Centre for Analytical Finance
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Pitfalls in Estimating Jump-Diffusion Models

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Working Paper Series No. 18 **November 1998**

Pitfalls in Estimating Jump-Diffusion Models[1]

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First version August 1994

January 24, 1998

[1]This is a revised version of the paper, 'Bernoulli Diffusion Model, Estimation & Option Pricing'.

[2]I acknowledge helpful comments from Anders Holst Andersen, Tom Engsted, Bjarne Astrup Jensen, Jesper Lund, Jørgen Aase Nielsen, Carsten Tanggaard, and participants at the EDEN Doctoral Tutorial in Finance 1994 and Bonn-Aarhus Seminar 1995, and I thank Hans Ole Mikkelsen for providing the S&P500 data used in the paper. Of course, I remain responsible for any errors.

Pitfalls in Estimating Jump-Diffusion Models

Abstract

In this paper we show that it is invalid to use standard maximum likelihood procedures in estimating jump-diffusion models. The reason is that in jump-diffusion models the log-return is equivalent to a discrete mixture of N normally distributed variables, where N goes to infinity. Thus, from the mixture-of-distributions literature we know that the likelihood function can be unbounded which causes inconsistency. In the paper we derive a method which provides consistent and asymptotically normally distributed estimator. The method is applied to some of the most actively traded New York Stock Exchange (NYSE) stocks and several stock indices. The implication of the estimated jump-diffusion models for option prices is examined.

Keywords: Jump-Diffusion Model, Profile Log-Likelihood Function, Option Pricing.
JEL Codes: C13, C22, G12, G13.

1 Introduction

Jump-diffusion models arise frequently in finance. One well-known example is Merton's (1976) option pricing model. In the empirical jump-diffusion literature, such models are usually estimated with standard Maximum Likelihood (ML). In the present paper we show that this approach is invalid, and we derive a more suitable procedure which gives consistent estimates of the model parameters. The standard ML procedure is invalid because in jump-diffusion models the log-return is equivalent to a discrete mixture of N normally distributed variables, where N goes to infinity. Thus, from the mixture-of-distributions literature [Kiefer (1978) and Hamilton (1994)] we know that the likelihood function for some parametric specifications is unbounded which causes inconsistency of standard ML.

The finance literature has considered different models for asset-price dynamics in order to account for various empirical regularities, while at the same time attaining a simple procedure for pricing contingent claims. The work can be categorised into continuous-time models and discrete-time models. Examples of the former include Black and Scholes (1973), Merton (1976), Hull and White (1987) and Bates (1996a, 1996b), and of the latter the ARCH models of Engle (1982), Bollerslev (1986) and Duan (1995). Black and Scholes assume that log-returns are normally distributed with constant volatility, resulting in a closed-form pricing formula for the plain-vanilla options. However, this model does not capture the often documented excess kurtosis that characterises log-returns. This excess kurtosis is accounted for by a jump-diffusion model like Merton's, where the Black-Scholes model is extended with a jump component. In Hull and White (1987) the Black-Scholes volatility is stochastic. Thus, their model exposes volatility clustering. Bates (1996a, 1996b) combines the Merton and Hull and White models. Unfortunately, the implication of building a more realistic model is increased complexity of option pricing and estimation. The Black-Scholes model is straightforward to estimate, as the log-returns are assumed to be normally distributed. Estimation of jump-diffusion model, [e.g., the Merton model], is not as easy as it appears in the literature [see, for example, Beckers (1982) and Ball and Torous (1983,1985)], since the likelihood function is unbounded. We propose a solution to the problem, where the profile of the likelihood function with respect to the relative variances between the diffusion and jump part is used to obtain a consistent estimator. The stochastic volatility models cannot be estimated directly as the volatility is unobserved.

The paper is organized as follows. The general jump-diffusion model is presented in section 2. In section 3 we formulate a discrete-time version of the Merton model. The estimation problem in the jump-diffusion models and the empirical results of the discrete-time model are described in section 4. The empirical results are based on some of the most traded NYSE stocks and several indices. Section 5 is concerned with estimation of different parametric specifications of the jump-diffusion models. The outcome of a jump-diffusion model for options is examined in section 6. Finally, section 7 concludes.

2 The Jump-Diffusion Model

The stock price, S_t, is described by a continuous diffusion part and a discontinuous jump part, where the continuous part is responsible for the usual fluctuation in S_t and the jump part accounts for the extreme events. This can be formulated by the following stochastic differential equation (SDE):

$$\frac{dS_t}{S_{t_-}} = \alpha dt + \sigma dW_t + dI_t \tag{1}$$

where α is the drift term, σ is the volatility of the diffusion part, W_t is a Wiener process and I_t is the jump component. t_- denotes the nearest point of time preceding t. The dynamics of I_t is described by J Poisson processes, $N_{j,t}$, and J stochastic or deterministic jump amplitudes, $Y_{j,t}$. $N_{j,t}$ has a constant intensity, λ_j, for $j = 1, \dots, J$. Further, we assume that $Y_{j,t} > -1$ for all j, which ensures non-negative stock prices. Thus, I_t is described by the SDE:

$$dI_t = \sum_{j=1}^{J} Y_{j,t} dN_{j,t}.$$

Hence, there is an instantaneous jump in the relative stock price of size $Y_{j,t}$ conditional on an increment in $N_{j,t}$. Furthermore, all processes are assumed to be independent. The solution to (1) is:[1]

$$S_t = S_0 e^{(\alpha - \frac{1}{2}\sigma^2)t + \sigma W_t} \prod_{0<s\leq t} \prod_{j=1}^{J} \left(1 + Y_{j,s} dN_{j,s}\right).$$

In order to estimate the jump-diffusion model, it is necessary to make restrictions on the jump amplitudes. The approach that we follow is to make a distributional assumption for the Ys, such that likelihood estimation is attainable. In the next section we look at the Merton model, where the jump amplitude is log-normally distributed. An alternative estimation approach is the Generalized Method of Moments [Hansen (1982)].

3 The Bernoulli Diffusion Model

In this section we present a discretized version of the Merton model. The Merton model has $J = 1$, $dN_t \sim Po(\lambda dt)$ and the jump amplitude is log-normally distributed,

[1] This solution is obtained by use of Itô's formula for semi-martingales [Rogers (1987)] combined with the fact that $P(\sum_{j=1}^{J} dN_{j,s} > 1) = \mathcal{O}(dt^2)$, and hence may be ignored.

$\log(1+Y_t) \sim N(\mu, \delta^2)$. Thus, $\log(S_t)$ has the form

$$\log(S_t) \;=\; \log(S_0) + (\alpha - \frac{1}{2}\sigma^2)t + \sigma W_t + \sum_{0<s\leq t} \log(1 + Y_s dN_s). \qquad (2)$$

We use MLE to estimate the parameters $\Psi = (\alpha, \sigma, \lambda, \mu, \delta)$. S_t is observed at the discrete-time points $t_i = i\Delta$ for $i = 0, \ldots, T$ where Δ is the sampling frequency. To simplify the notation, let S_i denote an observation of S at time t_i. The density function for the log-return, $x_{i+1} = \log\left(\frac{S_{i+1}}{S_i}\right)$, is

$$p(x; \Psi) \;=\; \sum_{j=0}^{\infty} \frac{e^{-\lambda\Delta}(\lambda\Delta)^j}{j!} \phi(x; (\alpha - \frac{1}{2}\sigma^2)\Delta + j\mu, \sigma^2\Delta + j\delta^2) \qquad (3)$$

where $\phi(x; m, v)$ is a density function for a normally distributed stochastic variable with mean m and variance v. This is obtained by noting that the log-return is normally distributed conditional on the number of increments of the Poisson process. Thus, the density function is evaluated by an infinite sum as in the density function for a Poisson process.

It is natural to use the approach of Ball and Torous (1983,1985), where the solution (2) is discretized. Thereby, the density function consists of a finite number of terms, instead of (3) where the sum has to be truncated after the first N terms for a sufficiently large N. The discretization of the solution (2) takes the form

$$\log(S_i) \;=\; \log(S_{i-1}) + (\alpha - \frac{1}{2}\sigma^2)\Delta + \sigma\Delta W_i + \log(1 + Y_i \Delta q_i)$$

where $\Delta W_i \sim N(0, \Delta)$, $\log(1 + Y_i) \sim N(\mu, \delta^2)$, $\Delta q_i \sim b(1, \lambda\Delta)$ and $\lambda\Delta < 1$. The density function for the log-return can be found in equation (4) in the next section. This discrete-time model is referred to as the Bernoulli diffusion model (BDM). The approximation is based on the assumption that $\lambda\Delta$ is close to 0. This is explained by the fact that the approximation is only appropriate if $P(N_{(i+1)\Delta} - N_{i\Delta} > 1) \simeq 0$; otherwise λ fails to approximate the intensity in the Poisson process.[2] The BDM can also be seen as the Merton model in the limit, since a Poisson process with intensity λt can be constructed as the sum of n identically independent Bernoulli distributed variables with intensity $\lambda\frac{t}{n}$ where $n \to \infty$. For further details see Ball and Torous (1983).

4 The Estimation Problem

The density function for the Δ period log-return, x, in the BDM has the form:

$$p(x; \Psi) \;=\; (1 - \lambda\Delta)\phi(x; (\alpha - \frac{1}{2}\sigma^2)\Delta, \sigma^2\Delta)$$

[2] $P(N_{(i+1)\Delta} - N_{i\Delta} > 1) \simeq 0$ is equivalent to $P(N_{(i+1)\Delta} - N_{i\Delta} \leq 1) \simeq 1$ which can be restated as $1 + \lambda\Delta \simeq e^{\lambda\Delta}$. This last expression is only true for $\lambda\Delta \simeq 0$.

$$+ \quad \lambda\Delta\phi(x;(\alpha-\frac{1}{2}\sigma^2)\Delta+\mu,\sigma^2\Delta+\delta^2). \tag{4}$$

The log-likelihood function can now be written as:

$$l(x_1,\ldots,x_T;\Psi) \;=\; \sum_{i=1}^{T}\log p(x_i;\Psi). \tag{5}$$

Thus, normally we find the maximum likelihood estimates (MLE) by maximising (5) with respect to $\Psi \in \Theta$, where $\Theta = \mathbb{R}\times\mathbb{R}_+\times(0,\frac{1}{\Delta})\times\mathbb{R}\times\mathbb{R}_+$. It is, however, invalid to use standard ML estimation in the Bernoulli model. This is clarified by the argument in Kiefer (1978). To simplify the point the parameters in the density function (4) have been changed.[3]

$$p(x_i) = w\phi(x_i;m_1,s_1^2)+(1-w)\phi(x_i;m_2,s_2^2). \tag{6}$$

> [Kiefer (1978) p.428] If $\hat{m}_1$ is choosen so that x_i is exactly equal to m_1 for any i then as $\hat{s}_1$ goes to zero $p(x_i)$ increases without bound. Since the second term in p shields p away from zero at the other observations (the first term in p is zero whenever $x_i = m_2$), l is unbounded.

An interpretation of this could be that we think of the log-return in the BDM as a mixture of two normal distributions with different means and variances. Furthermore, as the weight, w, of the distributions is unknown, it is impossible to identify from which of the two normal distributions each observation originates. Hence, combined with the fact that the variances of the two normal distributions are different, the MLE does not exist. This is in contrast to the situations where the variances are known or equal, or it is known from which normal distribution each observation descends. This has apparently not been fully recognized in the empirical jump-diffusion literature. In Ball and Torous (1983,1985), Beckers (1981), Frost (1993), Jorion (1989) and Trautmann and Beinert (1995), the empirical results are based on standard ML. Thus, it is not surprising that they, in some situations, get negative variance estimates or other estimates which are outside the feasible parameter region. If MLE is based on maximising (5) without any further restrictions on Θ, the result can be that for a fixed $\hat{s}_1^2 = \hat{\sigma}^2 \gg 0$, $\hat{s}_2^2 = \hat{\sigma}^2\Delta + \hat{\delta}^2$ goes to zero. This causes $\hat{\delta}^2$ to be negative. Finally, from the above argument it is verified that the MLE of λ can be any possible value without effecting the likelihood function (5). Thus, some of the λ-estimates in the literature may be unreliable. However, it is still possible to obtain consistent and asymptotically normally distributed estimates by using the following procedure [see Hamilton (1994) chapter 22 for alternative estimation approaches]. The idea is to restrict the volatility parameters σ and δ to be in a compact

[3] The original parameters can be found by solving the equations: $m_1 = (\alpha-\frac{1}{2}\sigma^2)\Delta$, $s_1^2 = \sigma^2\Delta$, $m_2 = (\alpha-\frac{1}{2}\sigma^2)\Delta+\mu$, $s_2^2 = \sigma^2\Delta+\delta^2$ and $w = (1-\lambda\Delta)$.

set, $[v_l; v_u]^2$, which must include the true values. In the original situation we just had $(\sigma, \delta) \in \mathbb{R}_+^2$. Hence, Θ is reduced to $\bar{\Theta} = \mathbb{R} \times [v_l; v_u] \times (0, \frac{1}{\Delta}) \times \mathbb{R} \times [v_l; v_u]$. Kiefer (1978) and Hoffmann-Jørgensen (1992) confirm that the estimates obtained from maximising (5) with respect to $\Psi \in \bar{\Theta}$ are consistent and asymptotically normally distributed. For practical implementation we make a reparametrisation which makes it possible to obtain the estimates in $\bar{\Theta}$. Therefore, for a fixed positive $m \in M$ let $\delta^2 = m\sigma^2$ be the relative size of the volatilities is fixed, where M is a compact set on $\mathbb{R}_+$. Define a new log-likelihood function

$$l_m(x_1, \ldots, x_T; \Psi^*) \;=\; l(x_1, \ldots, x_T; (\alpha, \sigma, \lambda, \mu, \sqrt{m}\sigma)) \tag{7}$$

where the right hand side is from (5). Ψ is reduced to $\Psi^* = (\alpha, \sigma, \lambda, \mu) \in \Theta^* = \mathbb{R} \times [v_l; v_u] \times (0, \frac{1}{\Delta}) \times \mathbb{R}$ compared to what we have in the original log-likelihood, since the relative size of the volatilities is known. For a fixed m the *true* MLE of Ψ^* is found by maximising (7) with respect to $\Psi^* \in \Theta^*$, since $l_m(\cdot; \Psi^*)$ is bounded in contrast to $l(\cdot; \Psi)$. This can be verified by keeping the Kiefer (1978) discussion in mind. If σ goes to zero, δ goes to zero, or equivalently, if s_1 goes to zero, s_2 goes to zero. Thus, both terms in (6) tend to zero unless one of the means, m_1 or m_2, equals x_i, and in this latter situation $p(x_i)$ goes to infinity. Nevertheless, as the $p(x_i)$'s tend to zero or infinity the likelihood function reaches zero, because the unbounded $p(x_i)$'s are dominated by the other $p(x_i)$s, which tend to zero.[4] Remember that the likelihood function is the product of all the $p(x_i)$s. Let $\hat{\Psi}_m^*$ denote the MLE obtained from (7). Then, the consistent estimator of Ψ is obtained by choosing the m which maximises $l_m(\cdot; \hat{\Psi}_m^*)$.[5] In practice, the optimum is found by drawing the profile log-likelihood, $l_m(\cdot; \hat{\Psi}_m^*)$, for $m \in M$. The last step is to find the standard errors based on the Hessian matrix of $l(\cdot; \hat{\Psi})$. An example of the profile log-likelihood is drawn in Figure 1 to illustrate how to select $\hat{m}$.

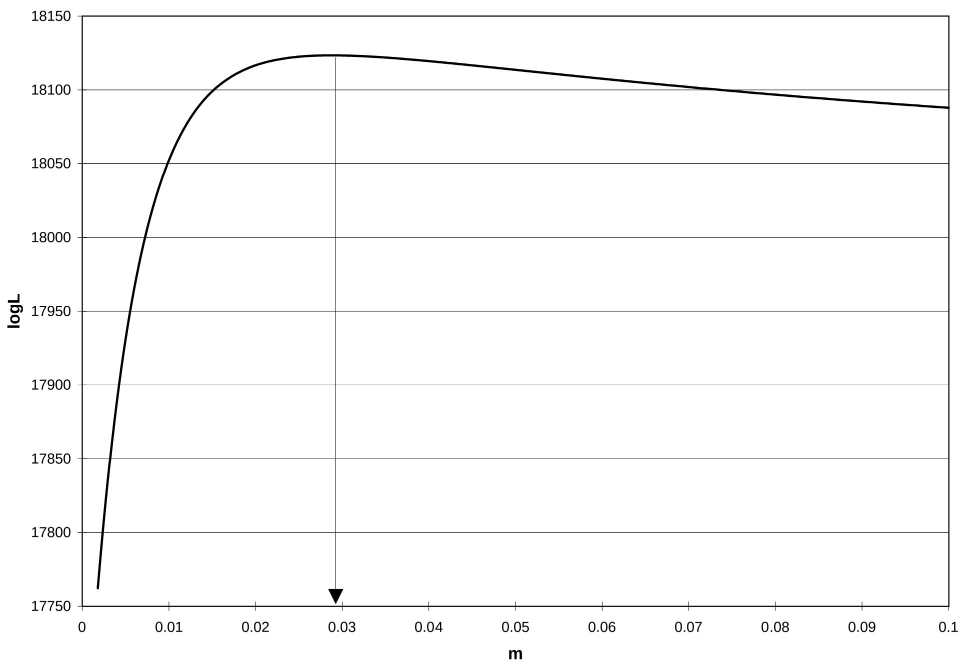
4.1 Estimation of the Bernoulli Diffusion Model

In this section we estimate the BDM for a wide range of stocks and indices. The method used to get the MLE is the one presented in the previous section. It is examined if the BDM is a good empirical approximation for the Merton model i.e., the empirical findings must support $\lambda\Delta$ being small.

We look at 18 very liquid NYSE stocks, each with daily observations in the period January 2, 1973 - July 8, 1997. The indices are DAX 100, FTSE 100, S&P 100, S&P 500 and KFX, each with daily observations in the periods January 1, 1973 - July 8, 1997, January 2, 1973 - July 8, 1997, Marts 5, 1984 - July 8, 1997, January 3, 1928 -

[4]We define that Y_1 dominates Y_2 if $Y_1(n) \to 0$, $Y_2(n) \to \infty$ and $Y_1(n)Y_2(n) \to 0$ for $n \to \infty$.

[5]We use $\hat{m}$ to find $\hat{\delta}^2 = \hat{m}\hat{\sigma}^2$.



The profile log-likelihood drawn in the Bernoulli diffusion model based upon the IBM data. $\hat{m} = 0.0286$. Note the log-likelihood function has only been calculated for a compact set of m.

Figure 1: The profile log-likelihood.

October 19, 1988 and December 4, 1989 - July 8, 1997, respectively. The data source is DATASTREAM except for S&P 500. A symbol list of the stocks and indices can be found in Appendix A. For estimation purposes let $\Delta = \frac{1}{261}$ since the data sets consist of daily log-returns. Hence, all the estimated parameters are of annualised sizes.

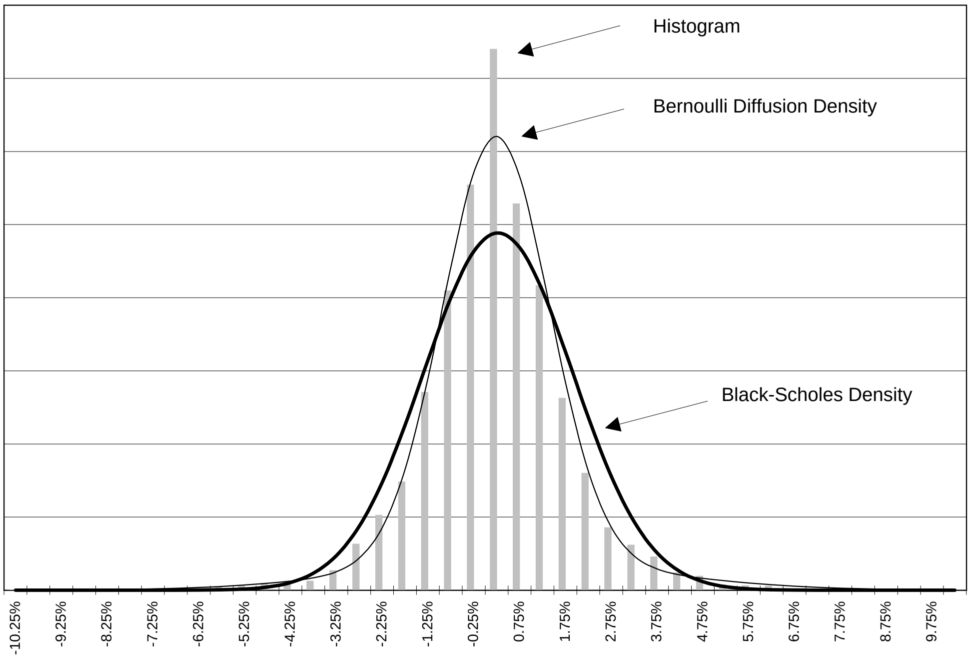
Plots of daily observations and log-returns are drawn in Appendix B. The descriptive statistics are reported in Appendix C. From these it is seen that the empirical distribution of the log-returns exhibits excess kurtosis. Figure 2 shows that the empirical distribution (normalized histogram) of the MOB log-return is leptokurtic, since it is badly fitted by the normal density, which underestimates the density of the numerically small and very large log-returns opposite the log-returns in the middle which are overestimated. Furthermore, Figure 2 verifies that the Bernoulli diffusion density gives a much closer fit to the empirical distribution.

The empirical results are summarized in the following. The expected jump amplitude, $E[Y] = e^{\mu+\frac{1}{2}\delta^2}-1$, is estimated to be between -0.62% and 0.62% with an average of 0.15%. A separate look at the stocks and indices indicates a different picture. For the stocks and indices the expected jump amplitudes are in the range of -0.07% to 0.62% and -0.62% to -0.15% with an average of 0.28% and -0.31%, respectively. For the indices there is a natural interpretation of this result. The BDM captures the often seen empirical result for equities that the volatility and log-return are negatively correlated. A jump in the Bernoulli

	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{\mu}$	$\hat{\delta}$
AXP	0.0138	0.2367	51.3085	0.0015	0.0332
	(0.0584)	(0.0033)	(0.0980)	(0.0013)	(0.0009)
CHV	0.0244	0.1740	71.8063	0.0010	0.0231
	(0.0375)	(0.0028)	(0.0453)	(0.0007)	(0.0005)
DD	-0.0337	0.1826	57.2233	0.0022	0.0216
	(0.0481)	(0.0026)	(0.0859)	(0.0009)	(0.0006)
DOW	0.0320	0.1944	54.2434	0.0010	0.0265
	(0.0323)	(0.0028)	(0.0387)	(0.0009)	(0.0007)
EK	-0.0501	0.2172	16.8542	0.0053	0.0421
	(0.0363)	(0.0024)	(0.0453)	(0.0024)	(0.0018)
GE	0.0423	0.1772	42.6189	0.0020	0.0237
	(0.0399)	(0.0023)	(0.0433)	(0.0010)	(0.0007)
GM	-0.1371	0.1902	59.9269	0.0028	0.0237
	(0.0725)	(0.0028)	(0.1084)	(0.0010)	(0.0006)
IBM	-0.0247	0.1942	21.5887	0.0036	0.0329
	(0.0358)	(0.0022)	(0.0680)	(0.0018)	(0.0013)
IP	-0.0045	0.2216	20.3257	0.0048	0.0347
	(0.0926)	(0.0025)	(0.0744)	(0.0023)	(0.0014)
KO	0.0947	0.1973	24.3613	0.0022	0.0337
	(0.0341)	(0.0023)	(0.0449)	(0.0017)	(0.0012)
MMM	-0.0272	0.1616	45.2876	0.0023	0.0239
	(0.0315)	(0.0022)	(0.0358)	(0.0009)	(0.0007)
MOB	-0.0219	0.1875	40.2220	0.0031	0.0291
	(0.0286)	(0.0024)	(0.0366)	(0.0012)	(0.0008)
MO	0.1362	0.1907	43.8275	0.0006	0.0277
	(0.0298)	(0.0025)	(0.0344)	(0.0011)	(0.0008)
MRK	-0.0454	0.1629	86.6403	0.0021	0.0191
	(0.0362)	(0.0028)	(0.0391)	(0.0006)	(0.0004)
PG	0.0661	0.1824	13.9918	0.0034	0.0346
	(0.0361)	(0.0020)	(0.0462)	(0.0023)	(0.0016)
S	-0.1171	0.2045	46.8044	0.0038	0.0276
	(0.0342)	(0.0028)	(0.0762)	(0.0011)	(0.0008)
T	-0.0181	0.1473	47.3143	0.0018	0.0223
	(0.0316)	(0.0021)	(0.0406)	(0.0008)	(0.0006)
XON	0.1343	0.1782	15.0312	-0.0012	0.0312
	(0.0388)	(0.0020)	(0.0517)	(0.0020)	(0.0014)
DAX	0.1583	0.1179	20.9262	-0.0038	0.0223
	(0.0256)	(0.0013)	(0.0583)	(0.0012)	(0.0009)
FTSE100	0.1565	0.1242	4.8844	-0.0068	0.0341
	(0.0355)	(0.0016)	(0.1580)	(0.0046)	(0.0035)
SP100	0.1762	0.1175	15.6630	-0.0024	0.0286
	(0.0356)	(0.0019)	(0.0871)	(0.0022)	(0.0017)
SP500	0.1213	0.1074	40.7286	-0.0018	0.0240
	(0.0163)	(0.0009)	(0.0506)	(0.0005)	(0.0004)
KFX	0.1840	0.0913	52.1461	-0.0022	0.0137
	(0.0453)	(0.0023)	(0.1830)	(0.0009)	(0.0006)

The estimation results are based on daily log-return and the estimates are in annualised sizes. The numbers in parentheses are standard errors.

Table 1: The Bernoulli diffusion model.



The histogram of daily log-return of MOB together with the estimated Black-Scholes and Bernoulli diffusion density based on the MLEs from Tables 1 and 3.

Figure 2: Estimated density functions.

process induces extra variance to the log-return from $\sigma^2\Delta$ to $\sigma^2\Delta+\delta^2$, and on average, it is expected that the index decreases when a jump occurs. The variance of the jump part, $\hat{\lambda}\hat{\delta}^2$, is estimated to be between 26.91% and 67.00% of the total variance, $\hat{\lambda}\hat{\delta}^2+\hat{\sigma}^2$, and with an average of 45.54%. There is no significant difference between this result for the stocks and indices. $\hat{\lambda}$ is between 4.88 and 86.64 and with an average of 38.86. For the stocks and indices the averages are 42.19 and 26.87, respectively. As mentioned earlier, it is necessary that $\lambda\Delta$ is small for the BDM to be a valid approximation of the Merton model. This is only attained for the FTSE 100 index where $\hat{\lambda} = 4.88$. Thus, we have to return to the Merton model, as the goal of the paper is to estimate a continuous-time model for the stock dynamics and not a discrete-time version like the BDM, since the estimates from the discrete-time BDM can not be converted into the corresponding parameters from a continuous-time jump-diffusion model. Consequently, the next section looks at estimating the Merton model and other parametric and distributional specifications of the jump-diffusion model.

5 Estimation of Jump-Diffusion Models

In the following 3 subsections we consider estimation of different parametric specifications of the jump-diffusion model. First, the Merton model is estimated after it is observed that the likelihood function is unbounded like the BDM likelihood function. Thus, the problem is solved in the same fashion as for the BDM. Second, a simplified version of the Merton model, where the jump amplitude is non-stochastic, is examined. Finally, we look at a jump-diffusion model which nests the previous two models.

5.1 The Merton Model

For estimation of the Merton model we have to approximate the density function (3) by the first N terms of the sum. The same problem arises, as in the BDM, namely that the likelihood function is unbounded. The reason for this is that the approximation of (3) by the first N terms corresponds to a discrete mixture of N normally distributed variables.[6] The j'th stochastic variable has mean $m_j = (\alpha - \frac{1}{2}\sigma^2)\Delta + j\mu$, variance $s_j^2 = \sigma^2\Delta + j\delta^2$ and with a weight of $w_j = e^{-\lambda\Delta}\frac{(\lambda\Delta)^j}{j!}$ in the mixture. The same procedure as in the BDM is used to obtain consistent and asymptotically normally distributed estimates. The profile log-likelihood only has to be calculated in one dimension, because all variances s_j's are described by the two parameters σ and δ. Finally, we have to use a sufficiently large N, such that the error imposed by the approximation is negligible. Note that the selected N depends on Δ and λ. Furthermore, recall that the BDM corresponds to $N = 1$, if $\lambda\Delta$ is close to zero. Numerical studies have shown that from $N = 20$ there is no significant difference in the estimates when using daily observations, (Appendix D). For the practical implementation the truncation of (3) has been done with $N = 100$.

The empirical results in the Merton model are as follows. The expected jump amplitude is estimated to be between -0.60% and 0.41% with an average of 0.06%. For the stocks and indices the estimated expected jump amplitudes are in the range of -0.01% to 0.41% and -0.60% to -0.02% with an average of 0.14% and -0.23%, respectively. The variance of the jump part is estimated to be between 26.71% and 77.93% of the total estimated variance with an average of 60.04%. For the stocks and indices the averages are 62.12% and 52.53%, respectively. $\hat{\lambda}$ is between 5.18 and 309.21 with an average of 128.19. For the stocks and indices, the averages $\hat{\lambda}$ are equal to 149.06 and 53.07, respectively. From the BDM to the Merton model the average $\hat{\lambda}$ has increased by 230% and does not decline for any of the series. This verifies that the BDM is a poor approximation to the Merton model. As we go from the BDM to the Merton model the estimates are only *unchanged* for FTSE 100.

[6]Note that the problem is present even if the density function can be calculated without any kind of approximation.

	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{\mu}$	$\hat{\delta}$
AXP	-0.0318	0.1940	143.2738	0.0008	0.0222
	(0.0338)	(0.0039)	(0.0380)	(0.0005)	(0.0005)
CHV	-0.0228	0.1272	228.2221	0.0005	0.0149
	(0.0352)	(0.0035)	(0.0336)	(0.0003)	(0.0003)
DD	-0.1249	0.1523	164.9792	0.0013	0.0147
	(0.0516)	(0.0033)	(0.0620)	(0.0004)	(0.0003)
DOW	0.0082	0.1627	141.8529	0.0005	0.0182
	(0.0458)	(0.0032)	(0.0472)	(0.0004)	(0.0004)
EK	-0.0848	0.2049	34.1198	0.0036	0.0299
	(0.0345)	(0.0025)	(0.0524)	(0.0013)	(0.0010)
GE	-0.0011	0.1623	86.1428	0.0015	0.0180
	(0.0351)	(0.0025)	(0.0410)	(0.0006)	(0.0005)
GM	-0.2402	0.1445	208.6694	0.0013	0.0150
	(0.0343)	(0.0037)	(0.0411)	(0.0003)	(0.0003)
IBM	-0.0502	0.1819	43.9948	0.0023	0.0241
	(0.0840)	(0.0023)	(0.1703)	(0.0012)	(0.0008)
IP	-0.1185	0.1855	92.7363	0.0022	0.0196
	(0.0381)	(0.0030)	(0.0676)	(0.0006)	(0.0005)
KO	0.0491	0.1815	53.1598	0.0018	0.0239
	(0.0306)	(0.0024)	(0.0393)	(0.0009)	(0.0007)
MMM	-0.1203	0.1118	200.1991	0.0010	0.0134
	(0.0328)	(0.0028)	(0.0366)	(0.0003)	(0.0002)
MO	0.1392	0.1666	105.5755	0.0002	0.0193
	(0.0391)	(0.0028)	(0.0439)	(0.0005)	(0.0004)
MOB	-0.0456	0.1613	101.1400	0.0014	0.0199
	(0.0331)	(0.0027)	(0.0416)	(0.0005)	(0.0005)
MRK	-0.1239	0.1123	309.2098	0.0008	0.0120
	(0.0377)	(0.0041)	(0.0503)	(0.0002)	(0.0002)
PG	-0.0556	0.1150	198.2615	0.0008	0.0128
	(0.0298)	(0.0029)	(0.0312)	(0.0002)	(0.0002)
S	-0.2942	0.1389	228.0275	0.0015	0.0155
	(0.0507)	(0.0038)	(0.0628)	(0.0003)	(0.0003)
T	-0.1179	0.0825	269.3887	0.0007	0.0114
	(0.0232)	(0.0027)	(0.0303)	(0.0002)	(0.0002)
XON	0.1083	0.1516	74.0497	0.0000	0.0167
	(0.0479)	(0.0022)	(0.0833)	(0.0006)	(0.0005)
DAX	0.1640	0.1153	28.1393	-0.0031	0.0192
	(0.0264)	(0.0013)	(0.0535)	(0.0009)	(0.0007)
FTSE100	0.1572	0.1240	5.1761	-0.0066	0.0329
	(0.0315)	(0.0016)	(0.0868)	(0.0043)	(0.0035)
SP100	0.1634	0.0905	87.6427	-0.0003	0.0128
	(0.0359)	(0.0020)	(0.0926)	(0.0005)	(0.0004)
SP500	0.1294	0.1004	62.1524	-0.0013	0.0191
	(0.0142)	(0.0009)	(0.0257)	(0.0004)	(0.0003)
KFX	0.1882	0.0858	82.2169	-0.0014	0.0114
	(0.0495)	(0.0024)	(0.1414)	(0.0006)	(0.0005)

The estimation results are based on daily log-return and the estimates are in annualised sizes. The numbers in parentheses are standard errors.

Table 2: The Merton model.

A desired feature of the Merton model is that λ should be low to describe extreme events, but this is not supported by the empirical findings. Instead, the jump component appears to approximate a second diffusion process. Thus, we want to investigate the improvement of including a jump component in the Black-Scholes model. For this purpose we estimate the Black-Scholes model

$$\frac{dS_t}{S_t} = \alpha dt + \sigma dW_t$$

where α is the drift term, σ is the volatility, and W_t is a Wiener process. Hence, the likelihood ratio test statistic (LR) can be calculated for the hypothesis of the standard diffusion model against the Merton model. Standard theory assumes that LR is asymptotically $\chi^2(3)$ distributed, since the dimension of Θ is reduced by 3. However, this is not the case, because the test is performed with λ on the border of Θ and μ and δ^2 unidentified. Hence, the LR is not a formal test. Nevertheless it is used as an indicator for which model is the more likely.[7] In Table 3 the estimates of the Black-Scholes model are displayed. Due to $\hat{\sigma}$ being lowest for the indices, it is seen again that the indices behave differently compared to the stocks. The LR is reported in Table 6 for testing the standard diffusion model against the Merton model and the large LR's indicate that the standard model is strongly rejected. This is not surprising, since as earlier shown, the Merton model (BDM) fits the empirical distribution much better than the Black-Scholes model, cf. Figure 2. Note that this is not a test to figure out whether S_t follows a continuous process or a discontinuous process like the Merton model. Aït-Sahalia (1997) presents a general test to examine whether a process follows a continuous-time Markov diffusion or not.

5.2 The Constant Merton Model

As mentioned earlier, the Merton model is a way of modelling extreme events. This is, however, not supported by the high $\hat{\lambda}$-estimates in Table 2. Thus, instead of including a stochastic jump amplitude we look at a model with constant jump amplitude, $\log(Y+1) = \mu$. This model will be referred to as the Constant Merton model (CMM). The density function of the log-return in this model is of the following form

$$p(x;\Psi) = \sum_{j=0}^{\infty} \frac{e^{-\lambda\Delta}(\lambda\Delta)^j}{j!} \phi(x;(\alpha - \frac{1}{2}\sigma^2)\Delta + j\mu, \sigma^2\Delta).$$

The likelihood function for the CMM is bounded, since the volatility is solely described by σ. Thus, the MLE always exists.

[7] Note, if LR is less than two times the reduction of the dimension of Θ, accepting the hypothesis is equivalent to use an applicable criteria like the Akaike Information Criterion (AIC).

	$\hat{\alpha}$	$\hat{\sigma}$
AXP	0.1201	0.3354
	(0.0350)	(0.0030)
CHV	0.1168	0.2618
	(0.0303)	(0.0023)
DD	0.1061	0.2455
	(0.0310)	(0.0022)
DOW	0.1059	0.2755
	(0.0351)	(0.0024)
EK	0.0547	0.2785
	(0.0325)	(0.0025)
GE	0.1384	0.2357
	(0.0296)	(0.0021)
GM	0.0483	0.2651
	(0.0531)	(0.0023)
IBM	0.0654	0.2477
	(0.0325)	(0.0022)
IP	0.1059	0.2721
	(0.0334)	(0.0024)
KO	0.1616	0.2583
	(0.0425)	(0.0023)
MMM	0.0902	0.2284
	(0.0291)	(0.0020)
MO	0.1804	0.2647
	(0.0308)	(0.0023)
MOB	0.1185	0.2639
	(0.0314)	(0.0023)
MRK	0.1527	0.2413
	(0.0361)	(0.0021)
PG	0.1220	0.2241
	(0.0336)	(0.0020)
S	0.0778	0.2793
	(0.0371)	(0.0025)
T	0.0790	0.2131
	(0.0273)	(0.0019)
XON	0.1231	0.2155
	(0.0302)	(0.0019)
DAX	0.0834	0.1568
	(0.0317)	(0.0014)
FTSE100	0.1261	0.1460
	(0.0397)	(0.0017)
SP100	0.1451	0.1634
	(0.0448)	(0.0020)
SP500	0.0615	0.1874
	(0.0211)	(0.0010)
KFX	0.0767	0.1356
	(0.0492)	(0.0022)

The estimation results are based on daily log-return and the estimates are in annualised sizes. The numbers in parentheses are standard errors.

Table 3: The Black-Scholes model.

The empirical results are reported in Table 4. $\hat{\lambda}$ has decreased dramatically to be in the range of 0.30 to 6.52 with an average of 2.08. The average $\hat{\lambda}$ is lower for the stocks, 1.64, than the indices, 3.67. The jump amplitude, $e^{\hat{\mu}} - 1$, is in the range of -9.00% to -2.44% with an average of -5.74% and for the stocks and indices -6.09% and -4.49%, respectively. Thus, the empirical findings support the idea that the jump component in the CMM can be used to describe the extreme events. Finally, the LR statistics for the hypothesis of the CMM against the Merton model is calculated. This is a test for $\delta = 0$, which is on the border of Θ. Thus, LR is asymptotically distributed as $\frac{1}{2}\chi^2(0) + \frac{1}{2}\chi^2(1)$ [Harvey(1989)]. The hypothesis of the CMM against the Merton model can not be accepted on the basis of the LR reported in Table 6.

5.3 The Extended Merton Model

We have seen that the CMM captures the extreme events but at the same time it cannot be statistically accepted compared to the Merton model. Hence, in this section we propose a model that nests the Merton model and the CMM. The jump component consists of two Poisson processes ($J = 2$) with intensities λ_1 and λ_2 and a stochastic jump amplitude and a deterministic jump amplitude, respectively. Thus, $\log(Y_{1,t} + 1) \sim N(\mu_1, \delta_1^2)$ and $\log(Y_{2,t} + 1) = \mu_2$. The model is referred to as the Extended Merton model (EMM). The density function for the log-return in the EMM is

$$\begin{aligned} p(x;\Psi) &= \sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty} \frac{e^{-\lambda_1\Delta}(\lambda_1\Delta)^{j_1}}{j_1!}\frac{e^{-\lambda_2\Delta}(\lambda_2\Delta)^{j_2}}{j_2!} \\ &\times \phi(x;(\alpha - \frac{1}{2}\sigma^2)\Delta + j_1\mu_1 + j_2\mu_2, \sigma^2\Delta + j_1\delta_1^2). \end{aligned}$$

This function has the same characteristics as the density function for the Merton model. This means that the likelihood function is unbounded, and the estimation method is the same as for the Merton model.

The empirical outcome of the EMM can be summarized as follows. The stochastic jump amplitude Y_1 is estimated to have an expected value between -0.42% and 0.06% with an average of -0.12%. For the stocks and indices it is in the range of -0.32% to 0.06% and -0.42% to -0.13% with an average of -0.07% and -0.27%, respectively. The corresponding intensity $\hat{\lambda}_1$ is between 3.80 and 257.16 with an average of 98.19. For the stocks and indices the averages are 103.18 and 80.26, respectively. Compared to the Merton model $E[Y_1]$ is numerically smaller on average, and $\hat{\lambda}$ has decreased on average with the exception that $\hat{\lambda}$ has increased for the indices from 53.07 to 80.26. The constant jump amplitude Y_2 is estimated to be between -1.16% and 3.07% with an average of 1.67%. For the stocks and indices the estimated constant jump amplitudes are in the range of 1.18% to 3.07% and -1.16% to -1.42% with an average of 1.96% and 0.60%, respectively.

	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{\mu}$
AXP	0.2887	0.3118	2.4754	-0.0712
	(0.0397)	(0.0029)	(0.0465)	(0.0028)
CHV	0.2497	0.2463	2.6653	-0.0513
	(0.0329)	(0.0023)	(0.0415)	(0.0024)
DD	0.1779	0.2348	1.2878	-0.0577
	(0.0499)	(0.0022)	(0.0688)	(0.0033)
DOW	0.2044	0.2610	1.6259	-0.0630
	(0.0458)	(0.0024)	(0.0412)	(0.0035)
EK	0.1254	0.2595	0.8759	-0.0866
	(0.0576)	(0.0023)	(0.0446)	(0.0033)
GE	0.2389	0.2219	1.9257	-0.0538
	(0.0314)	(0.0021)	(0.0379)	(0.0026)
GM	0.1460	0.2515	1.8357	-0.0551
	(0.0327)	(0.0023)	(0.0704)	(0.0027)
IBM	0.1214	0.2351	0.7544	-0.0783
	(0.0331)	(0.0021)	(0.0456)	(0.0038)
IP	0.1488	0.2598	0.4890	-0.0943
	(0.0360)	(0.0023)	(0.0393)	(0.0048)
KO	0.2669	0.2392	1.6916	-0.0651
	(0.0364)	(0.0022)	(0.0537)	(0.0027)
MMM	0.1549	0.2135	1.0352	-0.0657
	(0.0375)	(0.0019)	(0.0541)	(0.0030)
MO	0.3067	0.2449	2.2174	-0.0592
	(0.0339)	(0.0023)	(0.0463)	(0.0024)
MOB	0.2388	0.2458	2.0809	-0.0600
	(0.0307)	(0.0023)	(0.0458)	(0.0025)
MRK	0.2656	0.2296	2.6653	-0.0434
	(0.0452)	(0.0022)	(0.0609)	(0.0023)
PG	0.1782	0.2087	0.9792	-0.0608
	(0.0692)	(0.0019)	(0.1391)	(0.0022)
S	0.1540	0.2656	1.2014	-0.0665
	(0.1486)	(0.0024)	(0.2455)	(0.0032)
T	0.1676	0.1977	1.7907	-0.0512
	(0.0284)	(0.0019)	(0.2345)	(0.0030)
XON	0.2135	0.1998	1.9356	-0.0484
	(0.0296)	(0.0019)	(0.0372)	(0.0020)
DAX	0.2243	0.1377	4.6180	-0.0311
	(0.0264)	(0.0013)	(0.0638)	(0.0010)
FTSE100	0.1522	0.1371	0.2963	-0.0924
	(0.0341)	(0.0016)	(0.0632)	(0.0043)
SP100	0.2340	0.1396	2.1543	-0.0430
	(0.0406)	(0.0017)	(0.1333)	(0.0015)
SP500	0.2464	0.1617	4.7382	-0.0400
	(0.0176)	(0.0010)	(0.0299)	(0.0009)
KFX	0.2359	0.1195	6.5222	-0.0247
	(0.0429)	(0.0021)	(0.1588)	(0.0015)

The estimation results are based on daily log-return and the estimates are in annualised sizes. The numbers in parentheses are standard errors.

Table 4: The Constant Merton model.

	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\lambda}_1$	$\hat{\mu}_1$	$\hat{\delta}$	$\hat{\lambda}_2$	$\hat{\mu}_2$
AXP	-0.2479	0.1904	115.8832	-0.0008	0.0234	20.5337	0.0205
	(0.0467)	(0.0047)	(0.0374)	(0.0009)	(0.0006)	(0.0508)	(0.0031)
CHV	-0.1192	0.1299	186.8181	-0.0002	0.0157	16.0923	0.0155
	(0.0345)	(0.0038)	(0.0432)	(0.0004)	(0.0004)	(0.0417)	(0.0032)
DD	-0.3069	0.1562	88.8663	-0.0014	0.0168	29.2341	0.0179
	(0.0869)	(0.0034)	(0.1246)	(0.0013)	(0.0008)	(0.1269)	(0.0020)
DOW	-0.1929	0.1540	134.5514	-0.0003	0.0183	22.2525	0.0142
	(0.0352)	(0.0048)	(0.0439)	(0.0007)	(0.0004)	(0.0420)	(0.0031)
EK	-0.2129	0.2072	13.9242	-0.0027	0.0398	9.5139	0.0302
	(0.0459)	(0.0025)	(0.0404)	(0.0029)	(0.0021)	(0.0397)	(0.0019)
GE	-0.1786	0.1625	46.2382	-0.0019	0.0210	20.3432	0.0192
	(0.0439)	(0.0027)	(0.0672)	(0.0013)	(0.0009)	(0.0669)	(0.0017)
GM	-0.3876	0.1458	159.4666	0.0003	0.0160	22.8690	0.0162
	(0.0812)	(0.0047)	(0.0506)	(0.0009)	(0.0005)	(0.0878)	(0.0034)
IBM	-0.1970	0.1814	25.3271	-0.0004	0.0288	12.1062	0.0214
	(0.1620)	(0.0025)	(0.2538)	(0.0026)	(0.0012)	(0.3609)	(0.0020)
IP	-0.2174	0.2051	21.7205	-0.0027	0.0297	13.4279	0.0272
	(0.0311)	(0.0027)	(0.0425)	(0.0022)	(0.0015)	(0.0370)	(0.0018)
KO	-0.1866	0.1770	29.2927	-0.0029	0.0284	19.6179	0.0211
	(0.0346)	(0.0027)	(0.0423)	(0.0016)	(0.0012)	(0.0479)	(0.0014)
MMM	-0.1973	0.1172	140.9188	-0.0003	0.0146	19.1106	0.0159
	(0.0413)	(0.0027)	(0.0259)	(0.0005)	(0.0004)	(0.0670)	(0.0019)
MO	0.1094	0.1705	81.9168	-0.0013	0.0204	6.0938	0.0252
	(0.0365)	(0.0027)	(0.0416)	(0.0009)	(0.0007)	(0.0418)	(0.0052)
MOB	-0.0851	0.1684	70.2610	0.0003	0.0221	6.558	0.0243
	(0.0342)	(0.0026)	(0.0525)	(0.0009)	(0.0007)	(0.0491)	(0.0039)
MRK	-0.1606	0.1365	165.0237	-0.0002	0.0141	18.0358	0.0183
	(0.1139)	(0.0034)	(0.1301)	(0.0010)	(0.0006)	(0.0855)	(0.0037)
PG	-0.1441	0.1171	155.2489	0.0000	0.0135	17.7345	0.0140
	(0.0413)	(0.0031)	(0.0350)	(0.0005)	(0.0003)	(0.0455)	(0.0023)
S	-0.3979	0.1452	165.2492	0.0005	0.0168	20.3159	0.0178
	(0.0276)	(0.0039)	(0.0342)	(0.0005)	(0.0004)	(0.0330)	(0.0027)
T	-0.1891	0.0857	218.4134	0.0001	0.0121	19.7028	0.0117
	(0.02882)	(0.0026)	(0.0415)	(0.0003)	(0.0002)	(0.0544)	(0.0017)
XON	-0.0043	0.1551	38.0480	-0.0034	0.0196	12.8285	0.0191
	(0.0384)	(0.0023)	(0.1078)	(0.0012)	(0.0009)	(0.1110)	(0.0018)
DAX	0.0281	0.1091	27.1981	-0.0044	0.0190	17.0240	0.0098
	(0.0260)	(0.0018)	(0.0529)	(0.0010)	(0.0007)	(0.0483)	(0.0012)
FTSE100	0.3261	0.1167	3.8024	-0.0039	0.0376	16.2071	-0.0117
	(0.0331)	(0.0020)	(0.0801)	(0.0058)	(0.0044)	(0.0892)	(0.0011)
SP100	0.0123	0.0860	59.3845	-0.0027	0.0140	26.0608	0.0110
	(0.0349)	(0.0022)	(0.0798)	(0.0008)	(0.0006)	(0.1011)	(0.0009)
SP500	0.0821	0.0993	53.7418	-0.0024	0.0200	6.8216	0.0141
	(0.0162)	(0.0009)	(0.0334)	(0.0004)	(0.0003)	(0.0270)	(0.0010)
KFX	-0.0297	0.0349	257.1591	-0.0013	0.0071	64.8075	0.0066
	(0.0438)	(0.0023)	(0.1375)	(0.0003)	(0.0003)	(0.1301)	(0.0005)

The estimation results are based on daily log-return and the estimates are in annualised sizes. The numbers in parentheses are standard errors.

Table 5: The Extended Merton model.

$\hat{\lambda}_2$ is between 6.09 and 64.81, with an average of 19.01. For individual stocks and indices the averages are 17.02 and 26.18, respectively. Compared to the CMM, Y_2 has gone from negative to positive, except for FTSE 100, and λ_2 has on average increased dramatically from 2.08 to 19.01. The variance of the jump component is estimated to be between 28.30% and 91.41% with an average of 56.55%. For the stocks and indices the averages are 55.89% and 58.92%, respectively. This is more or less the same as for the Merton model. Finally, the LR statistics for the hypothesis of the Merton model against the EMM is calculated. Again the LR can only be used as an indication of the most likely model, because the test is performed with λ on the border of Θ and with δ^2 unidentified. However, for some of the stocks the LR is of such a size that it seems reasonable to accept the hypothesis of the Merton model against the EMM.

6 Option Pricing

The option prices from the jump-diffusion models and the Black-Scholes prices are compared in this section. The aim is to show that the empirically supported high λ-values do not lead to the wanted difference in the prices. This point is illustrated by calculating prices for a high intensity stock and a low intensity index.

The jump-diffusion model like Merton's gives rise to an incomplete market in contrast to the Black-Scholes model. This means that a portfolio which exactly replicates an option cannot be constructed. Hence, the risk neutral world is not uniquely determined, since a set of equivalent martingale measures, Q, exists, which precludes arbitrage.[8] It is beyond the scope of this paper to look at how to select the *correct* equivalent martingale measure. Thus, it is assumed that the risk-neutral measure exists and that we chose the correct one in pricing put options in the jump-diffusion models. We assume that the riskless interest rate, r, is constant for the selected time horizon. Furthermore, the stock/index pays a continuous dividend stream q.

In a risk-neutral world the partial integro differential equation (PIDE) for the price, C, of an option depending on $x = \log(S)$ is given as [Andreasen and Gruenewald (1996)][9]

$$\begin{aligned} rC &= \frac{\partial C}{\partial t} + (r - q - k\lambda^Q - \frac{1}{2}\sigma^2)\frac{\partial C}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2 C}{\partial x^2} \\ &+ \lambda^Q \int_{\mathbb{R}} \left(C(t, x+y) - C(t_-, x)\right) \phi(y; \mu + \frac{1}{2}\delta^2, \delta^2) dy \end{aligned} \tag{8}$$

where $\lambda^Q = (1-\theta)\lambda$ is the risk adjusted jump intensity and $\theta \leq 1$ is the risk adjusted function of the jump components. Finally, $k = \log E^Q(Y_t) = \mu + \frac{1}{2}\delta^2$. The PIDE looks like the partial differential equation that we obtain in the Black-Scholes model, except for

[8]The Q-measure denotes the state variables in a risk-neutral world.

[9]Consequently, the jump amplitude is transformed to $y = \log(1+Y)$.

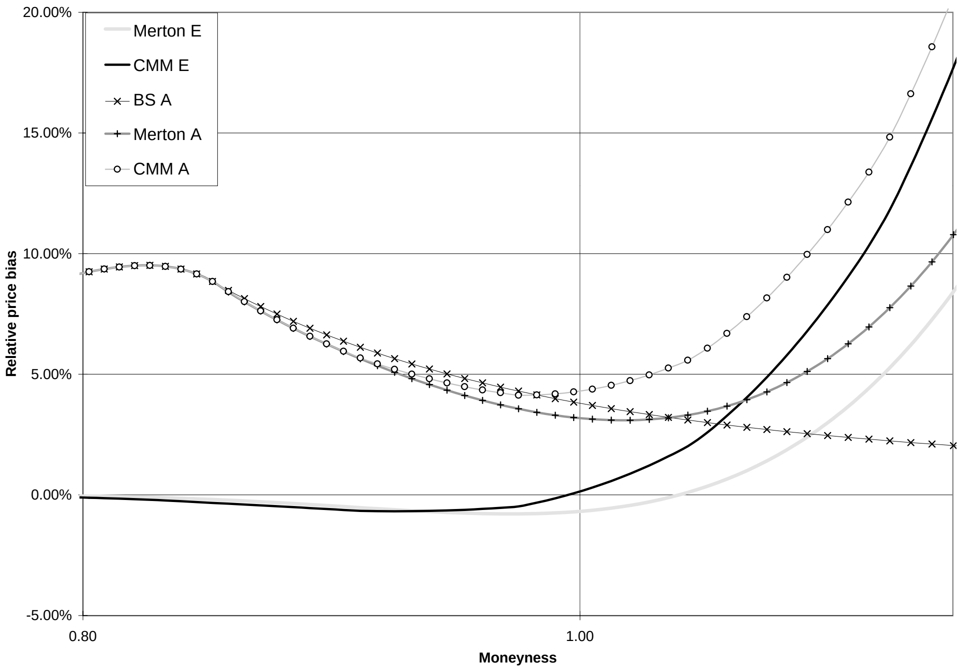
	BS - Merton	CMM - Merton	Merton - EMM
AXP	856.3877	468.5825	5.2646
CHV	737.5097	488.1717	2.4995
DD	526.9195	278.5661	9.3537
DOW	797.1449	491.2153	2.8874
EK	1254.3835	624.5579	23.2626
GE	688.0375	376.4906	14.8918
GM	644.5855	381.8080	3.4635
IBM	996.4858	563.6644	10.5974
IP	808.6205	385.4643	12.3159
KO	1095.6925	560.6751	26.4577
MMM	1074.7505	513.2556	5.4845
MO	917.7122	438.3309	1.5553
MOB	1085.7785	677.8284	1.4443
MRK	495.4647	346.6874	1.2756
PG	1135.1342	523.7390	3.5413
S	817.3667	491.5173	3.6528
T	1166.9780	658.9977	4.1436
XON	899.1684	398.5019	10.7124
DAX	1310.2088	771.9623	9.0629
FTSE100	651.8265	269.2482	15.3145
SP100	1172.8505	388.0618	20.9218
SP500	6321.6343	4085.0554	20.1583
KFX	317.4211	171.3849	22.8716

The likelihood ratio test statistics calculated for the three hypothesis; *i)* the Black-Scholes model against the Merton model, *ii)* the constant Merton model against the Merton model, and *iii)* The Merton model against the extended Merton model. At the 5% level of significance the critical LR value for the hypothesis of the CMM against the Merton model is 2.7055. Note the difference between *i)* and *ii)* is the likelihood ratio test statistics for then Black-Scholes model against the constant Merton model.

Table 6: The likelihood ratio test statistics.

the last term, caused by the jump component. This term is the instantaneous expected change in the option price due to the discontinuous stock price. Note that (8) is derived under the assumption that the Merton model is true.[10] The closed form solution for the European vanilla option is in Merton (1976).

The finite-difference method presented by Andreasen and Gruenewald (1996) is used to obtain prices for the American put options in the Merton model and the CMM. Hence, the Crank-Nicolson method is applied to the *normal* terms of the PIDE and the last term is approximated by the explicit finite-difference method. Finally, the Richardson extrapolation is used to speed up the convergence.[11]



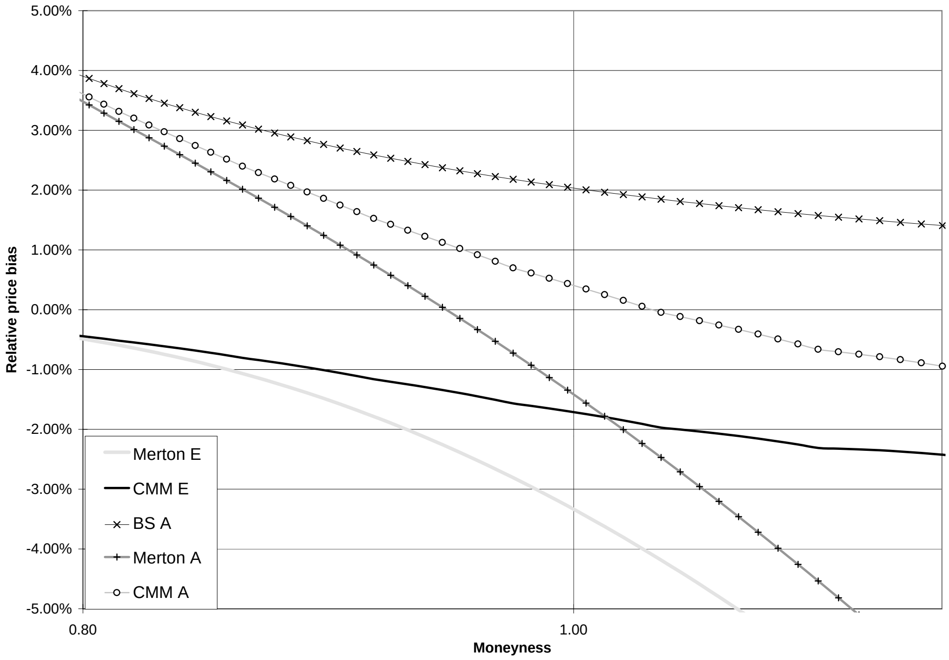
The graph displays the relative price bias to the European Black-Scholes put option as a function of moneyness, $\frac{S_t e^{r(T-t)}}{X}$. Merton E/A denotes the European/American put option in the Merton model. CMM E/A denotes the European/American put option in the CMM. BS A denotes the American put option in the Black-Scholes model. The put option expires in 6 months and with exercise price, $X = 100$. $r = 5\%$, $q = 2\%$ and no risk premia on the jump component, $\theta = 0$.

Figure 3: FTSE, low intensity index.

The numerical results for the put options are based on the assumption that $r = 5\%$, $q = 2\%$ and no risk premia on the jump component, $\theta = 0$. The put option expires in

[10]For the CMM the last term is replaced by $\lambda^Q \left(C(t, x + \mu) - C(t_-, x)\right)$. Hence, the PIDE is reduced to a partial differential difference equation (PDDE).

[11]The Richardson extrapolation is based on the fact that the applied finite-difference method has a convergence error of order one in the time dimension. Hence, let $C(h)$ denote the value obtained by dividing the time interval into h pieces. Then the extrapolated value is $\bar{C} = 2C(2h) - C(h)$.



The graph displays the relative price bias to the European Black-Scholes put option as a function of moneyness, $\frac{S_t e^{r(T-t)}}{X}$. Merton E/A denotes the European/American put option in the Merton model. CMM E/A denotes the European/American put option in the CMM. BS A denotes the American put option in the Black-Scholes model. The put option expires in 6 months and with exercise price, $X = 100$. $r = 5\%$, $q = 2\%$ and no risk premia on the jump component, $\theta = 0$.

Figure 4: MOB, high intensity stock.

6 months and with exercise price, $X = 100$. The European and American put options are evaluated in the Black-Scholes model, Merton model and CMM. The relative price bias to the European Black-Scholes put is calculated to ease the comparison. First of all, the analysis is done with the estimates obtained from the FTSE 100 index, since the $\hat{\lambda}$ is low in the Merton model. This comparison is valid since the estimated variance is of the same size for the different models combined with the assumption that $\theta = 0$, since the variance is unchanged under the Q-measure. The results are illustrated in Figure 3 where the x-axis is moneyness, which is defined as $\frac{S_t e^{r(T-t)}}{X}$. Thus, in-the-money options correspond to values less than one and out-of-the money options correspond to values greater than one.

For the European put options, the outcome is; *i)* prices of in-the-money options are more or less the same for all three models; *ii)* an out-of-the money option is much cheaper in the Black-Scholes model; and *iii)* the price obtained in the Merton model is smaller than in the CMM. The same features are found for the American put options. They only differ in their higher price level, but this is a result of the early exercise opportunity. To explain the much higher out-of-the money options in the Merton model and the CMM, we have to look at the estimated jump amplitude which is estimated to be negative on average. Hence, even though the option is far out of the money, it is likely to become in the money *instantaneously*, if a negative jump in the underlying process occurs. This is of course not the case for the Black-Scholes model, as the underlying process moves continuously over time. The Black-Scholes volatility smile can also be mimicked from the jump-diffusion models. Thus, the conclusion could be that a Merton model/CMM captures what is observed on the market. However, for realistic values of λ we establish that the conclusion is less significant. This is based on redoing the exercise for estimates obtained from MOB which is close to an average stock. The $\hat{\lambda}$ from the Merton model is about 20 times bigger than before. Figure 4 reports the results. Note that the scaling on the y-axis is different from before as the relative price biases are smaller. The Black-Scholes price is the highest for all kinds of moneyness, but as before the prices obtained in the Merton model are smaller than in the CMM. The interesting point is that the relative price biases obtained in the jump-diffusion models are numerically smaller than before. Hence, this indicates that these models can only partly explain the Black-Scholes volatility smile.

7 Conclusion

In this paper we have shown that in a jump-diffusion model the log-return is equivalent to a discrete mixture of N normally distributed variables, where N goes to infinity. Thus, we know from the mixture-of-distribution literature [Kiefer (1978)] that the likelihood function for some parametric specifications is unbounded. Hence, the estimation of jump-diffusion models must be carried out carefully, since the standard maximum likelihood estimates are invalid. A method has been proposed, where the profile of the likelihood function with respect to the relative variances between the diffusion and jump part is used to obtain a consistent estimator.

The empirical results, based on the presented method, indicate that the stocks/indices are insufficiently described by the Merton model, as the estimated arrival intensity of the jumps is of such a size that adding a jump component more looks like inserting a Wiener process. However, there is empirical evidence that adding a second jump component with a constant jump amplitude improves the results.

Finally, the implication of moving from the Black-Scholes model to the jump-diffusion model is examined for put option prices. The difference to the Black-Scholes price is a decreasing function in the intensity of the jump component. Hence, the jump-diffusion model gives less price difference than often expected, since the literature mostly assumes a very low jump intensity or even does the estimation using an invalid estimation method.

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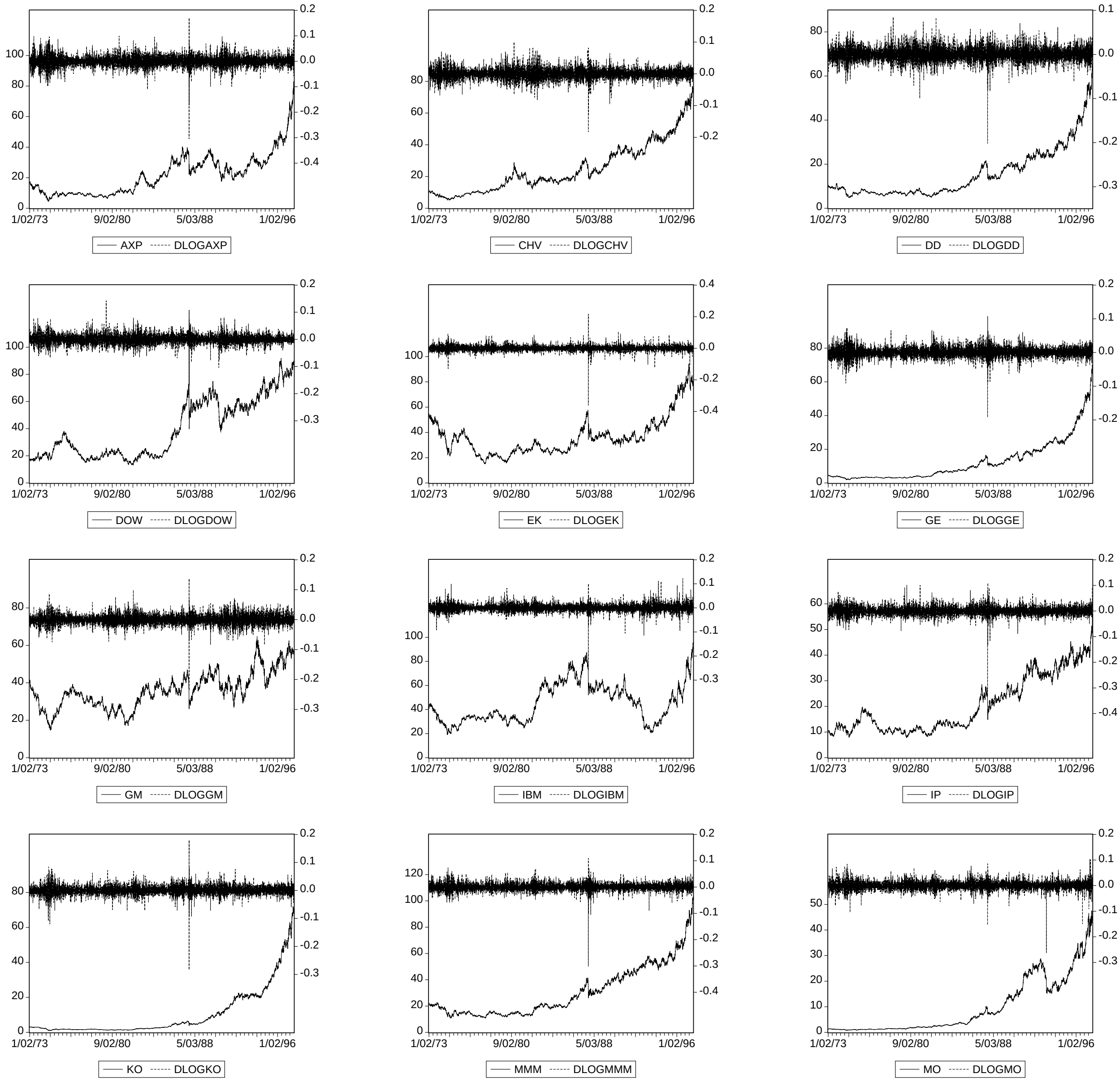
Appendix A
Sample of Securities/Indices

Symbol	Name	Sample Period
AXP	American Express	Jan 2, 1973 - July 8, 1997
CHV	Chevron	Jan 2, 1973 - July 8, 1997
DD	Du Pont	Jan 2, 1973 - July 8, 1997
DOW	Dow Chemicals	Jan 2, 1973 - July 8, 1997
EK	Eastman Kodak	Jan 2, 1973 - July 8, 1997
GE	General Electric	Jan 2, 1973 - July 8, 1997
GM	General Motors	Jan 2, 1973 - July 8, 1997
IBM	IBM	Jan 2, 1973 - July 8, 1997
IP	International Paper	Jan 2, 1973 - July 8, 1997
KO	Coca Cola	Jan 2, 1973 - July 8, 1997
MMM	3M	Jan 2, 1973 - July 8, 1997
MO	Philip Morris	Jan 2, 1973 - July 8, 1997
MOB	Mobil	Jan 2, 1973 - July 8, 1997
MRK	Merck	Jan 2, 1973 - July 8, 1997
PG	Procter & Gamble	Jan 2, 1973 - July 8, 1997
S	Sears Roebuck	Jan 2, 1973 - July 8, 1997
T	AT&T	Jan 2, 1973 - July 8, 1997
XON	Exxon	Jan 2, 1973 - July 8, 1997
DAX	DAX 100	Jan 1, 1973 - July 8, 1997
FTSE100	FTSE 100	Jan 2, 1984 - July 8, 1997
SP100	S&P 100	Marts 5, 1984 - July 8, 1997
SP500	S&P 500	Jan 3, 1928 - Oct 19, 1988
KFX	KFX	Dec 4, 1989 - July 8, 1997

Table 7: Symbol list.

Appendix B
Time-Series Plot



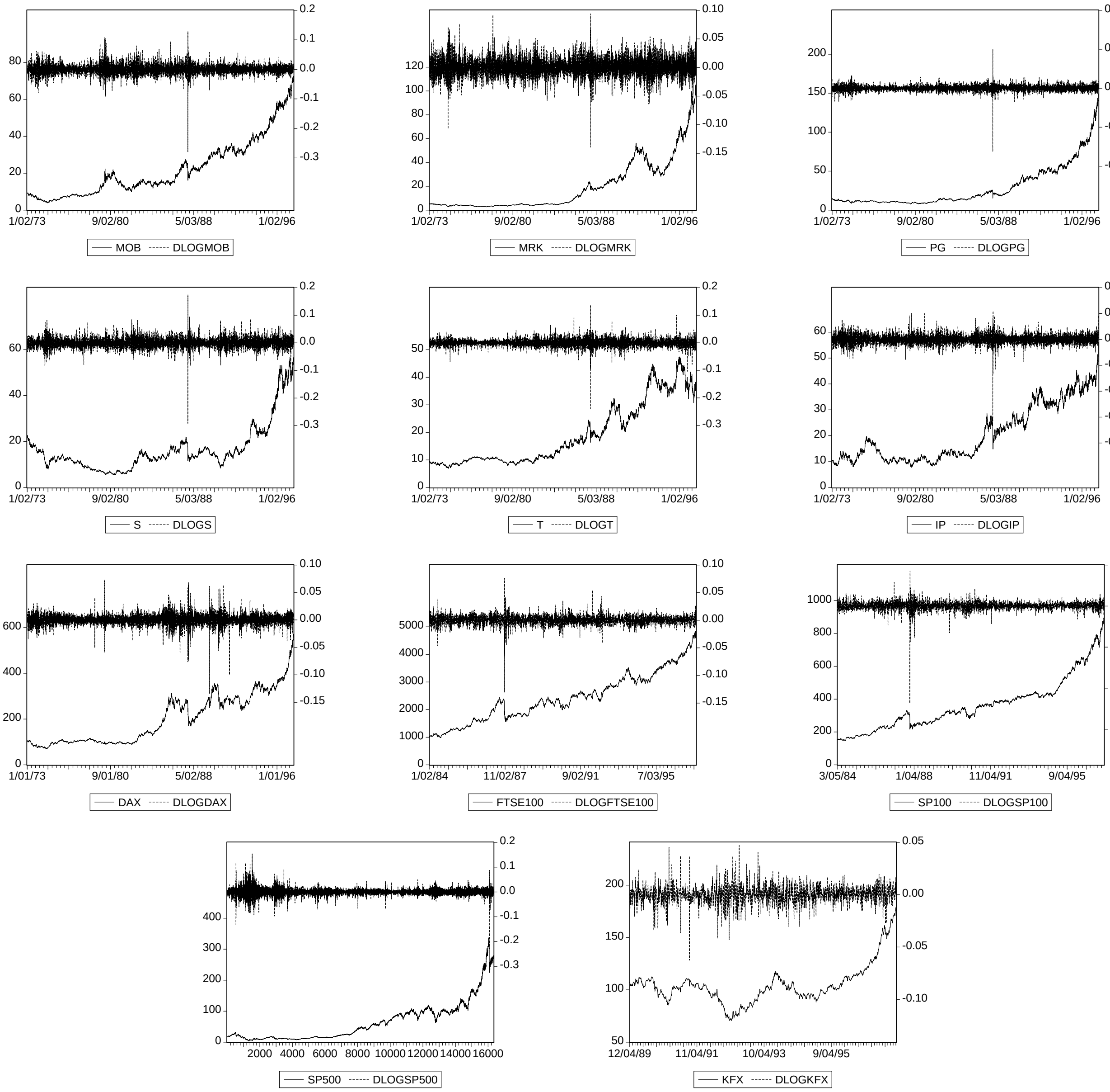


Figure 5: Plots of the daily levels and log-returns.

Appendix C

Descriptive Statistics

	Mean	Median	Max	Min	Std.Dev.	Skewness	Kurtosis	Obs
AXP	21.59	20.69	83.00	4.44	12.82	1.13	4.67	6396
CHV	24.81	19.62	76.88	5.09	15.50	0.98	3.32	6396
DD	15.52	9.69	65.06	4.79	11.20	1.55	5.27	6396
DOW	39.66	30.25	91.88	13.25	21.55	0.50	1.83	6396
EK	36.40	33.61	94.25	15.03	14.75	1.43	5.07	6396
GE	12.39	7.45	69.88	1.88	11.98	1.81	6.42	6396
GM	35.89	35.38	64.75	14.56	9.59	0.37	2.86	6396
IBM	45.23	42.47	95.75	19.00	16.06	0.50	2.30	6396
IP	20.52	14.87	56.13	7.78	10.89	0.71	2.12	6396
KO	10.05	2.98	71.88	0.96	13.66	2.12	7.34	6396
MMM	30.56	21.25	103.06	10.81	18.87	1.11	3.65	6396
MO	9.58	3.36	47.50	0.73	10.62	1.22	3.53	6396
MOB	22.06	15.94	72.63	3.87	15.32	1.09	3.51	6396
MRK	19.41	5.71	105.19	2.67	21.18	1.51	4.83	6396
PG	28.74	14.47	149.38	7.88	26.41	1.79	6.03	6396
S	16.39	13.77	56.63	5.46	10.15	2.04	7.08	6396
T	19.14	14.54	47.42	6.91	11.32	0.83	2.29	6396
XON	17.57	12.46	64.63	3.43	12.61	0.92	3.15	6396
DAX	199.45	168.25	571.38	70.61	107.66	0.60	2.38	6397
FTSE100	2428.24	2348.80	4831.70	978.70	875.29	0.41	2.48	3527
SP100	363.04	332.74	897.24	146.46	155.74	1.04	3.79	3482
SP500	63.02	43.04	336.77	4.40	62.07	1.63	5.92	16331
KFX	104.53	102.25	179.01	70.59	18.50	1.49	6.19	1982

Where skewness=$\frac{E[X-E[X]]^3}{[Var[X]]^{1.5}}$ and kurtosis=$\frac{E[X-E[X]]^4}{[Var[X]]^2}$.

Table 8: Descriptive statistics of the daily levels.

	Mean	Median	Max	Min	Std.Dev.	Skewness	Kurtosis	Obs
DLOGAXP	0.00	0.00	0.17	-0.30	0.0208	-0.45	13.58	6395
DLOGCHV	0.00	0.00	0.10	-0.18	0.0162	-0.15	8.36	6395
DLOGDD	0.00	0.00	0.08	-0.20	0.0152	-0.29	9.73	6395
DLOGDOW	0.00	0.00	0.14	-0.21	0.0171	-0.28	11.34	6395
DLOGEK	0.00	0.00	0.22	-0.36	0.0172	-1.06	39.85	6395
DLOGGE	0.00	0.00	0.11	-0.19	0.0146	-0.27	10.46	6395
DLOGGM	0.00	0.00	0.14	-0.24	0.0164	-0.26	12.04	6395
DLOGIBM	0.00	0.00	0.12	-0.26	0.0153	-0.49	20.68	6395
DLOGIP	0.00	0.00	0.11	-0.31	0.0168	-0.83	24.84	6395
DLOGKO	0.00	0.00	0.18	-0.28	0.0160	-0.65	24.46	6395
DLOGMMM	0.00	0.00	0.11	-0.30	0.0141	-1.42	38.02	6395
DLOGMO	0.00	0.00	0.10	-0.26	0.0164	-0.79	17.69	6395
DLOGMOB	0.00	0.00	0.13	-0.28	0.0163	-0.43	20.34	6395
DLOGMRK	0.00	0.00	0.09	-0.14	0.0149	0.01	6.33	6395
DLOGPG	0.00	0.00	0.20	-0.33	0.0139	-1.47	58.74	6395
DLOGS	0.00	0.00	0.17	-0.29	0.0173	-0.40	18.89	6395
DLOGT	0.00	0.00	0.14	-0.24	0.0132	-0.72	25.27	6395
DLOGXON	0.00	0.00	0.16	-0.27	0.0133	-0.99	33.61	6395
DLOGDAX	0.00	0.00	0.07	-0.13	0.0097	-0.80	16.56	6396
DLOGFTSE100	0.00	0.00	0.08	-0.13	0.0090	-1.52	26.37	3526
DLOGSP100	0.00	0.00	0.09	-0.25	0.0101	-3.85	95.96	3481
DLOGSP500	0.00	0.00	0.15	-0.23	0.0116	-0.49	25.75	16330
DLOGKFX	0.00	0.00	0.05	-0.06	0.0084	-0.32	7.85	1981

Where skewness=$\frac{E[X-E[X]]^3}{[Var[X]]^{1.5}}$ and kurtosis=$\frac{E[X-E[X]]^4}{[Var[X]]^2}$.

Table 9: Descriptive statistics of the daily log-returns.

Appendix D
Convergence of the Poisson Log-likelihood Function

In this section it is shown how the estimates in the Merton Model converges as a function of N, i.e. as the number of terms in the density function (3) increases. The numerical study has been carried out for two situations of $\hat{\lambda}$ in the BDM. First for a large value of $\hat{\lambda}$ which implies that the BDM is a bad proxy for the Merton model. Second, for the opposite situation where the BDM is a good approximation for the Merton model. This is verified by Tables 10 and 11.

	α	σ	λ	μ	δ	$-2\log L$
Bernoulli	-0.0247373	0.1941625	21.5886572	0.0036270	0.0329370	-36246.7613461
Poisson N						
1	-0.0120694	0.2013206	13.8913734	0.0048266	0.0384560	-36219.7593565
2	-0.0315926	0.1907254	27.7450255	0.0030455	0.0290412	-36266.9592174
3	-0.0429981	0.1852112	37.3907245	0.0025371	0.0257222	-36276.8552560
4	-0.0479416	0.1829339	41.8877242	0.0023728	0.0245664	-36279.1464381
5	-0.0497343	0.1821183	43.5728596	0.0023187	0.0241799	-36279.6460999
6	-0.0501326	0.1819373	43.9519413	0.0023070	0.0240965	-36279.7164502
7	-0.0501854	0.1819169	43.9947336	0.0023058	0.0240872	-36279.7221930
8	-0.0501804	0.1819171	43.9947924	0.0023058	0.0240871	-36279.7224948
9	-0.0501804	0.1819171	43.9947924	0.0023058	0.0240871	-36279.7225063
10	-0.0501802	0.1819169	43.9947960	0.0023058	0.0240871	-36279.7225066
15	-0.0501802	0.1819169	43.9947960	0.0023058	0.0240871	-36279.7225066
20	-0.0501802	0.1819169	43.9947960	0.0023058	0.0240871	-36279.7225066
25	-0.0501802	0.1819169	43.9947960	0.0023058	0.0240871	-36279.7225066
35	-0.0501802	0.1819169	43.9947960	0.0023058	0.0240871	-36279.7225066
50	-0.0501802	0.1819169	43.9947960	0.0023058	0.0240871	-36279.7225066
75	-0.0501802	0.1819169	43.9947960	0.0023058	0.0240871	-36279.7225066
100	-0.0501802	0.1819169	43.9947960	0.0023058	0.0240871	-36279.7225066

Table 10: IBM daily observations.

	α	σ	λ	μ	δ	$-2\log L$
Bernoulli	0.156470	0.124165	4.884408	-0.006816	0.034095	-23834.379902
Poisson N						
1	0.155859	0.124359	4.644752	-0.007031	0.034806	-23833.199876
2	0.157160	0.124006	5.163110	-0.006586	0.032897	-23834.774664
3	0.157160	0.124006	5.163112	-0.006584	0.032897	-23834.792295
4	0.157166	0.124004	5.164929	-0.006582	0.032871	-23834.792452
5	0.157166	0.124004	5.164937	-0.006583	0.032871	-23834.792452
6	0.157166	0.124004	5.164943	-0.006582	0.032871	-23834.792452
7	0.157167	0.124004	5.164951	-0.006583	0.032871	-23834.792453
8	0.157167	0.124004	5.164957	-0.006582	0.032871	-23834.792453
9	0.157167	0.124004	5.164965	-0.006583	0.032871	-23834.792453
10	0.157167	0.124004	5.164971	-0.006582	0.032871	-23834.792453
11	0.157167	0.124004	5.164979	-0.006583	0.032871	-23834.792453
12	0.157167	0.124004	5.164986	-0.006582	0.032871	-23834.792453
13	0.157168	0.124004	5.164993	-0.006583	0.032871	-23834.792453
14	0.157168	0.124004	5.164999	-0.006582	0.032871	-23834.792453
15	0.157168	0.124004	5.165007	-0.006583	0.032871	-23834.792453
16	0.157168	0.124004	5.165013	-0.006582	0.032871	-23834.792454
17	0.157168	0.124004	5.165020	-0.006583	0.032871	-23834.792454
18	0.157168	0.124004	5.165027	-0.006582	0.032871	-23834.792454
19	0.157169	0.124004	5.165034	-0.006583	0.032871	-23834.792454
20	0.157169	0.124004	5.165040	-0.006582	0.032871	-23834.792454
25	0.157169	0.124004	5.165048	-0.006583	0.032871	-23834.792454
35	0.157169	0.124004	5.165054	-0.006582	0.032871	-23834.792454
50	0.157169	0.124004	5.165062	-0.006583	0.032871	-23834.792454
75	0.157169	0.124004	5.165068	-0.006582	0.032871	-23834.792454
100	0.157169	0.124004	5.165069	-0.006582	0.032871	-23834.792454

Table 11: FTSE 100 daily observations.

List of CAF's Working Papers

1. O. E. Barndorff-Nielsen (November 1997), *Processes of Normal Inverse Gaussian Type*.

2. P. Honoré (November 1997), *Modelling Interest Rate Dynamics in a Corridor with Jump Processes*.

3. G. Peskir (November 1997), *The Concept of Risk in the Theory of Option Pricing*.

4. A. T. Hansen and P. L. Jørgensen (November 1997), *Analytical Valuation of American-style Asian Options*.

5. T. H. Rydberg (December 1997), *Why Financial Data are Interesting to Statisticians*.

6. G. Peskir (December 1997), *Designing Options Given the Risk: The Optimal Skorokhod-Embedding Problem*.

7. J. L. Jensen and J. Pedersen (December 1997), *A note on models for stock prices*.

8. M. Bladt and T. H. Rydberg (December 1997), *An actuarial approach to option pricing under the physical measure and without market assumptions*.

9. J. Aa. Nielsen and K. Sandmann (April 1998), *Asian exchange rate options under stochastic interest rates: pricing as a sum of delayed payment options.*

10. O. E. Barndorff-Nielsen and N. Shephard (May 1998), *Aggregation and model construction for volatility models.*

11. M. Sørensen (August 1998), *On asymptotics of estimating functions.*

12. A. T. Hansen and P. L. Jørgensen (August 1998), *Exact Analytical Valuation of Bonds when Spot Interest Rates are Log-Normal.*

13. T. Björk, B. J. Christensen and A. Gombani (September 1998), *Some Control Theoretic Aspects of Interest Rate Theory*.

14. B. J. Christensen and N. R. Prabhala (September 1998), *The Relation Between Implied and Realized Volatility*.

15. O.E. Barndorff-Nielsen and W. Jiang (August 1998), *An initial analysis of some German stock price series.*

16. C. Vorm Christensen and H. Schmidli (September 1998), *Pricing catastrophe insurance products based on actually reported claims*.

17. M. Bibby and M. Sørensen (September 1998), *Simplified Estimating Functions for Diffusion Models with a High-dimensional Parameter*

18. P. Honoré (November 1998), *Pitfalls in Estimating Jump-Diffusion Models*

19. P. Honoré (November 1998), *Panel-Data Estimation of Non-Linear Term-Structure Models.*

20. T. Engsted and K. Nyholm (November 1998), *Regime shifts in the Danish term structure of interest rates.*

21. T. Hviid Rydberg and N. Shephard (December 1998), *Dynamics of trade- by-trade price movements: decomposition and models*.

22. E. Hansen and A. Rahbek (December 1998), *Stationarity and asymptotics of multivariate ARCH time series with an application to robustness of cointegration analysis.*

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ISSN 1398-6163

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