APPLICATION OF STOCHASTIC TECHNIQUES TO THE PROSPECTIVE ANALYSIS OF THE ACCOUNTING IMPACT OF INTEREST RATE RISK

EXAMPLE ON THE EXPENSES RELATED TO A COMPLEX BOND DEBT

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SUMMARY

This article introduces an operational approach for the analysis of the interest rate risk from a medium term, economic and accounting point of view. This approach is developed in several stages: first of all we present the model and the stochastic variables selected, then we present the calibration of simulation techniques, and finally the obtained results. What makes the originality of the approach is the starting point which consists in deliberately leaving aside the risk-neutral simulation models to focus on the target: the realism of the simulated term structures. Retaining the Nelson-Siegel model's parameters of form as stochastic variables as well as jump processes as short rates' parameter would make complex a risk-neutral probability approach, but actually makes modelling under real probability easier.

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AGENDA

1. INTRODUCTION

The new Solvency 2 framework for quantitative financial risk control is based on two basic principles:

- The valuation of the provisions assuming arbitrage-free condition;
- The control of the probability of ruin over one year.

The practical consequences of this conceptual framework are important, since they result in having to consider dynamic risk factors in two dimensions: price determination and quantile calculation. Various approaches were developed: determination of replicating portfolio (Revelen [2009], Schrager [2008]), use of deflators (Descure and Borean [2006], Jouini and Al [2005] or Sijlamassi and Ouaknine [2004]) and joint use of historical and neutral risk probabilities (Devineau and Loisel [2009]).

The purpose of this article is to present how the application of simple techniques makes it possible to calculate medium term risk indicators based on accounting indicators. The logic of this framework is close to that of the *replicating portfolio*, in that it considers dynamic risk factors in the historical universe and it is based on closed formulas for the options' valuation.

The risk factor considered here is the interest rate risk, and we present a measurement of accounting risk over a multiannual horizon for the purpose of either managing debt or loans.

We start by describing the interest rate risk representation mode retained, and then we show how it impacts accounting figures. We finish with the presentation of an example and then we open up the debate on ways of improvement or possible extensions.

It should be noted that this approach was actually implemented in the real world within departments such as treasury or asset management. This work is therefore an empirical one based on existing techniques used in a pragmatic way in order to build effective risk indicators.

2. FINANCIAL MODELLING

The literature on the interest rate risk is abundant and many models were proposed. One can refer to Roncalli [1998] for a detailed analysis of the modelling of the term structure or Planchet and Al [2009] for a more synthetic presentation.

In the context of this paper we start with an initial term structure and expose it to random shocks, then measuring the impact on the value of the interest instruments composing the portfolio.

The shock modelling is carried out in two stages:

- Initially a parametric model is adjusted with the term structure and the adjusted term structure is reshaped;

- The relative differences between the adjusted term structures before and after reshaping are applied to the raw curve to reshape it in turn.

This approach provides a guarantee that the initial term structure is effectively directly used as a model parameter.

2.1. PARAMETRIC MODELLING OF THE SHOCKS APPLIED TO THE TERM STRUCTURE

The choice of the parametric model should allow rebuilding in a realistic way (as in historically observed statistical properties) the shocks on the full range of rates. The chosen reference model is the three factors of form and one scale factor model proposed by Nelson and Siegel (Nelson and Siegel [1987]). In this model it is assumed that the instantaneous spot rate is written (using Roncalli notations [1998]) as follows:

$$f_t(\tau) = \mu_1 + \mu_2 \, \mathbf{exp}\left(-\frac{\tau}{\tau_1}\right) + \mu_3 \frac{\tau}{\tau_1} \mathbf{exp}\left(-\frac{\tau}{\tau_1}\right)$$

which leads, as the zero-coupon rate $R_t(\tau)$ is calculated from $R_t(\tau) = \frac{1}{\tau}\int_0^\tau f_t(u)\,du$, to the following:

$$R_t(\tau) = \mu_1 + \mu_2 \frac{1 - \mathbf{exp}\left(-\frac{\tau}{\tau_1}\right)}{\frac{\tau}{\tau_1}} + \mu_3 \left(\frac{1 - \mathbf{exp}\left(-\frac{\tau}{\tau_1}\right)}{\frac{\tau}{\tau_1}} - \mathbf{exp}\left(-\frac{\tau}{\tau_1}\right) \right).$$

The use of the instantaneous spot rates has the advantage of leading to a simple necessary and sufficient condition: the arbitrage-free condition, which is the positivity of all spot rates (*cf.* Hull [1999]).

From now on, in order to optimise wording, we will note $\varphi(x) = \frac{1 - e^{-x}}{x}$ and $\psi(x) = \varphi(x) - e^{-x}$ so that:

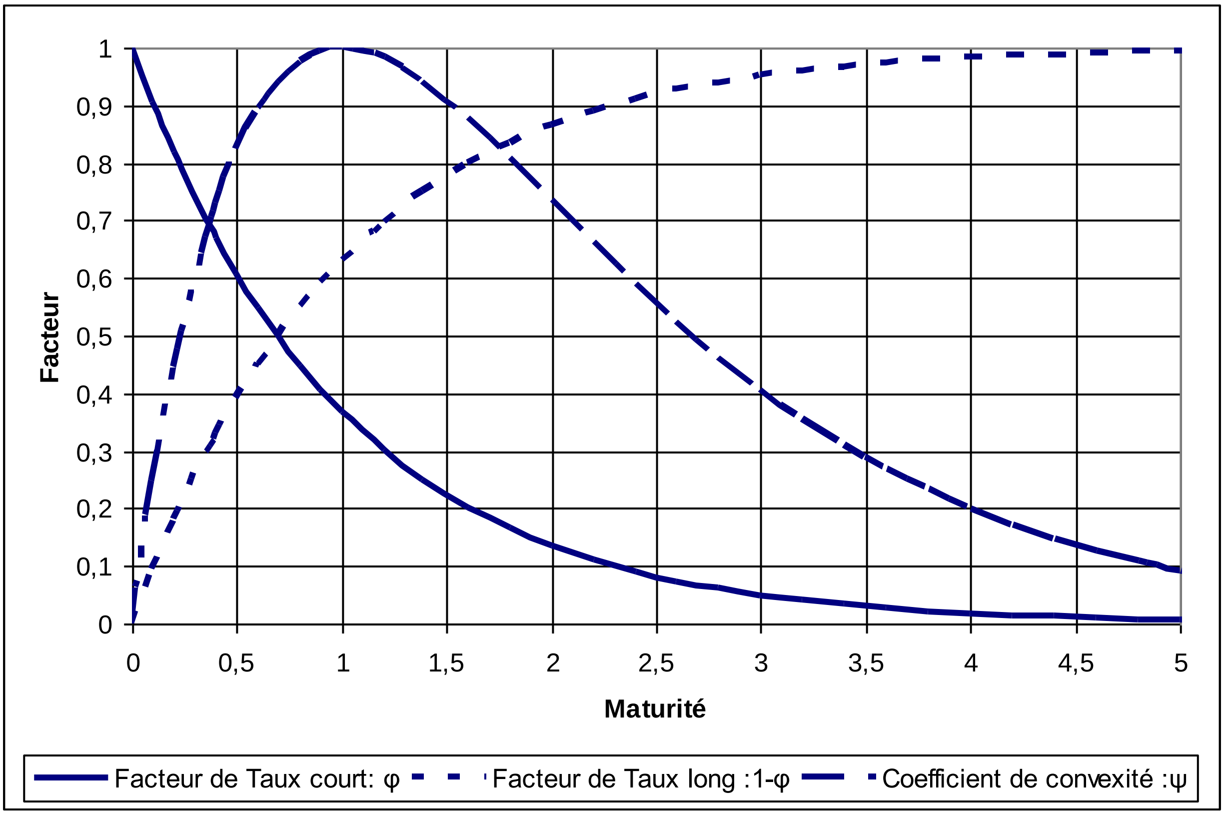
$$R_t(\tau) = \mu_1 + \mu_2 \varphi\left(\frac{\tau}{\tau_1}\right) + \mu_3 \psi\left(\frac{\tau}{\tau_1}\right)$$

We will now suppose that the form parameters depend on time and that the scale parameter τ_1 is constant (*cf. infra*) and we will note $\mu_1 = l(t)$, $\mu_2 = -s(t)$ and $\mu_3 = c(t)$ to refer to the interpretation of these values in the model, that is to say respectively the long rate, the spread and the convexity. To shock the term structure we use the following representation:

$$R_t(\tau) = r_0(t)\varphi\left(\frac{\tau}{\tau_1}\right) + l(t)\left(1 - \varphi\left(\frac{\tau}{\tau_1}\right)\right) + c(t)\psi\left(\frac{\tau}{\tau_1}\right)$$

where $r_0(t) = l(t) - s(t)$ is the instantaneous short rate. This equation has the advantage of revealing factors of determination of the zero-coupon rate that are easily readable: the short rate, the long rate and the convexity.

The functions φ, $1 - \varphi$ and ψ are represented below, their impact on the various segments of the term structure is immediate:



The modelling of the reshapings of the term structure is then carried out by proposing dynamics for the processes $l(t), s(t)$ and $c(t)$.

Before defining these dynamics we justify the choice of the constancy of the scale parameter τ_1. Modelling robustness came first, before a probably illusory precision, and this parameter was set to 2 which allows the function which represents the shocks of curve to be maximum. This choice is justified by the output of the principal component analysis (PCA) presented below and moreover is in line with the experience of experts for whom this point separates the short term market segments (deposit and futures) from the long term ones for swaps of maturity higher than two years.

The choice of the number of factors determining the shape of the term structure and their interpretation has been the subject of much work, a summary of which can be found in Roncalli [1998]. We conclude from these studies that the three factors $l(t), s(t)$ and $c(t)$ used here classically explain more than 95 % of the variance. We have nevertheless verified this result over a 10 year period covering years 2007 and 2008 marked by a crisis of the interbank market. The selected points correspond to 1 month, 3 months, 6 months and 12 months maturities for deposit rates and 2,3,4,5,7,12,15,20 and 30 years for swap rates, that is to say a total of 14 points. We find the following correlation matrix:

Table 1- Maturities correlation matrix

	1 mois	3 mois	6 mois	12 mois	2 ans	3 ans	4 ans	5 ans	7 ans	10 ans	12 ans	15 ans	20 ans	30 ans
1 mois	100%	64%	59%	56%	18%	15%	12%	10%	9%	7%	5%	3%	1%	-1%
3 mois	64%	100%	90%	74%	39%	34%	29%	26%	24%	19%	16%	12%	9%	6%
6 mois	59%	90%	100%	90%	57%	51%	45%	41%	38%	31%	27%	22%	18%	14%
12 mois	56%	74%	90%	100%	72%	67%	62%	58%	54%	47%	42%	37%	32%	28%
2 ans	18%	39%	57%	72%	100%	98%	94%	91%	85%	77%	72%	68%	62%	58%
3 ans	15%	34%	51%	67%	98%	100%	97%	96%	92%	84%	80%	76%	70%	66%
4 ans	12%	29%	45%	62%	94%	97%	100%	99%	95%	88%	84%	81%	75%	70%
5 ans	10%	26%	41%	58%	91%	96%	99%	100%	97%	91%	87%	84%	79%	74%
7 ans	9%	24%	38%	54%	85%	92%	95%	97%	100%	98%	96%	92%	88%	84%
10 ans	7%	19%	31%	47%	77%	84%	88%	91%	98%	100%	99%	97%	94%	90%
12 ans	5%	16%	27%	42%	72%	80%	84%	87%	96%	99%	100%	98%	97%	93%
15 ans	3%	12%	22%	37%	68%	76%	81%	84%	92%	97%	98%	100%	98%	96%
20 ans	1%	9%	18%	32%	62%	70%	75%	79%	88%	94%	97%	98%	100%	98%
30 ans	-1%	6%	14%	28%	58%	66%	70%	74%	84%	90%	93%	96%	98%	100%

On the basis of this correlation matrix a PCA provides the following results:

Table 2- Eigenvectors

Vecteurs propres	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0,063	0,429	0,479	0,754	0,061	0,063	0,044	0,070	0,002	0,001	0,009	0,006	0,000	0,005
2	0,118	0,489	0,194	-0,423	0,585	0,103	-0,107	-0,403	-0,028	0,013	-0,047	0,011	0,003	-0,009
3	0,164	0,479	0,026	-0,368	-0,155	-0,027	0,183	0,735	0,054	-0,024	0,075	-0,017	-0,011	0,017
4	0,212	0,403	-0,088	-0,059	-0,691	-0,247	-0,246	-0,425	-0,038	-0,004	-0,020	-0,016	0,012	-0,017
5	0,290	0,106	-0,393	0,117	-0,062	0,522	0,365	-0,080	-0,081	0,225	-0,432	0,240	0,061	-0,128
6	0,306	0,047	-0,329	0,127	0,064	0,252	0,184	-0,137	0,032	-0,219	0,693	-0,325	-0,152	0,041
7	0,310	0,001	-0,284	0,154	0,186	-0,075	-0,407	0,164	-0,036	0,078	-0,218	-0,167	0,109	0,688
8	0,312	-0,028	-0,229	0,147	0,217	-0,188	-0,464	0,205	0,000	0,045	-0,056	-0,029	-0,173	-0,673
9	0,319	-0,075	-0,048	0,051	0,122	-0,317	0,132	-0,037	0,204	-0,304	0,178	0,633	0,431	0,031
10	0,313	-0,125	0,117	-0,015	0,048	-0,348	0,348	-0,099	0,256	-0,007	-0,227	0,018	-0,692	0,146
11	0,307	-0,154	0,186	-0,044	0,020	-0,228	0,306	-0,067	0,114	0,216	-0,128	-0,577	0,510	-0,165
12	0,298	-0,182	0,237	-0,072	-0,028	-0,042	0,074	0,040	-0,870	-0,213	-0,009	0,039	-0,054	0,012
13	0,286	-0,208	0,313	-0,110	-0,105	0,186	-0,182	0,020	0,060	0,687	0,368	0,257	-0,056	0,075
14	0,274	-0,223	0,353	-0,131	-0,183	0,487	-0,273	0,032	0,325	-0,487	-0,197	-0,077	0,009	-0,025

with the following eigenvalues:

Table 3- Eigenvalues

Valeurs propres	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Valeurs	9,42838	2,9045	0,85815	0,40589	0,17532	0,09274	0,04516	0,03623	0,01931	0,01185	0,00901	0,00599	0,0048	0,00267
% expliqué	67%	88%	94%	97%	98%	99%	99%	100%	100%	100%	100%	100%	100%	100%

The first three eigenvectors explain 94 % of the total variance; we can also see that the first factor corresponds to a homogeneous deformation of the rates' level, the second to modifications of slope and the third to changes of convexity. These classical results are still observable to this date and validate *a posteriori* the satisfactory descriptive capacity of the Nelson and Siegel parametric model selected here.

Risk factors dynamics should now be specified.

2.2. DESCRIPTION OF RISK FACTORS DYNAMICS

Let us consider the classical model suggested in Hull and White [1994], which uses the mean reverting property of the classical model of Vasicek [1977]:

$$dr_0(t) = \mu_r(l_t - r_t)dt + \sigma_r dW_r(t)$$
$$dl(t) = \mu_l(l_\infty - l_t)dt + \sigma_l dW_l(t)$$

With this type of specification, short rates necessarily converge towards the asymptotic long rate, i.e. the asymptotic curve is necessarily, in mean, flat. If this behaviour can be justified in risk neutral probability, it is not the case any more in historical probability where the opposite should be true, the asymptotic curve *a priori* having, in mean, a normal form, i.e. increasing and with a slope given by $l_\infty - r_\infty$. We therefore move on to:

$$dr_0(t) = \mu_r(r_\infty - r_t)dt + \sigma_r dW_r(t) + S(t)dN(t)$$
$$dl(t) = \mu_l(l_\infty - l_t)dt + \sigma_l dW_l(t)$$

in order to reproduce this effect and to take into account jumps on the level of the short rate. We also add the following curve factor dynamics:

$$dc(t) = \mu_c(c_\infty - c_t)dt + \sigma_c dW_c(t).$$

This point can be seen like a switch from risk neutral probability to physical probability by change of trend, the long term return level being part of the trend.

The jump process is described by a Poisson process N with constant intensity λ. The sizes of the jumps are supposed to be described by a series of independent and identically distributed random variables following the law of Pareto of parameter (s, α) so that:

$$P(S > x) = \left(\frac{x}{s}\right)^{-\alpha}.$$

In terms of structure of dependence, the historically noted correlations are compatible with an assumption of independence. The correlation of Brownians is however easy to add in the process if a correlation is observed. Besides, the integration of dependences between the size of the jumps and/or their time of occurrence is possible *via* copulas (for the aspects related to the dependency structure of assets, one can for example consult Kharoubi-Rakotomalala C. [2009]) but was not implemented.

2.3. CALIBRATION OF THE PARAMETERS

The calibration of the parameters of the model is carried out in two steps: initially the parameters of the Nelson-Siegel model are adjusted starting from historical series of price, then, in a second step, evolution of the adjusted parameters is used to estimate the diffusion parameters of each factor.

A time *t* being fixed, τ_1 being supposed fixed and known, the values of $r_0(t)$, $l(t)$ and $c(t)$ are estimated by minimising the sum of the squares of the differences between the zero-coupon rates observed and those resulting from the model. The following loss function is thus used:

$$p(\tau_1) = \sum_{i \in I} \left(R_i - R_t(\tau_i) \right)^2$$

with $R_t(\tau) = r_0(t)\varphi\left(\frac{\tau}{\tau_1}\right) + l(t)\left(1 - \varphi\left(\frac{\tau}{\tau_1}\right)\right) + c(t)\psi\left(\frac{\tau}{\tau_1}\right)$ the rate resulting from the model, R_i the rate read on the market, for all $i \in I$, I being the set of available rates as at time *t*. As $R_t(\tau)$ is a linear function of the parameter to be estimated $\theta_t = \left(r_0(t), l(t), c(t)\right)$, the derivative $\frac{\partial}{\partial \theta} p(\tau_1)$ is calculated simply and we obtain an explicit solution function of τ_1. In the applications, we take as indicated *supra* $\tau_1 = 2$.

By carrying out this estimate on different dates, one rebuilds a time series $\left\{\theta_t, t \in T\right\}$. In practice a weekly step of estimate is used. By carrying out a Euler discretisation of dynamic factors, the obtained series' parameters can be estimated. The processes being independent, the estimate of the parameters is carried out separately for each dynamics. Concerning *L* and *C*, we are in the classical situation of a linear regression model whose estimators are well known (*cf* Planchet and al [2009]). The presence of the jump component in the short rate process imposes a slightly different approach, by isolating the jumps which are presumed associated with large deviations from standard values:

- The Pareto law threshold is estimated by comparing observed values with the theoretical median of the sample's maximum under the assumption of pure Brownian motion; it is determined from the law of the maximum;

- Classical estimates of the parameters are then carried out for the "no jump" part, while the maximum likelihood estimator of α is used for the jumps.

In the suggested numerical application, the rate factor was modelled using jump processes of two different types: processes with normal jumps or Pareto jumps. The calibration of the parameters was carried out on the 3rd and 4th moments in the case of jumps presumed normal, and by explicit identification of the jumps in the case of jumps following a law of Pareto, in accordance with the process described above.

The numerical application presented here is based on Euros and US Dollars swap term structures (source: Bloomberg). On the basis of this data as at year end 2007, the results for jump statistics are as follows.

Table 4- Jumps fitting

Sauts Normaux	Vol Brownien	Fréquence des sauts	Ecart type des sauts	Méthode	Commentaire
Calibrage simple	0.309%	4.613	0.117%	Vol brownien approchée par excès, moments 2 et 4 fittés	cf Etape I, manière simple pour prendre les sauts en compte; mais avec biais
Calibrage complet	0.242%	11.580	0.093%	Moments 2, 3 et 4 fittés	Intérêts de sauts mensuels sur un horizon de cinq ans ?

Pareto (Fat tails)	Vol Brownien	Fréquence des sauts	Alpha des sauts	Méthode	Commentaire
Sauts symétriques	0.291%	6.380	2.85	Identifiaction des sauts après estimation vol du brownien	Pas de moments d'ordre 3
Sauts asymétriques	0.291%	A droite: 3.50 A gauche: 2.88	A droite: 3.41 A gauche: 2.37	Identifiaction des sauts après estimation vol du brownien	Pas de moment d'ordre 3 à gauche Pas de moment d'ordre 4 à droite

The choice was made to carry out the fitting based on a history period dating from the introduction of the Euro (assuming the ECB's objectives will be stable over five years). The alternatives which could potentially be considered consist in:

- ✓ go back further in time in order to take into account the shocks of the 90's;
- ✓ go back further in time but eliminating the shocks of the early 90's, which relate to a specific moment in History (fall of the Berlin Wall).

One can note here that *a priori* the question of a possible liquidity premium not captured by the model arises. Taking into account the context of this work which is concerned with simulation of *euribor* indexed cash flows and with asset valuation, the liquidity premium issue does not present a major problem. In a different context, for example if it were about liability discounting, the liquidity should then be taken into account in the modelling.

3. BALANCE SHEET MODELLING

The modelling of the term structure shocks' impacts on the value of the assets under study must be supplemented by a description of their accounting, in order to assess their effects from the company's standpoint.

Within the framework of this study, one considers a balance sheet in which are the following instruments of credit:

- ✓ fixed or variable rates bonds, redeemable or *in fine*;
- ✓ inflation indexed bonds;

- ✓ loans and credit lines;
- ✓ traditional financing instruments: leasing, bank loan, etc.;
- ✓ caps and floors, plain vanilla or with barrier.

The valuation is carried out using closed formulas: discounting and spot rates calculation for the linear instruments, and Black formula for the options (*cf* Hull [1999] for the related formulas and the proofs):

Table 5- Options[2] related closed formulas

	Closed form solution
Cap	$L\delta_k e^{-rT}\left[F_k N\left(d_1^{'}\right)-R_K N\left(d_2^{'}\right)\right]$
Floor	$L\delta_k e^{-rT}\left[F_k N\left(-d_1^{'}\right)-R_K N\left(-d_2^{'}\right)\right]$
European Put	$e^{-rT}KN(-d_2)-S_0N(-d_1)$
European call	$S_0N(d_1)-e^{-rT}KN(d_2)$
Call down-and-in	$S_0\left(H/S_O\right)^{2\lambda}N(y)-e^{-rT}K\left(H/S_O\right)^{2\lambda-2}N\left(y-\sigma\sqrt{T}\right)$
Call up-and-out	$S_0N(x_1)-e^{-rT}KN\left(x_1-\sigma\sqrt{T}\right)-S_0\left(H/S_0\right)^{2\lambda}\left[N(-y)-N(-y_1)\right]$ $+Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}\left[N\left(-y+\sigma\sqrt{T}\right)-N\left(-y_1+\sigma\sqrt{T}\right)\right]$
Up-and-out Put	$-S_0\left(H/S_O\right)^{2\lambda}N(-y)+e^{-rT}K\left(H/S_O\right)^{2\lambda-2}N\left(-y+\sigma\sqrt{T}\right)$
Down-and-out Put	$-S_0N(-x_1)+e^{-rT}KN\left(-x_1+\sigma\sqrt{T}\right)+S_0\left(H/S_0\right)^{2\lambda}\left[N(y)-N(y_1)\right]$ $-Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}\left[N\left(y-\sigma\sqrt{T}\right)-N\left(y_1-\sigma\sqrt{T}\right)\right]$

With N the cumulative distribution function of a standard normal distribution and:

- ✓ cap and floor of principal L and ceiling R_k where F_k is the forward rate between t_k and t_{k+1} with:

[2] In case of zero dividend underlying stock.

$$d_1' = \frac{\ln\left(F_k / R_k \right) + \sigma_k^2 \, t_k / 2}{\sigma_k \sqrt{t_k}} \,,\ \delta_k = t_{k+1} - t_k \text{ and } d_2' = d_1' - \sigma_k \sqrt{t_k} \,.$$

- ✓ european call and put of maturity *T* with:

$$d_1 = \frac{\ln\left(S_0 / K \right) + \sigma^2 \, T / 2}{\sigma \sqrt{T}} \text{ and } d_2 = d_1 - \sigma \sqrt{T} \,.$$

- ✓ option with barrier *H* with:

$$y = \frac{\ln\left(H^2 / S_0 K \right) + \lambda \sigma^2 T}{\sigma \sqrt{T}} \,,\ \lambda = \frac{r + \sigma^2 / 2}{\sigma^2} \,,\ x_1 = \frac{\ln\left(S_0 / K \right) + \lambda \sigma^2 T}{\sigma \sqrt{T}} \text{ and } y_1 = \frac{\ln\left(H / S_0 \right) + \lambda \sigma^2 T}{\sigma \sqrt{T}} \,..$$

Implicit volatility by *tenor* is observed in the beginning, then incremented with the variation of historical volatilities specific to each scenario and each *tenor*. The difference between implicit volatility and historical volatility is therefore maintained constant during time. The accounting impact of the shocks can then be considered.

3.1. IAS 39

3.1.1. General principles

The purpose of IAS39 is the recognition and measurement of financial instruments. The concept of financial instrument was defined in France by law 96-597 of July 2nd, 1996 on the modernisation of the financial activities, article 1 – integrated in the monetary and financial law, article L. 211-1. This definition integrates in particular equities, titles giving access to capital, the shares or actions of OPC, term financial instruments and titles giving access to capital. In 1989 the IASB[3] extended this definition to derived financial instruments: "a financial instrument is a contract that gives rise to a financial asset of one entity and a financial liability or equity instrument of another entity". IAS 39 was the subject of a first publication in 1998. It has been re-examined many times, in particular in December 2003 and March 2004. IAS39 requires financial instruments to be classified in one of the following categories:

- ✓ *FVTPL*: financial instruments acquired or held for the purpose of selling in the short term. These financial instruments are measured at fair value with changes recognised through profit or loss;

[3] *International Accounting Stantards Board.*

- ✓ *HTM*: financial assets with fixed or determinable payments that an entity intends and is able to hold to maturity. They are measured at amortised cost according to the method of the effective interest rate, the flows and depreciations[4] of which are booked through profit or loss. By definition, the accounting value of this asset class is interest rate risk neutral;
- ✓ *L&R*: non-derivative financial instruments with fixed or determinable payments. They are measured at amortised cost according to the method of the effective interest rate, the flows and depreciations[5] of which are booked through profit or loss;
- ✓ *AFS*: financial assets which are not classified as any of the above three categories. They are measured at fair value with changes recognised through equity. It should be noted that in the event of objective indications of the depreciation of an asset, the cumulative loss that was recognised in equity is recognised in profit or loss – that is derecognition.

3.1.2. Accounting[6] for derivative, hybrid and hedging instruments

Derivative instruments:

Within the meaning of IAS39 a derivative instrument presents a low cost at emission and is settled on a future date at a value fluctuating with an underlying instrument. Examples of derivative instruments are: fixed term contracts, swaps, options, caps, floors and collars. Their accounting follows that of the FVPTL except in the case of a hedging instrument or an unquoted equity derivative for which fair value cannot be determined in a reliable way. It should be noted that in this last case and if the settlement of this derivative cannot be made other than by the physical delivery of equities, then the derivative can be booked at the cost or amortised cost.

Hybrid instruments:

In the case of hybrid instruments, the embedded derivative must be recognised at fair value. The IASB defines an embedded derivative as a feature within a contract, such that the cash flows associated with that feature behave in a similar fashion to a stand-alone derivative. If the embedded derivative must be measured at fair value, the accounting treatment of the hybrid instrument follows the rule of *split accounting*. This rule aims at assessing whether the embedded derivative should be separated from its host contract and accounted for as a derivative, which would be when:

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[4] Against credit risk.
[5] Against credit risk.
[6] In case of an acquisition and not of a sale.

- ✓ the economic risks and characteristics of the embedded derivative are not closely related to those of the host contract;
- ✓ the entire instrument is not measured at fair value with changes in fair value recognised in the income statement;
- ✓ a separate instrument with the same terms as the embedded derivative would meet the definition of a derivative.

Thus the following hybrid instruments will have to be split between host contract and embedded derivative:

- ✓ put or call embedded in an equity instrument;
- ✓ equity indexed interest or principal payments in host debt or insurance contracts;
- ✓ convertible bonds;
- ✓ credit derivative embedded in a debt instrument and which allows an asset's credit risk transfer.

Contrary, the following hybrid instruments should not be split:

- ✓ credit derivative related to an interest rate modifying the amounts of interests of the host contract (OATi type);
- ✓ caps or floors embedded in an instrument of debt if they do not present leverage and if they are emitted at market rate (to be specified);
- ✓ element of debt in a foreign currency;
- ✓ early redemption option.

Hedging instruments:

IAS 39 defines a hedging instrument as an instrument whose fair value or cash flows are expected to offset changes in the fair value or cash flows of a designated hedged item. The general principle of hedge accounting is to retain[7] the element of cover as the principal element, this last one being accounted for at fair value. In the case of a fair value hedge, the accounting treatment of the hedged element must follow that of the hedging instrument.

The accounting of profits and losses depends on the nature of the hedge, namely fair value hedge, cash flow hedge and hedge of a net investment[8] in a foreign operation:

- ✓ in the case of a fair value hedge, the gain or loss from the change in fair value of the hedging instrument and hedged item are recognised immediately in profit or loss;

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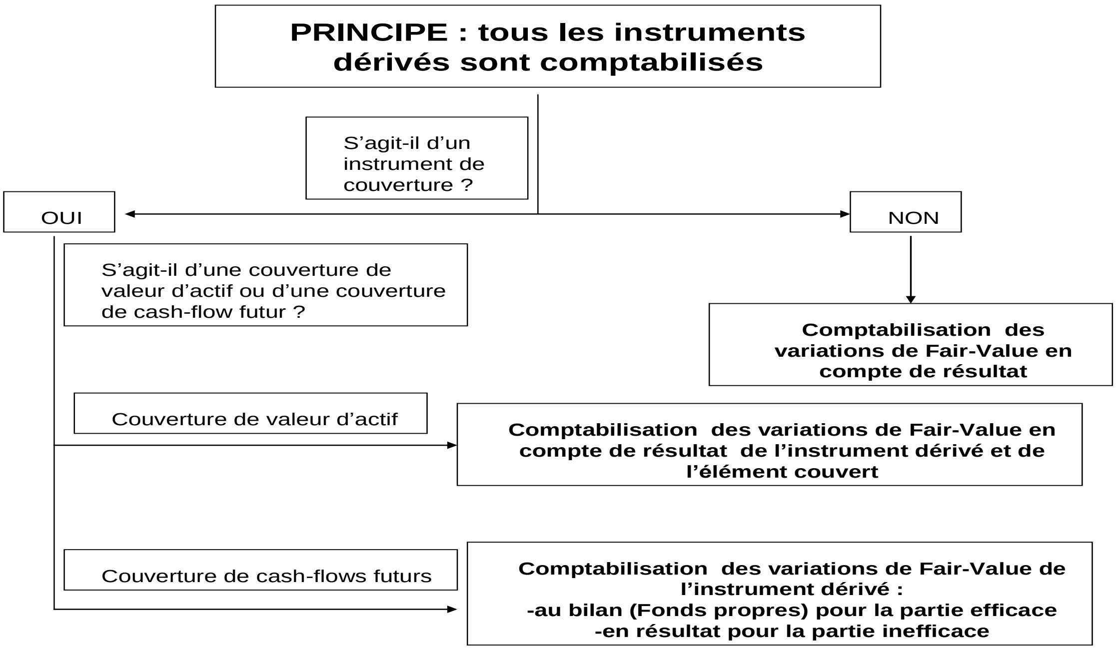
[7] In the case where the company is able to prove the effectiveness of the cover.

[8] We do not cover the accounting treatment of this type of hedge in this article.

- ✓ in the case of a cash flow hedge:
  - while waiting for the settlement of the hedged transaction, the portion of the gain or loss on the hedging instrument that is determined to be an effective hedge is recognised as prescribed by the accounting treatment of that instrument;
  - at the settlement of the hedged transaction, any gain or loss on the hedging instrument that was previously recognised directly in equity is 'recycled' into profit or loss in the same period(s) in which the financial asset or liability affects profit or loss[9].

In a synthetic way, one can summarise the principle behind accounting for derivative instruments *via* the following diagram:

Diagram 1- Principle behind accounting for derivative instruments



3.1.3. <u>Application to the balance sheet</u>

On the basis of the specific portfolio structure considered here, and by considering a traditional investment (e.g. no foreign currency accounting) as well as the principles set out above, the balance sheet accounting can be summarised as shown in Table 6 below. Note that the availability of line by line information could lead to a materially different assessment.

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[9] In case the settlement results in an asset or a liability, the gain or loss on the hedging instrument that was previously recognised in other comprehensive income is removed from equity and is included in the initial cost or other carrying amount of the acquired non-financial asset or liability.

Table 6- Balance sheet accounting

	No hedge	Fair value hedge	Cash flow hedge
Fixed rate bonds (redeemable or *in fine)*	Fair Value via equity or amortised cost	Fair Value via P&L	Fair Value via equity or amortised cost
Variable rate bonds	Fair Value via equity or amortised cost	Fair Value via P&L	Fair Value via equity or amortised cost
Caps and floors only	Fair Value via P&L	N/A	N/A
Caps and floors embedded in a hybrid asset	Accounting of host contract	N/A	N/A
Leasing	Fair Value via P&L	Fair Value via P&L	Fair Value via P&L
Bank loans	Fair Value via equity or amortised cost	Fair Value via P&L	Fair Value via equity or amortised cost
Plain vanillas (stand alone or embedded)	Fair Value via P&L	N/A	N/A
Option with barrier (stand alone or embedded)	Fair Value via P&L	N/A	N/A

3.2. THE FORTHCOMING IFRS 9, AFTER IAS 39

The IASB published on November 19th, 2009, the first version of new IFRS 9 which is to replace current IAS 39 in the long term.

The main new feature introduced by IFRS 9 is the riddance of the four asset categories set out in IAS 39. Henceforth, a financial asset or liability will have to be accounted for either at fair value via P&L, or at amortised cost. The classification rule is defined by the company's business models as well as by the contractual characteristics of its cash flows. Thus an asset can be accounted for at amortised cost if the cash flows are composed of interests and principal and if the company's business model plans on paying and receiving the contractually expected cash flows (interests and principal) generated during the emission or the detention of the asset. Cash flows must be determinable and without leverage effect. Contrary to the current IAS 39 the appreciation is not made instrument by instrument but globally.

Derivative instruments are still to be accounted for at fair value via P&L.

As per IFRS 9 the accounting treatment of hybrid instruments should follow that of its host contract. Thus the principle of separation as set out in IAS 39 does not exist anymore. Henceforth, hybrid instruments will be accounted for according to the nature of their host contract but only plain vanilla debt instruments including a derivative such as cap, floor, senior tranche of a structured debt (split in fair value via P&L and amortised cost under IAS 39) will be classified in the amortised cost category, excluding:

- ✓ non *senior* tranches of structured debt;
- ✓ convertible bonds;
- ✓ assets with indexed performance (except authorised OATis);
- ✓ swaps and forwards;
- ✓ bonds held in OPCVMs.

It should be noted that an option known as "OCI option" allows the accounting of asset at fair value via equity. However, the market does not seem to be willing to use it.

For the time being, the publication issued by the IASB does not deal with hedging instruments, but an exposure draft should be published at the beginning of 2010 on this subject.

4. *VaR* CALCULATION

Considering the complexity of accounting mechanisms, the use of an approach by simulations appears to be unavoidable for the behaviour of risk factors in the real-world, the prices of the assets being calculated by closed formulas, be it justified approximations of the value of the option.

Initially trajectories of $r_0(t)$, $l(t)$ et $c(t)$ are generated over the desired projection horizon, then estimators of the *VaR* are deduced from the obtained realisations of the distribution of the debt's future cash flows.

4.1. SIMULATION METHOD FOR THE TERM STRUCTURE

The Monte Carlo simulation implemented here is based on the following principles, with an underlying processes' monthly discretisation step h ($h=\frac{1}{12}$):

- ✓ simulation of the Brownians: 3 standard normal distributions are simulated and stored once and for all;
- ✓ simulation of the months with or without jumps:
  - drawing of a value U in a uniform law;
  - if $U > \lambda h$, $h=\frac{1}{12}$ there are no jumps;

- if $\frac{\lambda h}{2} \leq U < \lambda h$ there is 1 jump on the right;
- if $U \leq \frac{\lambda h}{2}$ there is 1 jump on the left;

✓ size of the jumps: the size of the jump is obtained by inversion of the cumulative distribution function of the Pareto law: $x = s \times (1-V)^{-1/\alpha}$ with V *a* realisation of a uniform law.

Realisations of uniform variables are generated using Sobol sequences (*cf* Thiémard [2000] or Planchet and Al [2005]) which allows the optimisation of the statistical indicators' convergence, whatever the sample size. This quasi-random method has the following advantages:

✓ quality control of the generated risk;

✓ cancellation of the sampling risk in the comparison of the results of two simulations on data of different market or management;

✓ computing performance.

Considering the return to average process, the parameters are initially simulated in absolute terms then the shocks are published in tables, in order to allow their application to term structures other than the initial one.

This method avoids generating shocks for each *VaR* calculation: only the starting portfolios and market data are modified.

4.2. ESTIMATING THE QUANTILE AND STATISTICS

Estimating a high order quantile based on a sample is not simple and works on this subject are numerous (one will refer for example to Christoffersen et Al [2001] and Jorion [2001]). We use the three following methods here:

✓ direct estimate of the quantile based on the simulated sample. This non-parametric method consists in ordering the sample in order to determine its quantiles. Thus in the case of a sample containing 1 000 values, the quantile at 95 % corresponds to the 950th largest value;

✓ Cornish-Fisher approximation (Cornish and Fisher [1937]) based on the moments of order 3 and 4. It is a semi-parametric estimator based on a normal distribution's quantile corrected by a development known as Cornish-Fisher to take into account the skewness and the kurtosis (moments of order 3 and order 4 of the distribution of returns). Its expression is:

$$VaR(1-\alpha) = \mu + \tilde{Z}_\alpha \sigma ,$$

$$\tilde{Z}_\alpha = Z_\alpha + \frac{1}{6}\left(Z_\alpha^2 - 1\right)S + \frac{1}{24}\left(Z_\alpha^3 - 3Z_\alpha\right)\left(K-3\right) - \frac{1}{36}\left(2Z_\alpha^3 - 5Z_\alpha\right)S^2$$

Where Z_α is the $1-\alpha$ quantile of a standard normal distribution, *S* is the skewness of the portfolio and *K* its kurtosis;

- ✓ Cornish-Fisher approximation based on the moments of order 3 and 4.

The elements presented above are now used for the determination of the *VaR* at 95 % of the expenses associated with a corporate debt.

5. APPLICATION TO THE EXPENSES RELATED TO A COMPLEX BOND DEBT

Management data used here are made up of a debt of an industrial company serving the public sector. This debt, presumed complex, exposes the company to having to pay cash flows which amounts are random, partly due to the presence of fluctuating rate instruments and derivative instruments. We consider the impact of interest rate fluctuations on two different amounts:

- the expenses as they appear in the income statement, possibly expressed as a ratio over outstanding debt (for a portfolio of assets, it would symmetrically be the accounting income);

- the equity movement (in million Euros), which corresponds to the variation of the cash flow hedging instruments.

Note that the second amount is materially less sensitive than the first one, the potential impact of the interest rate risk on equity being negligible in practice, contrary to the impact on the income statement.

The expenses are determined based on the following cash flows:

- coupons for fixed rate bonds (or fluctuating rates *swapped* for fixed rates if the hedge is accepted);

- a *euribor* interest rate + margin for fluctuating rate bonds;

- fair value variation for trading derivatives.

For the bonds indexed on inflation, by simplification, we fix the real rate at its starting level (coupon = inflation + fixed margin = nominal rate - real rate + fixed margin 1 = nominal rate + fixed margin 2). The behaviour is afterwards similar to that of classical variable rate bonds. This approximation is justified by the fact that the risk indicator is the accounting rate of the debt measured in nominal terms and also by the fact that the projection horizon is relatively short (5 years) with respect to inflation risk.

The *VaR* at 95 % of the expenses resulting from the two cash flows described above is computed by simulating 10 000 curves.

Table 7- Risk indicator - constant outstanding debt

Horizon		1	2	3	4	5
Outstanding debt (base 100 at year 0)		100	100	100	100	100
Statistical indicators of the expense rate	Median	5,097%	4,997%	5,090%	5,113%	5,047%
	Average	5,106%	5,001%	5,089%	5,111%	5,042%
	Standard deviation	0,116%	0,161%	0,199%	0,237%	0,283%
	Skewness	0,494	0,126	-0,097	-0,063	-0,056
	Kurtosis excess	0,572	0,373	0,023	-0,317	-0,418
95th centile		5,314%	5,267%	5,412%	5,500%	5,498%
Gaussian parametric estimate		5,297%	5,266%	5,416%	5,500%	5,507%
Corresponding centile		93,63%	94,87%	95,20%	95,05%	95,36%
Semi-parametric estimate (Cornish Fisher)		5,311%	5,271%	5,411%	5,498%	5,505%
Corresponding centile		94,86%	95,16%	94,87%	94,88%	95,29%
T VaR 95 %		5,379%	5,352%	5,489%	5,574%	5,596%

Table 8- Risk indicator - organic growth of the debt (turnover growth excluding the financing of acquisitions)

Horizon		1	2	3	4	5
Outstanding debt (base 100 at year 0)		105	114	118	117	118
Statistical indicators of the expense rate	Median	5,081%	4,969%	5,036%	5,057%	4,996%
	Average	5,091%	4,972%	5,036%	5,053%	4,992%
	Standard deviation	0,120%	0,195%	0,248%	0,286%	0,325%
	Skewness	0,485	0,112	-0,051	-0,054	-0,053
	Kurtosis excess	0,602	0,293	-0,018	-0,242	-0,384
95th centile		5,304%	5,296%	5,444%	5,518%	5,515%
Gaussian parametric estimate		5,288%	5,293%	5,444%	5,523%	5,527%
Corresponding centile		93,70%	94,81%	95,03%	95,20%	95,40%
Semi-parametric estimate (Cornish-Fisher)		5,303%	5,298%	5,441%	5,520%	5,524%
Corresponding centile		94,86%	95,11%	94,81%	95,09%	95,32%
T VaR 95 %		5,374%	5,392%	5,539%	5,615%	5,627%

The tables above allow the following observations.

- The uncertainty related to the accounting expenses, excluding the impact of the issuer's own credit risk, is about 12bp in the first year and stabilises around 30bp over five years.

- In a slightly counter-intuitive way, the risk is stable (even decreases) between the fourth and the fifth year. That is due to the presence in the portfolio of derivative instruments classified as *trading* over that period.

- The tails of distribution do not present strong leptokurtic characteristics, despite the presence of Pareto jumps on the short rates. The dispersion of future issuing dates and the smoothing induced by accounting rules can explain this apparent paradox.

- The excellent quality of the semi-parametric Cornish-Fisher approximations, despite the complexity of calculations. This means that a number of simulations which allows estimating the moments up to the order four is in fact also sufficient for estimating the quantiles at 95 %.

6. CONCLUSION

The model presented here allows *VaR* calculation for a portfolio of interest rate instruments. It is based on modelling as realistically as possible the deformations of the term structure and drawing consequences on the value of derivative instruments by an adjustment of the parameters which integrates the risk premium in order to avoid developing a "risk neutral" model on top of the historical model. This approach presents the advantage of allowing quantile calculation as well as pricing while avoiding heavy approaches such as "fan in fan simulations" or the replicating portfolio.

This approach was implemented in the relatively simple context of a corporate debt. It can also be used in an insurance context, for example on the hedging of annuities by a bond portfolio as suggested in Pierre [2010]. It indeed allows a measurement of the deformation of the term structure and of its impact with more finesse than by using an approach by duration.

Its wider use, for example within the framework of provisions calculation for saving contracts in Euro, is on the other hand not straight forward because of the difficulty to determine the cost of options and guarantees through closed formulas.

Moreover, a credit risk dimension should be included in the model in order to give a fair representation of the interest rate assets.

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