A proposal of interest rate *dampener* for Solvency II Framework introducing a three factors mean reversion model

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Abstract

The standard approach used by Solvency II for interest rate risk assessment is a simplification of the RiskMetrics approach, designed in 1989 by JPMorgan. Based on this approach, we demonstrate that the Solvency II directive, in its current state, leads to a biased assessment of risk but especially to a pro cyclical effect caused by a negative correlation between SCR and Net Assets. We introduce a proposal of correction, so called interest rate *dampener*, where the stress is time dependent through mean reversion modeling. To illustrate the counter cyclical property of our method we apply it on two fictive life and non-life insurance undertakings and we show that our proposal allows a stabilization of the Solvency ratio without affecting the level of prudency.

Keyword : Solvency II, interest rate risk, Value at Risk.

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Introduction

The Solvency II Directive is an EU directive that both reforms and harmonizes the EU insurance regulation. Each Insurance undertaking has to hold sufficient capital to face the 0.5% probability worst case annual losses. This main objective leads to three sub-objectives. The first sub-objective is setting a process of VaR determination which, according to Christoffersen (1999), has at least the same importance as the results themselves. The second is quantifying and identifying each risk factor, and thus introducing quantitative analysis in management. This is an historical practice in the banking industry but a significant change for insurance companies, especially for financial risk (see Ramosaj(2010)). The last, named Own Risk and Solvency Assessment (ORSA) is to integrate the risk measure in the decision process. Indeed, with the obligation of making provision for risk, thinking about capitalistic intensity for each business and for each investment portfolio becomes a key feature for growth (see Capozza and Li (1996)). Considering that risk has to be reserved (concept of risk margin, cost of options and warranties) this raises the issue of uncertainty about the risk itself (see Hugonnier and al. (2012)).

In this paper we focus on interest rate risk. We consider a Zero coupon bond portfolio and deterministic liabilities. This framework for risk analysis is a simplification of the *RiskMetrics* method designed by JPMorgan in 1989 (see RiskMetrics Technical Note 2006). This paper shows the inconsistencies of the standard formula approach and details the reason why these inconsistencies are incompatible with an optimized and prudent business management. Time dependencies of stress are introduced to correct this pitfall. Time dependency is modeled through the parameters of the Nelson Siegel (1987) curve fitting method. These parameters are the level, the slope and the convexity, according to Bonnin and al. (2011) Ornstein Uhlenbeck process are used. We calibrate them from historical data since the settlement of the euro currency. Calibration is then adjusted to fit the standard stress level as defined by the EIOPA. The average level of the 0.5 percentile VaR is not questioned considering that debates between EIOPA and industry are now closed. The purpose of this paper is to propose an improvement of the standard formula, compliant with the current framework as fixed in Solvency II directive. The proposed method allows a significant reduction in the pro-cyclical effect and of the solvency ratio volatility. The impact of this improvement is illustrated through the examples of two fictive insurance companies, one with non-life activity and the other with life activity.

1 The EIOPA's standard formula approach

The solvency II requests that a part of capital is held in order to cover the annual $VaR_{0.5\%}$, with no new business assumption (i.e. existing contracts or in-force contracts). The EIOPA risk assessment approach is a parametric modular method. QIS 5 technical specifications (CEIOPS 2010b) and calibration paper is a simplification of the *RiskMetrics* method designed by JPMorgan in 1989.

The annual $VaR_{99.5\%}$ of each risk factor is computed from the historical distribution estimated by year to year variations (overlapping). Therefore market consistent values are reassessed under stress on each risk factor. The gap between the VaR and the current situation (ΔNAV) are aggregated by risk factor through the strong assumption of normal copula, see Mittnik(2012) for a questioning of this assumption.

1.1 The EIOPA's standard formula approach

As previously mentioned we consider a simple insurance portfolio modeled as a series of zero-coupon bonds both on liabilities and assets. This assumption is logical for *property and casualty*, health, pension and annuities portfolios. Insurance portfolios with policyholder options are excluded from the scope but we can expect a similar behavior to the proposed model.

Let's therefore consider a T maturity zero-coupon bond which is valued as follows

$$P(T, r_T, s_{rtg}) = \frac{1}{(1 + r_T + s_{rtg})^T} \tag{1}$$

r_T is the risk free interest rate for the maturity T and s_{rtg} the spread for the rating rtg

Let's assume

$$\Delta NAV_{r_T} = P(T, r_T, s_{rtg}) - P(T, VaR_{r_T}, s_{rtg}) \tag{2}$$

$$\Delta NAV_{s_{rtg}} = P(T, r_T, s_{rtg}) - P(T, r_T, VaR_{s_{rtg}}) \tag{3}$$

The SCR.5.77 article of EIOPA technical specifications (CEIOPS 2010b) states that $P(T, r_T, VaR_{99.5\%}(s_{rtg})$ is computed with order one Taylor's development on the s_{rtg}. Considering the assumption that r_T and s_{rtg} are distributed according to a Gaussian copula, $VaR_{99.5\%}$ of $P(T, r_T, s_{rtg}))$ can be written as follows :

$$VaR_{99.5\%}(P(T, r_T, s_{rtg})) = P(T, r_T, s_{rtg}) + \sqrt{\Delta NAV_{r_t}^2 + \Delta NAV_{s_{rtg}}^2 + 2\rho_{rs}\Delta NAV_{r_T}\Delta NAV_{s_{rtg}}} \tag{4}$$

With ρ_{rs} is defined as Pearson correlation between r_T and s_{rtg} for extreme values (ranges : $[0.0\%, 5.0\%]$ and $[95.0\%, 100.0\%]$). This refinement overcomes the normal copula assumption.

For each risk factor the $VaR_{99.5\%}$ has to be assessed. This estimation can be performed by various methods, such as with time series theory, see Bollerslev and al.(2010), with market consistent diffusion process, see Hull and White (1990) or heterosedastic process see Huang(2009), the EIOPA chose an historic VaR approach. This approach considers a year to year variation of each risk factor and estimates the 99.5% and 0.5% percentile. This historical data used covers the period between August 1997 and May 2009 according to the EIOPA's calibration paper (CEIOPS 2010a). To ensure consistency we use a similar period (January 1999 - May 2009) but we demonstrate that results would be significantly different if we extend the period to 2013. Indeed since 2009 sub primes crisis, European central bank drives the interest rate to lower levels. The purpose of this paper is not to question the standard calibration in the Solvency II directive as this is a result of negotiations between EIOPA and the insurance industry. We focus on building a dampener effect on interest rates. For this exercise we use the same period as EIOPA and we give the results in the appendix.

We extract the zero-coupon bond price from the Euroswaps par rate and deduce the ZC actuarial rate. Euroswaps exchange annually a fixed leg on a 30/360 basis against a floating leg which six months Euribor, consequently the bootstrapping consists in finding df_i for each maturity i from one to thirty years with a one year step such as shown below :

$$1 = \sum_{i=1}^{N} (SWAP_N \cdot \Delta_i \cdot df_i) + df_N$$

Therefore

$$df_N = \frac{(1 - \sum_{i=1}^{N-1} SWAP_N \cdot \Delta_i \cdot df_i)}{(1 + SWAP_N \cdot \Delta_N)}$$

Where $SWAP_N$ is the par rate of the n-year swap, Δ_i is the length of the period $[i-1; i]$, in years considering a 30/360 day count fraction and df_i is the discount factor for that time period from zero to i. We then obtain the zero-coupon bond rate by the classical formula.

$$r_i = e^{(\frac{-\ln(df_i)}{T})} - 1$$

The missing $SWAP_N$ are computed with a cubic spline interpolation, according to Hagan (2006). The resulting curve is for a 30/360 days count with an actuarial rate.

We have back tested our method against the EIOPA method, comparing our results and the EIOPA risk free curve released for the 2009-01-01, 2010-01-01 and 2011-01-01 QIS exercise. The maximum gap between the two curves never exceeds 10.08 bps.

We use the EIOPA approach to assess the VaR for each maturity. This VaR is estimated empirically by a shifting of relative variation. As expected, on a similar calibration range, VaR are close to EIOPA QIS5 technical specifications (see table below). However if the range period for estimation is extended to January 2013 we can observe a significant rise on the downward stress for the long term rate. From 2009, because of ECB policy, the interest rate level has been maintained to lower levels. Debates on Solvency II parameterization are very unlikely to be reopened between EIOPA and the industry and using the statistical stress would be even more pro-cyclical and unrealistic in the current context.

	EIOPA 1997 to 2009		1999 to 2009		1999 to 2013	
T	Down	Up	Down	Up	Down	Up
1	-75%	70%	-66%	76%	-78%	78%
2	-65%	70%	-58%	73%	-72%	77%
3	-56%	64%	-49%	66%	-67%	67%
4	-50%	59%	-43%	60%	-61%	59%
5	-46%	55%	-39%	55%	-57%	54%
6	-42%	52%	-35%	52%	-53%	50%
7	-39%	49%	-33%	48%	-50%	47%
8	-36%	47%	-31%	47%	-47%	46%
9	-33%	44%	-29%	45%	-45%	44%
10	-31%	42%	-27%	44%	-44%	43%
15	-27%	33%	-24%	38%	-41%	38%
20	-29%	26%	-24%	34%	-42%	33%
30	-30%	25%	-26%	30%	-42%	28%

Table 1: VaR for r_T with the EIOPA approach

According to SCR.5.22 of EIOPA technical specifications (CEIOPS 2010b) the absolute change of interest rates should at least be one percentage point. Where the unstressed rate is lower than 1%, the shocked rate in the downward scenario should be assumed to be 0%.

The spread risk factor is computed with the EIOPA method and the Bank Of America Merrill Lynch Index (CEIOPS 2010a) is used as spread index proxy. This Index is an average of option adjusted spread, also called flat spread, of a market representative basket of bonds. The risk is summarized in the following table.

Rating	1999 to 2009
AAA	90bps
AA	110bps
A	140bps
BBB	250bps
BB	450bps
B or lower	750bps
Unrated or lower	300bps

Table 2: $(VaR_{99.5\%}(s_{rtg}) - s_{rtg})$ with the EIOPA estimation method

Based on this approach, we are able to apply the full EIOPA risk framework on Zero Coupon bonds and deterministic liabilities. As an example we consider 5Y AAA Zero coupon with an upward stress and we apply the standard framework to assess the $VaR_{99.5\%}$.

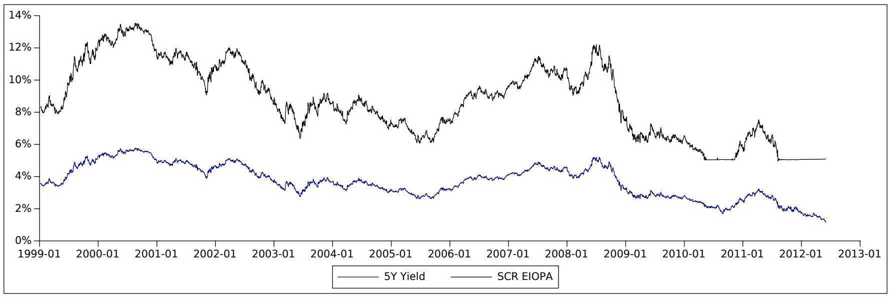

Figure 1: SCR Evolution of a 5Y Deterministic Liabilities (Modeled by a Zero Coupon)

The evolution shows some inconsistencies in the EIOPA risk assessment. First of all the risk assessment is correlated positively with interest rate levels. This means that when the level of interest rate increases, the risk of interest rate increases at the same time. This behavior leads to a significant pro-cyclical effect without increasing the predictability of the risk measure (see section 3.2 for a quantified analysis). This pro-cyclicality comes from a double impact. Indeed, companies exposed to the increase of interest rates are suffering from losses after a decrease of interest rate and they are sentenced to a non-consistent stress.

These facts shows that the EIOPA model does not integrate all the requirements of the Solvency II directive. Firstly a lack of predictability could lead to a lack of capital in a crisis period when the market liquidity is lower. This leads to the huge challenge of refinancing a company in this kind of period. Secondly this is not in line with the article 28 which asks to integrate, when it is possible and without prejudice on risk forecast accuracy, a counter cyclical effect. In order to consider the potential impact of a pro-cyclical risk model on the stability of the financial systems. Thirdly this is inconsistent with the integration of risk measure in decision process. Indeed if risk measures are inconsistent, decision process cannot be optimal and companies will therefore have to develop an internal model. These kinds of incentives would generate a competitive advantage for large companies that can afford for higher actuarial internal developments. Consequently, to avoid this, the standard formula has to be a trade-off between accuracy, simplicity and a reasonable recognition of counter-cyclicity.

2 The EIOPA extended approach

2.1 Model specification

As we previously mentioned, our goal is to set up a model that is not over-sophisticated but apprehends the main behavior of the yield curve. There are of course more realistic real world interest rate models see (see Norman(2009)) but our choice is to give priority to simplicity and to the possibility of giving economic justifications as well as getting an analytical expression of the quantile and avoiding the use of Monte Carlo simulations. This model is based on the Nelson Siegel regression in which the yield curve is expressed by the following regression (see Nelson and Siegel (1987))

$$R(t,u) = l_t - s_t\phi(\frac{u}{\tau}) + c_t\psi(\frac{u}{\tau}) \tag{5}$$

Where $\phi(x) = \frac{1-e^{-x}}{x}$, $\psi(x) = \phi(x) - e^{-x}$, t is the date, u is the maturity of the interest rate. l_t is the level, s_t the slope and c_t the curvature.

The choice of these risk factors is confirmed from the principal component analysis on the variation of the yield curve. Indeed we are able to obtain these factors by a combination of Eigen vectors (see Figure 14 in appendix and Bonnin and al. (2011)). Note that using l_t,s_t and c_t within Nelson Siegel framework leads to the loss of the orthogonality property between risk factors (in comparison to the PCA). However we obtain more explicit economic interpretation of them.

The cumulated inertia of the first three Eigen vectors is about 89% (see in appendix). This proves that the model is able to apprehend most of the interest rate curve movements, and that is confirmed by Diebold and al. (2008), where he demonstrates that diffusion on Nelson Siegel variation outperformed, in terms of predictability, other models like Fama-Bliss forward rate regression or Cocharane-Piazzesi forward curve regression.

The estimation of l_t, s_t, c_t and tau can be made by least-square regression, (see Nelson 1987) but this method shows instability (see Gili and al. (2010)). This makes the estimation of risk factors processes difficult. Consequently we choose the Diebold and al. (2006) method where $l(t)$, $s(t)$ and $c(t)$ are estimated by solving the following system for each date, from a maturities triplet:

$$\begin{cases} R(t,0.25) = l_t - \phi(\frac{0.25}{\tau})s_t + \psi(\frac{0.25}{\tau})c_t \\ R(t,10) \quad = l_t - \phi(\frac{10}{\tau})s_t + \psi(\frac{10}{\tau})c_t \\ R(t,30) \quad = l_t - \phi(\frac{30}{\tau})s_t + \psi(\frac{30}{\tau})c_t \end{cases} \tag{6}$$

According to Diebold and al.(2006) method we estimate the tau which minimizes the correlation between risk factors. We actually minimize the sum of square coefficients of correlation. On the studied period this optimum is reached for a value of tau equal to 2.6 (see appendix). The maturities triplet (0.25 Year, 10 Years and 30 Years) leads to a R^2 between model and observed curve equals to 98.60% over the calibration range 1999 to 2009.

We then come to set stochastic differential equations. These equations are chosen for the small number of parameters, for the possibility of providing with an economic interpretation, but especially for the existence of an analytical expression of the quantile.

$$dl_t \quad = k_l(\theta_l - l_t)dt + \sigma_l dW_t^l \tag{7}$$

$$ds_t \quad = k_s(\theta_s - s_t)dt + \sigma_s dW_t^s \tag{8}$$

$$dc_t \quad = k_c(\theta_c - c_t)dt + \sigma_c dW_t^c \tag{9}$$

Stochastic differential equations on l_t, s_t and c_t are integrated by applying the Ito Lemma and according to Brigo and Mercurio (2001) we obtain the following integrated form :

$$\begin{aligned} l_T &= l_0 e^{-k_l T} + \theta_l (1 - e^{-k_l T}) + \sigma e^{-k_l} \int_0^T e^{k_l}\, dW_l & (10) \\ s_T &= s_0 e^{-k_s T} + \theta_s (1 - e^{-k_s T}) + \sigma e^{-k_s} \int_0^T e^{k_s}\, dW_s & (11) \\ c_T &= c_0 e^{-k_c T} + \theta_c (1 - e^{-k_c T}) + \sigma e^{-k_c} \int_0^T e^{k_c}\, dW_c & (12) \end{aligned}$$

where T is the horizon of risk and equal to one year in Solvency II context. As we aim at estimating the risk on the interest rate distribution after a one year period, we compute the integrated variables conditionally to the information at t_0 ($|F_{t_0}$)

2.2 Analytical expression of the quantile

According to (10),(11) and (12), l_T, s_T and c_T follow the normal distribution presented below

$$\begin{aligned} l_T|F_{t_0} &\sim N(l_0 e^{-k_l T} + \theta_l (1 - e^{-k_l T})\ ,\ \sigma_l \sqrt{\frac{1 - e^{-2k_l T}}{2k_l}}) & (13) \\ s_T|F_{t_0} &\sim N(s_0 e^{-k_s T} + \theta_s (1 - e^{-k_s T})\ ,\ \sigma_s \sqrt{\frac{1 - e^{-2k_s T}}{2k_s}}) & (14) \\ c_T|F_{t_0} &\sim N(c_0 e^{-k_c T} + \theta_c (1 - e^{-k_c T})\ ,\ \sigma_c \sqrt{\frac{1 - e^{-2k_c T}}{2k_c}}) & (15) \end{aligned}$$

We are able to obtain an analytical expression of the quantile. To clarify all the following equations let

$$\begin{aligned} \alpha_l(T) &= l_0 e^{-k_l T} + \theta_l (1 - e^{-k_l T}) & (16) \\ \beta_l(T) &= \sigma_l \sqrt{\frac{1 - e^{-2k_l T}}{2k_l}} & (17) \\ \alpha_s(T) &= s_0 e^{-k_s T} + \theta_s (1 - e^{-k_s T}) & (18) \\ \beta_s(T) &= \sigma_s \sqrt{\frac{1 - e^{-2k_s T}}{2k_s}} & (19) \\ \alpha_c(T) &= c_0 e^{-k_c T} + \theta_c (1 - e^{-k_c T}) & (20) \\ \beta_c(T) &= \sigma_c \sqrt{\frac{1 - e^{-2k_c T}}{2k_c}} & (21) \end{aligned}$$

Therefore l_T, s_T and c_T are expressed as follows

$$\begin{aligned} l_T|F_{t_0} &\sim N(\alpha_l\ ,\ \beta_l) & (22) \\ s_T|F_{t_0} &\sim N(\alpha_s\ ,\ \beta_s) & (23) \\ c_T|F_{t_0} &\sim N(\alpha_c\ ,\ \beta_c) & (24) \end{aligned}$$

Please note, as expected we have indeed $\alpha_l = \theta_l$ if $l_t = \theta_l$, $\alpha_s = \theta_s$ if $s_t = \theta_s$ $\alpha_c = \theta_c$ if $c_t = \theta_c$ and this $\forall t \in \mathbb{R}^+$

We deduce from (5),(22),(23) and (24) the distribution for $R(T,u)|F_{t_0}$:

$$R(T,u)|F_{t_0} \sim N(\alpha_R \ , \ \beta_R) \tag{25}$$

With

$$\begin{aligned}
\alpha_R &= \alpha_l + \phi(\frac{u}{\tau})\alpha_s + \psi(\frac{u}{\tau})\alpha_c \\
\\
\beta_R^2 &= +\ \beta_l^2 \\
&\quad + \phi^2(\frac{u}{\tau})\beta_s^2 \\
&\quad + \psi^2(\frac{u}{\tau})\beta_c^2 \\
&\quad + 2\ \phi(\frac{u}{\tau})\ \beta_l\ \beta_s\ \rho_{dW^l,dW^s} \\
&\quad + 2\ \psi(\frac{u}{\tau})\ \beta_l\ \beta_c\ \rho_{dW^l,dW^c} \\
&\quad + 2\ \phi(\frac{u}{\tau})\ \psi(\frac{u}{\tau})\ \beta_c\ \beta_s\ \rho_{dW^c,dW^s}
\end{aligned}$$

ρ_{dW^l,dW^s}, ρ_{dW^l,dW^c} and ρ_{dW^c,dW^s} are the correlations between the three brownian motions.

Therefore we have an analytical expression for the Solvency II percentiles,

$$VaR_{99.5\%}(R(T,u)|F_{t_0}) = \beta_R U(99.5\%) + \alpha_R \tag{26}$$

$$VaR_{0.5\%}(R(T,u)|F_{t_0}) = \beta_R U(0.5\%) + \alpha_R \tag{27}$$

With $U(99.5\%) = 2.576$ and by symmetry $U(0.5\%) = -2.576$

2.3 Historical calibration with exact discretization

According to Gillespie (1995) the exact discretization of an Ornstein-Uhlenbeck process is :

$$X_{t+\Delta_t} = e^{-k_X \Delta_t} X_t + (1 - e^{-k_X \Delta_t})\theta_X + \sigma_X \sqrt{\frac{1 - e^{-2k_X \Delta_t}}{2k_X}} dW_t^X \tag{28}$$

For this calibration we are looking for the linear relation which minimizes the squares of residuals

$$X_{t+\Delta_t} = c_0^X + c_1^X X_t + \varepsilon_t^X \tag{29}$$

By identification we have:

$$k_X = -\frac{ln(c_1^X)}{\Delta_t} \tag{30}$$

$$\theta_X = \frac{c_0^X}{1 - e^{-k_X \Delta_t}} \tag{31}$$

$$\sigma_X = \sqrt{\frac{2k_X Var(\varepsilon_t^X)}{1 - e^{-2k_X \Delta_t}}} \tag{32}$$

Between January 4th 1999 (the settlement of euro currency) and May 1st 2009 we estimate on a daily frequency basis excluding market closed days, the risk factors $l(t)$, $s(t)$ and $c(t)$ from the linear system (6). We therefore obtain the following assessment [1]:

$$\begin{array}{lll} c_0^l = 1.009 \cdot 10^{-4} & c_0^s = 2.700 \cdot 10^{-5} & c_0^c = -1.543 \cdot 10^{-5} \\ c_1^l = 9.980 \cdot 10^{-1} & c_1^s = 9.988 \cdot 10^{-1} & c_1^c = 9.926 \cdot 10^{-1} \\ Var(\varepsilon_t^l) = 2.527 \cdot 10^{-7} & Var(\varepsilon_t^s) = 3.393 \cdot 10^{-7} & Var(\varepsilon_t^c) = 1.515 \cdot 10^{-7} \end{array}$$

So the assessment of the parameters of the stochastic differential equation is :

$$\begin{array}{lll} k_l = 51.04\% & k_s = 29.76\% & k_c = 185.96\% \\ \theta_l = 4.947\% & \theta_s = 2.269\% & \theta_c = -0.208\% \\ \sigma_l = 0.796\% & \sigma_s = 0.921\% & \sigma_c = 1.953\% \end{array}$$

Finally we deflate the deterministic drift from $l(t)$, $s(t)$ and $c(t)$ variations and we obtain the correlation between the three brownian motions.

$$\begin{aligned} \rho_{dW^l,dW^s} &= 36.81\% \\ \rho_{dW^l,dW^c} &= -3.34\% \\ \rho_{dW^c,dW^s} &= -1.77\% \end{aligned}$$

Our extended approach reveals dependence between the interest rate curve and the stress value. To compare the stress with the EIOPA standard formula we define a stationary state where stresses are not time dependent. In this state l_t, s_t and c_t are equal to their long term average (θ_l,θ_s,θ_c). A stationary curve is then defined (see in appendix) with an associated stress. We consider it as comparable to the EIOPA standard stress scenario on interest rates. However, in this state, the model stress does not exactly match with the EIOPA stress (see the graphic below).

[1] Data and source code are available upon request

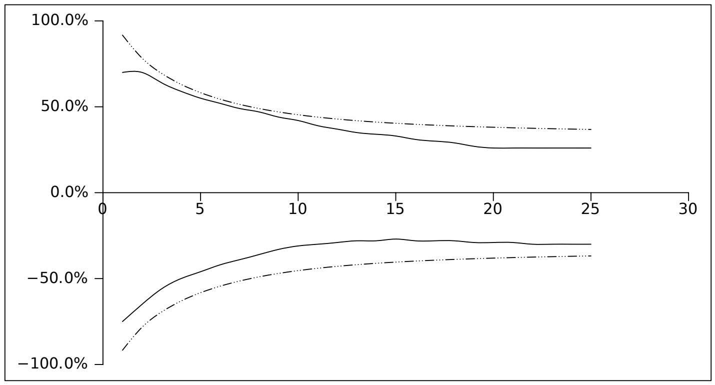

Figure 2: Comparison of EIOPA's funnel of doubt model (plain lines) against the proposed model's funnel of doubt (dotted lines) with an historical calibration

2.4 Difference of assessment: explanation and convergence

The mismatch between the two methods comes from the parameter's assessment methodology. Indeed, the EIOPA estimates directly the annual VaR on the series of overlapping annual returns on a daily basis. In the model proposed in this paper, we estimate the daily volatility, which is then annualized with consideration of one order auto-correlation. Then the annual VaR is deduced under a normal assumption (brownian process) (see (28) (29)). The difference of VaR assessment, for the 30 Years term rate is around 6% : with a VaR of 30% with EIOPA method versus a VaR of 36% with the proposed Nelson-Siegel / Ornstein-Uhlenbech model. In order to quantify the bias of each method we simulate an Ornstein Uhlenbeck process, in which the annual VaR is perfectly known, and we apply on it the two calibration method to evaluate the bias in the VaR assessment. Parameters of the process are those of the level process ($k = 51.04\%$, $\theta = 4.947\%$, $\sigma = 0.796\%$) and the starting point is the 30 year rate at January 4th 1999 ($X_0 = 4.87\%$). We simulate 10^4 processes over 10 years on a daily step. The average of the EIOPA's method VaR assessment is 35.31% with a standard deviation of 5.52%. The average of the auto regression method is 41.37% with a standard deviation of 0.59%. This has to be compared to the theoretical VaR of the simulated process which is 41.29%. These results make appear that our method is less biased than EIOPA's and consequently the EIOPA risk framework could have a lack of prudency. This behavior can be explained by the fact that the overlapping VaR assessment method considers only realized path opposed to the auto regression model which allows consideration of unrealized path and the extrapolation of the annual VaR. Nevertheless as we mentioned in introduction, we don't want to derive from the standard stress. As a consequence, we have chosen to estimate the volatility for the level risk factor with overlapping VaR assessment method. This leads to an assessment of 0.503% for the volatility and 27% for the VaR. In that way the proposed risk assessment method is fully consistent with EIOPA standard formula and funnels of doubt are very closed in average (see the graphic below).

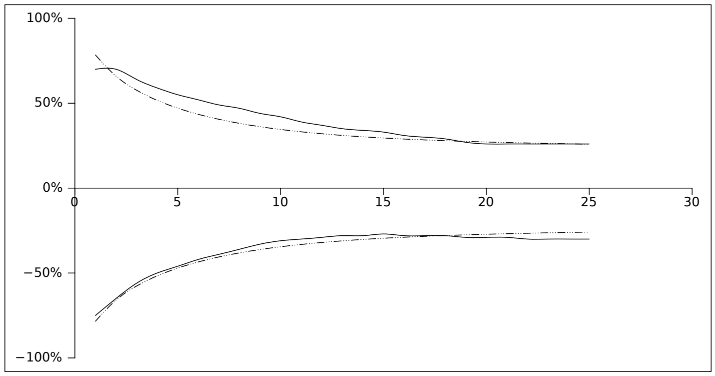

Figure 3: Convergence of EIOPA funnel of doubt model (plain lines) against model funnel of doubt (dotted lines) with EIOPA adjusted calibration

3 Model behavior and risk assessment improvements

3.1 Time adjusted interest rate stress

Thus according to the process as described above, we have estimated the risk factors values l_t, s_t and c_t. We then are able to get a time adjusted stress of interest rates, estimating the annual $VaR_{0.5\%}$ at each daily step. As expected up-stress is negatively correlated with interest rates levels and down-stress is positively correlated with interest rates levels, as the mean reversion property allows for a counter-cyclical stress.

As a proof of concept, we consider a simple asset modeled as a AAA 5Y Zero coupon, exposed to the upward stress and we compute its SCR with the EIOPA method and the proposed mean-reversion model between January 4th 1999 and May 1st 2012.

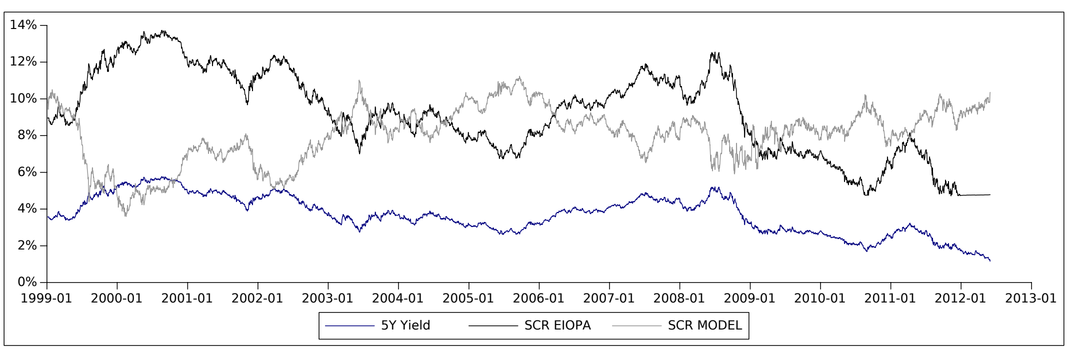

Figure 4: SCR Evolution of a 5Y AAA Asset (Modeled by a Zero Coupon) exposed to the increase of interest rate

As expected, the SCR with the EIOPA method increases when the interest rate increases whereas the SCR with the proposed model decrease when the interest rate increase. The method is consequently counter cyclical. Indeed, the model increases the assessment of SCR after gains (decrease in interest rate) and decreases it after losses (increase in interest rate).

We then consider a simple liability modeled as 5Y Zero coupon, exposed to the downward stress. and we compute its SCR with the EIOPA method and the proposed mean-reversion model between January 4th 1999 and May 1st 2012.

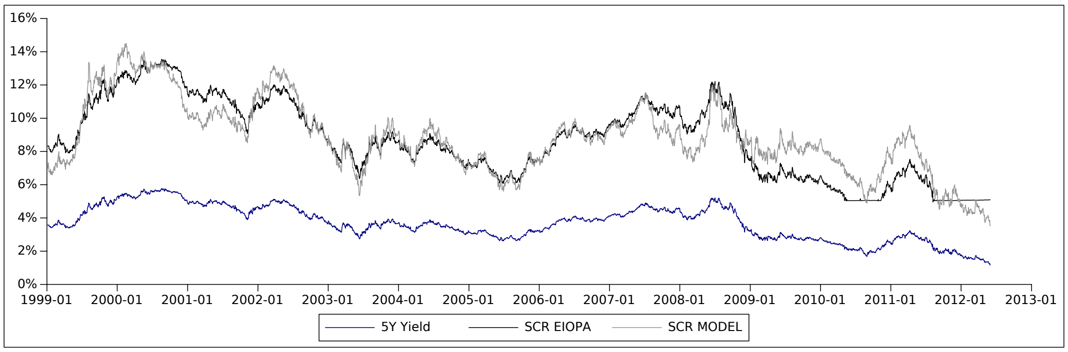

Figure 5: SCR Evolution of a 5Y Liabilities (Modeled by a Zero Coupon) exposed to the decrease of interest rate

We observe the same counter-cyclical behavior with a downward shock (on liability side). Indeed when interest rates increase, the value of liabilities drops (we realize a gain), and the proposed model IR SCR rises. On the contrary when interest rates decrease, this is an adverse shock on liabilities and the proposed model SCR is lower than EIOPA standard shock.

3.2 Benefits on the VaR forecast accuracy

According to the directive a counter-cyclical effect could be introduce (Article 28) but without prejudice to the accuracy of the VaR forecast which remains the main objective (Article 27). The proposed model should consequently be back tested in order to show that no deterioration of accuracy is introduced and it is consequently a practical alternative to the existing EIOPA model.

There is a large number of Value at Risk back testing procedure in literature which commonly test the unconditional proportion of violation and the independence between two successive violations (avoidance of clustering effect), through a study of the density function of the time between two VaR failures (see D.Campbell 2006 for a summary of back testing frameworks). Nevertheless, all these tests share the same weakness coming from the size of the sample. This weakness is particularly important in Solvency II because of the VaR settings (horizon of one year with alpha equal to 0.5%). Indeed with these settings the average time between two VaR violations is 200 years,therefore getting a significant number of VaR failures is fairly difficult.

The problem of significant number of VaR failure observations can be solved by changing the VaR horizon. Indeed in case of no significant autocorrelation the accuracy of a model is inherited from the short to the long term horizon.

The correlogram below of the daily variations shows erratic and very low autocorrelations and leads to the conclusion of the absence of significant autocorrelation between variations

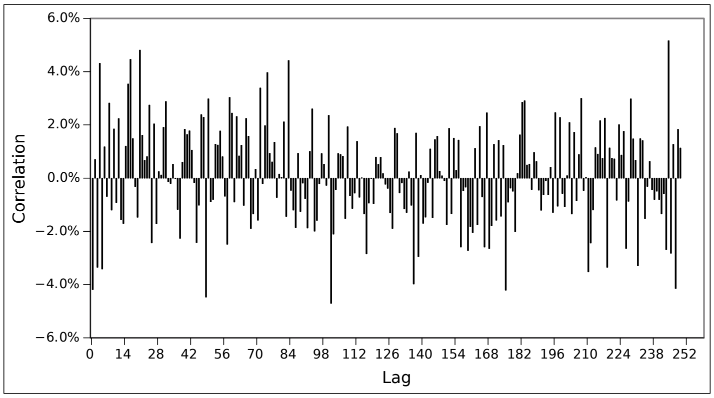

Figure 6: Correlogram of one day variations from 1 to 250 open days lag

In the absence of autocorrelation the one day horizon VaR in the standard formula is
$VaR(1day, 0.5\%) = \sqrt{\frac{1}{250}} VaR(1year, 0.5\%)$

The ex-ante $VaR(1day, 0.5\%)$ forecast for a 5Y Zero coupon bond with a nominal of 100EUR is then compared to its ex-post price variation to identify past events of VaR failure.

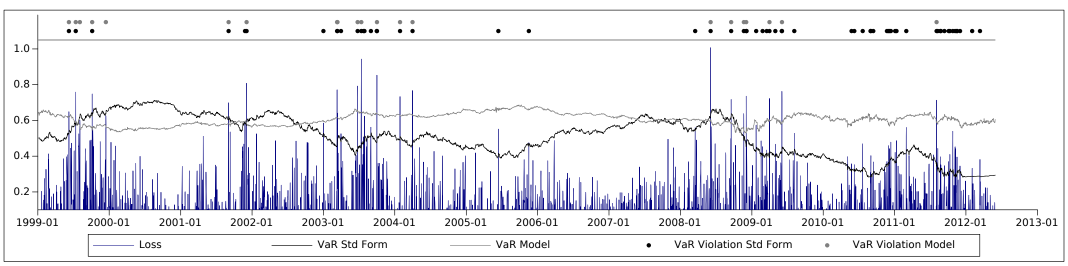

Figure 7: Historic of VaR and VaR failures events

VaR Forecast	[0.28, 0.71]	[0.28, 0.43[	[0.43, 0.57[	[0.57, 0.71]
#Observation	3498	921	1398	1178
#VaR Failures	62	34	21	7
VaR Failures frequency	1.77%	3.69%	1.50%	0.59%

Table 3: VaR failure with the EIOPA method

VaR Forecast	[0.53, 0.69[	[0.53, 0.58[	[0.58, 0.64[	[0.64, 0.71]
#Observation	3498	1013	1801	681
#VaR Failures	22	7	12	3
VaR Failures frequency	0.63%	0.69%	0.67%	0.44%

Table 4: VaR failure with the model method

The graph and data above show facts which attest the superiority of the proposed model in term of accuracy in VaR forecast. Firstly the hit ratio in the proposed model (0.63%) is closer to the target hit ratio (0.5%) than the standard formula approach (1.77%). Secondly, the increase or decrease of the ex-ante risk forecast does not introduce significant bias on the hit ratio unlike the EIOPA model (see VaR failures at the end of 2011 which come exclusively from low risk forecast). Finally when there is a VaR failure in the proposed model, there is also a VaR failure in the EIOPA model, whereas a VaR failure in the EIOPA model does not implied a systematic VaR failure in the model. The proposed model is consequently more prudent.

The clustering effect can be quantified by the construction of empirical density function of the time between two violations failures. For an unbiased model this function is an exponential distribution with the average equal to the VaR tolerance threshold (see Christoffersen 2004). The graph below shows that the proposed model fits more closely the theorical distribution than the EIOPA model.

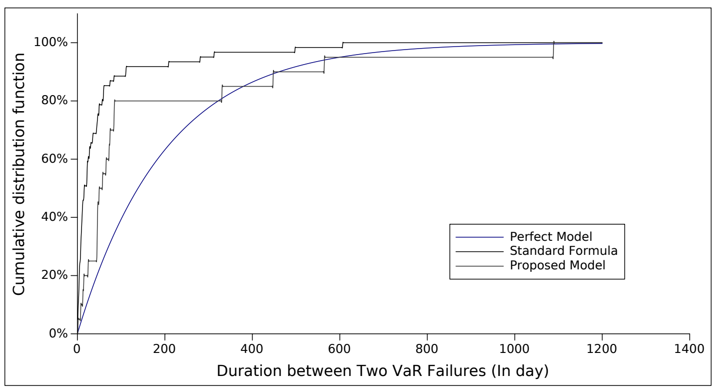

Figure 8: Empirical distribution of duration between two VaR failures

A slight clustering effect remains in the proposed model. It comes from the homoscedasticity hypothesis in the diffusion process used which is not always applicable. The one month shifted standard deviation shows rare high volatility period which coincide with each standard formula VaR failure cluster. The proposed model has a better resilience on this hypothesis than the EIOPA approach. In order to keep as much simplicity as possible we have chosen to neglect the diffusion of volatility. The model remains however more accurate than the EIOPA framework.

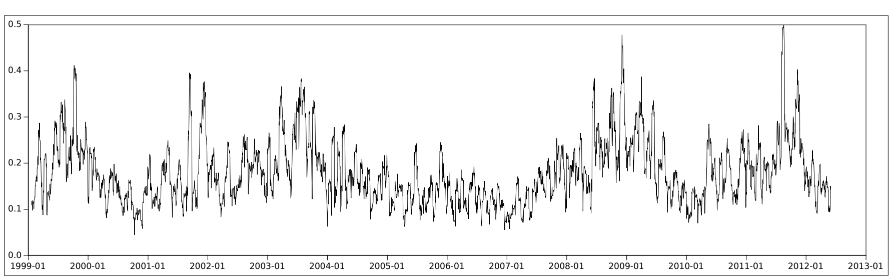

Figure 9: Volatility of 5Y Zero Coupon bond price variation

3.3 Benefits on the Solvency ratio

The following of this paper considers fictive insurance undertakings, one life (long liability duration), and one which similar to a healthcare entity (very short tail business) or short tail non-life. The paper will demonstrate how the proposed framework enables a stabilization of the Solvency ratio for any type of balance sheet and business. This counter-cyclical effect on SCR is called *interest rate dampener* model.

Let's consider first a short tail business undertaking, non-life of annual healthcare. This undertaking is exposed to the upward stress: assets longer than liabilities.

Settlement Date (t_0)	January 4th 1999
Asset Portfolio Value No discount factor (GA)	123.8MEUR
Best estimate of Liabilities No discount factor (GL)	103.2MEUR
Asset Portfolio Value (A)	100.0MEUR
Best estimate of Liabilities (L)	100.0MEUR
Risky Asset duration (u_A)	5Years
Risky Asset mean rating (rtg)	A
Liabilities duration (u_L)	1Year
Solvency Capital Requirement at t_0 (SCR)	9.3MEUR
Solvency Ratio (SR)	3.0
Own Funds (OF)	28.0MEUR

Table 5: Characteristics of the company

We then need to specify the main management rules. First of all we assume that the asset portfolio is a constant maturity and constant rating zero-coupon bond through time. Consequently there is no variation of risk level due to the portfolio aging, as it would be the case in a run-off assumption. For the same reason we consider that the liabilities is also modeled by a constant maturity zero-coupon bond. Our last assumption is that own funds are held on in cash, so they do not generate additional risk or revenues. We suppose a *steady-state business* where any technical or financial income is fully distributed at any time to equity holders, and where the assets and liabilities structure is only affected by the changes in interest rates and spread environment.

For this company $\forall t$:

$$
\begin{aligned}
A(t) &= \frac{GA}{(1+R(t,u_A)+s_{rtg})^{u_A}} && (33)\\
L(t) &= \frac{GL}{(1+R(t,u_L))^{u_L}} && (34)\\
VaR_r(t) &= \left[\frac{GA}{(1+R^{+}(t,u_A)+s_{rtg})^{u_A}} - A(t)\right] - \left[\frac{GL}{(1+R^{+}(t,u_L))^{u_L}} - L(t)\right] && (35)\\
R^{+}(t,u) &= R(t,u) + Max(S^{u}_{e_{up}}R(t,u), 100bp) && (36)\\
VaR_{spr}(t) &= A(t)u_A VaR_{spr}(rtg) && (37)\\
SCR(t) &= \sqrt{VaR_r(t)^2 + VaR_{spr}(t)^2 + 2\rho_{rs}VaR_r(t)VaR_{spr}(t)} && (38)\\
SR(t) &= OF(t)/SCR(t) && (39)
\end{aligned}
$$

Where $\rho_{rs} = 0.5$ when we consider a down stress and $\rho_{rs} = 0.0$ in the up stress We consider that Own Funds (OF) are determined recursively as follows

$$
\begin{aligned}
OF(t_0) &= 2SCR(t_o) && (40)\\
OF(t) &= OF(t-\Delta_t) + (A(t) - A(t-\Delta_t)) - (L(t) - L(t-\Delta_t)) && (41)
\end{aligned}
$$

After setting the company we are able to make an historical simulation of the solvency ratio between this date and May 1st 2012.

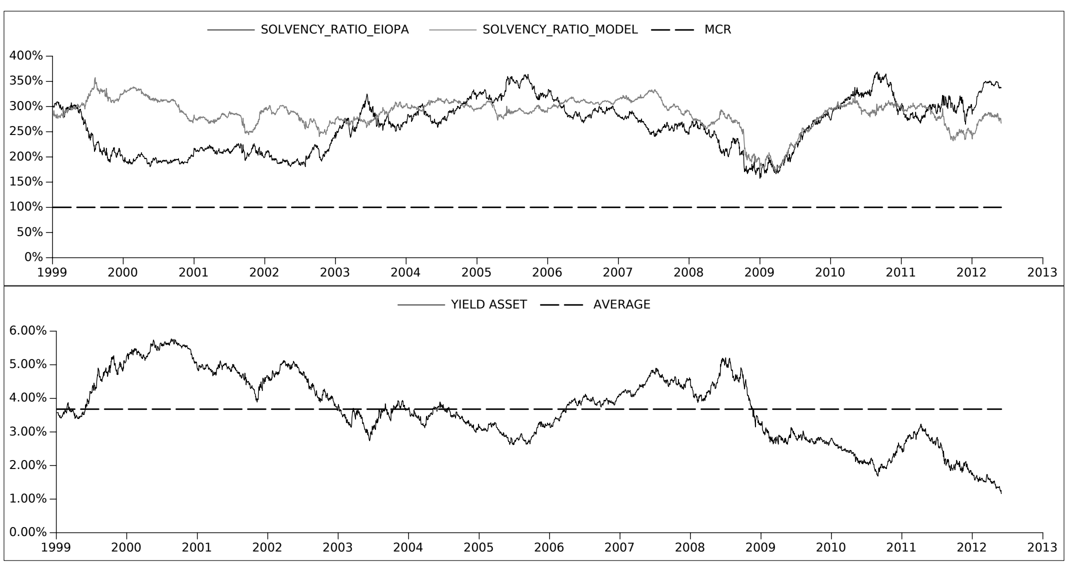

Figure 10: Evolution of Solvency ratio for a non-life company

	EIOPA Method	Proposed method
Average	262%	287%
Min	159%	172%
Max	369%	357%
Maximum Drawdown	211%	185%
Volatility	51%	30%

Table 6: Statistics about Solveny ratio

The historical simulation on the short tail insurance undertaking presented above shows that the solvency ratio computed with the proposed model is higher than the ratio computed with the EIOPA's standard formula in high interest rates period, and lower in low interest rates period. This means that the proposed framework smoothes the switching between bad and good market environment conditions. This improvement (so-called *interest rate dampener*) also decreases the volatility of the solvency ratio, 51% with the EIOPA method versus 30% with the proposed framework and decreases the sensitivity of the solvency ratio to the yield curve. As consequence it leads also to a simplification of the management by decreasing the probability of a SCR higher than available Own funds (ratio below 100%) due to the market environment. Notice that risk factors other than interest rates and spread are not considered in this article. By introducing solvency ratio inertia the framework helps insurance undertakings to survive in temporary difficult market conditions. Nevertheless the proposed framework has no effect about the spread variation and as we can observe in the 2009 spread crisis, the solvency ratio drops because of increase of spread whatever the used risk framework. This leads to the necessity of proposing a *spread risk dampener*. This dampener is one of the major topic addressed by EIOPA with the LTGA industry impact study, especially through the introduction of a counter-cyclical premium and Matching Adjustment.

The same exercise with similar results can be made on a life insurance undertaking, where liabilities are longer than assets, and with the following characteristics:

Settlement Date (t_0)	January 04th 1999
Asset Portfolio Value No discount(GA)	145.3MEUR
Best estimate of Liabilities No discount(GL)	251.3MEUR
Asset Portfolio Value (A)	100.0MEUR
Best estimate of Liabilities (L)	100.0MEUR
Risky Asset duration (u_A)	8Years
Risky Asset mean rating (rtg)	A
Liabilities duration (u_L)	20Year
Solvency Capital Requirement at t_0 (SCR)	25.1MEUR
Solvency Ratio (SR)	3.0
Own Funds (OF)	75.2MEUR

Table 7: Characteristics of the company

We are keep the same management rules as previously and all formula are the same except for the $VaR_r(t)$ for which we have

$$VaR_r(t) = \left[\frac{GA}{(1+R^-(t,u_A)+s_{rtg})^{u_A}} - A(t)\right] - \left[\frac{GL}{(1+R^-(t,u_L)+s_{rtg})^{u_L}} - L(t)\right] \quad (42)$$

$$R^-(t,u) = Max(R(t,u) + Min(S^u_{e_{down}} R(t,u), -100bp), 0) \quad (43)$$

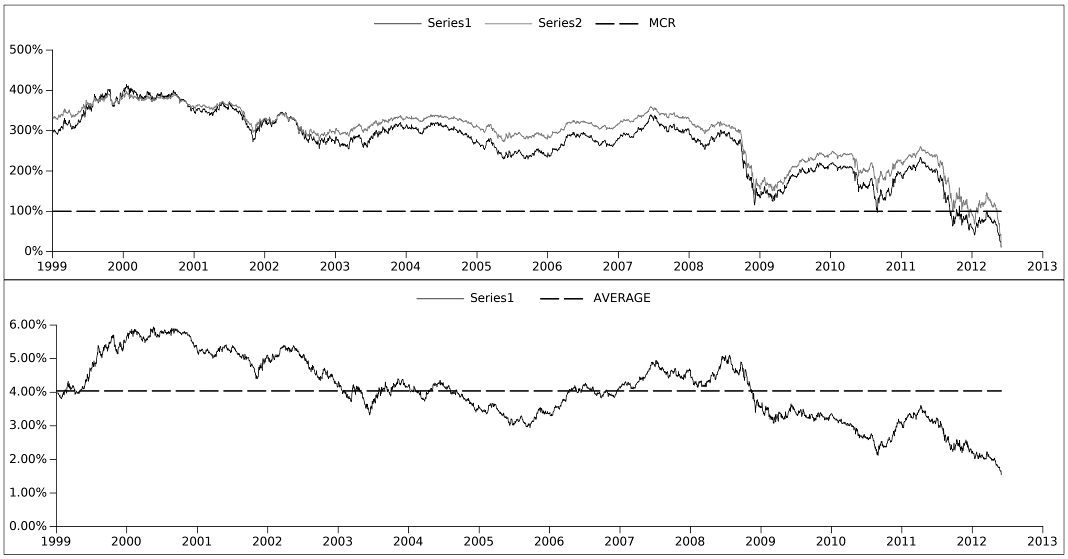

Figure 11: Evolution of Solvency ratio for a life company

	EIOPA Method	Proposed method
Average	267%	292%
Min	6%	14%
Max	415%	397%
Maximum Drawdown	408%	384%
Volatility	80%	69%

Table 8: Statistics about Solvency ratio

The historical simulation is run on the life insurance undertaking (long tail business) as described above. For long liability business, as a reminder, the crisis period are linked to low interest rates environment. The results above shows that the solvency ratio computed with the proposed model is higher than the ratio computed with the EIOPA's standard formula in low interest rates period, and lower in high interest rates period. This means that the proposed framework has the same smoothing effect as the non-life insurance undertaking. The decrease of the volatility of the solvency ratio is also effective, 80% with the EIOPA method versus 69% with the proposed framework as well as the decreases of the sensitivity of the solvency ratio to the yield curve. Notice that the 2009 credit crisis has less impact on the life insurance undertakings solvency ratio than a non-life undertaking solvency ratio. Indeed the main risk of life insurance undertaking is the decrease of interest rate rather than credit spread on the asset side. In a very low level of interest rate environment (like at the end of

2012), the impact on balance sheet on economic value basis is such that the solvency ratio goes below 100% (as if 300% in 1999) even with the recommended proposed model that gives a very low downward stress (-8%). Therefore the dampener effect is quite well obtained with the use of our SCR model, but doesn't equilibrate the first order shock on the NAV due to changes in market condition, in a crisis situation. Our *interest rate risk dampener* smooth the SCR shocks, but not the volatility of the Net Asset Value (NAV) itself in crisis situations, and doesn't address the issue of economic or fair value accounting assumption in Solvency II versus local GAAP rules.

Conclusion

Interest rates are an important source of risk exposure for insurance undertakings. The asset liability duration mismatch is often met in practical life of insurance undertakings. Especially when the liabilities are very long in Life, Long Term Care, annuities, P&C TPL, investing in long assets is made complicated by the lack of liquid or even existing markets for long maturity bonds. It appears that the standard risk approach is biased and results in an increase of the SCR requirement in a wrong timing. This inaccuracy in timing implies dangerous pro-cyclical effects and misleads the meaning of a management driven by the standard risk formula (especially investment management). This is a strong incentive to costly internal models development to the insurance industry in order to ensure meaningful decisions. This doesn't match with the initial purpose of the standard formula concept, and of Solvency II framework that should protect the market against competition distortion between small and big players. Our proposal, which is in line with the spirit of the standard approach, introduces simply but approved stochastic modifications, and through a modelization of the reverse to the mean effect, corrects the bias of the standard formula. This correction is effective for non-life and life insurance undertakings. The proposed framework can also be improved. The dampener mechanism can be apply to other risk factors and the modelization could integrate the state of the art of stochastic processes theory (heteroscedastic, jump, regime switching).

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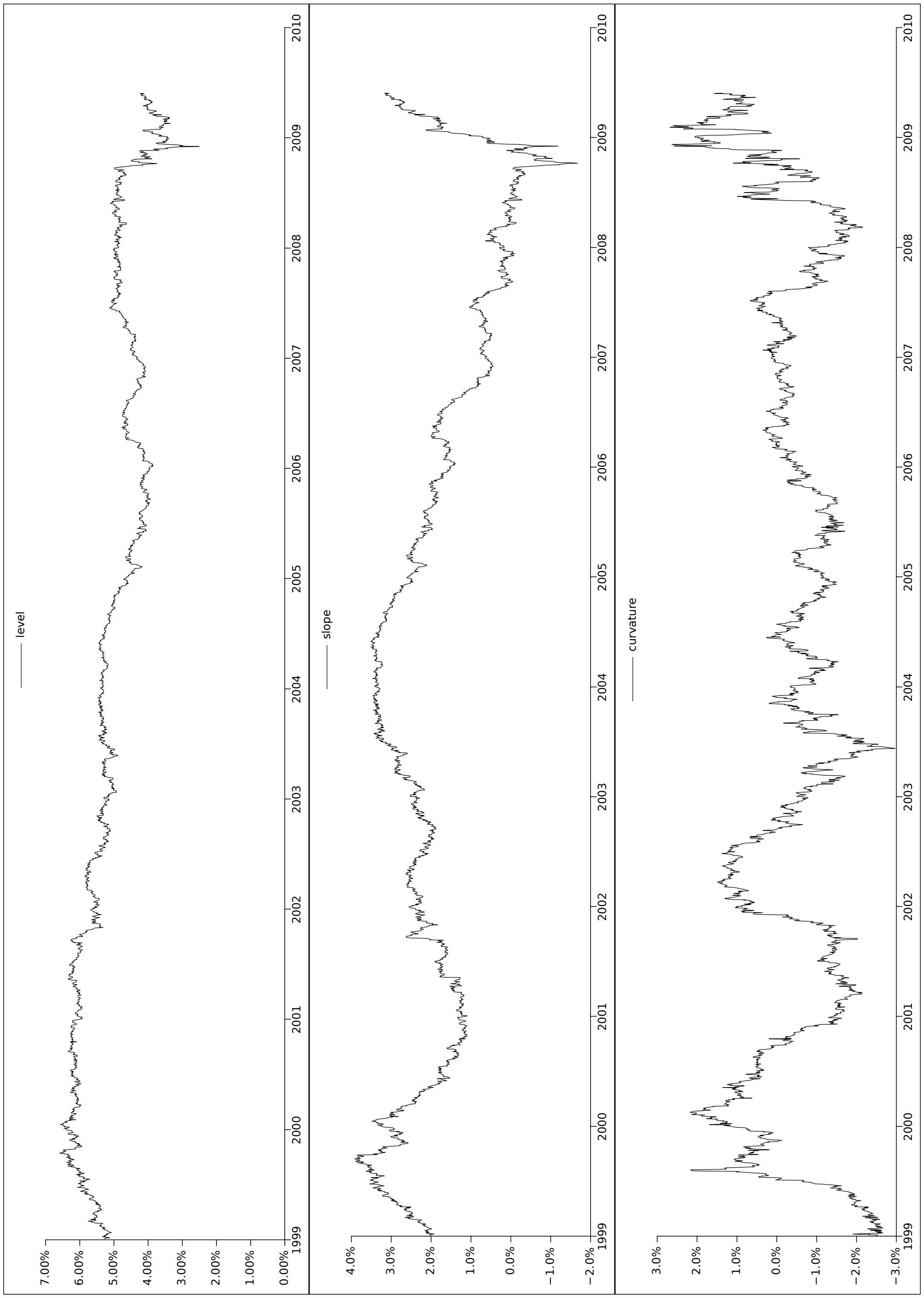

Figure 12: Historic of risk factors

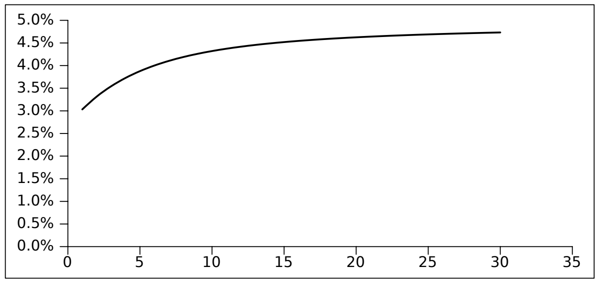

Figure 13: Yield Curve in the stationary state

T	**EIOPA down**	**EIOPA up**	**Model Down**	**Model up**
1	-75%	70%	92%	-92%
2	-65%	70%	78%	-78%
3	-56%	64%	69%	-69%
4	-50%	59%	63%	-63%
5	-46%	55%	58%	-58%
6	-42%	52%	54%	-54%
7	-39%	49%	51%	-51%
8	-36%	47%	49%	-49%
9	-33%	44%	47%	-47%
10	-31%	42%	45%	-45%
15	-27%	33%	40%	-40%
20	-29%	26%	38%	-38%
25	-30%	26%	37%	-37%
30	-30%	26%	36%	-36%

Table 9: Comparison of Model's VaR and EIOPA's VaR

T	**EIOPA down**	**EIOPA up**	**Model Down**	**Model up**
1	-75%	70%	78%	-78%
2	-65%	70%	66%	-66%
3	-56%	64%	58%	-58%
4	-50%	59%	52%	-52%
5	-46%	55%	47%	-47%
6	-42%	52%	43%	-43%
7	-39%	49%	41%	-41%
8	-36%	47%	38%	-38%
9	-33%	44%	36%	-36%
10	-31%	42%	35%	-35%
15	-27%	33%	30%	-30%
20	-29%	26%	27%	-27%
25	-30%	26%	37%	-37%
30	-30%	26%	25%	-25%

Table 10: Comparison of Model's VaR with EIOPA with level risk factor volatility adjustment

T	0.004	0.25	1	2	3	4	5	6	7	8	9	10	15	20	30
0.004	100%	15%	6%	4%	2%	2%	1%	1%	1%	1%	1%	1%	0%	0%	0%
0.25	15%	100%	30%	14%	11%	8%	6%	4%	5%	5%	4%	3%	0%	-1%	-3%
1	6%	30%	100%	82%	76%	72%	70%	68%	65%	63%	60%	60%	54%	50%	46%
2	4%	14%	82%	100%	92%	89%	86%	85%	81%	79%	77%	76%	69%	65%	61%
3	2%	11%	76%	92%	100%	89%	87%	86%	84%	82%	80%	78%	71%	68%	64%
4	2%	8%	72%	89%	89%	100%	96%	96%	94%	92%	91%	89%	82%	79%	75%
5	1%	6%	70%	86%	87%	96%	100%	97%	94%	93%	91%	91%	85%	82%	78%
6	1%	4%	68%	85%	86%	96%	97%	100%	97%	95%	94%	93%	88%	85%	81%
7	1%	5%	65%	81%	84%	94%	94%	97%	100%	98%	98%	96%	91%	88%	85%
8	1%	5%	63%	79%	82%	92%	93%	95%	98%	100%	99%	97%	91%	91%	86%
9	1%	4%	60%	77%	80%	91%	91%	94%	98%	99%	100%	97%	92%	93%	87%
10	1%	3%	60%	76%	78%	89%	91%	93%	96%	97%	97%	100%	95%	94%	90%
15	0%	0%	54%	69%	71%	82%	85%	88%	91%	91%	92%	95%	100%	94%	94%
20	0%	-1%	50%	65%	68%	79%	82%	85%	88%	91%	93%	94%	94%	100%	95%
30	0%	-3%	46%	61%	64%	75%	78%	81%	85%	86%	87%	90%	94%	95%	100%

Table 11: Interest rates correlation matrix

T	0.004	0.25	1	2	3	4	5	6	7	8	9	10	15	20	30
0.004	-1%	37%	87%	32%	0%	0%	0%	-1%	-1%	0%	0%	0%	0%	0%	0%
0.25	-2%	65%	-1%	-73%	19%	-7%	-4%	1%	-1%	0%	0%	1%	0%	0%	0%
1	-21%	42%	-24%	15%	-72%	39%	9%	-15%	4%	2%	1%	2%	0%	0%	1%
2	-26%	25%	-20%	27%	-3%	-38%	-11%	77%	1%	-6%	5%	3%	1%	2%	-1%
3	-26%	19%	-18%	25%	14%	-60%	35%	-53%	-9%	1%	-3%	3%	2%	0%	0%
4	-29%	8%	-8%	15%	24%	14%	-34%	-10%	12%	28%	-32%	-65%	-22%	-5%	6%
5	-29%	4%	-6%	10%	20%	14%	-47%	-21%	5%	-53%	32%	4%	40%	-4%	11%
6	-29%	0%	-3%	7%	21%	16%	-26%	-7%	5%	17%	-18%	72%	-42%	2%	-8%
7	-29%	-4%	2%	-1%	18%	21%	16%	6%	-12%	40%	12%	0%	46%	48%	-42%
8	-29%	-6%	4%	-5%	15%	19%	35%	12%	7%	7%	6%	1%	13%	-81%	-18%
9	-29%	-9%	7%	-8%	13%	18%	39%	13%	14%	-2%	-9%	5%	6%	23%	77%
10	-29%	-11%	9%	-11%	2%	9%	22%	3%	-17%	-31%	47%	-23%	-60%	18%	-20%
15	-28%	-18%	13%	-18%	-21%	-7%	-14%	3%	-79%	-8%	-35%	0%	9%	-8%	8%
20	-27%	-21%	17%	-23%	-22%	-16%	5%	0%	49%	-37%	-46%	-3%	7%	14%	-32%
30	-26%	-24%	19%	-25%	-35%	-34%	-28%	-9%	21%	45%	41%	1%	2%	-9%	18%

Table 12: Interest rates Eigen Vectors

Eigen Values	11.077	1.379	0.966	0.752	0.283	0.159	0.101	0.073	0.058	0.045	0.036	0.027	0.020	0.015	0.011
Cumulated Inertia	74%	83%	89%	94%	96%	97%	98%	99%	99%	99%	100%	100%	100%	100%	100%

Table 13: Interest rates Eigen values and cumulated inertia

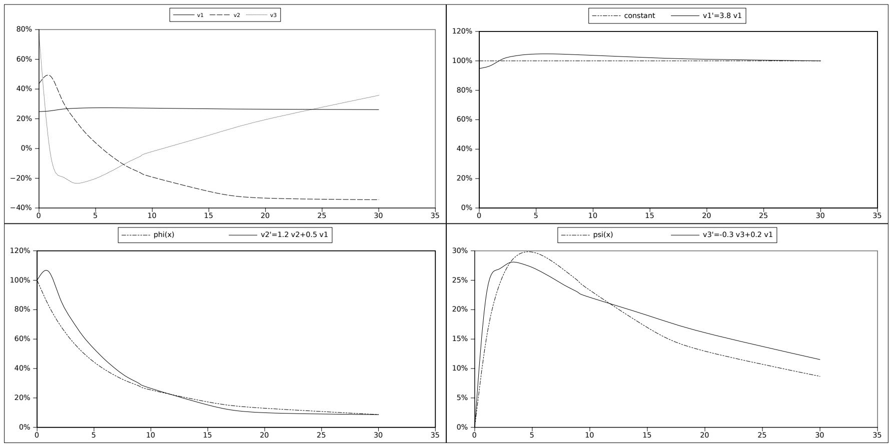

Figure 14: $v1$,$v2$,$v3$ are the first three eigen vectors and $v1'$,$v2'$,$v3'$ are linear combinations of them to fit (in the least square sense) Nelson-Siegel Functionals

τ	$Corr(l,s)$	$Corr(l,c)$	$Corr(c,s)$	Sum of squares
2.0	36.55%	-11.12%	-19.42%	18.36%
2.1	36.60%	-9.82%	-16.04%	16.93%
2.2	36.64%	-8.51%	-12.55%	15.72%
2.3	36.68%	-7.20%	-8.98%	14.78%
2.4	36.71%	-5.91%	-5.37%	14.11%
2.5	36.73%	-4.64%	-1.76%	13.74%
2.6	36.75%	-3.40%	1.83%	13.65%
2.7	36.75%	-2.19%	5.37%	13.84%
2.8	36.75%	-1.03%	8.84%	14.30%
2.9	36.73%	0.07%	12.21%	14.98%
3.0	36.71%	1.12%	15.47%	15.88%

Table 14: Correlation between risk factors in function of τ