Life Insurance in a Contingent Claim Framework: Pricing and Regulatory Implications

Eric BRIYS *

François de VARENNE **

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* Professor of Finance, Groupe HEC, France

** PhD Student, Groupe HEC, France and Research Fellow, FFSA, France

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Eric Briys

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Abstract

In this paper we develop a contingent claim model to evaluate the equity and liabilities of a life insurance company. The limited liability of shareholders is explicitly modelled. We focus on a specific type of life insurance policy, namely the profit-sharing policy. In this policy, the policyholder is entitled to a guaranteed interest rate and a percentage of the company's yearly financial revenues. The implicit equilibrium interest rate and profit sharing ratio are derived and analyzed. We finally discuss regulatory measures frequently encountered in the life insurance business such as rate ceilings, capital ratios and asset restrictions.

'I am not sure there are any serious issues confronting the life insurance industry these days, unless of course you consider solvency, liquidity, junk bonds, deteriorating mortgage and real estate portfolios, risk-based capital requirements, asset mix, separate accounts, credit risk, congressional inquiries, shrinking surplus, demutualization and more.'

Salvatore R. Curiale
Superintendant of New York
State Insurance Department

1. Introduction

The Savings and Loans debacle, the soaring number of financial institutions insolvencies have been a traumatic event for the United States. The roaring eighties, as they call them, have been a period where a lot of changes affected the US financial landscape. The rising interest rates in the early eighties led to a significant flow of consumers dollar into mutual funds. The competition for the savings dollar became very fierce among financial institutions. This pressure combined with regulatory mistakes (see White [1991]) had a perverse consequence on many financial institutions. They asumed new bets by reaching for riskier assets offering higher yields and/or by operating with less capital per dollar of assets. In that respect, the example of the life insurance business is very informative. Life insurance companies were forced to redesign their product lines and to migrate towards interest rate sensitive products (see Wright [1991]). The obvious consequence of this shift to investment oriented policies was a drastic change in investment practices. Assets were restructured to search for higher yields and to trigger asset and liabilities mismatching.

The end result was, as we know now, disastrous. Before 1987 less than ten life insurance companies insolvencies were to be counted. In 1987, nineteen companies went bankrupt. In 1989, that same number soared to a worrying 40. 1991 established a new record with 58 insolvent life insurance companies !

For instance First Executive Corporation ($ 19 billion of assets) and First Capital Holdings Corporation ($ 10 billion of assets) were trapped by junk bonds representing more than 40% of their assets. Baldwin United Corporation went bankrupt by mismanaging the interest rate exposure of its SPDA liabilities. Monarch Life ($ 4.5 billion of assets), a leader in variable life, went bankrupt because of its overconcentration in high risk real estate deals.

No need to say that this very costly turmoil has triggered a lot of regulatory actions. Lessons have to be drawn from this debacle to understand what went awry and to make sure that it never happens again. As Rep. Dingell of the State of Michigan puts it, 'Failed Promises' should no longer be allowed. The objective of the paper is to contribute to a better understanding of the driving forces of a life insurance company. Indeed, to assess the riskiness of life insurance operations, the dynamics of assets and liabilities have to be understood in the first place.

The primary purpose of this article is to evaluate the equity and liabilities of a life insurance company given the risk profile of its assets and its liabilities.

We specifically take into consideration four types of risk: asset risk, interest rate risk, leverage risk and default risk. To be able to do so, we use a model which is both utility free and market-value based. Although market value accounting is still debated (see Bernard, Merton and Palepu [1992], Beaver, Datar and Wolfson [1990], White [1991]) and may be difficult to put into practice, it is by far less misleading than the standard historical accounting approach. Our model is based on Merton's contingent claim approach to financial intermediaries [1977, 1978, 1990]. An obvious advantage of the contingent claim analysis is that it enables to capture the shareholders'option to walk away when things go wrong in their firm [1]. Only one type of life insurance policy is considered in the following. This

[1] The application of option pricing to insurance avoids some problems encontered with CAPM: especially the effect of insolvency on the shareholders'fair rate of return (see Doherty and Garven [1986], D'Arcy and Doherty [1988]).

is the most common one in France. It only contains a savings component and a single premium inflow. The rate to be served on the policy is fixed and guaranteed. However, regulation imposes a profit-sharing mechanism. Indeed, by law, french life insurers have to pay policyholders at least 85% of their net financial revenues, namely dividends, coupons, realized capital gains etc...

To sum up, policyholders benefit from a guaranteed interest rate and a percentage of the performance of the company's asset portfolio. This profit sharing mechanism is known in France under the name of 'Participation aux Bénéfices' and bears some analogy to the anglosaxon 'with profits' policies. These anglosaxon policies insure customers for a lowish basic sum, which is then topped up with discretionary bonuses (the 'profits'), depending on how the insurer's investments perform [2]. As a result two key inputs characterize such policies: the guaranteed interest rate and the participation level. Our model enables us to determine the fair interest rate or the fair participation level policyholders should require to fully compensate them for the risks they face. The structure of the policy is thus endogenous to the model. The whole model is driven by a competitive market assumption: life insurance companies are both rate and participation level takers.

As an outgrowth of the model, we also examine some current regulations dealing with rate ceilings, asset composition, capital ratios. We show that some of these regulations are either contradictory or redundant.

The paper is structured as follows. The section 2 introduces the model and its main assumptions. In section 3, we develop the model and derive the market values of equity and liabilities of the life insurance company. In section 4, the fair interest rate and participation level are computed and their properties are analyzed. We assess their sensitivity to changes in the various parameter affecting the life insurance company. Section 5 examines regulatory issues and draws some implications. A conclusion summarizes the main findings and suggests three further avenues of research.

2 According to a recent article published by The Economist (July 17th 1993), UK firms have started to cut the bonuses on 'with profits' policies. Indeed, a lot of UK insurers appear to be too 'vulnerable' to sustain high 'with profits' levels.

2. The model and its assumptions

We consider a life insurance company whose planning horizon extends over a given time interval $[0, T]$. Time $t = T$ can be considered as the time at which the company is subject to a comprehensive on site audit by regulators. In the United States, these audits usually occur every three to five years. Their primary purpose is to assess the net worth of the life insurance company and to check that it is solvent. Indeed, if at the time of the audit assets are found to be less than liabilities, the insurance company's assets are costlessly transfered to policyholders. If the company's assets exceed liabilities, the company is allowed to pursue its operations uninterrupted. We thus lend a rather passive role to regulators like Cummins [1988] and Doherty and Garven [1986]. They follow a simple intervention policy. It is true that in reality regulators have a more active role and follow intervention rules that are more complex (see for instance Cummins, Harrington and Niehaus [1993]). The purpose of the present paper is however not on regulation per se and only simple rules are considered.

Insurance and financial markets are assumed to be competitive. The life insurance company is a price-taker and therefore it will have to service policyholders on a market basis. At time $t = 0$, the insurance company acquires an asset portfolio A_0 and finances this portfolio with paid-in capital E_0 and a homogenous life insurance policy expiring at time T. If the company is not declared insolvent, new policies can be written for another period. The life insurance policy is structured as follows. The policyholder is guaranteed a fixed interest rate r^*. On top of this fixed interest rate, the policyholder is entitled to a share δ of the net financial revenues (dividends, net capital gains, coupons...) of the life insurance company. This policy is quite frequent in France where state regulation makes it compulsory for life insurance companies to pass on to policyholders at least 85% of their financial revenues (the so-called 'participation bénéficiaire').

Brennan [1993] shows why with profits insurance policies turn out to be inefficient under specific assumptions. Indeed, a participating or with profits policy will be 'costly or inefficient in the sense that there will exist another policy that does not involve ratcheting and provides the same distribution of final wealth while requiring a lower investment'. However, both Brennan [1993] and Merton [1989] account for the existence of such policies by arguing that transaction costs provide a 'raison d'être' for financial intermediation. The insurance contract can then be viewed as a product that is not directly available on the market. By issuing such a contract, the insurance company contributes to a more complete market.

The regulation on the participation level in France is dated back 1966. At that time, the insureds had still in mind the poor after-war performance of their life policies. Indeed, their policies were not protected against inflation. That is why regulators decided at the time to introduce some form of indexation through the so-called 'participation bénéficiaire'.

The company is thus obliged to tell policyholders the fraction δ ($\delta \geq 85\%$) that it is going to distribute them. Although inflation is claimed by regulators to be at the origin of the participation mechanism, there is another way of rationalizing it.

The guaranteed rate r^* is usually less than the market rate for a riskfree asset of the same maturity as the policy. The participation coefficient δ can be viewed as making up for the difference between the two rates and embodying the required risk premium by policyholders holding risky life insurance policies. Indeed, shareholders have a limited liability and, in the case the company is declared insolvent at time $t = T$, they simply walk away. Under that scenario, policyholders only receive what is left [3]. When the model is developed, this story will become quite clear.

At time $t = 0$, the company's balance sheet looks like as follows:

Assets		Liabilities and Equity	
Assets	A_0	Liabilities	$L_0 = \alpha A_0$
		Equity	$E_0 = (1 - \alpha) A_0$
Total	A_0	Total	A_0

[3] Ingersoll [1987] explains why the policyholders have no incentive to renegociate their contracts in case of default.

For the sake of simplicity, we normalize A_0 to the value \$ 1. $(1-\alpha)$ denotes the proportion of the initial assets A_0 financed by equity. This is a decision variable of the company. The initial portfolio of assets is assumed to be totally invested in risky assets (equity, risky bonds, real estate ...).

The first risk element of the balance sheet is interest rate risk. To capture the uncertainty in the term structure of interest rates, we use the Heath-Jarrow-Morton [1992] process, where the initial forward rate curve $f(0,t)$ is given. Under this assumption, the instantaneous risk free rate r_t at time t is given by (see Heath-Jarrow-Morton [1992, p. 90, eq. 29]):

$$r_t = f(0,t) + \frac{1}{2}\sigma_P^2 t^2 + \sigma_P W_t \tag{1}$$

where σ_P denotes the interest rate volatility and W_t a standard Wiener process.

From (1), one can write the price dynamics of a default free zero-coupon bond maturing at time T:

$$\frac{dP(t,T)}{P(t,T)} = r_t dt - \sigma_P(T-t)dW_t \tag{2}$$

We define the time $t=0$ yield-to-maturity $\overline{r}$ by:

$$P(0,T) = e^{-\overline{r}T}$$

or, by using the initial forward rate curve $f(0,t)$:

$$\overline{r} = \frac{1}{T}\int_0^T f(0,t)dt$$

The second risk element is asset risk, that is all risk affecting assets (equity, real estate...) other than the interest rate risk. To give a complete picture of the riskiness of the insurance company, the portfolio of assets is assumed to be affected by both the interest rate risk and the asset risk.

As a result, the portfolio of assets, A_t, is assumed to be governed by the following stochastic process:

$$\frac{dA_t}{A_t} = \mu dt + \sigma_A \left[\rho dW_t + \sqrt{1-\rho^2} dZ_t\right] \tag{3}$$

where μ and σ_A denote respectively the instantaneous expected return on assets and their instantaneous volatility. Z_t is a standard Wiener process independent of W_t capturing the asset risk other than the interest rate risk. The coefficient ρ, included between -1 and $+1$, represents the correlation between the total value of assets A_t and the interest rate r_t. In other words, ρ corresponds to the share of interest rate risk in the total risk of the assets.

More specifically, the total variance of assets σ_A^2 can be split into two parts:

- an interest rate risk component: $\rho^2\sigma_A^2$
- an asset risk component: $(1-\rho^2)\sigma_A^2$

Let us define $L_T^* = \alpha A_0 e^{r^* T}$ which is the guaranteed payoff to policyholders.

The policyholders payoffs are contractually defined as follows:

1st case: The insurance is totally insolvent: $A_T < L_T^*$. The time T value of assets is below the guaranteed liability. The company is declared bankrupt and the policyholders receive what is left:

$$L_T^* = A_T$$

2nd case: The company is able to fullfill its guaranteed commitment but unable to serve the 'participation bénéficiaire'. Assuming that the policyholders assets are earmarked, we define FR_T the financial revenues to policyholders after guaranteed commitments have been fullfilled:

$$\begin{aligned} FR_T &= \delta\,[\frac{L_0}{A_0}(A_T - A_0) - (L_T^* - L_0)] \\ &= \delta\,[\alpha A_T - L_T^*] \end{aligned} \tag{4}$$

where δ denotes the contractual participation coefficient. If FR_T is negative, it amounts to:

$$A_T < \frac{L_T^*}{\alpha}$$

That is:

$$L_T = L_T^*$$

3rd case: FR_T is positive, or $A_T \geq \frac{L_T^*}{\alpha}$. Assets generate enough value to match the guaranteed payment and the policyholders'participation. In such a case, the liabilities at time T are equal to:

$$\begin{aligned} L_T &= L_T^* + FR_T \\ &= L_T^* + \delta(\alpha A_T - L_T^*) \\ &= (1-\delta)L_T^* + \delta\alpha A_T \end{aligned} \tag{5}$$

To sum up, the first case corresponds to a case of total insolvency ; the second case is a partial insolvency in the sense that only guaranteed commitments are fullfilled ; the third case corresponds to a fully solvent scenario.

The value L_T^*/α delimiting the third case is not innocuous. Indeed, with our notation:

$$\frac{L_T^*}{\alpha} = \frac{L_0 e^{r^* T}}{L_0/A_0} = A_0 e^{r^* T}$$

The company starts to share its profits as soon as the rate of return on the assets is bigger than the guaranteed interest rate r^*.

Because of limited liability, the shareholders'stake is a residual stake. The final payoffs E_T are as follows:

1st case:	$E_T = 0$	if $A_T < L_T^*$
2nd case:	$E_T = A_T - L_T^*$	if $L_T^* \leq A_T < \frac{L_T^*}{\alpha}$
3rd case:	$E_T = (1-\delta\alpha)A_T - (1-\delta)L_T^*$	if $\frac{L_T^*}{\alpha} \leq A_T$

These payoffs suggest that equity has the features of a contingent claim written on the insurance company's assets. Indeed, cash-flows are truncated. Since the seminal works of Black and Scholes [1973] and Merton [1973], it is well-known that the limited liability equity of a levered firm can be valued as a contingent claim on the firm's underlying assets.

3. Equity and liabilities valuation

By applying the option pricing framework (see Merton [1990], Ronn and Verma [1986], Crouhy and Galai [1991]), the market value of both equity and liabilities can be assessed. As far as equity is concerned, section 2 indicates that the value of equity as of time T is given by:

$$E_T = Max\Big\{0, Min\Big[A_T - L_T^*, (1-\delta\alpha)A_T - (1-\delta)L_T^*\Big]\Big\}$$

Namely:

$$E_T = Max[0, A_T - L_T^*] - \delta\alpha\, Max[0, A_T - \frac{L_T^*}{\alpha}]$$

The equity position is an hybrid position and its value as of time t, $0 \leq t \leq T$, is given by:

$$E_t = C_E(A_t, L_T^*) - \delta\, C_E(\alpha A_t, L_T^*) \tag{6}$$

where $C_E(A_t, L_T^*)$ and $C_E(\alpha A_t, L_T^*)$ are both european calls maturing at time $T-t$ and with exercice price L_T^*.

From Heath-Jarrow-Morton [1992], it is easy to show that the value of the call $C_E(A_t, L_T^*)$ is given by:

$$C_E(A_t, L_T^*) = A_t N(d_1) - P(t,T) L_T^* N(d_2) \tag{7}$$

where

$$
\begin{cases}
d_1 = \dfrac{\ln(A_t/L_T{}^{*}P(t,T)) + \frac{1}{2}\overline{\sigma}^2\tau}{\overline{\sigma}\sqrt{\tau}} \\
d_2 = d_1 - \overline{\sigma}\sqrt{\tau} \\
\overline{\sigma}^2 = \sigma_A{}^2 + \rho\sigma_A\sigma_P\tau + \sigma_P{}^2\frac{\tau^2}{3} \\
\tau = T - t \\
P(t,T) = \text{the price at time } t \text{ of a riskless zero-coupon bond maturing at time } T \\
N(\;) = \text{the cumulative normal distribution}
\end{cases}
$$

In the same way, we can write:

$$C_E(\alpha A_t, L_T^*) = \alpha A_t N(d_3) - P(t,T)L_T^* N(d_4) \tag{8}$$

where

$$
\begin{cases}
d_3 = \dfrac{\ln(\alpha A_t/L_T{}^{*}P(t,T)) + \frac{1}{2}\overline{\sigma}^2\tau}{\overline{\sigma}\sqrt{\tau}} \\
d_4 = d_3 - \overline{\sigma}\sqrt{\tau}
\end{cases}
$$

With (7) and (8), equation (6) becomes:

$$E_t = A_t[N(d_1) - \delta\alpha N(d_3)] - P(t,T)L_T^*[N(d_2) - \delta N(d_4)] \tag{9}$$

Equity is a portfolio of two european calls. The first one is the traditional limited liability call. Shareholders have the option to walk away if things go wrong. The second call corresponds to a short position. Indeed, shareholders have written a call to policyholders by introducing a contractual asset based participation clause. Equity is thus made of a long call position and a short call position, the latter being weighted by the participation coefficient δ.

The figure 1 pictures the time T cashflow position of the company's shareholders.

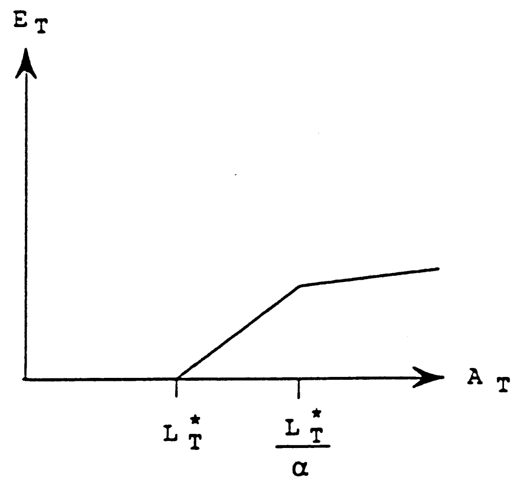

Figure 1 **Shareholders'final payoffs**

As far as liabilities are concerned, the final payoffs of section 2 indicate that they can be priced as follows:

$$L_t = L_T^* P(t,T) - P_E(A_t, L_T^*) + \delta C_E(\alpha A_t, L_T^*) \qquad (10)$$

where $P_E(A_t, L_T^*)$ denotes the price of the shareholders' put to default, that is to walk away from their guaranteed commitments.

The liabilities are thus made of a long position on a riskfree payoff, a short position on a put to default and a long position on a call on financial revenues.
More specifically, the first two terms in (10) represent the value of a risky policy without participation (i.e. a risky bond). The third term is a call option on the α fraction of the firm with the exercice price L_T^*.

Using the formula for the european put, equation (10) can be written as follows:

$$L_t = A_t[N(-d_1) + \delta\alpha N(d_3)] + P(t,T)L_T^*[N(d_2) - \delta N(d_4)] \qquad (11)$$

This combined position can be summarized by the figure 2 picturing liabilities cashflows as of time T.

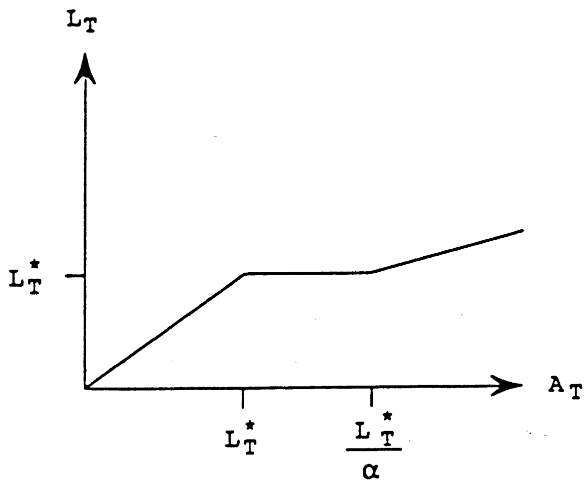

Figure 2 Policyholders'final payoffs

From equations (9) and (11), we can see that the relevant volatility parameter for pricing both equity and liabilities is $\overline{\sigma}$. $\overline{\sigma}$ is the volatility of the ratio $A_t/P(t,T)$, or in other words, the volatility of the firm assets using the default-free zero-coupon bond as the numeraire.

4. The required guaranteed interest rate and participation level

Since the life insurance company is assumed not to be (re)insured, policyholders do face the risk that their contract does not perform as initially planned. Shareholders can walk away if things go wrong. Policyholders are then paid on what is left. As a consequence, policyholders ask for a risk premium to compensate them for the risks they are carrying. Policyholders have two ways to be rewarded. For a given level of participation coefficient δ, they will adjust the rate r^* so that they get a fair rate of return on their savings. Or, for a given rate r^*, they will make sure that the participation coefficient δ is such that the insurance policy offers an ex-ante fair rate of return. Following Crouhy and Galai [1991], this will translate into an equilibrium condition such that the present value of equity is equal to the initial equity issuing price:

$$E_0 = PV(E_T) = (1 - \alpha)$$

avec $A_0 = 1$

or, equivalently,

$$C_E(1, L_T^*) - \delta C_E(\alpha, L_T^*) = 1 - \alpha$$

$$[N(d_1) - \delta\alpha N(d_3)] - \alpha e^{(r^* - \overline{r})T}[N(d_2) - \delta N(d_4)] = 1 - \alpha \tag{12}$$

where $\overline{r}$ is defined as $P(0,T) = e^{-\overline{r}T}$.

Indeed, on one hand, shareholders will never invest in the life insurance company if the present value of their investment is less than their initial outlay. On the other hand, policyholders will make sure that either the guaranteed rate r^* or the participation level δ is compatible with a fair risk-adjusted return on their claim.

As a result equation (12) gives either the guaranteed rate r^* or the participation level δ as the equilibrating variable. If δ is given, the guaranteed rate r^* remains to be determined so that equation (12) is satisfied. If r^* is given, equation (12) yields the equilibrating value of the participation coefficient δ. It is worthwhile noting that while r^* cannot be computed explicitly, an analytical expression for the participation coefficient δ can be derived:

$$\delta = \frac{C_E(1, L_T^*) - (1 - \alpha)}{C_E(\alpha, L_T^*)} \tag{13}$$

Replacing the respective calls by their closed form solutions in (13) yields:

$$\delta = \frac{\alpha[1 - e^{(r^* - \overline{r})T} N(d_2)] + N(d_1) - 1}{\alpha[N(d_3) - e^{(r^* - \overline{r})T} N(d_4)]} \tag{14}$$

where d_1, d_2, d_3 and d_4 are defined as above.

By applying the put-call parity, the participation level δ can be rewritten as a function of the shareholders'put to default:

$$\delta = \frac{\alpha - L_T^* P(0,T) + P_E(1, L_T^*)}{C_E(\alpha, L_T^*)} \tag{15}$$

We now examine the impact of the different parameters (volatility $\overline{\sigma}$, capital ratio α, etc...) on the equilibrating values of the guaranteed interest rate r^* and of the participation level δ.

Let us rewrite equation (12) as:

$$g(r^*, \overline{r}, \alpha, \overline{\sigma}, \delta) = [N(d_1) - \delta\alpha N(d_3)] \\ - \alpha e^{(r^* - \overline{r})T}[N(d_2) - \delta N(d_4)] - (1-\alpha) = 0 \tag{16}$$

This function g will be used in the following for computing the relevant comparative statics.[4]

- **The impact of total volatility on the level of the guaranteed interest rate**

To capture the response of the guaranteed interest rate r^* to a change in total volatility $\overline{\sigma}$, one implicitly differenciates the function g as defined in (16) which yields the following:

$$\frac{dr^*}{d\overline{\sigma}^2} = -\frac{\partial g / \partial \overline{\sigma}^2}{\partial g / \partial r^*} \tag{17}$$

It is fairly easy to show that the denominator is always negative. This in turn implies that:

$$sgn(\frac{dr^*}{d\overline{\sigma}^2}) = sgn(\frac{\partial g}{\partial \overline{\sigma}^2}) \tag{18}$$

4 The various computations and graphs have been performed through numerical methods available on Mathematica.

After computations, one can write:

$$\frac{\partial g}{\partial \overline{\sigma}^2} = \frac{\sqrt{T}}{2\overline{\sigma}}[N'(d_1) - \alpha\delta N'(d_3)] \tag{19}$$

Except for the case where the participation coefficient δ is nil, the sign of this expression is ambiguous.

The figure 3 depicts the guaranteed rate r^* as a function of total volatility $\overline{\sigma}$ for different participation levels δ.

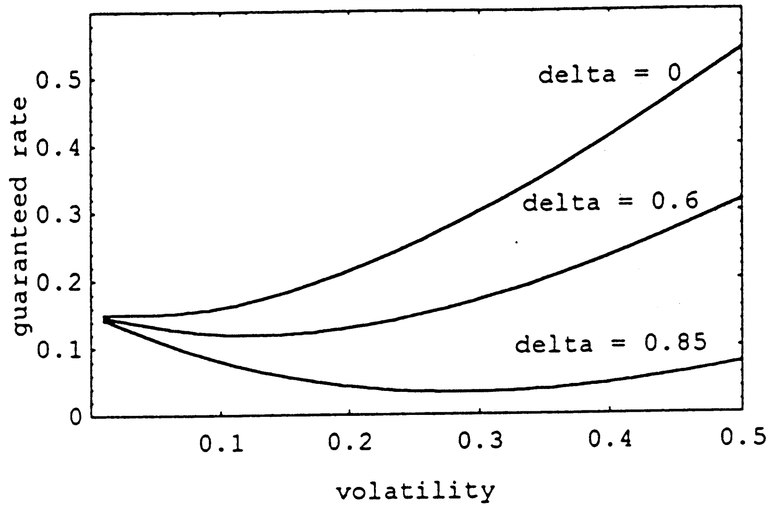

Figure 3 The guaranteed interest rate r^* as a function of total volatility $\overline{\sigma}$ (α=0.9, $\overline{r}$= 0.15, T=1)

The base case is the case where δ is equal to zero. Indeed, it corresponds to the situation analyzed by Crouhy and Galai [1991]. As indicated in (19), the interest rate r^* always increases when total volatility $\overline{\sigma}$ increases. This effect simply reflects an increasing risk premium required by policyholders for the growing risk they face.

The case of a non-zero δ is not as clear-cut. It can however be shown that the curve linking r^* to $\overline{\sigma}$ is U-shaped. For low volatility level the guaranteed rate r^* is decreasing. For high volatilty levels, r^* is increasing. Policyholders face a volatility dilemma. Indeed, they are both short a put to default and long a call to 'participate'. On one hand, an increase in volatility accentuates the depth-in-the-money of the put to default. On the other hand, it has a positive effect on their participation to financial revenues of the insurance company. For low level of volatility, this latter effect dominates. When the participation level δ increases, policyholders become more volatility prone.

- **The impact of total volatility on the level of the participation coefficient**

By differentiating (14) with respect to $\overline{\sigma}^2$, it obtains:

$$\frac{\partial\delta}{\partial\overline{\sigma}^2} = \frac{C_E(\alpha, L_T^*)N'(d_1) - \alpha[C_E(1, L_T^*) - (1-\alpha)]N'(d_3)}{C_E^2(\alpha, L_T^*)} \frac{\sqrt{T}}{2\overline{\sigma}} \tag{20}$$

whose sign is ambiguous.

Figure 4 depicts the relationship between δ and $\overline{\sigma}$ for various levels of the capital ratio α. The same pattern as previously applies. Policyholders are interested in volatility as long as it is not too high. Again this reflects the effect of their long call position.

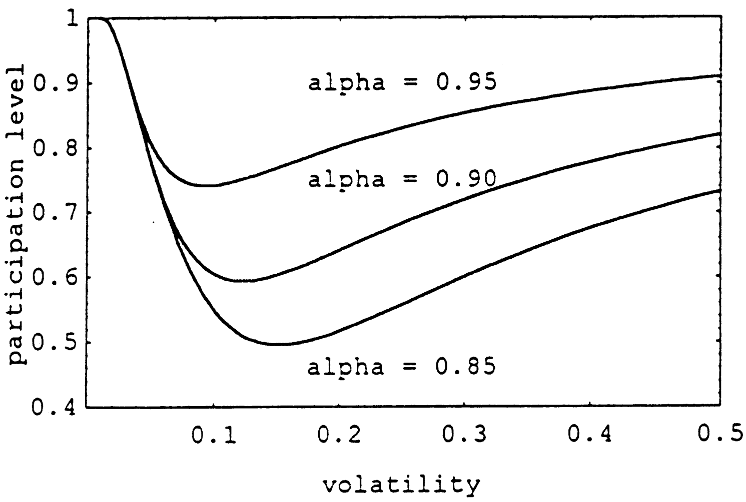

Figure 4 The participation coefficient δ as a function of total volatility $\overline{\sigma}$ ($\overline{r}$=0.15, r^*= 0.12, T=1)

It is worthwhile noting that the equilibrating participation coefficient δ may well be below the french regulatory threshold of 85%, as shown by figure 4 and table 1. Forcing the company to apply a minimum 85% level may have two consequences:

either the company reduces the riskiness of its assets (left shift) or it really increases it (right shift). The second solution is certainly not what is expected by regulators !

Panel A: r^*=0.1125

$\overline{\sigma}$ \ α	0.70	0.75	0.80	0.85	0.90	0.95	0.99
0.05	0.85	0.85	0.85	0.85	0.85	0.87	0.96
0.10	0.61	0.61	0.61	0.63	0.67	0.78	0.95
0.15	0.47	0.48	0.50	0.55	0.65	0.79	0.95
0.20	0.40	0.43	0.48	0.57	0.67	0.82	0.96
0.25	0.38	0.42	0.49	0.59	0.70	0.84	0.97
0.30	0.38	0.45	0.53	0.62	0.73	0.86	0.97

Panel B: r^*=0.0825

$\overline{\sigma}$ \ α	0.70	0.75	0.80	0.85	0.90	0.95	0.99
0.05	0.97	0.97	0.97	0.97	0.97	0.97	0.99
0.10	0.82	0.82	0.82	0.82	0.84	0.89	0.97
0.15	0.68	0.68	0.70	0.72	0.78	0.87	0.97
0.20	0.58	0.60	0.63	0.69	0.76	0.87	0.97
0.25	0.53	0.57	0.62	0.68	0.77	0.88	0.97
0.30	0.51	0.56	0.62	0.70	0.79	0.89	0.98

Table 1 The participation coefficient δ as a function of total volatility $\overline{\sigma}$, leverage α and guaranteed interest rate r^* ($\overline{r}$=0.15, T=1)

• The impact of the leverage on the guaranteed rate

By the same implicit differentiation token as above, it obtains:

$$sgn(\frac{dr^*}{d\alpha}) = sgn(\frac{\partial g}{\partial \alpha})$$

where:

$$\frac{\partial g}{\partial \alpha} = \frac{N(-d_1)}{\alpha}$$

which is obviously positive. Indeed, a greater financial leverage implies a higher financial risk which has to be compensated for. The figure 5 illustrates this effect.

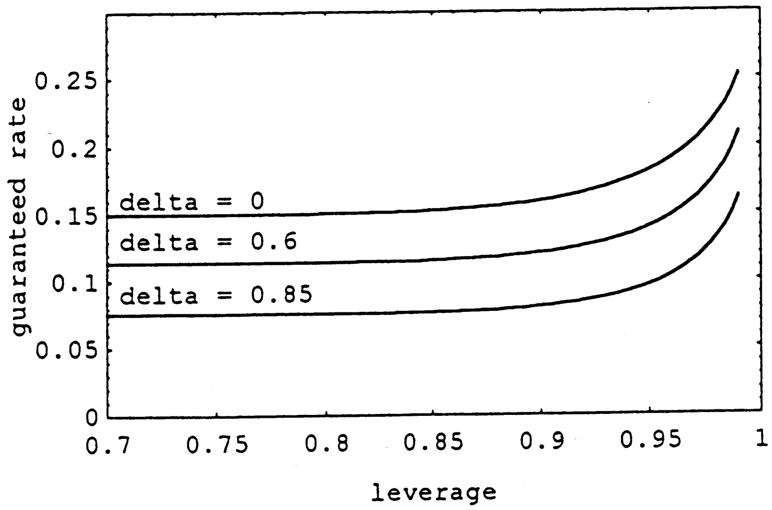

Figure 5 The guaranteed interest rate r^* as a function of the capital ratio α ($\overline{r}$=0.15, $\overline{\sigma}$= 0.1, T=1)

5. Regulatory implications

As mentioned by Klein [1993], 'the recent spate of insurer failures coming on the heels of the savings and loan disaster and problems in the banking industry has raised concerns about the adequacy of insurance regulation'. Many congressional investigations in the United States have indeed questioned whether the current regulatory system is able to effectively supervise and regulate insurance companies that are selling increasingly complex products. However, as stressed by Klein [1993], the current regulatory system is in place and will certainly be costly 'to junk and replace'.

As far as life insurance companies are concerned, regulators usually envisage three main courses of action. First, they impose a minimum capital requirement.They control asset compositions to prevent insurance companies from investing into risky ventures. Finally, they introduce ceilings on rates that can be guaranteed to policyholders. French regulators, as already mentioned, also impose a minimum level of participation (85%).

In the following, we investigate the effects of such restrictions dealing either with the asset side or the liability side. More specifically, we show that some of these regulations are contradictory.

One of the most popular solvency regulation is the one pertaining to the capital asset ratio. Even though the accurate amont of capital needed is still debated, the widespread belief is that a stringent capital asset ratio decreases the risk of bankruptcy. In our case, a stricter regulation on capital asset ratios entails a decrease in the participation coefficient δ. Policyholders carry less leverage risk and, as a consequence, lower their participation requirement as shown in figure 6.

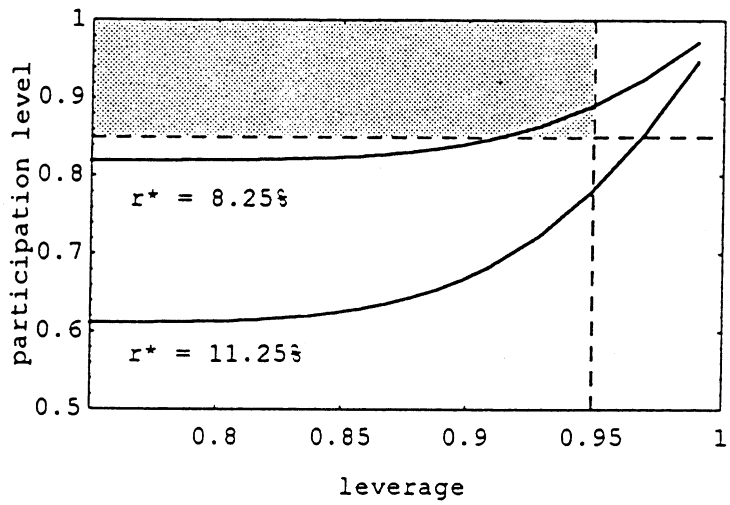

Figure 6 The participation coefficient δ as a function of the capital ratio α ($\overline{r}$=0.15, $\overline{\sigma}$= 0.1, T=1)

However, imposing at the same time a minimum level for δ (such as 85%) may lead to inconsistencies. In figure 6, the shaded area represents the feasibility area where both regulations, capital ratio (in France α_{max} is roughly 95%) and participation level (δ_{min} = 85%) are effective. For an interest rate r^* equal to 11.25%, no equilibrating values can be found. Moreover, a recent french regulation has introduced a ceiling on the interest rate r^*. It cannot exceed 75% of a rate average of Treasury Bonds. Regulators claim that such a ceiling lessens the incentives of companies to invest in risky portfolios. Assume in figure 6, that a company wants to operate at the ceiling ($r^* = 75\% \times 15\% = 11.25\%$). Obviously, it cannot meet the minimum level of participation δ. The solution is then to lower the rate r^* to 8.25%. In that case, there are points in the second curve which lie within the shaded area. But this means that the ceiling regulation is useless. The constraint on the participation level δ is so stringent that the ceiling is ineffective. In other words, the company is 'overregulated'.

The model enables us for instance to understand why with profits insurance policies are not widely sold in Germany. Indeed the minimum level of participation δ is 95%. From figure 6, we can see that such a level implies a very narrow shaded area which makes it rather difficult for German insurance companies to operate[5].

Regulators also control the riskiness of assets and often impose asset restrictions. In our model, this translates into a constraint on the maximum level of volatility. Figure 7 depicts the relationship between the participation level δ, the capital ratio α and total volatility $\overline{\sigma}$.

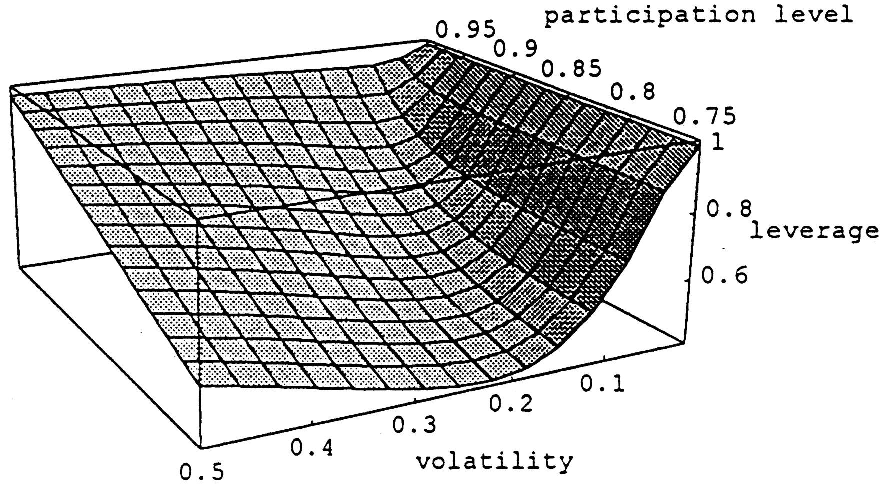

Figure 7 The relationship between the participation level δ, the capital ratio α and the total volatility $\overline{\sigma}$ ($\overline{r}$=0.15, r^*=0.1125, T=1)

Assume that the company is operating at a rate r^* which satisfies the ceiling constraint. Assume also that regulators impose a maximum volatility of 30%. One can see, from figure 7, that less capitalized companies (α close to 95%) have a greater room of manoeuvre. Indeed, there are two asset structures ($\overline{\sigma}$ < 30%)

[5] We are grateful to Diego Wauters for suggesting this interpretation.

for which the participation level δ is above its minimum. According to the well-known result of Galai and Masulis [1976], shareholders may then have an incentive to unexpectedly shift from the less risky structure to the riskier structure. For well capitalized companies there is only one choice which corresponds to a very low volatility level. At this low volatility level, they can enforce the minimum participation coefficient of 85%. Again here, some regulations are either inconsistent or redundant given market forces. Figure 4 and figure 7 also show that for some values of the relevant parameters, the asset structure constraint may be irrelevant. Indeed, assume that regulators impose a leverage constraint such that α has to be lower than 0.95 and that the asset restriction leads to a maximum volatility compatible with α equal to 0.95. Then it is obvious that the floor constraint δ is strong enough to force insurance companies to migrate towards low level of volatility (compatible with δ at least equal to 0.85).

6. Conclusion

Over the recent past, life insurance companies have been forced to redesign their product lines. The most obvious consequence of this reshuffling has been a shift towards interest rate sensitive policies. Life insurance companies have become more sensitive to interest rate movements than in the past. This trend occurs at a time where an unprecedented tide of financial insolvencies has raised growing concerns among politicians, claimholders and regulators.

By relying upon a contingent claim valuation framework, the model presented in this paper tries to capture the various risks that a life insurance carries. Asset risk, interest rate risk, default risk, leverage risk have been considered. Their respective effects on the company and its solvency have been assessed. More specifically, a competitive market assumption combined with the option pricing framework shows how assets and liabilities are intertwined. The implications of a specific asset structure, of a particular leverage ratio and of the level of default free interest rates for the contractual features of with profits policies have been drawn. Some insights on regulation have also been given.

The framework presented here is fairly general and should open at least three further avenues of research.

First, an immediate outgrowth of the model is to introduce so called embedded options. Indeed, most life insurance policies contain various types of options such as policy loan options, early lapsation options etc... For instance, when insureds are allowed to drop out early while being guaranted a fixed rate, their position is equivalent to a long position on a (zero-coupon) bond put option. It is to be expected in that case that the participation level δ should be lower ceteris paribus. However, imposing a floor on δ may have some unexpected consequences and imply that embedded options are mispriced.

The second area for future research relates to be immunization concept. From our model, interest rate sensitivities of both assets and liabilities can be computed and used to design immunization strategies.

Finally, the model can accomodate reinsurance contracts. Possible risk-shifting behaviour between the insurance and the reinsurance company can then be analyzed.

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