The economic evaluation of life insurance liabilities: pitfalls, best practices and recommendations for relevant implementation

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The "economic" valuation of liabilities under Solvency 2 corresponds to the *best-estimate* (discounted future cash flows) plus an adjustment to compensate for the immobilization of the solvency capital required for non-hedgeable risks. The valuation framework under IFRS 17 is similar, in that it proposes the valuation of liabilities as the sum of a *best-estimate (discounted* future cash flows) and an adjustment for non-financial risks.

The economic valuation approach for liabilities imposed by Solvency 2 and IFRS 17 can therefore only be fully applied when one has the ability to define: (1) the future cash flows of the liability, (2) an appropriate probability measure and (3) the discount rates.

The application of this approach to savings contracts in € (and more generally in the presence of a profit-sharing scheme) leads practitioners to the now classic modelling structure in which a "risk neutral" economic scenario generator feeds a flow projection model to allow an approximation by simulation of the value of the *best estimate*.

In this article, we present a critical analysis of the economic valuation process of life insurance liabilities. We discuss the modelling and probability measurement choices made by the market and their implications. We also provide recommendations for relevant modelling.

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Summary

1 The "economic" valuation of insurance liabilities

Over the past thirty years or so, the insurance industry has developed the notion of "economic" valuation of financial flows, meaning that the liability value associated with a replicable risk must be equal to the price of the hedge of this risk.

This notion was initially created since the end of the 1970s in finance with the development of derivatives markets, with the seminal work of BLACK and SCHOLES [1973], then Merton [1976], followed by HARRISSON and KREPS [1979] and HARRISSON and PLISKA [1981]. They link the price of a contingent asset to a risk management technique and this asset's flow replication to a portfolio of self-financed assets.

Contingent asset markets have developed considerably due to the active management of positions to limit risk[4], hedging imperfections themselves giving rise to a specific literature to optimize the rules for managing hedging assets (cf. NTEUKAM and al. [2011]).

In the early 1990s, BRIYS and de VARENNE [1994] noted the formal analogy of certain contract flows integrating profit sharing with vanilla option flows and, by analogy, proposed to use the no arbitrage price calculation framework for insurance contracts.

Implying that the insurer's assets should be valued at market value, this new vision of the balance sheet seduced the industry at the very beginning of the 2000s, as unrealized capital gains were significant and not recognized in the company statuary accounts.

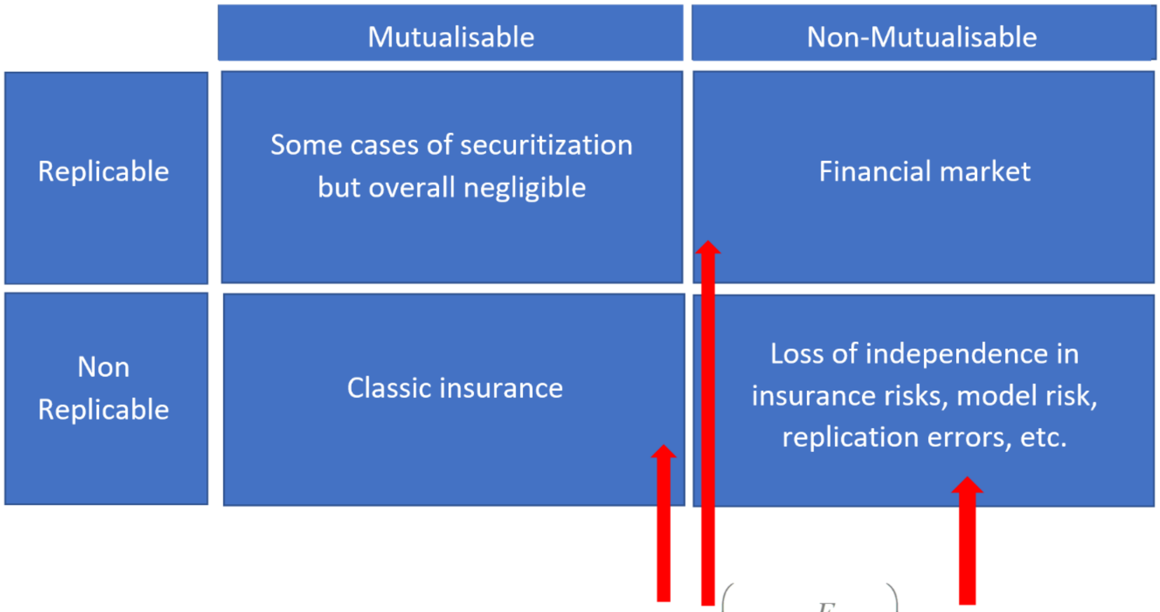
It was therefore adopted as the basic principle for balance sheet valuation in the Solvency 2 directive.

In this context, the "economic" valuation corresponds to the *best-estimate (*discounted future cash flows) and is, if necessary, supplemented by a risk margin to compensate for the immobilization of the solvency capital required for non-hedgeable risks (resulting in a capital cost for non-financial risks or for an imperfect hedge for financial risks).

The "economic" frameworks, Solvency 2 and IFRS 17, thus provide a framework for calculating insurance liabilities that can be summarized by the diagram shown in Fig. 1.

[4] And even, in the idealized framework of a complete market without frictions or costs to eliminate it.

Fig. 1: **Links between risk and valuation technique**



$$PT = BE + RM = E^{P^A \otimes Q^F}\left(\sum_{j\geq 0}\frac{F_j}{(1+R_j)^j}\right) + RM$$

In summary, mutualisable insurance risks are evaluated in a statistical framework, replicable financial risks are evaluated in a framework where there is no arbitrage opportunity, and any risks that do not fall into either of these two categories are included in the risk margin.

In addition, the valuation framework under IFRS 17 is similar to that of Solvency 2 in that it presents the valuation of liabilities as the sum of a *best-estimate* (discounted future cash flows) and an adjustment for non-financial risks.

Although the two standards, Solvency 2 and IFRS 17, present significant discrepancies, valuation principles under Solvency 2 discussed in this paper are naturally generalized to IFRS 17[5].

Issues related to risk margin calculations will not be addressed in this paper, i.e. we focus on the part of the flows relating to flow replication management, without considering the insurance risks and the hedging imperfections. In what follows, we will therefore not distinguish between the terms "best-estimate" and "value of liabilities".

Article 75 of Solvency 2 states that liabilities are "*valued* ***at the*** *amount for which they could be* ***transferred*** *or* ***settled*** *between* ***knowledgeable willing*** *parties in* ***an arm's length transaction***". Section 77 defines *best-estimate* as the "***probability*** *weighted average of* ***future cash flows,*** *taking account of the time value of money (expected present value of future cash flows), using the relevant risk-free* ***interest rate term structure***".

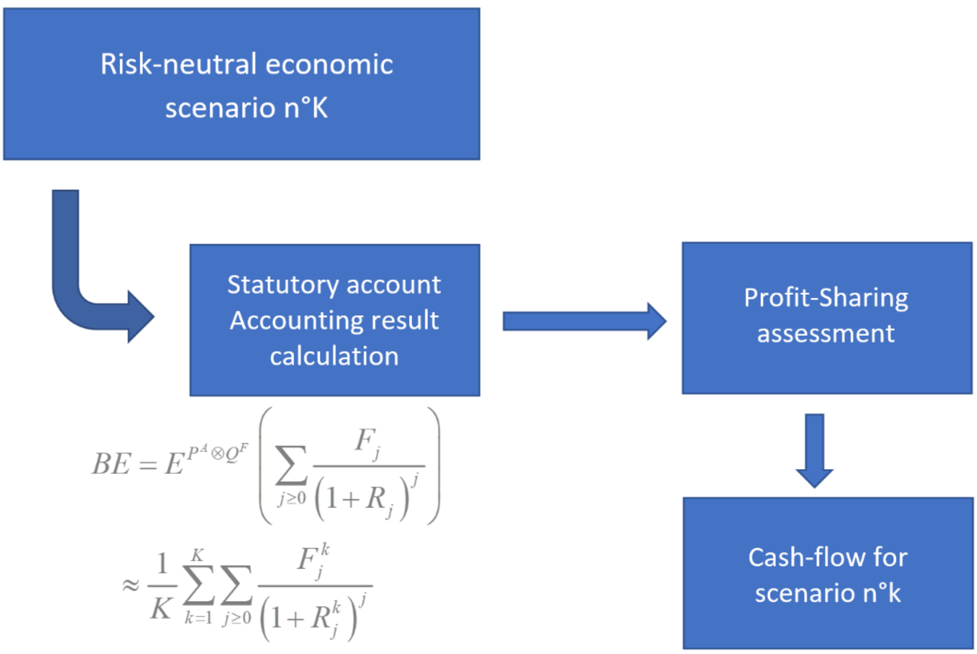
[5] It is worth recalling that under IFRS 17 the contractual service margin, estimating expected and unrealized profits, is not part of the IFRS 17 equity capital as is the value of future profits under Solvency 2. Under both standards, the insurer's commitments to policyholders consist of a *best-estimate* and an adjustment.

This approach, described as dynamic in WALTER and BRIAN [2008], is close to the Keynesian definition of fundamental value. Indeed, in his General Theory, Keynes states that "*the fundamental value of a firm corresponds to an estimate of the present value of the future income flows attached to the holding of its capital*". [6]

The economic liability valuation approach imposed by Solvency 2 can thus only be fully applied when there is the capacity to define: (1) liability's future flows, (2) an appropriate probability measure and (3) the discount rates.

The application of this approach to savings contracts in € (and more generally in the presence of a profit-sharing scheme) leads practitioners to the now classical modeling structure in which a "risk neutral" economic scenario generator (e.g. LEROY and PLANCHET [2013]) supplies a flow projection model in order to allow an approximation by simulating the best estimate value (see. PLANCHET [2015]) as shown in Fig. 2.

Fig. 2: Economic value calculation of a contract with asset/liability interactions



$$BE = E^{P^A \otimes Q^F}\left(\sum_{j \geq 0} \frac{F_j}{(1+R_j)^j}\right) \approx \frac{1}{K}\sum_{k=1}^{K}\sum_{j \geq 0}\frac{F_j^k}{(1+R_j^k)^j}$$

In what follows, we further examine each of the process' components in order to discuss implications in terms of modelling aiming to achieve the most relevant possible assessments of a life insurer's liability flows' "economic value".

This work is organized as follows:

- Section 2 reviews savings contracts' options and guarantees. It shows how future flows are defined for best-estimate calculation and presents the market practices;
- Section 3 discusses the relevance of the choice of the risk neutral probability measure for the savings liabilities valuation in euro;

[6] WALTER and BRIAN (2008) present a critical analysis of the fundamental value notion. This analysis is conducted as close as possible to the tension between the abstract calculation of financial phenomena and the concrete forms these phenomena take.

- Section 4 discusses the practices of generating risk-neutral economic scenarios, notably interest rates, for valuing liabilities in a Mark-to-Model framework. In this section, we also present best-estimate sensitivities regarding the choice of interest rate models and calibration data under risk-neutral measure. Finally, we present a best-estimate calculation under historical probability and its sensitivity to the choice of interest rate model calibration data;
- Section 5 concludes and summarizes our recommendations for relevant modeling.

2 Savings contracts, future flows, *best-estimate* and risk factors

2.1 The context of savings contracts

There are two main types of savings contracts on the French life insurance market: contracts in € and unit-linked contracts (including *Euro-croissance*).

At the end of 2018, savings contracts reserves represented €1,692 billion[7], or 70% of the French insurance market's investments (FFA [2019]). Savings contracts in euros represent €1,297 billion[8] or 54% of investments and unit-linked contracts represent €341 billion or 14% of investments.

In this article we examine the economic valuation of classic € savings contracts (see section 7for a detailed presentation of these contracts).

The options included in these contracts can be summarized in three categories:

- Financial options: the insurer commits to a minimum return on savings by guaranteeing a minimum rate of revaluation or a guaranteed profit sharing rate.
- Behavioural options: the insurer offers redemption options, euro - Unit-Linked arbitrage, free or scheduled payments, loyalty bonus, etc. The activation of these options is at the discretion of the policy-holder.
- Biometric options: they are options depending on the risk of mortality (or longevity) such as the insurer's proposal of deferred annuities.

The policy-holder therefore benefits from three financial options (see BRYS and DE VARENNE [1994]):

- The technical rate or guaranteed profit-sharing rate option, similar to a European vanilla option;
- The ratchet option, similar to an American put option;
- The forward rate option on free or scheduled contributions, similar to a *swaption*.

Thus, the risk factors to which the liabilities of classic € savings contracts are exposed are (see ARMEL and PLANCHET [2019] and LAURENT and al. [2016]):

[7] Including €54bn of provisions for profit sharing.
[8] Excluding provisions for profit sharing.

- Biometric and structural redemption risks that are non-replicable. They are nevertheless mutualizable;
- Risks related to policyholders’ cyclical (or dynamic) behaviour that are non-replicable (see sections 2.4 and 3). These behaviours represent policyholders’ reaction to economic and financial contexts and to revaluation rates (and therefore to the insurer's decisions). They may be reflected in surrenders, arbitrages or contributions;
- Financial market risks and in particular interest rate risk, which can be partly replicable.

The insurer's reaction to the return rates on assets and to the forecasts regarding policyholders' behaviour consists of management actions, using an accounting rate. The outcome of which is the served revaluation rate (see section 2.3 for a standard algorithm presentation for calculating the served revaluation rate practised by the market).

The next sections show how future flows of savings contract liabilities in € can be calculated in order to evaluate the best-estimate and provide a summary of market modeling practices.

2.2 Cash flow analysis [9]

Article 77 of the Solvency 2 Directive specifies that cash flows used in best estimate calculation must take into account all cash inflows and outflows needed to meet insurance and reinsurance liabilities over the liabilities' term.

Savings contracts' best-estimate in € calculated at a time t is written as follows (see ARMEL and PLANCHET [2019], LAURENT et al. [2016]):

$$BE(t) = E\left(\sum_{i=t}^{+\infty} F_i . exp\,(-i . r_i)\right)$$

where r_i is the risk-free forward rate at maturity i.

The F_i flow is the sum of payments made to policyholders and expenses minus premiums and loads :

$$F_i = Payements_i^{gross} - Premiums_i + Fees_i - Loadings_i$$

In practice, best-estimate evaluation is most often based on Monte-Carlo simulations (see Fig. 2) and flows' evaluation ends at a projection horizon T.

Future contracts are also excluded from the best-estimate valuation scope. Additionally, if savings contracts do not contain predetermined financial guarantees for all future

[9] For more details on cash-flows’ construction and best-estimate evaluation, the reader can refer to ARMEL and PLANCHET [2019].

payments, which is generally the case with traditional savings contracts, future premiums cannot be taken into account in the valuation scope (ACPR [2013]).

Charges and fees are functions of the surrender value (earned savings), the number of contracts (especially for fixed costs valuation) and inflation (see ACPR [2013]). Surrender value evaluation and the probability of the policyholder's presence in the portfolio, at the time during which the flow is calculated, enables the load and expense flows calculation.

A savings contract's surrender value (acquired savings) at time t+1, denoted VR(t+1), is written according to the surrender value at time t and the net revaluation rates of loads served at time t+1, denoted c_{t+1}, as follows (cf. ARMEL and PLANCHET [2019] and Bonnin et al. [2014]):

$$VR(t+1) = VR(t) \times exp\ (c_{t+1})\ = PM_0 \times exp\left(\sum_{i=0}^{t} c_{i+1}\right)$$

with $VR(0) = PM_0$ where PM_0indicates the mathematical provision at time 0.

This surrender value is conditional upon the insured's presence on the calculation date. The probabilized surrender value is the surrender value multiplied by the presence probability in the portfolio.

The savings' outflow in € can be explained by two factors: death or lapse (ratchet or € contract arbitrage with UL contracts)[10].

Let:

- q_t the mortality rate between t and $t+1$ and $q_{-1} = 0$.
- v_t the redemption rate between t and $t+1$ and $v_{-1} = 0$.

Then the probabilized surrender value (denoted $VRP(t+1)$) is:

$$VRP(t+1) = PM_0 \times \left(\prod_{j=0}^{t}(1-q_j)(1-v_j)\right) \times exp\left(\sum_{i=0}^{t} c_{i+1}\right)$$

And the outgoing cash flow net of loadings and fees at time $t+1$ is:

$$F_{t+1} = PM_0 \times \left(\prod_{j=0}^{t-1}(1-q_j)(1-v_j)\right)(q_t + v_t - q_t.v_t) \times exp\left(\sum_{i=0}^{t} c_{i+1}\right)$$

National guidelines (ACPR [2013]) specify that "In addition to the *structural surrenders that the insurer may observe in a "normal" economic context on euro savings life insurance contracts, the insurer must take into account cyclical surrenders. These occur in particular in a highly competitive context when the policyholder arbitrates their insurance contract in favour of other financial supports (insurance, banking or real estate products)*". ACPR recommends using experience or market tables to model structural surrenders. The

[10] The term "surrender" or "termination" will be used interchangeably.

modelling of cyclical (or dynamic) surrender presented in ACPR [2013] is discussed in section 2.4.

Redemption rates are consequently supposed to be written as the sum of a dynamic redemption rate, indicating financial arbitrage behaviour and a structural redemption rate independently from the economy (e.g. the withdrawal to finance holidays).

Therefore, by introducing the expectation contingent to market risks and based on the hypothesis of independence[11], we can directly use the death or structural redemption rates given by the tables, whether regulatory or experiential, without having to use simulation techniques in the process of best-estimate calculating.

Let's denote:

- q_t mortality rate between t and $t+1$.
- v_t surrender rate between t and $t+1$. This rate includes both dynamic and structural surrender.
- $R(t) = R(t) = \prod_{j=1}^{t-1}(1-v_j)$ and $R(0)=1$ is the unredeemed part of the savings between 0 and t.
- $\psi(t) = exp\ exp\left\{\sum_{i=1}^{t} c_i - \sum_{i=1}^{t} r_i\right\}$ with:
  - c_i the rate of revaluation of savings net of loadings at time i. This rate must be higher than the minimum guaranteed rate ;
  - r_i is the risk-free rate for the period between $i-1$ and i.

Then on a finite projection time horizon denoted T the discretized best-estimate net of loads and fees is written:

$$BE^{net}(0) = PM_0.E\left(\sum_{t=1}^{T}\frac{l_{t-1}}{l_0}.R(t-1).(q_{t-1}+v_{t-1}-q_{t-1}.v_{t-1}).\psi(t) + \frac{l_T}{l_0}.R(T).\psi(T)\right)$$

More generally (cf. ARMEL and PLANCHET [2019]), we can write the best-estimate in the following form:

$$BE(0) = PM_0.E\left(\sum_{t=1}^{T}\alpha_t.\psi(t)\right)$$

The factor α_t takes into account the probability of outflow in year t due to mortality or surrenders, the probability of being under contract at time t-1 and the charges or loads rates. This factor is stochastic because it depends on the state of the economy (it includes in particular dynamic surrenders).

In practice, this expectation is estimated by practitioners as a risk-neutral measure. It therefore requires risk-neutral economic scenario generation. The relevance of using this measure is discussed in section 3.

[11] A mortality catastrophe can nevertheless cause the shutdown of the economy.

It can also be noted that "inside the expectation", there are in particular two stochastic processes depending on the economy and the behaviour of agents (policyholders and insurers):

- The revaluation rate, which varies according to the state of the market and the insurer's investment policy. It is also the consequence of profit optimization under economic constraints and the policyholder behaviour.
- The dynamic lapse rate reflects a financial arbitrage behavior aimed at maximizing profitability. It can be negative, that is, policyholders buy back less than "usual". In particular, it can be negative when the "moneyness"[12] of the insurance contract is favorable to them. However, it can also be positive when more profitable risk-contingent investments, compared to insurance contracts, are available.

The following sections, 2.3and 2.4, summarizes practices used by the market to model the revaluation policy and policyholders' dynamic behaviour.

2.3 Revaluation algorithm: review of market practices

The options and guarantees provided in the euro savings contracts have as underlying securities, the euro fund assets managed by the insurer at its discretion.

Table 1 shows insurers' assets' distribution at market value at the end of 2016 (source FFA [2017a]).

Table 1: Insurance company investments at the end of 2016

Investments of insurance companies at the end of 2016	*€ Billions*	*Allocation*
Equity	401	17%
Corporate Bonds	907	39%
Bonds issued or guaranteed by States	773	33%
Real estate	97	4%
Monetary	123	5%
Other	49	2%
Total	*2,350*	*100%*
Life and mixed societies	2,114	90%
Non life / Casualty	236	10%

We can note that :

- The assets held by the insurers are simple: bonds, shares, real estate investments and money;
- Bond assets are prevalent, resulting from a policy of managing the asset-liability duration gap by partially "hedging" liability flows with bond flows;
- Liquidity is managed through equity investments and money;

[12] See ARMEL AND PLANCHET [2019] for a definition of the moneyness of a savings contract in euros.

- Performance management is conducted, in part, with equity investments, real estate and some bond assets.

While the asset composition of insurance companies may show the similarities mentioned above, management, allocation and stock-picking policies are specific to each insurer.

Liability valuation models take into account the insurer's asset management policy. This management translates in particular in strategic allocations, creation of unrealized capital gains or losses, buy-sell operations, etc.

At any given date, the insurer has financial income generated by assets and by its policy and can revalue savings contracts.

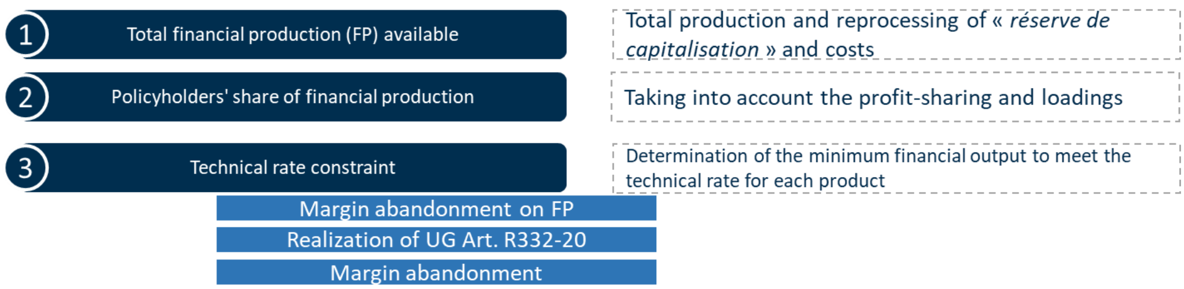
The algorithm for revaluing savings contracts in € can be presented as an optimization process under constraints. The insurer aims at optimizing its result (for example: margin, ROE, target dividend, internal rate of return, etc.) under economic, regulatory, and contractual constraints and policyholder behavioural constraints. Particularly, the following constraints can be listed: technical rate, minimum guaranteed rate, competition rate and minimum profit-sharing rate.

In liability valuation models, margin optimization is not only restricted to the revaluation algorithm, but also involves investment policy. Indeed, the insurer may consider modelling an optimal buy-sell policy for bond assets, for which the realized UGL endow the capitalization reserve (the so called "*réserve de capitalisation*" in French), which is similar to equity capital (and is not acquired by policyholders). The ACPR (ACPR [2013]) nevertheless specifies that the insurers' bond policy in asset-liability models must be consistent with bond policy practiced under the assumption of business continuity.

An examination of the revaluation algorithms of certain major players in the French euro savings market has enabled to draw up a standard diagram of the revaluation process. This diagram is presented below. It presents the steps for optimizing profitability (referred to as "margin" in the following sections) under the constraints implemented in the models and reflects the contract revaluation processes implemented in practice by insurers.

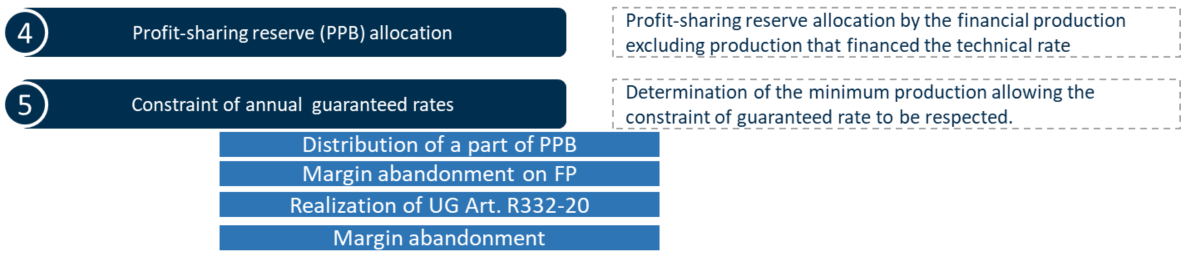
The first three steps aim to reassert the value of the contracts at the technical rate as shown in the next figure. It should be pointed out that the sub-step of margin abandonment on FP (financial products in the accounting sense) can occur for some companies after the UGL (Unrealized Gains or Losses) realization.

Fig. 3: Technical Rate Service



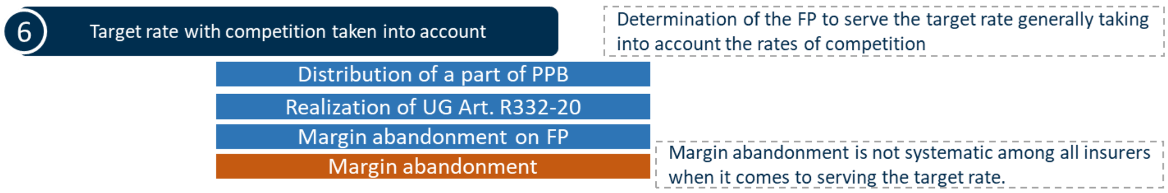
If the financial production is enough to serve the technical rates, the profit sharing reserve (designated in the following by PPB for "*Provision pour Participation aux Bénéfices*") is provided with the balance. The increased PPB is then used to serve the minimum guaranteed rates (some companies may give up their margin on FP after the UGL realization step).

Fig. 4: <u>Guaranteed Minimum Rate Service</u>



If the PPB is enough to pay the minimum guaranteed rates, we can then look at what is called the "target revaluation rate"[13]. When the wealth (financial production and PPB) is significant, some insurers realize unrealized losses to adjust the distributed wealth downwards and remove depreciated assets from the portfolio. If the wealth is significantly lower, we can observe a loss of margin on financial production or a realization of unrealised gains before considering a loss of margin on the result.

Fig. 5: <u>Service of the target rate</u>



The last step is to verify distribution constraints of mandatory minimum profits (including PPB that has been provisioned for more than 8 years).

Fig. 6: <u>Target rate correction to satisfy the minimum profit-sharing constraint</u>

In practice, there is little room for life insurance companies to revalue contracts using technical rates or minimum guaranteed rates. We observe little differences in financial products' generating models:

- On step 1 - financial production: systematic realisation of X% of UGL (systematic turnover on equities and real estate), reallocation of assets, etc.
- On steps 3 and 5: some insurers realise UGL before any margin abandonment on financial products (FP).

For step 6, heterogeneous approaches are observed on the market for the definition of the target revaluation rate. We usually distinguish between logics involving "a rate expected by the policyholder" and one or more references in the rate construction:

[13] See below for a definition of the target revaluation rate and a presentation of market practices.

- Interest rates possibly restated from the loadings rate (e.g. *TME*, 10-year *swap* or zero-coupon rate, weighted average of 1-year and 10-year *swap* rates, *Livret* A, 10-year *swap* rate plus volatility adjustment, etc.);
- Financial performance of an index (e.g., adjusting the CAC40 performance over 3 years);
- Internal benchmark (e.g., revaluation rate served to policyholders in year N or N-1);
- Competitive rate such as the rate published by the ACPR (ACPR [2018]) or the market average revaluation rate.

Further examples of references are provided by the French Institute of Actuaries ([2016], p. 42).

The majority of the approaches used in practice use one or two indicators, including, very often, an interest rate indicator. This logic is justified in particular by the close relationship between OAT rate and revaluations observed in the past (see BOREL-MATHURIN and al. [2018]).

For instance, some insurers assume that the rate expected by policyholders is a weighted average of a "memory effect" and a rate served by the supposed competition equal to the 10-year OAT rate:

$$Tx\_expected(t) = max\big(TMG, a \times tx\_Servi(t-1) + (1-a) \times OAT(t, 10ans)\big)$$

The final target rate corresponds to the expected rate minus a subjective $Spread$ that materializes product characteristics representing a brake on lapses, such as a rate guarantee or a particularly advantageous taxation.

$$Tx\_target(t) = max\big(TMG, Tx\_expected(t) - Spread(t)\big)$$

The final revaluation rate may be different (upward or downward) from the target rate defined in Step 6.

The difference between the revaluation rate and the rate expected by policyholders is used by practitioners as a dynamic lapse determinant variable. The following section summarizes market practices.

2.4 Dynamic behaviours of policyholders

Policyholders' dynamic behaviour is modelled by dynamic lapse. It is therefore assumed in market models that policyholders modulate their lapses upwards or downwards according to the financial arbitrage opportunities that occur.

According to ACPR [2013], dynamic lapses are commonly modelled by a function exclusively depending on the difference between the paid revaluation rate and a rate dependent on the economic environment, often referred to as the policyholder's expected revaluation rate (see section 2.3). The dynamic lapse rate should be added to the structural surrender rate.

If the served rate (TS) is lower than the expected rate (TA) by the policyholder, the latter will tend to withdraw more than indicated by the structural lapse curve.

Conversely, if policyholders are offered a higher rate than they expected, they will withdraw less in the following year than in the past.

The ACPR (ACPR [2013]) proposes to maintain in the models the dynamic lapses as a function of the gap $(TS - TA)$ inside a tunnel presented in Fig. 7.

The majority of organizations use the proposed legislation of ACPR (ACPR [2013]) to model dynamic lapse. This kind of model consists in assuming that the dynamic lapse is piecewise affine function of value (TS-TA).

Fig. 7: <u>Min-max tunnel proposed by ACPR for dynamic lapse modelling</u>

Therefore the dynamic lapse model implemented by the market explicitly assumes that the lapse decision results from a reasoning based on historical data (the served revaluation rates and the rates of the competition to date) and not on the policyholder's rational expectations (see section 3.1.3 for a discussion of the mathematical implications of this point).

The modeling discrepancies that can be seen between insurers concern the setting of the piecewise affine reaction function (expected rate, thresholds, etc.), but not the basic framework. On the academic level, the few existing references on the subject concern the rationalization of the parameters of the piecewise affine function or the study of explanatory variables for lapses (e.g. SURU [2011] and RAKAH [2015]). There are also some works proposing modeling using logistic regressions (cf. SAKHO [2018]).

In this section we have presented the process of creating euro savings contracts' future flows in order to evaluate the best-estimate. We have also summarized market modelling practices.

In the next section, we will discuss the relevance of the choice, by the market, of a risk-neutral probability measure for valuing liabilities and its consistency with the modelled and observed behaviours of agents (insurers and policyholders).

3 Probability measure and management rules

The necessary and sufficient conditions for the use of risk-neutral probability measure for best-estimate evaluation are:

- Absence of arbitrage opportunities (AAO);
- Market completeness

Indeed, the work of HARRISSON and KREPS [1979] and HARRISON and PLISKA [1981] has shown that, under the hypothesis of a complete market[14] and AAO, there is a single probability measure equivalent to the historical probability such that the discounted prices are, under this probability, martingales. This result reduces the calculation of the price of an asset to an expectation calculation.

More generally, it should be noted that discounting prices at the risk-free rate amounts to changing the numeraire. Instead of expressing the value of an asset in the current monetary unit, it is expressed in a particular unit, consisting of a capitalization bond at the risk-free rate. This approach has been generalized by GEMAN, EL KAROUI and ROCHET [1995], who show that it is possible to link any numeraire (a strictly positive measurable process) with a probability measure in which asset prices are martingales.

We discuss in the following section the two assumptions needed for the existence of a unique risk neutral probability measure in life insurance: i.e. AAO and completeness.

3.1 Arbitrage opportunities in the euro savings market

A market without arbitrage opportunities is a market where it is impossible to implement a financial strategy that, while involving no initial investment, ensures a non-zero expected gain.

In a liquid market, where there are neither transaction costs nor limitations on support assets' management (buy-sell), there are no arbitrage opportunities (EL KAROUI [2004]). The property of no arbitrage opportunity is satisfied in any perfect market in equilibrium.

In insurance, the presence and persistence of arbitrage opportunities can be explained at least by:

- Euro savings insurance market structure and the nature of the contracts;

[14] Within the idealized market framework, securities are perfectly divisible and short sales are allowed. Operators are rational and have the same ability to process information. They cannot, by individual action, move prices. There are no barriers to borrowing. Lending and borrowing rates are identical. There are no transaction fees or taxes.

- The information asymmetry and the limited rationality of the agents intervening in this market.

3.1.1 Some arbitrage opportunities related to the structure of the insurance market

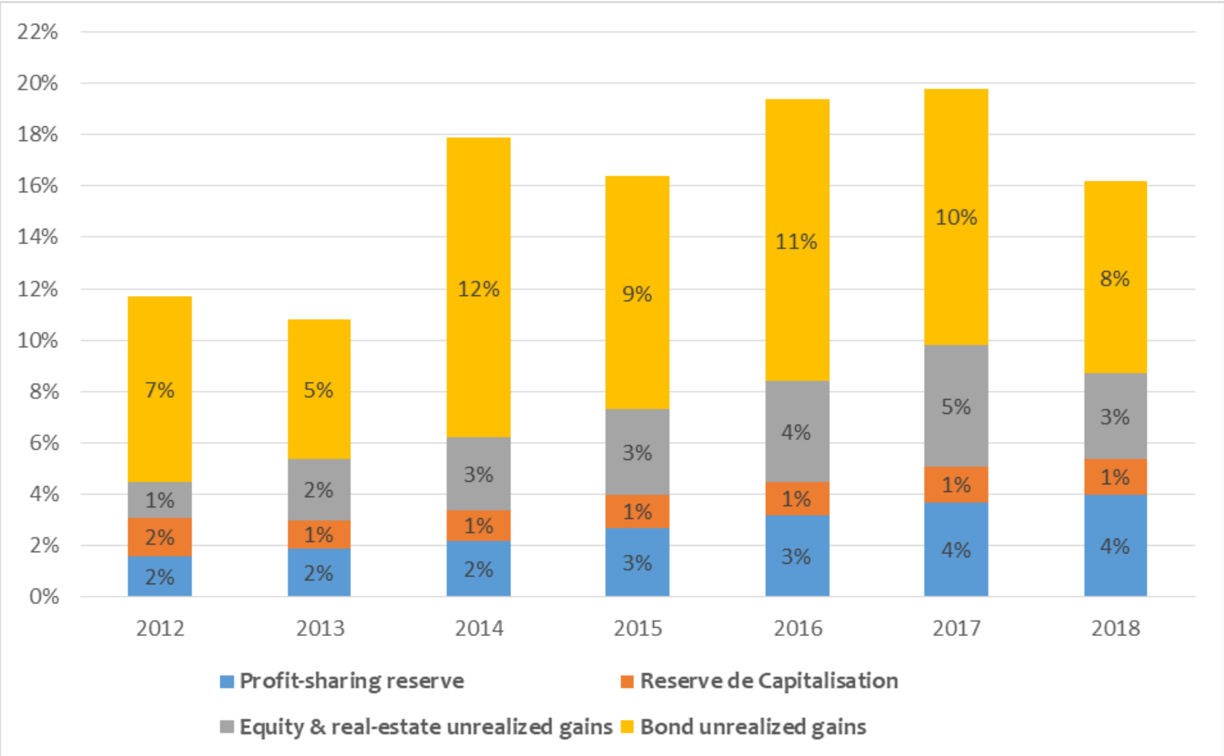
3.1.1.1 Example 1: Initial wealth and arbitrage opportunities

An investor wishing to invest an amount of $PM(0)$ in a € fund can benefit from the initial wealth accumulated by the insurer. This is as follows:

- The profit-sharing reserve, which is fully acquired by policyholders present at the date of its distribution (and not at its endowment). Its distribution is at the insurer discretion ;
- Euro-fund's assets' unrealized gains or losses. Remember that financial income is valued on the basis of assets book value ;
- Reserves allowing management of accounting returns such as the capitalization reserve (*reserve de capitalisation*), financial contingencies reserve (*provision pour aléas financiers*) and exigibility reserve (*provision pour risque d'exigibilité*). These reserves are at the discretion of the insurer.

Figure Fig. 8 illustrates the importance of the initial wealth accumulated by companies under the insurance code on the French market (see FFA [2017b] and FFA [2019]).

Fig. 8: Initial wealth of insurers on the French market from 2012 to 2018 as a % of non-unit-linked life insurance reserves



In 2018, the average maximum initial wealth of life insurers is at least 16.1% of reserves of funds in euros. This is comparable to three times the cumulative own funds (see Table 2).

The initial wealth acquired in full from the insured is 4.0%. Thus, the initial wealth that can be distributed by the insurer without regulatory obligation and at its discretion can theoretically vary within a range that represents 12.1% of the outstanding amounts in 2018. This is comparable to twice the cumulative equity of life insurers in 2018 as shown in the following table.

Table 2: Reserves and own-funds of life and mixed insurance companies (FFA [2019])

In € Billions	2013	2014	2015	2016	2017	2018
Gross technical reservess on the balance sheet	1559	1636	1713	1783	1845	1864
Own-funds*	63	66	71	73	77	79
Ratio: equity and unrealized capital gains / technical provisions (%)	11%	17%	15%	15%	15%	12%
Part of units linked in technical reserves on the balance sheet (%)	16%	16%	17%	18%	20%	19%
Unrealized capital gains	109	211	188	202	201	150

* Including the result of the financial year

This initial wealth creates an arbitrage opportunity. Indeed, it is acquired in part by the policyholders and will be distributed in future revaluation rates in addition to the asset performance. Thus, the present value of an investment of $PM(0)$ monetary units can be greater than or equal to $PM(0)$ as shown by ARMEL and PLANCHET [2019].

An investor can theoretically have a short position of $PM(0)$ on a risk-free asset and invest $PM(0)$ in an insurance contract (which is risk-free for investments of less than 70 k€, see section 7).

3.1.1.2 Example 2: Discretionary profit sharing and arbitrage opportunities

The insurer's policy has a significant impact on the insurance contract value. Indeed, let's consider two insurers A and B with identical characteristics. They share the same assets and liabilities, operate in the same economic environment and have the same initial wealth.

If insurer B decides in its revaluation policy to take a lower margin than insurer A, this implies that the revaluation rates served by insurer B are higher and the B's liability value is therefore higher than A's.

The short sale of A contracts and the purchase of B contracts generates a certain gain for the policyholder.

More generally, ARMEL and PLANCHET [2019] show that the savings contracts liabilities' value in euro in the Solvency 2 framework is not unique and not only represents the risk value, but at best, that of the risk conditionally on the insurer's revaluation policy.

The best-estimate can take all values in a closed interval whose bounds are independent of the models' choices for generating economic scenarios.

The interval's width can be significant and matches the difference between a minimum initial wealth, whose distribution is mandatory, and a maximum wealth available to the insurer[15] .

[15] For more details on the creation of this interval the reader can consult ARMEL and PLANCHET [2019].

Based on the statistics shown in Table 2, the width can be evaluated on average as the difference between total initial wealth and PPB. It reaches 12.1 % of reserves at the end of 2018 (comparable to twice insurers' own funds).

The two previous examples illustrate explicit cases where we can observe arbitrage opportunities in life insurance.

In addition, each year we can observe quite significant differences between euro savings contracts' revaluation rates on the French market[16] without this representing risk differences for policyholders, nor without this leading to significant savings transfers between insurers.

More generally, profit-sharing mechanisms and wealth mutualization (initial and generated during the euro fund life) between (1) the insurer and the policyholders via the profit sharing rules, capitalization reserve endowment and other reserves and (2) between same generation policyholders (the financial profit is acquired by all) and of different[17] generations via the PPB and other reserves, create in effect arbitrage opportunities. Indeed, we can see that:

- Insurers' discretionary policies create arbitrage opportunities because the insurance contracts' returns are heterogeneous despite the risks' similarity for the policyholder (zero for investments under 70 K€);
- The return mutualization between policyholders and accumulated wealth sharing (UGL, PPB, and other reserves) enables them to benefit (or avoid) from the asset's past performances without having invested in it initially.

The existence and persistence of these arbitrage opportunities can be explained at least by the savings market's structure, information asymmetry and agents' limited rationality.

3.1.1.3 The savings market in €: persistent arbitrage opportunities

Insurance arbitrage opportunities persist and it is not always possible to seize them. This can be explained by at least the following structural limitations of the savings market in €:

- Liquidity: transfers of savings portfolios in euros between insurers are infrequent and policy lapses by policyholders can be settled within 2 months ;
- Management limitations (buy-sell) :
    - o It is impossible to transfer savings contracts in € in France to another person. The only potential buyer of the contract is the insurer who sold it. The surrender value is not the economic value but the acquired savings' value at the surrender date ;

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[16] https://www.argusdelassurance.com/epargne/assurance-vie/assurance-vie-tous-les-rendements-2018-des-fonds-en-euros.139849

[17] HOMBERT and LYONNET [2017] study intergenerational risk sharing in euro-denominated life insurance policies. They show that revaluation rates are substantially less volatile than returns on the insurer's assets, which makes them more predictable, and show that inflows react only weakly to the predictability of these rates.

- A lapse transaction on a contract is not equal to a remit transaction on the same contract. The impact on payoffs can be significant and depends on the nature of the contractual clauses such as: interest rate guarantees, investment distribution between € and unit-linked products, deductions, fee structure, tax penalties, etc.
- Contract transfers between insurers are relatively rare.

- The cost in case of a lapse is significant. It is composed of a lapse penalty billed by the insurer and a tax penalty (see section 7).

These structural limits in the exchange of savings contracts in euros and their nature mean that the savings insurance market is significantly different from organized financial markets where agents' coordination by price is more efficient.

This efficiency on financial markets can be explained by the creation of a space that organizes and simplifies exchanges (liquidity, friction reduction, short selling, options clearing, penalization of certain information asymmetries such as insider trading, etc.).

Indeed, in the standard general equilibrium paradigm, it is assumed that the coordination of agents is ensured by prices. Given the assumption of rationality, contracts drawn up by agents are complete and optimal. They perfectly ensure their coordination and are therefore the only necessary means of coordination.

However, according to the economics of conventions (see BATIFOULIER & al. [2001] and EYMARD & al. [2006]), prices are indeed the efficient means of coordination when all assumptions of general equilibrium theory are verified. They no longer ensure perfect inter-individual coordination when, in particular, there is uncertainty and significant information asymmetries (cf. section 3.1.2.2).

Absent from the Walrasian theory, the convention would be an element of coordination complementary to the price mechanism. For conventionalist authors, price is therefore one means of coordination among others, effective in a particular space, i.e. the market. This market is structured by rules. SEARLE [1995] distinguishes two categories: regulatory rules and constituent rules (cf. BATIFOULIER & al. [2001]).

Practices are regulated by regulatory rules. These rules are like the price, a means of inter-individual coordination. They regulate behaviours that already existed beforehand, such as the mode of information exchange, i.e. prohibiting insider trading.

Constituent rules create the very possibility of behavioral coordination and institute it. They do not regulate an activity that existed previously, but define the space, the framework, the constraints and the interaction rules between agents. Let us take the example of the auctioneer (replaced by robots on stock exchanges). The auctioneer centralizes information, shouts out prices, applies the supply and demand law, and prohibits any exchange before equilibrium is reached. Without these rules, prices cannot ensure decisions' coordination. The market is therefore a constructed space. It is because there are rules that structure the financial market's coordination space that prices ensure coordination, as is clearly illustrated in MUNIESA [2000]. These rules are not simple means of coordination like prices are; they define the framework in which a means of coordination, here the price, operates.

In this section, we have shown the presence, in the € savings market, of arbitrage opportunities that the market structure and contracts' nature do not allow to seize.

The presence of arbitrage opportunities can be explained, in addition, by (1) markets' informational inefficiency and (2) agent's limited rationality (or irrationality) (policyholders and insurers). These two points are discussed in the following sections (3.1.2and 3.1.3).

3.1.2 Market efficiency and information asymmetries

What is the relationship between arbitrage and market efficiency? If the price structure on the markets is such that certain predictable elements of future prices are not reflected in current prices, then it becomes possible to profit without any initial investment and thus exploit an arbitrage opportunity. Assuming market efficiency, such arbitrage opportunities are not possible.

Informational efficiency is therefore a necessary condition for the existence of a risk-neutral probability.

In addition, Article 75 of Solvency 2 states that liabilities are "*valued at the amount for which they could be transferred or settled between **knowledgeable** willing parties in an arm's length transaction*".

This definition combines the "fair value" valuation of liabilities with the idea of well-informed contractors. Information is therefore considered to be a central assumption in measuring liabilities' fair value.

In addition to the now classic subject of financial markets' informational efficiency (cf. WALTER [1996], [2005] and [2013]), situations of information asymmetries in insurance can also be observed.

Section 3.1.2.1 discusses the notion of informational efficiency and its limitations. Section 3.1.2.2 presents some information asymmetries that can be observed in the € savings market.

3.1.2.1 Informational efficiency [18]

Formally introduced in 1965 by Fama (supplemented and amended between 1965 and 1976 by the same author), the idea of financial markets' efficiency represents the culmination of a century of financial theoretical thought. This secular reconstruction is presented in detail in WALTER ([1996], [2005] and [2013]).

It is simply said that there is efficiency when prices reflect all available information. WALTER [2013] proposes to speak of informational "effectiveness" and not of efficiency and in this case "the effectiveness of a market in the informational sense would be its capacity to transform information into price. From this point of view, a market can be more or less effective in the sense that the price can integrate more or less information" and considers the informational efficiency hypothesis as a stochastic convention.

[18] This section is inspired by the work of WALTER ([1996], [2005] and [2013]) and ORLÉAN [2008].

Moreover, the criticism of this hypothesis on financial markets can be organized into three parts:

- Internal inconsistencies, for example :
    - The rational bubbles' theory (for example: BLANCHARD and WATSON [1984], TIROLE [1982] and [1985]) which assumes that expectations relating to prices and not fundamentals are realized when players adhere to them. Price is therefore not the result of information but of a strategic vision of the players (example of HARRISON and KREPS [1978] where each investor is led to modify his evaluation according to the evaluation proposed by others).
    - The paradox of GROSSMAN and STIGLITZ [1980]: if price is efficient and information is costly, then it is rational not to obtain information directly. But if this is the case, with no one having any incentive to obtain information, price cannot be efficient.
- Behavioural finance has developed around cognitive psychology (notably by KAHNEMAN and TVERSKY [1979]) and bases its reasoning mainly on the hypothesis of correlation of agents' irrationalities. Since real arbitrage is risky, it strongly contests the neoclassical argument that states that the arbitrage of rational players will make prices converge towards their fundamentals.
- Self-referenced finance (ORLÉAN [2004]) aims to show that it is possible to design a theoretical framework that rejects the efficiency hypothesis, all the while refusing to make player's irrationality a central part of its understanding of speculative bubbles. The self-referential approach refuses to make the fundamental value an objective data, pre-existing to its calculation and likely to be known by all investors. Legitimate valuation is the result of the self-referential process itself, for which each individual seeks to position themselves in relation to the anticipation of others.

We can observe information asymmetries specific to life insurance in addition to the hypothesis' limits of financial market efficiency, which naturally extends to the French savings market. This point is discussed in the following section.

3.1.2.2 Information asymmetries in the savings market in €.

ALBRECHER [2016] lists asymmetrical information situations in insurance, elements which are listed below, completed and adapted to savings contracts in €.

Information asymmetries between agents and the "world".

This asymmetry concerns all life insurance market actors: insurers, policyholders, reinsurers, asset managers, etc. It consists in distinguishing uncertainty from risk. Indeed, Keynes asserts that no one can know all the events likely to affect their decision. The set of possibilities is not a prerequisite for choices; these are generally made on the basis of a limited knowledge of the facts. Thus, uncertainty can no longer be reduced to a calculation of probabilities on a given set of states of nature, but must be considered as radical.

Asymmetries of information between the policyholder and the insurer

The policyholder has more information about his investment objectives and actual risk profile than they express to the insurer.

The knowledge of the insurer of the reasoning process that can lead a policyholder to arbitrate their savings and their level of knowledge of the markets is limited. Moreover, the heterogeneity observed on the market in the definition of policy-holders' expected rates shows the lack of consensus on the characterization of policyholders' behaviour.

Also, the insurer does not have complete information on the policyholder's evolving financial situation, which may have an impact on their behaviour: tax situation, income, assets, career, etc.

Finally, the insurer is not in a position to fully anticipate the investment objectives of policyholders in a constantly changing economic and tax environment (e.g. the objective of succession or the use of the insurance contract as a short-term savings account given the drop in bank passbook rates[19]).

A surrender deprives the insurer of their future margin and may result in losses if the policyholder does not stay long enough to cover the costs incurred by the insurer to underwrite the policy. Symmetrically, when *moneyness* benefits the policyholder, lower surrenders cost the insurer.

Asymmetries of information between the insurer and the policyholder

The insurer's experience and their ability to better measure the contract's profitability and its risk (and to mutualize it) creates information asymmetries. A non-exhaustive list which is presented below:

- **Investment policy**: the insurer is fully aware of its assets (underlying the options sold to policyholders). The discretionary management policy makes it difficult for policyholders to anticipate future flows of the underlying over the life of the contract. Indeed, allocation and stock-picking policy is regularly updated by the insurer. Furthermore, the asset composition (and therefore of the underlying) may change at its discretion.
- **Wealth management policy**: the insurer can manage the unrealized capital gains (UG) of its bond portfolio with the capitalization reserve[20]. For example, in a negative interest rate context, an OAT bond purchased before the ECB's monetary quantitative easing policy is in UG and displays interesting coupons. If the bond is not sold, these coupons benefit policyholders in guise of financial products. If the bond is sold, UG funds the capitalization reserve (similar to equity capital) and enables the insurer to partially cover the risk of a rise in interest rates. However, the policyholder is deprived of a revaluation surplus.
- **Insurer's revaluation policy**: is at the insurer's discretion and is set neither in space (e.g., the decision to revalue some contracts more than others) nor in time (e.g., changing ROE targets).

[19] Cf. for example Les Echos [2020].

[20] The capitalization reserve ("*reserve de capitalisation*") is a technical provision admitted as a component of equity capital under Solvency 2.

- **The insurer's calculating capacity** and their better knowledge of assets and liabilities means that they are better informed about the contracts' moneyness[21]. The insurer can take advantage of this information to launch buy-back campaigns *of* contracts by encouraging policyholders to exit with bonuses. Therefore the insurer takes advantage of an arbitrage opportunity allowing them to pay less for their liabilities[22].
- **Competition knowledge**: the insurer also has a better competition knowledge and can consequently adjust its commercial policy, its investment management and its revaluation policy in order to have a better optimization of its margin.

Other information asymmetries

There is a natural asymmetry of information between competing insurers in the euro savings market. This concerns, for instance, insurers' risk/return profiles, market attitude and valuation between competitors, the nature of assets and liabilities, revaluation policies, etc.

In Solvency 2 valuation model, the insurer takes into account its expectations of its competitors positions by introducing a rate, called the competition revaluation rate, in the revaluation algorithm. However, there is heterogeneity between insurers in the choice and objectification of this rate (cf. section 2.4).

Finally, we can outline that there are naturally asymmetries of information between the insurer and the reinsurer as well as between the insurer and the investors given the insurer's better knowledge of their risk/return profile.

Information asymmetries observed in the insurance market systematically imply the presence of arbitrage opportunities. Moreover, EYRAUD-LOISEL [2019] shows that market incompleteness is due not only to a lack of replication assets, but also to a lack of information. The incompleteness of the € savings market can thus be explained at least by the inability to replicate flows and information asymmetry. The creation of a self-financed replicating portfolio to hedge liability options and guarantees can only be considered partially (see section 3.2).

3.1.3 The rationality of agents

Obviously, if all agents are rational, in the sense of financial theory, informational efficiency prevails. Indeed, if each agent is able to correctly evaluate prices, no valuation can deviate from the "fair" value of the contract. Rational agents are then price takers and their behaviour is coordinated by the latter. However, agents' irrationality can make prices inefficient and thus create arbitrage opportunities.

After a synthetic literature review based on the hypothesis of agent rationality, we present in this section some limitations specific to life insurance.

[21] See ARMEL and PLANCHET [2019] for a definition of this notion in the case of savings contracts in euros.
[22] See Les Echos [2016] for an example of a buy-back campaign.

3.1.3.1 *What is it like to be rational?*

The axiomatization completed by SAVAGE [1954] supposes that every individual, faced with a choice, knows all their possible actions, as well as the comprehensive list of circumstances (states of nature) likely to affect them. All they have to do is choose the action that will enable them to achieve the highest possible satisfaction, thus optimizing their utility.

Several empirical works refute the expected utility theory. Empirical criticism focuses in particular on individual cognition hypothesis (KAHNEMAN and TVERSKY [1974]). Expected utility theory indeed provides agents with important cognitive capacities. However, individuals cannot determine the optimal action because they have neither the capacity to collect and analyse information nor the necessary computational skills.

SIMON [1976] proposes a redefinition of rationality and opposes the concepts of substantial rationality and procedural rationality.

The hypothesis of substantial rationality is proposed by standard economic theory: the agent maximizes mathematical expectation of a given utility function. This conception is perceived as substantial because the rationality judgment only relates to the decision outcome. It is opposed to the procedural rationality hypothesis, which is concerned with the deliberation process that leads to a decision. A decision is then judged to be rational if the process that generated it is rational.

Moreover, limited rationality is a critical notion of maximizing expected utility. Before opposing substantial rationality and procedural rationality, SIMON [1947] opposes unlimited rationality and limited rationality. If substantial and unlimited rationality match one and only hypothesis, the one used in economics, then procedural and limited rationality are distinct.

Limited rationality implies that individuals have neither the information nor the computational capacity to be able to maximize a utility function. Once the agent's cognitive limits have been recognized, it is necessary to determine the decision-making procedures followed by agents, since these are then reflected in their behaviour: the search, evaluation and ordering of the various possible actions are the result of a deliberative process. Limited rationality is only a negative characterization of rationality, the positive aspect of which is procedural rationality (BATIFOULIER & al. [2001]).

In a Keynesian vision of coordination, the agent does not only calculate during the procedural deliberation that results in a decision. They interpret. Indeed, they select the information that they consider to be the most important in order to formulate their decision. This is called interpretative rationality (BATIFOULIER & al. [2001]).

3.1.3.2 *Agents' rationality on savings markets*

Due to the lack of available data, policyholders' behavioral characterization that generates financial arbitrage, in order to optimize utility, is a complex topic. As per in section 2.4, the majority of insurers assume that dynamic lapses depend on a satisfaction function measured as the simple difference between a served revaluation rate and an expected revaluation rate. The latter is not easy to characterize and its definition differs from one actor to another.

The risks of structural lapses and mass lapses in life insurance have been analysed in numerous actuarial publications such as MILHAUD & al [2010] and LOISEL and MILHAUD [2011].

To date, no publication has been able to perfectly characterize the phenomenon of dynamic lapses, motivated by a desire for financial arbitrage to optimize profit expectations.

Moreover, the parallel that can be drawn between bank panics and dynamic lapses is not entirely relevant insofar as the two phenomena have different motivations and the insurers levers are different from those of the banking sector. In fact, the significant lapses observed on the French market were localized, involving only a few insurers, and followed a damaging of their reputation (see BOREL-MATHURIN & al. [2018]).

SEJOURNE [2006] also shows that savers' behaviour (redemptions, arbitrages, payments, investments) does not correspond to that of rational (substantial rationality) and well-informed economic agents. He does not, however, characterize the process and rationality of policyholders leading to financial arbitrage in life insurance.

The inability of insurers to fully characterize the phenomenon of dynamic lapses complicate its anticipation. Insurers can only partially anticipate policyholders' behaviour, making dynamic lapse risk management imperfect.

Moreover, the dynamic lapse model adopted by the market does not assume that policyholders maximize their utility. Therefore the model introduces arbitrage opportunities. Indeed, the model presented in section 2.4 assumes that policyholders' decisions depend on a satisfaction function, which is the difference between the served revaluation rate and the expected revaluation rate. Based solely on historical analysis, the model does not involve market expectations in order to optimize utility.

Also, policyholders subscribing to the same fund do not necessarily have the same expectations because they do not have the same profiles (objectives, seniority, assets, taxation, etc.). The satisfaction function should be specific to each policyholder.

A rational policyholder, in the substantial meaning of the word, wishing to buy back their policy to reinvest it over an interest horizon T, should at least, like the insurer, evaluate the value of the payoff at time T by taking into account (1) its expectations of future (and not past) revaluations to be served by its insurer (2) its expectations of the tax penalty in the event of lapse (3) its future expectations of revaluations of competing savings products and their taxation and (4) its biometric risks. This may seem complex given the cognitive and computational capacity of the policyholder.

In conclusion, the euro savings market is a market where arbitrage opportunities are present and persistent. As expressed above, this can be explained, at least, by the nature of savings contracts and their optional structure, information asymmetries, the limited rationality (or irrationality) of agents and the savings market structure.

In the presence of arbitrage opportunities, there is no risk-neutral probability measure, making prices martingale.

Also, the replication of insurance liabilities can only be partial on the market and the risk-neutral probability measure, if it exists, is not necessarily unique. This point is discussed in the following section.

3.2 Cash-flows replication

3.2.1 Euro savings contracts cash-flows are partially replicable

The logic behind financial approaches in pricing relies on equivalence, in a world that preserves the absence of arbitrage opportunities, between the price and the hedging portfolio initial value. The hedging portfolio, when properly managed, has the property that it cancels the risk associated with uncertainty in the flow. The value at maturity of this portfolio is equal to the honored flow amount regardless of the world' s state. Of course, this situation is idealized and implies assumptions that are not met in practice, but it provides a framework for risk management (i.e., building and managing replicating portfolios) by controlling the imperfections' cost at the margin.

We can observe on this point that the founding article of Black and Scholes (BLACK and SCHOLES [1973], TANKOV [2011] and TOUZI and TANKOV [2012]) obtains the price of the derivative by explicitly building a strategy for flow replication.

In an insurance context, vanilla derivatives and guarantees' insurance contract, introduced in unit-linked contracts, are linked (cf. BRENNAN and SCHWARTZ [1976]). The first example, that French regulations used for Black-Scholes formula starting in the late 1990s, are "floor" guarantees (so called "*garantie plancher*" in French), guaranteeing at least the reimbursement of the amount invested in case of policyholder death. Actuaries then analysed the link between risk neutral probability and historical probability by looking at the risk management of this kind of contract (see MERLUS and PÉQUEUX [2000], FRANZ & al. [2003]) as well as the practical hedge management question (NTEUKAM & al. [2011]). Variable annuity contracts are the most direct generalization (BAUER & al. [2007], COLEMAN & al. [2005]). In all these cases, an explicit direct link is made between the commitment valuation and the way the asset representing this commitment is managed. In addition to the hedges' strictly financial imperfections, imperfect mutualization of insurance risks (mortality, longevity, lapse, etc.) induce fluctuations.

For savings contracts in €, however, the assets are neither segregated nor managed according to the options identified on the liabilities' side. Indeed, if in the case of guarantees on unit-linked contracts, the insurer actually puts in place financial hedge, it is not the same for € contracts. In the latter case, while there is a policy of ad hoc hedging for extreme situations, particularly in terms of the interest rate environment, no replicating portfolio is set up to hedge best estimate reserves. This would, moreover, be inefficient and costly and lead to a disconnection between the valuation logic (which provides the value of a hedge) and risk management (lack of effective hedge implementation). The formula for calculating a best estimate is therefore of a normative nature in this context.

Furthermore, technical risks (lapse, mortality, payments, etc.) are partially hedged by investing in bonds or by subscribing to reinsurance contracts.

Indeed, life insurers have *ALM*[23] models (historical probability calculations) that allow them to take into account portfolio management constraints (duration gap between assets and liabilities, cash-flow matching, target return, target margin, *etc.*) in order to build a strategic allocation that respects the insurer's risk appetite (the risk is not nil). The assets' alignment with a new strategic allocation can nevertheless be spread over several months (or even years).

The hedge of dynamic lapse cash-flows is complex because their characterization is partial and their anticipation by the insurer is limited (see sections 3.1.2.2and 3.1.3.2).

Also, the replication of insurers' management policy through assets and derivatives is partial because the profit-sharing process is subjective and is not fixed in space or time. It also involves accounting operations and reserves that make the implementation of financial hedging complex.

However, in a framework where dynamic lapses are constant, can liabilities be replicated if the insurer adopts financial management, by distributing the financial rate of return on its assets?

This question was discussed in ARMEL and PLANCHET [2019] and the following section summarizes it.

3.2.2 Savings contract in € and flow replication: example of ratchet options

In a framework where dynamic lapses are constant and where the insurer distributes the financial return on its assets, ARMEL and PLANCHET [2019] show that the best-estimate can be written as the sum of ratchet options (or cliquet-options) whose underlying is the insurer's assets and whose notional value includes, in particular, mathematical reserves, exit probabilities, fees and loads.

If the insurance risks of mortality and structural lapses are perfectly mutualized with low volatility, the hedge of options and guarantees of savings contracts in € amounts to a hedge of asset cash-flows on the financial market (see ARMEL and PLANCHET [2019]). This consists in building a replicating portfolio of ratchet options whose underlying is the insurer's assets.

In practice, this replication is complex. Insurers hold certain illiquid assets (such as private equities, private bonds and real estate) with significant maturities (such as bonds and infrastructure investments). However, they can put in place partial hedging mechanisms by studying the correlations of the assets held with financial market benchmarks.

3.3 Is it possible to use a risk neutral measure for the valuation of liabilities?

The practical use of risk neutral valuation is appropriate and should be limited to the derivatives market (see EL KAROUI & al. [2017]). This requires both hedging cash-flows and daily updating of all probability measures, models, prices and positions.

[23] ALM: Asset and Liability Management.

Indeed, it can be observed that this valuation approach assumes the absence of arbitrage opportunities and assumes that the contract's cash-flows are fully replicable (since the value of the hedge portfolio is calculated in this way at the origin). This proves to be inaccurate and logically leads to significant difficulties in implementing "replicating portfolio" techniques for this type of contracts.

As discussed above, arbitrage opportunities persist in the savings market and the replication of liability cash-flows can only be partial. The market is therefore incomplete. The use of a risk-neutral probability measure for the valuation of liabilities is therefore questionable.

Finally, from a practical point of view, the use of a risk-neutral measure causes an implementation difficulty faced by practitioners. The transition from "historical probability" to "risk neutral probability" consists in increasing the probability of the occurrence of unfavourable events for the investor in order to reflect their risk aversion. However, management rules implemented in valuation models are not adapted to situations in which rates reach levels considered unrealistic. The use of a stochastic deflator, which allows historical scenarios to be used, is one way of resolving this difficulty. This point is developed in section 5.

The following section (Section 5) first presents practices for generating risk-neutral economic scenarios, including interest rates, for valuing liabilities in a Mark-to-Model framework. Next, we present best-estimate sensitivities with interest rate models' choice and calibration data under risk-neutral measure. Finally, we present a best-estimate calculation under historical probability and its sensitivity to the interest rate model calibration data choice.

4 Generation of economic scenarios and yield curve

4.1 The economic scenarios generation

An Economic Scenario Generator (ESG) is a mathematical model reproducing the economic environment. It is used to produce simulations of the joint behaviour of financial market values and economic variables over a time horizon of interest.

In insurance, an ESG allowing the valuation of bonds, equities, real estate investments and money market securities covers 98% of French insurance companies assets and allows to simulate risk-free rates (see Table 1).

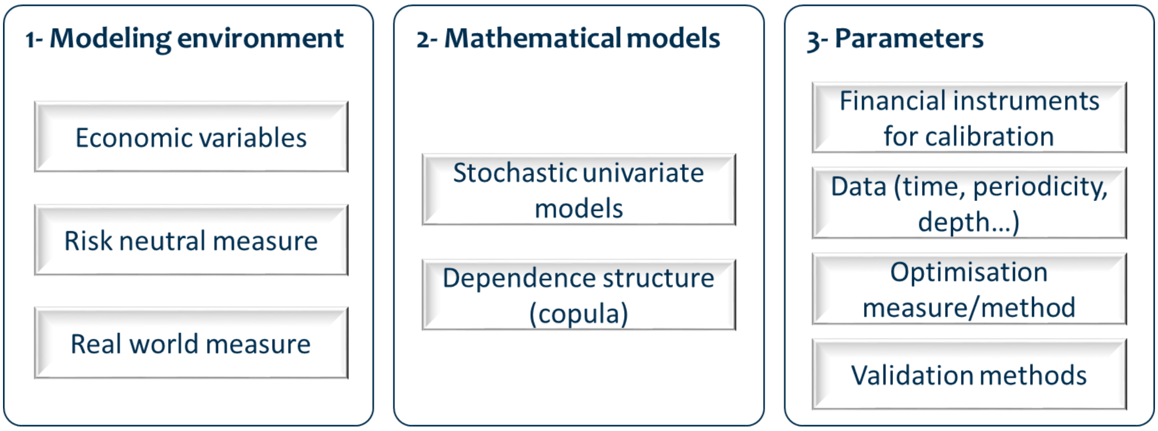
The process of generating economic scenarios intended for the evaluation of the best-estimate can be summarized in three steps (see ARMEL and PLANCHET [2018]):

1. The modelling environment: this involves choosing the probability measure and economic variables to be modelled.
2. Models: this involves building mathematical models of the interest variables. This consists in choosing models that will represent the individual dynamics of these variables and choosing the model that represents co-movement.

3. Parameters and calibration: this involves choosing financial derivatives for calibrations, data, methods for statistical estimation of model parameters and validation methods.

These steps can be schematized as shown in Fig. 9.

Fig. 9: Schematic illustration of ESG modelling and calibration choices



4.2 Economic scenario generation practices

4.2.1 A *Marked to Model* valuation framework [24]

Applying a Marked-to-Market approach to evaluate the *best-estimate,* for savings contracts in €, implies having the insurance policies' options and guarantees prices. As this information is not observable on an organized, deep and liquid market, the calculation is performed in a Marked-to-Model framework.

As a result, the calibration and validation of the economic scenario generator (ESG) by comparing the results of the simulations with the observed data, within the framework of a statistical approach, cannot be considered.

Solvency 2 nevertheless requires that the ESG should be Market Consistent[25], meaning, consistent with observed prices (see for example El Karoui & al. [2017] for a critical analysis of Market Consistency). The technical specifications QIS5 [2010] specify that a Market-Consistent calibration must be as follows (TP.2.97):

- The asset model must be calibrated to reflect the nature and duration of the liabilities, in particular liabilities incorporating guarantees and options;
- The asset model must be calibrated by taking into account risk-free interest rate curve used to discount cash flows;
- The asset model must be calibrated to a relevant volatility measure.

[24] The reader can refer to Armel and Planchet [2018] for a detailed presentation of the framework and the quantitative process for valuing the best-estimate of savings contracts in €.
[25] See for example Wütrich [2016].

4.2.2 The calibration convention for risk-free rate models

Effectively, practitioners simply calibrate and assess the ESG with reference to financial instruments (calls, puts, caps, floors, swaptions, etc.) derived from risk factors modelled without justifying the suitability of these instruments for liability options (see ARMEL and PLANCHET [2018] for presentation of this calibration process and regulatory requirements). This calibration is dynamic and is consistent with current requirements of the Solvency 2 regulation.

The choice of the term structure diffusion model is a central element in the construction of a "risk neutral" ESG. In order to respect regulatory constraints and in the absence of observable prices for euro-contract liabilities, a risk-free interest rate calibration convention emerged[26]. This convention can be summarized in four steps:

1. **Model and financial instruments derivative**: choice of interest rate model and choice of derivatives for its calibration: caps, floors, swaptions, etc.
2. **Strike price and implied volatilities**: choosing a strike price and extracting market volatilities[27]. These volatilities correspond to the implied volatilities of the derivatives selected in step 1. They are consistent with the market's risk-free yield curve.
3. **Valuation of derivative products** using the curve published by EIOPA: use of the Black model (if volatilities are implied by a log-normal model) or the Bachelier model (if volatilities are implied by a normal model) to price derivative instruments using the risk-free rate curve published by EIOPA. It is these prices that will play the role of "market prices" to calibrate the selected interest rate model.
4. **Calibration of the selected interest rate model** by minimizing a distance between: (1) prices re-estimated by using the EIOPA yield curve and market volatilities and (2) the theoretical prices of the interest rate model.

4.2.3 Limits of the interest rate model calibration convention

The best-estimate is an unobserved price. ESG is calibrated not on options and guarantees of the insurance contract but on financial products. The generation of economic scenarios for the valuation of the *best-estimate* implies 5 variables:

1. **The price of the derivative**: the ESG is calibrated on financial products (calls, puts, caps, floors, swaption...). This raises the question of the consistency of the option structure of the best-estimate and the financial product chosen for the ESG calibration. This question is discussed in ARMEL and PLANCHET [2019] which show, under certain conditions, that the best-estimate has a cliquet optional structure;
2. ***Strikes***: derivative prices depend on strikes. In order to offer ESGs' calibration consistent with best-estimate option structure, these ESGs should be calibrated on

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[26] See the Q&A of QIS 5, published by EIOPA, question 76 of the document. https://eiopa.europa.eu/Publications/QIS/CEIOPS-Q-and-A-document-20101104.pdf

[27] Volatility areas do not necessarily result from a direct price measurement but from a reconstruction by the price provider (e.g. via a SABR model for Bloomberg).

strikes consistent with the euro savings contract's options and guarantees exercise thresholds. It would therefore be relevant to study the liabilities' moneyness (see ARMEL and PLANCHET [2019]).

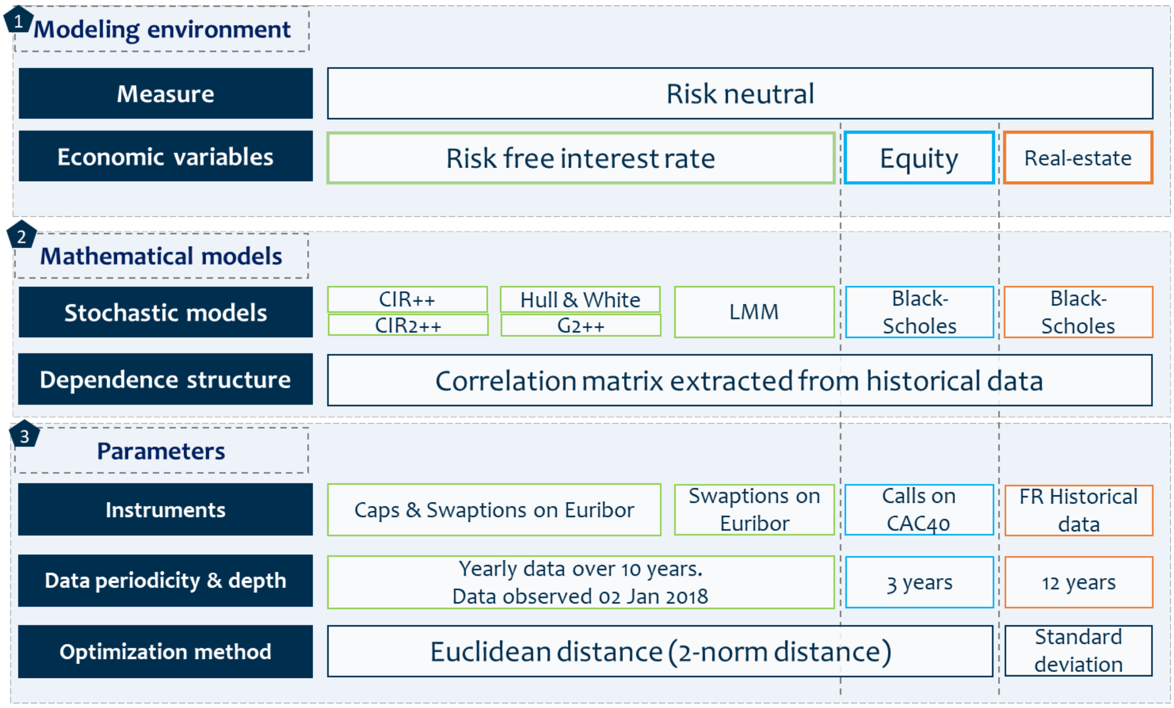
3. **The risk-free yield curve** is provided by EIOPA. It is a yield curve that is not observable in the financial market and is not subject to any market trade. A specific difficulty for the term structure of interest rates is also to extrapolate it to maturities for which there are not necessarily sufficient liquid instruments to have reference prices. EIOPA has chosen a macroeconomic reasoning to justify long-term rates (the UFR of the European prudential regulation), which is much discussed. The models used by central banks, often based on parametric models of the Nelson-Siegel type, can be interesting from this perspective (see CHRISTENSEN [2007]).
4. **The shift factor** is a factor introduced to allow log-normal models to take into account negative rates. This makes it possible to (1) reproduce market values by the Black model and calibrate interest rate models and (2) simulate log-normal models such as the LMM model. The shift factor depends on the risk-free rate curve used in the model, particularly on the minimum value of this curve, but its choice remains arbitrary;
5. **Implied volatilities** depend on the price of financial instruments, the strike price, the risk-free interest rate yield-curve and the shift factor (when the model is log-normal). When calibrating interest rate models for best-estimate valuation, it is these implied volatilities that are used in the Black or Bachelier models to replicate market prices with the yield curve reported by the EIOPA. It is noted that :
    a. The parameterization of the Black model by the EIOPA risk-free yield curve implies the introduction of a shift factor. In order to keep a certain consistency in the model, this factor should be identical to the one used to extract the implied volatilities from the market prices;
    b. Consistency between the EIOPA risk-free yield curve and the implied volatility is not systematic. Volatilities depend on the risk-free yield curve used to evaluate them;
    c. Implied volatilities depend on the financial instruments strikes used in the calibration process. Consistency between implied volatilities and options and guarantees on liabilities is not systematic.

4.3 Sensitivities of the best-estimate to "risk neutral" interest rate models

The choice of the term structure model is a central element in the construction of a "risk neutral" ESG. A comparison between the Hull and White, G2++ and LMM+ models was carried out in ARMEL and PLANCHET [2018] and ARMEL and PLANCHET [2019] and completed with the analysis of the CIR++ and CIR2++ models in ARMEL and PLANCHET [2020a].

A summary of the choices made for the construction of the economic scenario generators is presented in Fig. 10.

Fig. 10: Modelling choices



The *Table 3* shows the results of the best-estimate sensitivities to the choices of (1) the interest rate model, (2) the calibration data (cap and swaptions), and (3) the shift factor of the Black model used in the calibration process. The LMM model, calibrated on data observed on January 2, 2018, could not be retained however due to its divergence. The version used in sensitivity tests is an adjusted and convergent model, which is therefore no longer Market Consistent.

Table 3: Comparison of *best-estimates*

Best-estimate in M€	*Standard deviation*	*Min*	*Max*	*(Max-Min)/Mean*
CIR type models, market-consistant : CIR++and CIR2++	0.77%	91	92	2,00%
Normal type models, market-consistant (HW & G2 ++)	1.20%	87	91	4.30%
Market-consistant models : CIR++, CIR2++, HW & G2++	1.13%	87	92	5.38%
All models including adjusted LMM model	1.06%	87	92	5.38%

We can summarize this work by stating that the G2++ and CIR2++ models makes up performing solutions for best estimate values calculation, with a much higher degree of complexity for CIR2++ without major gain in terms of capacity to represent market prices. The shifted LMM+, used by some insurers, is unsuitable because of convergence problems and the arbitrary nature of the shift that needs to be introduced to take into account negative rates.

In general, it can be observed that, among the above models, those initially designed to avoid negative rates have been adapted to this new economic context by introducing a shift which, by a simple sliding of the origin on the abscissa axis, allows the model to remain unchanged. This adjustment was initially carried out as a matter of urgency, to allow the use of existing tools, which can take time to modify (BEINKER and STAPPER [2012] illustrate the position of financial consultants at the beginning of the period of negative interest rates). But the lack of theoretical justification for the shift and its arbitrary nature should lead to these models' exclusion in a more sustainable perspective.

Moreover, the impact of the choice of interest rate models on the value of the best-estimate may appear at first to be fairly contained. By retaining only *market consistent* interest rate models, the difference between minimum and maximum values represent 5.4% of the average value of best-estimates and 7.0% of mathematical reserves.

This impact is substantial if compared to own-funds and no unquestionable criterion allows at this stage to prefer one or the other of the above models, once the LMM model is excluded because of its lack of convergence. ARMEL and PLANCHET [2019] also show that the capacity of an interest rate model to reproduce the prices of floorlets and, by extension, the prices of caps, can be considered as a criterion for the choice of interest rate models for valuing the liabilities of savings contracts in euros.

Although it seems hard to question in the short term the calculation logic that has been imposed in recent years in the framework of the new prudential system, it may nevertheless be useful to think about alternative approaches, potentially usable in the ORSA framework. Indeed, the usual approach for best estimate calculation is not well suited to the ORSA, given its cumbersome implementation and the significant computing capacity it requires. Moreover, it may appear paradoxical that Solvency 2 application leads to reserves that are not linked to a risk management policy and, in this sense, are largely arbitrary (in any case, at least as arbitrary as current mathematical reserves).

One can therefore seek to analyse more precisely the articulation of the replicable and non-replicable components of the revaluation rate served by the contract. An approach of this type is, for example, proposed in BONNIN and al. [2014]. ESG development in this context is discussed in GOURIÉROUX and MONTFORT [2015].

We can also consider the use of deflators as a technical solution, which is imperfect, but which allows us to give meaning to the notion of "economic value" (see, for instance, ARMEL and PLANCHET [2020b], CHENG and PLANCHET [2019] and CISSÉ [2019]). While this approach does not change anything from a theoretical point of view (if the assumptions of AAO and completeness are verified, cf. section 3), it nevertheless presents several important practical interests:

- The economic scenarios used for the simulations are injected into an ALM model implementing accounting rules and discretionary decisions of the insurer (profit-sharing allocation) and the policyholder (lapses). The use of a deflator allows the injection into this ALM model of scenarios generated under historical probability, which allows to justify reaction functions (target revaluation rate calculation and determination of economic lapses in particular); this justification is indeed impossible with risk-neutral scenarios where one is led to give significant weight to scenarios that are very unlikely historically (see section 4.5);
- It allows, in the ALM model, a complete separation between valuation (price calculation), integrated in the discounting factor, and cash-flows' generation. The calibration of the discounting factor from vanilla derivatives is thus more legitimate than in the case of the use of a risk neutral measure.

The following section presents a study of best-estimate sensitivity to changes in the probability measure.

4.4 Historical probability pricing and sensitivity to calibration data [28]

ARMEL and PLANCHET [2020b] complete sensitivity tests carried out in ARMEL and PLANCHET ([2018], [2019] and [2020a]) by evaluating the impact of the choice of an economic scenario generator under the historical probability measure, whose interest rate model is the CIR++ model, on the best-estimate of savings contracts in €. Sensitivities to the choice of data and the Black model shift factor, used in the calibration process, were also carried out.

In order to assess these impacts, we have used the same models and parameters for assessing liabilities presented in ARMEL and PLANCHET [2019].

We also adopted a calibration approach that is (1) implied for the model parameters and (2) historical for the risk premium. This approach leads to a clear separation between the determination of options' cost, included in the deflator, and contract cash-flows' production. It also meets regulatory requirements.

Indeed, although the theoretical model assumes that historical and implied parameters are equal, in practice they are different. Parameters calibrated on historical data depend on data choices (index, size, frequency, etc.). The implied parameters depend on financial instruments' price, strike price, risk-free interest rate and shift factors used to allow models to take into account negative rates. Also, as aforementioned, the implied volatility surfaces do not necessarily result from a direct price measurement but from a reconstruction by the price provider (e.g. via a SABR model for Bloomberg). This point is discussed in REBONATO [2004].

Additionally, in Solvency 2 framework, economic scenarios used for best-estimate valuation must be consistent with market prices (Market-Consistent) and QIS [2010] technical specifications refer to calibration of models taking into account the implied volatilities[29].

Under historical probability (HP), we find that best-estimates are not very sensitive to Black valuation model shift factors and the choice of derivatives for calibration. Indeed, *Table 4* shows that the difference between minimum and maximum values represents 2% of best-estimates' average value.

[28] We would like to thank CHRISTIAN GIBOT for having contributed to the initiation of our research on the relevance of an approach by deflators for the valuation of savings contracts in €.

[29] Cf. the Q&A of QIS 5, published by EIOPA, question 76 of the document: https://eiopa.europa.eu/Publications/QIS/CEIOPS-Q-and-A-document-20101104.pdf

Table 4: Comparison of RN-HP *best-estimates*

Best-estimate in M€	*Standard deviation*	*Min*	*Max*	*(Max-Min)/Mean*
CIR++ model in historical probability (HP)	0.68%	85	87	1.99%
CIR++ and CIR2++ models in risque neutral probability (RN)	0.77%	91	92	2,00%
All market-consistant models (RN et HP) : CIR++, CIR2++, HW & G2++	2.42%	85	92	8.08%
All models including adjusted LMM	*2.32%*	*85*	*92*	*8.08%*

If we take into account sensitivity results presented in ARMEL and PLANCHET [2019] and ARMEL and PLANCHET [2020a], we observe in *Table 4* that the impact on the value of the best-estimate can appear to be fairly contained. If we retain only market-consistent interest rate models, the difference between minimum and maximum values represents 8.1% of the average value of *best-estimates* and 10.5% of mathematical reserves. This impact is, however, substantial if compared to equity capital. Indeed, in France, shareholders' equity represents on average 6.1% of savings contracts reserves in € at the end of 2018 (see Table 2).

4.5 Gap analysis between best-estimates *in historical and risk-neutral probabilities*

We present in Table 5 and Table 6 best-estimates evaluated by an ESG under historical probability and a risk-neutral ESG whose interest rate models are CIR++ (see ARMEL and PLANCHET [2020a] for a detailed study of this model).

We observe that the differences between best-estimates vary between 4% and 7% and are mainly explained by variations in best-estimates net of expenses[30]. Best-estimates of expenses (discounted future expenses) are indeed stable.

Table 5: Estimated Best-Estimate under Risk Neutral Probability - ESG with CIR++ IR Model

GSE - CIR++ under Q (M€)	*Calibration on ATM Caps*			*Calibration on ATM Swaptions*		
Shift of Black model for CIR++ calibration	*0.4%*	*1.0%*	*2.0%*	*0.4%*	*1.0%*	*2.0%*
BE	***91***	***91***	***91***	***91***	***92***	***92***
BE net of fees	*83*	*83*	*83*	*84*	*84*	*85*
BEG net of fees	*72*	*72*	*72*	*73*	*73*	*73*
FDB	*11*	*11*	*11*	*11*	*11*	*12*
Fees	*8*	*8*	*8*	*8*	*8*	*8*

Table 6: *Estimated* Best-Estimate under Historical Probability - ESG with CIR++ IR Model

GSE - CIR++ under P (M€)	*Calibration on ATM Caps*			*Calibration on ATM Swaptions*		
Shift of Black model for CIR++ calibration	*0.4%*	*1.0%*	*2.0%*	*0.4%*	*1.0%*	*2.0%*
BE	***87***	***86***	***85***	***86***	***86***	***86***
BE net of fees	*79*	*78*	*77*	*78*	*78*	*78*
BEG net of fees	*71*	*70*	*69*	*70*	*69*	*70*
FDB	*8*	*8*	*8*	*9*	*8*	*9*
Fees	*8*	*8*	*8*	*8*	*8*	*8*

All other parameters/inputs being equal (mortality rates, structural lapses, expense rates, loading rates, etc.), deflator approach use can have an impact on the exit probability and on the revaluation rate (cf. sections 2.2). The scenarios injected into the ALM model do not

[30] Section 4.5.1defines the *BEG* and *FDB*.

have the atypical characteristics of certain risk-neutral scenarios that disrupt the model's reaction functions. The differences observed can therefore be explained by the impacts on agents' behaviour, modelled in the liability valuation model, which are:

- Policyholders' behaviours: materializing by dynamic lapses meant to follow here a piecewise affine function of the difference between an expected revaluation rate and a served revaluation rate;
- Insurer behaviour, including its investment policy and its revaluation policy which integrates, among other things, the insurer's reaction function to the behaviour of policyholders and therefore, in our case, to the risk of dynamic lapses.

Risk premium introduction in the deflator approach increases the probability of favourable scenarios where the insurer has sufficient accounting returns to at least distribute revaluation rates expected by the policyholder.

Therefore, in the deflator approach, we would expect to see less reaction from policyholders. Dynamic lapses rates will therefore be lower and less extreme than when using risk-neutral scenarios. The following simplified example illustrates this point.

4.5.1 Dynamic lapse in historical probability and risk-neutral probability: illustration

Here we examine, over a one-year horizon, dynamic lapses rates distributions under historical probability and under risk-neutral probability.

To do so, let us suppose that the insurer's revaluation policy is to distribute positive financial return on its assets. Also let us assume that the asset price follows a Black-Scholes model and that the interest rate over one year is constant.

We also use the same characteristics than the dynamic lapses function used in the best-estimate valuation model used here (cf. section 2.3and ACPR [2013]).

Table 7: Parameters of the reaction function of policyholders

Alpha	*Beta*	*Gamma*	*Delta*	*RCMIN*	*RCMAX*
-5%	*-1%*	*1%*	*3%*	*-5%*	*20%*

Finally, we retain the modelling parameters presented in *Table 8*.

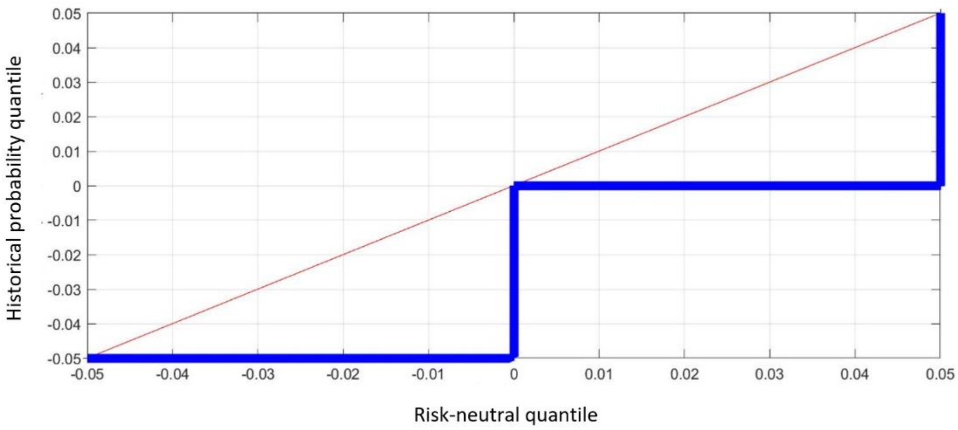
Table 8: Simplified ALM simulation parameters

One-year risk-free interest rate	*Insurer's asset volatility*	*Asset drift under historical probability*	*Revaluation rate expected by policyholders*
1%	*10%*	*3%*	*2%*

Figure Fig. 11 presents the quantile-to-quantile (*QQplot*) diagram of dynamic lapse rates evaluated under historical probability and under risk-neutral probability. We can see that the dynamic lapse rates evaluated under risk-neutral probability are higher than those evaluated under historical probability. All the points are indeed below the first bisector of the plan.

The differences observed between dynamic lapse rates are substantial and show that the policyholders' reaction is more pronounced when the assessment is carried out under a risk neutral probability.

Fig. 11: Quantile to Quantile diagram of the dynamic lapse evaluated under historical probability and under risk-neutral probability



4.5.2 Gap analysis

In order to analyse the differences observed in Table 5 and Table 6, we will focus on the best-estimate net of expenses and proceed in two steps:

- Isolation and analysis of policyholders' behaviour ;
- Identification of the impact of the insurer's behaviour.

Using the notations in section 2the out-flow at time t is written as a probability of exit multiplied by the initial investment plus the cumulative revaluation:

$$F_t = \alpha_t . PM_0 . exp\left(\sum_{i=0}^{t-1} c_{i+1}\right)$$

This flow can be split into two flows: $F_t = F_t^{gar} + F_t^{discr}$ with:

- F_t^{gar} takes into account the contractually guaranteed minimum revaluation rate. If $(tmg_i)_{i\in[\![1,T]\!]}$ are the guaranteed rates (including the initial PPB that matures), then:

$$F_t^{gar} = \alpha_t . PM_0 . exp\left(\sum_{i=0}^{t-1} tmg_{i+1}\right)$$

- F_t^{discr} represents the flow of the additional revaluation corresponding to the surplus that the insurer distributes at its discretion as profit sharing :

$$F_t^{discr} = F_t - F_t^{gar} = \alpha_t . PM_0 . \left(exp\left(\sum_{i=0}^{t-1} c_{i+1}\right) - exp\left(\sum_{i=0}^{t-1} tmg_{i+1}\right)\right)$$

The guaranteed best-estimate (BEG) is the expectation of the flows F_t^{gar}. The Future Discretionary Benefits (FDB) is the expectation of discounted flows F_t^{discr}.

The decrease in the best-estimate net of expenses, when a deflator approach is used, is explained by the decrease in the BEG and the FDB (see Table 5 and Table 6).

We can note that the probability of exit α_t is the only stochastic factor dependent on the economy of the guaranteed flow F_t^{gar} because it integrates dynamic lapses. Adopting a deflator approach has an impact on F_t^{gar} only through the factor α_t.

The decrease in BEG evaluated in historical probability is explained by the spread of lapses over the projection horizon following the decrease in dynamic lapses. When we adopt a deflator approach, we observe that the expected flows F_t^{gar} are lower over most of the projection time horizon and that their duration is longer, as shown in Table 9.

Table 9: Flow durations and lapses

Duration of guaranteed cash-flows in years	*Calibration on ATM Caps*			*Calibration on ATM Swaptions*		
Shift of Black model for CIR++ calibration	*0.4%*	*10%*	*2.0%*	*0.4%*	*10%*	*2.0%*
Under Q	*8.2*	*8.2*	*8.1*	*8.2*	*8.2*	*8.4*
Under P	*9,0*	*9.1*	*9.3*	*9.2*	*9.2*	*9.0*
Gap	*0.8*	*10*	*11*	*10*	*0.9*	*0.5*

We deduce that dynamic lapses are more important when liabilities' valuation is carried out under the risk-neutral probability. This finding is consistent with the simplified example presented in section 4.5.1.

In addition, the analysis of the variation in FDB flows involves studying variations in exit probabilities and revaluation rates, in particular the discretionary part.

If it is clear that the drop in exit probabilities α_t naturally implies a drop in *FDB* flows, it is difficult to have a more detailed analysis of the movements in discretionary revaluation rates given the large number of parameters involved in their evaluation.

Furthermore, under the historical probability measure, at each projection step, the financial income resulting from the insurer's assets accounting management is different from that recorded under the risk neutral probability. The insurer's buy and sale transactions are in fact different, the resulting asset is different and accounting reserves related to the asset are different (such as "*la réserve de capitalisation*", "*la provision pour aléas financiers*" and "*la provision pour dépréciation durable*").

Also, the decrease in the number of scenarios where policyholders' reaction is more pronounced, under the historical probability, means that the revaluation algorithm is less constrained to distribute a surplus of available wealth in order to reduce dynamic lapses. Finally, the decrease in lapses allows the insurer to improve its margin (when the moneyness *is* favourable to it). Having fewer lapses, the reserve's loadings base is greater, which results in a transfer of a part of the wealth to its own funds.

In conclusion, when economic scenarios are generated under risk-neutral probability, the latter reflecting investors' aversion to risk, a significant proportion of Monte-Carlo paths may present atypical levels for risk factors. Typically, since the emergence of negative rates, with investors being adverse to a sudden rise (or fall) in interest rates, paths with very high interest rate levels (in absolute value) appear. It is therefore difficult to justify reaction functions' behaviour reflecting actions of insurers and policyholders, over ranges never observed before (for example, a significant proportion of 10-year rates above 50%). Pragmatic adjustments are used by practitioners. They are sometimes very rustic and incorrect from a theoretical point of view, such as the introduction of thresholds on the level of spot rates or more subtle ones, such as the freeze, which consists of freezing the yield curve as soon as the one year forward rate exceeds a certain threshold (and this without deteriorating the martingality tests). The use of a deflator solves this difficulty, as the ALM model is provided with historical scenarios, thus making it easier to justify the reaction functions.

Moreover, in a largely normative valuation framework, it leads to a clear separation between options' cost determination, included in the deflator, and contract cash-flows' production. The use of scenarios produced in historical probability also makes it possible to produce indicators required by the supervisor, such as chronicles of average cash-flows.

These two points give the deflator approach a major advantage over the standard scheme of injecting risk-neutral scenarios into the ALM model. The work of ARMEL and PLANCHET [2020b] and CHENG and PLANCHET [2019] shows that numerical problems that have long been an obstacle to the practical use of deflators can be overcome and that the deflator approach is therefore operational for insurers and is not limited to an academic style exercise.

5 Conclusion

Setting up models for calculating best estimate reserves for savings contracts and, more broadly, contracts with a profit-sharing clause has led to a form of market consensus articulating a risk-neutral economic scenario generator with a cash-flow projection model.

This model structure has the advantage of allowing easy consideration of potentially complex management rules describing the management of the served revaluation rate according to market conditions on the one hand and the insurer's own situation on the other (particularly in terms of unrealized capital gains or losses).

However, it is based on the questionable approximation that cash-flows associated with these contracts are replicable in a market without arbitrage opportunities, which implies difficulties in implementing models and leads to a strong mismatch between the valuation principle and the risk management associated with the contract.

It is therefore useful to propose better adapted models that make the non-replicable component of these contracts explicit and thus provide a more appropriate framework for the relevant evaluation of associated liabilities.

Furthermore, in the presence of arbitrage opportunities in an incomplete market, the existence and uniqueness of a risk-neutral probability measure is no longer guaranteed. The use of this probability to value savings liabilities in euros seems therefore inappropriate and the valuation should therefore be carried out under historical probability.

The deflator approach seems relevant in this context. It allows for a better rationalization of economic valuations (notably agents' behaviour) and eliminates direct interactions between cash-flows' construction and prices' calculation. This separation between option valuation models and the creation of contract cash-flows' makes it possible to provide, for savings contracts, a framework similar to that used for variable annuity contracts (for option valuation) and the better quality of cash-flow structure makes the best *estimate* values more reliable.

In conclusion, the potential leads to build a valuation framework adapted to savings contracts in euros are the following:

- Use deflators to facilitate the justification of reaction functions in ALM models;
- Prefer models that naturally take into account the observed reality (negative rates) with an idea of parsimony and robustness guiding choices in a necessarily normative framework due to lack of directly observed prices;
- Ensure a form of consistency between the projection horizon used and the choices made in terms of calibration. Perhaps it is more consistent not to calibrate a model used to value flows over a 30 year period to the latest observed prices. However, it would be more appropriate to calibrate it to a set of price structures at different past dates (cf. LAÏDI and PLANCHET [2015] and EL KAROUI and al. [2017]);
- Reintegrate into the valuation process the idea that while exogenous financial risk is replicable, idiosyncratic financial risk is not, because of insurer's ability to influence the flows it serves (BONNIN and al. [2014]).

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7 Appendix: the context of savings contracts

On the French life insurance market, there are two main types of savings contracts: € contracts and unit-linked contracts (including *euro-croissance*).

At the end of 2018, savings contracts reserves represented €1,692 billion[31], or 70% of the French insurance market's investments (FFA [2019]). Savings contracts in euros represent €1,297 billion[32] or 54% of investments and unit-linked contracts represent €341 billion or 14% of investments.

Unit-linked contracts are contracts in which savings are invested directly in financial instruments and do not offer policyholders guarantees on the capital invested. They may, however, be covered by floor guarantees to limit losses (so called *guaranties plancher*).

Euro-croissance funds include a capital guarantee (total or partial) which is acquired by the policyholder only after a minimum holding period. This minimum holding period is set by the Insurance Code at 8 years starting from the first payment. Reserves remain insignificant compared to contracts in € and unit-linked policies.

Savings contracts in € offer a capitalization of the investment and the possibility to redeem the contract at any time (C. ass., Article R-132-5-3). Premiums collected by insurers are invested at their discretion. For the policyholder, the loss of capital can only occur in the event of bankruptcy of the insurer. In this case, the fund "*fonds de garantie des assurances de personnes (FGAP)*" may be seized. The loss is covered up to 70 K€.

The savings contracts' gains are subject to a social tax when they are recorded or when the contract is redeemed. The overall tax rate applied to income paid in 2019 is 17.2%. When the policyholder makes a partial or full lapse of their savings, the gains become taxable with income tax. The Table 10 presents a summary of the taxation structure of savings contracts in case of lapse[33] .

Table 10: Taxation structure of savings contracts

Age of the contract	Taxation for payments/ subscriptions before September 27, 2017	Taxation for payments/ subscriptions after September 27, 2017
Between 0 and 4 years	A choice : - Deduction of 35%+Social contributions - Income tax: inclusion in taxable income	12.8% flat-rate deductions +Social contributions
Between 4 and 8 years	A choice : - Deduction of 15%+Social contributions - Income tax: inclusion in taxable income	12.8% flat-rate deductions +Social contributions
After 8 years	A choice : - Deduction of 15%+Social security contributions - Income tax: inclusion in taxable income	For payments less than or equal to € 150,000: 7.5% deductions +social contributions (CSG and CRDS) For payments over € 150,000: 12.8% on capital gains +social contributions The annual allowance of € 4,600 for a single person or € 9,200 for a couple is applicable to all contracts. It is applied in priority to the part taxed at 7.5%

In addition, the French Insurance Code provides for a maximum period of two months for the availability of funds following a redemption request. In the event of death, the insurer must pay out the capital within one month. These deadlines can be reduced contractually in the general conditions.

[31] Including €54bn of profit sharing reserve.

[32] Excluding profit sharing reserve.

[33] For more details, the reader can refer to: https://www.impots.gouv.fr/portail/particulier/lassurance-vie-et-le-pea-0.

In this appendix we focus on classic € savings contracts.

7.1 The revaluation regulatory framework

The technical interests make up a minimum contractual revaluation of savings (C. ass., Article A-132-1). This revaluation is incremented with an additional payment: profit sharing. This represents the balance of the technical-financial profit and loss account after taking into account technical interests. It does not give any individual right to the policyholder and its distribution is at the discretion of the insurer.

The savings contracts in euros revaluation is composed of

- a minimum revaluation: technical rate or minimum guaranteed rate if applicable;
- and a discretionary revaluation through profit-sharing.

7.1.1 Technical rates

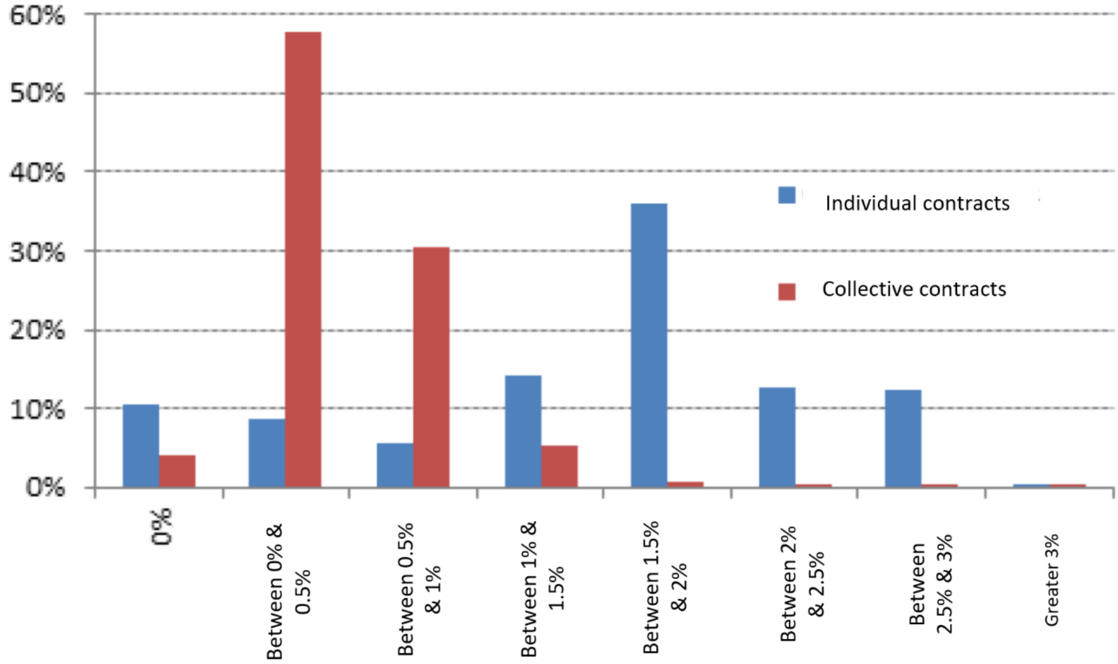
Article A132-1 of the French Insurance Code (applicable, in its current substance, since 1995) sets the maximum rate that an insurer can guarantee over the duration of the savings contract in euros on the subscription date. It stipulates that this maximum technical rate is set, depending on the duration of the commitment, at 60% or 75% of the average rates for French government bonds (*TME* for "*Taux Moyens des empreints d'État*") calculated on a half-yearly basis.

On average, technical rates were 0.51% for all life insurance contracts in 2015. However, this average covers very different situations. Indeed, the average of technical rates on individual contracts, which represent most of the technical reserves for contracts in euros (92%), is lower (0.42%) than that of collective contracts (*Madelin* type pension plan, articles 39, 82, 83 of the General Tax Code, popular retirement savings plan, additional professional pension plan) that is 1.5%[34].

Fig. 12: <u>Breakdown of entities by average technical rate in 2015 (by weight of mathematical reserves)</u> [35]

[34] See ACPR [2017].

[35] Source ACPR [2017].



In the context of negative rates currently observed on the market, the technical rate option cost is an important issue. The question of guaranteeing a negative technical rate (net of loadings) for new subscriptions naturally arises.

No interpretation of Article A132-1 can lead to exclude the guarantee of a negative technical interest rate for new subscriptions. However, such a guarantee would not be compatible with the objective of caution, implicit in this article, i.e. to guarantee, mostly a rate lower (60% or 75%) than the TME calculated on a half-yearly basis because, in a situation of negative rates, this maximum would be higher.

Also, Article A132-1-1 of French Insurance Code (applicable in its current substance since 1998) stipulates that the maximum technical interest rate applicable to contract-prices is set on a scale with 0 as the origin and 0.25 point steps, without going below 0. However, it does not prohibit the use of a rate lower than the maximum allowed.

7.1.2 Guaranteed minimum rates

Beyond the technical rate, the ability of insurers to take on additional commitments in the form of a minimum guaranteed rate (TMG in French for "*Taux Minimum Garanti*") is limited to 2 years and is in practice marginal among the main insurers (ACPR [2017]).

Article A132-2 of the Insurance Codes provides the possibility for an insurer to annually guarantee rates higher than the technical interest. Therefore the TMG consists of the technical interest rate and a part, capped by regulation, of the profit sharing.

Article A132-3 of the French Insurance Code sets out the methods for calculating these TMGs. It provides that:

- It is possible to annually guarantee a profit-sharing limited to a maximum equal to the difference, when positive, between 80% of the average accounting income on assets over two years and the sum of the technical interest attributed to the contracts;

- the TMG cannot exceed the minimum between (1) 150% of the maximum technical rate at 75% of the TMG calculated on a semi-annual basis and (2) the maximum between 120% of the maximum technical rate and 110% of the average served revaluation rates.

If the TMEs calculated on a semi-annual basis are negative, Article A. 132-3 therefore does not allow to guarantee positive annual TMGs.

7.1.3 Profit sharing

The minimum profit-sharing amount is the credit balance of the profit-sharing account which includes[36] :

- 90% of the underwriting technical result if it is a credit and 100% if it is a debit;
- 85% of the portion of financial income allocated to technical reserves;
- The profit-sharing account's debit balance for the previous year.

The Insurance Code therefore does not allow the possibility of recording a negative profit sharing regardless of the state of the economy.

Furthermore, profit-sharing does not give the policyholder any individual rights. It is either distributed immediately or allocated to the profit sharing reserve, which must be distributed within eight years of its allocation to the fund.

Policyholders therefore have two acquired reserves:

- Mathematical reserves which are determined individually and correspond to the savings acquired;
- The profit-sharing reserve, which is global and whose redistribution is at the discretion of the insurer.

The profit-sharing allows smooth revaluation in space (between the different contracts) and over time, and thus to drive the activity according to commercial constraints and financial market conditions. In addition to profit-sharing, other reserves are made by the insurer implying a smoothing of the accounting performance of the asset over time. These include the capitalization reserve (*reserve de capitalisation*), the financial contingencies reserve (*provision pour aléas financiers*) and the exigibility reserve (*provision pour risque d'exigibilité*).

Finally, the insurer also has unrealized wealth (difference between the market value and book value of the asset), which gives more flexibility in managing the revaluation of savings.

In this section we have examined the regulatory framework for the revaluation of savings contracts in €. The articles of the Insurance Code framing this revaluation will probably be

[36] See Articles A331-3 & seq. of the Insurance Code and Instruction No. 2016-I-15 of the ACPR.

amended to reflect the structural decline in interest rates. The following section presents a parallel between revaluation mechanisms and classic financial options.

7.2 Savings contract options and risk factors

The options included in classic € savings contracts can be summarized in three categories:

- Financial options: the insurer commits to a minimum return on savings by guaranteeing a minimum rate of revaluation or a guaranteed profit sharing rate.
- Behavioural options: the insurer offers options of redemption, euro - Unit-Linked arbitrage, free or scheduled payments, loyalty bonus... The activation of these options is at the discretion of the policy-holder.
- Biometric options: are options depending on the risk of mortality (or longevity) such as the insurer's proposal for deferred annuities.

The policyholder therefore benefits from three financial options (see BRYS and DE VARENNE [1994]):

- The option of technical rate or guaranteed profit-sharing rate, similar to a European vanilla option;
- The ratchet option, similar to an American put option;
- The forward rate option on free or scheduled contributions, similar to a *swaption*.

Thus, the risk factors for which classic € savings contracts liabilities are exposed are (see ARMEL and PLANCHET [2019] and LAURENT and al. [2016]):

- Biometric and structural lapse risks that are non-replicable. They are nevertheless mutualizable;
- Risks related to the cyclical (or dynamic) behaviour of policyholders that are non-replicable (see sections 2.4and 3). These behaviours represent policyholders' reaction to economic and financial contexts and to revaluation rates (and therefore to the insurer's decisions). They may be reflected in surrenders, arbitrages or contributions;
- Financial market risks and in particular interest rate risk, which can be partly replicable.

The insurer's reaction facing the rates of return on assets and the expectations of policyholders' behaviour makes up management actions, based on an accounting rate. This results in the served revaluation rate (see section 2.3 for a standard algorithm presentation to calculate the served revaluation rate practised by the market).