Modelling and Valuation of Guarantees in With-Profit and Unitised With Profit Life Insurance Contracts

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Abstract

The purpose of this paper is to develop suitable valuation techniques for the broad category of participating life insurance policies. The nature of the liability implied by these contracts allows treating them as options written on the reference portfolio backing the policy. Consequently, our valuation approach is based on the classical contingent claim theory; in particular, Monte Carlo techniques are used to compute the values of the so called "policy reserve", that is the guaranteed payoff and the reversionary bonus, and the terminal bonus. The numerical results obtained are used to investigate the sensitivity of the policy reserve and the terminal bonus to changes in the model parameters. The paper also addresses the issue of a fair contract design for with-profit life insurance policies. Bearing in mind that the parameters characterizing the financial market are in general not under the control of the life insurance office, the implemented valuation procedure is used to determine the feasible set of design parameters that would lead to a fair contract.

1 Introduction

The modelling, valuation and pricing of participating life insurance contracts are important subjects for consideration because of the need for internal financial risk management of a life insurer, the need to demonstrate solvency and hence the ability to pay benefits, the need to measure profitability and the need to offer customers a product at a "fair" price. A range of groups are interested in these issues viz managers, regulators, current and future shareholders, current and future policyholders. However the contracts offer a range of guarantees and option-like features that make the task of modelling difficult (see below).

Currently, there is considerable public concern about the financial health of life insurance companies, transacting participating life insurance business. There are five principal reasons for this concern. Firstly, market interest rates have fallen since 1998 to levels, which are low relative to the guaranteed rates implicit in many types of policy design - this has led to the collapse of Nissan Mutual Life in Japan in 1997 (see Grosen and Jørgensen, 2002) and the closing of Equitable Life in 2000 in the UK to new business (see Ballotta and Haberman, 2002, for further details). Secondly, equity markets have fallen in value since the start of 2000. Thirdly, deliberate mismatching has led companies to bear more "interest rate risk" and may have led to forced asset sales at a time when the market is falling. We note that, conventionally, insurance companies choose assets to match the characteristics of the liability assumed. Matching by duration is thus used to reduce the sensitivity of the company's funds to adverse changes in interest rates. However, deliberate mismatching in favour of equities has been adopted by many companies and the situation may have been worsened by issuers of bonds choosing short maturities when interest rates were high. Fourthly, there has been a move to transparency with consumers (and commentators) seeking more information about the intricacies of policy design and being less content with benefits which depend partly, in magnitude, on the discretion of the company's actuary. Fifthly, the international move towards market based, fair value accountancy standards is likely to affect the reporting of company results and the regulations of the life insurance industry.

Participating contracts make up a significant part of the life insurance market of many industrialized countries including the US, Japan and members of the European Union. Their origin can be traced back to the early 1800s when the early life insurance industry in the UK was using an inappropriate survival model (with mortality rates that were too high) for the calculation of the premium. This practice led to unanticipated profits which were shared among the policyholder by increasing their benefits, noting that most of the companies were mutual in terms of financial structure. So the origin comes from a form of retrospective pricing adjustment.

Given this pedigree of two centuries, we might ask why these contracts continue to be offered. Brennan (1993) considers this question and identifies a number of answers. Firstly, transaction costs may explain financial intermediation. Secondly, the stability of the reversionary bonus declared may be a desirable feature from the viewpoint of the policyholder. Thirdly, the benefits payable on early death (or surrender) may mean that time path as well as the terminal value are important to the policyholder. The participating contract may then be regarded as a product that is not directly available from the financial market and (in absence, for example, of a real risk free asset) may be contributing to a more complete market (Briys and de Varenne, 1994).

Following the pioneering work of Brennan and Schwartz (1976), most of the life insurance modelling literature has focused on unit-linked contracts, with minimum survival guarantees. Despite their historic and ongoing importance, participating contracts have been ignored because of their complexity and because the implicit guarantees seemed to be of minor significance in term of high interest rates and rising equity markets. As noted above, the economics environment has changed in recent years. The literature developed with single period models, which ignored the periodic build-up of the guarantees (Briys and de Varenne, 1994, 1997) but now focuses on multi-period models. Thus we would cite Bacinello (2002), Grosen and Jørgensen (2000, 2002), Hansen and Miltersen (2002), Jensen et al (2001), Miltersen and Persson (1999), Persson and Aase (1997) who have used market-based methodology, involving arbitrage free models, to investigate a range of different policy designs. Similarly, Wilkie (1987) and Hare et al (2000) have focused on UK designs but using a simulation-based asset model with arbitrage present.

Our approach is to consider and model the most common policy designs used in the UK for unitised with profits contracts and use a market-based methodology. These designs are common in many other European countries and Japan, where interest rate guarantees are offered.

The paper is organised as follows. Section 2 describes the valuation framework and Section 3 analyses in detail the numerical simulation-based results, with particular emphasis on a comparative statics sensitivity analysis in section 3.1 and consideration of the parameter choices consistent with the "fair value" principle in section 3.2.

2 Participating contracts and valuation framework

Participating life-insurance contracts are designed so that, in return for the payment of a fixed single or annual premia, they entitle the policyholder to a certain guaranteed benefit plus a regular, periodic reversionary bonus reflecting the individual policyholders' shares of the office's profits. The reversionary bonus rate is usually determined via a smoothing adjustment to the rate of return on the portfolio of assets backing the policy. The reversionary bonus, once added, becomes part of the guaranteed benefit. We describe the guaranteed payoff and the reversionary bonus as constituting the "policy reserve". At the claim date of the contract, a terminal bonus is also paid based on the final surplus earned by the insurance company.

The liability implied by these contracts is then linked to the investment profile of the insurer and it is composed of the fixed guaranteed benefit, the variable component added periodically to the benefit and based on the returns earned by the insurance company (the reversionary bonus), and the variable component based on the final surplus (terminal bonus). All three parts are payable on the maturity date or prior death of the policyholder (for an endowment assurance type of policy). The sources of risk associated with this type of contract therefore include the risk from the financial markets, from surrenders and from mortality. In this analysis, we focus only on the first type of risk and we ignore both the possibility that the policyholder sells back the contract to the insurer (the surrender option) and mortality, recognizing that these are both areas in which the analysis may be extended.

Consider the classical Black-Scholes economy, i.e. a frictionless market with continuous trading, no taxes, no transaction costs, and no restrictions on borrowing or short sales and perfectly divisible securities. A policyholder enters at time $t = 0$ a contract maturing in T years, paying a single premium, P_0. The insurance company invests the premium into an equity-based portfolio with market value[1] $A(0) = P_0$, and whose dynamics under the risk-neutral equivalent probability measure $\hat{\mathbb{P}}$ is described by the following stochastic differential equation:

$$dA(t) = rA(t)\,dt + \sigma A(t)\,d\hat{W}(t),$$

where $\left(\hat{W}(t) : t \geq 0\right)$ is a standard one-dimensional $\hat{\mathbb{P}}$-Brownian motion, $\sigma \in \mathbb{R}^+$, and $r \in \mathbb{R}^+$ is the risk-free rate of interest.

At the beginning of each period over the lifetime of the contract, the policy reserve, P, accumulates at rate r_P so that

$$P(t) = P(t-1)(1 + r_P(t)) \quad t = 1, 2, ...T.$$

Based on evidence from Needleman and Roff (1995), Chadburn (1998), and the results of the recent Asset Share Survey by Tillinghast-Towers Perrin

[1]We consider in section 3 the possibility that equityholder's capital is required at inception so that $A(0) > P_0$.

(2001), we consider three smoothing schemes commonly used by insurance companies in the UK for the building up of the benefit and the reversionary bonus in relation to the reference portfolio A. Let r_G be the annual guaranteed rate. Then $r_P(t)$ is determined as follows.

Scheme I The rate credited on the policyholder account is the greater of the guaranteed rate r_G and the arithmetic average of the last τ period returns on the reference portfolio, so that

$$r_P(t) = \max\left\{ r_G, \frac{\beta}{n}\left(\frac{A(t)}{A(t-1)} + ... + \frac{A(t-n+1)}{A(t-n)} - n\right)\right\},$$

where $\beta \in (0,1)$ denotes the participating rate and n is the length of the smoothing period chosen as

$$n = \min(t, \tau).$$

Scheme II The policy rate is now based on the geometric average of the last τ period returns on the reference portfolio. In other words

$$r_P(t) = \max\left\{ r_G, \beta\left(\sqrt[n]{\frac{A(t)}{A(t-n)}} - 1\right)\right\},$$

where β and n are defined as before.

Scheme III The last scheme considered in our analysis is based on the concept of a smoothed asset share. Let P^1 denote the unsmoothed asset share such that

$$\begin{aligned} P^1(t) &= P^1(t-1)(1 + r_P(t)) \\ r_P(t) &= \max\left\{ r_G, \beta\frac{A(t) - A(t-1)}{A(t-1)}\right\}; \end{aligned}$$

then the policy reserve is defined as the average of the value at time (t) of the unsmoothed asset share with weight α, and the value at time $(t-1)$ of the smoothed asset share, i.e. the policy reserve itself, with weight $(1-\alpha)$. In other words

$$P(t) = \alpha P^1(t) + (1-\alpha) P(t-1)$$

with $\alpha \in (0,1)$ playing the role of the smoothing parameter (and hence replacing n).

At maturity of the contract, T, the policyholder receives the terminal value of the policy reserve and any surplus generated by the reference portfolio over the benefit, i.e.

$$P(T) + \gamma R(T),$$

where

$$R(T) = (A(T) - P(T))^{+},$$

and $\gamma \in (0,1)$ is a second participation parameter, and the notation v^{+} denotes $\max(v, 0)$. As a consequence, the terminal payoff for the insurer is

$$\begin{cases} (1-\gamma) R(T) & \text{if } A(T) > P(T) \\ P(T) - A(T) & \text{if } A(T) < P(T). \end{cases}$$

We note the distinction between the two participating parameters, whereby β affects the annual credited rate $r_P(t)$, whereas γ only operates at maturity (or on earlier death). Hence, the potential default for the insurance company is

$$D(T) = (P(T) - A(T))^{+}.$$

As such, the payoff at maturity for both the policyholder and the insurance company can be regarded as contingent claims on the reference portfolio. The risk-neutral valuation principle implies that the arbitrage-free value of each component of the terminal payoffs is calculated as

$$\begin{aligned} V_P(0) &= \hat{\mathbb{E}}\left[e^{-rT} P(T)\right], \\ V_R(0) &= \hat{\mathbb{E}}\left[e^{-rT} R(T)\right], \\ V_D(0) &= \hat{\mathbb{E}}\left[e^{-rT} D(T)\right]. \end{aligned}$$

It is clear that

$$V_D(0) = V_P(0) + V_R(0) - A(0),$$

therefore the value of the potential default can be calculated as a residual element. As will be seen in section 3.2, we defer consideration of V_D until subsequent work.

3 Numerical results

Although explicit analytical results can be obtained for $V_P(0)$ in the cases of scheme I/II when $n = 1$ and when $P(t)$ is calculated according to Scheme III (see Ballotta and Haberman, 2003)), in this study we use Monte Carlo techniques to compute the values of the policy reserve, $V_P(0)$, and the terminal bonus, $V_R(0)$. Monte Carlo simulations are based on 10,000 iterations for contracts expiring in 20 years and monitored on annual basis, i.e. the time step in each iteration is 1 year. The antithetic variable technique is implemented to increase the accuracy of the estimates.

3.1 Pricing and comparative statics

In this section, we consider the results obtained for the arithmetic crediting scheme (scheme I), and the smoothed asset share crediting scheme (scheme III) only. The results and analysis concerning the geometric scheme (scheme II) are similar to those obtained for scheme I: further details are available from the authors. Unless otherwise stated, the benchmark set of parameters is as follows:

$$A_0 = P_0 = 100; \quad r = 6\%; \quad r_G = 4\%; \quad T = 20.$$

$V_P(0)$**: Scheme I & II** In Figure 1, we consider the effect on the value of the policy reserve of different lengths of the smoothing period. As intuition suggests, V_P, is a decreasing function of n, the parameter governing this length. In fact, extending the averaging period reduces the volatility of the rate of return credited to the policy reserve, which in our model specification plays the role of the underlying asset. Consequently, as standard option theory shows, the option premium reduces.

Figure 2 shows the sensitivity of the policy reserve to the volatility parameter, σ, for different values of β ranging from 0.1 to 0.9 in steps of 0.1. From the plot, we observe that the value of the policy reserve, V_P, is an increasing function of the participation rate, β, at any level of σ. This is due to the fact that, as participation rate, the parameter β controls how much of the asset return is credited to the policy. Also, we observe that, as for any fixed strike option, the policy reserve is an increasing function of the underlying asset volatility. However, the policy reserve appears not to be very sensitive to σ when the participation rate, β, is low. In fact, as previously observed, β controls how much of the asset return feeds into the policy, and in this sense it acts as a "rescaling factor" of the asset volatility parameter. In other words, if β is small, little of the asset return volatility is transferred from the reference portfolio to the policy; however, as β increases, the policy reserve "inherits" more and more of the volatility risk affecting the reference portfolio. Different profiles of the policy reserve as a function of the guaranteed rate r_G are represented in Figure 3, for different levels of σ, from 0.1 to 0.5. As intuition suggests, the value of the policy reserve is increasing as the minimum guaranteed rate of return is raised. We note that the profile is approximately linear.

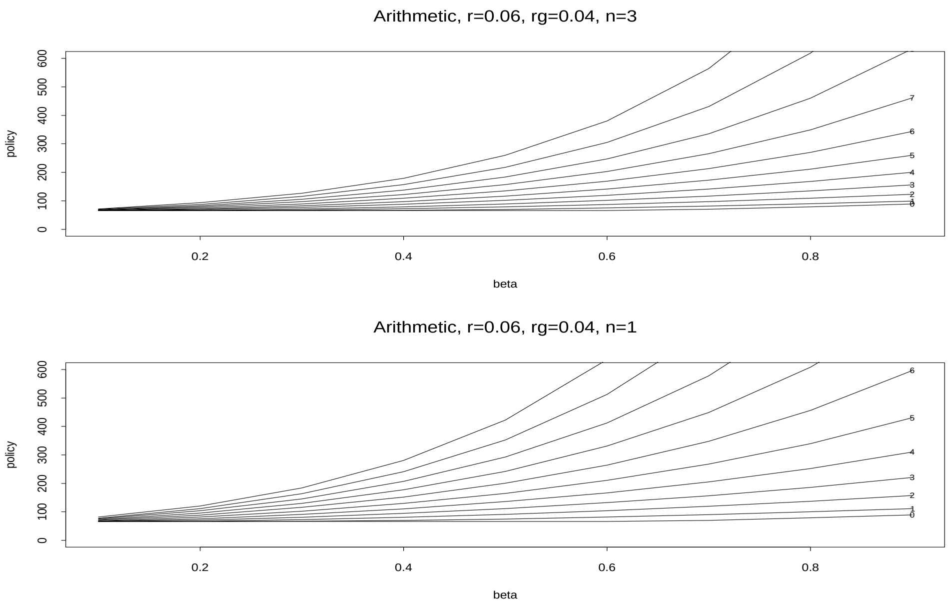

Figure 1: Arithmetic Scheme: Effect of the smoothing period on the policy reserve.

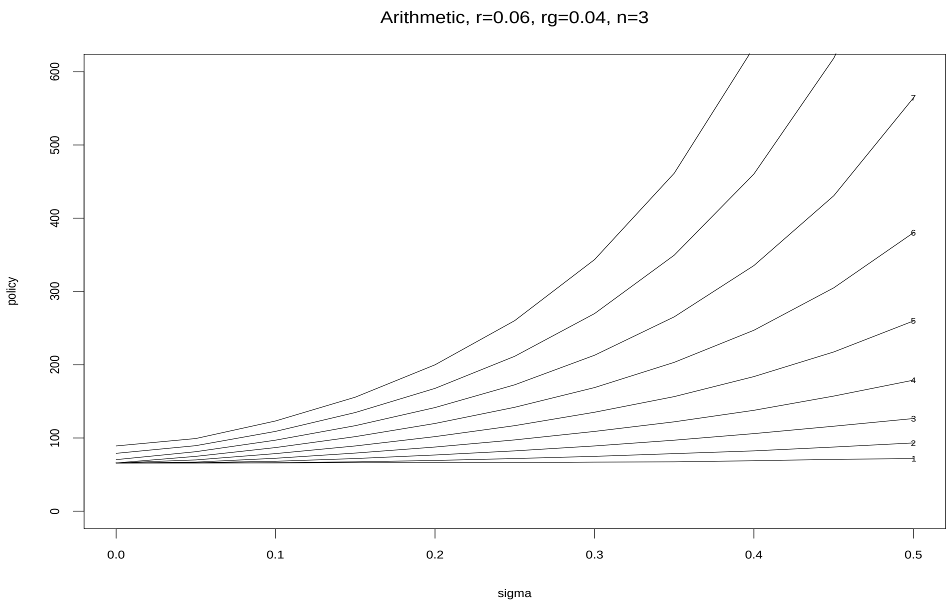

Figure 2: Arithmetic Scheme: Sensitivity of the policy reserve to the market volatility (the Vega).

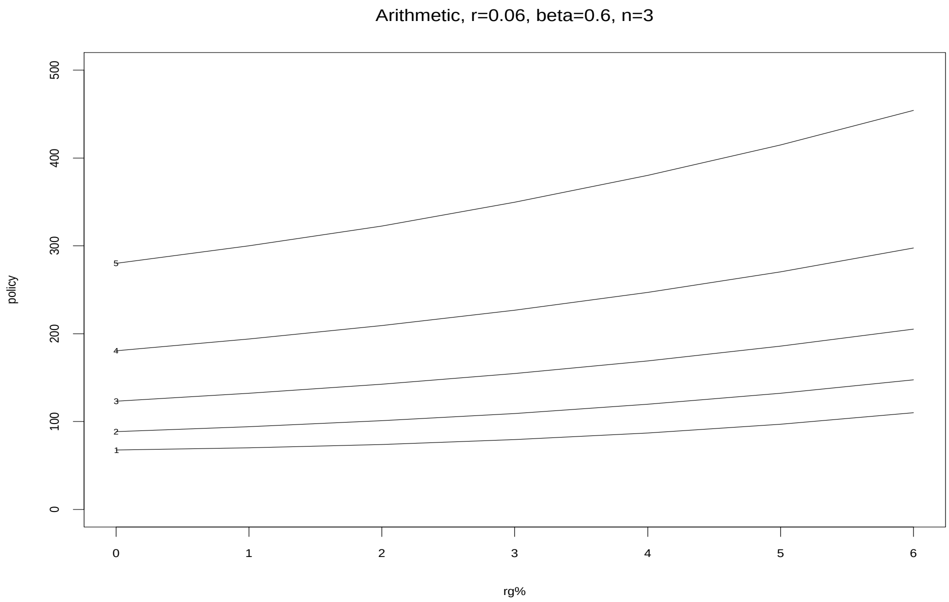

Figure 3: Arithmetic Scheme: Policy reserve vs the minimum guaranteed rate.

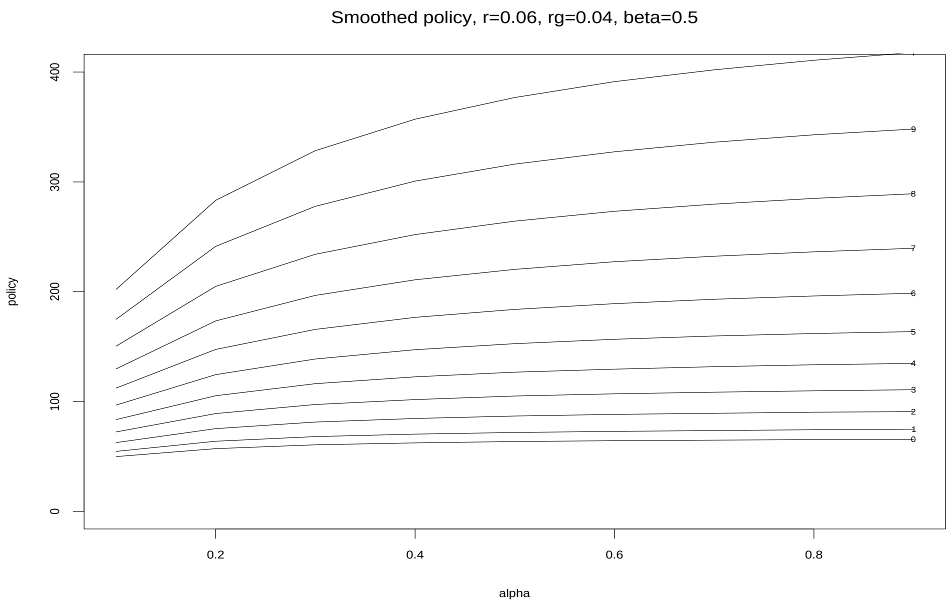

Figure 4: Smoothed Asset Share Scheme: Sensitivity of the policy reserve to the degree of smoothing.

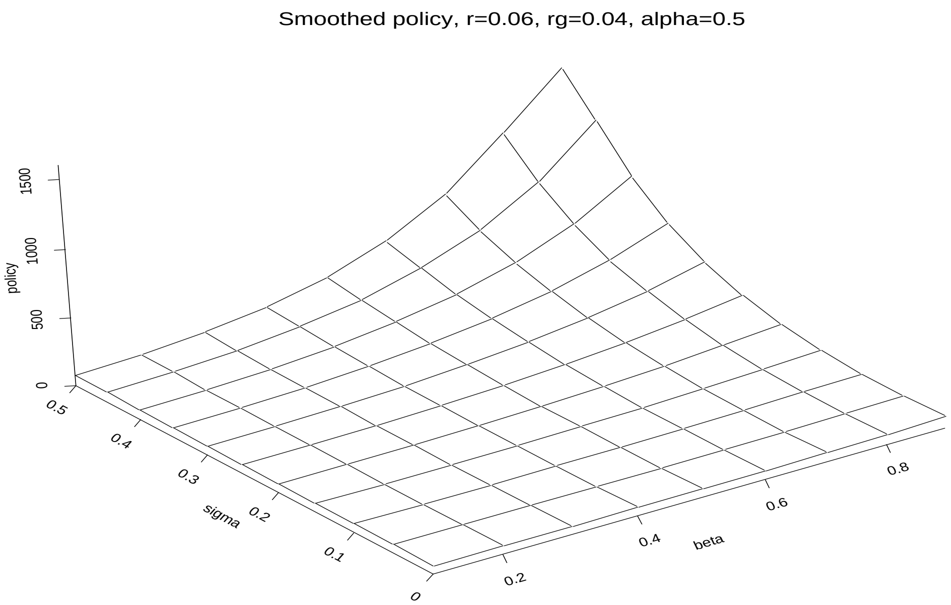

Figure 5: Smoothed Asset Share Scheme: The sensitivity of the policy reserve to changes in the volatility parameter and the participation rate.

$V_P\,(0)$**: Scheme III** Turning now to Scheme III, the smoothed asset share scheme, we observe that the results concerning the sensitivities of the policy reserve value to the main parameters considered, are similar to the ones obtained for Scheme I and II. Figure 4 shows the value of the policy reserve plotted against the smoothing parameter α, for different levels of the asset returns volatility, σ. We observe that V_P is an increasing function of α. In fact, α controls the degree of smoothing in the reversionary bonus rate, so that $\alpha \to 1$ corresponds to the case in which the full asset return over the current year is credited to the policy (hence no smoothing is being operated), whilst $\alpha \to 0$ corresponds to the heaviest degree of smoothing, as no returns originating from the reference portfolio are paid to the policy reserve. The profiles of the value V_P of the policy reserve as function of both σ and β (for a representative choice of α) and σ and α (for a representative choice of β) are represented in Figures 5 and 6 respectively. These plots show that V_P is a convex function of β and a concave function of α.

$V_R\,(0)$**:** In this section, we analyze the behaviour of the value of the terminal bonus as a function of the full set of parameters. It has to be noted that the parameters α, β, n, r_G are design parameters, related only to the structure of the policy reserve; as such they affect only $P\,(T)$, and hence V_P, but not the market value of the reference portfolio $A\,(T)$.

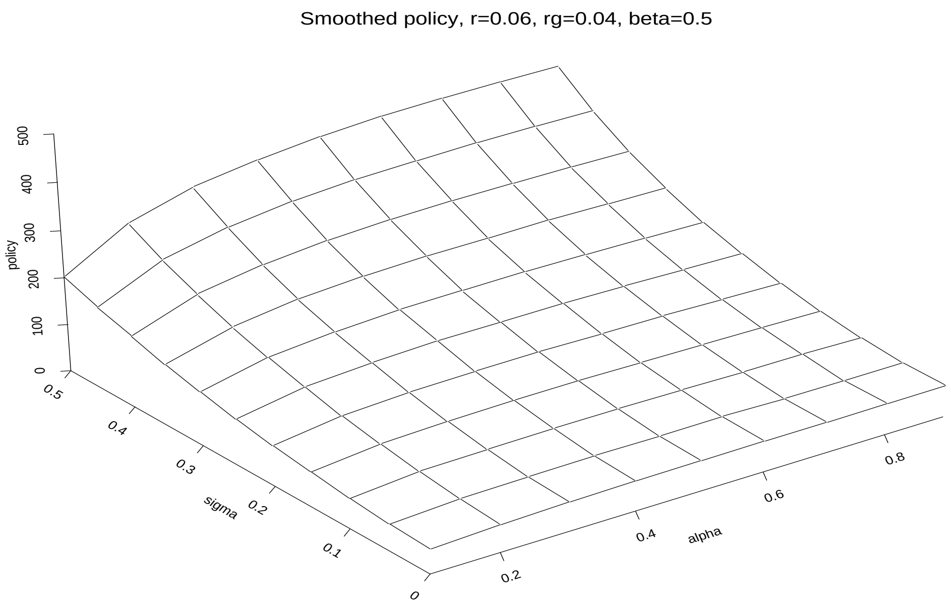

Figure 6: Smoothed Asset Share Scheme: The sensitivity of the policy reserve to changes in the volatility parameter and the degree of smoothing.

Consequently, the sensitivity of V_R to these parameters is opposite to that of V_P, although the shape is more complex since $R(T)$ is a convex function itself of $P(T)$. The return volatility, σ, however, affects both the reference portfolio (directly) and the policy reserve; and, as a result, the profiles that the value of terminal bonus exhibits are distinctive and particularly interesting.

Scheme I & II: Figure 7 shows that the value of the terminal bonus is a decreasing function of the participation rate, β. This is consistent with what has been previously observed: as seen in Figure 2, the policy reserve is more valuable as the proportion of the asset return which is credited to the policy is increased; at the same time the market value of the reference portfolio is independent of the design parameter β. However, the profiles we obtain for different levels of σ suggest a cross-over or inversion feature, which is particularly outlined in Figure 8. Here we have three corresponding graphs (for the same values of r, n, and r_G), but we show the value of the terminal bonus, V_R, plotted against σ for different values of β (ranging from 0.1 to 0.9, with $\beta = 0.1$ at the top and $\beta = 0.9$ at the bottom of each panel). As we can observe, the terminal bonus presents a different pattern depending on the value of the participation rate. For small values of β, V_R is an increasing function of

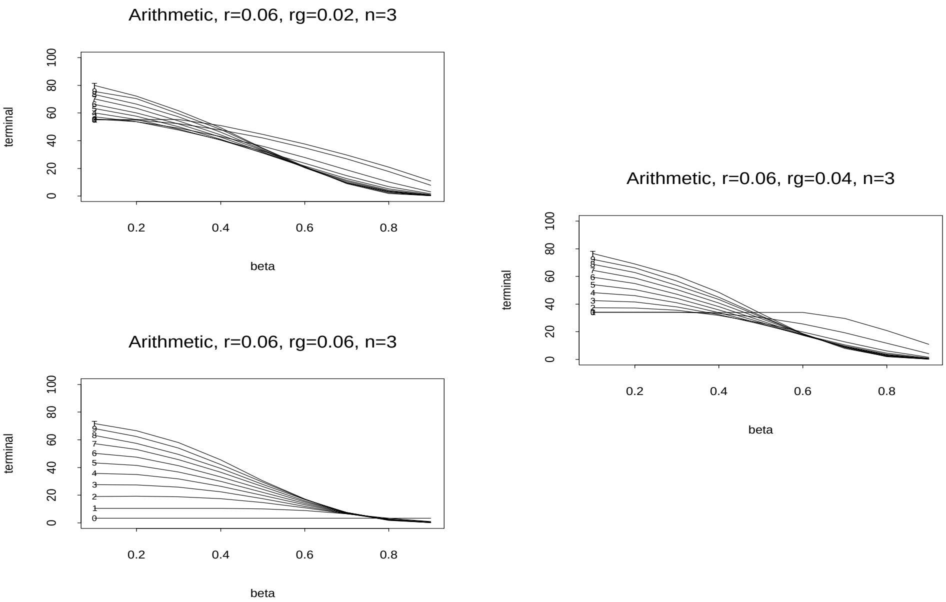

Figure 7: Arithmetic Scheme: the terminal bonus vs β profile.

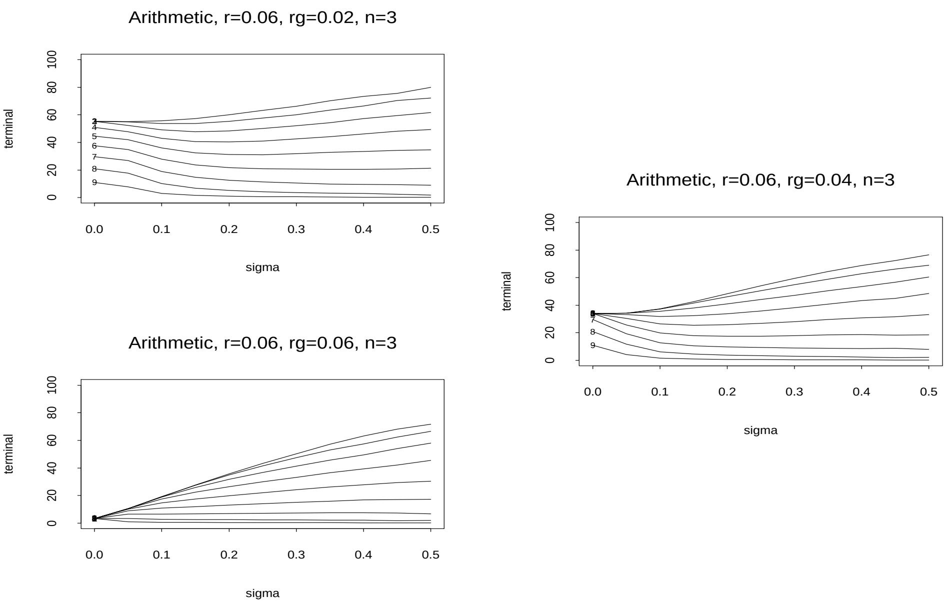

Figure 8: Arithmetic Scheme: the terminal bonus vs σ profile.

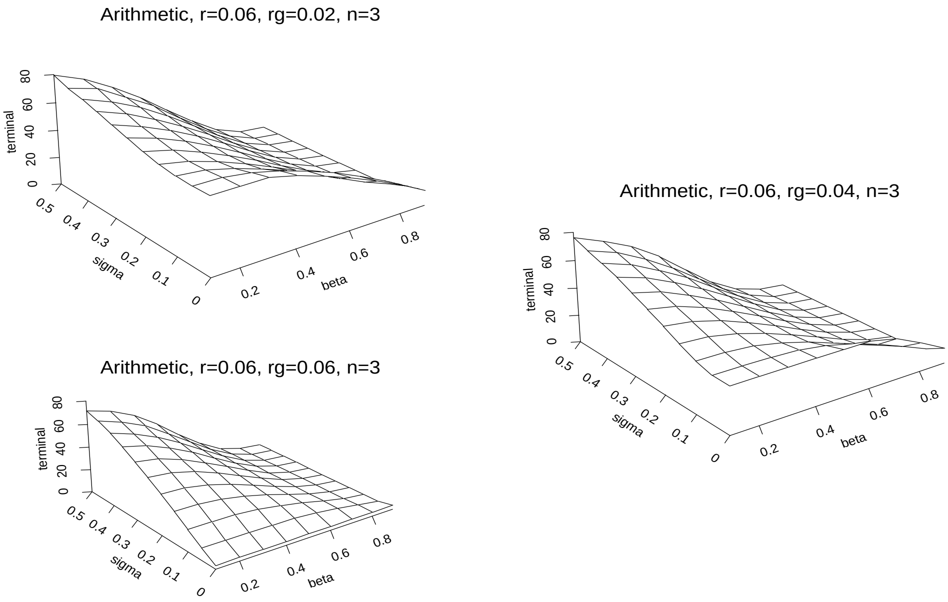

Figure 9: Arithmetic Scheme: the terminal bonus sensitivity to changes in the volatility and the participation rate.

σ, but as β is increased, the pattern of V_R shows an inversion of trend. We note that the effect almost disappears when the guarantee, r_G, equals the market interest rate (panel bottom left in Figure 8).

When the participation rate is low, the policy reserve is almost insensitive to σ, as we have seen before in Figure 2. This means that $P(T)$ is approximately constant. On the other hand, $A(T)$, the market value of the equity fund, is fully sensitive to changes in the volatility σ. Because of the downside protection offered by the guarantee, the terminal bonus $R(T)$ behaves like a conventional vanilla option and is an increasing function of σ. However, the higher the participation rate, β, the more of the volatility risk is transferred from the reference portfolio to the policy reserve. Consequently, as β increases, $A(T)$ and $P(T)$ both react similarly to changes in σ and the chance of exercising the terminal bonus option becomes smaller. V_R is effectively a premium for the probability mass in the tail of the distribution of $A(T)$, where the tail is defined by $P(T)$.

This phenomenon is attenuated for higher levels of the guarantee, as is shown in Figure 9. This Figure contains 3-dimensional pictures of V_R as function of σ and β for different choices of r_G. These plots show again the inversion feature in the trend of V_R with σ at high levels of the participation rate, and how the cross-over effect is affected by increases in the guarantee. As shown by the graphs, the higher is the guarantee, the higher the participation rate has to be in order to produce the inversion of trend in the value of the terminal bonus. In fact, as the guarantee rate r_G is increased, the policy reserve becomes less sensitive to the asset volatility and hence more stable, in the sense that it resembles accumulation at a fixed rate rather than in line with the reference portfolio.

Scheme III: Figure 10 shows the behavior of the terminal bonus as a function of the participation rate for different levels of the asset volatility in the case of the smoothed asset scheme (scheme III). The patterns are equivalent to the ones observed in Figure 7. Also in this case, we observe the existence of a cross-over feature which becomes less pronounced as the level of the guarantee is increased. This feature is shown in more detail in Figure 11 (for α fixed at 0.6). As in Figure 9, Figure 11 shows the terminal bonus of the smoothing scheme case as function of both σ and β for different choices of r_G. The inversion in the profile of V_R with respect to σ at different levels of the participation rate is shown as well as the effect of increases in the level of the guaranteed rate, r_G. The effect is emphasized further by the three following figures. In Figure 12 the participation rate is fixed at a low level ($\beta = 0.1$). The value of the terminal bonus is plotted against σ and α, where α controls the degree of smoothing, for different levels of the guarantee. The three panels show that V_R is an increasing function of σ and no unusual features are observed. Figure 13 shows the same profiles but for a medium level of the participation rate ($\beta = 0.5$). The value of the terminal bonus as function of σ has a U shape across the range of values of α. This shape tends to disappear when r_G is equal the guarantee. Finally, in Figure 14, β is fixed at a high level ($\beta = 0.9$), which lowers the value of the terminal bonus across all values of σ, α and r_G. V_R presents now a decreasing trend with respect to σ, as noted for Scheme I. The effect is more pronounced for low values of α, which correspond to the case of heavy smoothing. For $r = r_G$, the effect disappears only for α close to 1.

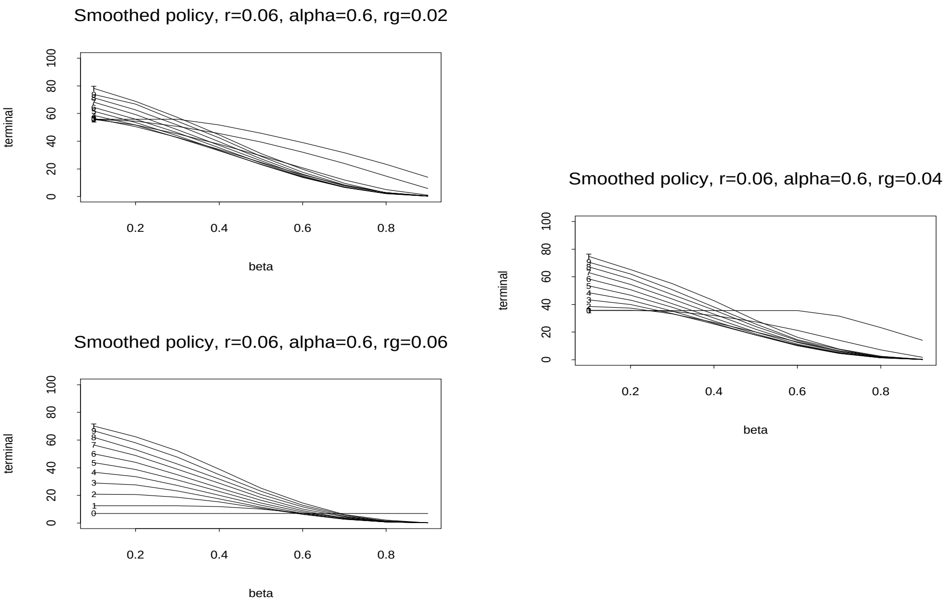

Figure 10: Smoothed Asset Share Scheme: the terminal bonus vs β profile.

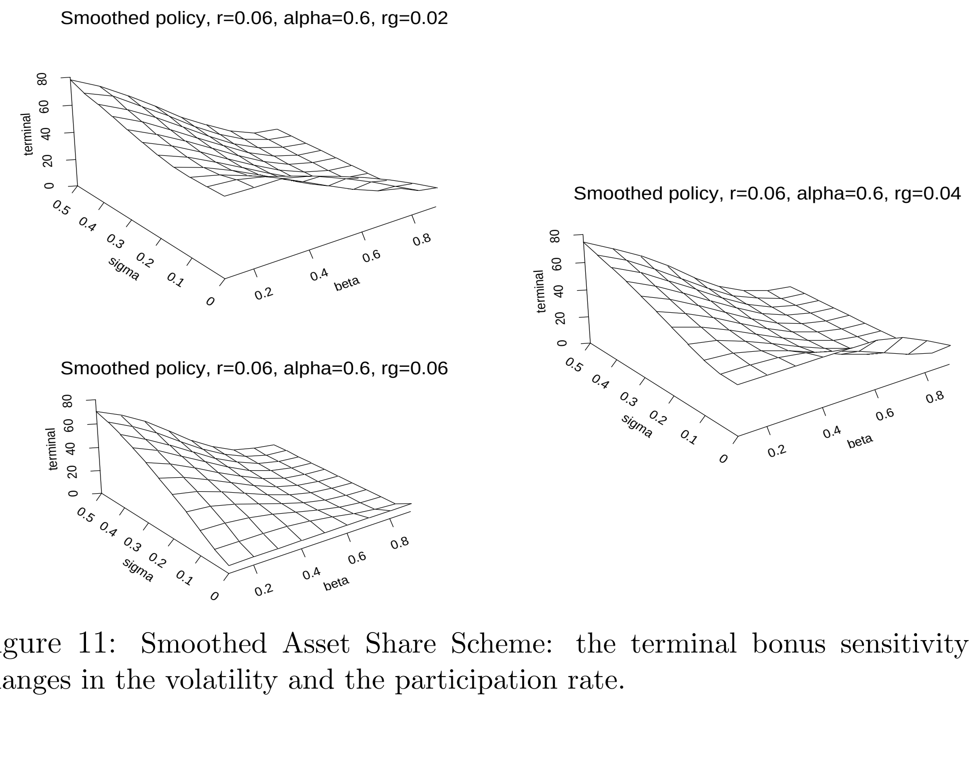

Figure 11: Smoothed Asset Share Scheme: the terminal bonus sensitivity to changes in the volatility and the participation rate.

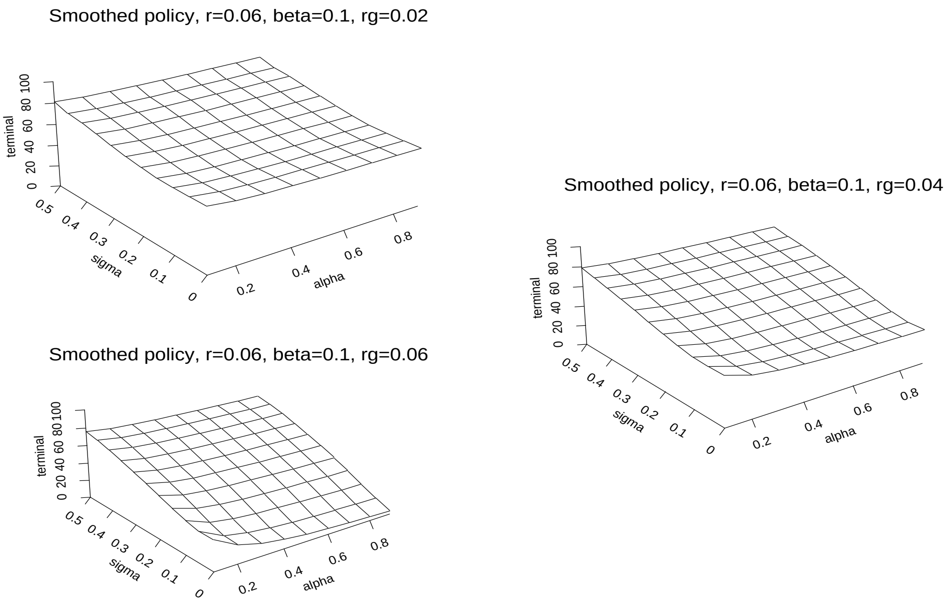

Figure 12: Smoothed Asset Share Scheme: the terminal bonus sensitivity to changes in the volatility and the degree of smoothing for minimum participation rate ($\beta = 0.1$).

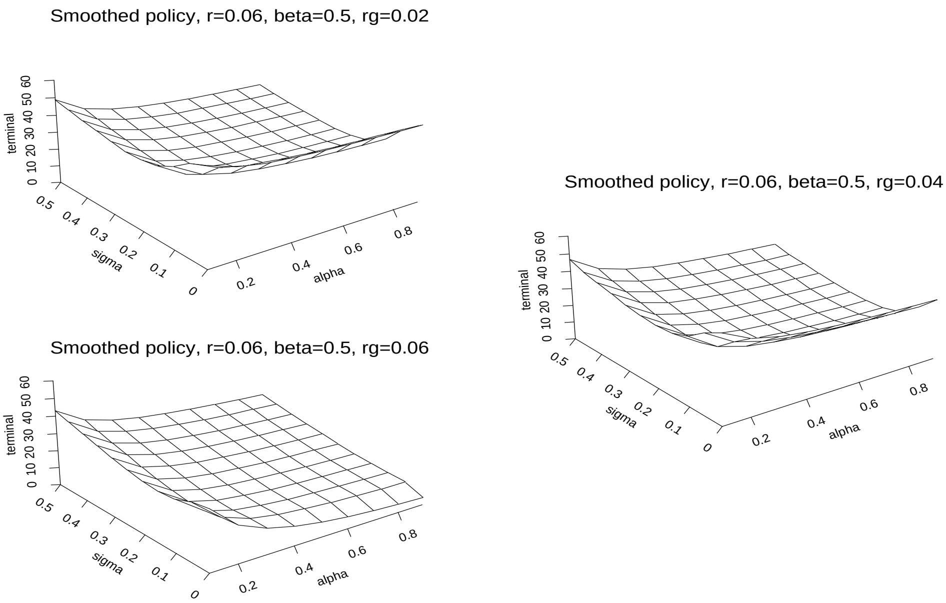

Figure 13: Smoothed Asset Share Scheme: the terminal bonus sensitivity to changes in the volatility and the degree of smoothing for medium participation rate ($\beta = 0.5$).

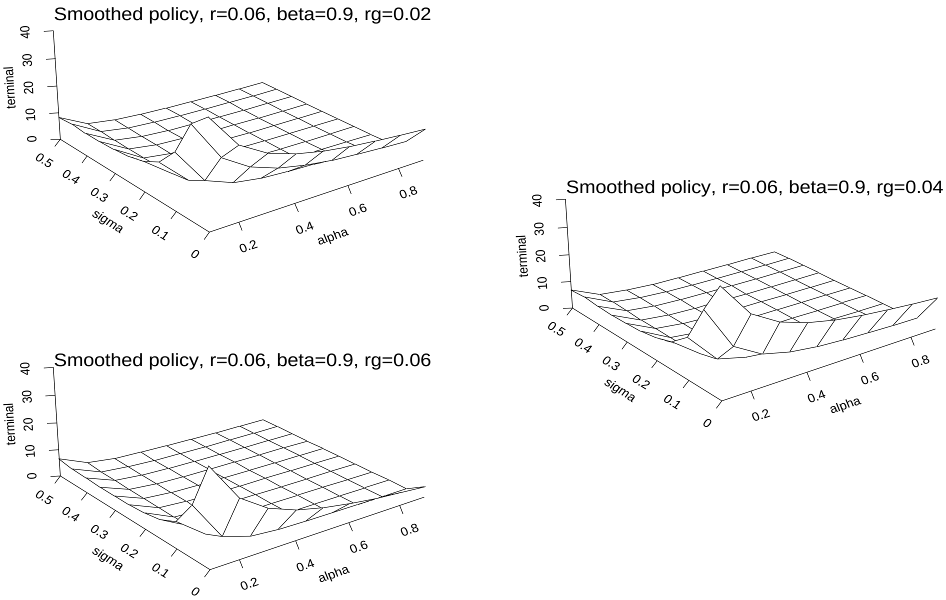

Figure 14: Smoothed Asset Share Scheme: the terminal bonus sensitivity to changes in the volatility and the degree of smoothing for maximum participation rate ($\beta = 0.9$).

3.2 Pricing and parameter selection: the "fair value" principle

So far we have considered the behaviour of the values of the policy reserve, V_P, and the terminal bonus, V_R, as the underlying model parameters are changed. In this section, we address the issue of a fair design for unitised with-profit life insurance contracts, i.e. the set of design parameters such that the value of the contract, as computed via arbitrage principles (see section 2), equals the initial premium paid by the policyholder. As seen in the previous sections, these contracts can be treated as financial derivative securities written on the reference portfolio. As such, their values depend on the specification of the contract design parameters: the level of the guaranteed return, r_G, the participation coefficients, β and γ, the smoothing parameters, n or α (according to which smoothing scheme is adopted for the reversionary bonus rate), and the term of the contract, T. The market parameters, like the reference portfolio volatility or the risk-free rate of interest, are also essential to complete the description of the contract. However, not every choice of these parameters determines an initially fair contract. Bearing in mind that the financial parameters are in general not under the control of the life insurance office, a possible guideline for the design of fair contracts may be obtained from an inspection of the insurer's balance sheet, here schematically

At time 0	Assets	Liabilities	
	A_0	$L_0 = P_0$	single premium
		E_0	equity capital
	A_0	A_0	

At time t	Assets	Liabilities	
	$A(t)$	$L(t)$	policyholder claim
		$E(t)$	equity capital
	$A(t)$	$A(t)$	

Table 1: Balance sheet for the policy contract.

represented in Table 1. As already mentioned, the policy represents a liability for the insurer since in return for the initial premium, P_0, the policyholder is entitled to a claim on future contingent payouts. Also, the equityholders add capital, E_0, so that the two together set up the total initial asset portfolio backing the insurance policy. Given P_0 and E_0, we can construct an equation for the equilibrium condition at the start of the contract. If we ignore the potential default for the insurance company, since the policyholders receives at maturity

$$L(T) = P(T) + \gamma R(T),$$

the insurer gets

$$\begin{aligned} E(T) &= A(T) - L(T) \\ &= A(T) - P(T) - \gamma R(T). \end{aligned}$$

Therefore

$$\begin{aligned} E(0) &= \hat{\mathbb{E}}\left[e^{-rT} E(T)\right] = A_0 - V_P(0) - \gamma V_R(0) \\ &= P_0 - V_P(0) - \gamma V_R(0). \end{aligned}$$

Since in our analysis we assume that shareholders do not contribute to the set up of the reference portfolio, i.e. $E(0) = 0$, then

$$P_0 = V_P(0) + \gamma V_R(0) \tag{1}$$

In the remaining of this paper, we focus on the determination of a feasible set of contractual parameters that satisfies equation (1) for each crediting scheme. We will return to the case of $E(0) \neq 0$ and allowing for the potential default in subsequent work.

3.2.1 Guarantees and participation rates: scheme I

In this section, we explore possible combinations of (r_G, β, γ) such that the equilibrium condition (1) is satisfied. We focus in particular on the arithmetic crediting scheme, as the results obtained for the geometric scheme and the smoothed asset share scheme are similar.

In Figure 15, we plot the set of feasible combinations of the minimum guarantee, r_G, and the participation rate, β, for different levels of the terminal bonus rate, γ, and of the market volatility, σ. As the four panels show, there is a trade-off between r_G and β: in fact, if the contract offers a high guarantee, the policyholder is in a sense less willing to ask for a high participation rate as compensation for the "equity risk", that is for the risk of low returns from the reference portfolio. We also observe that, as the market conditions become more and more uncertain (i.e. when σ increases), the range of feasible choices for both r_G and β becomes smaller. In other words, the insurance company needs to reduce the benefits paid to the policyholder in order to contain the risk exposure implied by the contract. (This trade-off between the participation rate, β, and σ has also been observed by Briys and de Varenne, 1994).

In Figure 16, we consider the possible combinations of the minimum guaranteed rate of return, r_G, and the terminal bonus rate, γ, for different levels of the participation rate, β, in three market volatility scenarios. As in the previous case, we observe a trade-off between r_G and γ. In fact, when r_G is low, we expect the policyholder to require a larger percentage of the insurance final surplus in return for the initial premium. However, as the participation rate β increases, i.e. as more of the asset return is credited to the policy reserve, the insurer has to reduce both the guaranteed rate and the terminal bonus rate, especially when the market is very volatile, to the extent that, for a participation rate β as high as 70%, there are no feasible contracts when the market volatility is higher that 15% per annum.

Similar trends can be observed in Figure 17, in which we analyze the feasible combinations of the two participation rates, β and γ, for different levels of r_G and in different market volatility scenarios. Again, a trade-off between the parameters β and γ is observed. This trend suggests that in order to maintain the initial premium fixed, the insurance company has to lower the terminal bonus rate, γ, when the participation rate in the company profits, β, is high. The bottom-left panel shows the case in which the rate r_G equals the market interest rate; as the plot shows, the largest feasible rates are about 20% for the terminal bonus rate, γ, and 45% for the participation rate β, both in corresponding of the lowest volatility scenario ($\sigma = 10\%$) considered here.

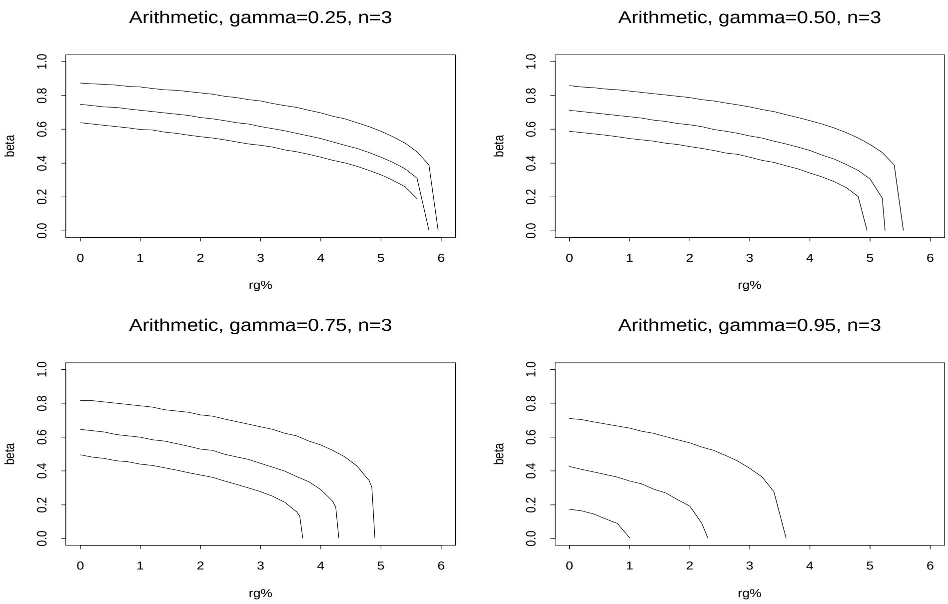

Figure 15: The trade-off between the minimum guarantee and the participation rate in the insurance profits.

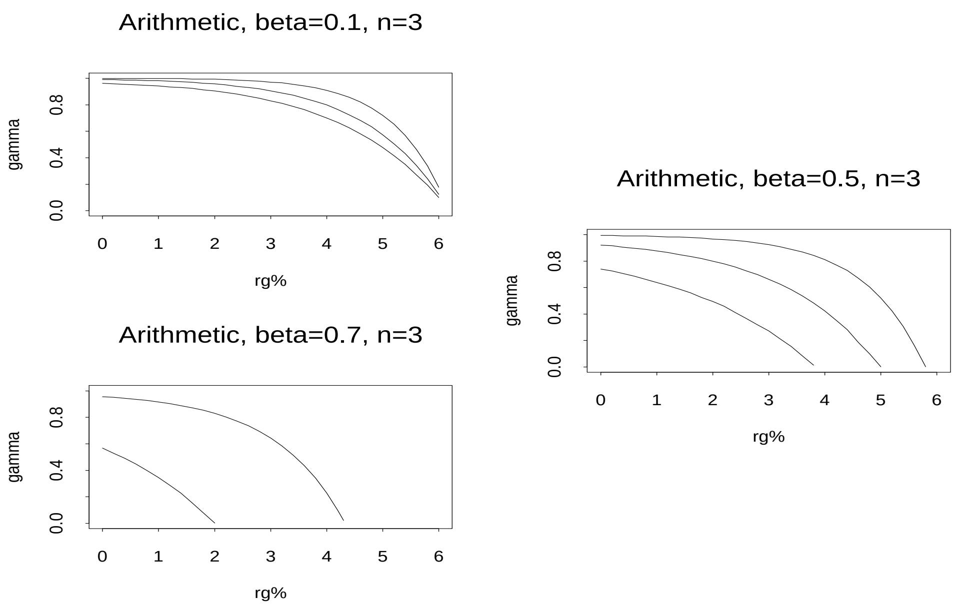

Figure 16: The trade-off between the minimum guarantee and the terminal bonus rate.

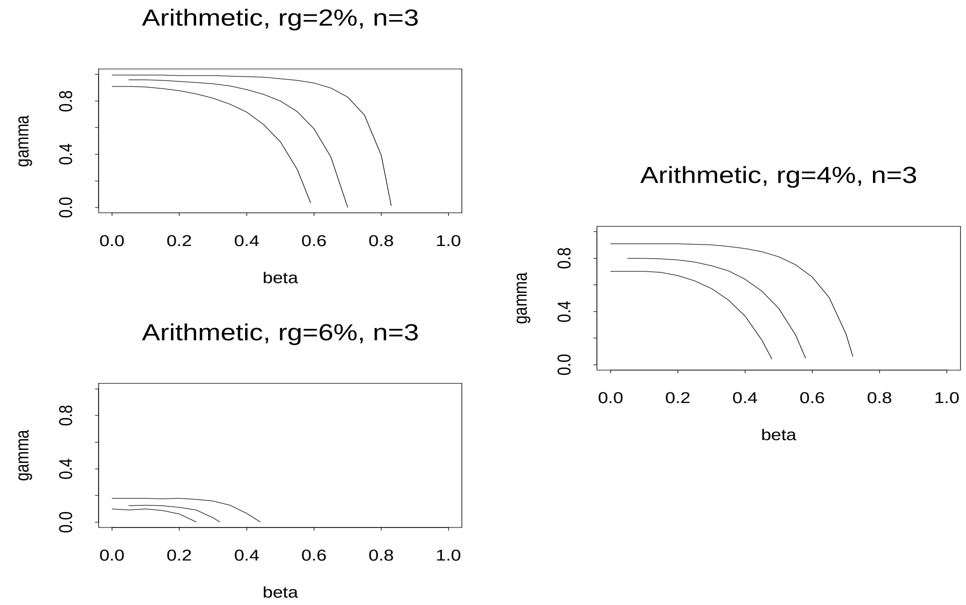

Figure 17: Feasible combinations of participation rates and terminal bonus rate vs the participation rate in the insurer profits.

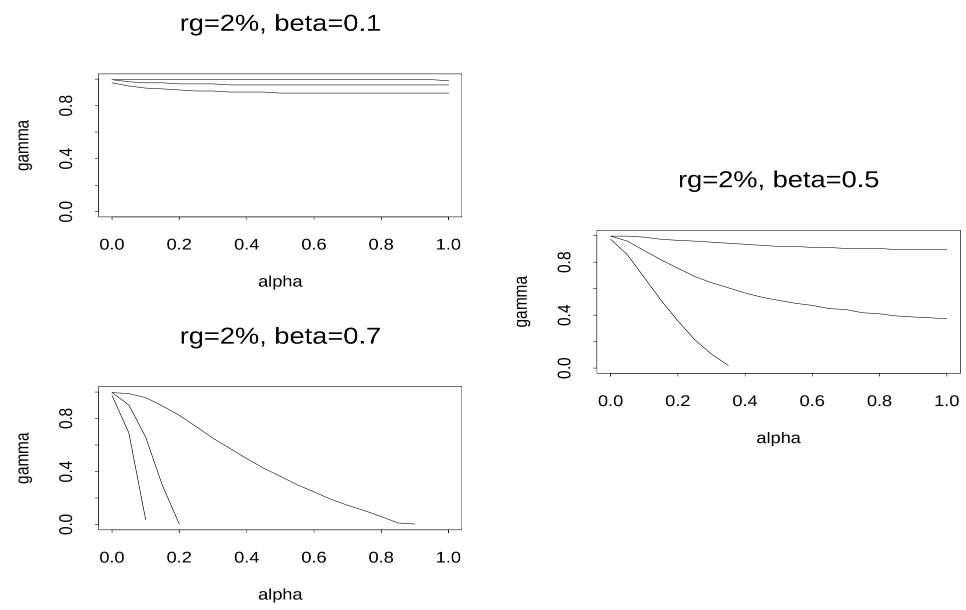

Figure 18: Smoothed Asset Share Scheme: the smoothing effect vs the terminal bonus rate. The case of a low guarantee.

3.2.2 The effect of smoothing: scheme III

In Figures 18-20, we focus on the smoothed asset share crediting scheme (scheme III), to analyze the effect of smoothing. In particular, in these plots we look at different combination of the smoothing parameter, α, and the terminal bonus rate, γ, bearing in mind that the case for $\alpha = 0$ corresponds to the heaviest degree of smoothing as no return from the reference portfolio backing the contract is credited to the policy reserve, whilst for $\alpha \rightarrow 1$, no smoothing is applied and the full rate of return from the investment portfolio is credited to the policyholder. In particular, the first panel of Figure 18 shows that, for a 2% guaranteed rate of return (compared to a 6% interest rate prevailing in the market), when the participation rate, β, is very low, reductions in the degree of smoothing do not affect the policy reserve. Hence, the policyholder requires a high terminal bonus rate to compensate for the low returns paid by the policy reserve. In fact, the first panel in Figure 18 shows that γ stays almost fixed at its maximum value no matter the degree of smoothing. As β increases, the smoothing parameter α affects more and more the value of the policy reserve, and therefore the terminal bonus rate has to be readjusted accordingly in order to respect the equilibrium condition (1). Figure 19 and 20 show the same combinations of parameters but for higher levels of the guaranteed rate r_G; as we observe from the plots, as r_G increases, a reduction in the degree of smoothing, α, reduces the terminal bonus rate, γ, even for lower values of the participation rate, β.

In Figure 21, we plot the set of feasible combinations of the guarantee, r_G, and the degree of smoothing, α, such that the initial premium is fixed for different levels of the participation rate, β. The panels show that, in general, the higher the weight assigned to the unsmoothed asset share component in the reversionary bonus, the lower is the minimum guaranteed rate offered by the insurer to the policyholder. Increasing the participation rate, β, affects the shape of the curves, which become almost flat for $\beta = 0.8$. It appears that, for a fixed terminal bonus rate, γ, when the insurer offers a high participation rate in the company's returns, the weight of the unsmoothed asset share has to be very small, no matter the level of the guarantee, r_G. This is particularly accentuated in high volatility scenarios, as the insurer has to cut both r_G and α in order to manage the increasing market uncertainty.

Figure 22 shows the feasible set of choices for the participation rate, β, and the smoothing parameter, α, for different levels of the terminal bonus rate, γ. From the plots, it appears that the insurer looks for protection against the increasing equity risk induced by a low degree of smoothing ($\alpha \rightarrow 1$), by reducing the participation in the returns of the reference portfolio. Also, in this case, the insurer faces the increasing uncertainty in the market by

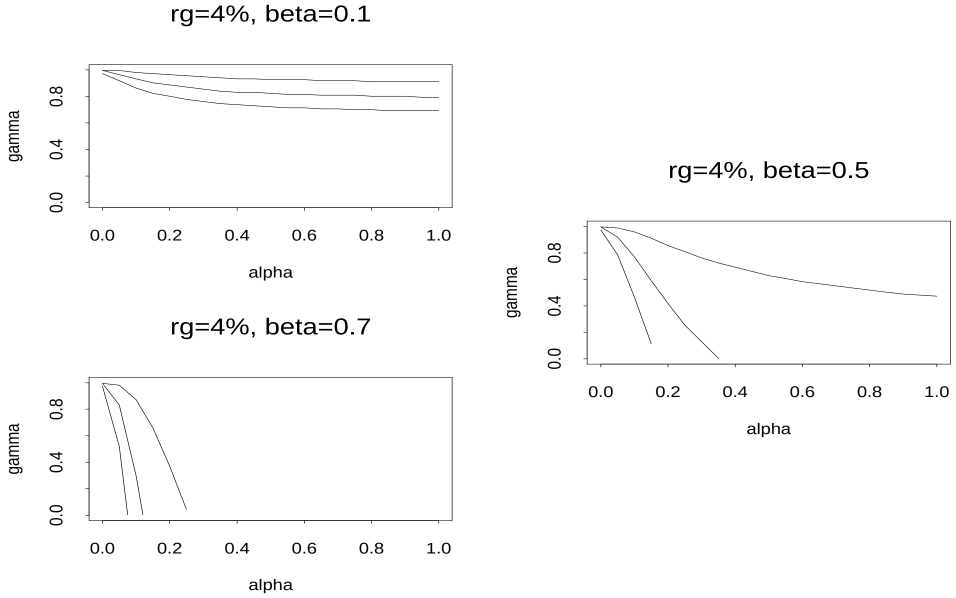

Figure 19: Smoothed Asset Share Scheme: the smoothing effect vs the terminal bonus rate. The case of a medium guarantee.

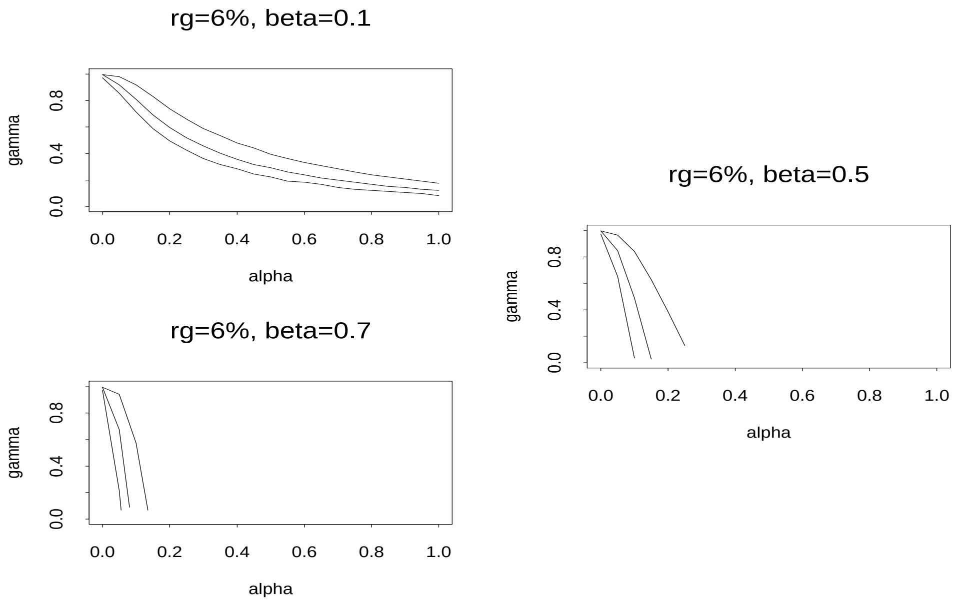

Figure 20: Smoothed Asset Share Scheme: the smoothing effect vs the terminal bonus rate. The case of guarantees equal to the market interest rate.

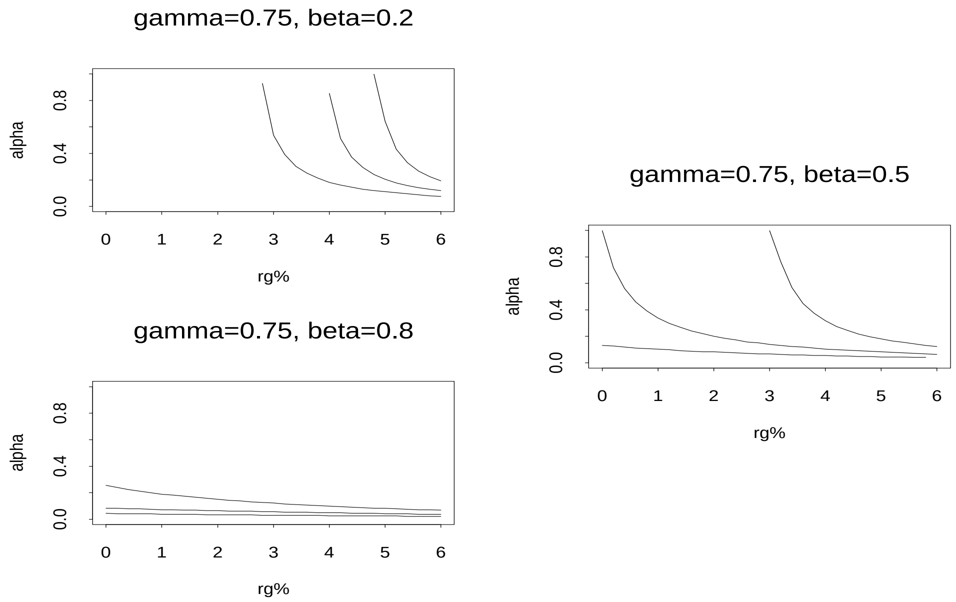

Figure 21: Smoothed Asset Share Scheme. Feasible set for the degree of smoothing and the guarantee.

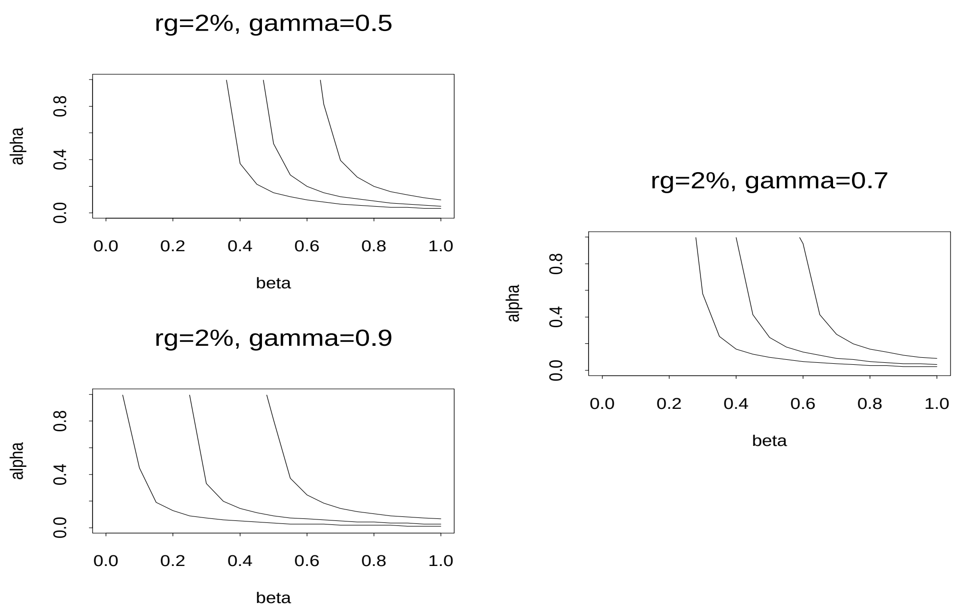

Figure 22: Smoothed Asset Share Scheme. Trade-off between the degree of smoothing and the participation rate in the insurer profits.

reducing the overall weight, α, of the policy reserve. For a fixed initial premium, the reduction allowed by the equilibrium condition (1) is larger for contracts offering a higher terminal bonus rate.

4 Conclusions

In this paper, we have introduced a market-based valuation framework for the most common unitised with-profits life insurance contracts with minimum guarantees used in the UK. These kinds of contract represent liabilities to the issuers implying that their value and the potential risk to the insurance company's solvency should be properly quantified. This raises the subject of their fair valuation, where by fair valuation is meant the definition of a pricing methodology consistent with the absence of arbitrage in the financial market.

These contracts can be decomposed into a riskless bond represented by the certain guaranteed benefit, and a sequence of embedded options made up by the periodic reversionary bonus and the terminal bonus. The proposed model focuses on these last two elements of unitised with-profits contracts, and exploits contingent claim valuation theory in order to determine the market value of the liabilities represented by the embedded options, when surrender opportunities, mortality and default risks are ignored.

Given the path-dependency affecting this class of contracts, which precludes the derivation of explicit valuation formulae, a Monte Carlo simulation procedure is implemented to perform the numerical analysis. Sensitivity analysis for the values of the reversionary bonus and the terminal bonus is presented.

The issue of a fair design for unitised with profit life insurance contracts is addressed; the numerical analysis performed shows how changes in market conditions can jeopardize the solvency of the issuer, if the design parameters of the contract are not set carefully and kept under review.

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