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Uncertainty on Survival Probabilities and Solvency Capital Requirement

Application to Long-Term Care Insurance

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Monitoring the risks, Solvency II and ORSA

- The solvency is a meaningful process of accountability in which the insurer must monitor its activities.
- The ORSA is the set of processes that contribute to the regular assessment of the overall internal solvency of the company.
- The ORSA allows to shift from a logic of retrospective risks control to a logic of steering by monitoring the risks which incorporates the solvency.

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LTC, a particular life insurance business

- LTC is a mix of social and health care provided on a daily basis to people suffering from a loss of mobility and autonomy in their activity of daily living.
- LTC insurance contracts are individual or collective and guarantee the payment of a fixed allowance, in the form of monthly cash benefit, possibly proportional to the degree of dependency.
- See Kessler (2008) and Courbage and Roudaut (2011) for studies on the French LTC insurance market.

Actuarial publications on this topic

Most of the publications focus on :

- The construction of models of projected benefits, see Deléglise *et al.* (2009).
- The assessment of transition probabilities to model the life-history of LTC patients, see Gauzère *et al.* (1999), Czado and Rudolph (2002) and Helms *et al.* (2005).
- The construction of the survival distribution of LTC insurance policyholders, see Tomas and Planchet (2013).

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Uncertainty on Survival Probabilities and SCR

- We have no exogenous information about the LTC claimants, in terms of gender, place, or level of care.
- We observe only the aggregated exposition and number of deaths over two dimensions :
  - The age of occurrence of the claim.
  - The duration of the care.
- LTC claimants belong only to one state of severeness : heavy claimants.
- The pricing and reserving are very sensitive to the choice of the mortality table adopted.
- We analyze the consequences of an error of appreciation of the survival probabilities of LTC claimants in terms of level of reserves.
- We describe a framework to measure the gap between the risk profile from the standard formula to a risk analysis specific to the insurer.

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Notation I

We consider a LTC insured population of n individuals,

- with the same level of severity (heavy claimant),
- with durations of care u_i (in months), .
- and ages of occurrence of the pathology v_i with $i = 1, \dots, n$.

We note

- Λ the random amount of the valuation of the insurer, the sum of the future cash flows discounted with monthly rate r.
- F_t the total cash flows payable at month t, the sum of the n individual care c_i delivered over the month t.

Notation II

- $T_{u_i}(v_i)$, the remaining lifetime of an individual i when the pathology occurred at age v_i with the duration of the care u_i :

$$\begin{aligned}\mathbb{P}\left[T_{u_i}(v_i) > t\right] &= \mathbb{P}\left[T(v_i) > u_i + t \middle| T(v_i) > u_i\right] \\ &= {}_t p_{u_i}(v_i),\end{aligned}$$

- and

$$\begin{aligned}\mathbb{P}\left[T_{u_i}(v_i) \leq t\right] &= \mathbb{P}\left[T(v_i) \leq u_i + t \middle| T(v_i) > u_i\right] \\ &= {}_t q_{u_i}(v_i) = 1 - {}_t p_{u_i}(v_i).\end{aligned}$$

- The expectation of the remaining lifetime is

$$\mathbb{E}\left[T_u(v)\right] = \int_{t\geq 0} t d_t q_u(v) = \int_{t\geq 0} {}_t p_u(v) dt.$$

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Reserves valuation

- The total cash flows payable at month t has the form :

$$F_t = \sum_{i=1}^{n} c_i \ \mathbb{I}_{]t;+\infty[}\big(T_{u_i}(v_i)\big).$$

- The sum of total discounted cash flows is then :

$$\begin{aligned}\Lambda &= \sum_{t=1}^{+\infty} F_t \ (1+r)^{-t} = \sum_{t=1}^{+\infty} (1+r)^{-t} \sum_{i=1}^{n} c_i \ \mathbb{I}_{]t;+\infty[}\big(T_{u_i}(v_i)\big) \\ &= \sum_{i=1}^{n} c_i \sum_{t=1}^{+\infty} \frac{\mathbb{I}_{]t;+\infty[}\big(T_{u_i}(v_i)\big)}{(1+r)^t} = \sum_{i=1}^{n} c_i \ \Psi_i.\end{aligned}$$

- The reserve has a simple form :

$$\mathbb{E}[\Lambda] = \sum_{t=1}^{+\infty} (1+r)^{-t} \sum_{i=1}^{n} c_i \ \mathbb{P}[T_u(v) > t].$$

Risk of random fluctuations and table risk

When provisioning the amount $\mathbb{E}[\Lambda]$, the insurer faces adverse deviations due to two distinct factors :

- The random fluctuations of the observed mortality rates around the relevant expected values.
  - Consequence of the finite size of the population exposed to the risk.
  - Its financial impact decreases, in relative terms, as the portfolio size increases.
- The inaccuracy of the underlying survival law, from which the probability $\mathbb{P}\left[T_{u_i}(v_i) > t\right]$ are derived, is called the table risk.
  - Parameter risk
  - Any other sources leading to a misinterpretation of the life table (evolution of medical techniques or a change in rules of acceptance)
  - In relative terms, its severity does not reduce as the portfolio size increases, since deviations concern all the insureds in the same direction.

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Risk of random fluctuations I

Construct a confidence interval of sum of the total discounted cash flows around its expected value.

- Assumption of independence between individuals
- Assume that the individual cash flows Ψ_i are bounded by a constant
- The limit distribution of Λ is gaussian :

$$\frac{\Lambda - \mathbb{E}[\Lambda]}{\sigma_\Lambda} \xrightarrow[n\to+\infty]{} \mathcal{N}(0,1),$$

- Λ falls in the random interval with approximately $(1-\alpha)$ coverage probability,

$$\mathcal{I}_\Lambda = \mathbb{E}[\Lambda] \pm \Phi^{-1}_{1-\alpha/2} \times \sigma_\Lambda,$$

Risk of random fluctuations II

- To assess the two first moments of Λ, we only need to know the expectation and variance of Ψ, since

$$\mathbb{E}[\Lambda] = \sum_{i=1}^{n} c_i \, \mathbb{E}[\Psi] \quad \text{and} \quad \mathbb{V}[\Lambda] = \sum_{i=1}^{n} c_i^2 \, \mathbb{V}[\Psi], \tag{1}$$

- With a continuous expression of Ψ,

$$\psi = \int_0^{+\infty} \exp(-\tau \, t) \, \mathbb{I}_{]t;+\infty[} \, dt \quad \text{with} \quad \tau = \ln(1+r).$$

It leads to

$$\mathbb{E}[\psi] = \int_0^{+\infty} \exp(-\tau \, t) \, S_v(t) \, dt \approx \sum_{t \geq 1} \exp(-\tau \, t) \, S_v(t). \tag{2}$$

$$\begin{aligned} \mathbb{E}[\psi^2] &= \frac{2}{\tau} \int_0^{+\infty} (1 - \exp(-\tau \, t)) \exp(-\tau \, t) \, S_v(t) \, dt \\ &\approx \frac{2}{\tau} \sum_{t \geq 1} (1 - \exp(-\tau \, t)) \exp(-\tau \, t) \, S_v(t). \end{aligned} \tag{3}$$

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Table risk I

- Add a disturbance on the logits of the adjusted probabilities of death, see Planchet and Thérond (2011).
- $\widehat{q}_u(v)$ and $q_u^a(v)$ the adjusted and disturbed probability of death at duration u for the age of occurrence v respectively, then,

$$\text{logit } q_v^a(v) = \ln\left(\frac{q_u^a(v)}{1-q_u^a(v)}\right) = \ln\left(\frac{\widehat{q}_u(v)}{1-\widehat{q}_u(v)}\right) + \epsilon, \qquad (4)$$

where ϵ is a variable centered which we suppose to be gaussian in the following.

- Equivalently,

$$q_u^a(v) = \frac{a \times \exp\big(\text{logit } \widehat{q}_u(v)\big)}{1 + a \times \exp\big(\text{logit } \widehat{q}_u(v)\big)}, \quad \text{with} \quad a = \ln \epsilon. \qquad (5)$$

Table risk II

- The disturbance is controlled by the volatility of ϵ, noted σ_ϵ.
- We will vary σ_ϵ from 1 to 20 %.
- We measure the uncertainty on the expectation of the remaining lifetime in computing the relative difference, δ, between the expectation and the 95 % quantile of the simulated remaining lifetime :

$$\begin{aligned}\delta &= \frac{\rho_{95\%}(\mathbb{E}[T(v)\,|\,a]) - \mathbb{E}\left[\mathbb{E}[T(v)\,|\,a]\right]}{\mathbb{E}\left[\mathbb{E}[T(v)\,|\,a]\right]} \\ &= \frac{\rho_{95\%}(\mathbb{E}[T(v)\,|\,a]) - \mathbb{E}[T(v)]}{\mathbb{E}[T(v)]}. \end{aligned} \tag{6}$$

Content

Application to the SCR I

- Solvency II life risk module have been specified in QIS5, CEIOPS (2010, pp.147-163).
- It prescribes a SCR which accounts explicitly for the uncertainty arising from the systematic deviations and parameters estimation.
- Our aim : measure the relevancy of the shocks described in the QIS5 specification with the specific risks supported by the insurer in an ORSA perspective.
- Compute the ratio between the 99.5 % quantile of the distribution of Λ and its expectation.
- Compare to a reduction of 20 % on the adjusted probabilities of death in term of level of reserve.
- However, computing the quantiles of the distribution of Λ gives a biased evaluation of the SCR.
  - Does not take in account the limitation of the projection (computed to infinity)
  - The risk margin

Application to the SCR II

- Following Guibert *et al.* (2010, Section 3), we use the general approximation

$$\mathrm{SCR} = \frac{\dfrac{\mathrm{VaR}_{99.5\,\%}(\chi)}{\mathrm{BEL}_0} - 1}{1 - \alpha\, D_0 \left(\dfrac{\mathrm{VaR}_{99.5\,\%}(\chi)}{\mathrm{BEL}_0} - 1 \right)} \times \mathrm{BEL}_0$$

$$\text{with} \quad \chi = \frac{F_1 + \mathrm{BEL}_1}{1 + R_1},$$

where F_1 denotes the cash flows payable at month 1, R_1 is the return on assets at month 1 and D_0 the duration of the liability. The best estimate of the reserve at month 0 and 1 are denoted by BEL_0 and BEL_1 respectively.

Application to the SCR III

- The law of the variable χ can be reasonably approximated by the sum of the discounted cash flows, i.e. Λ, which is, conditionally to the disturbance, (approximatively) gaussian, see Guibert *et al.* (2010, Section 3.2) :

$$F_\Lambda(x) = \mathbb{P}[\Lambda \leq x] = \mathbb{E}\left[\mathbb{P}[\Lambda \leq x | a]\right] \xrightarrow[n \to +\infty]{} \int \Phi\left(\frac{x - \mu(a)}{\sigma(a)}\right) F_a(da).$$

- In practice, we approximate this function by Monte Carlo simulations on the basis of a sample of the variable a :

$$F_\Lambda(x) \approx F_K(x) = \frac{1}{K}\sum_{k=1}^{K} \Phi\left(\frac{x - \mu(a_k)}{\sigma(a_k)}\right).$$

- Then, a quantile ρ is derived by solving the equation $F_K(x_\rho) = \rho$.

Application to the SCR IV

- The moments of Λ are derived in expression (1). If we consider that the monthly care costs 1 and a zero discount rate, it leads, for a portfolio of n LTC claimants to

$$\mu_a = \mathbb{E}[\Lambda|a] = n \times \mathbb{E}[\Psi|a] \text{ and } \sigma_a = \sigma_{\Lambda\,|\,a} = \sqrt{n \times \mathbb{V}[\Psi|a]},$$

$$\mathbb{E}[\Psi|a] \approx \sum_{t\geq 1} S_v(t\,|\,a)$$

$$\mathbb{V}[\Psi|a] \approx 2 \sum_{t\geq 1} t\; S_v(t\,|\,a) - (S_v(t\,|\,a))^2,$$

following expressions (2) and (3).

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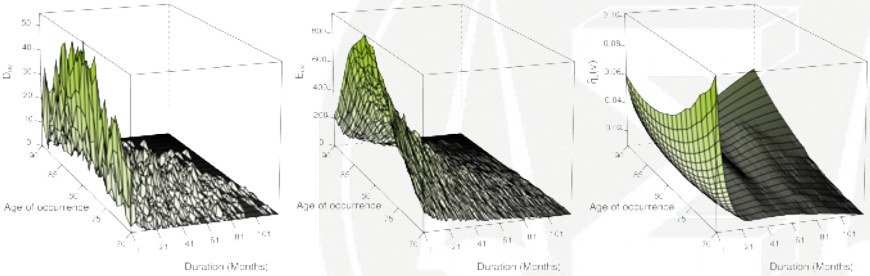
The data I

- Come from a portfolio of LTC heavy claimants of a French insurance company.
- The period of observation stretches from 01/01/1998 to 31/12/2010.
- Composed of a mixture of pathologies, among others, by dementia, neurological illness and terminal cancer.
- Consist for 2/3 of women and 1/3 of men.
- The computations are carried out with the help of the software R, R Development Core Team (2013).

The data II



(a) Number of deaths. (b) Exposure. (c) $\widehat{q}_u(v)$.

FIGURE: Number of death, exposure and $\widehat{q}_u(v)$ obtained in Tomas and Planchet (2013, Section 3.2).

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Uncertainty on the remaining life expectancy I

- We then apply model (5) and vary σ_ϵ from 1 to 20 %.
- The remaining life expectancy varies slightly with σ_ϵ, being around 41.4 months when the pathology occurred at age 80.
- We then measure the impact of uncertainty on the expected lifetime by computing δ, equation (6).
- From experts opinion, the remaining lifetime of LTC claimants when the pathology occurred at age 80 varies by 12 %.
- With a level of volatility $\sigma_\epsilon = 9\,\%$, the resulting uncertainty δ is approximatively 12 % when the pathology occurred at age 80.

Uncertainty on the remaining life expectancy II

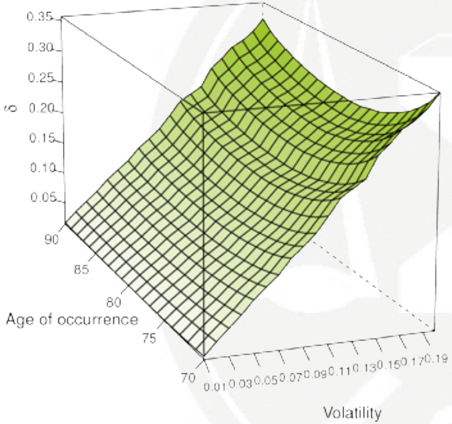

FIGURE: Relative difference δ between 95 % quantile of the simulated remaining lifetime and its expectation.

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Risk profile from the standard formula and risk analysis specific to the organism I

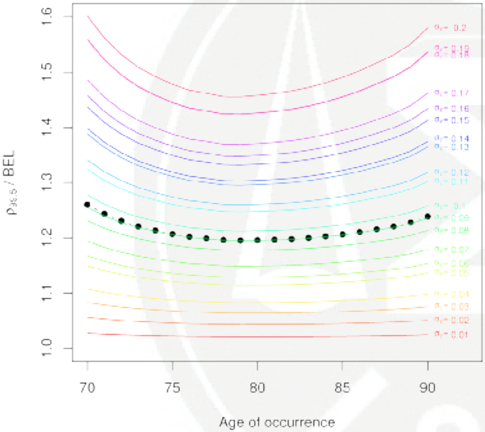

FIGURE: QIS5 disability / morbidity shock (black dotted line) and ratio between the 99.5 % quantile of the the distribution of Λ and its expectation (color lines).

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Ratio SCR and best estimate of the reserve as a function of the portfolio size I

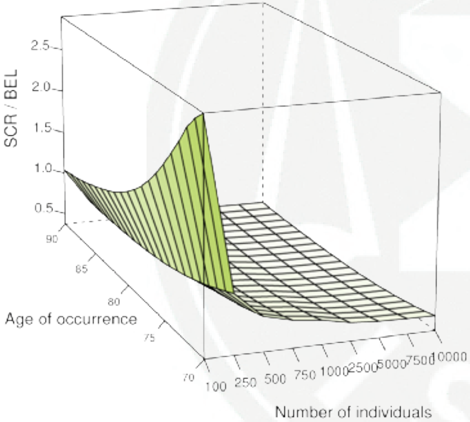

FIGURE: Ratio between the SCR and the best estimate of the reserve as a function of the portfolio size, with σ_ϵ of 9 % and α of 6 %.

Conclusions I

- The uncertainty associated to the underlying survival law has important consequences in terms of volatility of reserves.
- The general framework described in Guibert *et al.* (2010) allows us to assess the adequacy of the standard shocks described in the QIS5 specifications with a risk analysis specific to the insurer depending on the structure of the portfolio.
- We can then build a stochastic model taking into account the constraint of quantifying the uncertainty in a finite horizon and the effect of the risk margin.

Conclusions II

- This highlights the essential assessment of uncertainty associated to the underlying survival law as the milestone for a thorough evaluation of the insurer solvency.
- It leads to consider the implementation of a partial internal model for the underwriting risk.
- The approach presented shows that the level of underwriting SCR obtained is strongly associated to the precision of the assessment of the underlying survival law and in particular to the tail of distribution.

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