The Collective Risk Model Simulation Algorithm

The goal of the collective risk model is to describe, prospectively, the distribution of losses covered by an insurance policy. A good way to view the collective risk model is by a Monte-Carlo simulation.

For each line of insurance you need a claim count distribution, and a claim severity distribution.

Simulation Algorithm #1

The Collective Risk Model

For each line of insurance, i:

1. Select a random number of claims \( K_i \) from line i's claim count distribution.

2. Select \( K_i \) claim amounts at random from i's claim severity distribution.

3. Let X be the sum of all the claim amounts selected in Step 2 for all lines of insurance.

*CrimCalc* describes the distribution of X.

A more detailed description of the claim severity, and claim count distributions is given elsewhere.

The Calculation Method

For sizeable insureds, simulation can be very time consuming, so *CrimCalc* uses Fourier inversion, which is often a much faster way to calculate an aggregate loss distribution. Here is a quick description of Fourier inversion.

1. Calculate the Fourier Transform (FT) of each claim severity distribution.
2. Each line FT is a function of its count distribution and the FT of its severity distribution.
3. Multiply the line FT's to get the overall FT.
4. Numerically invert the overall FT to get the overall loss distribution.

*CrimCalc* provides two different methods of Fourier inversion.

1. The Heckman/Meyers algorithm. This method works for piecewise linear claim severity distributions. It does a direct inversion using numerical integration.
2. The Fast Fourier Transform (FFT). This method works on discrete claim severity distributions where the claim amounts are equally spaced.

The Heckman/Meyers algorithm has been used in *CrimCalc* from its inception back in 1986. The FFT was introduced in Version 2.2.

The FFT provides a nice complement to the Heckman/Meyers algorithm. The Heckman/Meyers algorithm is fastest when being used for pricing large accounts, or for DFA exercises with the aggregate loss distribution of an entire insurance company. The FFT is much faster than Heckman/Meyers in situations where there is a small probability of very large claims. This is typically the case when you want the aggregate loss distribution for a reinsurance treaty, or for catastrophes.

My experience to date is that at least one of the two methods will get nearly instantaneous results.

Correlation and Parameter Uncertainty

Recent work done by Shaun Wang for the CAS Committee on the Theory of Risk has brought attention to the problem of correlation between lines of insurance in the collective risk model. His report, titled "Aggregation of Correlated Risk Portfolios: Models and Algorithms" is available at the CAS Web Site.

http://www.crimcalc.com/computer.htm 22/07/2002
Drawing on Dr. Wang’s work, CrimCalc provides a way to incorporate correlation into the collective risk model by introducing parameter uncertainty. I have written a discussion of his paper that you can download. It appears in the Proceedings of the Casualty Actuarial Society.

Claim Count Distributions

Consider the case where we have the lines of insurance grouped into J covariance groups. Let the mean of the primary claim count distribution of the ith line in the jth covariance group be denoted by \( \lambda_{ij} \). All other parameters of the claim count distribution can be similarly subscripted. Our next version of the collective risk model is defined as follows.

Simulation Algorithm #2

The Collective Risk Model with Parameter Uncertainty in the Claim Count Distributions

1. For each covariance group j, select \( \alpha_j \) from a distribution with:

\[
E[\alpha_j] = 1 \text{ and } \text{Var}[\alpha_j] = g_j
\]

\( g_j \) is called the covariance generator for the jth covariance group.

2. For each line j in the covariance group:

2.1 Select a random number of claims, \( K_{ij} \), from a claim count distribution with mean \( \alpha_j \times \lambda_{ij} \)

2.2 Select \( K_{ij} \) claim amounts from the ij-th severity distribution

3. \( X \) is the sum of all claim amount selected in Step 2.2.

Since the claim counts for all lines in a covariance group will be associated by the common multiplier \( \alpha_j \), the claim counts will be correlated. CrimCalc gives the coefficients of correlation between the claim count distributions of the various lines.

Parameter Uncertainty in the Claim Severity

A parameter quantifying the uncertainty of the claim severity distributions is called the mixing parameter. An example of its use would be to account for uncertain inflation. We denote the mixing parameter by \( b \).

The mixing parameter can be interpreted by replacing Step 3 in the above simulation algorithm with Steps 3 and 4 in the following:

Simulation Algorithm #3

The Collective Risk Model with Parameter Uncertainty in the Claim Count and Claim Severity Distributions

1. For each covariance group j, select \( \alpha_j \) from a distribution with:

\[
E[\alpha_j] = 1 \text{ and } \text{Var}[\alpha_j] = g_j
\]

2. For each line j in the covariance group:

2.1 Select a random number of claims, \( K_{ij} \), from a claim count distribution with mean \( \alpha_j \times \lambda_{ij} \)
2.2 Select $K_{ij}$ claim amounts from the $ij$-th severity distribution


4. Let $X$ be the sum of all the claims selected in Step 2.2 over all lines of insurance, multiplied by $T$.

Note that this model of parameter uncertainty affects all severity distributions simultaneously. This generates correlation between all lines of insurance. *CrimCalc* gives the resulting correlations of the total losses between each line.

Parameter uncertainty in the claim severity distribution can only be used with the Heckman/Meyers algorithm.

Last updated on February 03, 2002