DYNAMIC MODEL FOR STOCK MARKET RISK EVALUATION

by

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ABSTRACT. A logistic mixture model of stock price dynamics is derived in which the process behaves like a random walk in one regime while it is like an error-correcting process in the other. The model has a useful interpretation - regime one produces bubble periods and regime two periods which bring prices back to fundamentals whose indicators are valuation ratios. The probability of a regime switch depends on exogenous inflation. The proposed model is a variant of the so called dynamic Gordon growth model.

When used as a stock market evaluation tool in the DFA the model has a desirable feature. It is mean reverting but has only a weak prediction power in the short term. An important property of the model is its good ability to produce shape-changing predictive distributions. These distributions carry information that is essential for the risk management. The model is applied to the U.S. stock market data. Some simulation experiments are conducted, with the result that risk depends heavily on the holding period, price-dividend ratio and inflation.

KEYWORDS. Value-at-Risk, Dividend-Price Ratio, LMARX Model, Time Varying Risk, Changing Return Distributions, Dynamic Gordon Model.

1. Introduction

The standard efficient market model with a constant discount factor claims that stock returns are unforecastable and Random Walk is the appropriate model. However, there is compelling statistical evidence that stock price volatility in comparison with the volatility of dividends is too large with

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respect to the standard efficient market model (Shiller (1981), West (1988)). This implies that in the long run the Random Walk model for the stock returns leads to too broad prediction interval, given that the variability of the stock price reverts to the variability of dividends over some time horizon. This phenomena has been exploited e.g. in the *Wilkie model* (Wilkie (1995)), where the relationship between stock price and dividend is essential. The so called *Gordon growth model* (Campbell (1997)) constitutes a theoretical background for the approach. According to the Gordon model, the stock price at time point t is determined by formula

$$P_t = (1 + G)D_t \, / (R - G)$$

where D_t is dividend, R is discount factor and G is expected growth rate of dividends. Gordon's growth model is based on the assumption that the expected growth rate of dividends G and the discount factor R are constant over time. A consequence of the Gordon growth model is that the dividend-price ratio D_t/P_t is constant. It is well-known fact that this assumption doesn't hold even approximately because the dividend-price ratio varies very much over time.

John Campbell and Robert Shiller have introduced so called *dynamic Gordon growth model* (Campbell (1997)), which allows that expected stock returns and expected growth rate of dividends are time-varying. This model is based on the loglinear approximation of dividend-price ratio. When lowercase letters p_t and d_t denote logs of uppercase letters P_t and D_t and Δ denotes difference, the dividend-price ratio can be expressed in the form

(1) $d_t - p_t = c + E_t \Sigma_{j=0}^{\infty} \rho^j [-\Delta d_{t+1+j} + r_{t+1+j}],$

where r_t is log return, c is a constant and

$\rho = 1/(1 + \exp(E(d - p))).$

The formula (1) presumes that

$P(\lim_{j \to \infty} \rho^j p_{t+j} = 0) = 1.$

On the basis of Formula 1 the log dividend-price ratio is stationary if the log difference of dividend Δd_t and return r_t are stationary. In other words

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log dividend and log price are cointegrated under the above conditions. This relationship is very important in the long horizon modelling. It means that

(2) $\quad \lim_{j\to\infty}[Var_t(p_{t+j} - p_t)/Var_t(d_{t+j} - d_t)] = 1.$

Hence long-term variability of the stock price reverts to the variability of the dividend. On the basis of formula (2) a low log dividend-price ratio means that in the future either log dividends grow fast or returns are low. The simple form of the efficient market theory based on the constant discount factor claims that returns in the future are unforecastable. According to the empirical study of John Campbell and Robert Shiller (Campbell (1997)), the log dividend-price ratio has only a weak power to forecast the growth of dividends over either a one or ten-year horizon. On the other hand, the log dividend-price rate has no power to forecast one year log price growth, but there is a significant positive relation between the dividend-price ratio and subsequent ten-year price growth.

Our objective is to contribute to a better evaluation of stock market risks by introducing a regime switching model in the dynamic Gordon model context. The proposed model is intuitively appealing, having a "bubble" regime and a "mean reversion" regime. Mathematically it is a special case of the recently proposed logistic mixture autoregressive model (LMARX) (Wong and Li (2001)), which allows exogenous variables and time-varying predictive distributions. We analyze and test the model with the U.S. stock market data. Then we apply the model to study risk characteristics of the stock market in different holding periods and initial values. Albrecht et al (2001) have analytically quantified the long-term risks of a stock investment in the traditional Random Walk context. This paper gives a different picture of some aspects of risks, because of the unequal underlying assumptions.

2. Nonlinear variant of the dynamic Gordon model

The historical stock market bubbles, which find extensive description in Burton Malkiel's popular best-seller *Random Walk down Wall Street* (Malkiel (1999)), show that in some time periods the behaviour of stock prices cannot be reduced to market fundamentals.

Based on this phenomenon and the dynamic Gordon model, we have ended up with a regime switching model, where in regime 1 the changes

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of stock price are independent of dividends and in regime 2 the stock price change depends on the dividend-price ratio. Following the principle of parsimony, we assume that in regime 1 the log stock price follows standard Random Walk

$$(3) \quad \Delta p_t = a_1 + \varepsilon_t^{(1)}$$
$$\varepsilon_t^{(1)} \sim N(0, \sigma_1^2).$$

and in regime 2 the change of the log stock price depends on (the negative of) the log dividend-price ratio y_t ($= -\log(D_t/P_t) = p_t - d_t$) through simple linear regression

$$(4) \quad \Delta p_t = a_2 - by_{t-1} + \varepsilon_t^{(2)}$$
$$\varepsilon_t^{(2)} \sim N(0, \sigma_2^2).$$

The model can be equivalently expressed in the form

$$y_t = (1 - z_t)(a_1 + y_{t-1} + \varepsilon_t^{(1)}) + z_t(a_2 + (1 - b)y_{t-1} + \varepsilon_t^{(2)}) - \Delta d_t,$$

where z_t is an unobservable indicator function, which is one in regime 2 and zero otherwise.

When seeking for the regime switching mechanism we build on the following observations. Franco Modigliani and Richard Cohn have advanced a hypothesis that people suffer from so called "inflation illusion" (Modigliani (1979)). According to this hypothesis, market participants discount real dividends by a nominal interest rate, which has a strong dependence on inflation. Secondly, Robert Shiller has studied by a survey method public attitudes towards inflation (Shiller (1996)). He concluded that people believe that some badly-behaving or greedy people cause prices to increase, increases that are not met with wage increases. Shiller called this the bad-actor-sticky-wage model. In other words, people misunderstand the relationship between nominal quantities and inflation. These studies imply that inflation is negatively related to the log dividend-price ratio. On the other hand, inflation is positively related to the nominal dividend growth.

The above findings indicate that inflation would be a suitable explanatory variable for regime switch. It is desirable that the regime switching is difficult to predict. Hence, we confine ourselves to dealing only with stochastic regime-shifting processes. A final requirement for the model is that there

exists in practise working estimation method. Hence we assume that the probability of regime 1 is given by

$$(5) \quad 0 \leq \alpha_t = f(i_t, i_{t-1}, ..., i_{t-n}) \leq 1$$

where f is a function of the past log inflation values $i_t, i_{t-1}, ..., i_{t-n}$. Now the model defined by formulas $(3-5)$ is a special case of logistic mixture autoregressive model with an exogenous variable (LMARX model) proposed by Wong and Li (Wong and Li (2001)). It can be estimated directly via the log-likelihood function, or estimation can be carried out by the EM algorithm. Van Norden and Schaller (1999) have also modelled market crashes by regime-switching model. The structure of their model differs from our model in many aspects.

When assessing investment risk it is important to take into account that stock returns are not normally distributed (see e.g. Embrechts et al (2001)). The return distribution has fat tails and it is asymmetric. The general practice in actuarial modelling is to use a non-normal probability distribution. Here we have chosen another approach. We think that fat tails and asymmetry of an empirical probability distribution are indication of nonlinearity in the data generating process defined by $(3-5)$. This approach includes the assumption that the risk of the share investment is not constant over time.

3. Data

We apply the model defined by formulas $(3-5)$ to the U.S. quarterly stock data until 1995. The data which we use is from Standard & Poor and it is available on the home page of Robert Shiller. The stock price and dividend indices are value weighting indices of the 500 largest companies of the USA. The inflation rate is calculated from the Consumer Price Index of the USA. Next we explain why we don't include observations after 1995 in the estimation period.

The previous study includes the assumption that the log dividend-price ratio is stationary in the long run. But is it a realistic assumption? John Carlson, Eduard Peltz and Mark Wohar (2001) have tested by the Bai-Perron test for structural change in the mean of the dividend-price ratio using the Standard & Poor 500 index. Using annual data from 1872 to 2001 they found two structural breaks in 1955 and in 1982. Using quarterly data from 1945 to 2001 they found one breakpoint in 1992. They concluded that the

change of the dividend-price ratio is permanent. One explanation of recent low dividend-price ratios is share repurchases. Share repurchases by S & P 500 companies have risen sharply in recent years (Liang (1999)). Share repurchases is a tax-favoured alternative to transfer cash to shareholders. Share repurchase benefits shareholders, because future dividend payment will be dividend among fewer shares. On the other hand, companies issue shares for the satisfaction of employee stock option exercises. Issuing of shares has naturally an inverse effect on the share price than repurchases.

Nellie Liang and Steven Sharpe (1999) have studied the effect of share repurchases and emission. Their sample includes the 144 largest firm of the S & P 500 index. Their estimates of net repurchases as percentile of the market value from 1994 to 1998 are represented in table 1. The Long-Term average is estimated from the period $1946 - 1998$.

Year	1994	1995	1996	1997	1998	Long-Term
Net Repurchases	1.19	1.34	1.56	1.98	1.49	0.57

Table 1. Net repurchases (%) of shares after a structural break and long-term average.

The stock market data period includes several divergent observations. The three most divergent values are 2/1962, 3/1974 and 4/1987. The divergent observation of 1962 is related to the so called *tronic boom* in $1959 - 1962$ (Malkiel (1999)). At that time people invested in companies, whose name sounded as electronics, even if the company had nothing to do with the electronics industry. A consequence of this bubble was a sharp decline in stock prices in the second quarter of 1962. The second divergent observation in 1974 is related to the oil crises. The divergent observation in 1987 is a consequence of the market crash in October 1987.

4.   Estimation

Dividends

Dividend is the leading factor of our model. Dividends are calculated in each quarter from dividends which are distributed during the previous 12

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months. We model the log difference of dividends by the AR-ARCH model, which can be represented in the form

$$y_t = c + \Sigma_{i=1}^p \alpha_i y_{t-i} + \varepsilon_t$$
$$\varepsilon_t | I_{t-1} \sim N(0, h_t)$$
$$h_t = k + \Sigma_{i=1}^h \phi_i \varepsilon_{t-i}^2$$
$$Cov(\varepsilon_t, \varepsilon_i) = 0 \text{ for all } i, j.$$

Figure 1 shows that there seems to be a structural break in the variance of the dividend series in the turn of the $1950s$ and $1960s$ (see Appendix). Hence we limited the estimation period from 1959 up to 1994. The resulting process is AR(2)-ARCH(4)

(6) $\Delta d_t = 0.0043 + 0.374\Delta d_{t-1} + 0.315\Delta d_{t-2} + \varepsilon_t$
$\varepsilon_t \sim N(0, 0.000038 + 0.9\varepsilon_{t-4}^2)$

whose mean is $E(\Delta d_t) = 0.0138$. First we e tested the serial correlation of residuals e_t by the Ljung-Box Q-test (Box (1994)). We started by testing in one lag, second including the first four lags, third including the first ten lags, fourth including the first twenty lags. In all cases, serial correlation was insignificant at the 5% significance level.

Secondly, we tested the normality of standardized residuals $z_t = e_t / \sqrt{h_t}$ by the Jarque-Bera test (Spanos (1993)). The null hypothesis of normality was rejected at the 5% significance level. The skewness of standardized residuals was 0.24 and the kurtosis was 6.09.

Inflation

We model the log inflation by the AR(4)-model using the log difference of dividend of the previous quarter as explanatory variable. The resulting process is

(7) $i_t = 0.307i_{t-1} + 0.327i_{t-3} + 0.170i_{t-4} + 0.139\Delta d_{t-1} + \varepsilon_t$
$\varepsilon_t \sim N(0, 0.000031).$

Using the same tests as for dividends we concluded that the serial correlations of residuals are insignificant at the 5% level. Normality of standardized residuals was not rejected at the same level.

Linear model for the dividend-price ratio

For the sake of comparison we model the log dividend-price ratio by ARMAX$(1,0)$ with explanatory variable log inflation. By the conditional maximum likelihood technique (we assume that the first observation is fixed) we got the following model for the log dividend-price ratio

(8) $\quad y_t = 0.215 + 0.944 y_{t-1} - 1.545 i_t - \Delta d_t + \varepsilon_t,$
$\qquad \varepsilon_t \sim N(0, 0.068^2).$

This model can be expressed more illustratively in the error-correction form

$$\Delta p_t = 0.215 - (1 - 0.944)(p_{t-1} - d_{t-1}) - 1.545 i_t + \varepsilon i_t$$

$$= 0.215 - 0.066(p_{t-1} - d_{t-1}) - 1.545 i_t + \varepsilon_t.$$

The serial correlation of residuals is insignificant at the 5% level, excluding lag 20. Then the p-value is 0.0281. We tested the conditional heteroskedasticity of residuals by Lagrange's multiplier test. It implies that the conditional heteroskedasticity of residuals is insignificant at 5% level. The normality of residuals rejected clearly. The skewness of residuals is -0.916 and the kurtosis is 5.690.

Nonlinear dynamic Gordon model

Finally we study the LMARX model defined by formulas $(3-5)$. We estimate it directly via the log-likelihood function using standard numerical estimation techniques. The resulting parameter values of the model are

Regime 1:
(9) $\quad \Delta p_t = 0.027 + \varepsilon_t^{(1)}$
$\qquad \varepsilon_t^{(1)} \sim N(0, 0.052^2)$

Regime 2:
(10) $\quad \Delta p_t = 1.078 - 0.357 y_{t-1} + \varepsilon_t^{(2)}$
$\qquad \varepsilon_t^{(2)} \sim N(0, 0.077^2).$

When inflation is denoted by i_t the probability of regime 2 is

$$(11) \quad \alpha_t = \Phi(-2.120 + 333.440(i_t + i_{t-1} + i_{t-2} + i_{t-3})^2).$$

Regime probabilities are illustrated in figure 2. It shows that the probability for the process being in regime 2 is much larger during high inflation periods.

5. Testing and diagnostics

Comparison with the linear model

Unfortunately there does not exist any straightforward way to test the null hypothesis of a linear model against two regime alternatives. The reason for this is that when the null hypothesis holds and parameters are the same in both regimes nuisance parameters c_1 and c_2 are not identified. In other words, under the null hypothesis, values of the likelihood function are independent of these parameters. The consequence of this is that standard asymptotic properties of a likelihood-ratio test do not hold.

We compared the linear model and the regime-switching model by model selection criterion AIC and BIC (Box et al (1994)). Akaiken's information criterion AIC is defined by

$$\text{AIC}(\beta) = -2\log(L(y;\beta)) + 2p,$$

where $L(y;\beta)$ is the likelihood function of observation vector y in respect to parameter β and p is the length of parameter vector β. The Bayesian (also called Schwarz-Rissanen) information criterion BIC is defined by

$$\text{BIC}(\beta) = -2\log(L(y;\beta)) + p\log(n),$$

where n is the number of observations. This model selection criterion is a conservative criterion in the sense that it punishes more the complexity of the model than AIC in large samples. Table 2 shows that the regime-switching model is clearly better than the linear model by both criteria.

Model	AIC	BIC
Linear	-844.4742	-831.4028
Regime-switching	-875.7054	-852.8304

Table 2. The values of AIC and BIC for the linear and nonlinear model.

Testing against more general alternative

We tested the LMARX model $(3-5)$ by a likelihood ratio test against an alternative:

Regime 1:
$$\Delta p_t = a_1 + b_1 y_{t-1} + \varepsilon_t^{(1)}$$
$$\varepsilon_t^{(1)} \sim N(0, \sigma_1^2).$$

Regime 2:
$$\Delta p_t = a_2 + b_2 y_{t-1} + \varepsilon_t^{(2)}$$
$$\varepsilon_t^{(2)} \sim N(0, \sigma_2^2).$$

The null hypothesis of the simpler model cannot be rejected at the 5% level.

Model diagnostics

For diagnostics we studied quantiles v_t of the conditional distribution. If the nonlinear model $(9-11)$ is true, v_t are independent and approximately standard uniform. In the case of the linear model (8) the most divergent observation is the market crash in 1987, whose studentized value is -4.24. If the linear model (8) is true, least so severe observation occurs on an average every 22 800 year. In the case of the regime-switching model the lowest quantile is 0.0052 in 2/1962. Hence, if the true data generating process is the nonlinear model, S&P 500 collapses so dramatically once in every 48 years. The market crash in 1987 has the quantile value 0.0122 and at least so severe observation occurs on an average every 20 year. These calculations

show that linear regime-switching model is much better in modelling the short-term risk of share investment than the linear model or an estimation period including extremely unusual observations.

Quantile residuals (Dunn and Smyth 1996) u_t are based on the fact that the inverse normal distribution transformations of standard uniform variables

$$u_t = \Phi^{-1}(v_t)$$

are standard normal variables. Figure 3 demonstrates how well this assumption holds. Because many regression diagnostic tests are based on the assumption of normality, it is reasonable to use quantile residuals in diagnostic study. We tested the serial autocorrelation and the conditional heteroskedasticity of quantile residuals in the same way as for the linear model. The conclusion of these tests is also the same as in the case of the linear model: there is no clear evidence of serial correlation or conditional heteroskedasticity.

6. Interpretation of the parameters

The nonlinear model $(9-11)$ operates much more often in regime 1 than in regime 2. This means that most of the time the stock price does not react to the information on dividends. This is coherent with the short run unforecastability of the stock price. A regime switch from regime 1 to regime 2 can cause a jump in the stock price. The sign and magnitude of the jump is determined by the log dividend-price ratio of the previous quarter y_{t-1}. Formula (10) implies that if $y_{t-1} < 3.02$, a jump is more likely positive than negative and vice versa. For parameter a_1 in regime 1 it holds that $a_1 = 0.027 > E(\Delta d_t) = 0.014$. Hence, the stock price grows faster than the dividend most of the time. It means that the jump of the process after a switch from regime 1 to regime 2 is more likely negative (causing market crash or bubble burst) than positive.

The stationary regime 2 has a crucial role in our model. It keeps the dividend-price ratio stable in the long run. A long run simulation presented in figure 5 demonstrates how it works. The dividend-price ratio may reach very high values, but after some time period it reverts back to its normal level.

The relationship between inflation and stock price in the nonlinear model $(9-11)$ is totally different from that in the linear model (8). In both models inflation is statistically strongly significant. In a low-inflation period the probability of a regime switch is approximately constant. The inflation raises the regime-switching probability substantially only when it is clearly higher than average. In a very high-inflation period the probability of regime 2 is almost 1. In other words, only a large change in inflation has a clear effect on the stock price.

A clear difference between the regime-switching model and the linear model is how the stock price behaves under a high-deflation (negative inflation) period. The used data does not include any high deflation period. According to the linear model, the dividend-price ratio is high during a deflation period. Under the regime-switching model this relationship is negative, because the probability of regime 2 depends on the square of inflation. It is well known that deflation is often related to general economic depression. It is unreasonable to assume that the stock price is overvalued under that kind of circumstances. It is an interesting further question to check how well the nonlinear model fits the data that includes high-deflation periods (e.g. in Japan in the 1990s).

7. Risk evaluation

In this section we make some observations regarding the long-term risk in stock investment and in particular the risk imposed by changing predictive distributions. To evaluate the risk we conduct simulation experiments by the proposed nonlinear model $(9-11)$. Here we consider merely real returns including dividends, which are reinvested into the index at the end of each year. The factors whose effect we study are holding period, inflation and dividend-price ratio. We make comparisons with the linear model (8).

Predictive distributions

We generated 10^3 observations from the nonlinear model with varying holding periods and initial values. The resulting simulated predictive distributions are given in figures 6 - 10. Figures 6 and 7 illustrate how the one-year predictive distribution transforms dramatically when the initial dividend-price ratio is changed from a high 3.8 level to a normal 3.2 level. The initial inflation is 4 percent in both cases.

The shape of the predictive distribution changes over time. This is shown in figures $8 - 10$ where forecasting horizon varies from one-year to ten-year. The initial value of the inflation is 4 percent and dividend-price ratio 3.8. The one-year predictive distribution is negatively skewed, the five-year distribution is rather symmetric, but ten-year predictive distribution is positively skewed. These observations are important since a negatively skewed distribution imposes much higher risk than a symmetric one and the other way around.

Albrecht et al (2001) analyze the probabilistic characteristics of stock investment risk. They model log returns by independent normal distribution (geometric Brownian motion model). Hence they assume, contrary to us, that the risk depends only on the forecasting (holding) period. Further, If the log returns are independent and normally distributed then the scaling property says that standard deviation should be proportional to the square of elapsed time. However, empirical studies suggest that standard deviation increases less than the scaling property claims (see e.g. Campbell et al (1997)). The mean and standard deviation of the one year returns are 0.066 and 0.162 in the data period. We compare a normal distribution model with these parameters to the nonlinear model. When forecasting horizon is extended to 30-year the standard deviation of the normal distribution model increases to $0.162\sqrt{30} = 0.89$. But the standard deviation of the nonlinear model reaches a lower level 0.54 because of the mean reversion. Hence, at least in the case of a moderate dividend-price ratio our approach indicates lower variation in the long horizon than Albrecht et al.

Value-at-Risk

Here we make use of Value-at-Risk (VaR), which is a widespread risk measure in financial risk management. But it is important to emphasize that this measure has severe limitations in the case of changing distributions. If the profit-loss distribution of X be F(x) then at 1 percent level Value-at-Risk is defined as

$$VaR_{0.01}(X) = \inf\{x \mid F(x) \geq 0.01\}.$$

In order to assess the impact of the distributional characteristics on the risk we compare the proposed nonlinear model with the linear model. The simulated value-at-risk figures for the stock indices are shown in tables 3 and 4. The simulation results can be summarized as follows. The risk depends to

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a considerable extent on the present dividend-price ratio and the risk persists over a very long period. High inflation is an important risk factor, but it plays a much lesser role in the long horizon. The outcomes of the nonlinear and linear model are basically very similar but nonlinear model holds clearly higher extreme risk in one-year and five-year horizon. Luckily, simultaneously high inflation and dividend-price ratio is not a likely occurrence in the real economy since it would be the most dangerous combination.

Years	1	1	1	5	5	5
$(\mathbf{p-d})/\mathbf{i}$	0	0.04	0.1	0	0.04	0.1
2.8	93/96	91/89	93/80	125/133	118/115	109/92
3.2	89/87	82/81	69/72	92/96	83/82	75/66
3.6	79/79	68/73	50/65	63/68	57/60	49/47
4	68/72	54/72	36/59	43/51	38/44	33/35

Table 3.   Value-at-Risk (%) at 1 percent confidence level in one and five-year horizon.

Years	10	10	10	20	20	20
$(\mathbf{p-d})/\mathbf{i}$	0	0.04	0.1	0	0.04	0.1
2.8	173/166	148/153	128/130	236/236	216/222	187/203
3.2	107/112	94/101	86/85	145/145	134/143	126/123
3.6	68/73	63/65	57/56	98/95	83/91	81/80
4	43/50	41/44	38/37	58/63	54/58	55/53

Table 4.   Value-at-Risk (%) at 1 percent confidence level in ten and twenty-year horizon.

Conclusion

The proposed regime-switching model has the following attractive properties as a tool for dynamic financial analysis (DFA):

- Simple basic structure
- Good fit for the data
- Intuitively appealing

According to the proposed model the stock investment risk is time varying. It is much larger when the dividend-price ratio is high. The inflation works as an exogenous trigger variable. It is an interesting question whether the price-earnings ratio and an interest rate would work in this modelling context.

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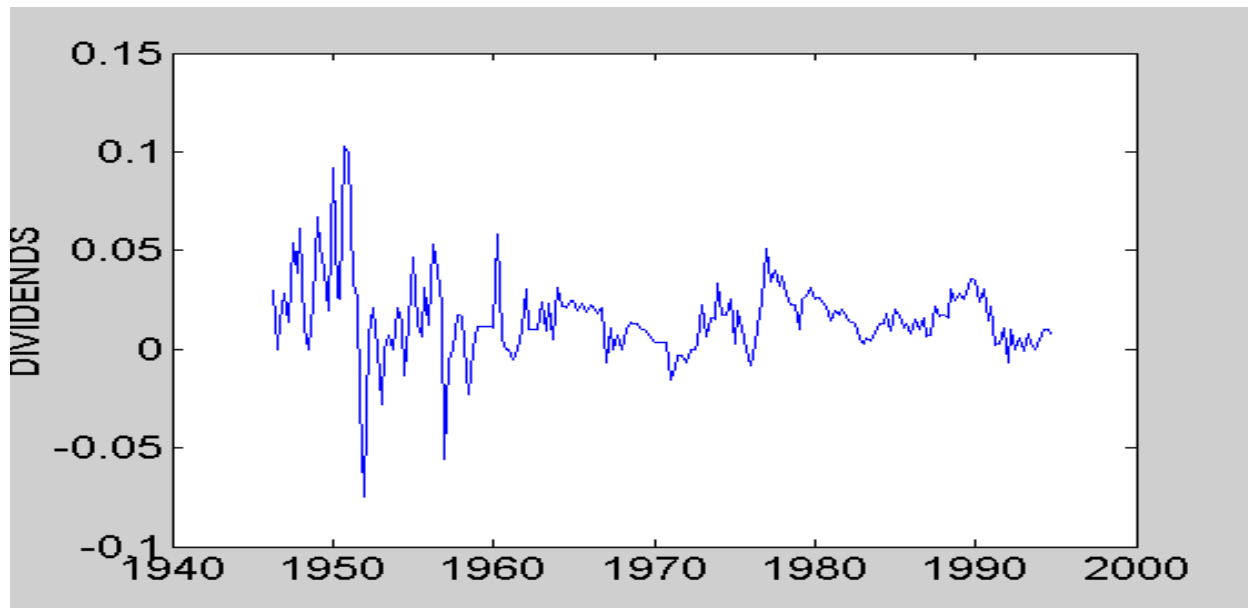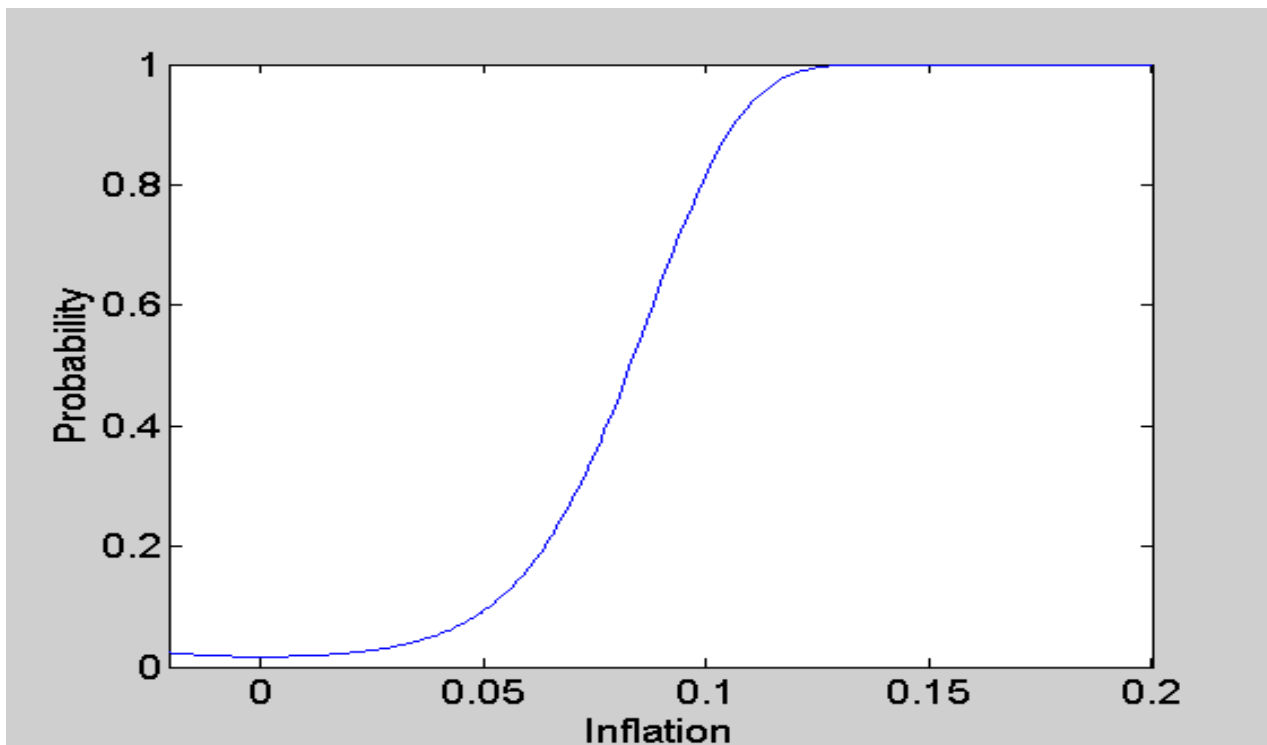**Fig. 1.** The log difference of dividends 1946-1994.

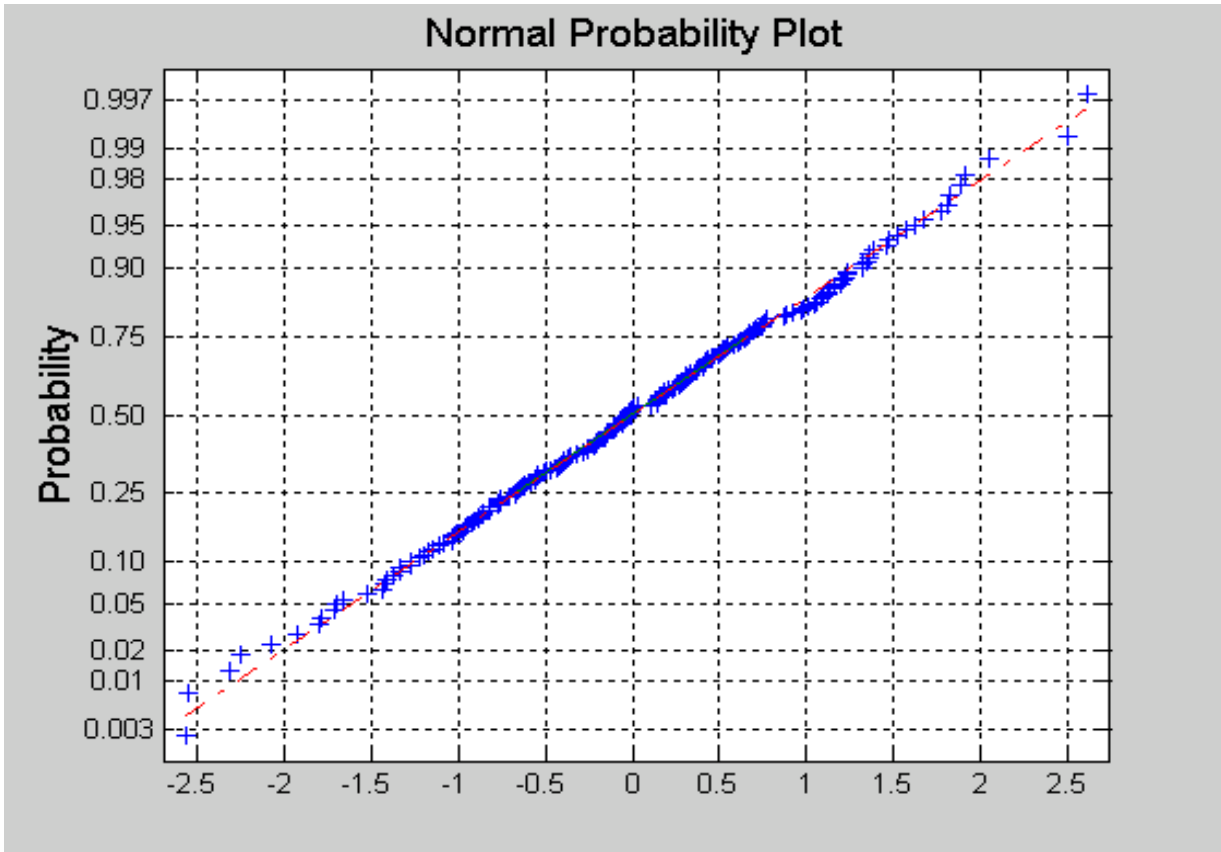**Fig. 2.** The probability of regime 2 as a function of inflation
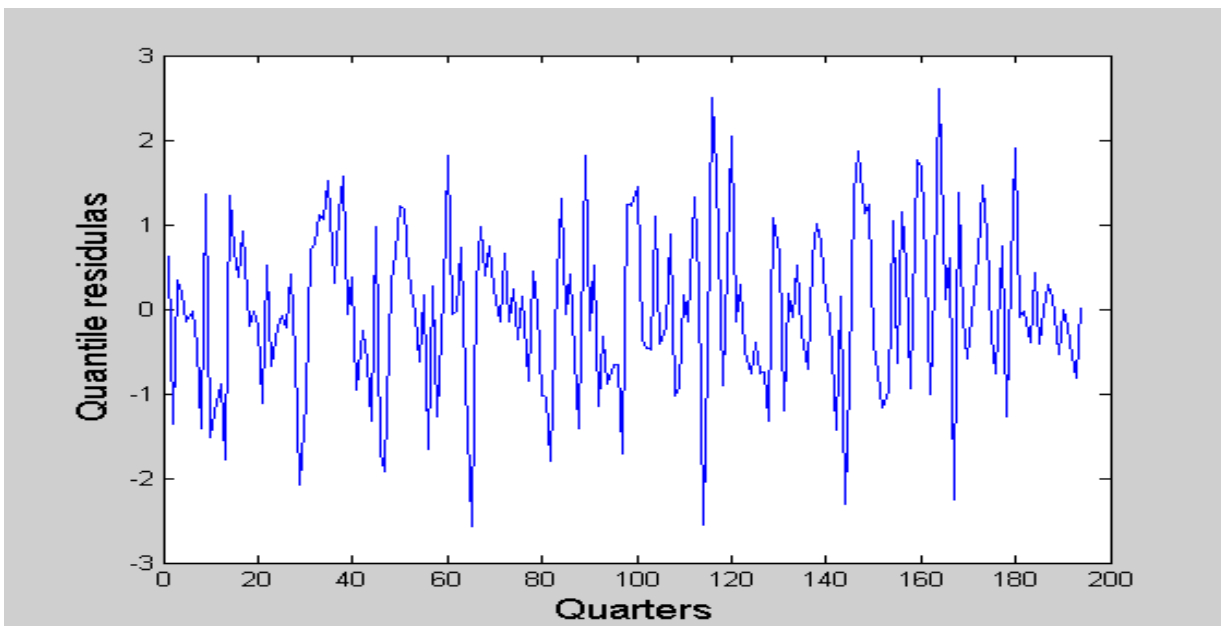
Fig. 3. QQ-plot for model (9-11) versus data.

Fig. 4. The quantile residuals.
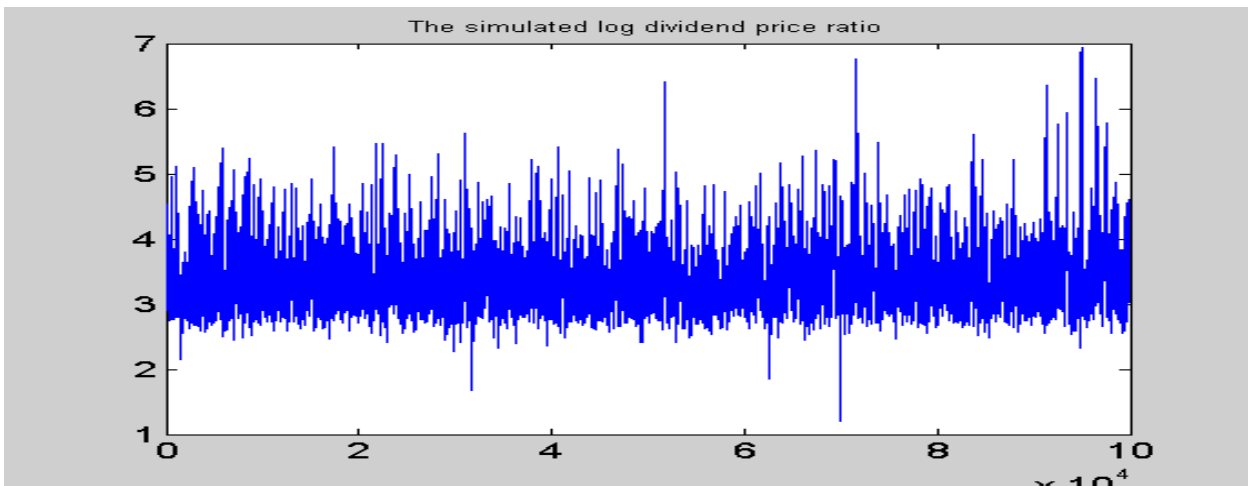
Fig. 5. Long-run simulations (200 quarters).
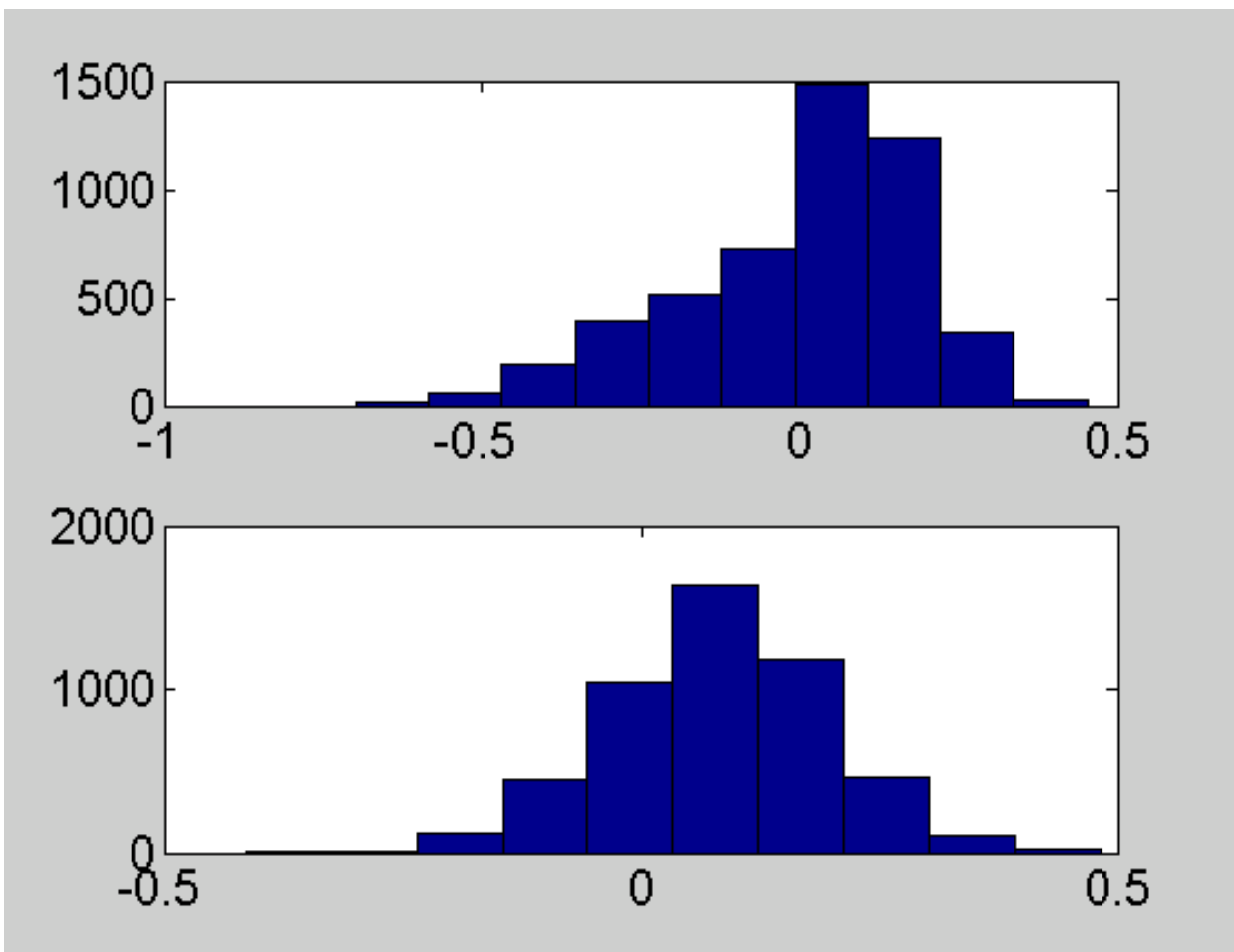
Fig. 6-7. The 1-year predictive return distribution. The initial dividend-price ratio 3.2 (above) and 3.8 (below).
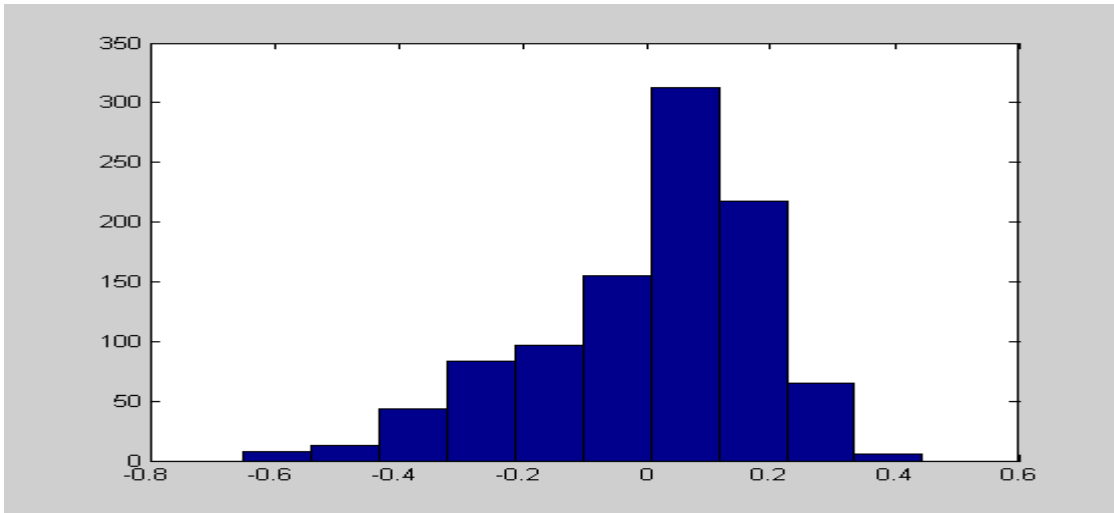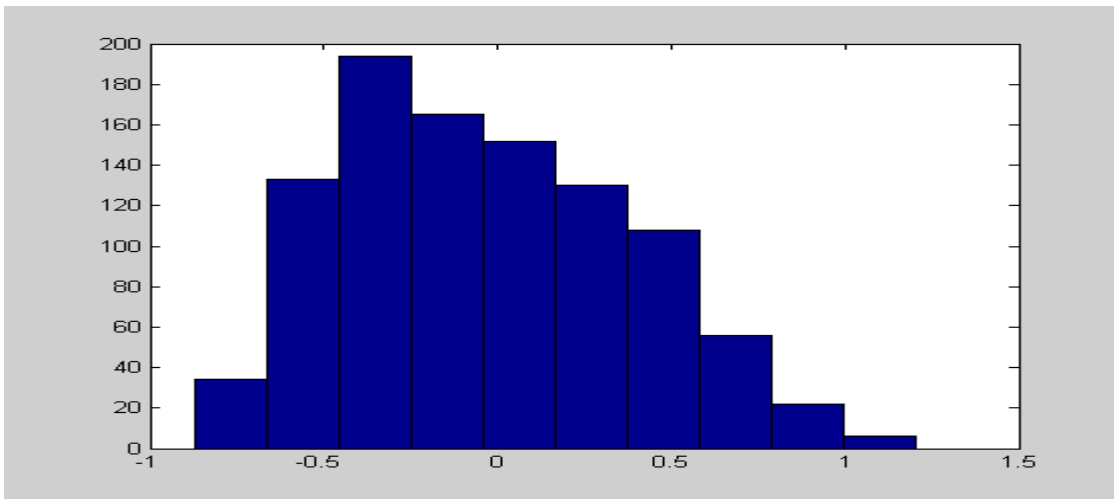
Fig. 8. One-year predictive return distribution.
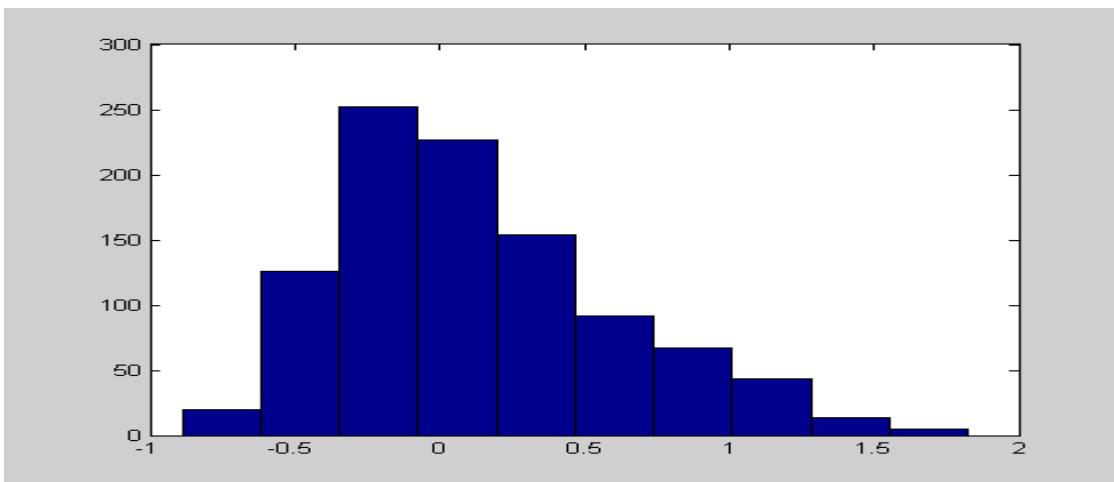

Fig. 9. Five-year predictive return distribution.

Fig. 10. Ten-year predictive return distribution.