurer and may be nil.

The minimum contractual best-estimate

- Minimum contractual best-estimate: the sum of discounted future guaranteed cash flows. It corresponds to the scenario where the insurer pays each year *i* the minimum guaranteed rate tmg_i.
- In this case one can prove that:

$$BE_{contr}(0) = PM(0).\sum_{t=1}^{T} f_t^{(tmg_i)}\, exp\left(\sum_{i=1}^{t} tmg_i\right) \boldsymbol{P^M(0,t)}$$

- The contractual best-estimate depends on observed market bonds prices and does not need an ESG model.
- The contractual best-estimate represents the minimal commitment of the insurer. It indicates in particular the level of the guarantee when the promised rates are significant.

The best-estimate is bounded

- Let's note:

$$Wealth_{min}^{init}(0) = PPB(0) + PMVL_{min}(0) + PR_{min}(0)$$

$$Wealth_{max}^{init}(0) = PPB(0) + PMVL_{max}(0) + PR_{max}(0)$$

- The minimum value of the best-estimate is written:

$$\boldsymbol{BE_{min}(0)} = max\left(PM(0), BE_{contr}(0), PM(0) + Wealth_{min}^{init}(0)\right)$$

- Assuming that the insurer maximizes its utility and is careful to preserve its own funds, the maximum value of the best-estimate is written as:

$$\boldsymbol{BE_{max}(0)} = max\left(PM(0), BE_{contr}(0), PM(0) + Wealth_{max}^{init}(0)\right)$$

- If $\boldsymbol{BE_{max}(0) = BE_{min}(0) = BE_{contr}(0)}$ this means that the insurer will mobilize a part of its capital to honor its commitments conditionally to the state of the world on the date 0.

The best-estimate value depends on the policy of the insurer

- **The best estimate can reach its bounds as well as any other value between them depending on the policy of the insurer.**
- Let's consider two insurers A and B in good financial health with identical characteristics:
  - The same assets and liabilities.
  - In the same economic environment.
- Having the same portfolio structure, the best-estimates of both insurers have the same maximum and minimum limits.
- Let $\boldsymbol{BE^A(0)} < \boldsymbol{BE_{max}(0)}$ be the best-estimate of company A.

Whatever the state of the economy, the insurer B can choose a target best-estimate such as:

$$\boldsymbol{BE^A(0) < BE_{target}(0) < BE_{max}(0)}$$

- In fact : B can valuate the invested capital by adding a spread "$\boldsymbol{a}$" solution of the next equation :

$$\boldsymbol{BE_{target}(0)} = \mathbf{PM(0)}.\boldsymbol{E}\left(\sum_{\mathbf{t=1}}^{\mathbf{T}} \mathbf{f_t}.\,\boldsymbol{\psi}_{\boldsymbol{a}}\mathbf{(t)}\right)$$

- $\psi_a(t) = \exp\{\sum_{i=1}^{t} c_i - \sum_{i=1}^{t} r_i + t.\boldsymbol{a}\}$
- c_i : the rate of revaluation of the savings on the date i paid by insurer A. This rate must be higher than the guaranteed minimum rate.

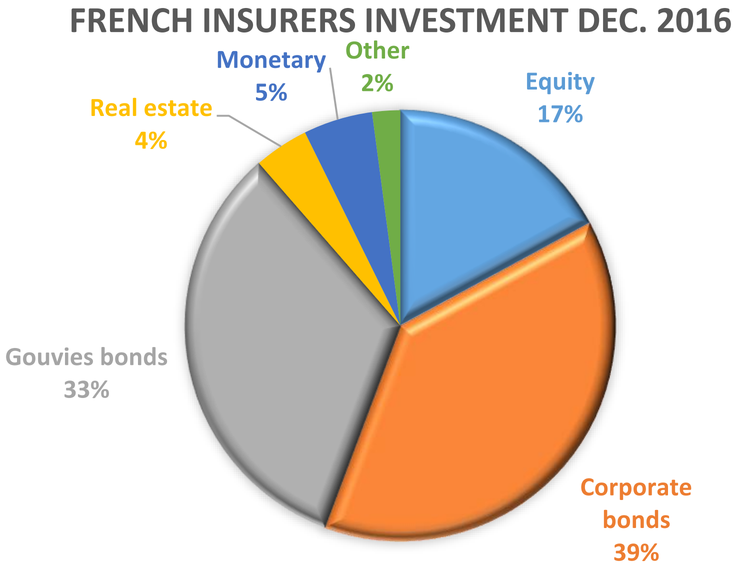
Summary

Conclusions (1/3)

- The policy of the insurer can have a significant impact on the best-estimate and this regardless of the universe of biometric, behavioral or economic risks.
- The best estimate of a savings portfolio in € is not unique. The instantaneous transfer of the portfolio from one company to another instantly changes the amount of the best-estimate if the management policies are different, even if the economic environment remains unchanged.
- There is no unique “economic valuation” of an insurance liability as it is defined in Solvency 2 frame work but an infinity of possible values.
- Each best-estimate represents the economic value of the liabilities conditionally to the policy of the insurance company.
- The calibration and validation of the economic scenario generator (ESG), used to evaluate the best-estimate, by comparing the simulations to the observed data as part of a statistical approach, cannot be considered for two reasons:
  - The best-estimate is not observed in a deep and liquid market.
  - We can derive different best-estimate values for the same underlying risks: a deep and liquid market for the best-estimate as it is defined in Solvency 2 frame work cannot exist.

Conclusions (2/3)

- The best estimate is bounded and there is no need for an ESG model to evaluate its bounds.
- whatever the choice of the ESG model, one can drive the revaluation rates to reach a predefined target best-estimate in the min-max interval.
- It is questionable to link an ESG to a best-estimate without specifying precisely the insurer management policy of the revaluation rates.
- It is questionable to use some specific zero-rates derivatives to calibrate the ESG models without establishing a link between those derivatives and insurer policy & assets.
- As we can see, an ESG that models bond price, equity and real-estate covers 98% of the insurers assets.



Conclusions (3/3)

- In the specific case when :
  - The insurer revaluate the capital with the financial return rate of the assets.
  - Lapses are stable.
- We can wright the best-estimate as a sum of cliquet options on the insurer asset :

$$BE(0) = PM(0).\left(\sum_{t=1}^{T} B_t . Y_t\right)$$

- B_t is the notional of the option and depends on underwriting parameters
- Y_t is the price of a cliquet option with a strike F and maturity t:

$$Y_t = E^Q\left(\exp\left(\sum_{i=1}^{t} \max(s_i, F) - \sum_{i=1}^{t} r_i\right)\right)$$

- The best-estimate has a cliquet-optional structure and can be likened to a series of vanilla options.
- A cliquet option is a volatility asset.
- Choosing an IR model in a normal or log-normal frame work to evaluate the best-estimate is equivalent to chose a volatility model since the mean is given by the regulator.

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