Between the individual and collective models, revisited

François Dufresne *
Ecole des HEC
University of Lausanne

August 14, 2002

Abstract

We show that the aggregate claims distribution of a portfolio modelled by a mix of the individual and collective models can be obtained with a single recursion (under some conditions). This seems to have gone unnoticed in the literature. In fact, it is an application of "De Pril transforms", an appellation introduced by Sundt (1992). We discuss why the collective model is not necessarily an approximation of the individual model in the context of pension funds, for example. An application to a Swiss pension fund is presented. This paper is practical and pedagogical in nature.

Keywords

Individual model, collective model, recursion, aggregate claims distribution, De Pril transform, Kornya's approximation.

*This paper was written while the author was visiting the Department of Statistics of the University of Toronto.

1. Introduction

The individual model is a natural construct for a life insurance portfolio or a pension fund. At a given time, the insureds of a portfolio are well known, the pension fund's members as well. Their characteristics, sex, age, face amounts, etc., are also available as are good estimates of the needed biometric functions (probability of death, etc.). However, there is an implicit assumption underlying the use of an individual model in these contexts: the group is closed.

The collective model has been suggested as an approximation to the individual model. The usual approximation consists of replacing the individual model by a "close" compound Poisson model. This was standard enough to be included many years ago in actuarial education of aspiring SOA's members, see chapter 13 of Bowers *et al* (1986) or chapter 4 of Klugman *et al* (1998).

Frequently in practice, the calculation of the aggregate claims distribution arises in the determination of stop-loss premiums. Although stop-loss reinsurance is mainly used in non-life insurance, it is routinely employed by pension funds in some European countries, and especially in Switzerland. Swiss pension funds can provide substantial disability and death benefits, consequently individual *net amounts at risk* (NAAR) can be very large. This explains the popularity of stop-loss reinsurance among small and medium size Swiss pension funds.

There is then a real need for a model as close as possible to "reality", in order to have a best estimate of the stop-loss premium (net or loaded according to a premium calculation principle). In section 3, we will discuss briefly some possibilities in regard of what is the probable evolution of the pension fund considered (or insurance portfolio). We will do it after recalling the basics of the individual model, the collective model, and mixes of both, in the next section. In-between models have been suggested by Goovaerts and Kaas (1988).

We will show in section 4 that the distribution of such a mixed model can be calculated with a single recursion. Many recursive formulas for the individual and the collective models have been developed over the last decades following the pioneering works of Panjer (1981) and Kornya (1983). After rediscovering a few formulas known in computer science (De Pril 1985, 1986), De Pril (1988, 1989, 1994 with Dhaene) made a very important contribution to the individual model efficient calculations literature. The list of papers here is not exhaustive, these references are the most important in regard to practical use in the opinion of this paper's author. A recent survey of the topic is Sundt (2002). A less important but practice oriented paper is Dufresne (1996) where a formula, which is a special case of a De Pril's formula, is derived in the lines of Kornya. Kornya's approach will be used again in the following. The relation of the single (mixed) recursion to De Pril transforms, introduced by Sundt (1992, 1995, 1998), will be discussed.

Next, we consider a small pension fund and apply the hybrid model to it. Finally, we conclude with some remarks.

2. The models

In the individual model of Risk Theory, the aggregate claims random variable (r.v.) S^{Ind} is defined by

$$S^{Ind} = X_1 + X_2 + \ldots + X_m \tag{1}$$

where $X_1, X_2, ..., X_m$ are mutually independent random variables. The random variable X_k gives the total claim amount of the insured number k of the portfolio for a given period of time, $k = 1, 2, ..., m$.

To simplify the presentation, we will assume that an insured can have at most one claim per period of time, and that the claim amounts are positive. This last assumption is generally not satisfied in the case of pension funds (at least, Swiss ones).

One can model the individual claim amounts X_k by setting

$$X_k = I_k B_k \tag{2}$$

where I_k is an indicator random variable and B_k is the claim amount random variable, given that a claim occurs. We can assume that I_k and B_k are independent. Let $p_k = Pr[I_k = 0]$ and $q_k = Pr[I_k = 1] = 1 - p_k$. The distribution of I_k is a Bernoulli of parameter q_k. Instead of writing X_k as a product of two independent random variables, one can alternatively write it as the following sum

$$X_k = \sum_{i=1}^{I_k} B_i \tag{3}$$

with the usual convention that the value of an empty sum is zero. This last representation will be useful to define the collective model approximation to the individual one.

Throughout this paper, let denote the probability function (discrete case) or the probability density function (continuous case) of a random variable Z by $f_Z(\cdot)$.

In the collective model, the aggregate claims random variable S^{Coll} is defined by

$$S^{Coll} = Y_1 + Y_2 + \ldots + Y_N \tag{4}$$

where $Y_1, Y_2, \ldots$, the individual claim amounts, are mutually independent random variables and are independent of N. The random variables $Y_1, Y_2, \ldots$ are also identically distributed.

The collective model approximation to the individual model can be defined by the replacement of I_k in $\sum_{i=1}^{I_k} B_i$ by a Poisson r.v., say N_k, with the same expected value: $\lambda_k = \mathrm{E}[I_k], k = 1, 2, \ldots, m$. Then

$$X_k^{CA} \equiv \sum_{i=1}^{N_k} B_i \quad \sim \quad \text{compound Poisson}(\lambda_k, f_{B_k}(\cdot)), \tag{5}$$

where the superscript CA means "collective approximation". By a well known property of the compound Poisson distribution, see, for example, Theorem 11.1 of Bowers *et al* (1986), the following random variable

$$S^{CA} = X_1^{CA} + X_2^{CA} + \ldots + X_m^{CA} \tag{6}$$

has a compound Poisson distribution with parameters

$$\lambda = \lambda_1 + \lambda_2 + \ldots + \lambda_m \tag{7}$$

and

$$f(x) = \sum_{i=1}^{m} \frac{\lambda_i}{\lambda} f_{B_i}(x) \tag{8}$$

where $\lambda_k = q_k = \mathrm{E}[I_k], k = 1, 2, \ldots, m$.

Consequently, S^{CA} can be written as

$$S^{CA} = Y_1 + Y_2 + \ldots + Y_N \tag{9}$$

where $Y_1, Y_2, \ldots$ are independent and identically distributed according to (8) and independent of N which has a Poisson distribution of parameter λ given by (7).

If the X_k^{CA} are "close" (in distribution) to the original X_k, the distribution of S^{CA} will be close to the distribution of S^{Ind}. This happens when every N_k has almost the same distribution as the corresponding I_k, and this is when the probability, q_k, of a claim is "small". This last assertion is simply the celebrated Poisson approximation to the binomial distribution. It is easy to show that the collective approximation preserves the first moment but not the variance of the original distribution. The variance of S^{CA} is greater than the one of S^{Ind}, but very slightly in usual applications. Errors bounds for the approximation of the individual model by the collective models have been developed, see, for example, Gerber (1984).

3. Practical considerations: mixed model

In practice, calculation of the aggregate claims distribution is always done with a specific time horizon in mind. For example, stop-loss premium contracts for pension funds usually have a one year duration, sometimes a longer period is chosen, e.g. three years. During that period there will be new entrants in the pension fund (or insurance portfolio) as well as withdrawals. Some of the new entrants will come in as replacement for disabled or deceased members during the given period of time. Nevertheless, a stop-loss contract will cover everyone: people who were there at the beginning of the period as well as those who arrive during the period. Also, early withdrawals do not represent full risk exposures.

Consequently, even if the data as they are laid on paper (or computer file) speak for an individual model, a collective model might be, in fact, "closer" to reality. In the following, we assume that the data correspond to and individual model and that the random variables X_k are well defined, $k = 1, 2, \ldots, m$

If we account for the following facts:

- death and disability are more likely to strike older people,
- if people are replaced, they usually will be so by younger persons,
- some people will not be replaced at all (and this can be already known by the management staff),
- older people have larger net amounts at risk, younger have smaller ones.

Note: The argumentation found in this section has sometimes to be adjusted for insurance portfolios. Swiss pension funds are our main example. The list above is not exhaustive.

We conclude that a model "in-between" could best reflect the features of a pension fund. Goovaerts and Kaas (1988) did suggest the use of such a model. They proposed to model the riskiest part of a portfolio by an individual model and the rest of it by a collective model. They were motivated by computational efficiency since they considered the collective approximation just as a computational device allowing to avoid brute force convolution. Here we suggest a similar modelling on different grounds: to best reflect the dynamics of a pension fund (or insurance portfolio).

Thus, we split the pension fund in two groups:

1. members who are unlikely to be replaced at all [1],
2. members who are likely to be replaced by new entrants.

Let denote by V the set of indexes of pension fund members (or "risks") who are assigned to group 1.

The claim number frequency of group 1 is decreasing over time (assuming, for example, a uniform distribution of decrements). The one of group 2 will be assumed constant (since there are replacements). Then we use the individual

[1] Or who would be replaced by new members with very small NAAR.

model for the aggregate claims of group 1, S_1, and a collective approximate model for group 2, S_2:

$$S_1 = \sum_{k \in V} X_k \tag{10}$$

and

$$S_2 = \text{CA}^{\star}[\{X_k\}_{k \notin V}] \tag{11}$$

where CA[$\cdot$] means collective approximation based on the random variables inside the brackets. The star refers to a possible adjustment of the claim frequency as described below. We assume that S_1 and S_2 are independent. Thus, the pension fund aggregate claims, S, is given by

$$S = S_1 + S_2. \tag{12}$$

Parameters of the two submodels are chosen in the usual way, except for the claim number parameter, λ, of the second one. The expected number of claims under the collective model should be adjusted to take into account the replacements, other new entrants and the withdrawals. It should be noted that we implicitly assume, in the collective submodel, that the distribution of individual claim amounts is the same for current members, new entrants and persons withdrawing.

Once the two submodels are set up, it remains to compute the distribution of S. In the next section, we show that it can be obtained with a single recursion.

4. Recursive calculation

To simplify the presentation, we assume that the net amounts at risk are all non-negative integers. In the case of a pension fund, the distributions of the X_k are tri-atomic (no claim, mortality, disability). The p.f. of X_k can be written as

$$f_{X_k}(x) = \begin{cases} p_k & : \quad x = 0 \\ q_k^{(m)} & : \quad x = m_k \\ q_k^{(d)} & : \quad x = d_k \\ 0 & : \quad elsewhere \end{cases} \tag{13}$$

where m_k is the mortality NAAR, and d_k is the disability NAAR. For the given period of time, the probabilities of death and disability for individual number k are $q_k^{(m)}$ and $q_k^{(d)}$, respectively. It follows that the p.f. of B_k is

$$f_{B_k}(x) = \begin{cases} q_k^{(m)}/q_k & : \quad x = m_k \\ q_k^{(d)}/q_k & : \quad x = d_k \\ 0 & : \quad elsewhere. \end{cases} \tag{14}$$

From our assumptions, the probability of no claims is

$$\begin{aligned} \Pr[S=0] &= \Pr[S_1=0] \cdot \Pr[S_2=0] \\ &= e^{-\lambda} \prod_{k \in V} p_k. \end{aligned} \tag{15}$$

If we denote by $\varphi_Z(s)$ the probability generating function (p.g.f.) of a r.v. Z, then we have

$$\varphi_S(s) = \varphi_{S_1}(s)\varphi_{S_2}(s). \tag{16}$$

Now, the p.g.f. of S_2 can be written as

$$\varphi_{S_2}(s) = e^{\lambda(\varphi_Y(s)-1)} \tag{17}$$

where $\varphi_Y(s)$ is a polynomial.

By definition of S_1, its p.g.f. is

$$\varphi_{S_1}(s) = \prod_{k \in V} \left(p_k + q_k^{(m)} s^{m_k} + q_k^{(d)} s^{d_k}\right) \tag{18}$$

$$= (\prod_{k \in V} p_k) \cdot \prod_{k \in V} \left(1 + \tilde{q}_k^{(m)} s^{m_k} + \tilde{q}_k^{(d)} s^{d_k}\right) \tag{19}$$

with $\tilde{q}_k^{(m)} = q_k^{(m)}/p_k$, and $\tilde{q}_k^{(d)} = q_k^{(d)}/p_k$.

In the spirit of Kornya (1983) and De Pril (1988)[2], one can also write the p.g.f. of S_1 as

$$\varphi_{S_1}(s) = e^{G(s)} \tag{20}$$

where

$$G(s) = g_0 + g_1 s + g_2 s^2 + g_3 s^3 + \ldots \tag{21}$$

[2]Although De Pril used to take the logarithm of the p.g.f.

under mild assumptions. Of course, the value of g_0 is easily determined:

$$g_0 = \ln f_{S_1}(0) = \ln \Pr[S_1 = 0] = \ln \prod_{k\in V} p_k. \qquad (22)$$

If $q_k^{(m)} + q_k^{(d)} < 1/2$, which is usually the case in practice, the other coefficients g_x in $G(s)$ are given by

$$g_x = \sum_{k\in V} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sum_{j=1}^{n} \binom{n}{j} (\tilde{q}_k^{(m)})^j \cdot (\tilde{q}_k^{(d)})^{n-j} \cdot 1_{\{x=m_k j+d_k(n-j)\}} \qquad (23)$$

for $x = 1, 2, \ldots$ See Dufresne (1996) for an elementary derivation of (23) or De Pril (1989)[3]. Under the aforementioned assumption, the infinite sum converges. Thus, a (very) good approximation of g_x is obtained by truncating the infinite sum at some value r of the index n. This value r is called the order of the approximation and can be chosen to achieve any desired degree of precision. The reader is referred to De Pril (1988) for the general formula to use in selecting r. The approximate value of g_x resulting from such truncation will be denoted by $g_x^{(r)}$.

Now, we have all the ingredients to compute the probability function of S at any desired degree of precision. The p.f. of S, $f_S(x) = \Pr[S = x]$, is simply given by the coefficient of s^x in $\varphi_S(s)$ which can be computed easily and efficiently as we shall see in what follows.

If we substitute (17) and (20) in (16), we get

$$\begin{aligned} \varphi_S(s) &= e^{G(s)+\lambda(\varphi_Y(s)-1)} \\ &= e^{C(s)} \end{aligned} \qquad (24)$$

where

$$C(s) = c_0 + c_1 s + c_2 s^2 + c_3 s^3 + \ldots \qquad (25)$$

is a power series in s with $c_x = g_x + \lambda f_Y(x), x = 1, 2, \ldots$, and $c_0 = \ln \Pr[S = 0]$. Since all the coefficients in $C(s)$ can be determined, we can use the following theorem mentioned in Kornya (1983) to obtain the (approximate) distribution of S.

[3] The case of "translated rescaled Binomial" distributions for the B_k's is not directly treated in De Pril's papers, as far as the present author knows. His general formula would lead to (23)

Theorem: *If $A(s)$ and $B(s)$ are power series given by*

$$A(s) = \sum_{i=0}^{\infty} a_i s^i \qquad \text{and} \qquad B(s) = \sum_{j=0}^{\infty} b_j s^j, \tag{26}$$

such that

$$A(s) = e^{B(s)} \tag{27}$$

then

$$A(0) = a_0 = e^{b_0} \tag{28}$$

and

$$a_x = \frac{1}{x} \sum_{k=1}^{x} k b_k a_{x-k}, \qquad x = 1, 2, 3, ... \tag{29}$$

Proof: For the proof, one considers the identity $A'(s) = B'(s)A(s)$ and compares the coefficient of s^{x-1} on both sides.

Replacing in the theorem $A(s)$ by $\varphi_S(s)$ and $B(s)$ by $C(s)$, and rewriting slightly we get

$$\begin{aligned} f_S(x) &= \frac{1}{x} \sum_{k=1}^{x} k c_k f_S(x-k) \\ &= \frac{1}{x} \sum_{k=1}^{x} k \left(g_k^{(r)} + \lambda f_Y(k) \right) f_S(x-k), \quad x = 1, 2, 3, \ldots \end{aligned} \tag{30}$$

Formula (30) with $f_S(0) = \Pr[S = 0]$ given by (15) constitutes a recursive formula.

Naturally, if the set V is empty, formula (30) is the standard compound Poisson recursive formula[4]. On the other hand, if $V = \{1, 2, 3, \ldots, m\}$ we recover the recursive formula for this particular individual model since $\lambda = 0$. Otherwise, we have an in-between model and a mixed recursive formula.

In view of the De Pril transform (DPT) introduced by Sundt (1992), recursive formula (30) is not surprising at all. To oversimplify a bit, the DPT of a random variable (or a distribution) is x times the coefficient of s^x in $B(s)$ when one writes a p.g.f. as $\exp(B(s))$, see the above theorem. The

[4]The value of an empty sum is zero (with probability 1).

sequence of coefficients $\{c_x\}_{x=1}^{\infty}$ multiplied by x is the DPT of S. If we denote by $\mathcal{D}$ the De Pril transform operator, we have

$$\mathcal{D}[S] = \{xc_x\}_{x=1}^{\infty}. \tag{31}$$

It can be seen that it is the sum of the DPTs of S_1 and S_2 since their DPTs are $\{xg_x\}_{x=1}^{\infty}$ and $\{\lambda x f_Y(x)\}_{x=1}^{\infty}$, respectively. The following DPT's properties justify these claims:

- the DPT of the sum of independent random variables[5] is the sum of their respective DPTs;
- if its probability mass at zero, $f_Z(0)$, and its DPT are known, the probability function of a r.v. Z can be calculated recursively with the following formula:

$$f_Z(x) = \frac{1}{x}\sum_{k=1}^{x} \mathcal{D}[Z](x) f_Z(x-k), \qquad x = 1, 2, 3, ... \tag{32}$$

  where $\mathcal{D}[Z](x)$ is the x-th element in the DPT sequence.

If we look at (23), we see that it is the sum of individual DPTs of the X_k, $k \in V$, divided by x. The DPTs of these X_k could also be calculated numerically by a recursive formula, see Sundt (1992, 1995) for example. Since these numerical calculations cannot go indefinitely, by stopping at an adequate point one gets the approximation $g_x^{(r)}$ (after dividing by x). We do not go any further on this topic; the interested reader is referred to Sundt's various papers on the subject.

5. Illustration

For a numerical illustration, we will use the data of the pension fund considered by Held (1982). Today, these data are still typical of a Swiss pension fund. They could be adjusted to reflect the time value of money but that would only represent a change of monetary unit. The reader should keep in mind that the amounts of money involved are expressed in Swiss francs of

[5] Or of the convolution of distributions.

1982. Adjusting the values mentally should be enough to appraise current values. We are mostly interested in relative differences. It is also for this reason that the fact that Swiss “technical bases” have evolved is immaterial.

Held's data contain some negative net amounts at risk. Although it may be important in practice to take into account these values, it is not the point we want to exemplify. Consequently, we set all the negative values equal to zero. Since, in this context, stop-loss reinsurance covers death and disability benefits, the data set pertains to a group of 230 *active* members of a given pension fund. The data set gives only the essential information: 230 quadruplets, each of which consists of the death NAAR, the disability NAAR, the (one-year) probabilities of death and of disability.

Since this is all the information we have about these active members, we will build our mixed models in a somewhat artificial way. We will sort the 230 quadruplets in descending order of the probability of disability[6]. We will consider three mixed models. The first will assigned the first 120 sorted risks to an individual model and the rest to a collective model. We proceed in the same way for the second mixed model but retain the first 80 risks for the individual model. For the third one, only the first 40 risks with the highest probability of disability go in the individual model, the others in the collective one. So, the cardinality of V is 120, 80, and 40 in the first, second, and third mixed models.

No adjustments to the claim frequency parameter is made in the collective models since we want to keep the examples simple. Moreover, we do not have much information.

In addition to the mixed models, pure individual and collective models are applied to the data. Thus, we have the two extreme cases and three in-between ones.

The calculations have been performed with the recursive formula presented in section 4. The order r of approximation for the individual model and submodels has been chosen high enough to ensure that the calculated values of $g_x^{(r)}$ are equal to the exact ones in view of the precision of the computer. The remaining error is only due to computer roundoff[7].

[6]Generally, the probability of disability is much higher than the probability of death.

[7]The recursion is stable, at least with this data set.

Tables 1 and 2 below present the results. The values in their first column are thousands of Swiss francs. The (pure) individual model is used as benchmark.

The second column of Table 1 gives the individual model's distribution function (d.f.) at fourteen selected points. The remaining columns give the ratios of the d.f. of the other four models to the one of the individual model. These ratios are expressed as percentages. One can see that the differences between the models are small. Nevertheless, their cumulative effect on the stop-loss premiums is non-negligible, as can be seen from Table 2. From Table 1 we can conclude that the level of the stop-loss deductible selected in this case would be about the same[8] whatever the model adopted. The stop-loss deductible is generally one of the higher quantiles.

Table 1. Three mixed models and the collective approximation relative to the individual model. Ratios of distribution functions.

	Ind	$\#V = 120$	$\#V = 80$	$\#V = 40$	*Coll*
x	$F_S(x)$	%	%	%	%
0	0.28725	100.40	100.70	100.70	100.71
3	0.30730	100.08	100.38	100.38	100.39
19	0.41256	99.49	99.79	99.79	99.80
33	0.50090	99.12	99.41	99.41	99.42
54	0.59968	98.96	99.25	99.26	99.26
88	0.69846	98.98	99.23	99.23	99.23
127	0.80091	98.95	99.18	99.18	99.18
144	0.85095	98.97	99.01	99.01	99.02
176	0.89999	98.94	98.87	98.87	98.87
237	0.95045	98.97	98.84	98.84	98.84
289	0.97491	98.98	98.88	98.88	98.88
363	0.98996	98.99	98.95	98.95	98.94
422	0.99514	99.00	98.97	98.97	98.97
537	0.99900	99.00	98.99	98.99	98.99

In its third column, Table 2 gives the individual model's net stop-loss premium (SLP) in thousand of Swiss francs. Column 2 repeats the quantiles

[8]In addition, stop-loss deductibles are normally round numbers.

values for easier reference. Table 2's four last columns show the increases in the net stop-loss premiums that result from switching from the individual model to one of the other models. The increases are expressed as percentages of the individual model's net SLPs.

From Table 2, one can see that the increases in the net SLPs are generally non-negligible. For quantiles that matter, the increase is about 5% for the last three models. One should note that the expected value of the aggregate claims is the same in all five models. Therefore, the absence of increase in the net stop-loss premium with deductible equal to zero is not accidental.

It should be also kept in mind that the Held's data set in not particularly "dangerous". It is also relatively small. Much more dangerous such data sets are encountered in practice and would magnify the phenomena observed in our examples. Current stop-loss gross premiums being about two times the net SLP, the differences in premiums could amount from several hundreds to a few thousands of Swiss francs, depending on which models is adopted.

Table 2. Three mixed models and the collective approximation relative to the individual model. Increases in the net Stop-loss premium.

	Ind	*Ind*	$\#V=120$	$\#V=80$	$\#V=40$	*Coll*
x	$F_S(x)$	$SLP(x)$	%	%	%	%
0	0.28725	66.4782	0.00	0.00	0.00	0.00
3	0.30730	64.3453	0.02	0.02	0.02	0.02
19	0.41256	53.8825	0.11	0.15	0.15	0.15
33	0.50090	46.1711	0.18	0.26	0.26	0.27
54	0.59968	36.7360	0.24	0.44	0.44	0.44
88	0.69846	24.9138	0.32	0.84	0.84	0.86
127	0.80091	15.3621	0.48	1.76	1.77	1.81
144	0.85095	12.2489	0.54	2.37	2.39	2.44
176	0.89999	8.2675	0.66	3.33	3.36	3.44
237	0.95045	3.9069	0.68	4.75	4.79	5.00
289	0.97491	1.9882	0.76	5.05	5.10	5.54
363	0.98996	0.7840	0.77	5.33	5.38	6.26
422	0.99514	0.3505	0.83	5.72	5.79	7.27
537	0.99900	0.0697	1.03	7.15	7.23	10.62

Observe that, currently, reinsurers that offer stop-loss coverages for pension funds tend to use the collective model for obvious reasons... Stop-loss premiums are negotiable. The models and approach presented in this paper can give the pension fund's management some arguments in the negotiation process.

6. Conclusion

We discussed why a collective model or some model "between" it and the individual model may be better suited to pension funds (and, possibly, some life insurance portfolios). We proposed the use of a recursive formula for the calculation of the aggregate claims distribution. This recursion is only valid when all the net amounts at risk are nonnegative. The general case still requires some numerical convolutions. Our examples showed that the differences between the different models can be non-negligible in practice.

References

Bowers, N.L., H.U. Gerber, J.C. Hickman, D.A. Jones and C.J. Nesbitt(1986) *Actuarial mathematics*, 1rst ed., Society of Actuaries, Itasca (Illinois).

De Pril, N. (1985) Recursion for convolutions of arithmetic distributions. *ASTIN Bulletin* **15**, 135-139.

De Pril, N. (1986) On the exact computation of the aggregate claims distribution in the individual life model. *ASTIN Bulletin* **16**, 109-112.

De Pril, N. (1988) Improved approximations for the aggregate claims distribution of a life portfolio. *Scandinavian Actuarial Journal*, 61-68.

De Pril, N. (1989) The aggregate claims distribution in the individual life model with arbitrary positive claims. *ASTIN Bulletin* **19**, 9-24.

Dhaene, J. and N. De Pril (1994) On a class of approximative computation methods in the individual risk model. *Insurance: Mathematics and Economics* **14**, 181-196.

Dufresne, F. (1996) An extension of Kornya's method with application to pension funds. *Bulletin of the Swiss Association of Actuaries*, 171-181.

Gerber, H.U. (1984) Error bounds for the compound Poisson approximation. *Insurance: Mathematics and Economics* **3**, 191-194.

Goovaerts, M.J. and R. Kaas (1988) Between individual and collective model for the total claims. *ASTIN Bulletin* **18**, 169-174.

Held, R. (1982) Zur rekursiven Berechnung von Stop-Loss Prämien für Pensionkassen. *Bulletin of the Swiss Association of Actuaries*, 67-89.

Klugman, S.A., H.H. Panjer and G.E. Willmot (1998) *Loss Models: From Data to Decisions*, Wiley, New York.

Kornya, P.S. (1983) Distribution of the aggregate claims in the individual Risk Theory model. *Transactions of the Society of Actuaries* **35**, 823-836. Discussion 837-858.

Panjer (1981) Recursive evaluation of a family of compound distributions. *ASTIN Bulletin* **12**, 22-26.

Sundt(1992) On some extensions of Panjer's class of counting distributions. *ASTIN Bulletin* **22**, 61-80.

Sundt(1995) On some properties of De Pril transforms of counting distributions. *ASTIN Bulletin* **25**, 19-31.

Sundt(1998) A generalisation of the De Pril transform. *Scandinavian Actuarial Journal*, 41-48.

Sundt(2002) Recursive evaluation of aggregate claims distributions. *Insurance: Mathematics and Economics* **30**, 297-322.