Ambiguity Aversion and the Puzzle of Own-Company Stock in Pension Plans*

Phelim Boyle† Raman Uppal‡ Tan Wang§

This version: February 2004

*We thank Suleyman Basak, Francisco Gomes, Dietmar Leisen, and Anthony Neuberger for their extensive suggestions. We also acknowledge comments from Ron Anderson, Lorenzo Garlappi, Juan-Pedro Gomez, Gur Huberman, Vasant Naik, Hongjun Yan and from participants at seminars at Lehman Brothers (London), McGill University, Northwestern University, Oxford University, University of Southern Switzerland, University of Toronto, Wilfrid Laurier University, Yale University, the CEPR Conference on Financing Retirement in Europe, the Northern Finance Association and the International Symposium on Insurance and Finance at Bergen.

†University of Waterloo, School of Accountancy, Hagey Hall Room 175B Waterloo, Canada ON N2L 3G1 Tel: 1-519-888-4567 ext. 6513; Email: pboyle@uwaterloo.ca.

‡CEPR and London Business School, IFA, 6 Sussex Place Regent's Park, London, England NW1 4SA; Tel: 44-20-7706-6883; Email: ruppal@london.edu.

§CCFR and Sauder School of Business, University of British Columbia, Faculty of Commerce, 2053 Main Mall, Vancouver, Canada V6T 1Z2, Canada; Tel: 1-604-822-9414; Email: tan.wang@commerce.ubc.ca.

Ambiguity Aversion and the Puzzle of Own-Company Stock in Pension Plans

Abstract

In a defined contribution pension plan, employees make the investment decisions since they ultimately bear the investment risks. Empirical studies find that whenever the firm's own stock is one of the available assets, many employees invest a significant fraction of their discretionary contributions in the stock of their employer. Moreover, the proportion allocated to own-company stock increases as the volatility of the company's stock decreases. We analyze this puzzle using a framework based on ambiguity aversion when making decisions in the presence of model misspecification. Based on this framework we derive a simple model where it is optimal for agents to invest in own-company stock. We use this model to evaluate quantitatively the extent of model misspecification required to generate the observed investment in own-company stock. Our calibration results indicate that if the investor thinks that the expected return on own-company stock will outperform other firms in the market by just 1% to 2%, then this will lead to an investment in own-company stock of about 20% to 30%. And, consistent with the empirical evidence, this allocation increases as the volatility of the company's stock decreases.

Keywords: Investment, portfolio choice, uncertainty, robust control, defined contribution
JEL Classification: G11, G12, G23, D81

1 Introduction

John Maynard Keynes viewed the best investment strategy as one where you put all your money in the stock about which you feel most favorably; thus, if you were favorably disposed toward a railroad company or the railways sector, he recommended that the entire portfolio be invested in that particular stock or sector.[1] Markowitz (1952), on the other hand, showed that it is inefficient to put a large holding in *any* single stock because the risk of doing so can be quite high from the lack of diversification.[2] Such risk was dramatically illustrated with the collapse of Enron where retirement funds were heavily invested in Enron's *own* stock.[3]

Even though Markowitz's idea of diversification has been accepted as one of the most fundamental theoretical insights in modern financial economics, empirical evidence suggests that investors do not hold diversified portfolios but rather invest in only a few assets, typically those with which they are familiar (see, for instance, Coval and Moskovitz (1999), Grinblatt and Keloharju (1999), and Huberman (2001)). The empirical evidence on pension funds in particular is that, whenever the firm's own stock is one of the assets available for investment,[4] many employees invest a significant fraction of their discretionary contributions in the stock of their employer; for instance, Mitchell and Utkus (2002) and Meulbroek (2002) find that the percentage of assets representing company stock in defined contribution plans is around twenty-nine percent, and Benartzi (2001) finds in a sample of S&P 500 firms that about one third of the assets in retirement plans are invested in company stock and of the discretionary contributions about a quarter are invested in company stock.[5]

[1] Bernstein (1992, p. 48) gives the following quote from a letter written by Keynes: "I am in favor of having as large a unit as market conditions will allow . . . To suppose that safety-first consists in having a small gamble in a large number of different [companies] where I have no information to reach a good judgement, as compared with a substantial stake in a company where one's information is adequate, strikes me as a travesty of investment policy." Keynes was not alone in holding such a view. Loeb (1950) advocates that, "Once you obtain confidence, diversification is undesirable; diversification [is] an admission of not knowing what to do and an effort to strike an average."

[2] Tobin (1958) extends the work of Markowitz (1952) by showing that in a static setting agents should invest in only two funds: a riskless asset and a fund containing only risky assets, and Sharpe (1964) shows that in equilibrium the fund containing only risky assets must be the market portfolio.

[3] In the case of Enron, it has been estimated by VanDerhei (2002) that 58% of the company's 401(k) assets were invested in Enron stock, whose market value fell by 98.8% during 2001. Bodie, Hammond and Mitchell (2001) discuss the systematic and unsystematic components of retirement risks.

[4] Mitchell and Utkus (2002) estimate that firms offering this investment option account for 42% of all defined contribution plan participants and 59% of total plan assets. In total twenty-three million employees have this investment option available and the total assets of these plans is approximately $1.2 trillion. See Choi, Laibson, Madrian and Metrick (2001) for other details on the pension choices offered to employees.

[5] See Holden and VanDerhei (2001a), (2001b) and VanDerhei (2002) for additional details.

Our objective in this paper is to develop a model of portfolio choice that incorporates the views of *both* Keynes and Markowitz and to use this model to examine *quantitatively* one of the most puzzling aspects of defined contribution pension plans: the large proportion of assets of these plans that are invested in the stock of the sponsoring company.

In recent years, there has been a worldwide trend from defined benefit plans to defined contribution plans.[6] Gale, Papke and VanDerhei (1999) document this trend in the United States. Mitchell and Utkus (2002) estimate that there are 700,000 corporate defined contribution plans in the U.S. covering fifty-six million workers while there are 56,000 defined benefit plans covering twenty-three million employees. The total assets of these defined contribution plans are approximately two trillion dollars. The pressures to switch from defined benefit plans to defined contribution plans have also affected public sector pension plans which were traditionally defined benefit plans and in the U.S. a number of states have converted their public sector employee plans from defined benefit plans to defined contribution plans. For example, in 2002, the State of Florida gave its 600,000 public sector employees the option of converting into a defined contribution plan (Mitchell and Lachance, 2002). Several European countries have also modified their government-sponsored pension plans from a pure defined benefit structure to accommodate individual retirement accounts which correspond to defined contribution plans; see Feldstein and Siebert (2002).

The investment choices of defined contribution pension plan members are of interest not only because of the large volume of assets involved at the aggregate level but also because of the puzzling behavior with regard to the own company stock. The empirical evidence on how employees invest their contributions documents persistent and significant deviations from the predictions of the orthodox versions of modern portfolio theory. Thus it is not surprising that the investment choices made in defined contribution plans and the own company stock puzzle in particular have attracted considerable interest in the last few years. Most of the explanations of how defined contribution plan participants make investment decisions are based on ideas from behavioral economics and behavioral finance. These ideas have provided valuable new insights on how investment decisions are actually made. Mitchell and Utkus(2003) provide a summary of the application of these ideas to the defined contribution plan investment decision. In a similar vein Cohen(2003) suggest a

[6] There are two main types of employer sponsored pension plans. Under a *defined benefit* plan the pension benefit at retirement is based on the employee's years of service and "final" average salary, usually based on the salary for the last three to five years prior to retirement. Thus, in a defined benefit plan the employer makes the investment decisions and assumes the investment risk. In a typical *defined contribution* plan regular contributions, based on the employee's salary, are made to an investment account and the retirement benefit depends on the investment performance of this account. In a defined contribution plan the employee generally makes the investment choices and bears the investment risk.

loyalty based explanation for the over investment in own company stock. There have also been a number of empirical studies on the investment choices of employees. Recently Huberman, Iyengar and Jiang(2003) document and analyze the investment and participation choices of a large sample of defined contribution plan participants. They explore the sensitivity of these choices to various individual and plan level variables. Huberman and Sengmüller(2002) analyze the determinants of employees' changes to the own company stock component of their 401(k) investments.

While the papers discussed above have proposed several explanations for the own-company-stock investment puzzle, in the absence of analytical models, these studies are qualitative in nature. In particular, these papers do not address the question of how to quantify factors such as familiarity and loyalty, and whether these factors, once properly quantified, can actually induce the magnitude of biased holding of own-company stock that has been found empirically. Our contribution is to show how one can evaluate quantitatively the holding of own-company stock using an analytic framework where there is ambiguity in the true distribution of stock returns and investors are averse to this ambiguity. Such a framework is relevant for investment decisions given the finding of Heath and Tversky (1991) that ambiguity aversion is particularly strong in cases where people feel that their competence in assessing the relevant probabilities is low. Moreover, Fox and Tversky (1995) show that this effect is even stronger when people are reminded of their incompetence, either through comparison with other bets in which they have more expertise, or by comparison with other people who are more qualified to evaluate the bet.

The main feature of our framework is that it allows investors to distinguish between their ambiguity about the distribution of returns on familiar assets (own-company stock) and their ambiguity about returns on other assets in the economy.[7] We show that if investors are averse to ambiguity then two fund separation breaks down; in this case, while an investor will still seek diversification, he will hold not just a combination of the riskfree asset and the market portfolio but also own-company stock. This is true even when the investor is ambiguous also about the return on own-company stock. We show analytically that the model also has the following implications, which are consistent with the stylized facts documented empirically: the proportion of wealth allocated

[7]Our framework is closely related to the growing literature on ambiguity and its implications for asset pricing. Gilboa and Schmeidler (1989), Dow and Werland (1992), Epstein and Wang (1994), Chen and Epstein (2000), Epstein and Miao (2000), Anderson, Hansen and Sargent (1999), Hansen, Sargent and Tallarini (1999), Hansen and Sargent (2000a), Maenhout (1999) and Uppal and Wang (2003) proposed several classes of models that can be used to study ambiguity and investors' aversion to ambiguity and its effect on asset prices. The relation between these classes of models is discussed in Epstein and Schneider (2002), Hansen and Sargent (2001a) Hansen and Sargent (2001b), and Schroder and Skiadas (2002).

to own-company stock relative to the market portfolio increases with a decrease in ambiguity of own-company stock returns; and, the proportion of wealth allocated to own-company stock relative to the market increases with a decrease in the volatility of own-company stock returns.

We then calibrate our model to data on stock returns in order to evaluate the conditions under which the magnitude of the overinvestment in own-company stock generated by the model is consistent with that observed empirically. We show how one can specify the model so that it is parsimonious in terms of the number of parameters that need to be estimated in order to take it to the data and how one can gauge the magnitude of the subjective parameters that determine the ambiguity of the investor toward the returns of particular assets either in terms of an adjustment to the mean or the volatility of the return on these assets. This allows one to evaluate whether the parameter values for which one gets substantial holding of own-company stock are reasonable. We find that even for fairly conservative parameter values, the fraction of wealth to be invested in own-company stock according to the model is consistent with the empirical observations.

The remainder of this paper is organized as follows. In Section 2 we provide a brief preview of the results. In Section 3, we describe the framework we will use and derive in closed-form the implications of this model for investing in own-company stock in the presence of ambiguity about the distributions for stock returns. In Section 4, we demonstrate using a simple calibration exercise that the model delivers portfolio holdings that are consistent with the observed patterns of own-company stock ownership in defined contribution pension plans. We discuss, in Section 5, how one can evaluate the effects of human capital on the results of our basic model. Our conclusions are presented in Section 6.

2 A Preview

Before we present the details of our model that we will develop in the next section, it will be helpful to preview the main empirical implication of the model.

The main message of our model can be described as follows. Let π_j denote the portfolio weight of the employee in their own company stock and π_m be that in the market portfolio. In a standard model with constant investment opportunity set and constant relative risk averse (power) utility, the optimal portfolio holdings would be given by

$$\begin{bmatrix} \pi_j \\ \pi_m \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{(\mu_m - r)}{\gamma \sigma_m^2} \end{bmatrix}, \tag{1}$$

where γ is the employee's risk aversion parameter, and μ_m and σ_m are the expected return and volatility of the return of the market portfolio. If the market has a large number of stocks and each stock is small relative to the market, the absolute amount of holding of own company stock should be close to zero. In essence, the own-company stock puzzle is that, against the prediction of the standard theory that $\pi_{j=0}$, employees tend to hold a large percent, on the order of 20% to 35%, of their discretionary investment in their own company[8] stock. This behavior contradicts the very idea of diversification that is a fundamental principle underlying the existing theories of portfolio choice and asset pricing.

Now suppose that the employee is ambiguous about the true model generating returns and that he is averse to this ambiguity.[9] The following table provides a summary of our results when we explicitly model the employee's aversion to ambiguity about stock returns, and where we assume that in the market there are a large number of stocks traded that are identical in terms of their expected return, volatility, and correlation with the market.

Row#	Adjustment to expected returns	Relative weights $\frac{\pi_j}{\pi_j+\pi_m}$	Relative weights $\frac{\pi_m}{\pi_j+\pi_m}$
Row 1	0.000	0.00	1.00
Row 2	0.012	0.20	0.80
Row 3	0.018	0.33	0.67
Row 4	0.021	0.43	0.57
Row 5	0.023	0.50	0.50

This table says that if an employee by working for a particular company is more confident about the stock return of this company relative to the market, then relative to the standard theory, he will over invest in his own-company stock. For example, Row 2 reports that if the employee is more confident about his own company stock so that he thinks the expected return of the stock of his company is 1.2% higher than the estimate based only on publicly available data, then 20% of his

[8]When own company stock is one of the available asset classes under the plan rules.

[9]There is overwhelming evidence that expected stock returns are very difficult to estimate precisely (Merton (1980) and French and Porterba (1991) among others). French and Porterba show that the standard deviation of the estimate of the expected return of US stock market is about 2%.

total investment in risky assets will be in the stock of his own company. Similarly, Row 3 says that if the employee expects his own company stock to have an expected return that is 1.8% higher than the estimate based only on publicly available data, he will hold 33% of his total investment in risky assets in the stock of his own company. Because the standard error of the estimate of expected returns for a typical stock is more than two percent (see for instance French and Porterba (1991)), this table suggests that it takes only a modest amount of ambiguity for the worker/investor to favor significantly his own company stock.

This table summaries briefly what we intend to demonstrate in this paper, namely ambiguity about the returns model and an employee's/investor's aversion to this ambiguity can help us to understand the own-company stock puzzle. In the next section, we will demonstrate this more formally with a model of portfolio selection. Section 4 provides the portfolio holdings generated when one calibrates this model to data on stock returns.

3 Portfolio choice in a model with ambiguity

In the first part of this section, we describe a model of portfolio choice that incorporates investors ambiguity about the true distribution of asset returns. In the second part of this section, we analyze the optimal portfolio holdings of risky assets implied by this model.

3.1 The model

We start by defining the processes for stock returns, which are standard. We then describe the preferences of agents in the presence of aversion to ambiguity. We conclude this section by explaining the process for the evolution of wealth over time.

3.1.1 The processes for stock returns

We consider an economy with N firms, where N is a large number. The process for the stock price, P_{it}, of each firm is characterized by the following

$$\frac{dP_{it}}{P_{it}} = \mu dt + (\sigma_S dZ_{St} + \sigma_U dZ_{it}), \quad i \leq N, \tag{2}$$

where Z_{St} and Z_{it}, $i \leq N$, are one-dimensional Brownian motions that are uncorrelated with each other, and that the Z_{it}, $i \leq N$, are independent so that the stocks are not redundant. In the equation above, μ represents the expected stock return, σ_S captures the systematic risk, and σ_U captures the unsystematic risk of the production process.[10] Given the large number of firms, the idiosyncratic risk of each company stock can be diversified away. Hence, all stocks have the same expected returns, μ and the total volatility of stock returns is $(\sigma_S^2 + \sigma_U^2)^{1/2}$. In addition to the risky stocks, we assume that there is a riskless asset in zero net supply with a constant rate of return r.

3.1.2 Preferences of agents who are averse to ambiguity

Without loss of generality, we assume that the agent we are considering works for firm j. We use the subscript j to indicate the variables for the own-company stock, and the subscript "$-j$" to denote the variables for the $N-1$ stocks other than j.

We assume that agents do not have precise knowledge of the distribution of the returns of the stocks in the economy. For simplicity, we assume that all agents have the same level of uncertainty regarding the return distribution of the stocks of companies they work for, given by ϕ_j. We also assume that all agents have the same level of uncertainty regarding the returns of the stock of the other companies for which they do not work, which is denoted by ϕ_{-j}. However, these two levels of uncertainty need not necessarily be the same; that is, ϕ_j need not be equal to ϕ_{-j}.

To reflect the aversion to ambiguity in the presence of uncertainty, we assume that the infinitely-lived agents have preferences represented by

$$V_{jt} = \frac{c_{jt}^{1-\gamma}}{1-\gamma}\,\Delta + e^{-\rho\,\Delta} \inf_{\xi} \left\{ \psi\left(E_t^{\xi}[V_{jt+\Delta}]\right) \sum_{k=\{j,-j\}} \frac{1}{\phi_k} E_t^{\xi_k}\left[\ln \frac{\xi_{kt+\Delta}}{\xi_{kt}}\right] + E_t^{\xi}\left(V_{jt+\Delta}\right) \right\} \tag{3}$$

rather than the standard preferences,

$$V_{jt} = \frac{c_{jt}^{1-\gamma}}{1-\gamma}\,\Delta + e^{-\rho\,\Delta} E_t\left(V_{jt+\Delta}\right). \tag{4}$$

[10]It would be straightforward to nest the above in an equilibrium setting, such as the production economy considered in Cox, Ingersoll and Ross (1985), but instead of a single firm we would have N firms, each having a similar production technology characterized by the following process for output, Y_{it},

$$dY_{it} = \mu Y_{it} dt + Y_{it}(\sigma_S dZ_{St} + \sigma_U dZ_{it}), \quad i \leq N,$$

and with each firm having a stock traded on this production process.

Comparing equations (3) and (4), we see that there are several components in (3) that are new relative to the standard intertemporally additive expected utility. The first new feature is the minimization (infimum) in the definition, which arises from the agent's aversion to uncertainty, as described in Gilboa and Schmeidler (1989). The basic intuition here is that since investors do not have precise knowledge of the true probability distribution of returns, they use a reference probability distribution P, resulting for example from analysis of the data, in conjunction with other alternative probability distributions Q, which for example are those that cannot be ruled out as being the true probability distribution according to the data analysis. The ξ in the term $E^{\xi}[V_{jt+\Delta}]$ inside the infimum sign in (3) is the density function of Q with respect to P. Thus $E^{\xi}[V_{jt+\Delta}]$ is the expected utility under the probability measure Q. Because the investors cannot distinguish clearly Q from P and they are averse to the ambiguity, they look at the worst case scenario of their expected utility under all the alternative Q measures.

The other term in (3) that is new relative to standard intertemporally additive expected utility is

$$\psi\left(E_t^{\xi}[V_{jt+\Delta}]\right)\sum_{k=\{j,-j\}}\frac{1}{\phi_k}E_t^{\xi_k}\left[\ln\frac{\xi_{kt+\Delta}}{\xi_{kt}}\right],$$

which is present for the reasons given in Anderson, Hansen and Sargent (1999) and Uppal and Wang (2002). The variable ξ_j in the term $\sum_{k=\{j,-j\}}\frac{1}{\phi_k}E_t^{\xi_k}[\ln\frac{\xi_{kt+\Delta}}{\xi_{kt}}]$ represents the density function of the *marginal* distribution of the own-company's return under the alternative probability measure Q with respect to the *marginal* distribution under the reference probability P; and ξ_{-j} is the density function of the marginal (joint) distribution of the other $N-1$ company's returns under the alternative probability measure Q with respect to the marginal (joint) distribution under the reference probability P. Thus $E_t^{\xi_j}\left[\ln\frac{\xi_{jt+\Delta}}{\xi_{jt}}\right]$ is the log likelihood ratio under Q of the marginal distribution of the own-company's return under Q to that under P; and $E_t^{\xi_{-j}}\left[\ln\frac{\xi_{-jt+\Delta}}{\xi_{-jt}}\right]$ has a similar interpretation for the other $N-1$ stocks. Since agent j does not know the true probability distribution, he relies on these log likelihood ratios as indication of whether a particular Q is likely to be the true probability distribution of the returns. Clearly when any one of these ratios is large, Q is significantly distinguishable from P. With the data available for example, the investor may then be able to tell that this Q is unlikely to be the true return distribution, and therefore would not take it as a serious alternative to P as the true distribution. When $E_t^{\xi_k}\left[\ln\frac{\xi_{kt+\Delta}}{\xi_{kt}}\right]$, $k=\{j,-j\}$ are large, Q is unlikely to be the solution of the minimization problem. Therefore, this term acts as a penalty function that keeps the investor away from choosing some Q as true return distributions. The other component, $\psi\left(E_t^{\xi}[V_{jt+\Delta}]\right)$, is a normalization factor (see Maenhout (1999) and Uppal and Wang (2002)) that converts the penalty to units of utility so that it is consistent with the units of $E_t^{\xi}(V_{jt+\Delta})$; the particular functional form of $\psi(\cdot)$ is often chosen for analytical convenience.[11] Finally, the two parameters ϕ_j and ϕ_{-j} are subjective parameters that indicate the investor's aversion to ambiguity about own-company stock returns and ambiguity about all other stock returns.

Thus, agents are identical in every respect of their preferences, except in their uncertainty regarding the expected return for the various stocks. This heterogeneity is reflected in the difference in the penalty functions.

3.1.3 Wealth dynamics

Since all agents are identical except for their uncertainty about the return of the stock for the company where they work, symmetry implies that agent j's holdings of companies m and n, π_{jm} and π_{jn}, must be such that $\pi_{jm} = \pi_{jn}$ for all m, $n \leq N$, as long m and n are not equal to j. Since we have assumed that the agents have CRRA utility functions and that the investment opportunity set is constant over time, these portfolio weights are constant over time. Furthermore, agent j's holdings of companies m and n are equal to those of agent k's (who works for company k) as long as m and n are not equal to k . We denote this common portfolio weight by π_{-j}.

Now consider two agents, j and k, working for two different companies. Let π_j and π_k be their portfolio weights for their own-companies. Again symmetry implies that $\pi_j = \pi_k$ for j, $k \leq N$. Denote the common value of the proportion invested by agent j in own-company stock by π_j. Then, using the same approach as the one developed in Merton (1971), the cumulative return on the optimal portfolio of agent j can be written as

$$\begin{aligned} dR_j &= \pi_j \mu dt + \pi_j(\sigma_S dZ_{St} + \sigma_U dZ_{jt}) + \sum_{i \leq N, i \neq j} \pi_i \mu dt + \sum_{i \leq N, i \neq j} \pi_i(\sigma_S dZ_{St} + \sigma_U dZ_{it}) \\ &\quad + \left(1 - \pi_j - \sum_{i \leq N, i \neq j} \pi_i\right) r dt \end{aligned}$$

[11]Note that the utility function $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ is unique only up to a positive affine transform. As can be verified, when ψ is a linear function, the preference defined by equation (3) remains unchanged when $u(c)$ is replaced with $au(c) + b$.

$$
\begin{aligned}
&= \left[\pi_j \mu + (N-1)\pi_{-j}\mu + \left(1 - \pi_j - (N-1)\pi_{-j}\right) r\right] dt \\
&\quad + \pi_j \left(\sigma_S dZ_{St} + \sigma_U dZ_{jt}\right) + (N-1)\pi_{-j}\left(\sigma_S dZ_{St} + \frac{\sigma_U}{N-1}\sum_{i\le N, i\ne j} dZ_{it}\right). \qquad (5)
\end{aligned}
$$

This suggests that for agent j, the original $N+1$ asset problem can be reduced to a problem of investing in three funds. One fund consists of the stock of the company for which he works; the second fund is the equally-weighted portfolio of all the other $(N-1)$ stocks; and, the third fund is the riskless asset. The covariance matrix of the returns of the first two funds is given by:

$$
\Omega = \begin{bmatrix} \sigma_S^2 + \sigma_U^2 & \sigma_S^2 \\ \sigma_S^2 & \sigma_S^2 + \sigma_U^2/(N-1) \end{bmatrix}. \qquad (6)
$$

and let us, with a slight abuse of notation, denote the vector of expected returns by $\mu = (\mu, \mu)^\top$. Then, the evolution of the investor's wealth, for a given investment decision $\pi = (\pi_j, (N-1)\pi_{-j})^\top$ and consumption decision c is given by

$$
dW_t = W_t dR_j - c_t dt. \qquad (7)
$$

In the next section, we explain how the agent chooses the optimal portfolio and consumption policies.

3.2 The optimal portfolio weights

Given the preferences in equation (3) and the wealth dynamics in (7), the resulting Hamilton-Jacobi-Bellman equation for agent j, from Uppal and Wang (2002), is

$$
\begin{aligned}
0 &= \sup_{c,\pi}\inf_{v}\left\{u(c) - \rho V + V_t + W V_W\left[r + \pi\left(\mu - r\mathbf{1}\right) - \frac{c}{W}\right] + \frac{W^2}{2} V_{WW}\pi\Omega\pi^\top \right. \\
&\qquad \left. + V_W W \pi v + \frac{\psi(V)}{2} v^\top \Phi v\right\}, \qquad (8)
\end{aligned}
$$

where

$$
\Phi \equiv \begin{bmatrix} \frac{1}{\phi_j(\sigma_S^2+\sigma_U^2)} & 0 \\ 0 & \frac{1}{\phi_{-j}(\sigma_S^2+\sigma_U^2/(N-1))} \end{bmatrix}.
$$

In equation (8), the last term, $\frac{\psi(V)}{2} v^\top \Phi v$, corresponds to the penalty term in (3). The second last term, $V_W W \pi v$, is due to the change of probability distribution from P to Q.[12] The other terms,

[12] In a continuous-time setting, a change of probability measure characterized by the density function ξ corresponds to a drift adjustment process v. Thus the ξ in equation (3) corresponds to the v in (8). See Appendix A.

all of which appear on the first line of the equation, are identical to those in the standard HJB equation. Given that the risk free rate r is a constant, one can show that under the specification where $\psi(V) = \frac{1-\gamma}{\gamma}V$, there is an explicit solution for the value function, $V(W) = \kappa_0 W^{1-\gamma}/(1-\gamma)$, where κ_0 is a constant. This then allows us to obtain the following result.

Proposition 1 *The optimal portfolio of agent j is given by*

$$\begin{bmatrix} \pi_j \\ (N-1)\pi_{-j} \end{bmatrix} = \frac{1}{\gamma} A^{-1}\Omega^{-1}(\mu - r\mathbf{1}), \tag{9}$$

where

$$A = \left(I + \Omega^{-1}\Phi^{-1}\right). \tag{10}$$

An immediate implication of the theorem above is that the optimal portfolio weights are constant and that $(\pi_j, \pi_{-j}) \gg 0$.

Given our specification of the N stock-return processes, in the standard model where it is assumed that agents have perfect knowledge of the underlying return distribution, this would lead to $\pi_j = \pi_{-j}$, so that the market portfolio consists of equal shares of all assets and the return on the market portfolio is given by

$$\begin{aligned} dR_m &= \mu dt + \sigma_S dZ_{St} + \frac{\sigma_U}{N}\sum_{i=1}^{N} dZ_{it} \\ &\approx \mu dt + \sigma_S dZ_{St}. \end{aligned} \tag{11}$$

where the second step follows from the fact that when N is large, the variance of $\frac{\sigma_U}{N}\sum_{j\leq N} dZ_{jt}$ is approximately zero due to diversification. On the other hand, in our model with aversion to ambiguity, the return on agent j's portfolio is

$$dR_j = \pi_j\left(\mu dt + \sigma_S dZ_{St} + \sigma_U dZ_{jt}\right) + (N-1)\pi_{-j}\left(\mu dt + \sigma_S dZ_{St} + \frac{\sigma_U}{N-1}\sum_{i\leq N, i\neq j} dZ_{it}\right)$$

When N is large, the variance of $\frac{\sigma_U}{N-1}\sum_{i\leq N, i\neq j} dZ_{it}$ is also approximately zero. That is, the return on agent j's portfolio is

$$dR_j \approx \pi_j(\mu dt + \sigma_S dZ_{St} + \sigma_U dZ_{jt}) + (N-1)\pi_{-j}(\mu dt + \sigma_S dZ_{St}).$$

Let W_{jt} be the wealth of agent j at time t and $W_{mt} = \sum_{i=1}^{N} W_{it}$, which, when multiplied by the number of agents in each class, gives the total market wealth invested in all companies. In our economy, the total wealth invested in company j is given by

$$W_{jt}\pi_j + \sum_{i \neq j} W_{it}\pi_{-j} = \pi_j W_{mt} + W_{jt}(\pi_{-j} - \pi_j),$$

multiplied by the number of agents in a class. Because each class has the same number of agents, when N is large and each agent's wealth is small relative to the market wealth, the market portfolio in our economy consists of approximately $1/N$ invested in each stock. Thus, in light of (11), agent j's portfolio can be viewed as consisting approximately of the market portfolio and some additional weight on his own-company's stock.

Note that when N tends to infinity, π_{-j} tends to zero. In other words, as the number of companies increases, the portfolio weight on any one of the non-own-company stocks decreases to zero. However, the limit of $(N-1)\pi_{-j} \approx N\pi_{-j}$ exists. In light of the discussion above that $N\pi_{-j}$ is approximately the weight on the market portfolio, we will denote the limit of $(N-1)\pi_{-j}$ by π_m.

Below, we analyze the closed-form expressions for the optimal portfolio weights, and following this we calibrate the model to examine the magnitude of these portfolio weights in Section 4.

In the absence of any ambiguity, the weights one would get from the standard portfolio model would be:

$$\begin{bmatrix} \pi_j \\ (N-1)\pi_{-j} \end{bmatrix} = \begin{bmatrix} \frac{1}{\gamma}\frac{\mu - r}{N\sigma_S^2 + \sigma_U^2} \\ \frac{1}{\gamma}\frac{(\mu - r)(N-1)}{N\sigma_S^2 + \sigma_U^2} \end{bmatrix} \tag{12}$$

Observe that if $N = 2$ then equation (12) reduces to portfolio weights in the two assets that are identical to one another:

$$\begin{bmatrix} \pi_j \\ \pi_{-j} \end{bmatrix} = \begin{bmatrix} \frac{1}{\gamma}\frac{\mu - r}{2\sigma_S^2 + \sigma_U^2} \\ \frac{1}{\gamma}\frac{(\mu - r)}{2\sigma_S^2 + \sigma_U^2} \end{bmatrix} \tag{13}$$

In the limit as $N \to \infty$, the optimal portfolio weights in equation (12) reduce to the Merton portfolio weights, with a zero investment in own-company stock:

$$\begin{bmatrix} \pi_j \\ \pi_m \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{\gamma}\frac{\mu - r}{\sigma_S{}^2} \end{bmatrix} = \pi_{\text{Merton}}. \tag{14}$$

The expressions above gives us the benchmark portfolio weights from the standard model where the investor has no ambiguity about stock returns ($\phi_j \to 0; \phi_{-j} \to 0$).

On the other hand, in the presence of ambiguity we have the following expression for the optimal portfolio weights for the case where $N = 2$

$$\begin{bmatrix} \pi_j \\ \pi_{-j} \end{bmatrix} = \begin{bmatrix} \frac{(\mu-r)\left(\sigma_U{}^2+\left(\sigma_S{}^2+\sigma_U{}^2\right)\phi_{-j}\right)}{\gamma\,(2\,\sigma_S{}^2\,\sigma_U{}^2\,(1+\phi_{-j})\,(1+\phi_j)+\sigma_U{}^4\,(1+\phi_{-j})\,(1+\phi_j)+\sigma_S{}^4\,(\phi_j+\phi_{-j}\,(1+\phi_j)))} \\ \frac{(\mu-r)\left(\sigma_S{}^2\,\phi_j+\sigma_U{}^2\,(1+\phi_j)\right)}{\gamma\,(2\,\sigma_S{}^2\,\sigma_U{}^2\,(1+\phi_{-j})\,(1+\phi_j)+\sigma_U{}^4\,(1+\phi_{-j})\,(1+\phi_j)+\sigma_S{}^4\,(\phi_j+\phi_{-j}\,(1+\phi_j)))} \end{bmatrix}$$

and when $N = 2$ and also $\phi_j = \phi_{-j} = \phi$, then the above expression reduces to:

$$\begin{bmatrix} \pi_j \\ \pi_{-j} \end{bmatrix} = \begin{bmatrix} \frac{\mu-r}{\gamma\,(\sigma_U{}^2\,(1+\phi)+\sigma_S{}^2\,(2+\phi))} \\ \frac{\mu-r}{\gamma\,(\sigma_U{}^2\,(1+\phi)+\sigma_S{}^2\,(2+\phi))} \end{bmatrix}. \tag{15}$$

The above expression shows that if there were only two assets and there was equal ambiguity about each asset, $\phi_j = \phi_{-j} = \phi$ then the proportion of wealth allocated to them would be the same, $\pi_j = \pi_{-j}$.

The following proposition shows that the investor will hold own-company stock only if there is some ambiguity about the return on the market portfolio, $\phi_{-j} > 0$. That is, the model can generate holdings in own-company stocks only in the presence of some ambiguity about the returns on the stocks of other firms.

Proposition 2 *If there is no ambiguity about the return on the market portfolio, $\phi_{-j} = 0$, then the optimal portfolio is given by the standard Merton portfolio with zero holding of own-company stock:*

$$\begin{bmatrix} \pi_j \\ \pi_m \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{\gamma}\frac{\mu-r}{\sigma_S{}^2} \end{bmatrix}. \tag{16}$$

But when there is ambiguity about market returns, then the investor will invest in own-company stock unless there is extreme ambiguity about own-company stock, $\phi_j \to \infty$.

Proposition 3 *When N is large, $\phi_{-j} > 0$ and $\phi_j \to \infty$, then the optimal holding of own-company stock is zero, while the holding in the market portfolio is scaled by the factor $1/(1 + \phi_{-j})$ to reflect the ambiguity about the overall market return.*

$$\begin{bmatrix} \pi_j \\ \pi_m \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{(1+\phi_{-j})}\frac{1}{\gamma}\frac{(\mu_2-r)}{\sigma_S{}^2} \end{bmatrix} = \frac{1}{(1+\phi_{-j})}\,\pi_{\text{Merton}}. \tag{17}$$

¿From Propositions 2 and 3 we have the following important consequence of the model.

Proposition 4 *The investor will always find it optimal to hold own-company stock as long as there is some ambiguity about the return on the market portfolio, $\phi_{-j} > 0$, and the investor is less than perfectly ignorant about own-company stock, $\phi_j < \infty$.*

We also have the following comparative static results about the holding of own-company stock.[13]

Proposition 5 *Keeping all else constant, the proportion of wealth invested in own-company stock, π_j, decreases with (a) an increase in the ambiguity about this stock, given by ϕ_j, (b) an increase in the volatility of own-company stock, which is given by $\sigma = \sqrt{\sigma_S^2 + \sigma_U^2}$.*

In this section, we have established some qualitative properties of the optimal portfolio weights in the presence of ambiguity about stock returns. In the next section, we calibrate the model to US stock returns to determine the magnitude of own-company holdings in order to evaluate the conditions under which the model can generate portfolio weights that are similar to the ones observed in the data.

4 Results based on calibration of the model to stock returns

In this section, we calibrate the model described above to data on US stock returns in order to examine whether the magnitude of the portfolio share allocated to own-company stock is consistent with empirical data. In order to determine whether the parameter values are reasonable we provide two interpretations for the optimal portfolio weights derived above—one in terms of an adjustment to expected stock returns and the other in terms of an adjustment to the riskiness of stock returns.

In order to reduce the number of parameters that need to be estimated and limit the degrees of freedom available to the model, we study the ratio of the weight allocated to own-company stock relative to the total investment in risky assets, $\pi_j/(\pi_j+(N-1)\pi_{-j}) \approx \pi_j/(\pi_j+\pi_m)$, rather than the absolute weight, π_j. The advantage of studying this ratio is that it is independent of the investor's

[13]It should be noted that in our formulation risk and ambiguity are not separable. More specifically, in our formulation, how ambiguity, described by ϕ_j and ϕ_{-j}, affects the investor's portfolio decision, depends on the variance-covariance matrix. Because the number of stocks in the two asset-classes is not the same, the variance of the returns on the two asset-classes is not the same. In Appendix B we explain the consequence of this, and in Appendix C we provide an alternative formulation with the N stocks modeled individually rather than in terms of two asset classes. This model provides further insights into how ambiguity affects an investor's portfolio choice and shows that our results are not driven by the fact that we have modeled two asset classes rather than considering explicitly the N stocks.

risk aversion and also of the equity risk premium; hence, our calibration results will not depend on estimates of these two quantities about which there is much controversy in the literature.

For the benchmark Merton portfolio, that is, the portfolio an investor would hold in the absence of ambiguity, this ratio is

$$\frac{\pi_j}{\pi_j + (N-1)\pi_{-j}} = 1/N, \tag{18}$$

and, in the limit as $N \to \infty$, we get that:

$$\frac{\pi_j}{\pi_j + \pi_m} = 0. \tag{19}$$

Thus, the benchmark model predicts zero holding of own-company stock.

In the model with ambiguity,

$$\frac{\pi_j}{\pi_j + (N-1)\pi_{-j}} = \frac{{\sigma_U}^2 + \left((N-1)\ {\sigma_S}^2 + {\sigma_U}^2\right)\ \phi_{-j}}{(N-1)\ {\sigma_S}^2\ (\phi_{-j} + \phi_j) + {\sigma_U}^2\ (N + \phi_{-j} + (N-1)\ \phi_j)} \tag{20}$$

and in the limit as $N \to \infty$, this ratio is

$$\frac{\pi_j}{\pi_j + \pi_m} = \frac{{\sigma_S}^2\, \phi_{-j}}{{\sigma_U}^2\ (1 + \phi_j) + {\sigma_S}^2\ (\phi_{-j} + \phi_j)}. \tag{21}$$

As Proposition 2 would indicate, we can see from equation (21) that the ratio $\frac{\pi_j}{\pi_j+\pi_m}$ is 0 when there is no ambiguity about the market return, $\phi_{-j} \to 0$; and, from Propositions 3 and 4 it follows that the ratio will be strictly positive as long as the investor is not extremely ambiguous about own-company stock returns, $\phi_j < \infty$.

¿From equation (21), we see that the weight in a particular risky asset *relative* to the total investment in all risky assets is independent of the magnitude of the risk premium, $\mu - r$, and also of the degree of risk aversion, γ, even though the *total* proportion of wealth in risky assets depends on the size of both. The relative weights depend only on the systematic volatility of the market, σ_S, the unsystematic volatility of the individual stock, σ_U, and the parameters indicating the ambiguity about return distributions of own-company stock and the market, ϕ_j and ϕ_{-j}, respectively. In addition to limiting the number of parameters to be estimated, studying the relative weights has the added advantage that volatility, in contrast to expected returns, can be estimated fairly accurately (see Merton (1980)); in particular, σ_S can be measured by estimating the volatility of the return on the market portfolio, and σ_U can be determined by estimating the volatility of individual stock returns and using the relation that the volatility of individual stock returns is equal to $(\sigma_S^2 + \sigma_U^2)^{1/2}$.

Thus, in order to evaluate whether the model can generate the observed investment in own-company stock in pension plans with reasonable parameter values, we need to judge only whether ϕ_j and ϕ_{-j} are reasonable. Even though ϕ_j and ϕ_{-j} are unobservable, we now show that the choice of these parameters can be interpreted either in terms of an adjustment of expected stock returns or in terms of an adjustment of the volatility of stock returns. That is, the optimal portfolio in (9) can be expressed using the standard Merton expression for portfolio weights but where the true expected return on stocks is reduced or volatility is increased to reflect the investor's concern for ambiguity.

We first provide an interpretation of the optimal portfolio weights in terms of an adjustment to the volatility of stock returns. Observe that the expression for the optimal portfolio weight can also be written as

$$\pi = \frac{1}{\gamma} (\Omega A)^{-1} (\mu - r\mathbf{1}) \tag{22}$$

and so one can interpret the effect of ambiguity being an adjustment of the variance-covariance matrix, Ω, by the matrix A. It turns out that the matrix product ΩA has a form that is intuitively appealing:

$$\begin{aligned} \Omega A &= \begin{bmatrix} (\sigma_S^2 + \sigma_U^2)(1+\phi_j) & \sigma_S^2 \\ \sigma_S^2 & (\sigma_S^2 + \sigma_U^2/(N-1))(1+\phi_{-j}) \end{bmatrix} \\ &\approx \begin{bmatrix} (\sigma_S^2 + \sigma_U^2)(1+\phi_j) & \sigma_S^2 \\ \sigma_S^2 & \sigma_S^2(1+\phi_{-j}) \end{bmatrix}. \end{aligned} \tag{23}$$

From equation (23) we see that ambiguity about the returns on the market portfolio can be interpreted as leading the investor to increase the volatility of the market return from σ_S to $\sigma_S(1+\phi_{-j})^{1/2}$ and to increase the volatility of own-company stock from $(\sigma_S^2 + \sigma_U^2)^{1/2}$ to $(\sigma_S^2 + \sigma_U^2)^{1/2}(1+\phi_j)^{1/2}$. Thus, we can define $\eta_{-j} \equiv (1+\phi_{-j})^{1/2} - 1$ as the percentage increase in the volatility of the return on the market portfolio and $\eta_j \equiv (1+\phi_j)^{1/2} - 1$ as the percentage increase in the volatility of own-company stock returns.

The above interpretation is particularly relevant given the perceptions that employees appear to have about the relative riskiness of their own company stock. For example

> . . . a recent survey of national defined contribution plan participants showed that participants systematically err in assessing the risks of their company stock (Figure 1), rating employer stock as less risky than a diversified equity mutual fund. Moreover, that survey showed that participants properly rated "individual stocks" as more risky than an equity mutual fund, but they considered their employer's stock as less risky (in effect they perceived their own company stock as less risky other individual stocks). Despite the fact that average volatility of an individual stock is at least twice the volatility of a diversified market portfolio, participants rated individual stocks as only slightly more risky.
>
> Mitchell and Utkus (2002, pp. 22-23)

These are not isolated findings; similar results are reported in a survey conducted by John Hancock Financial Services (1999) and Benartzi (2001). We will shortly see that our interpretation in terms of an adjustment to the riskiness of stocks provides an intuitive interpretation of the results obtained when we use data to calibrate the model to the volatility of stock returns.

Next, we present the interpretation of the optimal portfolio weights in terms of an adjustment to expected stock returns. Denoting the adjustment to the expected return by $\nu \equiv (\nu_j, \nu_{-j})^\top$, equation (9) can be rewritten as:

$$\begin{aligned} \pi &= \frac{1}{\gamma} A^{-1} \Omega^{-1} \Big(\mu - r\mathbf{1} \Big) \\ &= \frac{1}{\gamma} \Omega^{-1} \Big([\mu - r\mathbf{1}] - \nu \Big) . \end{aligned} \tag{24}$$

where

$$\nu = (I - \Omega A^{-1} \Omega^{-1})(\mu - r\mathbf{1}). \tag{25}$$

Thus, equation (24) can be interpreted as the Merton model but with the expected return on the assets being $\mu - \nu$ instead of μ. Hence, to judge whether the choice of ϕ_j and ϕ_{-j} is reasonable, we can consider instead the ν_j and ν_{-j} implied by the choice of ϕ_j and ϕ_{-j}; then, we can ask the question whether the difference in the adjustment to own-company expected returns and other stock returns, $\nu_j - \nu_{-j}$, is reasonable or not. An unappealing characteristic of this interpretation is that the expression for the adjustment to mean returns in equation (25) depends on the risk premium, even though the proportion invested in own-company stock relative to the market does not; in our calibrations, we compute the adjustment to the mean return assuming that the risk premium is 7% p.a.

For our empirical work we assume that the volatility of the market, σ_S, is 20% p.a. For the estimates of the volatility of individual stock returns we rely on the work of Chan, Karceski and Lakonishok (1999), who find that the average volatility of large firms is 28.3%, the average volatility of the average firm is 34.3%, and that the average volatility of small firms is 46.6%. We use these estimates to characterize three different company profiles by selecting the following values for the unsystematic volatility, $\sigma_U = \{20\%, 30\%, 40\%\}$, which then imply that the three corresponding values for own-company volatility, $(\sigma_S^2 + \sigma_U^2)^{1/2}$ are in line with the estimates of Chan, Karceski and Lakonishok (1999): $\{28.3\%, 36.1\%, 44.7\%\}$.

Our calibration results are presented in Tables 1, 2, and 3. Each table gives the proportion of wealth allocated to own-company stock, π_j, and that allocated to the market, π_m, for a particular choice of volatilities, $\{\sigma_S, \sigma_U\}$, and ambiguity parameters, $\{\phi_j, \phi_{-j}\}$. In all three tables, $\sigma_S = 20\%$, while $\sigma_U = 20\%$ in Table 1, 30% in Table 2, and 40% in Table 3. We allow ϕ_j and ϕ_{-j} to range from 0 to 1 in increments of 0.25. Each table also reports the adjustment to the volatility of own-company stock and the other stocks, η_j and η_{-j}, that correspond to each particular choice of $\{\phi_j, \phi_{-j}\}$. and the *relative* adjustment, $\eta_j - \eta_{-j}$. Finally, each table includes the adjustment to expected stock returns, ν_j and ν_{-j}, that correspond to the choice of $\{\phi_j, \phi_{-j}\}$, and the *relative* adjustment, $\nu_j - \nu_{-j}$.

Table 1 considers the case where $\sigma_U = 20\%$, which corresponds to own-company volatility being $(\sigma_S^2 + \sigma_U^2)^{1/2} = 28.28\%$. In the first panel, $\phi_j = 0.00$. From the first row of the first panel we see that when $\phi_{-j} = 0.00$, the entire holdings of risky assets are in the market and there is zero investment in own-company stock. The second row of the first panel shows that when $\phi_{-j} = 0.25$. the investment in the market drops to 80%, and that in own-company stock increases to 20%. The table also reports that a $\phi_j = 0.00$ and $\phi_{-j} = 0.25$ correspond to the investor viewing the market volatility as being 11.80% more risky than its true volatility. Equivalently, these values of ϕ_j and ϕ_{-j} correspond to the investor reducing the expected return on own company stock by 0% and on the market by 1.17%, implying that the excess return expected on own company stock relative to the market is 1.17%. These numbers indicated that a small deviation from the standard model without ambiguity is sufficient to generate substantial holdings of own-company stock.

The third row of the first panel of Table 1 shows that as ambiguity about market returns increases with ϕ_{-j} increasing from 0.25 to 0.50, holding of own-company stock increases from 20% to 33%. In this case, $\phi_j = 0.00$ and $\phi_{-j} = 0.25$ correspond to the investor viewing the market volatility as being 22.47% more risky than its true volatility or viewing the expected return on own-company stock to exceed the expected market return by 1.75%. The other rows of the first panel illustrate the same pattern: as ϕ_{-j} increases, the investment in own-company stock increases.

The second panel of Table 1 shows the same quantities but now for the case where there is ambiguity about own-company stock, given by $\phi_j = 0.25$. Again, we see the same pattern of results: when $\phi_{-j} = 0.00$ (first row of this panel), the investor holds only the market portfolio, but when there is ambiguity about the market return, then the investor holds also own-company stock. For instance, from the second row of this panel where $\phi_{-j} = 0.25$, we see that the investor holds 14.29% in own company stock and 85.71% in the market; and, the $\phi_j = 0.25$ and $\phi_{-j} = 0.25$ correspond to the investor viewing the market volatility and own-company stock volatility as being 11.80% higher than their true volatility or viewing the expected return on own-company stock to exceed the expected market return by 0.82%. Thus, even though ambiguity about own-company stock and the market is the same, $\phi_j = \phi_{-j} = 0.25$, and the implied adjustment to the volatility of the market and own-company stock is the same, the small change in the excess expected return of 0.0082 leads the investor to hold 14.29% of own-company stock.

Similarly, from the third panel where $\phi_j = 0.50$, we see that for third row where $\phi_{-j} = 0.50$ the investment in own-company stock is 20%; and $\phi_j = \phi_{-j} = 0.50$ corresponds to expecting the mean return on own-company stock to exceed the market return by just 1% while the adjustment to volatility of own-company stock and market returns is the same.

Reading down the panels, we see that as ϕ_j increases while keeping all else the same, the holding of own-company stock declines. For example, comparing the second panel where $\phi_j = 0.25$ to the first one where for $\phi_j = 0.00$, we see that for the row where $\phi_{-j} = 0.25$ the holding of own-company stock in the first panel has was 20% and in the second panel it has decreased to 14.29%. This is true for all the rows of the second panel compared to the corresponding rows of the first panel, and is a pattern that is repeated in the other panels of the table as well. However, even when the parameter driving ambiguity about own-company stock returns is greater than that for market returns, $\phi_j > \phi_{-j}$, as long as there is some ambiguity about market returns, $\phi_{-j} > 0$, the investor holds some company stock. For example, consider the last panel where $\phi_j = 1$ and look at the row where $\phi_{-j} = 0.75$. Even though $\phi_j > \phi_{-j}$ the investor invests 20% in own-company stock; this is a consequence of expecting mean stock returns to exceed the market return by 0.0088. But this small difference in expected returns is sufficient to lead to a significant investment in own-company stock.

Table 2 analyzes the model when unsystematic volatility is $\sigma_U = 30\%$ rather than 20%, and Table 3 considers the case where $\sigma_U = 40\%$. As unsystematic volatility increases, the volatility of own-company stock, $(\sigma_S^2 + \sigma_U^2)^{1/2}$, increases while the volatility of market returns, σ_S, stays the same. Consequently, we find that holding of own-company stock is smaller when unsystematic volatility is larger. For instance, comparing the second row of Table 1 with that of Tables 2 and 3, we see that the holding of own-company stock of 20% in Table 1 drops to 10% in Table 2 and to 5.88% in Table 3. This is consistent with the empirical finding that the fraction invested in own company stock is lower for small firms which typically have higher volatility (see Chan, Karceski and Lakonishok (1999)).

We conclude this section by noting that for reasonable values of ϕ_j and ϕ_{-j}, where the reasonableness is judged based on the adjustment implied by these parameters to the mean or volatility of returns the model can generate substantial holding of own-company stock that is is consistent with the empirical observations reported in Benartzi (2001), Meulbroek (2002) and Mitchell and Utkus (2002).

5 Effect of human capital on holding of own-company stock

In the model described above, we have not considered the human capital that each employee has invested in the firm. The main reason for this omission is that it is difficult to get closed-form solutions once the model includes human capital (see, for instance, Viceira (2001)). In this section, we explain how one can consider the effect of human capital on the optimal holding of own-company stock in our framework without modeling human capital explicitly.

To quantify the effect of human capital on the bias toward own-company stock, we need information on the magnitude of human capital as a proportion of wealth and also the correlation between the returns on human capital with the returns on own-company stock. While we have some estimates about the magnitude of human capital, we do not have reliable estimates for the correlation between the returns on human capital and the returns on own-company stock.[14] Thus, we formulate the problem of portfolio choice with human capital in such a way that one can calibrate the model to a broad range of values in order to evaluate the conditions under which human capital will have a significant effect on our conclusions.

To understand the effect of human capital on the holding of own-company stock, suppose that the total human capital of an employee consists of two parts: one part that is uncorrelated to the returns on all stocks, and a second part that is correlated with the returns on stocks; in particular, let this second part to be perfectly correlated to the returns on own-company stock. Denote the magnitude of the unsystematic part of human capital as a percentage of financial wealth by H^U and the systematic component by H^S. The part of human capital whose returns are idiosyncratic, H^U, will have no effect on the allocation of wealth between own-company stock and the market portfolio, even though it will lead to a reduction in the total share of aggregate wealth allocated to risky assets (see Pratt and Zeckhauser (1987), and Kimball (1990, 1993)).

Now consider the other part of human capital, H^S, the returns on which are perfectly correlated to returns on own-company stock. The optimal *total* investment (that is, investment in stocks *and* the investment in human capital) in the employee's own-company will be given by the fraction that we had determined earlier from our model without human capital, π_j, multiplied by $(1 + H^S)$. Given that the employee is constrained, by definition, to invest all her human capital in the firm, the employee will reduce her holding of own-company stock by H^S; that is, in the presence of human capital the optimal portfolio share, $\pi_j^{H^S}$, in own-company stock as a proportion of financial wealth, is

$$\pi_j^{H^S} = \pi_j(1 + H^S) - H^S. \tag{26}$$

The expression in equation (26) tells us how π_j, the portfolio weight from our model without human capital, needs to be adjusted in order to obtain the portfolio weight in the presence of human capital. This adjustment depends on only a single parameter H^S which represents that part of human capital that is perfectly correlated with the returns on own-company stock.

[14] Estimates of the correlation between the returns on human capital and the returns on stock returns at an *aggregate* level are close to zero (see, for instance, Cocco, Gomes and Maenhout (2002) and Davis and Willen (2000a, 2000b)). Massa and Simonov (2003) estimate the correlations between an individual's labor income and the return on the market portfolio, which they estimate to be negative, and between labor income and the investor's portfolio, which they find to be between 0.025 and 0.053; they do not compute the correlation between labor income and own-company stock.

Heaton and Lucas (2000, Table V) report capitalized labor income as a proportion of total assets for U.S. households based on the 1992 Survey of Consumer Finances. This corresponds to the *sum* of H^U and H^S in our model. They report this data by age and by the net worth of individuals. For individuals older than 65 years, capitalized labor income is about 2 percent of total assets. For individuals under 65 years who have a net worth ranging between ten thousand and one hundred thousand dollars, capitalized labor income as a percentage of total assets has a mean of 0.662 and a standard deviation of 0.308; for individuals with a net worth between one hundred thousand and one million dollars, the mean is 0.482 and the standard deviation is 0.290; and, for individuals with a net worth exceeding one million dollars the mean is 0.211 and the standard deviation is 0.200. Based on the numbers reported in Heaton and Lucas (2000), and given that we do not have any data on the relative magnitude of H^U and H^S, we consider H^S ranging from 0.100 to 0.500. Our results from this experiment are reported in Table 4, which corresponds to the very first panel of Table 4, that has now been extended to allow for different levels of H^S.

¿From Table 4, we see that when the human capital that is perfectly correlated to own-company stock returns is equal to 10% of financial wealth, the effect of human capital on the bias toward own-company stock is not big enough to offset the effect of ambiguity. For instance, in Row 3 of the first panel, the investment in own-company stock decreases from 0.3333 for the case without human capital to 0.2667 for the case with human capital. As H^S increases, it offsets increasingly the effect of ambiguity. For example, in Row 3 of the second panel where $H^S = 0.20$, the investment in own-company stock declines from 0.3333 to 0.2000, and in Row 3 of the last panel, where $H^S = 0.50$, the investment in own-company stock generated by ambiguity is reduced from 0.3333 all the way down to 0. But recall that an adjustment of 0.023 to expected returns corresponds to only *one* standard error of the estimated expected return, and even for this case investment in own-company stock when $H^S = 0.50$ (last panel) is 0.25; for more reasonable values of H^S, the investment in own-company stock arising from aversion to ambiguity is still significant.

6 Conclusion

In this paper, we evaluate the large holding of own-company stock in defined contribution pension plans even though this violates the most fundamental tenets of classical finance theory: diversification. That is, even though portfolio theory dictates that investors should invest in only a single fund of risky assets, which in equilibrium is the market portfolio, there is substantial evidence that rather than holding just the market portfolio, employees invest a substantial amount in the stock of their own company. While it is inefficient to invest a large proportion of one's wealth in the stock of *any* single company, the cost of doing so is even greater when the stock is that of the firm where one works.

We develop a formal model for decision making in which investors are averse to ambiguity about the true distribution of asset returns. The main feature of this model is that it allows investors to distinguish their ambiguity about one class or assets relative to others. We analyze the optimal portfolio in this model and show that if agents are ambiguous about the returns on the market portfolio, then they will invest in own-company stock. This is true even when investors are ambiguous also about the returns on own-company stock.

We show analytically that the model has the following implications, which are consistent with the stylized empirical observations: (i) In the presence of ambiguity about returns on the market portfolio, the investor holds own-company stock; (ii) The proportion of wealth allocated to own-company stock increases with an increase in ambiguity about market returns; (ii) The proportion of wealth allocated to own-company stock increases with a decrease in the volatility of own-company stock returns.

An attractive feature of the model is that to determine the relative allocation to own-company stock and the market portfolio one needs to estimate only the: (i) volatility of returns on the market and own company stock, and (ii) the parameters dictating the investor's ambiguity about the returns on the market and own-company stock. Moreover, the volatility of returns is observable and it is well known that this can be estimated quite precisely; the parameters determining ambiguity are unobservable but we show how these parameters can be related to an adjustment of either the mean or the volatility of market returns and own-company returns, which allows one to gauge whether the values chosen for these parameters are reasonable. Calibration of the model to stock returns shows that for reasonable parameter values the model is capable of generating the magnitude of investment in own-company stock that is observed in the data.

A Identifying the relation between ξ and v

In this section, we show the relation between the ξ used to specify preferences in the discrete time setting and the v that appear in the Bellman equation when time is continuous.

Define Z_{-jt} by

$$\frac{\sigma_U}{\sqrt{N-1}} dZ_{-jt} = \frac{\sigma_U}{N-1} \sum_{i \leq N, i \neq j} dZ_{it}$$

Then Z_{-jt} is a one-dimensional Brownian motion. The return on the portfolio of agent j can be written as

$$\begin{aligned} dR_j &= [\pi_j \mu + (N-1)\pi_{-j}\mu + (1 - \pi_j - (N-1)\pi_{-j})\, r]\, dt \\ & +(\pi_j, (N-1)\pi_{-j}) \begin{bmatrix} \sigma_S & \sigma_U & 0 \\ \sigma_S & 0 & \frac{\sigma_U}{\sqrt{N-1}} \end{bmatrix} \begin{bmatrix} dZ_{St} \\ dZ_{jt} \\ dZ_{-jt} \end{bmatrix}. \end{aligned}$$

The covariance matrix of the own-company stock and the portfolio of other stocks is given by

$$\Omega = \begin{bmatrix} \sigma_S^2 + \sigma_U^2 & \sigma_S^2 \\ \sigma_S^2 & \sigma_S^2 + \sigma_U^2/(N-1) \end{bmatrix}.$$

Now according to Uppal and Wang (2002),

$$a_j^\top = -\frac{v_j}{\sigma_S^2 + \sigma_U^2} (\sigma_S, \sigma_U, 0)$$

and

$$a_{-j}^\top = -\frac{v_{-j}}{\sigma_S^2 + \sigma_U^2/(N-1)} (\sigma_S, 0, \frac{\sigma_U}{\sqrt{N-1}})$$

Let $dZ_t = (dZ_{St}, dZ_{jt}, dZ_{-jt})$. Then, according to Uppal and Wang (2002),

$$\begin{aligned} \xi_{jt} &= \exp\left\{ \int_0^t a_j^\top dZ_s - \frac{1}{2} \int_0^t \frac{v_j^2}{\sigma_S^2 + \sigma_U^2} ds \right\}, \\ \xi_{-jt} &= \exp\left\{ \int_0^t a_{-j}^\top dZ_s - \frac{1}{2} \int_0^t \frac{v_{-j}^2}{\sigma_S^2 + \sigma_U^2/(N-1)} ds \right\}. \end{aligned}$$

To summarize, define Z_{-jt} by

$$\frac{\sigma_U}{\sqrt{N-1}} dZ_{-jt} = \frac{\sigma_U}{N-1} \sum_{i \leq N, i \neq j} dZ_{it}.$$

and

$$a_j^\top = -\frac{v_j}{\sigma_S^2+\sigma_U^2}(\sigma_S,\sigma_U,0)$$
$$a_{-j}^\top = -\frac{v_{-j}}{\sigma_S^2+\sigma_U^2/(N-1)}(\sigma_S,0,\frac{\sigma_U}{\sqrt{N-1}}).$$

Let $dZ_t=(dZ_{St},dZ_{jt},dZ_{-jt})$. Then, ξ and v are related through,

$$\xi_{jt} = \exp\left\{\int_0^t a_j^\top dZ_s - \frac{1}{2}\int_0^t \frac{v_j^2}{\sigma_S^2+\sigma_U^2}ds\right\},$$
$$\xi_{-jt} = \exp\left\{\int_0^t a_{-j}^\top dZ_s - \frac{1}{2}\int_0^t \frac{v_{-j}^2}{\sigma_S^2+\sigma_U^2/(N-1)}ds\right\}.$$

B Understanding the drift adjustment for own-company stock and the market

In this appendix, we show that the specification of the variance-covariance matrix Ω in equation (6) implies that when there is ambiguity about the marginal distribution of returns for own-company stock j ambiguity about the (joint) marginal distribution of returns for all the other stocks, then the drift adjustment for the return on the portfolio of all the other stocks will always be larger than that for the return on own-company stock. Consequently, investor j will always over-invest in stock j, except in the two extreme cases where there is no ambiguity about the return on the market, $\phi_{-j}=0$, or there is extreme ignorance about own-company stock returns, $\phi_j=\infty$. This result is due to the particular specification of Ω, as will be show in this appendix and Appendix C.

Start by considering a setting with a more general variance-covariance matrix

$$\Omega = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$$

and

$$\Phi = \begin{bmatrix} \frac{1}{\phi_1\sigma_1^2} & 0 \\ 0 & \frac{1}{\phi_2\sigma_2^2} \end{bmatrix}.$$

For this formulation, the drift adjustments are

$$\begin{bmatrix} \nu_j \\ \nu_{-j} \end{bmatrix} = (\mu - r)\begin{bmatrix} \frac{\phi_1(\sigma_2(1+\phi_2)-\rho\sigma_1)}{\sigma_2(1-\rho^2+\phi_2+\phi_1(1+\phi_2))} \\ \frac{\phi_2(\sigma_1(1+\phi_1)-\rho\sigma_2)}{\sigma_1(1-\rho^2+\phi_2+\phi_1(1+\phi_2))} \end{bmatrix}$$

Thus, the difference of the drift adjustments is,

$$\nu_{-j} - \nu_j = \frac{\sigma_1\,(\rho\,\sigma_1 - \sigma_2)\,\phi_1 + \sigma_2\,(\sigma_1 - \rho\,\sigma_2)\,\phi_2}{\sigma_1\,\sigma_2\,(1 - \rho^2 + \phi_2 + \phi_1\,(1+\phi_2))} \times (\mu - r)$$

Thus, the expression above shows that the sign of the difference in drift adjustments depends on the sign of the numerator,

$$\sigma_1\,(\rho\,\sigma_1 - \sigma_2)\,\phi_1 + \sigma_2\,(\sigma_1 - \rho\,\sigma_2)\,\phi_2,$$

which, in general, can be either positive or negative.

In our case, we see from the specification of Ω in equation (6) that $\sigma_1^2 = \sigma_S^2 + \sigma_U^2$, and $\sigma_2^2 = \sigma_S^2$ while $\rho\sigma_1\sigma_2 = \sigma_S^2$ so that $\rho\sigma_1 = \sigma_2$. Making these substitutions in the expression for the difference in drift adjustments gives

$$\nu_{-j} - \nu_j = \frac{{\sigma_U}^2\,\phi_2}{{\sigma_U}^2\,(1+\phi_1)\,(1+\phi_2) + {\sigma_S}^2\,(\phi_2 + \phi_1\,(1+\phi_2))} \times (\mu - r) > 0,$$

where the expression in the numerator is always positive. This shows that under our specification of the variance-covariance matrix Ω in equation (6), the difference in drift adjustments, $\nu_{-j} - \nu_j$, will always be positive.

C An Alternative Formulation

This section provides an alternative formulation of the investor's ambiguity about expected stock returns. The objective is to caution interpretations of comparative static analysis under any particular formulation of ambiguity.

In this section, we assume that the infinitely-lived agents have preferences represented by

$$V_{jt} = \frac{c_{jt}^{1-\gamma}}{1-\gamma}\,\Delta + e^{-\rho\,\Delta} \inf_{\xi} \left\{ \psi \left(E_t^{\xi}[V_{jt+\Delta}] \right) \sum_{k=1}^{n} \frac{1}{\phi_k} E_t^{\xi_k} \left[\ln \frac{\xi_{kt+\Delta}}{\xi_{kt}} \right] + E_t^{\xi}\left(V_{jt+\Delta}\right) \right\} \tag{27}$$

Comparing with (3), we see that the investor's ambiguity about each individual stock is described separately, reflected in the penalty function. Here ξ_k is the density function of the marginal distribution of stock k under the alternative probability measure.

The return equation is

$$dR_j = \sum_{i=1}^{n} \pi_i \mu dt + \sum_{i=1}^{n} \pi_i (\sigma_S dZ_{St} + \sigma_U dZ_{it}) + \left(1 - \sum_{i \le N} \pi_i\right) r dt.$$

The covariance matrix of the returns of all stocks is given by:

$$\Omega = \begin{bmatrix} \sigma_S^2 + \sigma_U^2 & \sigma_S^2 & \cdots & \sigma_S^2 \\ \sigma_S^2 & \sigma_S^2 + \sigma_U^2 & \cdots & \sigma_S^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_S^2 & \sigma_S^2 & \cdots & \sigma_S^2 + \sigma_U^2 \end{bmatrix}. \tag{28}$$

The wealth equation of the investor is again

$$dW_t = W_t dR_j - c_t dt. \tag{29}$$

Given the preferences in equation (27) and the wealth dynamics in (29), the resulting Hamilton-Jacobi-Bellman equation for agent j, from Uppal and Wang (2003), is

$$\begin{aligned} 0 \;=\; \sup_{c,\pi} \inf_{v} \Bigg\{ & u(c) - \rho V + V_t + W V_W \left[r + \pi \left(\mu - r\mathbf{1} \right) - \frac{c}{W} \right] + \frac{W^2}{2} V_{WW} \pi \Omega \pi^\top \\ & + V_W W \pi v + \frac{\psi(V)}{2} v^\top \Phi v \Bigg\}, \end{aligned} \tag{30}$$

where

$$\Phi \equiv \begin{bmatrix} \frac{1}{\phi_1(\sigma_S^2+\sigma_U^2)} & 0 & \cdots & 0 \\ 0 & \frac{1}{\phi_2(\sigma_S^2+\sigma_U^2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\phi_N(\sigma_S^2+\sigma_U^2)} \end{bmatrix}, \tag{31}$$

where $\phi_i = \phi_1$ for $i \neq j$.

Proposition 6 *The optimal portfolio of agent j is given by*

$$\pi = \frac{1}{\gamma} A^{-1} \Omega^{-1} (\mu - r\mathbf{1}), \tag{32}$$

where

$$A = \left(I + \Omega^{-1} \Phi^{-1} \right). \tag{33}$$

Note that because of the diagonal structure of Φ,

$$A^{-1}\Omega^{-1} = \begin{bmatrix} (1+\phi_1)(\sigma_S^2+\sigma_U^2) & \sigma_S^2 & \cdots & \sigma_S^2 \\ \sigma_S^2 & (1+\phi_2)(\sigma_S^2+\sigma_U^2) & \cdots & \sigma_S^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_S^2 & \sigma_S^2 & \cdots & (1+\phi_N)(\sigma_S^2+\sigma_U^2) \end{bmatrix}^{-1}. \tag{34}$$

So if $\phi_i = \phi_1$ for all i, then $\pi_1 = \pi_i$ for all i. In terms of drift adjustment,

$$\nu = (I - \Omega A^{-1}\Omega^{-1})(\mu - r\mathbf{1}). \tag{35}$$

As can be seen clearly that the functional form of the portfolio weights (32) in this section is exactly the same as the one in the main text. The only difference is in Φ given in (31). Notice that all parameters associated with ambiguity are embedded in Φ, which affects the investor's portfolio weights through $A^{-1}\Omega^{-1}$ in (34). In this formulation, because of the completely symmetric treatment of all stocks, when $\phi_1 = \phi_2 = \cdots = \phi_N$ which *can* be interpreted as equal ambiguity towards all stocks, the investor will hold equal amount of all stocks.

Table 1: Asset allocations for various levels of ambiguity when $\sigma_U = 0.20$

This table displays the proportion of wealth allocated to own-company stock ($\pi_j/(\pi_j + \pi_m)$) and to the market portfolio ($\pi_m/(\pi_j + \pi_m)$) as the degree of ambiguity about own-company stock returns (ϕ_j) and the market (ϕ_{-j}) varies. In order to judge whether the level of ambiguity is reasonable, the table reports the percentage increase in own-company volatility, η_j implied by ϕ_j, the percentage increase in market volatility, η_{-j} implied by ϕ_{-j}, and the difference between the two, $\eta_{-j} - \eta_j$. The table also reports the adjustments to expected stock returns $\{\nu_j, \nu_{-j}\}$ corresponding to ϕ_j and ϕ_{-j}, and the net adjustment, $\nu_{-j} - \nu_j$. The equity market premium, $\mu - r$ is assumed to be 0.07. It is assumed that the volatility of market returns is 0.20 and that idiosyncratic volatility is 0.20, so that the total volatility of individual stock returns is 0.2828.

	Risk adjustment			Drift adjustment			Relative weights	
ϕ_{-j}	η_j	η_{-j}	$\eta_{-j} - \eta_j$	ν_j	ν_{-j}	$\nu_{-j} - \nu_j$	$\frac{\pi_j}{\pi_j+\pi_m}$	$\frac{\pi_m}{\pi_j+\pi_m}$
Panel with $\phi_j = 0.00$								
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.0000	0.1180	0.1180	0.0000	0.0117	0.0117	0.2000	0.8000
0.50	0.0000	0.2247	0.2247	0.0000	0.0175	0.0175	0.3333	0.6667
0.75	0.0000	0.3229	0.3229	0.0000	0.0210	0.0210	0.4286	0.5714
1.00	0.0000	0.4142	0.4142	0.0000	0.0233	0.0233	0.5000	0.5000
Panel with $\phi_j = 0.25$								
0.00	0.1180	0.0000	-0.1180	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.1180	0.1180	0.0000	0.0041	0.0124	0.0082	0.1429	0.8571
0.50	0.1180	0.2247	0.1067	0.0064	0.0191	0.0127	0.2500	0.7500
0.75	0.1180	0.3229	0.2048	0.0078	0.0233	0.0156	0.3333	0.6667
1.00	0.1180	0.4142	0.2962	0.0088	0.0263	0.0175	0.4000	0.6000
Panel with $\phi_j = 0.50$								
0.00	0.2247	0.0000	-0.2247	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.2247	0.1180	-0.1067	0.0064	0.0127	0.0064	0.1111	0.8889
0.50	0.2247	0.2247	0.0000	0.0100	0.0200	0.0100	0.2000	0.8000
0.75	0.2247	0.3229	0.0981	0.0124	0.0247	0.0124	0.2727	0.7273
1.00	0.2247	0.4142	0.1895	0.0140	0.0280	0.0140	0.3333	0.6667
Panel with $\phi_j = 0.75$								
0.00	0.3229	0.0000	-0.3229	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.3229	0.1180	-0.2048	0.0078	0.0130	0.0052	0.0909	0.9091
0.50	0.3229	0.2247	-0.0981	0.0124	0.0206	0.0082	0.1667	0.8333
0.75	0.3229	0.3229	0.0000	0.0154	0.0256	0.0102	0.2308	0.7692
1.00	0.3229	0.4142	0.0913	0.0175	0.0292	0.0117	0.2857	0.7143
Panel with $\phi_j = 1.00$								
0.00	0.4142	0.0000	-0.4142	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.4142	0.1180	-0.2962	0.0088	0.0131	0.0044	0.0769	0.9231
0.50	0.4142	0.2247	-0.1895	0.0140	0.0210	0.0070	0.1429	0.8571
0.75	0.4142	0.3229	-0.0913	0.0175	0.0263	0.0088	0.2000	0.8000
1.00	0.4142	0.4142	0.0000	0.0200	0.0300	0.0100	0.2500	0.7500

Table 2: Asset allocations for various levels of ambiguity when $\sigma_U = 0.30$

This table displays the proportion of wealth allocated to own-company stock ($\pi_j/(\pi_j + \pi_m)$) and to the market portfolio ($\pi_m/(\pi_j + \pi_m)$) as the degree of ambiguity about own-company stock returns (ϕ_j) and the market (ϕ_{-j}) varies. In order to judge whether the level of ambiguity is reasonable, the table reports the percentage increase in own-company volatility, η_j implied by ϕ_j, the percentage increase in market volatility, η_{-j} implied by ϕ_{-j}, and the difference between the two, $\eta_{-j} - \eta_j$. The table also reports the adjustments to expected stock returns $\{\nu_j, \nu_{-j}\}$ corresponding to ϕ_j and ϕ_{-j}, and the net adjustment, $\nu_{-j} - \nu_j$. The equity market premium, $\mu - r$ is assumed to be 0.07. It is assumed that the volatility of market returns is 0.20 and that idiosyncratic volatility is 0.30, so that the total volatility of individual stock returns is 0.3606.

	Risk adjustment			Drift adjustment			Relative weights	
ϕ_{-j}	η_j	η_{-j}	$\eta_{-j} - \eta_j$	ν_j	ν_{-j}	$\nu_{-j} - \nu_j$	$\frac{\pi_j}{\pi_j+\pi_m}$	$\frac{\pi_m}{\pi_j+\pi_m}$
Panel with $\phi_j = 0.00$								
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.0000	0.1180	0.1180	0.0000	0.0129	0.0129	0.1000	0.9000
0.50	0.0000	0.2247	0.2247	0.0000	0.0203	0.0203	0.1818	0.8182
0.75	0.0000	0.3229	0.3229	0.0000	0.0252	0.0252	0.2500	0.7500
1.00	0.0000	0.4142	0.4142	0.0000	0.0286	0.0286	0.3077	0.6923
Panel with $\phi_j = 0.25$								
0.00	0.1180	0.0000	-0.1180	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.1180	0.1180	0.0000	0.0035	0.0131	0.0097	0.0755	0.9245
0.50	0.1180	0.2247	0.1067	0.0056	0.0210	0.0155	0.1404	0.8596
0.75	0.1180	0.3229	0.2048	0.0070	0.0263	0.0193	0.1967	0.8033
1.00	0.1180	0.4142	0.2962	0.0080	0.0301	0.0221	0.2462	0.7538
Panel with $\phi_j = 0.50$								
0.00	0.2247	0.0000	-0.2247	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.2247	0.1180	-0.1067	0.0056	0.0133	0.0077	0.0606	0.9394
0.50	0.2247	0.2247	0.0000	0.0090	0.0215	0.0125	0.1143	0.8857
0.75	0.2247	0.3229	0.0981	0.0113	0.0270	0.0157	0.1622	0.8378
1.00	0.2247	0.4142	0.1895	0.0130	0.0310	0.0180	0.2051	0.7949
Panel with $\phi_j = 0.75$								
0.00	0.3229	0.0000	-0.3229	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.3229	0.1180	-0.2048	0.0070	0.0134	0.0064	0.0506	0.9494
0.50	0.3229	0.2247	-0.0981	0.0113	0.0218	0.0105	0.0964	0.9036
0.75	0.3229	0.3229	0.0000	0.0143	0.0275	0.0132	0.1379	0.8621
1.00	0.3229	0.4142	0.0913	0.0164	0.0316	0.0152	0.1758	0.8242
Panel with $\phi_j = 1.00$								
0.00	0.4142	0.0000	-0.4142	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.4142	0.1180	-0.2962	0.0080	0.0135	0.0055	0.0435	0.9565
0.50	0.4142	0.2247	-0.1895	0.0130	0.0220	0.0090	0.0833	0.9167
0.75	0.4142	0.3229	-0.0913	0.0164	0.0278	0.0114	0.1200	0.8800
1.00	0.4142	0.4142	0.0000	0.0190	0.0321	0.0131	0.1538	0.8462

Table 3: Asset allocations for various levels of ambiguity when $\sigma_U = 0.40$

This table displays the proportion of wealth allocated to own-company stock ($\pi_j/(\pi_j+\pi_m)$) and to the market portfolio ($\pi_m/(\pi_j+\pi_m)$) as the degree of ambiguity about own-company stock returns (ϕ_j) and the market (ϕ_{-j}) varies. In order to judge whether the level of ambiguity is reasonable, the table reports the percentage increase in own-company volatility, η_j implied by ϕ_j, the percentage increase in market volatility, η_{-j} implied by ϕ_{-j}, and the difference between the two, $\eta_{-j}-\eta_j$. The table also reports the adjustments to expected stock returns $\{\nu_j, \nu_{-j}\}$ corresponding to ϕ_j and ϕ_{-j}, and the net adjustment, $\nu_{-j}-\nu_j$. The equity market premium, $\mu - r$ is assumed to be 0.07. It is assumed that the volatility of market returns is 0.20 and that idiosyncratic volatility is 0.40, so that the total volatility of individual stock returns is 0.4472.

	Risk adjustment			Drift adjustment			Relative weights	
ϕ_{-j}	η_j	η_{-j}	$\eta_{-j}-\eta_j$	ν_j	ν_{-j}	$\nu_{-j}-\nu_j$	$\frac{\pi_j}{\pi_j+\pi_m}$	$\frac{\pi_m}{\pi_j+\pi_m}$
				Panel with $\phi_j = 0.00$				
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.0000	0.1180	0.1180	0.0000	0.0133	0.0133	0.0588	0.9412
0.50	0.0000	0.2247	0.2247	0.0000	0.0215	0.0215	0.1111	0.8889
0.75	0.0000	0.3229	0.3229	0.0000	0.0271	0.0271	0.1579	0.8421
1.00	0.0000	0.4142	0.4142	0.0000	0.0311	0.0311	0.2000	0.8000
				Panel with $\phi_j = 0.25$				
0.00	0.1180	0.0000	-0.1180	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.1180	0.1180	0.0000	0.0032	0.0135	0.0103	0.0455	0.9545
0.50	0.1180	0.2247	0.1067	0.0052	0.0219	0.0167	0.0870	0.9130
0.75	0.1180	0.3229	0.2048	0.0066	0.0277	0.0211	0.1250	0.8750
1.00	0.1180	0.4142	0.2962	0.0076	0.0320	0.0243	0.1600	0.8400
				Panel with $\phi_j = 0.50$				
0.00	0.2247	0.0000	-0.2247	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.2247	0.1180	-0.1067	0.0052	0.0136	0.0084	0.0370	0.9630
0.50	0.2247	0.2247	0.0000	0.0085	0.0222	0.0137	0.0714	0.9286
0.75	0.2247	0.3229	0.0981	0.0108	0.0281	0.0173	0.1034	0.8966
1.00	0.2247	0.4142	0.1895	0.0125	0.0325	0.0200	0.1333	0.8667
				Panel with $\phi_j = 0.75$				
0.00	0.3229	0.0000	-0.3229	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.3229	0.1180	-0.2048	0.0066	0.0136	0.0070	0.0313	0.9688
0.50	0.3229	0.2247	-0.0981	0.0108	0.0224	0.0115	0.0606	0.9394
0.75	0.3229	0.3229	0.0000	0.0138	0.0284	0.0147	0.0882	0.9118
1.00	0.3229	0.4142	0.0913	0.0159	0.0329	0.0170	0.1143	0.8857
				Panel with $\phi_j = 1.00$				
0.00	0.4142	0.0000	-0.4142	0.0000	0.0000	0.0000	0.0000	1.0000
0.25	0.4142	0.1180	-0.2962	0.0076	0.0137	0.0061	0.0270	0.9730
0.50	0.4142	0.2247	-0.1895	0.0125	0.0225	0.0100	0.0526	0.9474
0.75	0.4142	0.3229	-0.0913	0.0159	0.0286	0.0127	0.0769	0.9231
1.00	0.4142	0.4142	0.0000	0.0184	0.0332	0.0147	0.1000	0.9000

Table 4: Asset allocations with and without human capital

The table reports the portfolio weights in the presence of ambiguity about asset returns for two cases: one, where there is no human capital, and two, where the human capital that is perfectly correlated to own-company stock returns as a proportion of financial wealth is given by $H^S > 0$. The table essentially extends the first panel in Table 1 (for the case $\phi_j = 0.00$) to allow for human capital. This table displays the proportion of wealth allocated to own-company stock and to the market portfolio in the absence of human capital, $\{\pi_j/(\pi_j+\pi_m), \pi_m/(\pi_j+\pi_m)\}$, and when human capital is present, $\{\pi_j^{H^S}/(\pi_j^{H^S}+\pi_m), \pi_m/(\pi_j^{H^S}+\pi_m)\}$ as the degree of ambiguity about own-company stock returns (ϕ_j) and the market (ϕ_{-j}) varies. In order to judge whether the level of ambiguity is reasonable, the table reports the net adjustments to expected stock returns, $\nu_{-j} - \nu_j$, corresponding to ϕ_j and ϕ_{-j}. The equity market premium, $\mu - r$ is assumed to be 0.07. It is assumed that the volatility of market returns is 0.20 and that idiosyncratic volatility is 0.20, so that the total volatility of individual stock returns is 0.2828.

Row#	Adjustment to expected returns	Relative weights without human capital		Relative weights with human capital	
		$\frac{\pi_j}{\pi_j+\pi_m}$	$\frac{\pi_m}{\pi_j+\pi_m}$	$\frac{\pi_j^{H^S}}{(\pi_j^{H^S}+\pi_m)}$	$\frac{\pi^m}{(\pi_j^{H^S}+\pi_m)}$
		Panel with $H^S = 0.10$			
Row 1	0.000	0.0000	1.0000	-0.1000	1.1000
Row 2	0.012	0.2000	0.8000	0.1200	0.8800
Row 3	0.018	0.3333	0.6667	0.2667	0.7333
Row 4	0.021	0.4286	0.5714	0.3715	0.6285
Row 5	0.023	0.5000	0.5000	0.4500	0.5500
		Panel with $H^S = 0.20$			
Row 1	0.000	0.0000	1.0000	-0.2000	1.2000
Row 2	0.012	0.2000	0.8000	0.0400	0.9600
Row 3	0.018	0.3333	0.6667	0.2000	0.8000
Row 4	0.021	0.4286	0.5714	0.3143	0.6857
Row 5	0.023	0.5000	0.5000	0.4000	0.6000
		Panel with $H^S = 0.30$			
Row 1	0.000	0.0000	1.0000	-0.3000	1.3000
Row 2	0.012	0.2000	0.8000	-0.0400	1.0400
Row 3	0.018	0.3333	0.6667	0.1333	0.8667
Row 4	0.021	0.4286	0.5714	0.2572	0.7428
Row 5	0.023	0.5000	0.5000	0.3500	0.6500
		Panel with $H^S = 0.40$			
Row 1	0.000	0.0000	1.0000	-0.4000	1.4000
Row 2	0.012	0.2000	0.8000	-0.1200	1.1200
Row 3	0.018	0.3333	0.6667	0.0667	0.9333
Row 4	0.021	0.4286	0.5714	0.2000	0.8000
Row 5	0.023	0.5000	0.5000	0.3000	0.7000
		Panel with $H^S = 0.50$			
Row 1	0.000	0.0000	1.0000	-0.5000	1.5000
Row 2	0.012	0.2000	0.8000	-0.2000	1.2000
Row 3	0.018	0.3333	0.6667	0.0000	1.0000
Row 4	0.021	0.4286	0.5714	0.1429	0.8571
Row 5	0.023	0.5000	0.5000	0.2500	0.7500

References

Anderson, E., L. Hansen and T. Sargent, 1999, Robustness detection and the price of risk, Working paper, University of Chicago.

Benartzi, S., 2001, Excessive Extrapolation and the Allocation of 401(k) Accounts to Company Stock, *Journal of Finance* 56 (5), 1747-1764.

Bernstein, P.L., 1992, *Capital Ideas: The Improbable Origins of Wall Street*, Free Press, New York.

Bodie, Z., P. B. Hammond and O. S. Mitchell, 2001, New Approaches to Analyzing and Managing Retirement Risks, *Benefits Quarterly*, 72-83.

Chan, L. K. C., J. Karceski and J. Lakonishok, 1999, On Portfolio Optimization: Forecasting Covariances and Choosing the Risk Model, *Review of Financial Studies* 12.5, 937-974.

Chen, Z. and L. Epstein, 2002, Ambiguity, risk and asset returns in continuous time, *Econometrica* 70, 1403–1443.

Choi, J. J., D. Laibson, B. Madrian and A. Metrick, 2001, Defined Contribution Pensions: Plan Rules, Participant Choices, and the Path of Least Resistance, NBER Working Paper Series 8655.

Cocco, J.F., Gomes, F.J. and Maenhout, P.J., 1999, Consumption and Portfolio Choice over the Life-Cycle, Working Paper, London Business School.

Cohen L., 2003, Loyalty Based Portfolio Choice, Working Paper, Graduate School of Business, University of Chicago.

Coval, J. D. and T. J. Moskowitz, 1999, Home Bias at Home: Local Equity Preference in Domestic Portfolios, *Journal of Finance* 54 (6), 2045-2073.

Cox, J., J. Ingersoll and S. Ross, 1985, An Intertemporal General Equilibrium Model of Asset Prices, *Econometrica* 53, 363-384.

Davis, S. and Willen, P., 2000a, Occupation-Level Income Shocks and Asset Returns, Mimeo.

Davis, S. and Willen, P., 2000b, Using Financial Assets to Hedge Labor Income Risk: Estimating the Benefits, Mimeo.

Ellsberg, D., 1961, Risk, ambiguity and the Savage axioms, *Quarterly Journal of Economics* 75, 643–669.

Epstein, L. and J. Miao, 2002, A two-person dynamic equilibrium under ambiguity, forthcoming, *Journal of Economic Dynamics and Control*.

Epstein, L. and M. Schneider, 2002, Recursive multiple-priors, forthcoming, *Journal of Economic Theory*.

Epstein, L. and T. Wang, 1994, Intertemporal asset pricing under Knightian uncertainty, *Econometrica* 62, 283–322.

Feldstein, Martin and Horst Siebert, 2002, *Social Security Pension Reform in Europe*, The University of Chicago Press.

Fox, C. and A. Tversky, 1995, Ambiguity Aversion and Comparative Ignorance, *Quarterly Journal of Economics* 110, 585-603.

French, K. R. and J. M. Poterba, 1991, Investor Diversification and International Equity Markets, *American Economic Review* 81 (2), 222-226.

Gale, W., L. Papke and J. VanDerhei, 1999, Understanding the Shift Toward Defined Contribution Plans, in *A Framework for Evaluating Pension Reform*, Brookings Institution/TIAA-CREF/Stanford University, forthcoming.

Gilboa, I. and D. Schmeidler, 1989, Maxmin Expected Utility Theory with Non-Unique Prior, *Journal of Mathematical Economics* 18, 141-153.

Grinblatt, M. and M. Keloharju, 2001, How Distance, Language, and Culture Influence Stockholdings and Trades, *Journal of Finance* 56 (3), 1053-1073.

John Hancock Financial Services, 1999, The Sixth Defined Contribution Plan Survey.

Hansen, L. and T. Sargent, 2001a, Acknowledging misspecification in macroeconomic theory, *Review of Economic Dynamics* 4, 519–535.

Hansen, L. and T. Sargent, 2001b, Robust control and model uncertainty, *American Economic Review* 91, 60–66.

Hansen, L. and T. Sargent, T. Tallarini, 1999, Robust permanent income and pricing, *Review of Economic Studies* 66, 873–907.

Heath, C. and A. Tversky, 1991, Preference and Belief: Ambiguity and Competence in Choice under Uncertainty, *Journal of Risk and Uncertainty* 4, 5-28.

Heaton, J. and D. Lucas, 2000, Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk, *Journal of Finance* 55, 1163-98.

Holden, S. and J. VanDerhei, 2001a, 401(k) Plan Asset Allocation, Account Balances, and Loan Activity in 1999, *Perspective* 7(1), January.

Holden, S. and J. VanDerhei, 2001b, Contribution behavior of 401(k) plan participants, *Perspective* 7(4), October.

Huberman, G., 2001, Familiarity Breeds Investment, *Review of Financial Studies* 14 (3), 659-680.

Huberman, G., and P Sengmüller, 2002, Company Stock in 401(k) Plans, Working Paper, Columbia Business School.

Huberman G., S. Iyengar and W. Jiang, 2003, Defined Contribution Pension Plans: Determinants of Participation and Contribution Rates. Working Paper, Columbia Business School.

Kimball, Miles, 1990, Precautionary savings in the small and in the large, *Econometrica* 46, 1429-1446.

Kimball, Miles, 1993, Standard risk aversion, *Econometrica* 61, 589-611.

Loeb, G.M., 1950, *The battle for investment survival*, John Wiley & Sons.

Maenhout, P., 1999, Robust portfolio rules and asset pricing, Working paper, Harvard University.

Markowitz, H., 1952, Portfolio Selection, *Journal of Finance* 7, 77-91.

Massa, M. and A. Simonov, 2003, Hedging, Stock Market Participation and Portfolio Choice, Mimeo.

Merton, R., 1971, Optimum consumption and portfolio rules in a continuous-time model, *Journal of Economic Theory* 3, 373–413.

Merton, R.C., 1980, On estimating the expected return on the market: An exploratory investigation, *Journal of Financial Economics* 8, 323–361.

Meulbroek, L. K., 2002, Company Stock in Pension Plans: How Costly Is It?, Harvard Business School Working Paper 02-058.

Mitchell, O. and M.-E., Lachance, 2002, Understanding Individual Account Guarantees, NBER Working paper 9195.

Mitchell, O. and S. Utkus, 2002, Company Stock and Retirement Plan Diversification, Pension Research Council Working Paper 202-4.

Mitchell, O. and S. Utkus, 2003, Lessons from Behavioral Finance for Retirement Plan Design, Pension Research Council Working Paper 2003-6.

Pratt, John W. and Richard J. Zeckhauser, 1987, Proper risk aversion, *Econometrica* 55, 143-154.

Schroder, M. and C. Skiadas, 2002, Optimal Lifetime Consumption-Portfolio Strategies under Trading Constraints and Generalized Recursive Preferences, Working Paper No. 285, Department of Finance, Kellogg School of Management.

Sharpe, W.F., 1964, Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk, *Journal of Finance* 19, 425-442.

Tobin, J., 1958, Liquidity Preference as Behavior Toward Risk, *Review of Economic Studies*, 67, 65-86

Tversky, A. and C. Heath, 1991, Preferences and Beliefs: Ambiguity and Competence in Choice Under Uncertainty, *Journal of Risk and Uncertainty* 4, 5–28.

Uppal, R. and T. Wang, 2003, Model Misspecification and Underdiversification, *Journal of Finance*, 58, 6, 2465-2486.

VanDerhei, J., 2002, Picking up the Enron Pieces, Testimony Before the Senate Finance Committee, Retirement Security, February 27.

Viceira, Luis, M., 2001, Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income, *Journal of Finance* 56, 433-70.