18TH International AFIR Colloquium

Financial Risk in a Changing World

Rome, September 30th - October 3rd, 2008

Author: Frédéric Planchet / Marc Juillard/ Pierre-E. Thérond

Extreme disturbances on the drift of anticipated mortality
Application to annuity plans

Introduction – the framework

We consider now the “global” systematic risk associated with the temporal fluctuations of mortality.

The stochastic models of mortality provide a tool well-adapted to this analysis. They suggest that the future death rate itself is a random variable, and thus $\mu(x,t)$ is a stochastic process (as a function of *t* for a fixed age *x*).

The mortality rate is effectively observed for an age and thus a given year is the realization of a random variable.

Introduction – the framework

Through construction of a prospective mortality table by the method of Lee & Carter (1992)

$$\ln \mu_{xt} = \alpha_x + \beta_x k_t + \varepsilon_{xt}$$

or one of its derivations (cf. Planchet & Thérond (2006)), we are led to estimate the future tendency through the modelling of coefficient

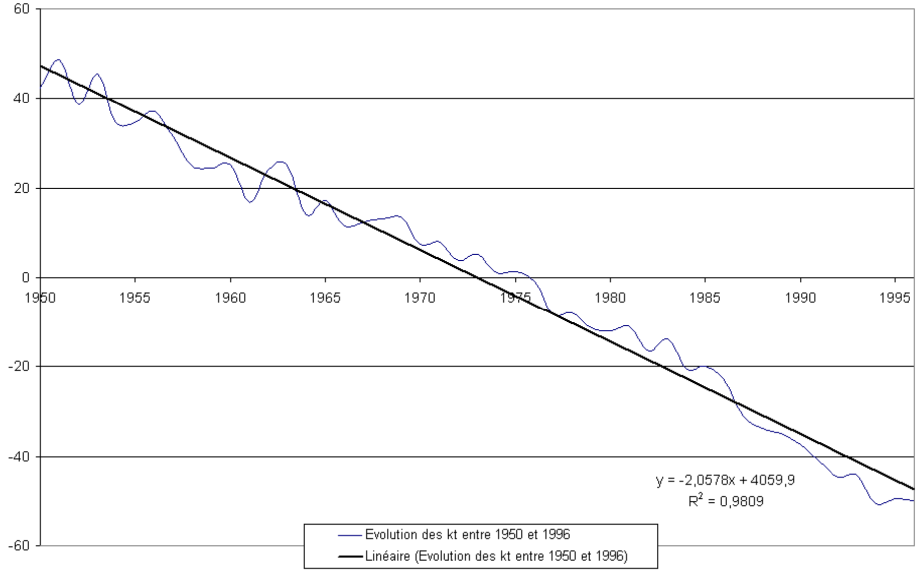
$$\left(k_t, t \geq t_M\right)$$

This model suggests an obvious linear tendency in general.

Introduction – the framework

This will be modelised simply by assuming that:

$$k_t^* = at + b + \gamma_t$$



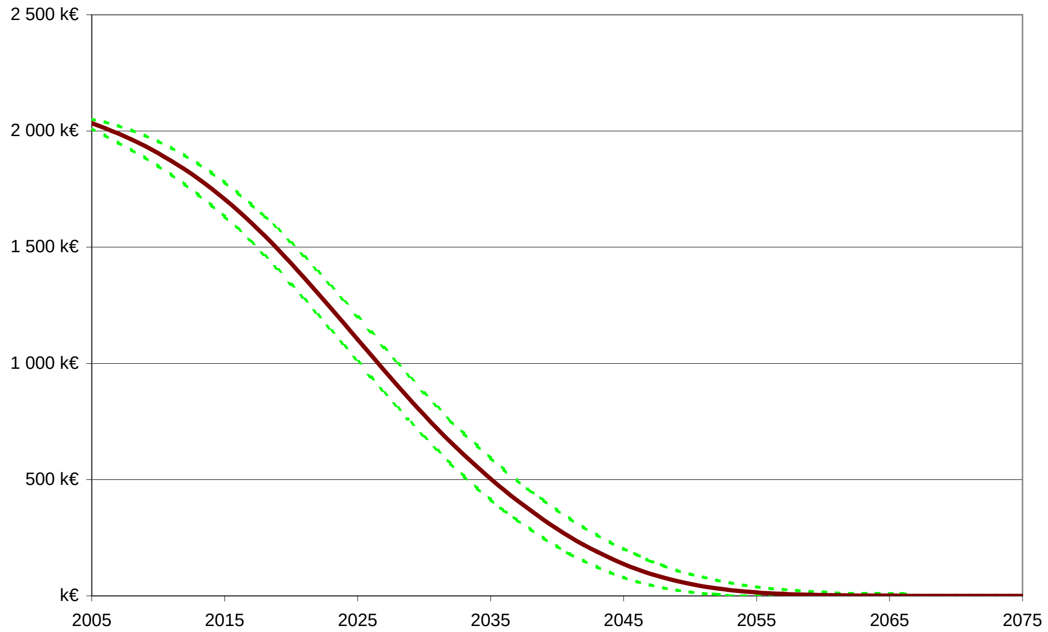
For example one can put:

$$\gamma_t \approx N\left(0, \sigma_\gamma^2\right)$$

This is a systematic risk.

Introduction – the framework

To quantify this risk, we consider a scheme of annuities according to the benefit amount year by year:



Introduction – the framework

The liabilities are easy to compute, and the stochastic variable of interest is:

$$\Lambda = \sum_{t=1}^{\infty} \tilde{F}_t (1+i)^{-t} = \sum_{t=1}^{\infty} \frac{1}{(1+i)^t} \sum_{j \in \mathbf{J}} r_j * \mathbf{1}_{]t;\infty[}\left(T_{x(j)}\right)$$

The best estimate is given by:

$$L_0 = E(\Lambda) = \sum_{t=1}^{\infty} F_t (1+i)^{-t}$$

Introduction – the framework

One can show that if you are using simple description of the noise about $(k_t, t \geq t_M)$ for example:

$$k_t^* = at + b + \gamma_t \qquad \gamma_t \approx N\left(0, \sigma_\gamma^2\right)$$

or the more sophisticated $k_t^* = \hat{a}t + \hat{b}$ with $\left(\hat{a}, \hat{b}\right)$ a gaussian vector, the systematic risk is very weak. So, the question is : **is it possible to build a model, coherent with the observations and that leads to significant part of systematic risk ?**

The model - description

Nevertheless, while in common approaches, this noise is assumed Gaussian or to follow an ARIMA process, we assume here that it is represented by a conditional model, which differentiates three proportions among the residuals of the adjustment of the empirical (k_t)

- the proportion p_- of the smallest residuals,
- the proportion p_+ of the largest residuals; and
- the $1 - p_- - p_+$ centered residuals, weak in absolute values.

The model - description

Considering the small amount of available data (about fifty observations) and due to the fact that we are interested here in big variations in comparison with tendency (weak deviation without practical impact), we fix arbitrarily

$$p_{-} = p_{+} = 0,43$$

Moreover, we suppose that the positive γ_t^+ deviation in relation to the tendency value follows a Pareto distribution with parameters (m_+, α_+). The same assumption is made for the negative deviation, with specific parameters.

The model - estimation

The Pareto distribution is $S_{m,\alpha}(x) = \left(\frac{x}{m}\right)^{-\alpha}$

The parameters as estimated by maximum likelihood

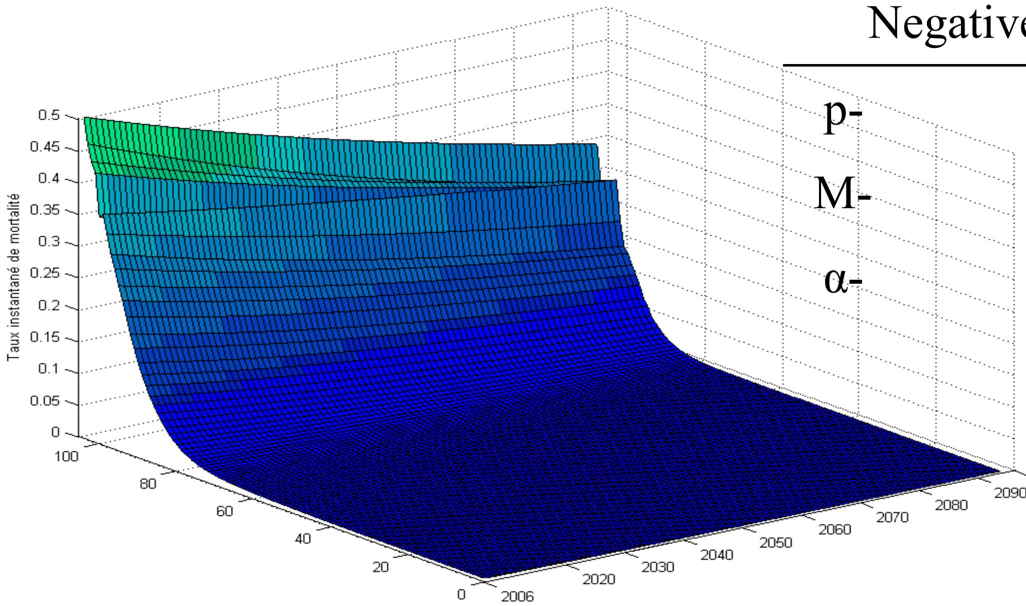
$$\hat{m}_+ = \mathbf{min}\left\{\gamma_{(i)}; i \geq n \times p_+\right\} \quad \hat{m}_- = -\mathbf{max}\left\{\gamma_{(i)}; i \leq n \times p_-\right\}$$

$$\hat{\alpha}_+ = \frac{\left[n \times p_+\right]}{\sum_{i=1}^{n_+} \mathbf{ln}\left(\frac{\gamma_i}{\hat{m}_+}\right)} \qquad \hat{\alpha}_- = \frac{\left[n \times p_-\right]}{\sum_{i=1}^{n_-} \mathbf{ln}\left(\frac{-\gamma_i}{\hat{m}_-}\right)}$$

NB : the tendancy is simply linear with least square estimation.

The model - description

With french mortality data (INED 1946-1996) we obtain:

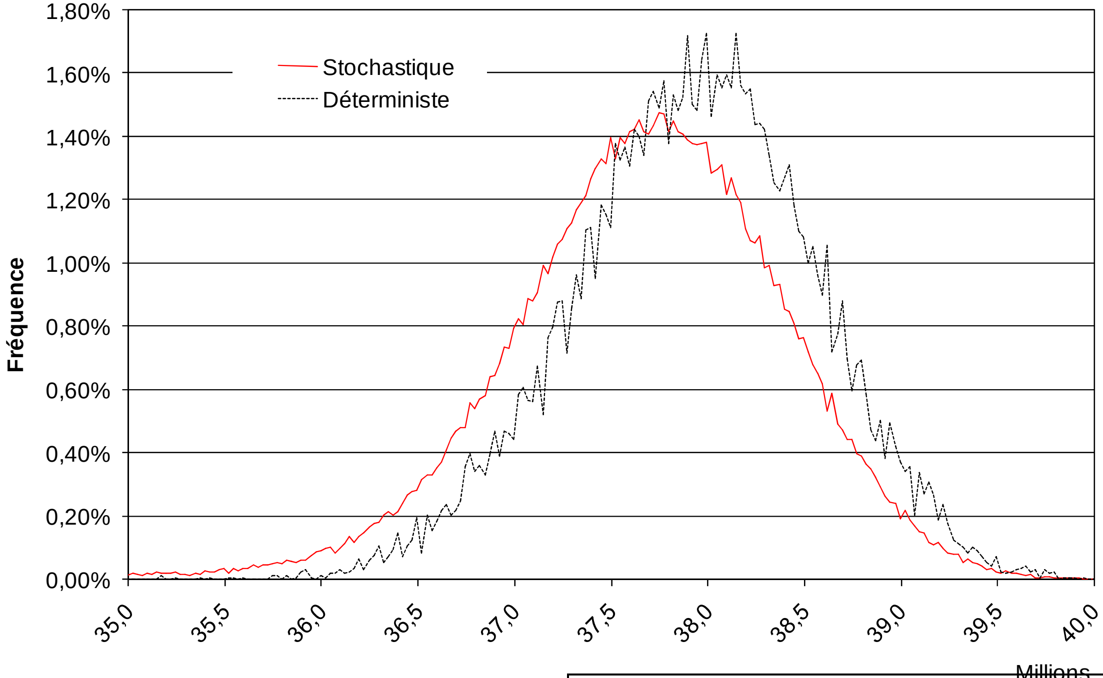


Negative Residuals		Positive Residuals	
p-	42.9 %	p+	42.9 %
M-	1.4385	M+	1.3878
α-	1.13	α+	1.1105

The model - Application to an annuity plan

In the following, for numerical applications we will use a portfolio consisting of 374 female annuitants with an average age of 63.8 at the end of the experiment. The annual mean income comes to €5.5 k. With the provision of bank rate of 2.5 %, initial policy reserve comes to €37.9 M with the determined prospective table supra.

The model - Application to an annuity plan



	Deterministic	Stochastic
Expectation	37 937 707	37 380 862
Standard Deviation	626 918	2 418 408
Lower Bound of Confidence Interval	36 625 000	34 295 073
Upper Bound of Confidence Interval	39 075 000	38 945 073
Coefficient of variation	1.65 %	6.47 %

The model - Application to an annuity plan

Even for a portfolio of small size, the impact of stochastic mortality on the structure of undertaking is important. We state two effects:

- the coefficient of variation of engagement is multiplied by 3 with respect to the situation of not taking account the systematic risk;
- the mean undertaking diminishes, due to the impact of shocks increasing with the decrement rates.

The model - Application to an annuity plan

We therefore compare the consequences in terms of risk management: the best estimate vision of undertaking is seen decreasing, but the presence of a systematic risk leads to calibrate a risk margin taking into account the strongest volatility. In total, it is not certain that the sum changes significantly, but its decomposition (in terms of expectation and risk margin) in prudent logic (Solvability 2) and accounting (IFRS insurance) is modified in a sensible way.

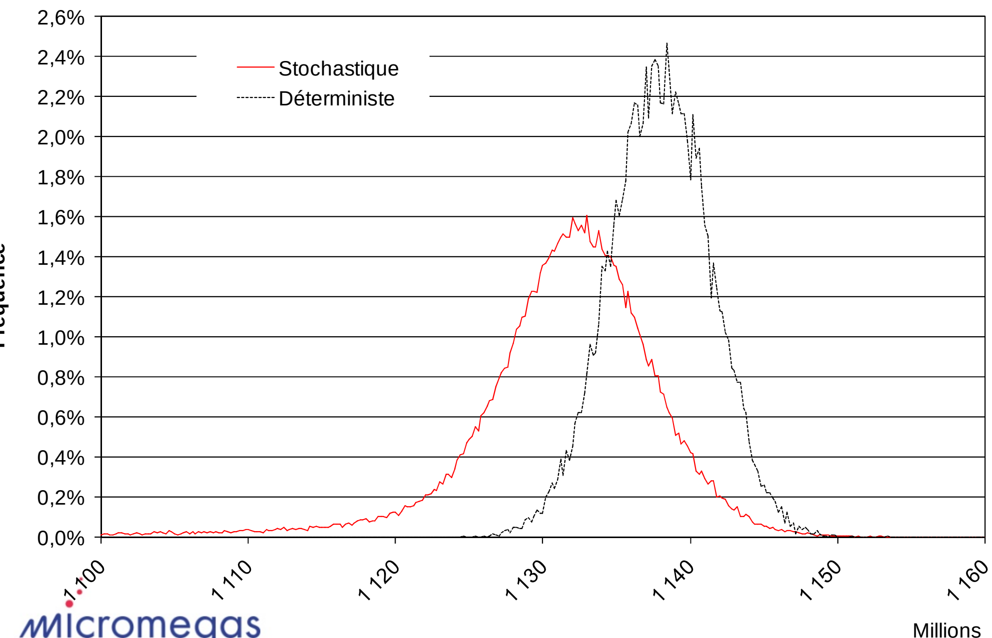
The model - Application to an annuity plan

However as show in Planchet et al. (2006) the size of the portfolio is an important parameter to take into account. In fact, if the absolute level of systematic risk does not depend on the size of the portfolio, it also does not depend on mutualisable risk. The part of variance explained by the stochastic component of mortality therefore increases with the size of the portfolio

The model - Application to an annuity plan

With size x30

	Deterministic	Stocha
Expectation	1 138 008 113	1 121 52
Standard deviation	5 592 212	69 93
Lower bound of confidence interval	1 131 130 658	1 032 81
Upper bound of confidence interval	1 144 480 658	1 141 81
Coefficient of variation	0,30 %	6



Conclusion

We propose a model used in calculating the reserve of life annuities, which inserts uncertainty explicitly in determining the long-term tendency through adjustment of this tendency.

If the level of reserve is not sensibly impacted by this change, the structure of reserve changes: the best estimate is seen again in decrease and risk margin in increase due to this systematic risk. This model seems to us in fact better considering the risk carried by the regime of income by allowing a more appropriate segmentation of the sum of undertaking between different risk sources.

References

- BROUHNS N., DENUIT M., VERMUNT J.K. [2002] « A Poisson log-bilinear regression approach to the construction of projected lifetables », *Insurance: Mathematics and Economics*, vol. 31, 373-393.
- LEE R.D., CARTER L. [1992] « Modelling and forecasting the time series of US mortality », *Journal of the American Statistical Association*, vol. 87, 659–671.
- LEE R.D. [2000] « The Lee–Carter method of forecasting mortality, with various extensions and applications », *North American Actuarial Journal*, vol. 4, 80–93.
- PLANCHET F., JUILLARD M., FAUCILLON L. [2006], « Quantification du risque systématique de mortalité pour un régime de rentes en cours de service », *Assurance et gestion des risques*, Vol. 75.
- PLANCHET F. [2007] « Prospective models of mortality with forced drift – Application to the longevity risk for life annuities », *Proceedings of the 11th IME Congress*
- PLANCHET F., THÉROND P.E. [2006] *Modèles de durée – applications actuarielles*, Paris : Economica.
- SITHOLE T., HABERMAN S., VERRALL R.J. [2000] « An investigation into parametric models for mortality projections, with applications to immediate annuitants and life office pensioners », *Insurance: Mathematics and Economics*, vol. 27, 285–312.

18TH International AFIR Colloquium

Istituto Italiano degli Attuari

Financial Risk in a Changing World

Rome, September 30th - October 3rd, 2008