Pay-As-You-Go—A Relict from the Past or a Promise for the Future?

A contribution to the Hannes Androsch Prize 2011 on
"The Design of a Social Security System Which Can Withstand the Dual Threat of Demographic and Financial Market Risk."

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Preface

The global economic crisis that started in 2007 has uncovered the fragility of the financial architecture and has caused a massive worldwide increase in government debt. The necessity to reduce public expenditures has in turn lead to various proposals to reform the pension system which has been accompanied by intensive public debates and also—in some countries—by protests, demonstrations and strikes. In fact, such events have been familiar companions of most pension reform discussions, before and after the crisis. It is likely, that the topic of how to design and restructure pension systems will not disappear in the near future—not least because the ongoing demographic developments will ensure an ongoing debate.

Against this background the Hannes Androsch Prize 2011 has asked a timely and important question: How to design a "social security system which can withstand the dual threat of demographic and financial market risk"? This is — without doubt — a big question and in answering it in a short monograph one faces a number of Scyllan and Charybdian choices. The analysis must strike a balance between being abstract (high-in-the-sky) and concrete (down-do-earth), between dealing with general principles of ideal and with details of existing pension systems, between an international perspective and the close look at the situation in one country, between focusing on a few aspects of the topic and dealing with a wide range of issues in a rather loose way, between giving an overview of the current state of knowledge and offering novel results and insights. I have tried my best to navigate through these antagonistic cliffs. I will provide principled reflections but also discuss a number of actual policy proposals and while my main emphasis will be on the presentation of new and hitherto unpublished results I am also going to embed them in and relate them to the existing literature.

If I had to summarize the contents of the following pages in one sentence then this would be: "Pay-as-you-go (PAYG) systems are better than their reputation." Despite all their well-known (and less well-known) deficiencies PAYG system are capable to deal with demographic and financial market risk in an appropriate (and sometimes even congenial) manner. This is not to say that PAYG schemes are perfect, neither in their ideal, text-book form nor as implemented in the real-world. There is always room for improvement and some problematic properties will remain even for the best of all designs. In this respect the PAYG system is equal to its major antagonist—the fully funded system—that has also negative features that cannot be completely eliminated by regulation. In a good part of this monograph I will in fact focus on a direct comparison between funded and unfunded (i.e. PAYG) systems. This will involve a discussion of some claims that are often put forward in support of the funded system: that it is associated with a higher rate of return and that it is more robust to demographic changes. These claims are, however, either completely wrong or not true in the stated generality as will be shown in this book. I do not want to picture this comparison in a black-and-white fashion, as the fight between the forces of light and darkness. The analysis will show that there exist good arguments for a "mixed system" that contains both, a funded and an unfunded pillar. As the analyses also show, however, the two pillar should not be mixed in equal proportions but the main part should be reserved for a reasonably designed PAYG system as this is better able to cope with the two-sided challenge of demographic and financial market risk. In the later parts of the monograph I will also deal with the question how to shape the details of a PAYG system such that it is in line with generally accepted notions of intergenerational fairness and I will discuss various important design choices that arise in the context of the currently most fashionable variety of PAYG systems: the notional defined contribution (NDC) plan.

The Hannes-Androsch-Prize has also asked the submissions to "provide proposals for an alternative design, which would optimise the magnitude and stability of pensions over time, and confront the double challenge of demographic developments and financial market risk." My answer to this task is that for a sufficiently crisis-proof system one does not need to come up with a completely new approach but one can rely on a framework with a strong and an appropriately designed PAYG pillar. In fact, if PAYG did not exist, one had to invent it since it is an integral component to meet the desired requirements. This does not imply that there is a ready-made design available that is suitable for every country under all circumstances. A viable pension system has to respect the national and historic contingencies in the same way as it has to adapt to the permanent changes in its environment. In order to be promising and effective these accommodations should, however, be guided by an understanding of the main mechanisms at work and the main principles involved. The current monograph aims at contributing to this knowledge.

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Chapter 1

Introduction

The two-sided challenge

Pension systems worldwide face a double-sided challenge. On the one hand they have to deal with demographic fluctuations and the ubiquitous process of aging, on the other hand they have to cushion various macroeconomic and financial market shocks.

The demographic challenge itself has also two dimensions—people get older and they have fewer children. In many industrialized countries the baby boom of the 60ies was followed by a decrease in birth rates in the 70ies and the total fertility rate is now at a level (1.52 for the EU-27) that is clearly below the replacement level. This process is accompanied by a steady increase in longevity. For the EU-countries, e.g., life expectancy at birth is projected to increase over the next 50 years by about 8.5 years (for men) and 6.9 years for women. Taken together, these demographic trends are forecasted to lead to a considerable increase in the old-age dependency ratio, for the EU from 25.4% in 2008 to 53.5% in 2050 (all data from Economic Policy Committee and European Commission 2009).

For individuals these developments are no issues of concern. The decrease in mortality is a source of pleasure and the fluctuating fertility rates a consequence of their personal choices. But for the societies at large the two-fold demographic development can have severe effects on their labor, goods and capital markets (cf. Bloom & Canning 2004, Poterba 2004) and in a crucial way on their pension systems, in particular when they are organized on a Pay-As-You-Go (PAYG) basis. Some people have argued that these systems are not apt to deal with the demographic changes and that one should either reduce their scope or abolish them altogether and move to funded systems.

The funded systems themselves, however, are also not without problems and they are particularly exposed to the second challenge—financial market risk. The huge volatility of asset returns has become quite salient in the course of the financial crisis: "The financial crisis reminds us [...] that the pensions payable under a private investment account system are highly variable from one year to the next unless workers invest in a very conservative portfolio" (Burtless 2010, p. 324). The OECD has analyzed the performance of private pensions during the crisis and has concluded that "private pension funds have been dealt a heavy blow: in the calendar year 2008, their investments lost 23% of their value on aggregate, or some USD 5.4 trillion" (OECD 2009, p. 25).

This brief portrait of the state of affairs already suggests that it is impossible to organize the provision for old age in such a way that it is immune against all kinds of shocks. What is also evident, however, is that different pension frameworks are susceptible to different types of risk. The PAYG system is commonly said to be more pressured by unfavorable demographic developments while the funded system is particularly exposed to financial market risk. Given that there is no panacean solution it seem suggestive to choose a mix between the two main contestants. This corresponds, in fact, to the famous multipillar proposal that was laid out by the World Bank in its report from 1994 on *Averting the Old Age Crisis*. As attractive as this solution to the two-sided challenge might seem it only elevates the problem to a higher level since the mere concept of multiple pillars does not reveal anything about the right proportion of mixing and does not answer the crucial question how large the unfunded pension pillar should be in relation to the funded pillar. The equal sharing rule would suggest a mixing of 50 : 50 but nothing guarantees that what is right to deal with children's quarrels over a toy is also the correct approach to decide how to organize a mature pension system. In the latter case a host of different considerations—based on concepts of portfolio theory, risk-sharing and fairness—come into play. This multidimensionality of perspectives under which the choice between (and the mixing of) a funded and an unfunded pension system can be viewed might also be responsible for the diverse answers and heated debates that are present in the academic literature and in the public arena. One can find brave defenders of the traditional Bismarckian PAYG system, fervent advocates of a complete transition to a funded system and supporters of various kinds of mixed or multiplillar systems with different recipes to combine the ingredients. Often one gets the impression that the opinions on these matters are highly entrenched and it is quite common to have one side of the debate accuse the other of misinterpretation (of evidence), misunderstanding (of facts) and misguidance (of policymakers or the public). It seems to be no coincidence that there is a long list of publications that explicitly sets out to debunk the (alleged) myths of the other side, might they be supporters of the funded or the unfunded paradigm (e.g. World Bank 1994, Orszag & Stiglitz 1999, Barr 2000, Breyer 2000). One person's truth seems to be another person's myth. Given these rather fixed positions it comes as no surprise that even the global financial crisis seems to have no strong effect on the opinions of the supporters of a dominant funded pillar and make them reconsider their views.[1]

[1] E.g.: "Die Finanz-und Wirtschaftskrise 2008/09 gibt keinen Anlass zu einer grundsätzlichen Neuordnung des [...] Mehrsäulenmodells der deutschen Altersvorsorge. [... Sie hat] an den beiden zentralen Gründen, die für einen Teilumstieg von Umlage- zum Kapitaldeckungsverfahren sprechen, nichts geändert: erstens der höhere Kapitalstock, der in der langen Frist ein höheres Bruttosozialprodukt ermöglicht, weil die steuerähnliche Belastung der Arbeitnehmer sinkt und die Gesamtfaktorproduktivität steigt; zweitens die gleichmäßigere Belastung der Generationen angesichts des dramatischen demographischen Wandels" (Börsch-Supan & Gasche 2010, p. 2, p. 15). Also Kotlikoff (2010) in his most recent book on the financial crisis sees no reason to change his blueprint for a pension reform as laid out in his drastic *The Coming*

In view of this rather antagonistic situation this monograph sets out to take a fresh look at the question whether the funded or the unfunded pillar should be the major fundament of a sustainable and reasonable pension system for the future. In order to guide and organize my discussions I will try to follow closely the question that has been asked in the announcement of the *Hannes Androsch Prize*: which system is more promising to be able to "withstand the dual threat of demographic and financial market risk"? The literature on these topics is vast and it is impossible to treat it in full depth. Instead of giving an encompassing overview of the literature I will focus on the crucial issues related to the prize-question.[2] Rather than presenting a mere summary of established results my main aim is to contribute novel ideas and insights to the existing stock of knowledge. Although the results are meant to reveal some basic truths about the working of pension systems, I am not going to give a definite answer to how to organize a perfect pension system in every detail. In fact, such a blueprint very much depends on the peculiarities of the time and the place and an analytical treatise can only aspire to offer some general guiding principles. Furthermore, the choice of old-age provision is not a mere technical issue but also involves personal or societal judgments that have to do with the weighting of evidence and the prioritizing of objectives and values.

Objectives of pension systems

Before coming to a detailed summary of the structure of this monograph it is useful to take up the last point and recall the main objectives of pension systems. Most institutions and pension experts single out two primary objectives: the smoothing of lifetime consumption and the prevention of old-age poverty. This is, e.g., summarized by the OECD:

> "Pension policy-making involves balancing two objectives. The first is to provide adequate levels of retirement incomes to ensure that people are not at risk of poverty in their old age. [...] The second objective is to ensure that pension incomes do not depart from the living standards individuals achieved during their working lives" OECD (2009, p. 55).

Similar views about the main objectives can also be found in publications of the World Bank, the European Commission and also by individual pension experts.[3] The attrac-

Generational Storm that is built around a complete transition to a funded system.

[2] At the end of each chapter I will present a small section with related literature. This is not meant to offer a complete survey of the field but just as a direction for further reading and to closely related papers.

[3] "The main objectives of pension systems [are] poverty alleviation and consumption smoothing" (Holzmann & Hinz 2005, p. 1); "[The three main objectives of pension programs are] poverty relief, con-

tiveness of funded and unfunded system differs according to the weight one puts on the two objectives. A high-earning, high-skill individual will assign a small probability to falling into poverty in old age and attach a greater weight to a high-return (and high-risk) pension provision. An already retired individual, on the other hand, will be very much in favor of a system that guarantees a stable income stream. The design of a pension system must take these different preferences and objectives into account. "Pension systems have multiple objectives, including consumption smoothing, insurance, poverty relief, and redistribution, all of which cannot be fully achieved at the same time. Thus policy has to optimize—not minimize or maximize—across a range of objectives" (Barr & Diamond 2009, p. 23).

In addition to these main objectives one can also go much more into detail and list a large number of additional goals. The OECD, e.g., in the same study that has been quoted above, mentions six objectives of retirement income provision:

> "Coverage of the pension system, by both mandatory and voluntary schemes; adequacy of retirement benefits; financial sustainability and affordability of pensions to taxpayers and contributors; economic efficiency: minimising the distortions of the retirement-income system on individuals' economic behaviour, such as labour supply and savings outside of pension plans; administrative efficiency: keeping the cost of collecting contributions, paying benefits and (where necessary) managing investments as low as possible; and security of benefits in the face of different risks and uncertainties" OECD (2009, p. 85).[4]

Even here one important aspect is missing: the pension system should be widely accepted by the population. And this will only be the case if the population thinks that the system gets the weighting of objectives more or less right and if it is in line with its views about an intergenerationally fair system.

Structure of the monograph

In the following I want to give a short outline of the structure of this monograph and a summary of the main results.

In chapters 2 to 4 I am going to present a direct comparison of the funded and unfunded system. I organize the analysis around the two demographic challenges—demographic risk

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sumption smoothing, and insurance (the last in respect, for example, of the longevity risk)" (Barr 2000, p. 39).

[4]A similar list of primary and secondary objectives of pension system can be found in chapter 2 of Barr & Diamond (2009)

and financial market risk—that have marked the discussions and that have also been singled out in the announcement of the *Hannes Androsch Prize*. In the course of my inquiry I will frequently refer to the different objectives of pension systems mentioned above and I will deal with questions of sustainability, consumption smoothing, adequacy and poverty-prevention, implementability and intergenerational fairness.[5] The starting point of my analysis is a very strong claim about PAYG systems that can be often heard in discussions how pension systems should deal with the current challenges. In particular, many people argue that the PAYG system is not capable to deal with the two-fold demographic pressure: the fluctuations in fertility rates and the steady increase in life expectancy.

This claim comes in two variants. In a "strong" form it is asserted that the PAYG system is simply not able to cope with the demographic fluctuations and that it is impossible to reform it in a way that its budgetary sustainability is guaranteed. I will refute this claim in *chapter 2* where I will show that there exist straightforward methods to make a PAYG pension system restistent to the two-sided economic challenge. Contrary to the strong claim about the inevitable collapse of the PAYG scheme I will show that there exists a variety of modifications and demographic adjustment factors that are simple and transparent enough to be actually incorporated into existing PAYG systems. In particular, I will show there that a combination of a "sustainability factor" and a "life expectancy factor" can be implemented in order to guarantee that the system is balanced in every period. The "life expectancy factor" stipulates that increases in life expectancy should be countered with proportional increases in the retirement age while fluctuations in cohort size and pari passu in the dependency ratio could trigger the "sustainability factor" that regulates how to change the contribution rate and the pension level in order to cushion these shocks. I will discuss the working of these factors in detail and I will also show how to react if there are simultaneous changes along both demographic dimensions.

There exists, however, also a second variant of the argument about PAYG systems and demographic fluctuations which is the topic of *chapter 3*. In this "weak" form it is argued that even if a PAYG can be immunized against demographic fluctuations it is more "susceptible" than a funded system and will lead to much larger and more chaotic adjustment processes. A funded system, on the other hand, is able to smooth the necessary adjustments. In chapter 3 I will show that also the "weak form claim" has to be

[5] Some chapters of the monograph are based on my work in this field over the last years: chapters 2 and 3 on Knell (2010*a*), chapter 4 on Knell (2010*b*) and chapter 6 on Knell (2005) and Knell et al. (2006). The content of the chapters, however, typically goes much beyond the results of these articles and I often vary and considerably extent the original works.

dismissed. How a funded system reacts to fluctuations in cohort sizes depends on many elements, including the availability of assets, the assumptions about individual behavior and the social structure of society. Under some (not implausible) assumptions I can show that the intergenerational effects of demographic shocks (as measured by the distribution of the internal rate of return) are almost completely identical for unfunded and funded systems. And even for more general assumptions the adjustment process and the intergenerational sharing of the adjustment burden is fairly similar. Many existing papers on this topic do not mention the ambivalence of the results since at the outset they make the "passepartout" assumption of "using a standard model". I show that a slightly more sophisticated and fractured model (with heterogeneous agents, different types of assets and with the possibility of non-optimizing behavior) gives rise to more subtle results where a superiority of the funded system to deal with demographic fluctuations cannot be taken for granted.

In *chapter 4* I will turn to the second challenge for pension systems: financial market risk. This is an issue that is particularly important with respect to poverty prevention and the provision of an adequate level old-age income. I will show that a strong PAYG pillar also recommends itself in order to deal with financial risk. The main reason for this result is that a properly designed PAYG system leads to smaller fluctuations both in the *absolute* income of pensioners and in their *relative income*. Since poverty is a relative concept this implies that a PAYG system will be better able to guarantee old-age incomes above the poverty line for most retirees. I use empirical data on the risk-return profile of funded and unfunded systems in order to get a rough estimate for the "optimal mix" between the two pillars. Depending on the assumptions about individual risk aversion and about the importance of relative standing the optimal size of the funded pillar comes out rather small—typically below 20% and often below 10%. This suggests that a reasonable pension system should consist of a large PAYG system and a small funded pillar—exactly the opposite of what is often suggested in public debates.

In *chapter 5* I look at the choice between different pension systems form yet another angle that is often not at the core of the analysis: intergenerational fairness. Ultimately, only a system that is regarded as more or less fair will also endure and withstand known and—for the moment—unknown risks and challenges. Despite the fact that in public discussions one can often hear arguments that refer to notions of fairness there does not exist a broad literature that deals with the issue of intergenerational fairness and pension systems in a systematic way. I discuss this topic by extending the four principles of fairness that can be found in the literature ("exogenous rights", "compensation", "reward" and "fitness") to the intergenerational context. I will show, e.g., that the use of the "exogenous rights" or "fitness" principles suggests the use of a PAYG variant that is more in between a defined contribution and a defined benefit system while the emphasis on the "reward" principle implies the defined contribution scheme as the appropriate scheme. The latter conclusion is based on the observation that the increase in the internal rates of return that follows a surge in fertility is more highly concentrated among the potential parent generations of these large cohorts if one uses a defined contribution scheme. I also show that a constantly increasing life expectancy gives rise to a second "biological interest rate" that is completely analogous to the first "biological interest rate" related to increases in population growth. The use of a life expectancy factor will distribute these extra returns more evenly across generations than the use of changing contribution rates or pension levels.

For the investigations of chapters 2 to 5 I use rather stylized PAYG systems that suffice, however, to make the main arguments about adjustment factors, sustainability and intergenerational distribution. In fact, if a pension scheme shows strange properties in these artificial worlds one cannot expect that it will manage actual demographic changes in a reasonable manner. In *chapter 6* I focus more thoroughly on the working, the design and the implementation of real-world PAYG systems. The center of attention in this chapter is the notional defined contribution (NDC) system since it offers a useful framework to discuss various principles and open issues of pension design and also because it has become quite prominent over the recent years. Most of the design principles of NDC systems are well known, but others are still not firmly established and I will mainly concentrate on the latter ones. In particular, I will discuss the appropriate choice of the notional interest rate, the right concept to determine the annuitization at the beginning of the retirement span and the accurate regulation of reductions or supplements for early or late retirement. It will be shown that the growth rate of the wage sum is a better choice for the notional interest rate although it will only lead to a balanced budget over time and not in every period of time (as is the case for the use of the "sustainability factor"). Furthermore, both methods of calculating the annuity (the use of historic life tables and the use of forecasted data) will lead to an unbalanced budget, even in present value terms. The first method is too "generous" causing persistent deficits while the second method is too "harsh" leading to ongoing surpluses and it might thus be reasonable to use a mixture of both concepts. Finally, I document that the standard "actuarially fair" determination of reductions and supplements is only viable if the *average* retirement age stays constant or if the notional interest rate is related to the wage sum.

Overall, the NDC system provides a workable and transparent organization of a PAYG system. It is, however, neither a ready-made solution with a clear and immovable structure nor the only framework to design a sustainable, accepted and viable PAYG system. In fact, one can observe a large variety of (PAYG) pension systems where the best exemplars are built on similar ideas and principles while using a different vocabulary and sometimes different dialects in specifying the details. The intention of this monograph is to foster the understanding of the basic grammar of successful PAYG schemes and to show how they can be further adapted in order to make them robust to the current challenges and to future shocks and crises.

Chapter 2

How to Make PAYG Pension Systems Stable Despite Demographic Changes

2.1 Introduction

As discussed in chapter 1 the frequent reference to the "demography challenge" entails two concerns: the long-run trends and short-run fluctuations in fertility and the steady increase in life expectancy. In order to make a PAYG pension system sustainable in the presence of these two-sided demographic challenge it is, however, important to base the proposals on the correct assumptions about the present and future progress of these developments. In particular, it matters whether one thinks of them as stationary or constantly changing (i.e. constantly increasing or decreasing) magnitudes or as processes that are characterized by discontinuous developments (e.g. fluctuations around a constant level or one-time changes). All of these patterns are in principle possible but the choice of assumptions affects the feasibility of adjustment strategies and also their intergenerational impact. It is, e.g., highly controversial whether population growth (or shrinkage) and life expectancy growth should be modeled as a constantly ongoing process. Isn't there an upper limit of people this planet is able to support? And isn't there a biological fixed maximum age a human can reach? Even if one answers these rhetorical questions in the affirmative the use of the assumption of constant growth rates might still be defended as being a short-cut for the most probable developments for the upcoming years and decades (and maybe even centuries).

In order to clearly separate these two demographic trends and to carve out the workings of the different automatic adjustment mechanisms I will use in the following a step-by-step approach. After presenting the general set-up of the framework that is used to model the PAYG system I will first deal with the case where life expectancy is constant and where only the cohort size fluctuates. Then I will turn things around and I will deal with situations where life expectancy increases while the cohort size is fixed. And only in the last part I will combine both processes and talk about situations where one faces changes along both demographic dimensions.

I will present some simple adjustment factors that can do the trick. In particular, it will be shown that one can use a "sustainability factor" (that adjusts the contribution rate and the pension payment in accordance with the development of the dependency ratio) and a "life expectancy factor" (that adjusts the retirement age in lockstep with increases in life expectancy at birth). A combination of these two adjustment factors leads to a constellation where the budget is balanced in every period.[1] These factors are not the

[1] This is also a consequence of the deterministic set-up, while for a stochastic set-up the budget will typically only be balanced over time (or in expected value).

only mechanisms that are able to guarantee a balanced budget. In fact, there exist real-world PAYG systems that are based on different automatic adjustment mechanisms and that also imply a balanced budget (at least over the medium turn; see chapter 6). The adjustment factors discussed in this chapter have been chosen because they are intuitive and they suffice to refute the claim that PAYG systems are inherently unstable. I do not suggest that these factors are the most advantageous and favorable ones, either from the perspective of risk-sharing or intergenerational fairness or from a political economy point-of-view. The latter is related to the question of how easy it is to administer, understand, communicate and adapt the system.

2.2 General framework

For the most part of this book I use a set-up in discrete time. Only for the case of changing life expectancy I will switch to a model in continuous time (since in this case one has to deal with non-integer values of time periods). Furthermore, in this and in the next chapter I am going to work within an entirely deterministic framework. The developments of cohort sizes and of life expectancy are assumed to follow non-stochastic processes that are known by all agents (including the government). This assumption is not completely absurd since the observed increase in average life expectancy follows a rather constant trend and an exceptionally large or small cohort is visible at least some decades in advance before it enters first the labor market and later the ranks of retired people. The use of a deterministic framework allows to focus on the pure mechanics of PAYG pension systems and to carve out the workings and the main properties of the proposed adjustment factors.

2.2.1 Households

I use a multi-period OLG model for this as for most of the other chapters. This is important since results that are derived in a two-period model do not always carry over to the multiperiod context (cf. e.g. Lindbeck & Persson 2003, p. 87). In fact, despite its prominence in the related literature the two-period model is quite special since it assumes that work and retirement are of the same length. Furthermore, in the two-period-case there are no generational overlaps in the working and in the pension period and accordingly also no possible counteracting effects of specific cohort sizes. The multiperiod model offers a more complex and more complete picture that is intimately related and immediately comparable to real-world situations and can be used to analyze specific developments like boom and bust cycles.

In particular, I assume that there is a large mass of worker-households that are organized as multi-period, overlapping generations. The cohort born in period t has a life expectancy of Y_t years and works for X_t periods after which it enters retirement. The size of each cohort is given by N_t. The three variables N_t, Y_t and X_t can change over time but all are assumed to be exogenously given and independent of any household decisions. In particular, this means that I abstract from endogenous labor supply, fertility and retirement decisions and also from investments into health care. I do this in order to focus on the crucial topic of this chapter—the design of a PAYG system that is immune to demographic fluctuations. Individuals earn a wage W_{t+a-1} during each of their working periods ($1 \leq a \leq X_t$) and they receive a pension payment P_{t+a-1} in each period of retirement ($X_t + 1 \leq a \leq Y_t$).[2] Wages grow at a constant rate g, i.e. $W_t = (1 + g)W_{t-1}$. During their working life households pay contributions to the PAYG system at rate τ_t. In this chapter I will only study the workings of the PAYG system and I will take the development of the factor prices as given. In chapter 3, however, I will present a complete microfounded model with a detailed specification of utility and production functions and endogenous factor prices. This model will include a PAYG system along the lines of this chapter as one building block.

2.2.2 The budget of the pension system

In order to be able to distinguish clearly between the viewpoint of generation t (i.e. the one born in t) and the outlook of the pension system in period t I introduce two further variables. E_t denotes the number of working years of the generation that retires in period t and F_t stands for the highest age observed in this period. Cross-sectional life expectancy (or life expectancy at birth) for members of generation t is thus given by F_t while their *forecasted* (or longitudinal) life expectancy is equal to Y_t. Note that in general it will hold that $X_t \neq E_t$ and $Y_t \neq F_t$.

Furthermore, I assume that life expectancy is non-decreasing and that if generation t works in some period then also all generations that are younger than t work as well. This allows to write the total number of workers L_t and the total number of retired persons R_t

[2] I thus assume that there is no seniority structure, i.e. at a certain point in time, all workers and all pensioners receive the same payment.

at time t as:

$$L_t = \sum_{a=1}^{E_t} N_{t-a+1} \tag{2.1}$$

$$R_t = \sum_{a=E_t+1}^{F_t} N_{t-a+1} \tag{2.2}$$

The dependency ratio z_t is defined as the quotient of pensioners to workers:

$$z_t = \frac{R_t}{L_t}, \tag{2.3}$$

while the (relative) pension level q_t is specified as:

$$q_t = \frac{P_t}{W_t}. \tag{2.4}$$

The pension level thus indicates which fraction of average income the representative member of a generation receives in a given year of his or her retirement.

Finally, I assume that the budget of the pension system is balanced in every period. The income of the pension system in time t is given by $W_t \tau_t L_t$ while the total expenditures are given by $P_t R_t = W_t q_t R_t$. Thus the balanced budget condition (BBC) can be written as:

$$\tau_t L_t = q_t R_t. \tag{2.5}$$

A "demographic steady-state" is defined as a situation where all demographic variables are constant and where also the retirement age is fixed, i.e. $N_t = N, Y_t = Y, X_t = X$, for all t. From (2.1), (2.2) and (2.5) it follows that the BBC reduces to: $\tau_t X = q_t(Y - X)$. The steady state dependency ratio for a constant population, a constant retirement age X and a constant life expectancy Y comes out as $\hat{z} = \frac{Y-X}{X}$. Denoting the steady state level of the contribution rate and the pension level by $\widehat{\tau}$ and $\widehat{q}$, respectively the BBC (2.5) implies the following relation:

$$\widehat{\tau} = \widehat{q}\hat{z}. \tag{2.6}$$

The values for $\widehat{\tau}$ and $\widehat{q}$ can be freely chosen as long as (2.6) is fulfilled. Their precise size will depend on the life-cycle patterns of needs and necessities, on preferences of a country, on historic developments, on political bargaining etc.

For later numerical examples I will often use the benchmark parametrization that $X = 45$ and $Y = 60$ and thus $\hat{z} = 1/3$. These values broadly conform to the case of the “standard pensioner” that is often invoked to assess properties of PAYG systems. The standard pensioner is a fictitious person which enters the labor market at the age of 20, works at the average wage for exactly 45 years and retires at the statutory retirement age of 65 (cf. OECD 2009). The value of $Y = 60$ then implies a life expectancy of 80 years which is close to the average current value for OECD countries.[3] In addition, I will use a steady state contribution rate of $\hat{\tau} = 1/4$ and a pension level of $\hat{q} = 3/4$. The parameters values have the property that in a stationary economy ($g = 0$) the after-contribution wage $(1 - \tau)W$ is equal to the pension payment $q \times W$.[4]

2.3 The sustainability factor and fluctuating cohort sizes

In this section I will abstract from increases in life expectancy and I will focus on situations where just the cohort size fluctuates. In the next section I will turn things around and I will study the case of increasing life expectancy and in section 2.5 I will then combine both developments.

A thorough analysis of the case of cohort size fluctuations is useful since in public and even academic discussions one can quite often hear the statement that in the presence of baby booms and busts or a steady decline in fertility rates PAYG systems are doomed to fail because of their inability to deal with these fluctuations. The purpose of this section is to show that this claim is simply wrong. In a situation of constantly fluctuating cohort sizes there still exists a continuum of adjustment possibilities that stipulate in different ways how the contribution rate and the pension payment have to be adapted in order to keep the system's budget balanced according to (2.5). This is the topic of this section. In the following section I will deal with the case of increasing life expectancy and I will show how the retirement age has to be adjusted in order to preserve sustainability. By using the central levers of a PAYG system —the contribution rate, the pension level and the retirement age— it is thus pretty straightforward to design a system that remains financially stable even in the presence of distinctive and unforeseen demographic fluctuations. An additional (and more difficult) question then is which of the numerous adjustment possibilities one should choose. This involves issues of risk-sharing, intergenerational fairness

[3]The most recent average numbers for the life expectancy at birth for OECD countries are: 81.7 (women) and 77.2 (men) (see OECD 2009). The average life expectancy at birth is thus around 79.

[4]Furthermore, these benchmark values are again fairly similar to the OECD averages. The average contribution rate is 21% while the average gross (net) replacement rate is 59% (70%) (see OECD 2009).

and social welfare that I will briefly touch upon in chapter 5.

For a discussion of the adjustment to fluctuations in the cohort size one can follow the logic of the German "sustainability factor" that has been introduced in 2003. It states how the contribution rate and the pension level have to be adapted when the dependency ratio z_t changes. The factor can be expressed formally in the following way:

$$\tau_t = \widehat{\tau}\left[1 + (1-\alpha)\left(\frac{z_t}{\hat{z}} - 1\right)\right], \tag{2.7}$$

$$q_t = \hat{q}\left[1 + \alpha\left(\frac{\hat{z}}{z_t} - 1\right)\right]. \tag{2.8}$$

Equations (2.7) and (2.8) state how the adjustment to an increase in the dependency ratio z_t is borne by workers and pensioners. If $\alpha = 0$ the pension level is held constant at $\hat{q}$ and thus the full burden of adjustment to demographic fluctuations is shouldered by the working population via changes in the contribution rate. This corresponds to the defined benefit (DB) case. The reverse is true for $\alpha = 1$ where the contribution rate stays constant at $\hat{\tau}$ and solely the pension level is varied to achieve a balanced budget. This refers to the defined contribution (DC) case. In general, α determines how the "demographic burden" is shared between contributors and pensioners. In Germany the relative adjustment weight was set equal to $\alpha = 0.25$ (cf. Kommission für die Nachhaltigkeit in der Finanzierung der Sozialen Sicherheitssysteme [KNFSS] 2003, Börsch-Supan et al. 2003).

A pension system that is characterized by (2.7) and (2.8) leads to a constantly balanced budget. This follows immediately from noting that $\tau_t L_t = \widehat{\tau}\left[\alpha L_t + (1-\alpha)\frac{R_t}{\hat{z}}\right]$, $q_t R_t = \hat{q}\left[(1-\alpha)R_t + \alpha \hat{z} L_t\right]$ and that $\widehat{\tau} = \hat{q}\hat{z}$ (from (2.6)). There is only one caveat to this statement. The parameter values as stipulated in (2.7) and (2.8) might not be viable options, since they are either "technically" or "politically" infeasible. Technical feasibility requires that the contribution rate must stay below 100% ($\tau_t < 1$) and that the pension level cannot turn negative ($q_t > 0$). The latter condition is guaranteed by (2.8) but for certain cohort size developments the contribution rate might exceed its upper limit. In the following we will assume that N_t develops in such a way that this technical constraint is never violated. On the other hand, it should also be noted that not any adjustment policy that is feasible in a strict technical sense is also feasible in a practical sense. Such politically problematic policies might, e.g., involve excessive contribution rates, inadequate pension levels or strange patterns in the development of the crucial parameters over time. I will not deal with these problems explicitly in this chapter (see, however, chapter 5) but they should be kept in mind.

In Figure 2.1 I show how a change in the adjustment weight α implies a different reaction of the pension parameters to a demographic shock. The latter is modeled by assuming that in period $t = 0$ there is a jump in the cohort size (by +100%) that only lasts for one period. In particular:

$$\begin{aligned} N_t &= \hat{N} \qquad \text{for } t \neq 0 \\ N_t &= \chi \hat{N} \qquad \text{for } t = 0. \end{aligned} \tag{2.9}$$

I assume that in the period before the shock the PAYG system has already reached its steady state of maturity. This assumption is kept throughout this monograph where I disregard the issue of the introductory generations. As seen in Figure 2.1a the implied changes in the contribution rate τ_t are larger for smaller α while the reverse is true for the pension level q_t (as shown in Figure 2.1b). In section 3.3 I will show that the intergenerational distribution of the costs and benefits of this one-time demographic shock depends heavily on the value that is chosen for α.

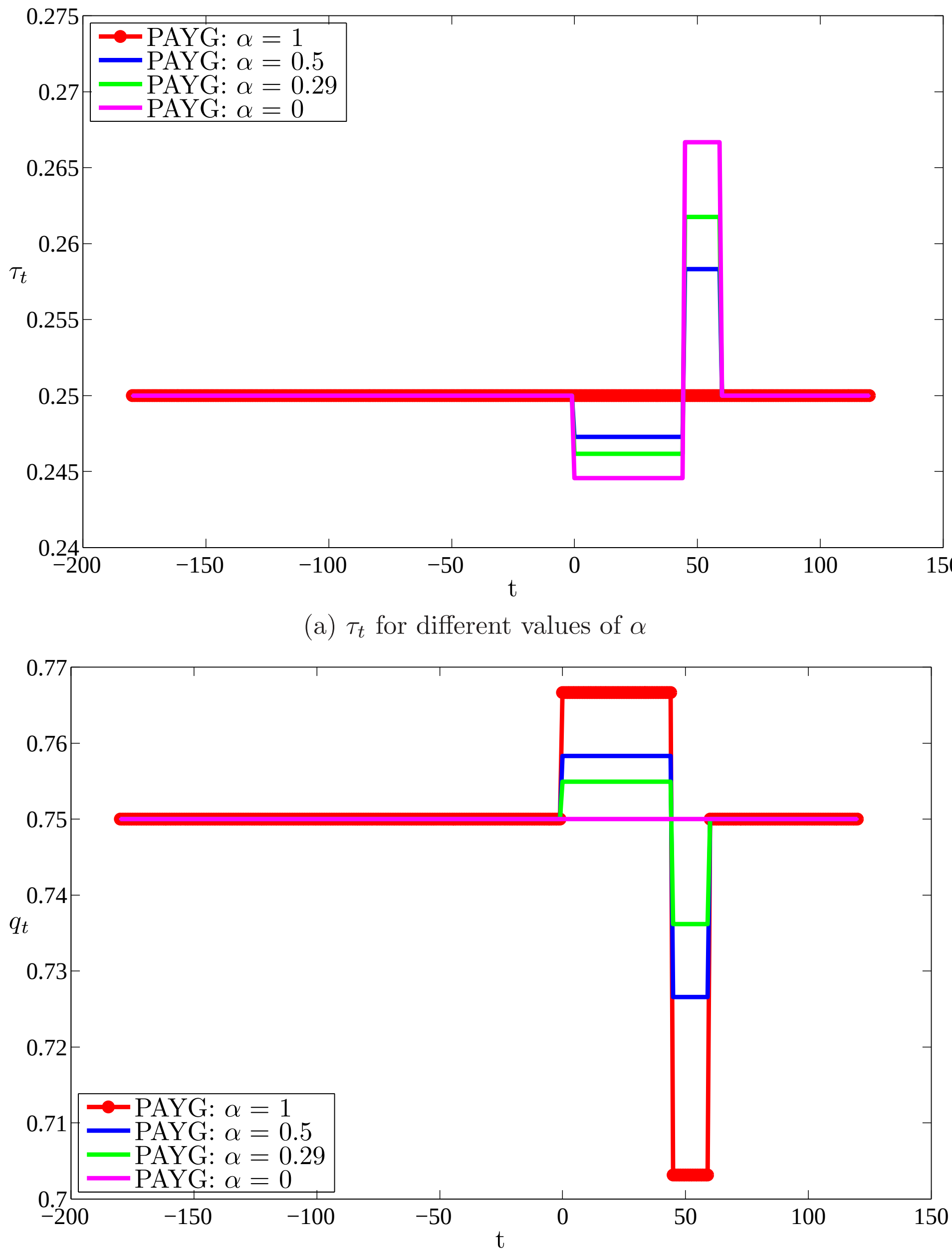
`Insert Figure 2.1 about here`

Although the German sustainability factor closely mirrors equations (2.7) and (2.8) there also exist some differences between the latter expressions and its real-world pendant. These differences have to do with *intra*generational variations in the calculation of the pension level, with the fact that the German sustainability factor is formulated in terms of changes and not in terms of levels and with the fact that its formulas involve a "modified gross earnings indexation" where changes in the contribution rate also enter the calculations for the pension level. In Knell (2010*a*) I discuss these differences more extensively and I show that the third element implies a "de facto" weighting parameter of $\alpha = 0.36$ instead of $\alpha = 0.25$. This is important if one wants to compare the results of the theoretical model with the real-world situation.

The sustainability factor is particularly interesting for the realistic case of discontinuous changes in N_t. I want to emphasize, however, that the PAYG system can also be designed in a stable way if the cohort size is constantly changing, e.g. if it grows or shrinks according to:

$$N_t = (1+n)N_{t-1}. \tag{2.10}$$

In this case the steady state dependency ratio is given by $\hat{z} = \frac{\sum_{a=X+1}^{Y} \frac{1}{(1+n)^{a-1}}}{\sum_{a=1}^{X} \frac{1}{(1+n)^{a-1}}}$. Note that $\frac{\partial \hat{z}}{\partial n} < 0$ and in stipulating the contribution rate and the pension level one must take the



(a) τ_t for different values of α

(b) q_t for different values of α

Figure 2.1: The contribution rate τ_t and the pension level q_t for a PAYG pension system and four different values of α. In addition life expectancy is $Y = 60$, workers retire at age $X = 45$ and $\hat{\tau} = 1/4$, $\hat{q} = 3/4$ and $\chi = 2$.

size of n into account. For the numerical example of above ($Y = 60$, $X = 45$) a balanced budget would be guaranteed by setting $\hat{\tau} = 0.18$ and $\hat{q} = 0.75$ or by choosing $\hat{\tau} = 0.25$ and $\hat{q} = 1.02$ if $n = 0.01$. For $n = -0.01$, on the other hand, $\hat{\tau} = 0.34$ and $\hat{q} = 0.75$ or $\hat{\tau} = 0.25$ and $\hat{q} = 0.59$ would be sustainable combinations.[5] Note that one could again use the sustainability factors (2.7) and (2.8) to deal with eventual fluctuations in cohort size around this (increasing or decreasing) trend n.

The analysis of this section has shown that it is not too difficult to design a PAYG system that is stable in the presence of constantly changing or randomly fluctuating cohort sizes. The claim that this is not possible can thus be refuted. In fact, there exists a whole continuum of possible adjustment schemes and a policy-maker can choose among them taking societal preferences, political constraints and principles of fairness into account. The sustainability factors presented in this section are more than a purely theoretical construct since the German pension system has employed a closely related concept. There exists, however, an even larger variety of ways to make a PAYG system stable. Some of them also lead to a balanced budget in every period others are only associated with a balanced budget in expected value (see, e.g., Hassler & Lindbeck 1997). I will say more on this in chapter 6 when I will talk about the NDC system in detail.

2.4 The life expectancy factor and increasing life expectancy

I turn now to the opposite case where the cohort size is assumed to be fixed and life expectancy changes. I regard this separation of the demographic effects as useful in order to understand the mechanisms at work and the logic of the proposed adjustment factors. In the following section I will then combine both trends and study a situation where the demography changes along both dimensions at the same time.

In this section I have to switch to a setting in continuous time (cf. Bommier & Lee 2000). This is necessary since I now deal with changes in life expectancy and retirement age that normally involve non-integer values. As a consequence one has to redefine the variables and the main equations. In particular, $q(t) = \frac{P(t)}{W(t)}$, $L(t) = \int_0^{E(t)} N(t-a)da$, $R(t) = \int_{E(t)}^{F(t)} N(t-a)da$, $z(t) = \frac{R(t)}{L(t)}$ and $W(t) = W(0)e^{gt}$. The balanced budget condition of the pension system is now given by: $\tau(t) = q(t)z(t)$. As said above, for this section I assume a constant cohort size ($N(t) = N$) and therefore the dependency ratio can

[5] Remember that for $n = 0$ we had $\hat{\tau} = 0.25$ and $\hat{q} = 0.75$ as a sustainable combination.

be written as: $z(t) = \frac{F(t)-E(t)}{E(t)}$. As far as the modeling of life expectancy is concerned I assume as the benchmark case that the maximum age (and thus life expectancy) increases linearly over time:

$$Y(t) = Y(0) + \gamma \cdot t, \tag{2.11}$$

where $0 \leq \gamma < 1$. The assumption about the linear increase in $Y(t)$ is in line with empirical results. Lee (2003), e.g., refers to a number of studies that have found a linear trend in life expectancy for a number of industrial countries and different time spans. The empirical estimates imply that life expectancy increases by between 0.15 and 0.25 years of life per calendar year of time, thus giving a range for the likely values of γ.[6]

The maximum age $F(t)$ that is observed in period t can be calculated from $Y(t - F(t)) = F(t)$:

$$F(t) = \frac{Y(t)}{1+\gamma}. \tag{2.12}$$

It is possible to doubt whether the assumption about a constantly increasing life expectancy makes any sense, since—taken literally—such an increase without bound seems inconceivable. On the other hand, however, demographers have repeatedly underestimated the increase in life expectancy (cf. Barr & Diamond 2006, p. 27) and even though one would not believe that the increase in life expectancy can go on forever, the development over the last decades and the forecast over the next 50 years is in fact best described by the assumption of a linear increase as in (2.11). At the end of the section I will also discuss briefly the case where life expectancy reaches a maximum level.[7]

In order to keep the budget of the pension system balanced in this situation the retirement age has to be determined in the following way:

$$E(t) = \frac{F(t)}{1+\hat{z}}. \tag{2.13}$$

In this case the dependency ration is stabilized at $z(t) = \hat{z}$.[8] Although this result appears almost trivial at first sight it has a number of interesting and not immediately evident

[6] In real life not everybody will reach the generation-specific maximum age $Y(t)$. Rather there exists a probability $s(t, t+a)$ that a member of generation t survives until period $t+a$. In this book, however, I abstract from the existence of premature deaths and I assume that everybody reaches the age $Y(t)$.

[7] In the literature that deals with PAYG systems the issue of rising life expectancy is mostly neglected, i.e. it is implicitly assumed that $\gamma = 0$. Some papers discuss the case of a (discontinuous) one time jump in life expectancy (Kifmann, 2001; Breyer and Kifmann, 2002), but this case is neither contrasted with the assumption of a constantly increasing life expectancy nor analyzed under the perspective of how it interacts with various assumptions about fertility development.

[8] This can be seen by noting that $z(t) = \frac{F(t)-E(t)}{E(t)}$ and by rearranging (2.13).

implications. First, an increase in life expectancy does not require an equal-sized but only an equal-proportionate increase in the retirement age. This can be illustrated by using again the numerical example with constant $Y = 60$ and $X = 45$ (and therefore also $Y = F$ and $X = E$). In this case $\hat{z} = 1/3$ and the system is in balance. Now imagine a different society with a higher life expectancy of $Y = 64$. If one wants to keep the particular values of $\hat{\tau}$ and $\hat{q}$ unchanged this means that also the dependency ratio has to be fixed at 1/3. In order to achieve this the higher life expectancy does not require a higher retirement age by the full four years but only an increase from 45 to 48, as stated in (2.13). The gain in life expectancy is shared between working and retirement in the same proportion (i.e. 3 : 1) that could also be observed in the benchmark situation.

Second, (2.13) gives a precise indication which notion of life expectancy should be used in the calculation of the statutory retirement age. Contrary to some real-world proposals it should not be the forecasted life expectancy $Y(t)$ for the generation that enters retirement in period t. In order to keep the PAYG system in continuous balance it is sufficient to base the calculation of the actual retirement age on the newborn generation t's life expectancy *at birth* (i.e. on $F(t)$).

Various authors have proposed an introduction of "life expectancy factors" (cf. Barr & Diamond 2006, Fehr & Habermann 2006). None of them, however, is really used in existing pension systems. The closest existing equivalent is probably the Swedish notional defined contribution system, which contains as a central element the provision that at the time of retirement the notional capital (that has been accumulated during the working years) is transformed into an annuity. This annuitization is done by dividing the notional capital by the (current) remaining life expectancy. Each increase in life expectancy will thus automatically lead to a decrease in the pension level. This mechanism is, however, different from (2.13) and it does not lead to a constant balance of the system, not even in the case of a constant and deterministic development (see chapter 6).

Other countries with automatic life expectancy factors include Finland, Italy, Poland and Latvia. To the best of my knowledge, none of these countries has, however, implemented an explicit life expectancy factor of the form (2.13). Latvia is an interesting case, since it uses an adjustment factor that is based on forecasted cohort life expectancy (cf. Lassila & Valkonen 2007). As discussed above, this is the wrong concept, at least when judged within the realms of the presented model. Of course, for the linear process of life expectancy one can transform the different concepts into one another.[9] Nevertheless, the

[9]Note that $E(t) = \frac{F(t)}{1+\hat{z}} = \frac{Y(t)}{(1+\hat{z})(1+\gamma)}$.

formula in (2.13) is the most basic formulation that is perhaps also easier to communicate since it only requires "known values" (current life expectancy $F(t)$ and the "target dependency ratio" $\hat{z}$) and not the uncertain magnitudes $Y(t)$ and γ. This is also the opinion of Palmer (2006, p. 28) and Lassila & Valkonen (2007) and also the suggestion of Barr & Diamond (2006, p. 27): "Automatic adjustments may function better—and politically more easily—if adjustments are based on actual mortality outcomes rather than projections."

One might now ask whether the policy given by (2.13) is the only ultimately sustainable policy in the case of a permanently increasing life expectancy or whether one could also use changes in the other two adjustment parameters— the contribution rate and the pension level. The answer is that for constant cohort sizes the policy (2.13) is in fact the only sustainable mechanism.[10] This can be seen by looking at the case where the retirement age is fixed at $E(t) = E$ which implies a dependency ration of $z(t) = \frac{F(t)}{E} - 1$. From this it follows that for $\gamma > 0$ the dependency ratio $z(t)$ increases without bound. In this case the use of the sustainability factors (2.7) and (2.8) (in their continuous-time versions) leads to an ever increasing contribution rate (for $\alpha < 1$), to an ever decreasing pension level (for $\alpha > 0$) or to both (for $0 < \alpha < 1$). Such a policy is clearly infeasible in the long-run since it will either disrupt the system (explosive contribution rate) or erode its task as old-age income support (shrinking pension levels). Only a policy that is able to stabilize the dependency ratio is an ultimately feasible policy in this demographic constellation. This corresponds to the use of (2.13).

It is clear that the latter statement depends crucially on the assumption about the development of life expectancy. If one assumes that its increase will stop at some maximum age Y^{max} then of course a whole variety of alternative adjustment policies are possible and potentially feasible. One should keep in mind, however, that the supposed convergence to a maximum age is a highly uncertain outcome where both the exact size and the exact time of its achievement are unknown. If the assumptions turn out to be wrong then the necessary additional adjustment measures might be quite disruptive and might cause a highly unequal treatment of different generations (cf. chapter 5).

On the other hand, the policy described by (2.13) is quite flexible as regards the speed and the eventual slowdown of the process of aging. If some generation $\hat{t}$ does in fact reach a maximum age Y^{max} then this is completely manageable within the framework of the life expectancy factor (2.13). Until the year where generation $\hat{t}$ reaches Y^{max} everything

[10] For a constantly increasing cohort size other policies might be possible (see section 2.5).

is as regulated and $E(\hat{t}) = \frac{F(\hat{t})}{1+\hat{z}} = \frac{Y^{max}}{1+\hat{z}}$. The next generation, however, is not getting older than Y^{max} which is recognized by the policy scheme in that the retirement age is also frozen at $E^{max} = \frac{Y^{max}}{1+\hat{z}}$. The formulation of the life expectancy factor is in this respect advantageous since it is only based on *current* and not on *anticipated* variables. Therefore the system might not lead to possible complaints of generations that had to work longer in expectation of a development that did not materialize. For this end it also seems imperative to have an automatic periodic (preferably annual) adjustment factor and no discretionary adjustments from time to time.

2.5 Dealing with fluctuating cohort sizes and increasing life expectancy at the same time

The life expectancy factor in (2.13) is based on a situation where the two-sided demographic challenge is shrunk to one dimension and where the cohort size is assumed to be constant. In this section I look at the case where the demography changes along both dimensions. A natural starting point is the situation where both developments are characterized by constant trends, i.e. life expectancy increases linearly with rate γ (see (2.11)) and cohort sizes change multiplicatively. In the discrete time version of the model this was captured by (2.10) while here I assume that $N(t) = N(0)e^{nt}$. The dependency ratio in this case if given by:[11]

$$z(t) = \frac{e^{-nE(t)} - e^{-nF(t)}}{1 - e^{-nE(t)}}. \tag{2.14}$$

One can now ask again how $E(t)$, the retirement age in period t, has to be adjusted in order to stabilize the dependency ratio at the targeted level $\hat{z}$. The solution is given by setting (2.14) equal to $\hat{z}$. It comes out as $E_(t) = F(t) + \frac{1}{n} \ln \left[\frac{1+\hat{z}}{1+e^{nF(t)}\hat{z}} \right]$. This can be linearized around $\gamma = 0$ in order to derive:

$$E(t) = \frac{F(t)}{1+\hat{z}} \left[1 - \frac{\hat{z}}{1+\hat{z}} nF(t) \right]. \tag{2.15}$$

Note that (2.15) collapses to (2.13) for $n = 0$. This expression is again interesting and points to a fact that is not immediately evident and maybe surprising. This fact is that if a pension system only relies on changes in the retirement age in order to stay in balance

[11] This follows from: $z(t) = \frac{R_t}{L_t} = \frac{N(0)\int_{E(t)}^{F(t)} e^{n(t-x)} dx}{N(0)\int_0^{E(t)} e^{n(t-x)} dx}$.

then the life expectancy factor must also take the changes in the population size into account. Expression (2.15) shows that in the case of a growing population ($n > 0$) the increase in $E(t)$ that stabilizes $z(t)$ is smaller than in the case with $n = 0$ ($\frac{\partial E(t)}{\partial n} < 0$). An increasing population ceteris paribus decreases the dependency ratio and thus (partly) counteracts the effects of population aging. By the same token a shrinking population size ($n < 0$) necessitates an increase in retirement age that is more than proportional to the increase in life expectancy. In order to get a feeling for the values involved let's assume that $\gamma = 0.2$. For the generation t with $Y(t) = 60$ one can use equation (2.15) (and a number of intermediate calculations) to derive that $X(t) = 42.9$ (for $n = 0$), $X(t) = 39.4$ (for $n = 0.01$) and $X(t) = 46.4$ (for $n = -0.01$). The interactions between the constantly changing population and the constantly growing life expectancy are not minor. As shown by the numerical example they can change the regulated retirement age for a generation with a specific life expectancy by two to three years. This fact does not seem to be well-known and it is also absent from the basic provisions in PAYG systems that account for changes in life expectancy (like in NDC systems, see chapter 6).

As an aside I want to note that in the case of a constantly increasing population ($n > 0$) a policy that holds the retirement age constant at $E(t) = E$ can also be feasible since fast population growth might counterbalance the increase in life expectancy. In fact, for $n < 0$ equation (2.14) implies that $\lim_{t\to\infty} z(t) \to \infty$ while for $n > 0$ it holds that $\lim_{t\to\infty} z(t) \to \frac{1}{e^{nE}-1}$. This limiting value of $z(t)$ is compatible with a contribution rate below 100% (i.e. $\lim_{t\to\infty} \tau(t) < 1$) if $n > \tilde{n} \equiv \frac{1}{E} \ln\left(1 + \frac{\hat{\tau}(1-\alpha)}{\hat{z}(1-\alpha\hat{\tau})}\right)$.

As said above, one might argue that the case of a permanent increase (or decrease on that matter) in cohort size and life expectancy is maybe a nice textbook example but a rather strange and almost pathological case for real life. On practical matters, I would argue that the quite compact life expectancy factor (2.13) is ambitious enough to be taken as a benchmark process. If a constant change in the size of the labor force starts to appear in the data then one could use discretionary adjustments in the other parameters to deal with this event.[12]

[12] In a way this would be similar to the institution of a "leap year" which is an "automatic adjustment factor" that keeps the calendar year synchronized with the astronomical year. In order to deal with "higher-order effects" there are also additional adjustments ("century leap years" etc.) that are akin to "discretionary changes" on top of the normal automatic adjustment factor.

2.6 Conclusion

In this section I have shown that a PAYG system can be designed in a way that its budget is constantly balanced even in the presence of permanently fluctuating cohort sizes and of ongoing increases in life expectancy. In the (empirically most relevant) case of boom and bust cycles in the size of cohorts and a linearly increasing life expectancy, an appropriate policy would, e.g., involve the use of the life expectancy factor (2.13) to adjust the retirement age and the use of the sustainability factors (2.7) and (2.8) to deal with the fluctuations in the labor force.[13] The choice between the different policies will depend on political feasibility, bargaining and the influence of interest groups, on potential path dependencies and also on preferences and on principles of fairness (see chapter 5). The aim of this chapter was to use particularly simple adjustment factors to refute the claim that a PAYG system cannot withstand the demographic challenge. In as far as the actual implementation is concerned this combination of two factors is of course not the only way how to construct a sustainable PAYG system. In the real world one can observe alternative adjustment strategies that lead to a balanced budget (at least over the medium run). The most prominent of these—the NDC system—will be discussed in chapter 6.

2.7 Related literature

There exists a number of survey articles and books that provide good overviews on the topic of pension systems. These include Homburg (1988), Breyer (1990), Breyer (2000), Lindbeck & Persson (2003), Diamond (2004), Holzmann (2004), Barr and Diamond (2006, 2009).

A comprehensive and systematic summary of the main properties and the outlook of the pension systems in developed countries can be found in the regular OECD publication *Pension at a Glance* (see OECD, 2006, 2009). The European Policy Committee also publishes in irregular intervals projections of the economic consequences of aging on future public expenditures (Economic Policy Committee and European Commission, 2006, 2009).

The German sustainability factor is discussed in Börsch-Supan et al. (2003) and in Knell (2010*a*). Life expectancy factors are proposed in Shoven & Goda (2008) and Heeringa & Bovenberg (2009). Shoven & Goda (2008) look at various ways to index

[13] Note that the use of this adjustment strategy would lead to paths for τ_t and q_t that look exactly like in Figure 2.1 (not shown).

the legislated retirement age to increases in life expectancy. Heeringa & Bovenberg (2009) contains not only a discussion of various life expectancy factors but also a proposal to use "non-fertility taxes" in response to negative fertility shocks (or, similarly, "non-participation taxes" in case of negative labour-force participation) to counter fluctuations in the size of (birth and labor force) cohorts. This is related to the "sustainability factors" presented in this chapter. On NDC systems see chapter 6 and the literature cited there.

Chapter 3

Unfunded vs. Funded Systems—The Case of Demographic Fluctuations

3.1 Introduction

In the last chapter I have shown that the claim that it is impossible to design a PAYG system that is capable to deal with demographic fluctuations without leading to severe budgetary problems is wrong. As mentioned in the introduction, there exists, however, also a weaker form of this argument that claims that a PAYG system (even if designed in a sustainable way) will lead to larger, more uncontrolled or even chaotic fluctuations when confronted with demographic shocks while a funded system will provide for a smoother adjustment process. This chapter deals with this argument. In particular, I will investigate the consequences of fluctuating cohort sizes on the economic fate of different generations. The main focus will be on the differences between pension systems that are organized as PAYG or as funded systems. This topic is of particular interest since the alleged immunity of funded pension systems to demographic fluctuations is often mentioned as one of the major advantages of these systems. E.g.:

> The funded system is more stable and financially resilient because under Paygo the required contribution rate depends on the rate of growth of payrolls. [...] With a fully-funded system, instead, the payroll tax that must be levied is unaffected by changes in demographic structure, because the pension is not paid by the contributions of the younger workers but by the capital accumulated by the pensioner (Modigliani et al. 2000, 2).

In a similar way this argument is also commonly met in discussion in the German speaking area:

> Der Hauptgrund für einen (Teil-)umstieg vom Umlageverfahren auf das Kapitaldeckungsverfahren [liegt nicht] in der höheren Rendite des Kapitaldeckungsverfahren [...], sondern in der Tatsache, dass nur ein Kapitaldeckungsverfahren die Lasten der demographischen Entwicklung über die Generationen verteilen kann (Börsch-Supan & Gasche 2010, p. 7).

Wilke uses a parallel argument when talking about the perspectives of the introduction of a NDC system in Germany.

> It should be clear that an NDC system does not solve the demographic problems but that it simply copes with them in a different way than conventional PAYG systems. [...] Thus, an NDC system can only offer an optimisation of the first pillar but does not replace the necessity to supplement the public pension system by a funded second and third pillar in order to prepare for the future demographic changes (Wilke 2008, p. 35).

A funded system—so the gist of the argument in these quotations—is better able to share the burden of necessary adjustments among a larger number of different generations. The main channel behind this (hypostasized) property is that funding allows for the accumulation and decumulation of real capital which helps to smooth the consequences of demographic fluctuations. This argument, however, is again controversial. Although not wrong in a strict sense (as has been the case for the refuted claim of the last chapter) it depends on many hidden assumptions about the saving and investment process that are unlikely to be fulfilled in real-world economies. In reality, e.g., it is not clear whether the introduction of a funded system will actually increase the amount of available savings, whether these additional savings will be channeled towards productive investments and whether these investments will ultimately increase the volume of output. In the simplest (and commonly used) model all of these crucial properties are automatically fulfilled but it is less clear whether this will be so in the real world of pension saving. A long quotation of Barr and Diamond emphasizes the fragility of this point of view.

> The relationship between funding and growth is neither simple nor automatic. [...] Does funding increase saving? [...] It will fail to do so if an increase in mandatory pension saving is offset by a decline in voluntary saving or a decline in the saving of government elsewhere in the budget. Thus saving may or may not increase—the amount of increased national savings has a complex relationship with the amount of increased funding. [...] How much will an increase in saving increase output? The simplest argument is that a move to funding (a) increases savings, which (b) increases investment, which (c) increases output by the marginal product of capital. These links hold in many circumstances, but not always or necessarily and not with a simple connection. As noted, a move to funding does not necessarily increase saving. The link between an increase in saving and increased investment is complex—some savings will simply increase prices of existing assets. Part of increased saving can drive up the prices of assets in limited supply, such as urban land. An increase in investment may not increase output by much. Inefficiencies in capital markets may make the marginal product of investment low (Barr & Diamond 2009, 71).

In the rest of this chapter I take the baseline argument and the mentioned counterarguments as my starting point and I will investigate whether and when a funded system does in fact lead to a smoother reaction to demographic shocks. A particular focus will be laid on two assumptions that are crucial to assess the effects of funding in this context. First, the type of assets in which the contributions to the funded pillar are invested (e.g., accumulable vs. fixed factors of production) and second the behavioral rules that guide these investment decisions (e.g., forward-looking behavior vs. rules-of-thumb). I will show that in the case where the pension savings are invested into an asset that is non-productive and in fixed supply and where individuals follow fixed savings rules the effect of demographic shocks on the intergenerational sharing of the burden (as measured by the internal rate of return) is basically identical in unfunded and funded systems. Allowing for investments into productive assets and for optimal behavior might, on the other hand, lead to smoother or wilder fluctuations of the funded systems. The reaction depends here on the parameters of the utility and the production function. Overall, the effects of funding on the smoothness of adjustment might go in either direction and it would be a drastic simplification to base the argument for a full or partial transition to a funded pension system on these ambiguous results.

3.2 The model

I start by presenting the general structure of the production process of the economy and turn then to the behavior of households.

3.2.1 The production side

I assume that there are two types of assets in the economy: a produced and accumulable factor of production K_{t-1} (say capital) and a fixed stock of a non-producible factor $H_t = \bar{H}$ (say land/houses). Aggregate output is produced by combining these two factors with labor L_t:

$$Y_t = (A_t)^{1-a_1}(K_{t-1})^{a_1}(\bar{H})^{a_2}(L_t)^{1-a_1-a_2}, \tag{3.1}$$

where A_t is a labor-augmenting productivity term that grows with rate g, i.e.: $A_{t+1} = A_t(1+g)$. The timing of variables follows the "end-of-period" convention, i.e. stock variables are timed in the period when they are determined.

Capital depreciates at rate d and the aggregate capital stock thus accumulates as:

$$K_t = Y_t - C_t + (1-d)K_{t-1}, \tag{3.2}$$

where C_t is aggregate consumption. The output good can either be used for consumption or for the accumulation of the capital stock and it is used as the numéraire good. The relative price of land is denoted by P_t^H.

I assume that all markets are competitive and the factor prices are thus given by their marginal products:

$$r_t^K = a_1 \frac{Y_t}{K_{t-1}} \tag{3.3}$$

$$r_t^H = a_2 \frac{Y_t}{\bar{H}} \tag{3.4}$$

$$W_t = (1 - a_1 - a_2) \frac{Y_t}{L_t}. \tag{3.5}$$

Throughout this chapter I will deal with situations where the population is ultimately stationary (although it might fluctuate in the short-run).[1] Productivity, however, is allowed to grow over time. In order to be able to study the behavior on a balanced growth path I will use the following normalization:

$$\widetilde{Y}_t = \frac{Y_t}{A_t}, \widetilde{K}_{t-1} = \frac{K_{t-1}}{A_t}, \widetilde{W}_t = \frac{W_t}{A_t}, \widetilde{r}_t^H = \frac{r_t^H}{A_t}, \widetilde{P}_t^H = \frac{P_t^H}{A_t}. \tag{3.6}$$

The production function and the factor prices can then be written as:

$$\begin{aligned} \widetilde{Y}_t &= (\widetilde{K}_{t-1})^{a_1} (\bar{H})^{a_2} (L_t)^{1-a_1-a_2}, \\ r_t^K &= a_1 \frac{\widetilde{Y}_t}{\widetilde{K}_{t-1}}, \ \ \widetilde{r}_t^H = a_2 \frac{\widetilde{Y}_t}{\bar{H}}, \ \ \widetilde{W}_t = (1 - a_1 - a_2) \frac{\widetilde{Y}_t}{L_t}. \end{aligned}$$

On a balanced growth path the wage rate W_t grows with rate g and the same is also true for the relative price P_t^H and the rate of return r_t^H of the fixed factor of production.

An interesting special case occurs when $a_2 = 0$ where the fixed factor $\bar{H}$ does not play a role for production and can only be used as a store of value. In this case one can think of it as gold (or as paper money).

3.2.2 The social structure of households

For the modeling of the household side I use a slightly unusual (although not implausible) framework. In particular, I assume that there exist two classes of households that differ with respect to their risk preferences, their planning horizons, their investment opportunities and their asset holdings.

[1] The extension to situations with a constant development of the population (see (2.10)) would be straightforward. Such an assumption, however, does not correspond to the observable case of demographic boom-and-bust cycles. Furthermore, a constant change of the population affects all generations in the same way and it thus would not involve the more interesting situation of a pattern where different generations are treated differently.

On the one hand, there is a large mass of households that are organized in a formation of multi-period, overlapping generations, following the structure that has been presented in chapter 2. I assume again that each cohort has a (fixed and deterministic) life expectancy of Y and that it works for X years earning an annual wage of W_t. For the remaining $(Y-X)$ years individuals are retired and they have to live on pension income and on private savings. The size of each cohort is N_t and this size is allowed to fluctuate over time as in section 2.3.

The intertemporal utility function of the representative member of generation t is given by:

$$U_t = \sum_{a=1}^{Y} \beta^{a-1} \frac{C_{a,t+a-1}^{1-\rho}}{1-\rho}, \tag{3.7}$$

where $\beta \leq 1$ is the time discount factor, $\rho \geq 1$ is the coefficient of relative risk aversion (or the inverse of the intertemporal elasticity of substitution) and $C_{a,t+a-1}$ stands for the level of consumption at age a of the generation that is born in period t. I assume that these "working class" households are only holding one type of asset (if at all): the non-produced fixed factor $\bar{H}$. I will discuss below the rationale behind this assumption.

These households' period budget constraints is then given by:

$$H_{a,t+a-1} P_{t+a-1}^{H} = I_a^W W_{t+a-1} + H_{a-1,t+a-2} \left(P_{t+a-1}^{H} + r_{t+a-1}^{H}\right) - C_{a,t+a-1}, \tag{3.8}$$

where $H_{a,t+a-1}$ stands for the total stock of land held by generation t at the end of period $t+a-1$ when they are a years old.[2] I^W is an indicator variable that is 1 for $1 \leq a \leq X$ and 0 for $X+1 \leq a \leq Y$. The agents enter the world without any asset holdings (i.e. $H_{0,t-1} = 0$) and from the assumption of a known year of death and the absence of a bequest motive it follows that they die without any asset holdings, i.e. $H_{Y,t+Y-1} = 0$.

Besides this quantitatively dominating group of "common people"—the "working class"—there also exists a tiny upper class of society that is in many dimensions the exact opposite of the majority. First, members of this upper class are assumed to have a dynastic perspective and they thus take the destiny of their offsprings into account. Second, they are less risk-averse than the ordinary people and I will work with the limiting case that they are completely risk-neutral (i.e. $\rho^{uc} = 0$). Third, they hold the entire stock of accumulated capital K_{t-1}. Fourth, they do not work and just live on capital income (i.e. $W_t^{uc} = 0$). Fifth, they are a very small group and they are not subject to the fertility

[2] At the beginning of time the entire stock of $\bar{H}$ is owned by the retired population.

fluctuations that characterize the rest of the society.[3] Due to their tiny size they do not influence the general trends in the development of the population and I will abstract from their small number when specifying the cohort size N_t and the size of the total population $\sum_{a=1}^{Y} N_{t-a+1}$.

Admittedly, this is a caricature of the existing social structure. Nevertheless, it has a realistic flavor that captures with a rather broad brush a dividing line in economic and non-economic life styles that is present in many economies. The "superrich" live a life of their own that is quite different from the lives of the "common people". They own a large part of total real wealth (if one abstracts from owner-occupied housing), their labor market income is negligible compared to their capital income, they have distinct reproductive patterns and often take a quite dynastic view and they are arguably also more risk-neutral than the rest of the population.[4]

Summarizing, I assume that one can write the intertemporal utility function of this upper class as:

$$U_t^{uc} = \sum_{a=1}^{\infty} (\beta^{uc})^{a-1} \frac{\left(C_{t+a-1}^{uc}\right)^{1-\rho^{uc}}}{1-\rho^{uc}}, \tag{3.9}$$

where the assumption of risk-neutrality ($\rho^{uc} = 0$) implies that (3.9) becomes a linear function of consumption. The period budget constraint of the upper class is given by:

$$K_{t+a-1} = -C_{t+a-1}^{uc} + R_{t+a-1}^{K} K_{t+a-2}, \tag{3.10}$$

where $R_t^K = (1 + r_t^K - d)$ is the gross rate of return on capital between periods t and $t+1$ and K_0 is given. I also assume that the transversality condition for capital holds. The assumptions underlying (3.10) of course beg the question why the (huge) rest of the economy does not hold any capital but only the non-accumulable asset $\bar{H}$. This might have many reasons, including limited knowledge, financial constraints or institutional obstacles. It might, e.g., be the case that investments into real capital must have a certain minimal size in order to be profitable and that this threshold is beyond the means of the working class. Furthermore, one must note that the model is set in a deterministic world and the assumed differential portfolio choices might also capture — as a short-cut — differences in risk-aversion. In the presence of uncertainty, any eventual differences in the returns

[3] In fact, due to their special position it is not unreasonable to assume that they are exempt from the normal sources that lie behind the booms-and-bust in cohort sizes: war casualties, diseases, changing social norms etc. The support of nannies, domestic help etc. might also contribute to the immunity to societal trends.

[4] This, in fact, might exactly be the reason why they are rich in the first place.

r_t^K and $\widetilde{r}_t^H$ should be adjusted for their different risk and the dichotomous structure of asset holding might simply reflect the different degrees of risk aversion and the associated class-specific investment behavior.

Maximization of (3.9) leads to the first-order condition: $\left(\frac{C^{uc}_{t+a}}{C^{uc}_{t+a-1}}\right)^{\rho^{uc}} = \beta^{uc} R^K_{t+a}$. Using $\rho^{uc} = 0$ this leads to the equation $R^K_{t+a} = \frac{1}{\beta^{uc}}$. The assumption of risk-neutral superrich thus leads to a situation where the rate of return on capital is constant over time and given by:

$$\bar{r}^K = \frac{1-\beta^{uc}}{\beta^{uc}} + d. \tag{3.11}$$

The superrich households are willing to let their levels of consumption fluctuate widely between periods in order to smooth the real rate of return.[5] Note that for the limiting case where $\beta^{uc} \rightarrow 1$ it holds that $\bar{r}^K = d$. This is equal to the golden rule allocation where aggregate consumption is at its maximum.

Given $\bar{r}^K$ one can then use $r_t^K = a_1 \frac{\widetilde{Y}_t}{\widetilde{K}_{t-1}}$ and the production function to calculate the level of capital K_{t-1} for every period (since $\bar{H}$ and L_t are also known in every period). It comes out as:

$$K_{t-1} = A_t \left(\frac{a_1}{\bar{r}^K}\right)^{\frac{1}{1-a_1}} \left(\bar{H}\right)^{\frac{a_2}{1-a_1}} \left(L_t\right)^{\frac{1-a_1-a_2}{1-a_1}}$$

Using this in the production function (3.1) one can write:

$$Y_t = BA_t(\bar{H})^{\tilde{a}_2}(L_t)^{1-\tilde{a}_2}, \tag{3.12}$$

where $B \equiv \left(\frac{a_1}{\bar{r}^K}\right)^{\frac{a_1}{1-a_1}}$ and $\tilde{a}_2 \equiv \frac{a_2}{1-a_1}$. Using the normalization for productivity growth (3.6) this can also be expressed as $\widetilde{Y}_t = B(\bar{H})^{\tilde{a}_2}(L_t)^{1-\tilde{a}_2}$. From the perspective of the working class the economy thus appears as a world where output is produced with labor and with a non-producible factor that is in fixed supply. For the case where $a_2 = 0$ equation (3.12) reduces to the linear production function $Y_t = BA_tL_t$.

On the whole, I regard this dichotomous structure of society as a useful set-up to study the question at hand.[6] In case one prefers a more traditional approach, however, one

[5] In this respect the present model is very different from a number of papers (cf. Krueger & Ludwig 2007, Fehr et al. 2008) that have tried to evaluate the impact of demographic changes on the global macroeconomy and in particular on factor prices. Most approaches have come to the conclusion that these demographic trends will lead to lower interest rates. The model presented here, however, shows that if there is a sufficient degree of heterogeneity between individuals and if the risky capital stock is primarily hold by the rather risk-neutral class, then the interest rate is fixed at $\bar{r}^K$. Demographic fluctuations might thus have a smaller effect on factor prices than is normally implied by these traditional analyses.

[6] One could easily add some further assumption to increase its degree of realism. E.g. one could allow for a bequest motive of the working class. This would result in a situation where each generation is born with a small endowment of land. The new-born thus might feel richer, but apart from that it would

could also go back to the frequently used small-open-economy assumption that has similar consequences as the model presented above. In particular, for a small, open economy the interest rate is given by the world interest rate r^W which is assumed to be constant. If one would assume in addition that the total capital stock is held by foreigners than the reduced form of this framework would closely correspond to my model. Underneath these similarities there are, however, crucial differences. First, the small-open-economy model has nothing to say about the determinants of the world interest rate while it is given by (3.11) in my model. Second, it always remains a partial equilibrium model which makes it difficult to study demographic changes as a *global* phenomenon. Third, it is harder to motivate why the capital is only owned by foreigners than to argue why it is the exclusive property of the superrich upper class. Therefore I will use the dichotomous framework as my benchmark model. In section 3.7 I will, however, lift this assumption and also study the case where there is just one class of representative households that only differ with respect to their age.

3.2.3 The internal rate of return

Before turning to the discussion of different pension systems I have to specify how I am going to measure the impact of demographic changes on the economic treatment of different cohorts. There exist various measures of intergenerational distribution that can be used to evaluate the distributional properties of existing pension schemes and the effects of proposed pension reforms. Four widely used measures are the internal rate of return, the present value ratio, the net present value and the implicit tax rate (cf. Geanakoplos et al. 1999, Fenge & Werding 2003). In this chapter I focus on the internal rate of return (IRR) since it is the most commonly used indicator in this context.

As in the context of finance (where it is used in capital budgeting to compare the profitability of investments) the IRR is defined as the annualized effective compounded discount rate that makes the net present value of all cash flows from a particular investment equal to zero. Or, equivalently, the IRR is the discount rate at which the net present value of costs (negative cash flows) and the one of benefits (positive cash flows) is equal. In the context of pension systems the costs are the contributions paid into the system

not change the fact that taken together the working class does not accumulate wealth and does not hold any savings in form of capital. On the other hand, in order to break the static structure of society, one could also allow for some amount of mobility, where each period a small fraction of the working and the upper class swaps position and exchanges assets. While these extensions would increase realism (and the notational demands) they would not change the main structure of the model.

while the positive cash flows are the pension benefits. In other words, the internal rate of return δ_t for generation t is implicitly given by:

$$\sum_{a=1}^{Y} CF_{a,t+a-1} \left(\frac{1}{1+\delta_t} \right)^{a-1} = 0, \tag{3.13}$$

where $CF_{a,t+a-1}$ is the (positive or negative) cash flow that accrues for generation t when it is a years old. Using the normalization $\widetilde{CF}_{a,t+a-1} = \frac{CF_{a,t+a-1}}{A_{t+a-1}}$ and (for small g and δ_t) the linear approximation $\left(\frac{1+g}{1+\delta_t}\right)^{a-1} \approx 1 + (a-1)(g - \delta_t)$ the IRR can be written as:

$$\delta_t = g + \frac{\sum_{a=1}^{Y} \widetilde{CF}_{a,t+a-1}}{\sum_{a=1}^{Y} \widetilde{CF}_{a,t+a-1}(a-1)}. \tag{3.14}$$

I will use these expressions for the IRR in the following to compare the intergenerational distribution of demographic shocks in various pension systems.

In an additional step one can then also ask whether these cohort-specific patterns of the IRR look "reasonable". This amounts to tackling the difficult task to find a possible justification for the unequal treatment of different generations by referring, e.g., to deeper principles like the ability to pay, the fault principle or to concepts of fairness. I will come back to this important topic in chapter 5.

3.3 A PAYG system with a sustainability factor

As a starting point of the discussion it is useful to look at the case of a PAYG pension scheme. In order to fix ideas I abstract here from the existence of land, i.e. I assume that $a_2 = 0$ and $\bar{H} = 0$. Using a normalization such that $B = 1$ equation (3.12) leads to $Y_t = A_t L_t$. The pension system is defined by two magnitudes: the contribution rate τ_t and the pension payment P_t, which is assumed to be identical for all retired cohorts. The budget constraint of the households as stated in (3.8) then reduces to:

$$\begin{aligned} C_{a,t+a-1} &= (1 - \tau_{t+a-1}) W_{t+a-1} \qquad \text{for } 1 \leq a \leq X \\ C_{a,t+a-1} &= P_{t+a-1} \qquad \text{for } X+1 \leq a \leq Y. \end{aligned} \tag{3.15}$$

The households are in this case exempt from any meaningful economic decision and their utility (given by (3.7)) is just a function of the publicly determined PAYG system. This decision-free environment is a useful benchmark for later comparisons.

For the determination of τ_t and $q_t = \frac{P_t}{W_t}$ I again assume that they are set according to a sustainability factor as described in (2.7) and (2.8) with some chosen adjustment weight α. As shown in chapter 2.3 this set-up guarantees that the budget of the PAYG pension system is balanced in every period and condition (2.5) holds.

3.3.1 General properties of the internal rate of return in a PAYG system

One can use the budget constraint (3.15) in the definition of the IRR (3.13) to derive:

$$\sum_{a=1}^{X} \tau_{t+a-1} W_{t+a-1} \left(\frac{1}{1+\delta_t}\right)^{a-1} = \sum_{a=X+1}^{Y} q_{t+a-1} W_{t+a-1} \left(\frac{1}{1+\delta_t}\right)^{a-1}. \tag{3.16}$$

The expression in (3.16) is just another way to say that the IRR is the discount rate that equalizes the net present value of costs (pension contributions) and benefits (pension payments). Equation (3.16) cannot be solved explicitly. One can use, however, the approximation (3.14) (which is valid for small g and δ_t) to get a closed form solution for the IRR:

$$\delta_t = g + \frac{\sum_{a=X+1}^{Y} q_{t+a-1} - \sum_{a=1}^{X} \tau_{t+a-1}}{\sum_{a=X+1}^{Y} q_{t+a-1}(a-1) - \sum_{a=1}^{X} \tau_{t+a-1}(a-1)}. \tag{3.17}$$

In Knell (2010a) I show that one can insert (2.7) and (2.8) into (3.17) to derive an approximation of δ_t of the form

$$\delta_t = g + \frac{2}{Y} \sum_{s=-(Y-1)}^{Y-1} \eta_s \frac{N_{t+s} - \hat{N}}{\hat{N}}, \tag{3.18}$$

where η_s is a piecewise linear function of s that can be calculated in an explicit way. These expression are stated in the appendix of Knell (2010a) and it is shown there that they depend on X, Y and α and that it holds that $\sum_{s=-(Y-1)}^{Y-1} \eta_s = 0$. Equation (3.18) shows how the size of the own (N_t) and of different past (N_s for $s < 0$) and future (N_s for $s > 0$) cohorts affects the internal rate of return of generation t. In fact, the coefficient η_s can be interpreted as the (semi-)elasticity of the internal rate of return δ_t with respect to the deviation of cohort size N_{t+s} from the reference cohort size $\hat{N}$. A value of $\eta_{20} = 0.02$ indicates, e.g., that (for $Y = 60$) the internal rate of return increases by 0.67 percentage points if the size of cohort N_{t+20} is 10% larger than $\hat{N}$.

Before turning to the case of irregular movements in N_t it is interesting to consider the case of a monotonous development of cohort sizes, i.e. $N_t = (1+n)N_{t-1}$ (cf. (2.10)). As is shown in the appendix of Knell (2010*a*) one can then calculate that $\delta_t = g + n$. In this case the internal rate of return equals the growth rate of the wage bill $g + n$. This is in fact the famous result, first derived by Samuelson (1958), that population growth in a PAYG system works like a "biological interest rate". The empirically more relevant situations, however, involve cohort sizes that change from year to year in an irregular manner (e.g. "baby boom and bust cycles"). For a discussion of this situation the linear approximation in (3.18) is convenient since it allows a direct comprehension of the distributional consequences of fluctuating cohort sizes. The explicit expressions of η_s are also useful to derive a number of additional results. It can be shown, e.g. (see Knell 2010*a*), that the sum of the absolute values of the weights has a minimum at a value of $\alpha = \frac{Y-X}{X}$ where it is given by $\sum_{s=-(Y-1)}^{Y-1} |\eta_s| = 1$. These results can be used to assess the impact of fluctuating cohort sizes on the intergenerational fluctuations of δ_t. Furthermore, one can calculate the adjustment weight α that leads to the minimum variance of fluctuations. As shown in Knell (2010*a*) it comes out as:

$$\alpha^{MinVar} = \frac{Z}{2Y} \frac{2Z^2\left(3Y^2 - Z^2\right) + Y^2 - 7Z^3Y + 2Z^2 + 2ZY}{2Z^2\left(Y^2 - Z^2\right) - 2Z^4 + 3Z^2 + Y^2 - ZY}, \tag{3.19}$$

where $Z = Y - X$ is the length of the retirement span. I will use the expression for α^{MinVar} in some of the following numerical exercises. In general, however, I will not employ the approximate expression for δ_t as given in (3.18) but I will use numerical methods to calculate the exact values. I do this in order to have a better comparisons to the cases of funded pension systems.

3.3.2 Patterns of the internal rate of return in PAYG systems

The impact of fluctuations in the cohort size on the intergenerational pattern of internal rates of return can best be illustrated by the use of a specific numerical example. This is done in Figure 3.1 for 5 different values of α and where the rest of the assumptions corresponds to the benchmark parameter values used in chapter 2.[7]

In order to get a clear picture of how the size of a specific cohort influences the internal

[7] The use of alternative parameter values does not change the general picture. Different values for life expectancy and the retirement age (e.g., $X = 40$ and $Y = 60$ or $X = 35$ and $Y = 55$) lead to very similar qualitative results and the effect of employing different values for $\hat{\tau}$ and $\hat{q}$ is even smaller since their first-order effects on δ_t are zero (see (3.18)).

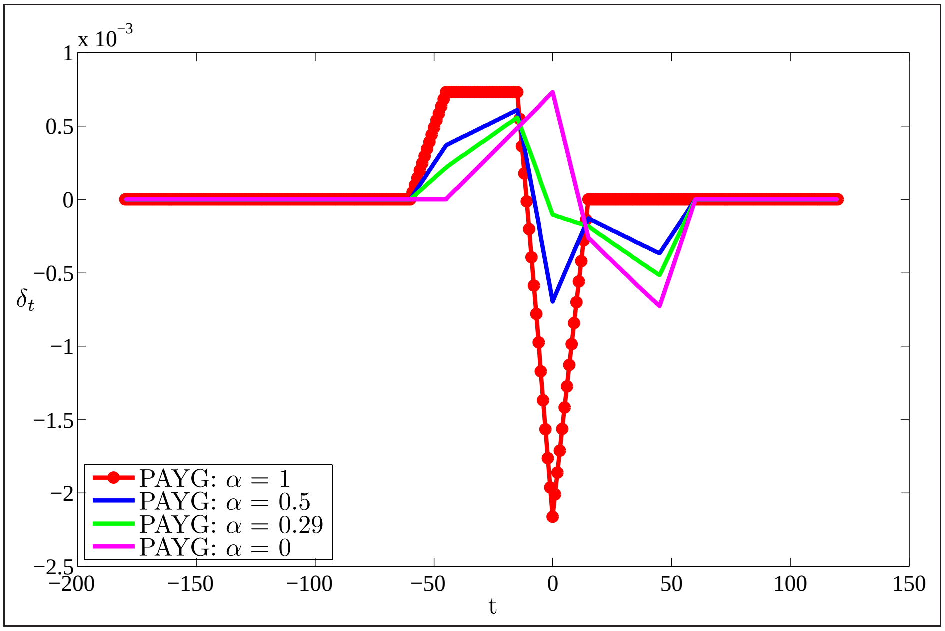


Figure 3.1: The IRR for a PAYG pension systems and four different values of α when life expectancy is $Y = 60$, workers retire at age $X = 45$ and $\hat{\tau} = 1/4$, $\hat{q} = 3/4$ and the demographic development follows (2.9) with $\chi = 2$, i.e. $N_t = 1$ for $\forall t \neq 0$ and $N_t = 2$ for $\forall t = 0$.

rate of return for another cohort I use again the simple assumption of a one-time jump (cf. (2.9)). The curves in Figure 3.1 can thus be interpreted as "impulse response functions". They document how the IRR of generations $\ldots, -2, -1, 0, 1, 2, \ldots$ react to an increase by the factor χ in the size of generation N_0.[8] Note, however, that the shock is perfectly anticipated and in later sections (when I will consider optimal behavior of forward-looking agents) this will lead to changes in crucial variables even before $t = 0$.

`Insert Figure 3.1 about here`

The one characteristic of these figures that immediately catches the eye is that the patterns look quite different and that the intergenerational distribution seems to depend heavily on the parameter α. Every pension scheme thus involves some underlying (explicit or implicit) "cohort dependence structure". An important distinguishing characteristic of the different curves is that they cut the zero line at different values. This means that the adjustment parameter α determines whether the impact of the exceptionally large cohort size N_0 is positive or negative for a particular generation t.

[8]In Knell (2010*a*) I focus on the question of how the IRR of one specific generation t is influenced by the size of the cohorts ranging from $-(t - Y + 1)$ to $(t - Y + 1)$. In this chapter I look, however, at the inverse question of how the size of one cohort influences the IRR of different cohorts.

In order to understand the intuition behind the pattern of δ_t for different generations it is useful to discuss the two benchmark cases with $\alpha = 1$ and $\alpha = 0$ more extensively. For the DC case ($\alpha = 1$) the contribution rate is fixed at $\widehat{\tau}$ for all times and the insured do not care about the dependency ratio while they are working. This changes, however, once they retire since then the relative sizes of L and R have an impact on their pension levels. The generations that are born before $t = -60$ are dead when the large cohort enters the labor market and they are thus unaffected and get the steady state IRR of $\delta_{ss} = 0$. For generation -59 this is, however, different since it has one year of retirement (at the age of 79) where its pension level is larger than normal ($q_0 = 0.77$ instead of $q_{ss} = 0.75$, see Figure 2.1b). This is reflected in the slightly higher IRR for this generation. The more years a generation spends in retirement while the large cohort is working the higher its IRR will rise. The generations between -45 and -15 are particularly favored since they receive the higher pension level for the whole period of their retirement. Starting with generation -14, however, this advantage is slowly turned upside down since these cohorts share some retirement years with the large cohort which depresses its pension level ($q_s = 0.7$ for $45 \leq s \leq 59$). The most "unfortunate" generation is the large cohort itself since it receives the low pension level for the entire duration of its retirement. This is reflected in the fact that δ_0 is the lowest value among all internal rates of return. Also the "neighboring" cohorts, however, are quite disadvantaged since they overlap for almost their total retirement span with the baby-boom generation. The IRR only returns to normal when a generation does not share any years in pension with this large cohort. The first generation for which this is the case is generation $+15$.

For the opposite case with $\alpha = 0$ (DB) the pension level is fixed at $\widehat{q}$ for all periods and the complete adjustment is done by changing the contribution rate. In order to be affected by the large cohort it must therefore be the case that a generation spends some years in the labor force together with the large cohort. The first generation for which this is true is generation -44 that shares one common working period with the boom generation and it benefits from the lower contribution rate (see Figure 2.1a) which slightly increases its IRR. The benefits increase for the following generations until they reach a maximum for the large cohort itself. It has the advantage of a lower contribution rate for their entire working life while it does not suffer lower pension levels due to the assumption of a DB system ($\alpha = 0$). The larger size of cohort 0 means, however, that the total pension expenditures increase once this cohort is in retirement and this higher costs are shouldered by the later generations. The generations that have been born shortly after the baby-boom cohort still are net-beneficiary from the demographic shock since their periods with lower-than-normal contribution rates outweigh the periods where they are larger. But this balance quickly changes sign and for all generations between +12 and +59 the IRR is negative. The "loser" is in this case generation +45 that has to pay the higher contributions rate for the maximum number of years while spending the smallest number of years on the labor market together with the large cohort. The intuition for the other cases of α follows a similar logic.

The empirical observable pattern of cohort sizes follows of course a more irregular and less stylized pattern. The example of a one-time jump in cohort size and the associated impact on the IRR of different generations as shown in Figure 3.1 gives, however, the underlying, inherent cohort dependence structure of the various regimes. This sets the stage for comparing the IRRs for the funded and the unfunded system and for investigating the claim that the funded system is associated with a smoother adjustment process.

Before turning to this comparison I want to add a few remarks on the internal rate of return. As noted at the beginning of this section, the IRR is a popular and commonly used measure of intergenerational distribution. It is, however, not related to the individuals' utility functions. As a complement to the IRR I want to briefly look at the relation between δ_t and $U(t)$. Inserting (3.15), (2.7) and (2.8) into the utility function (3.7), using the approximation $\beta(1+g)^{a-1} \approx 1-(1-\beta(1+g))(a-1)$ and setting $\beta = 1$ one can derive that:

$$\frac{U(t)}{W(t)} = X\left(1+g\frac{X-1}{2}\right)+\left(\sum_{a=X+1}^{Y} q_{t+a-1}(a-1)-\sum_{a=1}^{X}\tau_{t+a-1}(a-1)\right)\delta_t. \qquad (3.20)$$

Utility is thus proportional to the internal rate of return δ_t but the factor of proportionality is not constant over time but changes itself with the demographic development. The correspondence between the two concepts is thus not exact but it can be shown that for the numerical examples the correlation is in fact quite high.

3.4 A funded system with fixed rules and investments in gold

In order to offer a systematic comparisons of the properties of the unfunded system with the ones of its funded pendants I use various environments that differ with respect to the type of assets that are available and with respect to the strategies that guide peoples behavior. In this section I start with the most simple (but — as I will argue — not necessarily implausible) example of a case where the factor in limited supply plays no role in the production process ("gold") and where the contribution rate to the unfunded pillar and the rules of annuitization are fixed by the pension system.

3.4.1 Set-up

It is assumed that the fixed factor is useless for production, i.e. $a_2 = 0$ and therefore also $r^H_{t+a-1} = 0$. Furthermore, the contribution rate of the funded system is assumed to be the same as in the DC-version of the unfunded system ($\alpha = 1$), i.e. $\tau_t = \hat{\tau}, \forall t$. Consumption (cf. (3.8)) is thus given by:

$$C_{a,t+a-1} = (1-\hat{\tau})W_{t+a-1} \qquad \text{for } 1 \leq a \leq X. \tag{3.21}$$

The contributions to the pension fund buy $\frac{\hat{\tau}W_{t+a-1}}{P^H_{t+a-1}}$ units of gold. At the end of the working life the total stock of gold is given by $H_{X,t+X-1} = \hat{\tau}\sum_{a=1}^{X}\frac{W_{t+a-1}}{P^H_{t+a-1}}$. The level of consumption in old age is determined by the rules that govern the annuitization of this total pension capital. I assume that the disinvestment of the remaining capital stock $DI_{a,t+a-1}$ is given by:

$$\begin{aligned} DI_{a,t+a-1} &= \frac{\mu^{Y-a}}{Y-a+1}H_{a-1,t+a-1} && \text{for } X+1 \leq a \leq Y \\ C_{a,t+a-1} &= P^H_{t+a-1}DI_{a,t+a-1} && \text{for } X+1 \leq a \leq Y. \end{aligned} \tag{3.22}$$

For $\mu = 1$ the formulation in (3.22) means that the pension system decumulates the capital stock in a linear fashion such that in each retirement period the capital stock is spread evenly over the remaining years. If $\mu > 1$ the disinvestment is "front-loaded" while for $\mu < 1$ it is "back-loaded". Note, e.g., that for the case where $X = 4$ and $Y = 6$ a value of $\mu = \frac{4}{3}$ means that the second (and last) pension payment is exactly half of the first pension. The annuitization in (3.22) guarantees that the capital stock is completely depleted (i.e. $H_{Y,t+Y-1} = 0$).

The asset price P^H_t follows from equalizing asset supply and asset demand, i.e.:

$$\bar{H} = \sum_{a=1}^{Y} H_{a,t+a-1}N_{t-a+1}. \tag{3.23}$$

In the demographic steady state where $N_t = \hat{N}, \forall t$ the normalized gold price $\widetilde{P}^H_{ss} = \frac{P^H_t}{A_t}$ is constant. For the case $\mu = 1$ it holds that:

$$\bar{H} = \hat{N} \left(\sum_{a=1}^{X} \hat{\tau} \sum_{s=1}^{a} \frac{1}{\widetilde{P}_{ss}^{H}} + \sum_{a=X+1}^{Y} \hat{\tau} \frac{Y-a}{Y-X} \sum_{s=1}^{X} \frac{1}{\widetilde{P}_{ss}^{H}} \right).$$

We can solve this to calculate:

$$\widetilde{P}_{ss}^{H} = \frac{\hat{\tau} X Y \hat{N}}{2\bar{H}}. \tag{3.24}$$

As one would have expected, the equilibrium price of the asset in fixed supply increases with the size of the steady-state cohort $\hat{N}$, life expectancy Y, the retirement age X and the contribution rate $\hat{\tau}$ and it decreases with the amount of the available stock $\bar{H}$.

3.4.2 The pattern of the IRR

For the case of fluctuating cohort sizes one can use (3.23) to derive the time series of the asset price and the rest of the equations to determine the levels of consumption in working life and in retirement. As an example, say that the system is in the steady state for $t < 0$ with $N_t = \hat{N}$ but that in $t = 0$ the cohort size changes to $N_0 = \chi \hat{N}$. Before this shock period the asset price has been given by $\widetilde{P}_{ss}^{H}$ (see (3.24)). The new equilibrium price in period $t = 0$ can now be calculated from:

$$\bar{H} = \frac{\hat{\tau}}{\widetilde{P}_{0}^{H}} (\chi + X - 1) + \hat{N} \sum_{a=2}^{X} \sum_{s=1}^{a-1} \frac{\hat{\tau}}{\widetilde{P}_{ss}^{H}} + \hat{N} \sum_{a=X+1}^{Y} \frac{Y-a}{Y-X} \sum_{s=1}^{X} \frac{\hat{\tau}}{\widetilde{P}_{ss}^{H}}.$$

The asset price comes out as $\widetilde{P}_{0}^{H} = \widetilde{P}_{ss}^{H} \frac{X+\chi-1}{X}$. A positive demographic shock ($\chi > 1$) will thus push up the asset price since there are now more people around who want to invest. The following development of the asset price can be calculated in a recursive manner although the analytical expressions get more complicated. Figure 3.2 shows the equilibrium path of $\widetilde{P}_{t}^{H}$ for the same numerical example that has been discussed above for the PAYG system (see Figure 3.1).[9] These solutions of $\widetilde{P}_{t}^{H}$ can then be used in the definition of the internal rate of return (3.13) to calculate the patterns of δ_t for different generations. This is shown in Figure 3.3.

`Insert Figures 3.2 and 3.3 about here`

[9]The figure also contains the asset price curves for other assumptions concerning asset availability and saving behavior that will be discussed in later sections.

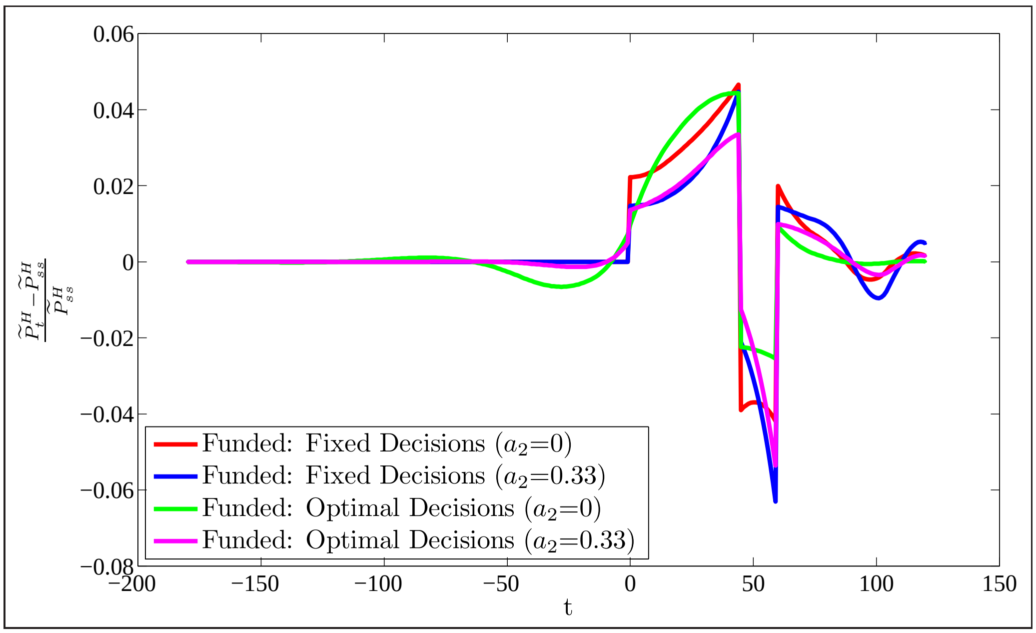
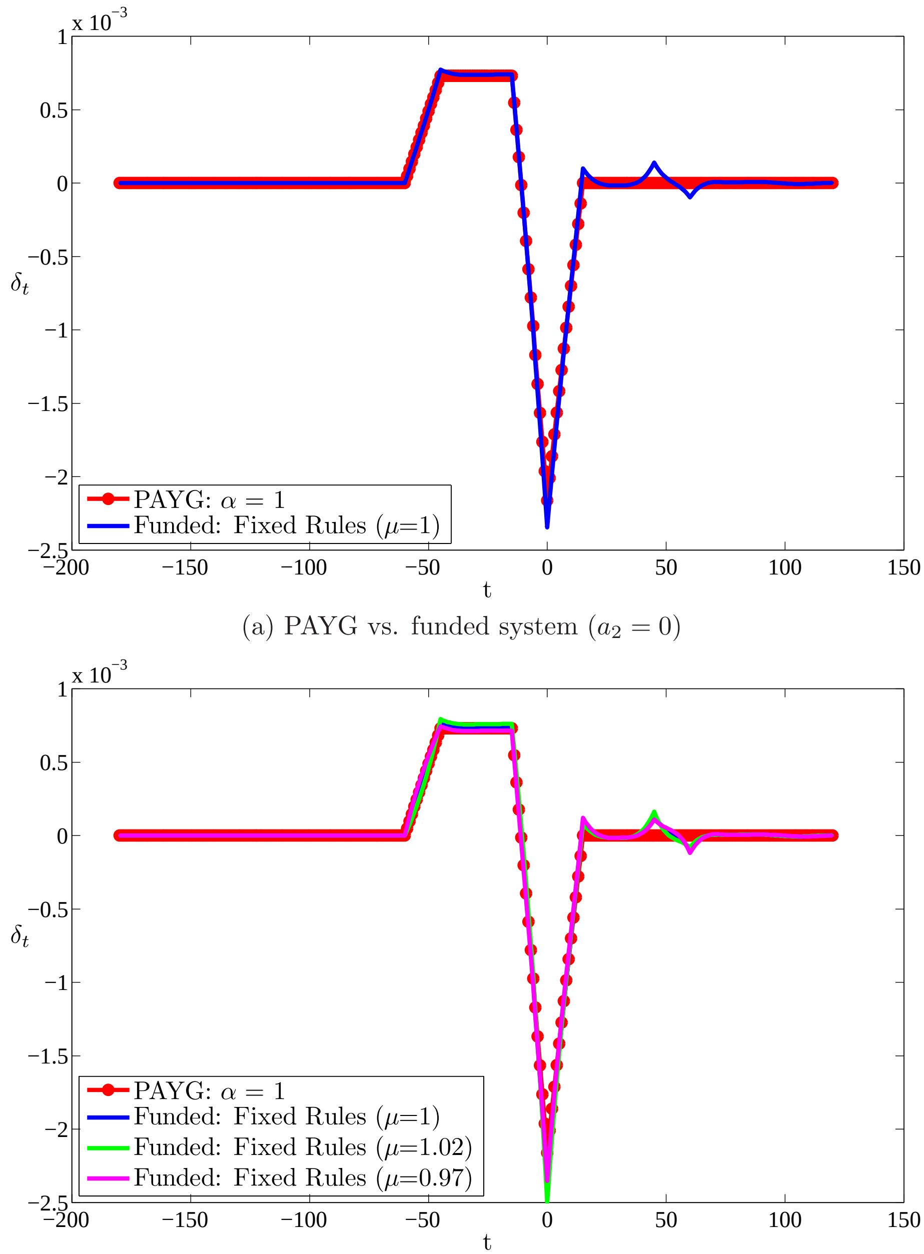


Figure 3.2: The figures show the percentage deviation of $\widetilde{P}_t^H$ from its steady state value under various assumptions concerning asset availability and investment behavior. The first case ($a_2 = 0$ and fixed decisions) is discussed in this section, the second ($a_2 = 1/3$ and fixed decisions) in section 3.5 and the two cases with optimal decisions in section 3.6. The rest of the parameter values are: $Y = 60$, $X = 45$, $g = 0$, $\chi = 2$.



(a) PAYG vs. funded system ($a_2 = 0$)

(b) PAYG vs. funded system ($a_2 = 0$) — Different annuitization rules

Figure 3.3: The figures show the IRR for a funded system where $a_2 = 0$ and where the pension fund has a fixed contribution rate $\hat{\tau} = 1/4$ and follows fixed annuitization rules. In panel (a) this rules is defined by $\mu = 1$ while panel (b) shows three different assumptions ($\mu = 1$, $\mu = 1.02$ and $\mu = 0.97$). For the sake of comparison the figures also show the pattern of the IRR for a DC-PAYG pension system ($\alpha = 1$). The rest of the parameter values are: $Y = 60$, $X = 45$, $g = 0$, $\chi = 2$.

Figure 3.3a contains a number of interesting results. The most striking observation is that the pattern of the generation-specific internal rates of return δ_t of the funded system is basically identical to the one of the PAYG system with $\alpha = 1$. The reason for this is as follows. At the time when the large cohort enters the labor market ($t = 0$) the price of the asset jumps upwards by about 2% since there is now a larger group of workers that want to save and thus a large volume of funds chases the given amount of available assets that are sold due to the fixed annuitization rules. This is clearly visible in Figure 3.2 (where the red line shows the normalized price $\widetilde{P}_t^H$ for the assumption $a_2 = 0$ and fixed savings rules). This price increase evidently has a positive effect on the rate of return of the persons who are already retired. Even the pensioner that is in the last year of his life (i.e. generation -59) gets a small increase in its IRR. In the next period ($t = 1$) the stock of assets in the pension fund is smaller than in the period before. The reason for this is that the cohort that has just retired (i.e. generation -44) has already faced one period of its working life (the previous period $t = 0$) where the asset price $\widetilde{P}_t^H$ has been higher and where the fixed volume of savings $\hat{\tau} W_0$ has bought less pieces of gold than in the steady state constellation. Given the annuitization rule (3.22) of the pension fund this means that the first pension installment of generation -44 is lower and thus the total supply of assets is also lower. The total demand for assets, however, is as large as in period $t = 0$ since the labor force has the same size as before ($L_0 = L_1 = 44 \times \hat{N} + 1 \times 2\hat{N}$) and there is a fixed contribution rate given by (3.21). This disparity of asset supply and asset demand further drives up the asset price (see Figure 3.2) which itself increases the mismatch in the following period $t = 2$ and so on up to period $t = 45$. All generations that are lucky enough to be in retirement or enter retirement in these periods of price hikes benefit from these developments and face a higher IRR. Most privileged are the cohorts -45 to -15, each of which has faced full 15 periods of retirement with asset prices that have been higher than normal. These generations differ in the number of working periods with inflated asset prices and also in the exact magnitude of these elevated prices. It is all the more astonishing that the resulting internal rates of return are very similar and there is only a slight decrease from generation -45 (for which $\delta_{-45} = 0.000772$) to generation -15 (where $\delta_{-15} = 0.000741$).

In period $t = 45$ the large cohort N_0 retires and this leads to a collapse in the asset price (by 8%). This "asset meltdown" is due to the fact that the demand for capital (by the remaining workers) is back to normal while the asset supply is hugely increased by the large cohort of new pensioners. The IRR of generation -14 shrinks by quite a bit since its last period of retirement coincides with the period of this price decrease. In the years that follow the asset price shows a little hump shape. This follows from the interplay of two effects. First, the cohorts that die have less and less capital (since they have faced steadily increasing asset prices when accumulating). This alone would increase asset prices. On the other hand, when cohort +1 retires it had at least one working period with low asset prices (namely period $t = 45$) in which it could accumulate a higher pension capital. These effects interact and result in the hump-shaped pattern.

When the large cohort N_0 dies in period $t = 60$ the asset price jumps up again. This is due to the fact that the cohorts that remain in retirement had to pay for the larger part of their working lives higher prices for their assets and have thus accumulated a smaller stock of capital. These generations, however, also die away and with this process also the asset price decreases until period $t = 98$. The influence of asset prices during accumulation (lower prices = a larger stock of assets) and during decumulation (lower prices = a lower return) creates a number of "echo effects" of this one-period shock that are reflected both in the path of the asset price P_t^H and in the pattern of δ_t. In fact, these variables will oscillate around the steady state without reaching it again in finite time. Note that in the case of the PAYG system these oscillation do not occur and the steady state is again attained in period $t = 15$.

Despite the slight differences in the pattern of the internal rates of return it is still surprising how close the two lines in Figure 3.3a are even though they are based on completely different calculations. One pattern follows from the stipulations of the "sustainability factor" and the other from the ups and downs of asset prices. The logic behind both patterns is, however, basically the same. The generations -60 to -15 are the clear "winners" of the baby-boom cohort N_0. They receive higher pensions in the unfunded and in the funded system where in the first case this is due to the advantageous fluctuations in the dependency ratio z_t and in the second case due to equally advantageous fluctuations in the asset price P_t^H. Both systems are also characterized by the fact that the boom generation itself is the "loser" that shows the lowest IRR among all generations ($\delta_0 = -0.0024$). The doubling of the own cohort size thus reduces the IRR by about a quarter of a percentage point. Although this does not look large one has to take into account that in the exercise we only change the size of one cohort (among 60 cohorts). Noting that the development of fertility is typically rather persistent (and there are well-known echo effects) the difference in the IRR among generations can be quite large.

3.4.3 The role of annuity factors

The exact method how the pension capital is annuitized does not play an important role. In Figure 3.3b I show the result for two different values of μ ($\mu = 1.02$ and $\mu = 0.97$). The first of these alternative mechanisms leads to a pattern of pension payments where in steady state the first pension (at age 66) is about 80% larger than the last pension (at age 80). For $\mu = 0.97$, on the other hand, the first pension is only 47% of the last pension while for the benchmark case with $\mu = 1$ the two pensions are equal. Despite these considerable difference in annuitization Figure 3.3b shows that the patterns of the IRR are almost identical for all three annuitization schemes. For the benchmark case with $\mu = 1$ consumption in the last pension period is not only equal to consumption in the first pension period but also equal to all levels of consumption during working life. This follows from our assumption that $\hat{\tau} = \frac{1}{4}$. It can be shown, however, that the pattern of δ_t is completely independent of the assumptions concerning $\hat{\tau}$ (and $\hat{q}$).[10]

3.4.4 Gold vs. bonds

Another way to make sense of the results in Figure 3.3 is to recognize that for $a_2 = 0$ the non-produced factor ("gold") could alternatively also be viewed as a perpetual bond. The bond does not pay an explicit coupon but its rate of return depends on the development of the price P_t^H. In steady state the rate of return is thus given by the growth rate g. In fact, it is a well-known fact that a PAYG system can be exactly replicated by a bond (e.g. Geanakoplos et al. 1999, Lindbeck & Persson 2003).[11] This was, e.g., underlined by Valdés-Prieto (2005) who has proposed to transform the implicit PAYG into explicit government bonds and who has argued that this will not have any substantive effect on the payoff structure of the PAYG system. An easy way to see this equivalence is by simply noting that in both cases the government (or the public pension fund) collects contributions that are invested into non-productive assets: in bonds or in gold. The system does not accumulate any new capital and it therefore also leaves the capacity of production unchanged.[12] When the pension is due, the government arranges for the

[10] To see this, one can substitute (3.15) into (3.13) or (3.14) and observe that $\hat{\tau}$ drops out.

[11] "[...] Government bonds do not provide a demographic reserve since they are claims on future tax payers just as pay-as-you-go pension claims are claims on future contribution payers. In fact, financing a pension system through domestic government bonds is macroeconomically equivalent to a pay-as-you-go system" (Börsch-Supan, quoted in Barr & Diamond (2009, p. 63)).

[12] This also means that in this case we are dealing with a world where the "Mackenroth thesis" is true. The method of pension financing has no impact on the pension payments since in both cases they have to be covered by national income which is unaffected by the organization of the pension system (cf.

payments (either by redeeming the bond or by selling the gold). The cash flows involved are basically identical. The interesting result of Figure 3.3, however, is to see that the equivalence between an unfunded DC-PAYG system and a funded system is also present when there are no bonds involved but some other non-productive asset in fixed supply. The equivalence is not perfect since the demographic shock causes fluctuations in the asset price that only vanish as times goes to infinity. In practical terms, however, the differences to the PAYG system are small and the effects on the intergenerational distribution are almost indistinguishable. In fact, it would also be possible to construct a government bond or a pension fund invested in gold that has exactly the same payment streams as the DC-PAYG system. For this, however, one would need to define an annuitization scheme that is more complicated than the one described by (3.22) and that also involves some redistribution among the retired cohorts.

3.4.5 How relevant is this case?

One could argue that the case that has been discussed in this section is rather special and that it is only interesting as an extreme example and not as a relevant template to explain real-world events. I would argue on the contrary that this case is more realistic and more revealing than one might think, especially when comparing it to the situation that is emphasized by the proponents of a transition to a funded system. The latter mostly assume (implicitly or explicitly) that such a transition will not only increase saving but that these new savings will also lead to investments in new capital stock.[13] As has been emphasized quite forcefully by Barr and Diamond (2008, 2009) this argumentation is flawed and every element in this chain of reasoning is disputable. In particular, it is not at all clear that the pension funds will be invested in new capital. Casual observation and empirical evidence has in fact shown that a considerable fraction of these resources is channeled into assets that share many characteristics of an asset in fixed supply: into housing ("property is the best investment for retirement"), bonds (where pension funds are sometimes even required to hold a sizable amount of these riskless assets) and, sure enough, gold. In fact, there have also been examples where the introduction of a funded pillar has only increased the price of existing equity without leading to an accompanying increase in stocks, an event that might be termed an "equity price bubble". In all these cases the extra resources of the funded system do not lead to fresh investments and do

Homburg 1988).

[13](Cf. eg. Sinn 2009, p. 230).

not increase the production possibilities of the economy. Certainly, not all of the funds will be invested in such assets in fixed supply and the case discussed in this section is an extreme scenario. Nevertheless, the assumption which dominates the literature that all savings are channeled into new capital investment is equally extreme and in reality one will observe a mixture of both events.

Finally, also the fixed contribution rate $\hat{\tau}$ must not necessarily be related to the stipulations of a central pension fund but can also be interpreted as a short-cut for less-than-fully-rational behavior. In section 3.6 I will discuss the case where the savings decisions are not associated with a fixed contribution rate but follow from explicit optimizing behavior. Although this is the standard approach taken in economic research it is not at all clear whether it is the most realistic description of human intertemporal decision making. In the presence of considerable risk and uncertainty and the ever-lurking influence of myopia and procrastination, a fixed saving rule has often been suggested as an optimal remedy ("nudge") towards these weaknesses. The constant contribution rate $\hat{\tau}$ can thus be understood as such a fixed commitment device.

3.4.6 Summary

The results of Figure 3.3 demonstrate that the often-heard claim that a PAYG system is more receptive and vulnerable to demographic fluctuations and that a funded system can dampen or even counteract these shocks (eg. Modigliani et al. 2000, Börsch-Supan & Gasche 2010) is not correct. This conclusion depends (at least) on the underlying assumptions concerning the availability of assets and the assumptions concerning portfolios choices and investment behavior. Which assumptions are more accurate and plausible is ultimately an empirical questions. The scenario that has been discussed in this section (where the difference between funding and non-funding basically vanishes) is certainly not the only possibility but it contains plausible features and is therefore more than a pathological case.

3.5 A funded system with fixed rules and investments in land

As the next step in carving out the role of funding I now deal with the case where the asset in fixed supply is not only used as a store of value but also plays some role in the production process (i.e. $a_2 > 0$). As a consequence, its rate of return will not only depend on the development of the asset price P_t^H but also on the marginal productivity r_t^H (see (3.8)). It is best to think of $\bar{H}$ in this case as a fixed stock of land that is needed to build factories, to accommodate offices and workers etc.

3.5.1 Set-up

The basic difference to the case where $a_2 = 0$ lies in the fact that now a demographic shock will also change the wage rate (see (3.5)) while before the wage has been fixed at $W_t = A_t$. With substitutable factors of production an increase in the size of one cohort will also increase the size of the labor force thereby lowering its marginal product and thus the wage rate. On the other hand, such a development will make the second factor of production relatively more scarce which will increase its value. As shown in chapter 2, a pension system that is characterized by a sustainability factor as specified in (2.7) and (2.8) is, however, also stable in this constellation.

Turning to the funded system, the central equations are basically the same as before. The only difference is that in the budget constraint (3.8) now $r_t^H \neq 0$ and the annuitization equation (3.22) has to be adapted to:

$$\begin{aligned} DI_{a,t+a-1} &= \frac{\mu^{Y-a}}{Y-a+1} H_{a-1,t+a-1} \left(1 + \frac{r_{t+a-1}^H}{P_{t+a-1}^H}\right) \qquad &&\text{for } X+1 \leq a \leq Y \\ C_{a,t+a-1} &= P_{t+a-1}^H DI_{a,t+a-1} \qquad &&\text{for } X+1 \leq a \leq Y. \end{aligned} \tag{3.25}$$

The contribution rate is again assumed to be $\hat{\tau}$ and thus the equation (3.21) that determines the consumption of young workers is also the same as before. The path of the asset price is still described by equation (3.23), i.e. by the value P_t^H that equates asset demand and asset supply. The factor price r_t^H, on the other hand, is given by (3.4).

3.5.2 The pattern of IRR

The change in factor prices will change the pattern of δ_t for both the funded and the unfunded system. The impact on the latter might at first sound surprising. It follows, however, from the simple fact that now the development of W_t is affected by the change in cohort size while for $a_2 = 0$ the wage rate had been independent of demographics. This is visible in panel (a) of Figure 3.4 where I show the consequences of a one-time demographic shock on the path of the IRR for a defined contribution PAYG system ($\alpha = 1$) for both

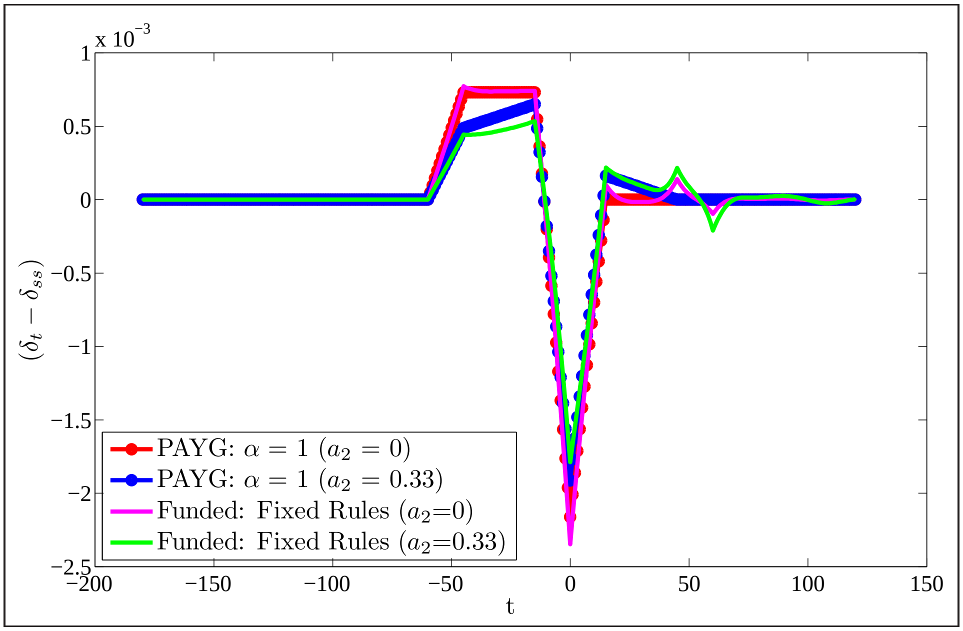


Figure 3.4: The figure compares the development of the IRR for funded systems where either $a_2 = 1/3$ or $a_2 = 0$ and where the pension fund has a fixed contribution rate $\hat{\tau} = 1/4$ and a fixed annuitization rule with $\mu = 1$. Since the steady state values for the IRR depend on a_2 the lines show $\delta_t - \delta_{ss}$, where $\delta_{ss} = 0$ (for $a_2 = 0$) and $\delta_{ss} = 0.034$ (for $a_2 = 1/3$). The other parameter values are the same as in Figure 3.3.

$a_2 = 0$ and $a_2 = \frac{1}{3}$. The main difference is that for $a_2 = \frac{1}{3}$ the IRR increases for the generations between -60 and -15 (while for $a_2 = 0$ it has been flat for generations -45 to -15) and it "overshoots" for generations $+15$ to $+45$. The reason for this is that the boost in the pension level of the earlier generations (the ones born between -60 and -15) is weaker than for $a_2 = 0$ due to the fact that the wages of the working population decrease during the years when generation 0 is part of the labor force. For later generations (the ones born after $t = 15$) the development of the wage rate has a positive impact since their pension level is based on the wage that has returned to the (higher) normal level while their contributions had been a constant fraction of the lower wage level.

`Insert Figure 3.4 about here`

The results for the funded system are also shown in Figure 3.4, both for $a_2 = \frac{1}{3}$ and (as a reference point) for $a_2 = 0$ (cf. Figure 3.3a). One observes that the path of IRR for the funded system again follows the pattern of the PAYG system although the differences are now slightly larger than for $a_2 = 0$. Nevertheless, for all practical purposes the pattern of the two curves is so close that the differences between the two systems can be regarded as negligible. The mechanism behind the pattern of δ_t for the funded system is that the change in W_t has a negative impact on earlier generations (they have less to invest) and a positive impact on later generations (they face a more "solvent" group of buyers). The oscillations that are visible in Figure 3.4 are again the consequence of counteracting price effects and their echoes on the accumulation and decumulation of funds. The rest of the intuition is the same as for the case of gold ($a_2 = 0$).

3.5.3 Summary

The differences between the case where workers invest in an unproductive (gold, $a_2 = 0$) or productive (land, $a_2 = \frac{1}{3}$) asset are qualitatively similar. Although the precise intergenerational pattern of the IRR depends on the role the fixed factor plays in production, the differences to the unfunded pension system are in both cases small. This is the main result related to the general topic of this chapter: funding does not lead to "smoother" reactions to demographic changes than PAYG pension systems. If anything, the patterns in Figure 3.4 suggest that the fluctuations of the funded system are somewhat *larger* than for the unfunded.[14]

3.6 A funded system with optimal investment behavior

So far I have assumed that the contributions to and dissipation of the pension capital follow fixed rules associated with the contribution rate $\hat{\tau}$ (for accumulation) and the annuitization rate μ (for decumulation). As I have argued above this can be due to the institutionally determined stipulations of a funded pension system or it might be a short-cut that captures the rules-of-thumb used by feeble and less-than-fully-rational individuals that try to commit themselves. There exists ample empirical evidence that supports the assumption that people use rules-of-thumb for savings decisions and that they are not behaving in a completely forward-looking and rational way. Nevertheless, it is also interesting to compare the results of the previous section with the case where individuals choose their consumption plans in an optimal way by maximizing their intertemporal utility function (3.7).

[14] I have also looked at the development of δ_t for different annuitizations parameters μ. The results are again similar (as has been the case in Figure 3.3b).

3.6.1 Set-up

The only difference to the cases discussed in sections 3.4 and 3.5 is that the consumption levels are not longer given by (3.21) and (3.22) or (3.25) but rather by the Euler equation:

$$C_{a+1,t+a} = \left(\beta \frac{P^H_{t+a} + r^H_{t+a}}{P^H_{t+a-1}}\right)^{\frac{1}{\rho}} C_{a,t+a-1}, \tag{3.26}$$

where $\frac{P^H_{t+a}+r^H_{t+a}}{P^H_{t+a-1}}$ can be regarded as the total rate of return when one unit of the asset is held from period $t+a-1$ to period $t+a$.[15] In addition, it is still true that the individual does not own anything when he or she is born ($H_{0,t-1} = 0$) and that at the end of life — due to the absence of any bequest motive — the stock of assets will be completely depleted. The rest of the equations (especially (3.4), (3.5) and (3.23)) are the same as before.

3.6.2 The pattern of IRR

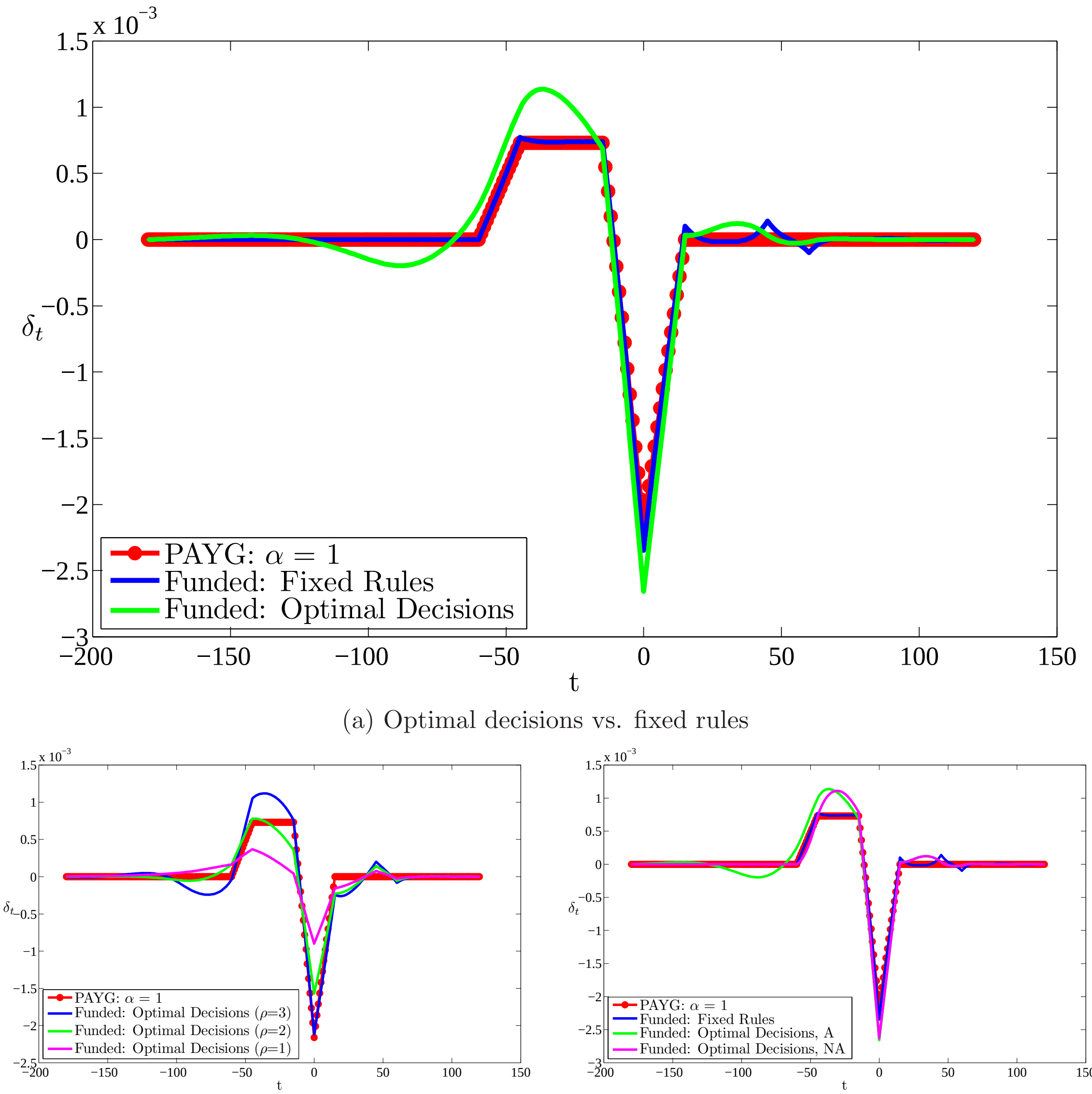
In Figures 3.5 and 3.6 I show the pattern of δ_t for the assumption of optimizing behavior for $a_2 = 0$ and $a_2 = \frac{1}{3}$, respectively. Besides the main results in panel (a) both figures also contain the solutions for some alternative specifications in panels (b) and (c). For the preference parameters I use $\rho = 3$ and $\beta = 0.97$. These magnitudes are in line with assumptions in the related literature (de Menil et al. 2006, Kulish et al. 2010).

`Insert Figures 3.5 and 3.6 about here`

The first thing to note by looking at Figure 3.5 is that optimizing behavior by the agents has no effect on the general nature of how the demographic shock is distrubuted among the different generations. The IRR is highest for the generations ranging from about -45 to -15 and lowest for the "shock generation" 0 and the generations in its temporal vicinity. There are, however, also a number of differences.

First, the forward-looking optimal behavior has the effect that even generations that do not share a single common period with the large cohort are affected by the demographic shock since they adjust their behavior in anticipation of the expected changes in the asset price. Depending on the assumed elasticity of intertemporal substitution the early-born generations will either increase ($\rho = 1$) or decrease ($\rho = 3$) their asset demand in reaction

[15] In terms of the normalized variables, (3.26) reads as: $\widetilde{C}_{a+1,t+a} = \left(\beta \frac{\widetilde{P}^H_{t+a}+\widetilde{r}^H_{t+a}}{\widetilde{P}^H_{t+a-1}}\right)^{\frac{1}{\rho}} (1+g)^{\frac{1-\rho}{\rho}} \widetilde{C}_{a,t+a-1}$.

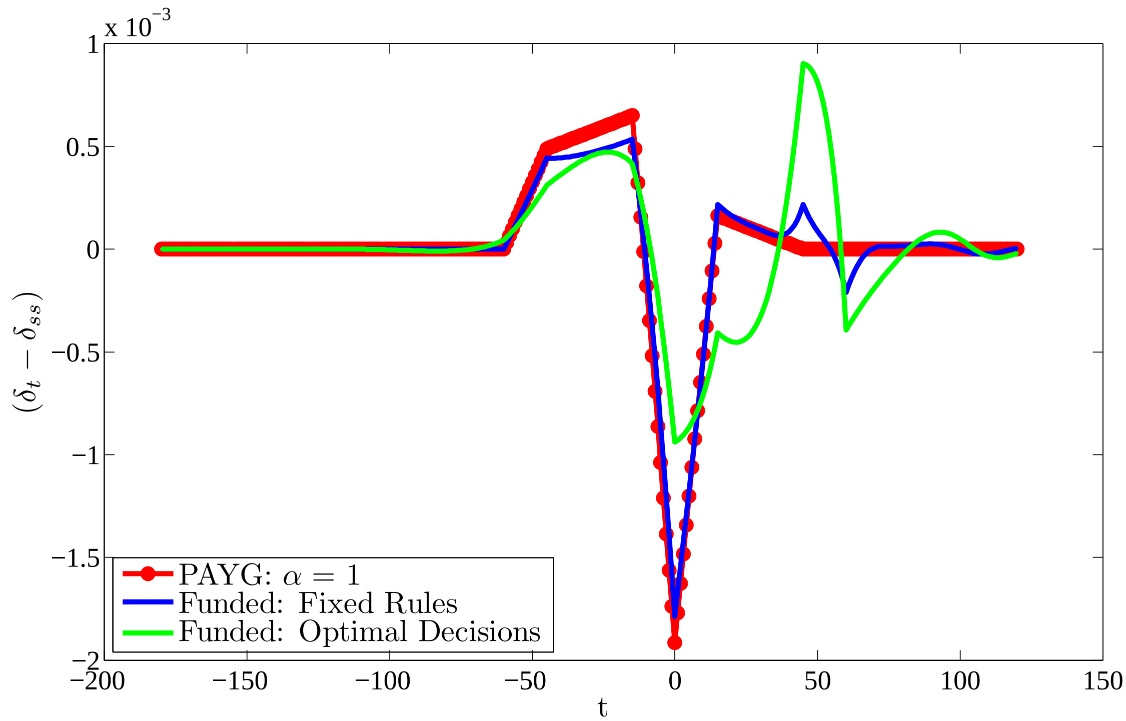


(a) Optimal decisions vs. fixed rules

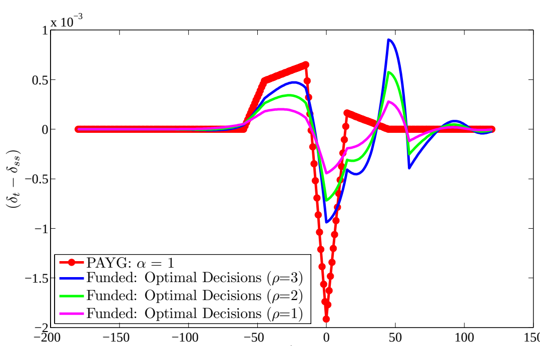
(b) Different values for ρ

(c) Anticipated vs. non-anticipated shocks

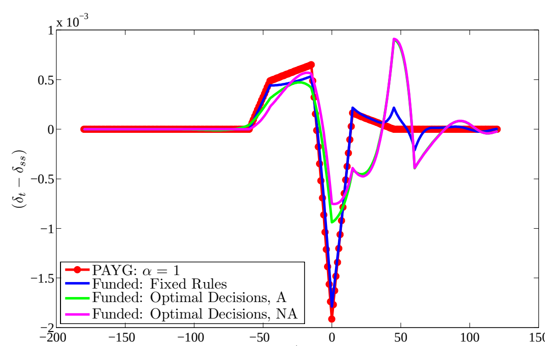
Figure 3.5: The figures show the IRR for a funded system where $a_2 = 0$ and where investment behavior is determined by utility maximization with $\rho = 3$ and $\beta = 0.97$. In panel (a) this is compared to the case of fixed investment rules (for $\hat{\tau} = 1/4$ and $\mu = 1$) and the case of a DC-PAYG system. Panel (b) shows the results for two alternative values of ρ and panel (c) looks at the case where the demographic shock is not anticipated but only recognized in period $t = 0$. The other parameter values are the same as in Figure 3.3.



(a) Optimal decisions vs. fixed rules



(b) Different values for ρ



(c) Anticipated vs. non-anticipated shocks

Figure 3.6: The figures report the development of the IRR for a funded system where $a_2 = 1/3$ and where investment behavior is determined by utility maximization with $\rho = 3$ and $\beta = 0.97$. For the sake of better comparison the lines show $\delta_t - \delta_{ss}$, where $\delta_{ss} = 0.034$ (for fixed decisions) and $\delta_{ss} = 0.063$ (for optimal decisions and $\rho = 3$). In panel (a) this is compared to the case of fixed investment rules (for $\hat{\tau} = 1/4$ and $\mu = 1$) and the case of a DC-PAYG system. Panel (b) shows the results for two alternative values of ρ and panel (c) looks at the case where the demographic shock is not anticipated but only recognized in period $t = 0$. The other parameter values is the same as in Figure 3.3.

to the price hike that will happen when the large cohort enters the labor market. This this early reaction has an effect on the asset price even before period $t = 0$ as shown in Figure 3.2. These price movements are reflected in the shape of the IRRs. Second, the amplitude of the pattern of δ_t also depends on the assumptions about the intertemporal utility function as shown in Figure 3.5b. For $\rho = 3$ the fluctuations are larger than in the case where the investment decisions followed fixed rules. This stems from the fact that with higher ρ people are less willing to substitute consumption between periods and they are thus more exposed to the price changes. On the other hand, a higher willingness to shift consumption between periods also means that prices will fluctuate less since the adjustment is more concentrated on changes in asset holdings. A similar, although less pronounced results holds for the rate of time preference β where a higher degree of patience leads to smaller fluctuations in P_t^H and δ_t (not shown). For certain combinations of ρ and β (see the case for $\rho = 2$ in Figure 3.5b) the pattern of δ_t is in fact very close to the one with fixed decision rules (which itself is almost indistinguishable from the DC-PAYG case). Third, the IRRs for the generations -45 to -15 are no longer (almost) flat but rather show a hump-shape, where the highest IRR is for generation -37 ($\rho = 3$) or -45 ($\rho = 1$). Interestingly, these humps reflect to some extent the empirically observed distribution of fertility ages (where the peak, however, normally occurs around the age of 30). The pattern of the IRR could thus be regarded as a "reward" for the respective reproductive behavior of different cohorts. I will say more on this in chapter 5 when I will deal with the topic of intergenerational fairness.

For the case where the non-accumulated factor is used for production ($a_2 = \frac{1}{3}$) the results are qualitatively the same. What stands out in Figure 3.6 are, however, the large swings in δ_t, in particular for the generations born after $t = 0$. The combination of changing asset prices P_t^H, rates of return r_t^H and wages W_t (all of which are affected by the demographic shock for $a_2 > 0$) together with the possibility to intertemporally substitute consumption leads to surprisingly large fluctuations in the IRR. The amplitude of these swings depends again on the structural parameters of the utility function and they are less pronounced for $\rho = 1$ (see Figure 3.6b).

The model on which Figures 3.5 and 3.6 are based assumes that all individuals know the future development of cohort sizes and that they react to these expected events in an optimal forward-looking way. In fact, this is a common assumption in these multiperiod OLG-models à la Auerbach-Kotlikoff and as a consequence prices and quantities (and thus also δ_t) change way before the jump in the cohort size. For demographic shifts it does not seem completely unreasonable do assume some form of forward-looking behavior. A smaller labor force cohort is at least 20 years visible before it enters the labor market. The extreme farsighted behavior that underlines the patterns in Figures 3.5 and 3.6 is, however, rather peculiar. It implies that even the generation born 100 years before the large cohort takes this century-later change into account when determining its consumption plans. As a contrast one can also look at the other polar case where the demographic change takes all cohorts completely by surprise and it is first noticed in $t = 0$. This is shown in Figures 3.5c and 3.6c. As can be seen in these pictures, the IRR for the generations born before $t = -59$ are now unaffected by the change in the size of cohort 0 and the impact on the generations born between $t = -59$ and $t = -15$ is slightly "shifted to the right". For later generations, however, the pattern of the IRR is fairly similar to the case where the demographic change is perfectly anticipated.

3.6.3 Summary

In order to assess the impact of funding on the intergenerational distribution of demographic shocks and to compare it to the case of an unfunded system one also has to form a judgment about people's behavior. In this section I have looked at the case of utility-maximizing intertemporal behavior while keeping the assumption that workers only invest in the asset in fixed supply. The results are sensitive to the choice of the preference parameters and to the importance of the factor in the production function. If individuals are quite risk-neutral and patient and if the asset is more like gold then the fluctuations in the IRR might be smoother than for the case of a PAYG system (although qualitatively the patterns are still similar). For other assumptions (larger ρ, larger a_2) this conclusion is, however, turned around and the IRR shows much wilder fluctuations for the funded system. Given our ignorance about the degrees of relative risk aversion or the intertemporal elasticity of substitution (both as far as their distribution among the population and as far as appropriate average values for a "representative individual" are concerned) and also about the basic principles of intertemporal behavior in the first place, the results of this section again cannot confirm the claim that funding is unambiguously associated with smoother adjustments.

3.7 A funded system and investments in capital

So far I have assumed that in society there are two types of classes: an "upper class" of superrich people that holds the entire stock of real, accumulable capital K_{t-1} and a "working class" that only transfers resources between different periods of life by buying and selling units of the non-accumulable stock but that—as a whole—does not accumulate any wealth besides $\bar{H}$. In section 3.2.2 I have discussed this structure in more detail and I have argued why I think that it is a useful and informative framework. In this section, however, I drop this assumption in order to study how the results change in the more standard framework of homogeneous households.

3.7.1 Set-up

In particular, I now assume that real capital is the only available asset, i.e. $a_1 > 0$ and $a_2 = 0$.[16] Under these assumptions the model corresponds to a typical one-good economy where produced output can be either consumed or accumulated and where the relative price of capital is thus fixed at 1. The period budget constraint (3.8) can thus be easily transformed to the present case by substituting "K" for "H" and by noting that $P_t^K = 1$. It comes out as:

$$K_{a,t+a-1} = I_a^W W_{t+a-1} + K_{a-1,t+a-2} R_{t+a-1}^K - C_{a,t+a-1}, \tag{3.27}$$

where $R_t^K = 1 + r_t^K - d$ is the rate of return, d is the depreciation rate and r_t^K is the marginal product of capital given by (3.3). $K_{a,t+a-1}$ are the units of capital owned by generation t at age a. As before I use the end-of-period notation for capital and it holds that workers enter and leave their lives without any capital holdings (i.e. $K_{0,t-1} = K_{Y,t+Y-1} = 0$). The aggregate capital stock is given by:

$$K_{t-1} = \sum_{a=1}^{Y} K_{a,t-1} N_{t-a+1}. \tag{3.28}$$

The rest of the model is the same as in the previous section. I will again deal with the cases where there are fixed savings rules (described by the parameters $\hat{\tau}$ and μ) and where saving is chosen in an optimal manner.

As far as the case with fixed decisions is concerned, consumption while young and old is given by expressions similar to (3.21) and (3.25). In particular: $C_{a,t+a-1} = (1-\hat{\tau})W_{t+a-1}$ for $1 \leq a \leq X$ and $C_{a,t+a-1} = \frac{\mu^{Y-a}}{Y-a+1} K_{a-1,t+a-1} R_{t+a-1}^K$ for $X+1 \leq a \leq Y$.

[16] Of course one could also deal with cases where both assets are available at the same time. I would guess, however, that the result will turn out to be basically a mixture of the benchmark cases discussed here. The same is also likely to be true for a model where a PAYG system exists alongside the other assets.

3.7.2 Pattern of IRR

I have used the same parametrization as before (plus $d = 0.05$) to solve the model numerically and to calculate from this the path of the IRR for different generations. The results are shown in panel (a) of Figure 3.7 together with the curves for the case of a DC-PAYG system and for the case where workers can only invest in a non-accumulable factor. In order to make these cases comparable I assume that in both cases with funding the coefficient of the asset in the production function is one third and that in the case of the PAYG system the stock of assets ($\bar{H}$ or K) is constant.

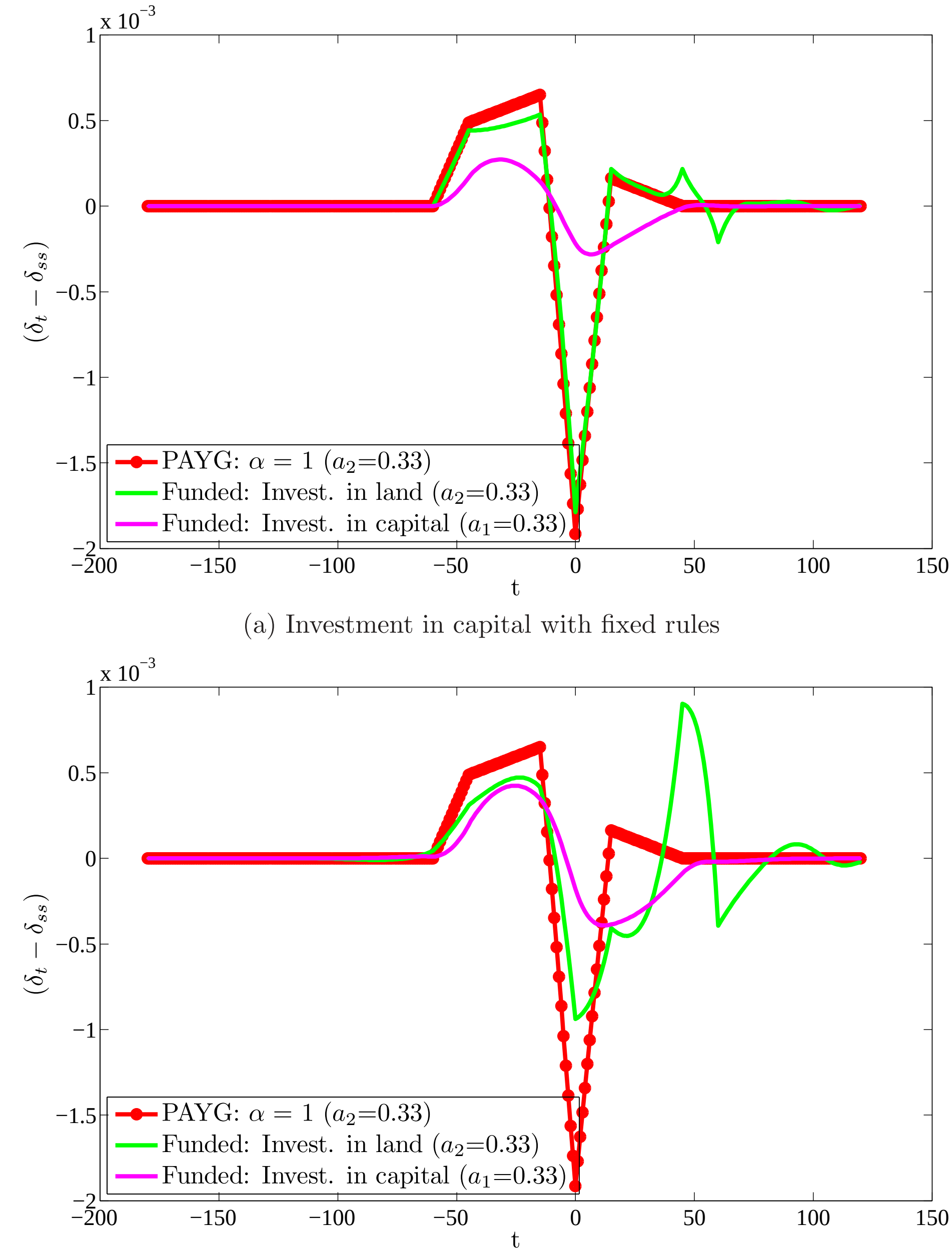
`Insert Figure 3.7 about here`

The pattern for the IRR is considerably smoother in the case where the pension fund invests in accumulable capital rather than in non-accumulable land. The reason for this outcome is that by accumulating and decumulating capital the pension fund (and this means the economy at large) can dampen the fluctuation in factor prices that would occur otherwise and can thus also mitigate the differential impact of the demographic shock on different generations.

As far as optimal behavior is concerned the Euler equation (3.26) has to be slightly adapted to: $C_{a+1,t+a} = \left(\beta R^K_{t+a-1}\right)^{\frac{1}{\rho}} C_{a,t+a-1}$. The rest of the model is again the same as in section 3.6. Figure 3.7b illustrates the pattern of IRR for this case. It is qualitatively similar to the pattern with fixed decision rules which can also be seen in Figure 3.8. Optimizing behavior does seem to be associated with larger swings in δ_t, a phenomenon that could already be observed in section 3.6. This result, however, depends again on the degree of relative risk aversion. Small values of ρ will bring the shapes of the curves in Figure 3.8 closer to each other (not shown).[17]

The availability of real capital thus seems to finally confirm the claim that has been the starting point of this section, namely that funding leads to a smoother adjustment process. It is certainly true that investments in capital and the possibility to accumulate help to dampen the ups and downs of asset prices, rates of returns and the associated IRRs. This is clearly visible in Figure 3.7 where, e.g., the drop in the IRR of generation 0 is much less pronounced than before. So far we have, however, only looked at the case of a defined contribution PAYG system ($\alpha = 1$). For other parameter values the

[17] One might think that in the case of optimal behavior the fluctuations in the utility level of different generations is smaller than for fixed rules, even for the case where $\rho = 1$. I have calculated the intertemporal pattern of U_t (not shown) and the conjecture is not confirmed. The pattern of U_t is almost parallel to the one of δ_t.



(a) Investment in capital with fixed rules

(b) Investment in capital with optimal decisions

Figure 3.7: The figures report the development of the IRR for a funded system where the funds are invested into real capital with $a_1 = 1/3$ (and $a_2 = 0$). For reasons of comparisons the figures also contain the lines for the DC-PAYG system and the case where the funds are invested into land (i.e. $a_2 = 1/3$) and they document $\delta_t - \delta_{ss}$. Panel (a) looks at the case with fixed investment rules (with $\hat{\tau} = 1/4$ and $\mu = 1$) while (b) studies the situation where investment behavior is determined by utility maximization with $\rho = 3$ and $\beta = 0.97$. The depreciation rate is set equal to $d = 0.05$ and the other parameter values are the same as in Figure 3.3.

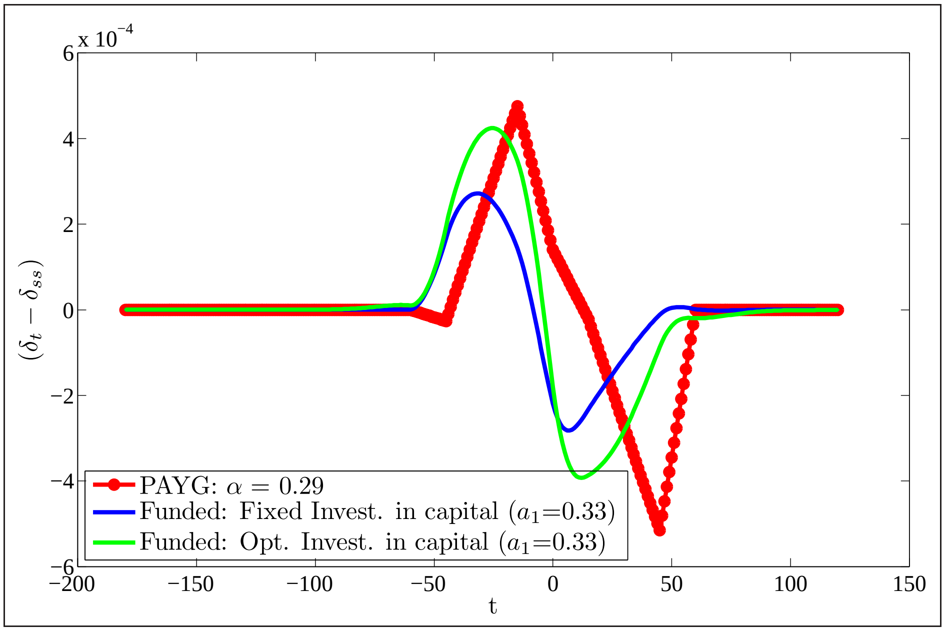


Figure 3.8: The figure compares the development of the IRR for funded systems with investment in capital ($a_1 = 1/3$) with either fixed decisions rules or optimal behavior and a PAYG system with an adjustment parameter of $\alpha = 0.29$ that leads to the smallest intergenerational fluctuations. The other parameter values are the same as in Figure 3.7.

intergenerational patterns of δ_t look quite differently as has been shown in Figure 3.1. As discussed there the parameter that leads to the smallest intergenerational fluctuations in the IRR is given by $\alpha = \alpha^{minvar}$ as stated in (3.19). A comparison of this variant of the PAYG system with the one based on investments in capital is shown in Figure 3.8. In this case the amplitudes of the fluctuations look rather similar, although the curves differ in how strongly different generations are affected.

`Insert Figure 3.8 about here`

3.7.3 Summary

The results of this section have brought mixed results for the claim that a funded system smooths the process of adjustment to demographic shocks. If compared to a defined contribution system the fluctuations are in fact smaller, both for the case with fixed rules and for the case with optimal behavior. Accumulation of real capital dampens fluctuations. The use of PAYG system with smaller values of α (e.g. $\alpha = \alpha^{MinVar}$), however, also reduce the intergenerational differences in δ_t for a PAYG system and for this case the differences in the smoothness of adjustment between funded and unfunded systems look much smaller.

3.8 Conclusion

Advocates of the funded system often claim that even a well-designed PAYG system will necessarily be less able to deal with demographic shocks than a funded system since it will cause large intergenerational differences in the mix of contributions and pension payments and in the internal rate of return and it will thus lead to a less equitable outcome. In this chapter I have shown that this assertion cannot be confirmed in a model that allows for heterogeneous individuals, for different types of assets and different determinants of consumption plans. To the best of my knowledge, the interplay between these factors has so far not been studied systematically in the related literature. As a by-product to the model-based comparison of funded and unfunded systems I have also introduced a new approach to capture the "dichotomous structure" of societies where the stock of capital (firms, machinery, equity etc.) is typically held by a tiny minority whose behavior and life-style are rather distinct from the rest of society. Due to their small size and their wealth the existence of this class is more or less irrelevant for decisions about the design of the pension system. It plays, however, an important indirect role since it influences the path and the fluctuations of factor prices.

The findings of this chapter suggest that the intergenerational pattern of δ_t in funded systems and its difference to unfunded systems crucially depends on a number of factors. In particular:

- **The social structure of society**. Can the economy be described by a sequence of "representative generations" that only differ from each other with respect to their age? Or is there a heterogeneous, dichotomous structure with a "working class" of OLG households and an "upper class" of infinitely lived dynasties?

- **The types of asset in which people invest**. Will saving workers invest in non-productive assets in fixed supply ("gold"), in productive assets in fixed supply ("land") or in productive, accumulable assets ("capital")? The latter is often assumed in the literature but it is not necessarily the most accurate description of reality. The case of gold is sometimes useful as an interesting benchmark.

- **The choice of the intertemporal consumption plan**. Do individuals decide in an optimal way or are they using rules-of-thumb? Extending the analysis of this chapter one could also discuss additional assumptions about investment behavior (e.g. the case of adaptive expectations).

- **Assumptions about individuals' preferences**. The size of the structural parameters ρ and β is important in the case of optimal decision-making. The more impatient and the less willing to intertemporally substitute the larger the fluctuations in IRR will be.

- **Anticipation of future events**. To what extent are demographic changes anticipated or foreseen by individuals? This assumption is again relevant in the case of optimal decisions. In a broad sense it is related to the phenomenon of myopia as a common trait of human behavior.

Table 3.1 summarizes the main results of this chapter.

`Insert Table 3.1 about here`

This list just refers to the factors that have been included within the framework of the formal model of this chapter. In reality, however, there exists a much larger list of factors and assumptions that have to be taken into account when talking about the likely effects of a funded system on the smoothness of adjustment to demographic shocks. For example, even in the case where individuals' savings are invested into real capital one has to think about the kind of capital in which this is done. In the simple deterministic one-good model the investment decision is a rather mechanistic process. Accumulation is here instantaneous and the success immediate. In reality, however, there is a large amount of uncertainty involved. Apparently reasonable capital investments will turn out to be useless and almost without value just a couple of years later. The generations that have invested into these dubious capital goods will face a detrimental shock to their IRR.

In ending this chapter I want to point out, that so far I have not talked about whether "smoothness of adjustment" is in fact a desirable property of pension systems or whether it is a valuable goal in itself. Larger fluctuations might be justified by taken principles of distributive justice or maxims of fairness into consideration. I will deal with this important issue in chapter 5.

Table 3.1: Summary of the main results of this chapter

Production Side Investment Asset	**Household Side** Investment Behavior	**Result—Funded vs. Unfunded**
Gold ($a_2 = 0$)	Rule-of-Thumb	The patterns of the IRR are almost the same for the funded and for the DC-PAYG system.
Land ($a_2 > 0$)	Rule-of-Thumb	The patterns of the IRR for the two systems are very close to each other.
Gold ($a_2 = 0$)	Optimal	The fluctuations in the IRR are qualitatively similar in the two systems. They might be larger (e.g. $\rho = 3$) or smaller (e.g. $\rho = 1$) in the funded system. For some values of ρ and β they are quite similar.
Land ($a_2 > 0$)	Optimal	The differences in the pattern of IRR are more pronounced and they depend on ρ and β. In general they are larger for the funded system.
Capital ($a_1 > 0$)	Rule-of-Thumb	The fluctuations in the IRR are smaller for the funded system than for DC-PAYG.
Capital ($a_1 > 0$)	Optimal	The fluctuations in the IRR are again smaller in the funded system (although larger than for rule-of-thumb behavior). The differences between the two systems are smaller for different variants of PAYG (e.g. $\alpha = 0.29$).

3.9 Related literature

The different version of the model presented in this chapter show a clear dependence of the asset price on demographic variables. It is thus a closely related to the literature on the "asset meltdown hypothesis". A discussion of this hypothesis and empirical tests can be found in Mankiw & Weil (1989), Barr (2000), Brooks (2002), Poterba (2004), Börsch-Supan (2004), Geanakoplos et al. (2004).

Different measures of intergenerational distribution are studied by Geanakoplos et al. (1999) or Fenge & Werding (2003). Empirical estimations of the IRR for the German system are provided by Wilke (2005).

Chapter 4

Unfunded vs. Funded Systems—The Case of Financial Market Risk

4.1 Introduction

So far I have shown that a PAYG system can be designed in a way that it can resist the demographic storm that blows from two directions (chapter 2) and I have argued that a PAYG system does not necessarily lead to a wilder adjustment process in response to demographic shocks than a funded system (chapter 3). The handling of demographic fluctuations is, however, not the only dimension along which pension systems differ. Another crucial dimension are the return-risk profiles that are characteristic for unfunded and funded systems.

The analysis of expected returns is often the starting point for comparing the different forms of financing the pension system. As shown in section 3.3 the internal rate of return of a PAYG system for an economy on a balanced growth path is given by the growth rate of the wage sum ($\delta_t = g + n$) which is lower than the real rate of interest if the economy is in a dynamically efficient state (cf. Abel 1989, Feldstein 1996).[1] The return advantage of funded systems is also reflected in empirical data where the average stock market return exceeds the growth rate of wages. For 16 countries, the average rate of return on equity over the century from 1900 to 2000 has been 5.1% while the real growth rate of GDP (as a proxy for the growth rate of wages) over the same time period has amounted to only 2% (own calculations based on data from Dimson et al. (2002) and Maddison (2003)).

The sole focus on the average rates of return is, however, flawed and—as stressed by Nicholas Barr—"does not compare like with like. A full analysis needs to include (a) the costs of the transition from PAYG to funding, (b) the comparative risks of the two systems, and (c) their comparative administrative costs" (Barr 2000, p. 26). The main focus of this chapter is the relative risk in different pension systems.[2] In the same data sources quoted above one finds, e.g., that the average annual standard deviation of equity returns in the sample of 16 countries is 22.7% while for the growth rate of national income

[1] In fact, this was also the outcome of the numerical examples of chapter 3 where the internal rates of return of the unfunded systems came out smaller than the ones for the funded systems as long as the latter included investments into productive assets. This was not immediately visible from the diagrams in the chapter where I have normalized the IRRs by plotting the difference $\delta_t - \delta_{ss}$. In Figure 3.4, e.g., the respective steady state value were $\delta_{ss} = 0.034$ for the funded and $\delta_{ss} = 0$ for the unfunded system (since the example has used $g = n = 0$).

[2] On the issue of administrative costs see the survey by d'Addio et al. (2009) that comes to the conclusion: "A reasonable assumption for the administrative costs of annuitising retirement capital in defined-contribution pension plans is around 5-10 percentage points of the accumulated balance. This is equivalent to a 0.25-0.50 percentage point reduction in annual returns" (d'Addio et al. 2009, p. 23ff.). On the problem and the costs of the transition of a PAYG to a funded system see Breyer (1989) or Sinn (2000).

it amounts to only 5.1% (Dimson et al. 2002, Maddison 2003).

The different variability of pension payments is therefore an important factor when talking about pension provision. Old people are often not able anymore to smooth income fluctuations by varying their labor supply and as a consequence they are therefore fully exposed to return shocks. If these turn our to be exceptionally large and/or if they come in a sequence of negative values then there exists the clear danger that retired individuals might fall below the poverty line. Excessively volatile pension incomes thus come into conflict with one of the central objectives of pension systems: the prevention of poverty. This is stressed by international organizations like the OECD or the World Bank and by individual researchers alike as quoted in chapter 1. In chapter 3 the focus was primarily on the issue of consumption smoothing (in particular with respect to the smoothing *between* generations) while the issue of poverty has only appeared indirectly in referring to the fact that the pension level q should not become too low. In this chapter I will put particular emphasis on the objective of "poverty prevention".

It is a standard result of portfolio theory that in the presence of a number of assets with different return-risk profiles it is optimal to invest in all of them in different proportions. The same suggestions is also frequently made in the context of pension provision where the main catchword is the "optimal mix" between the two pillars. In fact, the original pension strategy of the World Bank has been based on the multipillar framework. "The study suggests that financial security for the old and economic growth would be better served if governments develop three systems, or 'pillars' of old age security: a publicly managed system with mandatory participation and the limited goal of reducing poverty among the old; a privately managed, mandatory savings system; and voluntary savings. The first covers redistribution, the second and third cover savings, and all three coinsure against the many risks of old age." (World Bank 1994, p. xiv). The report was criticized on various accounts, not least because of the problematic experiences of some countries that have followed the steps outlines in the document. In a 10-years-after assessment the World Bank officially upheld the old multipillar strategy while striking a considerably more cautious tone ("a benchmark, not a blueprint"): "The Bank continues to perceive advantages in multipillar designs that contain some funded element when conditions are appropriate but increasingly recognizes that a range of choices can help policymakers to achieve effective old-age protection in a fiscally responsible manner [...] The proposed multipillar design is much more flexible and better addresses the main target groups in the population. Advance funding is still considered useful, but the limits of funding in some circumstances are also seen much more sharply" (Holzmann & Hinz 2005, p. 1ff). A similar approach is followed by the European Commission or by the OECD: “In the face of risk, it is easy to show that the best approach for an individual — and, by extension, for a government seeking to do the best thing for its citizens — is to use a mixture of sources of retirement incomes. This is why the OECD has long advocated a *diversified approach* to retirement-income provision, arguing that ‘diversity has many virtues’ ” (Whitehouse et al. 2009, p. 47).

These quotations show that there has been a remarkable convergence of opinions of the large international organizations. What is, however, also apparent in these public statements of the advocates of a “mixed” or ”multipillar” system is that they often remain surprisingly vague about the specifics of the preferred model. The notion of a “mix” might suggest that the two pillars should be implemented in equal proportions. But this is, of course, not the only possible combination and one could as well plead for a system with a dominating funded core and a small PAYG-financed component that guarantees a minimum pension. Or one could support a strong PAYG system that is amended by a small funded pillar as a mere supplement.[3]

In general, the “optimal mix” will depend on the comparative return-risk profile of capital-market-based and PAYG-financed systems. There exists a small number of papers that studies how the presence of risk might change the relative attractiveness of funded and unfunded systems (Dutta et al. 2000, Matsen & Thøgersen 2004, de Menil et al. 2006). One property of PAYG systems, however, is not fully taken into account in the existing literature. I will argue that this often overlooked attribute contains a hidden advantage of PAYG systems that further challenges the alleged superiority of funded systems. The property I am talking about is the fact that in an appropriately designed defined contribution PAYG system the average pension is closely (and in principle perfectly) tied to the level of average current wages. The sustainability factor of chapter 2 fulfills the requirement of such a well-designed system and in chapter 6 I will present another set-up (the notional defined contribution system) that also falls into this category. In these appropriate systems one will observe that if wages grow fast then pension benefits will increase at the same speed and if they show a disappointing performance then pensions will also be adjusted in tandem. In other words, the *relative* pension level (the ratio of the average pension to the average wage level) is held constant in these systems. Pensions might be

[3]The Dutch sometimes refer to their pension system as the “cappuccino-mix” that provides coffee as the basic pension for all, milk as the occupational pension for workers and cocoa as the private pension from additional investments. This metaphor shows that the proportions in which the ingredients are combined are not easily determined. Some people prefer a cappuccino while others choose a Café au lait, caffé latte, latte macchiato or any of the other customary coffee specialties.

(and in reality will be) lower than the after-contribution wage of the active population but this relative position itself is deterministic over time, irrespective of the degree of wage uncertainty. This can be seen by looking at the sustainability factors introduced in chapter 2. For the present example I assume that cohort sizes and life expectancy are constant and that the stochastic wage W_t is the only source of risk in the economy. Given the stationary demographic structure, the pension system is completely defined by the three parameters $\hat{\tau}$, $\hat{q}$ and X (and it is in this case independent of α). The pension payment, given by $\hat{q}W_t$, is risky since it depends on the ex-ante unknown value of W_t. At the same time, however, the *relative* pension level—the ratio of the pension payment to the current wage—is completely deterministic and given by $\hat{q}$. If people are not only concerned about absolute consumption but also about their level of consumption relative to some reference group (that includes active workers) then this property will strengthen the advantages of an unfunded system where both the future absolute pension benefit *and* the future relative position are less uncertain than in a funded system.[4] This is an advantage of the PAYG system that is often overlooked in public discussions where supporters of the PAYG system mainly refer to the smaller amount of risk that is contained in these systems. In fact, it should be noted that the "PAYG asset" also differs from government bonds (the epitome of a safe asset) with respect to its risk properties.[5] In particular, the PAYG system really eliminates the uncertainty about the future relative position. Risk-free government bonds, on the other hand, are characterized by a deterministic absolute future payoff but they cannot promise a future return that is directly (and proportionally) related to future wages. If bond markets and labor markets diverge then a pension income that is based on a pure bond portfolio could turn out to be associated with an unfavorable position vis-à-vis active workers. The perfect correlation of pension benefits with the income of active workers is a unique feature of PAYG assets.

In this chapter, I analyze and discuss this argument based on relative consumption preferences in greater detail. First, I develop a simple model in order to present the main

[4]There exists ample psychological, sociological and economic evidence that relative consumption motives and the related phenomenon of habit formation are important human traits (eg. Frank 1985, Weiss & Fershtman 1998, Clark et al. 2008). The consequences of this innate concern for relative standing has been studied in a variety of economic fields, ranging from the analysis of savings and consumption behavior (Frank 1985, Carroll 1998), tax policy (Boskin & Sheshinski 1978, Abel 1990) the relation between inequality and growth (Knell 1999) to the equity premium puzzle (Abel 1990, Campbell & Cochrane 1999).

[5]Geanakoplos et al. (1999) argue that the results of modern portfolio theory imply that a true measure of risk-adjusted financial market return would exactly be the risk-free rate which is of course much lower than the average rate of return on stock markets.

logic of the argument in an intuitive manner. I drop the assumption of a deterministic framework that I have used so far in this monograph. Since allowing for risks makes the model more complicated I also reduce the number of periods. In particular, I work with a two-period OLG model where individuals' utility is assumed to depend on a weighted average of absolute own consumption and the difference between absolute consumption and the consumption level of a reference group. The pension system is assumed to collect contributions that are used for an unfunded and a funded pillar and the latter invests solely in productive and accumulable capital. This simple framework suffices to derive the crucial results and the use of a more sophisticated structure (including, e.g., more periods and a second, non-productive, non-accumulable asset) would add nothing to the main point. I compute the optimal total contribution rate to the pension system and the optimal share of funding (i.e. the percentage of total contributions that should be invested into the funded pillar) that maximize individuals' (ex-ante) expected utility. The model allows for closed form solutions for the optimal parameters of the pension system. It comes out that for risk-averse individuals the optimal degree of funding is lower if asset returns are more volatile and if people are more concerned about their relative standing. The neglect of risk and of social comparison motives will thus overstate the case for a funded system.

The model used in this part has the nice feature that it can be immediately related to the topic of poverty and poverty prevention. The reason for this is that poverty is typically defined as a relative concept (cf. Sen 1983) and the consumption level of the reference group can be understood as a socially defined minimum consumption threshold that defines the poverty line. In industrialized countries this minimum subsistence level is mostly defined as some percentage of median or mean income (typically between 50% and 60%).[6]

In the second part of this chapter I construct a three-period version of the model that is calibrated to real-world data on asset returns and GDP growth rates (as a proxy for the returns of the PAYG pillar). The estimations use return data for the US and for a group of nine industrialized countries (called the "Group of Nine" (G9)) that are based on a long

[6] "For the 'purposes' of international comparison, the OECD treats poverty as a relative (rather than an absolute) concept. It is relative in two senses of the word. First poverty is measured against a yardstick dependent on median household incomes. Secondly, the poverty thresholds are country-specific, so poverty is measured against prevailing norms for living standards in a particular country at a particular time. [...] Most of the analysis [...] sets the threshold for poverty at 50% of median, equivalised household disposable income. People with incomes below this level are counted as 'income poor' " (OECD 2009, p. 63).

time span (1900-1999). The different risk-return-profiles between equity returns and GDP growth rates imply that even in the absence of social comparison motives it is optimal that the pension system contains a sizable unfunded pillar. As soon as the existence of a concern for relative standing is taken into account these shares fall, however, to considerably lower levels, as predicted by the theoretical model. For a moderate concern for relative standing (one that implies a poverty line of 30% of average income) the optimal share of funding does not exceed 20% for any of the cases considered and it is typically even lower than that. A number of robustness tests (most of them are contained in Knell (2010*b*)) confirm this result.

4.2 The model

Individuals are assumed to live for two periods — young and old. In the language of the model of chapter 2 this means that $Y = 2$. They supply a fixed amount of labor (normalized to 1) in the first period and earn a wage W_t. The contribution rate to the PAYG pension system is denoted by τ^U. Consumption in the first period is then given by $C_{1,t} = (1-\tau^U)W_t - S_{1,t}$, where $S_{1,t}$ stands for the private savings of the young. In the second period of their lives ($a = 2$) individuals do not work anymore and their consumption $C_{2,t+1}$ is provided by the payments of the PAYG pension system and by the revenues from their investments. The rate of return on these investments in the financial market is denoted by r_{t+1}. The pension benefit P^U_{t+1} is determined by the balanced budget condition of the PAYG pension system: $\tau^U W_{t+1}N_{t+1} = P^U_{t+1}N_t$, where N_t is the size of the generation born in period t. Under the assumption of a constant population the pension benefit comes out as: $P^U_{t+1} = \tau^U W_{t+1}$ (cf. (2.5)). The contributions $\tau^U W_t$ paid by generation t thus have a return ("notional interest rate") of $g_{t+1} = \frac{W_{t+1}}{W_t} - 1$ and the consumption in old age is given by:

$$C_{2,t+1} = \tau^U W_t(1+g_{t+1}) + S_{1,t}(1+r_{t+1}). \tag{4.1}$$

Instead of assuming that the pension system only runs a PAYG scheme and that only individuals invest in the financial market one could as well assume that the pension system takes the investment decisions on behalf of the individuals. In particular, I will assume that the pension system collects an additional fixed percentage τ^F_t of gross wage W_t that exactly corresponds to the individuals' optimal amount of saving, i.e. $S_{1,t} = \tau^F_t W_t$. These contributions are invested into the financial market where they earn the same rate of return r_{t+1} that can be achieved by private investments. Furthermore, it is assumed that the contribution rate to the funded pillar is constant over time (i.e. $\tau_t^F = \tau^F, \forall t$) which is in parallel to the constancy of the contribution rate to the unfunded pillar. Under these assumption one can use (4.1) to write the total pension benefit P_{t+1} as:

$$P_{t+1} = C_{2,t+1} = W_t \left[\tau^U(1+g_{t+1}) + \tau^F(1+r_{t+1})\right] = \tau W_t \left[(1-\lambda)(1+g_{t+1}) + \lambda(1+r_{t+1})\right], \tag{4.2}$$

where $\tau \equiv \tau^U + \tau^F$ denotes the total contribution rate to the pension system and $\lambda \equiv \frac{\tau^F}{\tau}$ stands for the share of the total contribution rate that is devoted to the funded pillar (i.e. to investments into the financial market). This is the specification of the problem that will be used in the following. In particular, I will take the viewpoint of a social planner who has to decide on the optimal values of τ and λ, while the individuals just consume what is determined by the pension system. This formulation is, however, completely equivalent to the situation where the social planner only decides about the optimal contribution rate τ^U to the PAYG system while taking individuals' optimal savings decisions into consideration.[7]

In contrast to other papers in the literature, I allow for the possibility that individuals care not only about absolute consumption but also about the level of their own consumption relative to the one of a reference group. In particular, it is assumed that the utility function for the representative individual of generation t is given by:

$$U_t = \frac{1}{1-\rho}\left(C_{1,t} - \theta D_{1,t}\right)^{1-\rho} + \frac{\beta}{1-\rho}\left(C_{2,t+1} - \theta D_{2,t+1}\right)^{1-\rho}. \tag{4.3}$$

Here $D_{1,t}$ ($D_{2,t+1}$) denotes the consumption of generation t's reference group when they are young (old). The parameter $\theta \geq 0$ measures the importance of the concern for relative standing, β stands for the time discount factor and ρ measures the curvature of the utility function. For $\theta = 0$ equation (4.3) reduces to the usual CRRA function as has been, e.g., also used in chapter 3 as (3.7).[8]

There exists a long literature on the question what constitutes the most appropriate specification of people's reference groups. A standard assumption in this context is that it consists of the average consumption level in a society and I will employ this assumption

[7] Note that in chapter 2 I have only looked at cases for which $\lambda = 0$ and I have assumed a particular value for $\hat{\tau}$.

[8] It is assumed in the following that the parameter values of the crucial variables are always determined in a way such that $C_{1,t} \geq \theta D_{1,t}$ and $C_{2,t+1} \geq \theta D_{2,t+1}$. If these conditions were not fulfilled then the utility function would not be well defined.

for the later empirical estimations. For the example in the theoretical part I will, however, use the simplifying assumption that comparisons are only relevant in old age and that therefore young individuals do not have a reference group while pensioners compare themselves solely to the active population. Expressed in the language of the model this assumption thus implies that $D_{1,t} = 0$ and $D_{2,t+1} = C_{1,t+1} = (1-\tau)W_{t+1}$. This assumption is not very realistic but it allows for closed form solutions and is therefore useful to develop some intuition.

Finally, it is assumed at the moment that wages are constant and deterministic ($g_t = 0, \forall t$) and that the rate of asset return can only take on two values: high and low. In particular, r_t has an equal probability to take on the values $r_H = \mu_r + \varepsilon_r$ and $r_L = \mu_r - \varepsilon_r$, respectively. It holds that $E(r_t) = \mu_r$ and $\sigma_r^2 = \varepsilon_r^2$. It is assumed that $r_L < 0$, i.e. that the bad asset market outcome is worse than the constant wage growth. Without this condition the asset market would always dominate the PAYG system under the assumption of a two-point distribution. In this simple model the volatile interest rate is thus the only source of uncertainty. In the empirical section the assumption of constant wages will of course be lifted.

4.3 The optimal design of the pension system

In order to analyze the relative attractiveness of funded and unfunded pensions systems I compute the optimal size of the pension system τ^* and the optimal share of the funded pillar λ^* that maximize expected utility.

Inserting the assumptions about the reference groups and asset returns into the utility function (4.3) one can derive an explicit expression for expected utility:

$$E[U_t] = -\frac{W_t^{1-\rho}}{\rho - 1} \left\{ (1-\tau)^{1-\rho} + \frac{\beta}{2} \left[\begin{array}{c} [\tau(1+\lambda r_H) - \theta(1-\tau)]^{1-\rho} + \\ [\tau(1+\lambda r_L) - \theta(1-\tau)]^{1-\rho} \end{array} \right] \right\} \tag{4.4}$$

It is assumed that $0 \leq \lambda^* \leq 1$, i.e. that short sales of the financial asset are excluded. The optimal contribution rate and the optimal share of funding can now be calculated from the first-order conditions. In the appendix of Knell (2010*b*) I derive the solution for the general case with $\rho \geq 1$. Here I want to focus on the case of log utility ($\rho = 1$). The optimal contribution rate τ^* and the optimal share of funding λ^* come out as:

$$\tau^* = \frac{\beta + \theta(1+\beta)}{(1+\theta)(1+\beta)} \tag{4.5}$$

$$\begin{aligned}\lambda^* &= \min[1, \hat{\lambda}] \text{ where}\\ \hat{\lambda} &= \frac{\beta(1+\theta)}{\beta+\theta(1+\beta)}\frac{\mu_r}{\varepsilon_r^2-\mu_r^2} \geq 0.\end{aligned} \tag{4.6}$$

From expression (4.5) and (4.6) it follows that: $\frac{\partial\tau^*}{\partial\beta} \geq 0$, $\frac{\partial\tau^*}{\partial\theta} \geq 0$, $\frac{\partial\hat{\lambda}}{\partial\beta} \geq 0$, $\frac{\partial\hat{\lambda}}{\partial\theta} \leq 0$, $\frac{\partial\hat{\lambda}}{\partial\mu_r} \geq 0$ and $\frac{\partial\hat{\lambda}}{\partial\varepsilon_r} \leq 0$.

These formulas have a number of interesting implications. Starting with the optimal contribution rate, the proposition indicates that τ^* increases in both β and θ. If people care more about the future (high β) and/or if they are more concerned about their relative standing during retirement (high θ) then the social planner will secure a higher and more stable income in old age and implement a higher contribution rate τ. In the absence of social comparisons this optimal contribution rate is simply given by $\tau^* = \frac{\beta}{1+\beta}$. If in addition to log-utility one also assumes a high degree of patience ($\beta = 1$) then $\tau^* = \frac{1}{2}$. Note that in this case the after-contribution wage $(1-\tau)W$ is equal to the pension payment $q \times W$. I have also used this case for the benchmark numerical example in chapters 2 and 3 (where the contribution rate was $\hat{\tau} = 1/4$ since there $Y = 60$ and $X = 45$).

For the discussion of the optimal share of funding I will concentrate on the case where $\lambda^* < 1$ (i.e., where the short sales constraint is not binding). First, one can observe that a higher return on equity ceteris paribus increases the advantage of the funded pillar, i.e. $\frac{\partial\hat{\lambda}}{\partial\mu_r} \geq 0$. In particular, if one assumes that the worst asset return r_L is the same as the return of the PAYG system (i.e. $r_L = g = 0$ or $\mu_r = \varepsilon_r$) then $\lim\limits_{\varepsilon_r \to \mu_r} \hat{\lambda} = \infty$ and the pension system only uses the funded pillar. On the other hand, however, the attractiveness of the funded pillar decreases with the riskiness of asset returns ($\frac{\partial\hat{\lambda}}{\partial\varepsilon_r} \leq 0$). The return advantage of the funded system ($\mu_r > 0$) is thus increasingly counteracted by higher return uncertainty. For sufficiently high levels of asset return uncertainty the pension system will in fact use exclusively the unfunded pillar, i.e. $\lim\limits_{\varepsilon_r \to \infty} \hat{\lambda} = 0$. Furthermore, a higher concern for relative standing is also associated with a lower optimal share of funding ($\frac{\partial\hat{\lambda}}{\partial\theta} \leq 0$). The same asset return uncertainty is more cumbersome if individuals care a lot about the level of consumption of their reference group.

The intuition behind this result is straightforward. Risk-averse individuals dislike situations with uncertain payoffs since the disutility of a bad state outweighs the additional utility of an equally sized favorable outcome. If people are concerned about their relative standing then they fear that a bad shock might drive them near to the reference standard $\theta D_{2,t+1}$, which can be interpreted as the socially defined subsistence level (or the poverty line). The more they approach this subsistence level the larger the disutility will get.

A higher concern for relative standing (measured by θ) therefore means that the dislike for fluctuating outcomes will become stronger and that the optimal share of funding will decrease. Individuals will increasingly prefer the unfunded pillar that promises lower but at the same time less uncertain returns. Put differently, an increase in θ has the same qualitative effect as an increase in ρ since both changes will increase the degree of relative risk aversion which for the functional form used in (4.3) is given by $RRA = \frac{\rho C}{C-\theta D}$ (see Meyer & Meyer 2005).

The crucial feature that underlies the main result is that a PAYG pension system fixes the relative position in old age. This is in fact not a consequence of the specific assumptions on which this proposition is based but it is rather more general. In particular, it also holds for uncertain wage growth and for different reference groups. This can be seen most easily for the case where the PAYG system provides the only source of old-age income (i.e. $\tau^F = \lambda = 0$). In this case a defined contribution pension system implies that for a stationary demographic development the pension benefit is proportional to the income of active workers. To see this, note that under these circumstances the consumption of young individuals (workers) in period t is given by $C_{1,t} = (1-\tau)W_t$ while consumption of the old is given by $C_{2,t} = \tau W_{t-1}(1+g_t) = \tau W_t$. Thus for any combination of the two consumption levels the reference consumption levels $D_{1,t}$ and $D_{2,t}$ will be directly proportional to W_t. Therefore the relative position of young and old individuals vis-à-vis their respective reference groups will be constant over time even if the growth rate g_t fluctuates.

4.4 Empirical estimation of the optimal mix

4.4.1 Model set-up for the empirical part

In order to get some feeling for the empirical implications of the model and to get a rough estimate for the optimal mix of real-world pension systems I use a three-period model (i.e. $Y = 3$). In particular, instead of (4.3) I use:

$$U_t = \sum_{a=1}^{3} \beta^{a-1} \frac{1}{1-\rho} \left(C_{a,t+a-1} - \theta D_{a,t+a-1}\right)^{1-\rho}, \tag{4.7}$$

where $C_{a,t+a-1}$ is the consumption of generation t at age a and $D_{a,t+a-1}$ stands for the consumption of generation t's reference group in this period. Furthermore, it is assumed that the cohort size is constant and that all generations work for two period ($X = 2$) and receive a pension payment for the remaining period of their lives. The rest of the set-up is the same as above. Working individuals earn a wage W_t and pay a contribution rate τ. A share λ of total contributions is used for the funded pillar and a share $(1 - \lambda)$ for the unfunded pillar. The assumption $Y = 3$ and $X = 2$ corresponds to a situation where one period lasts for 20 years and where individuals start to work at the age of 20, retire at the age of 60, and die at the age of 80—a "life-cycle" that is more or less in line with the actual pattern in many countries (and close to the benchmark assumption used in chapters 2 and 3).

The pension payment for generation t in the retirement period is denoted by $P_{3,t+2} = C_{3,t+2} = P^U_{3,t+2} + P^F_{3,t+2}$, where $P^U_{3,t+2}$ ($P^F_{3,t+2}$) is the pension benefit from the unfunded (funded) pillar. The unfunded pension is calculated like in a NDC system (see chapter 6) with the notional interest rate set equal to the growth rate of wages. In this case the PAYG pension is always strictly proportional to the current wage level as has already been the case in the 2-period model. The funded pension is equal to the accumulated capital from the contributions to the funded pillar. Details of the model and its calibration can be found in the appendix of Knell (2010*b*).

For the benchmark case I assume that the reference standard is now given by society-wide average consumption:

$$D_{a,t} = \bar{C}_t \quad \text{for } a = 1, 2, 3, \tag{4.8}$$

where $\bar{C}_t = \frac{1}{3}\sum_{a=1}^{3} C_{a,t}$ stands for average consumption in period t. Given the structure of the model lifetime utility of generation t depends on asset returns up to period $t+2$ (the last period of his or her life). In the other direction, however, the members of generation t are also influenced by economic variables that materialized before they were even born in as far as these variables will affect consumption of older generations and thus also the average consumption level. In particular, the reference standard $\bar{C}_t$ in generation t's initial period of life includes the consumption of generation $t-2$ and thus generation t is affected by macroeconomic variables that affect this generation's asset returns.

For the simulations one has therefore to deal with the important question of which utility concept should be used to derive the optimal design of the pension system. In particular, under the assumption of society-wide comparisons (cf. (4.8)) expected lifetime utility of generation t depends not only on the initial wage W_t and the future returns r_{t+1}, r_{t+2}, g_{t+1} and g_{t+2}, but also on the present and past returns r_t, r_{t-1}, g_t and g_{t-1} (which are known when generation t starts to work). There exist two possibilities for calculating expected utility. Either one treats W_t and the past and present returns as given which means that the future rates of return are the only sources of uncertainty in the model. Or one can fix W_t and calculate expected utility under the assumption that *all* (past, present and future) rates of returns are uncertain. In the related literature these different concepts of expected utility are treated under different names, e.g.: traditional vs. Rawlsian (Matsen & Thøgersen 2004), true vs. ex-ante (Hassler & Lindbeck 1997) or ex-post vs. ex-ante (Wagener 2003) risk-sharing. For my simulations I use the concept of ex-ante (or Rawlsian) risk-sharing since I want to look at the choice between funded and unfunded pension systems from the perspective of an intertemporal social planner who evaluates utility for a generation behind the veil of ignorance (see chapter 5). The problem of such a planner can be described as a situation where he has to consider *all* possible histories of returns that might have lead to some given wage level W_t and where he has to choose the optimal levels of τ and λ given these possible histories.

4.4.2 The data

For the data on asset returns and wage growth I make use of two sources. Dimson et al. (2002) provide data on real equity returns for a number of countries and 10 non-overlapping decades from 1900-1999. I take these decades data to derive the geometric mean and the standard deviation (SD) of 20-year returns based on 9 overlapping periods (1900-1919, 1910-1929, ..., 1980-1999). These summary statistics are reported in Table 4.1 for the US and for a pooled group of nine large economies included in the dataset (US, Japan, Germany, France, UK, Italy, Spain, Canada and Australia).[9]

`Insert Table 4.1 about here`

[9]In calculating the long-run averages it is assumed that the equity returns for this group of countries are realizations from the same underlying stochastic process. The summary statistics for this pooled country group are thus based on all 81 observations of 20-year returns for the nine individual countries. In Knell (2010*b*) I also report and discuss the return data for a number of additional countries and country groups.

Table 4.1: Summary statistics on equity return and on GDP growth rates

	Real return on equities over 20 years		Real per capita GDP growth over 20 years		
Country	Mean	SD	Mean	SD	Corr.
	1900-1999				
US	243.6%	263.6%	46.8%	11.6%	0.073
Group of Nine (G9)	152.3%	386.3%	47.3%	50.5%	0.484
	1950-1999				
US	302.9	337.3%	56.5%	8.6%	-0.751
Group of Nine (G9)	255.9%	493.6%	74.6%	59.6%	0.416

Source: Own calculations based on Dimson et al. (2002) and Maddison (2003). The (geometric) means are calculated from 20 year periods, for the upper panel periods between 1900 and 1999 and for the lower periods between 1950 and 1999. The (sample) standard deviations (SD) and correlations (Corr.) are based on the same data of 20-year returns. For the summary statistics of the country group "Group of Nine (G9)" the data for nine large economies (US, Japan, Germany, France, UK, Italy, Spain, Canada and Australia) are pooled.

Using data from Maddison (2003) one can also calculate the growth rates of real per capita GDP for the same sequence of overlapping 20 year intervals. Since data on real wage growth are not available for most countries over the time span from 1900 to 1999 it is assumed that per capita GDP is a reasonable proxy for wage developments. Geometric averages and standard deviations of real growth are replicated in the third and forth column of Table 4.1. The last column of the table reports the coefficient of correlation between equity returns and GDP growth rates.

It has been argued that the data from 1900 to 1999 cover some exceptional periods (years of depression, two world wars etc.) and that they are therefore not a reasonable guide for future investment returns and growth rates. In order to account for this (not completely uncontroversial) argument I also construct summary statistics for the post-war period starting in 1950 and shown in the lower part of Table 4.1.

The table shows that equity returns are higher but at the same time much more volatile than GDP growth. For the US, e.g., the mean equity return over the average 20-year period was 243.6% (corresponding to an average annual return of roughly 6.4%) with a standard deviation of 263.6%. There exist, however, considerable differences in the return-risk profile between different countries. The Anglo-Saxon countries (US, UK, Canada and Australia), e.g., typically show a higher average equity return than the European countries or Japan. These cross-country differences (especially during the first part of the last century) are reflected in the summary statistics for the country-group G9 that shows a large standard deviation (386.3%). The differences between countries (and pooled countries) are less pronounced in the data sample 1950-1999. It is remarkable, however, that this shorter period is in general characterized by higher 20-year returns (255.9% for the G9) and a higher standard deviation (493.6%) than the longer period.

The data for real per capita GDP, on the other hand, show a different picture. Real GDP in the US, e.g., has increased by only 46.8% over the average 20-year period between 1900 and 1999 (corresponding to an average annual growth rate of roughly 1.9%). The fluctuations in GDP growth are, however, also considerably less pronounced than the volatility of equity returns. The standard deviation is typically lower than the mean (and often considerably lower) while the contrary is true for equity returns. The last column of Table 4.1 contains the correlation between the growth rates of GDP and of equity returns. The results here are not very consistent which might also be a consequence of the fact that they are based on a small number of observations.

4.4.3 Calibration and Simulation

In order to simulate the model one has to calibrate a number of parameters that are related to individual preferences: ρ, β and θ. For the curvature of the utility function I set again $\rho = 3$. This is the same as in chapter 3 and close to the values chosen in the related literature (Matsen & Thøgersen 2004, de Menil et al. 2006, Feldstein & Ranguelova 2001). For the discount factor I choose a value of $\beta = 0.55$ which corresponds to an annual discount rate of 3% (again the same value as in chapter 3).

There exists less literature on the appropriate values for the strength of the concern for relative standing θ. It is, however, possible to gauge plausible values by referring to two classes of evidence. The first stems from the experimental choices between hypothetical societies conducted by Johansson-Stenman et al. (2002). In particular, in the experiments individuals are asked to make repeated choices between two societies that differ with respect to the average income and the income of the respondent's hypothetical grandchildren. For an assumed utility function one can then get an indication of an individual's concern for relative standing by looking at the societies between which the respondent is indifferent. The overall results in Johansson-Stenman et al. (2002) show some differences depending on the specific characteristics of the choice experiments and the assumed form of the utility function. For most cases, however, the value of the concern for relative standing is in the range between 0.2 and 0.5.

As a second source of evidence for plausible parameter values of θ one can use the literature on the poverty line. The poverty line is defined as the minimum level of income that is deemed necessary in order to achieve an adequate standard of living and to participate in social life. Most developed countries use a poverty line that is specified in terms of relative income. Institutions like the OECD and the EU employ a threshold of 50% or 60% of national median equivalised household income (see footnote 6). The poverty line in the model of this paper is given by $\theta\bar{C}_t$, where the reference standard refers, however, to the mean and not to the median. Since the median income is typically lower than the mean income the values of 50% or 60% would overstate the true weight of the relative component. In order to correct for this bias one can make a crude adjustment by assuming that median income is about 80% of mean income.[10] 50% of median income thus corresponds to 40% of mean income.

The results from the experimental choices and the definition of the poverty line thus suggest that parameter values for θ between 0.2 and 0.4 can be regarded as reasonable. For the sake of comparison the estimations will report results for the optimal pension design for values of θ between 0 and 0.5.

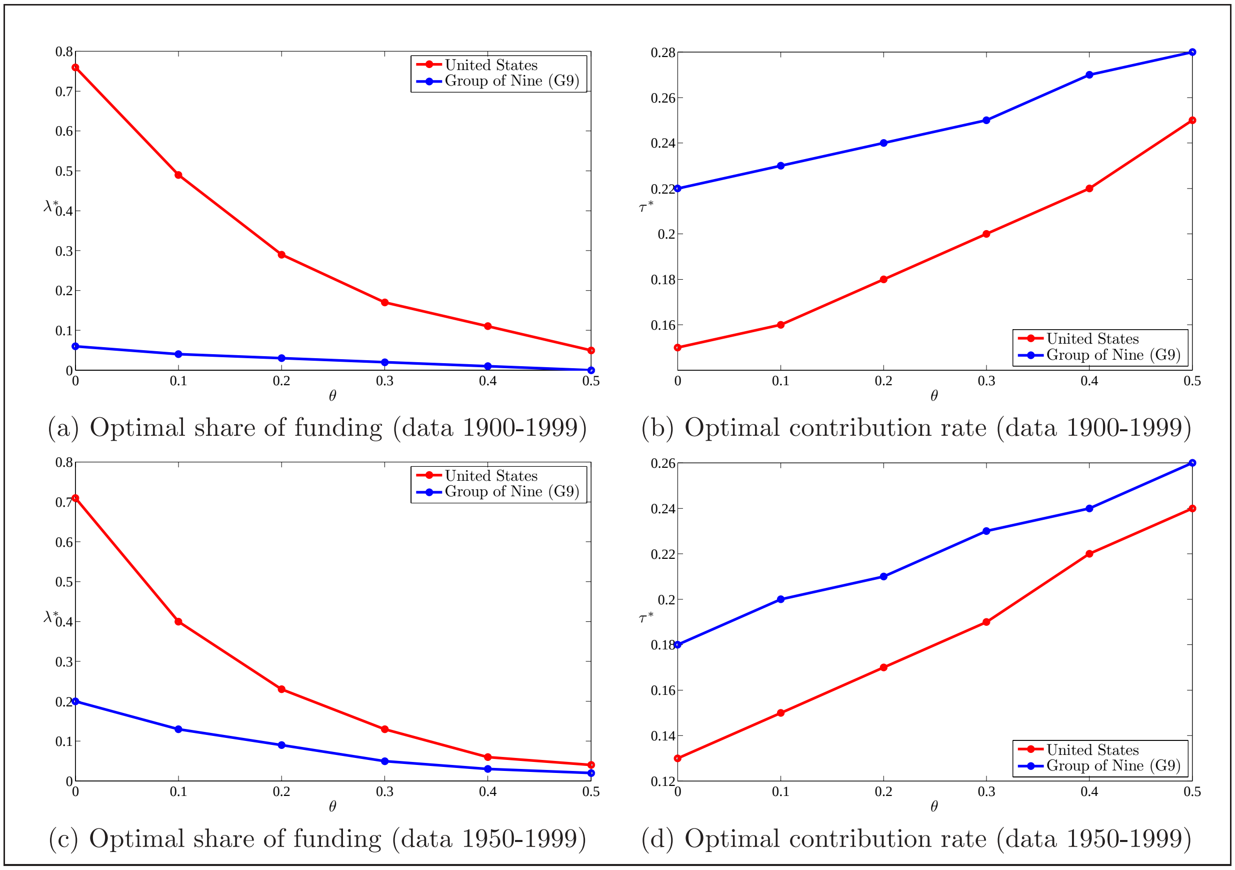
For the simulation of return data it is assumed that $R \equiv 1 + r$ and $G \equiv 1 + g$ are jointly lognormally distributed where the expected values, variances and covariances are derived from the data in Table 4.1.[11] For each simulation run 5 data points are drawn both for equity return and for GDP growth. For a given set of simulated data points lifetime utility for generation $t = 3$ (and for a fixed level of initial income W_3) is evaluated for various values of τ, λ and θ. This measure corresponds to the concept of "Rawlsian expected utility" (see above and Matsen & Thøgersen (2004)). The "optimal" (i.e. utility-maximizing) values of τ and λ for country i and a concern for relative standing θ are denoted by $\tau_i^*(\theta)$ and $\lambda_i^*(\theta)$. Further details of the simulation can again be found in Knell (2010*b*).

4.4.4 Benchmark results

Panels (a) and (b) of Figures 4.1 show the pattern of $\lambda_i^*(\theta)$ and $\tau_i^*(\theta)$ for the country-data based on the period 1900-1999 while panels (c) and (d) do the same thing for the data

[10] "The median [...] is typically between 80 and 85% of mean earnings" (OECD 2009, p. 130).

[11] For the US 1900-1999 sample this means, e.g., that $E(R) = 3.44$, $SD(R) = 2.64$, $E(G) = 1.47$, $SD(G) = 0.12$ and $Cor(R, G) = 0.07$.



(a) Optimal share of funding (data 1900-1999) (b) Optimal contribution rate (data 1900-1999)

(c) Optimal share of funding (data 1950-1999) (d) Optimal contribution rate (data 1950-1999)

Figure 4.1: The optimal share of funding λ^* and the optimal contribution rate τ^* as a function of the concern for relative standing θ for the US and for a group of nine large countries (G9).

based on the shorter period.

`Insert Figure 4.1 about here`

Disregarding the presence of social comparisons ($\theta = 0$) one observes that for the US the optimal share of funding λ^* is rather large (76% for the long period and 71% for the short period). For the country G9 that pools the return data for all large economies the optimal share of funding is, however, considerable lower (20% using data from 1950-1999 and even lower for the long time span). These results indicate that even without the consideration of preferences for relative consumption the return advantage of equity can be considerably thwarted by the higher risk of stock market investments.

If the existence of reference standards and relative poverty is taken into account ($\theta > 0$) the optimal share of funding is reduced for both countries. In fact, as illustrated in Figure 4.1, the simulation results indicate that for all examples shown the optimal share of funding decreases monotonically (i.e. $\frac{\partial \lambda_i^*(\theta)}{\partial \theta} < 0$) and in a non-negligible way. If one defines, e.g., the social reference standard as 20% of average wages (a rather modest definition) the optimal share of funding for the US falls from 76% (71%) to only 29% (20%). For the G9, on the other hand, the optimal share is reduced to a values between 3% and 9%. A reference standard weight of $\theta = 0.4$ is associated with even lower values for λ^*, typically around or below 10%.

For $\tau_i^*(\theta)$ (the optimal total contribution rate to the pension system) the differences between the various data samples seem to be less distinct. The optimal rates range between 0.13 and 0.22 (for the case where $\theta = 0$). In general it holds that the optimal level of the total contribution rate is lower when the optimal share of funding is higher. This is of course the expected result since a larger proportion of investments into the financial market allows to achieve a given level of expected old-age income with a smaller amount of savings. An increase in the concern for relative standing increases the optimal level of the contribution rate. For $\theta = 0.3$ the optimal contribution rate is now typically between 19% and 25%. As was the case for the optimal share of funding the differences in the optimal contribution rates between the various data samples are smaller for higher values of θ.

The results of Figure 4.1 should only be understood as approximate calculations that are based on a number of assumptions. They assume, e.g., that the portfolio consists exclusively of equity and that the administrative costs of managing the portfolio of the funded pillar are zero. Furthermore they are based on specific parameters for people's preferences and on a specific concept of reference groups. In Knell (2010*b*) I report the results of a large number of alternative simulations in order to analyze the robustness of the results to changes in some of the underlying assumptions. The robustness exercises show that these changes have typically rather moderate effects on the optimal total contribution rate but that they can sometimes have a considerable impact on the estimated optimal shares of funding. This is in particular true for the assumptions concerning the curvature of the utility function, the correlation between GDP growth and equity returns and for the assumption of alternative reference groups. In every case, however, one could observe that an increase in the concern for relative standing is associated with large reductions in these optimal shares. In fact, for medium-ranged assumptions about the reference group dependence (θ is around 0.3) the sensitivity to changes in the basic assumptions seems to be less pronounced. For almost none of the robustness exercises the optimal share of funding was found to exceed 20% when the strength of the relative comparison motive is assumed to be $\theta = 0.3$.

4.5 Conclusion

In this chapter I have dealt with a factor that is of crucial importance when comparing funded and unfunded pension system—their respective return-risk profiles. I have argued that for this comparison it is not only important to note that individuals are highly risk-averse but that—in addition— they also care about their economic standing relative to a reference group. Relative consumption is an important factor in order to evaluate the well-being of pensioners, in particular when one is concerned about one of the main objectives of pension systems: the prevention and relief of old-age poverty. I have developed a formal model that includes a consumption reference standard for old individuals which is defined as a certain percentage of labor income and that has the natural interpretation as a socially defined subsistence level (or as a poverty line).

It was shown that the optimal total contribution rate increases with the strength of the concern for relative standing while the optimal share of funding decreases with this parameter. This is an implication of the fact that in properly designed defined contribution PAYG systems pensions are perfectly tied to future wages. Accordingly, in PAYG systems there exists only uncertainty about the level of future wages (and thus about future absolute consumption) but not about the future relative position. For funded systems, on the other hand, both the absolute and the relative positions are uncertain and thus the attractiveness of funding is diminished by higher levels of equity return risk and/or a stronger concern for relative standing.

In the final part of the chapter I have used the model to get a rough estimate of the "optimal mix" between the two pillars using data for different country groups and time periods. For all cases considered the optimal share of funding drops considerably if one assumes a reasonable strength of the concern for relative standing. For $\theta = 0.3$, e.g., the optimal share of funding is typically below 20%. Overall one can thus conclude that the high risk of equity taken together with people's concern for relative consumption lowers the attractiveness of a funded pension system that is associated with its return advantage.

It goes without saying that the topic of risk in pension systems is comprehensive and the connection between a concern for relative standing, poverty and the choice between unfunded and funded systems describes only one aspect of this broad field. There exists, e.g., a literature on risk-sharing in different types of pension systems with a particular focus on productivity risk (Gordon & Varian 1988, Shiller 1999, Bohn 2001, Bohn 2009, Wagener 2004).[12] Furthermore, Edward Whitehouse from the OECD has written

[12] E.g. "Thus – crucially – a system with a PAYG element allows intergenerational risk sharing; this

(partly together with co-authors) a sequence of papers on various types of risk that a pension system is facing: life expectancy risk (Whitehouse 2007), purchasing-power risk (Whitehouse 2009), investment risk (Whitehouse et al. 2009, d'Addio et al. 2009), social and labour-market risks (Zaidi & Whitehouse 2009), myopia risk and policy risk. He concludes: "No one wants to bear risk, but, in most cases, someone has to. Risks in pension systems have, in the past, been poorly measured or even just ignored" (Whitehouse 2007, p. 4).

In addition to these different types of risk one must, however, note that pension systems often have to deal not only with quantifiable stochastic processes ("risk") but also with a large class of events that cannot be captured with the use of probability measures and that are sometimes subsumed under the heading of "Knightian uncertainty" or that are—more recently—referred to as "Black Swans" (Taleb 2007). Nobody knows what the future will bring and one can think of many events that can have a large effect on the society: a climate catastrophe, a war, a pandemia (or—to mention something positive—a medical breakthrough that leads to a jump in life expectancy), a global electricity blackout or some dangerous scenario that today is not even conceivable. Such a "Knightian event" will also have an impact on the pension system and it is likely that different age groups and socio-demographic strata will be affected to different degrees. One could argue that a PAYG system is in a better position to deal with these unforeseeable events (cf. Tichy 2005). First, because in a well-designed PAYG system the fate of workers and pensioners is closely tied to each other and second, because the centralized structure of national PAYG systems will make changes easier than in the typically more fragmented private funded system where policy-makers will face coordination and information problems. It is probable that also in a funded system the state will intervene and secure a livable income for retired people if the pension funds get into trouble. The associated bail-outs and the restructuring of pension rights and obligations will, however, be easier, quicker and better coordinated in a system that is based on a PAYG pillar.

This is not to say that funded pillars have no place at all in integrated pension systems. On the contrary, they will be important for voluntary savings, especially if—in a well-designed PAYG system—people have the freedom to choose their retirement age ad libitum (probably above some minimum retirement age) and will face actuarial neutral reductions in their pension payments. Private savings will then often be necessary to compensate the income gap if individuals choose an early retirement age. The unfunded

does not happen in a fully-funded system of individual accounts." (Barr & Diamond 2008, p. 13).

pillar should, however, be large enough such as to provide a pension payment above the poverty line for low-income households that retire at the normal age.

Having said that, there even exist good reasons to stipulate a *mandatory* funded pillar. There are at least two arguments for this—one related to financial constraints and the other to financial literacy. The first argument is expressed by Geanakoplos et al. (1999, p. 124): "The fundamental rationale for social security investment in the stock market rests on the existence of people who are currently constrained from holding equities. It is interesting to note that those who would benefit the most from social security investments in equity are probably the poor, since this group is least likely to hold stocks now". The second argument refers to the fact that the control over a certain amount of funds that can be invested into the financial market might increase people's financial literacy. In this, as in many other fields, learning by doing is an important method to acquire knowledge. Without the means to actually make portfolio choices individuals might simply lack the incentive to learn the basics of investment theory and to get informed about the working of financial markets. "Social security reforms associated with financial market deepening (e.g., the creation of private pension funds), by raising the incentive to acquire financial knowledge, eventually will lead also to improvements in economic literacy" (Jappelli 2010, p. F447). The absence of resources and incentives might be one reason why "financial literacy programs" often have such a hard time to achieve any measurable success. Needless to say that a mandatory funded pillar has to be designed in an appropriate way. This includes the number and types of funds available in which people are allowed to invest, the specifities of the "default fund" in which the money is channeled if individuals make no explicit choice, the possible existence of a clearing house and other important regulatory and supervisory aspects. On the whole, the Swedish system can be taken as a possible benchmark, both in as far as the size and the regulation of the funded pillar are concerned. In Sweden, the total contribution rate on earnings for pension system is 18.5% of which 16 percentage points are used for the PAYG component (organized as a NDC system) and the remaining 2.5 percentage points are a mandatory funded component.[13] Interestingly, the relative size of the funded pillar is 2.5/18.5 or 13.5% which is within the range of values that have been suggested by the theoretical model above.

[13] In fact, this could be termed the "Scandinavian solution" since many countries of this region have a small mandatory funded pillar as part of their pension system: "The mandatory DC schemes in the Nordic countries have relatively small contribution rates: 1% in Denmark, 2% in Norway and 2.5% in Sweden " (OECD 2009, p. 161).

4.6 Related literature

There exists a small number of papers that contain results that are related to my estimations, in particular the three papers by Dutta et al. (2000), Matsen & Thøgersen (2004) and de Menil et al. (2006). Although these papers do not take a possible concern for relative standing into account they have undertaken (in one way or another) empirical estimations for the optimal share of funding that can be related to the results of the paper for the case with $\theta = 0$. A direct and detailed comparison between the four papers is quite difficult since the studies show considerable differences concerning the analyzed countries and time periods and the used methods and data. It is interesting to note, however, that the paper by de Menil et al. (2006) (which is most closely related to my approach) leads to similar results despite considerable differences in the estimation procedure.

An extensive treatment of various aspects of risk-sharing in different types of pension systems can, e.g., be found in Gordon & Varian (1988), Shiller (1999), Bohn (2001), Bohn (2009), Wagener (2004), Bovenberg & Uhlig (2008).

Chapter 5

Intergenerational Fairness and the Choice between Different Pension Systems

5.1 Introduction

After having looked at a comparison between unfunded and funded pillars under the perspective of risk I want to redirect the focus of attention again more narrowly on the PAYG system. In chapter 2 I have shown that a PAYG system can be designed in a way such that it is resistant to demographic shifts and that, furthermore, in the case of fluctuations in the cohort size there exist many variants how to achieve a permanent balance of the system. This finding almost automatically leads to the question as to how to choose among the alternative, equally sustainable schemes.

In order to answer this question one has to take a number of factors into consideration. First, one has to return to the central objectives of pension systems (see chapter 1) and analyze how the different schemes fare with respect to the different objectives. This has been a central focus in the last chapter where I have argued that the unfunded system has important advantages in as far as the prevention of old-age poverty is concerned. I have focused there on the defined contribution variant of a PAYG system ($\alpha = 1$) and I have not dealt with the implications of different choices for α. In fact, for constant cohort sizes (as has been assumed in chapter 4) the different sustainability factors are identical.[1] In this chapter I want to discuss a second perspective under which the choice between different pension schemes can be scrutinized: the viewpoint of intergenerational fairness. Even in the absence of risk, every pension policy entails different mechanisms how to react to short-run imbalances and these schemes affect different generations in a different way. A crucial factor in comparing the alternatives or in choosing one before the other is to look at potential justifications for the "unequal treatment of equals" and thus to enter the realm of ideas of fairness and distributive justice.

The reason why I devote an entire chapter to this question is twofold. First, questions of intergenerational fairness appear in many forms when thinking about the "optimal design" of PAYG systems. Second, and in particular, the topic itself seems to be "under-researched". This lack of research is surprising given that public discussions (especially surrounding pension reforms) abound in references to all kinds of fairness. One policy measure is criticized because it puts the adjustment burden on the shoulders of the young (an "intergenerational fairness" argument), another piece of legislation gets a bad press because it seems disfavorable to the employment history of women or of manual work-

[1] There exist, however, other pension schemes that are not encompassed by the formulas in (2.7) and (2.8) (e.g. defined benefit schemes in absolute terms) and that have different implications for risk-sharing and poverty prevention in the presence of productivity shocks (see Bohn 2001, Wagener 2003, Wagener 2004).

ers (an "intragenerational fairness" argument), a third one because it regulates a steep increase in the retirement age thereby undermining the principles of predictability and reliability of the system for the people who are close to retirement (a " fairness in transition" argument). Given the centerpiece of the topic of fairness in public discussions it is all too surprising to observe the sparse research in this field. There exist some papers that deal with questions of intergenerational fairness from a social choice perspective but they mainly follow in the tradition of axiomatization and they are rather mute on the topics that are discussed in the public arena.[2]

I cannot close this gap here and offer a thorough and systematic treatment of the issue of intergenerational fairness in the context of pension systems. I just want to concentrate on some topics that have either already arisen over the course of the previous chapters or that are particularly prominent in the public discussions. The following pages are meant to give some flavor of the important and intriguing issues involved and they should not be regarded as a substitute for more serious and systematic research—ranging from theoretical studies to surveys and experiments—that are hopefully coming forward in the next years.

5.2 Different principles of fairness

Looking at the public discussions on pension reform and the frequent references to the notion of fairness, it does not require a particularly cynic observer to detect a certain amount of self-serving bias in the respective use of different definitions. As a counterbalance it is helpful to approach this topic in a sober, systematic way. A good point of departure is Hervé Moulin's excellent book on *Fair Division and Collective Welfare* in which he introduces four different concepts of fairness: "Four elementary ideas are at the heart of most nontechnical discussions of distributive justice. They organize neatly our thinking about conflictual interpretations of fairness. The four ideas are exogenous rights, compensation, reward and fitness" (Moulin 2003, p. 1). Moulin illustrates the different principles with the simple example of "the division of the parent's estate between siblings. Compensation suggests to give more to the poorest sibling, who needs more extra cash. The devoted child who took care of the parents' business deserves to be rewarded by a bigger share. Finally, strict equality of the shares, no matter what, is a popular and haggle-free division method according to exogenous rights [...] Fitness says to give the

[2] A good survey of this research is the book edited by Roemer & Suzumur (2007). Another reference is Roemer (1996).

resources to whomever makes the best use of it" (Moulin 2003, p. 2). Although Moulin does not talk explicitly about the issue of intergenerational fairness the four principles also appear almost naturally in this area and they support different choices and conclusions in various situations.

In the context of PAYG systems one can, e.g., often hear people talking about the "intergenerational contract", which—of course—is not a real, legally binding and enforceable contract but rather a thought construct, a metaphor for an intergenerationally interlocked support system. In this respect it is maybe the closest real-world-equivalent to the Lockean idea of a social contract or to the Rawlsian image of an agreement behind the veil of ignorance. The entire PAYG system as such is thus deeply rooted in the *exogenous rights* approach to fairness. Every generation has the duty to provide for its preceding cohorts' consumption in old age whereby it earns the right to an appropriate alimentation once it itself reaches retirement age. Having fulfilled one side of the contract (paying contributions during work) while being afterwards denied the other side of the contract (receiving a pension payment in old age) would be regarded as highly unfair by most people, as a blatant breach of the intergenerational contract. This is one reason why a sudden transition of an unfunded to a funded system is almost unthinkable since it would inevitable treat a rather small number of generations in a very inequitable manner.

The intergenerational contract in its abstract form does not say anything about the specific streams of contributions and pension payments that are deemed appropriate. Here is where the other fairness principles enter the stage. On the one hand, many people agree that life is partly a game of luck and that unfortunate members of society should not be thrown into misery when their abilities to work dwindle and in case their private savings and acquired pension rights are not enough to secure a decent standard of living. The central objective of pension systems—the "prevention of poverty"—can thus be understood as part of the agreement behind the veil of ignorance, as an insurance against bad, unforeseeable and only partly influenceable life events. Or—in a similar vein—the supplementary pension payments ("Ausgleichszulage") could also be interpreted as a *compensation* for the "involuntary, morally unjustified differences in individual characteristics" (Moulin 2003, p. 21) that have led to different life chances, achievements and pension claims.

On the other hand, however, many PAYG systems also use a *reward-based* structure, in particular in as far as the objective of consumption smoothing is concerned. A person that has contributed more to the pension system when working should also be granted a higher pension in retirement. In fact, this principle is often deep-seated in the construction of PAYG systems and it is related to the concepts of "actuarial" or "quasi-actuarial" fairness (cf. Lindbeck & Persson 2003) and to the German notion of "Teilhabeäquivalenz" (fairness within cohorts). The latter principle means that if an individual has always earned (and contributed) twice as much as another, otherwise identical individual then the first individual should also receive a pension that is twice as high. This is a clear evocation of the "reward principle of fairness" and it entails a notion of proportionality that is close to the fairness maxim by Aristotle: "Equals should be treated equally, and unequals unequally, in proportion to relevant similarities and differences" (*Nicomachean Ethics*).[3]

Finally, in pension discussions one can sometimes hear the claim that expected budgetary shortfalls should be financed by raising present and future contribution rates since the future generations will be more productive and thus also richer than the present generations of pensioners. This is a reference to the *fitness principle* of fairness, even if the proponents of this view might not always be aware of this connection.

As this brief discussion has shown, different principles of fairness have shaped the structure of PAYG systems from the very beginning and they accompany every process of restructuring and reform. It is important to remember that in this, as in many other arenas it is almost impossible to select one fairness principle that is dominant in all situations and for all issues involved. It is rather the case that fairness is a context-dependent and also partly a subjective notion and that there is always a potential conflict between different principles of justice (see Sen 2009, Sandel 2009). This is probably the reason why discussions on fairness and on moral reasoning in general are such interesting and important and at the same time difficult and never-ending endeavors.

5.3 Intergenerational fairness and fluctuations in fertility

In this section I start the discussion with the case of fluctuating cohort sizes while in the next section I will talk about some aspects of the situation with increasing life expectancy.

[3]This concept of proportionality is also a guiding principle for the modern form of "equity theory" (cf. Adams 1963) and other proposed sharing rules (see Konow 2003). According to equity theory an allocation or distributional rule will be regarded as fair if for two persons A and B the ratio of input to outcome is identical, or —expressed in terms of the "equity formula"—if $\frac{O_A}{I_A} = \frac{O_B}{I_B}$. "Inputs [$I$] are usually thought of as a participant's contributions to an exchange and outcomes [O] as the consequences, potentially positive or negative, that a participant has incurred in this connection." Konow (2003, p. 1211).

As shown in section 3.3, PAYG schemes that are characterized by different parameter values α imply specific patterns of δ_t when the economy is hit by a demographic shock. This is a convenient example to illustrate the perspectives of the different fairness principles and how they can be used to justify different conclusions and policy measures.

First, one could argue that every generation is identical (behind the imagined veil of ignorance). In particular, all of them have the same life expectancy, the same retirement age and they face the same source of risk—the development of cohort size N_t which under this interpretation is regarded as strictly exogenous (like the consequence of a natural disaster or a war).[4] In this case the equal individuals behind the veil of ignorance or the benevolent social planner acting on their behalf should try to minimize intergenerational fluctuations and spread the consequences of the demographic changes over as many different shoulders as possible. A similar conclusion would arise if one looks at the same situation from the perspective of the "fitness principle". A risk-averse decision maker with a concave utility function cares about the mean and the variability of the IRR. Since the expected value of the IRR is independent of the adjustment parameter α (cf. (3.14)) he or she will prefer the adjustment policy that delivers the smallest variability given the stochastic process of population growth.

The variability of δ_t can be captured by various concepts and I will focus here on two commonly used measures: the variance of δ_t and the absolute deviations around the mean. As far as the variance is concerned it is shown in Knell (2010*a*) that one can use the approximation (3.14) to derive a closed form of the variance-minimizing value of α. The resulting expression for α^{MinVar} is stated in chapter 3 as equation (3.19). On the other hand, I also show in Knell (2010*a*) that the sum of the absolute deviations of λ_s, i.e. $\sum_{s=-(Y-1)}^{Y-1} |\lambda_s|$ has a minimum at $\alpha \equiv \alpha^{MinAbs} = \frac{Y-X}{Y}$. Turning to the benchmark numerical values ($Y = 60$, $X = 45$) this means that $\alpha^{MinVar} = 0.295$ and $\alpha^{MinAbs} = 0.25$. Under alternative assumptions concerning life expectancy and the retirement age one would get $\alpha^{MinVar} = 0.37$ and $\alpha^{MinAbs} = 0.33$ (for $Y = 60$, $X = 40$) and $\alpha^{MinVar} = 0.39$ and $\alpha^{MinAbs} = 0.36$ (for $Y = 55$, $X = 35$). Under this perspective of "ex-ante equality of all generations behind the veil of ignorance" together with a "fitness principle" of fairness the social planner would thus choose a parameter between $\alpha = 0.25$ and $\alpha = 0.39$

[4]So far I have treated the development of N_t as deterministic. In the present analysis it is better to regard it as a stochastic process where only the basic properties of this process (like the mean and the variance) are known. In particular, assume that $N_t = \hat{N}(1 + \nu_t)$ where ν_t is a stochastic variable that is distributed with zero mean and variance σ_ν^2. Note that one can still use Figure 3.1 to gauge the impact of demographic shocks on the intergenerational distribution.

(depending on the used concept of variability and the parameters for life expectancy).[5]

These considerations, based on an "exogenous rights" or a fitness conception of fairness, are not uncontroversial. In particular, they are based on the assumption that the development of cohort sizes is the result of an exogenous process that hits the economy like a meteroit from outer space. In reality, however, conception is not immaculate and non-attributable. On the contrary, the size of one cohort is the result of the reproductive behavior of its preceding cohorts. Looking at the situation from this angle leads to a stronger emphasis on the "concept of fair rewards".[6] Cohorts that have not succeeded in full reproductive behavior are "responsible" for the baby bust and should be the main financiers of the demographic burden. Similarly, a generation that has produced a "baby boom" should be "rewarded" in comparison to a generation with normal or below-average reproductive behavior. There exists in fact an ongoing debate whether individuals with a smaller number of descendants should shoulder a larger part of the demographic burden, both because they are partly responsible for the drop in the size of birth cohorts and because they are in a better (financial) situation to make good for the shortfall through private provisions. This argument amounts on an individual level to link the contributions and benefits to a person's number of children.[7]

Looking at the treatment of different cohorts as a collective the reward-based perspective suggests that the IRR should reflect as strongly as possible the number of children a cohort has produced. The parent generations should bear the negative financial consequences of a declining cohort size and — vice versa — should also be accredited with gains from a possible increase. The empirical evidence on fertility rates suggests that they can be approximated quite well with the assumption of a triangular distribution. In particular, for a number of European countries the most recent data (see European Commission 2008) suggest that a triangular distribution that ranges from 15 to 45 with

[5] I want to note a remarkable application of this analysis. In particular, the choice of the adjustment weight of the German sustainability factor (the de iure weight of $\alpha = 0.25$ or the de facto weight of $\alpha = 0.36$, see Knell (2010*a*)) is supported by these principled considerations based on minimum variability. This looks surprising since it does not seem to be the case that the choice has been primarily motivated by such reflections but rather by the pragmatic objective of keeping the contribution rate below some threshold value. E.g.: "The point of departure for the reform proposals was to achieve a politically pre-defined contribution target. [...] The sustainability factor with a weighting α of 0.25 corresponds most closely to the targeted contribution rates. [...] [and it] will achieve the contribution rate targets of 20 percent in 2020 and 22 percent in 2030" (Börsch-Supan et al. 2003, p. 20ff.). This shows that actual decisions are more often made under the political constraints of the moment rather than with reference to longstanding, "everlasting" principles of fairness.

[6] "Differences in individual characteristics are morally relevant when they are viewed as voluntary and agents held responsible for them. They justify unequal treatment" (Moulin 2003, p. 22).

[7] See, e.g., Sinn (2005) and Werding (1999) and below.

a peak at the age of 30 fits the empirical fertility data quite well.[8]

Based on this assumption one would thus view generations −45 to −15 as the potential parent generations of generation 0. If the size N_0 of this cohort is exceptionally large (as has been the assumption in chapter 3) then these generations should also be the main beneficiaries of this baby boom, not least because they have financed this increase by reducing their own consumption.[9] Viewed from this perspective, the DC variant of the PAYG system with $\alpha = 1$ looks quite attractive (see Figure 3.1). In this case generations −59 to −12 have higher-than-normal IRRs while the impact for later-born generations up to generation +14 are negative. What is more, the IRR is in fact largest for exactly the range of generations −45 to −15 that are the potential parent generations (at least according to the stylized triangular fertility distribution). These extra returns can thus be regarded as a compensation for their expenditures of child-rearing and supplying the extra number of workers. The correspondence is, however, not perfect since all of these cohorts get the same elevated IRR which does not reflect the triangular fertility distribution. The large negative IRR for generation N_0 can be justified by noting that its members have not reproduced themselves but that instead the cohort size falls back to the level that could be observed before the shock. The boom generation is mostly responsible for the following bust and the DC system makes them to pay for it. More worrisome, however, seems the fact that the generations −11 to −1 and +1 to +14 face negative IRRs since they do not seem to be responsible for the return of the cohort size to its original level. This defense of the "neighboring cohorts" is, however, not completely justified since according to the triangular distribution it is not only generation 0 that is responsible for the baby bust.

In order to see this more clearly I have reproduced the intergenerational pattern of the internal rates of return for a different population structure and a different shock. In particular, I assume a triangular distribution with $N_t = \sum_{a=-15}^{-45} \chi_{t+a-1}\psi_a N_{t+a-1}$, where ψ_a is the "normal" number of children a generation has at age a while χ_t is a cohort-specific fertility shock. The triangular fertility distribution implies the following values: $\psi_a = \frac{4(a-FER^{min})}{(FER^{max}-FER^{min})^2}$ for $FER^{min} \leq a \leq FER^{mean}$ and $\psi_a = \frac{4(FER^{max}-a)}{(FER^{max}-FER^{min})^2}$ for $FER^{mean} \leq a \leq FER^{max}$, where FER^{min}, FER^{max} and FER^{mean} are the minimum, maximum and mean fertility age, respectively. For the calculation I use the values $FER^{min} = 15$, $FER^{max} = 45$ and $FER^{mean} = 30$. While in Figure 3.1 I have looked

[8]For the Netherlands, e.g., this is also documented in Heeringa & Bovenberg (2009). According to the OECD (2009) the age of first time mothers across 16 OECD countries has been 27.7. Including all mothers and also fathers would also move the average age of parenting closer to 30.

[9]Note that in the model I do not include any child subsidies or other measures of public support for children.

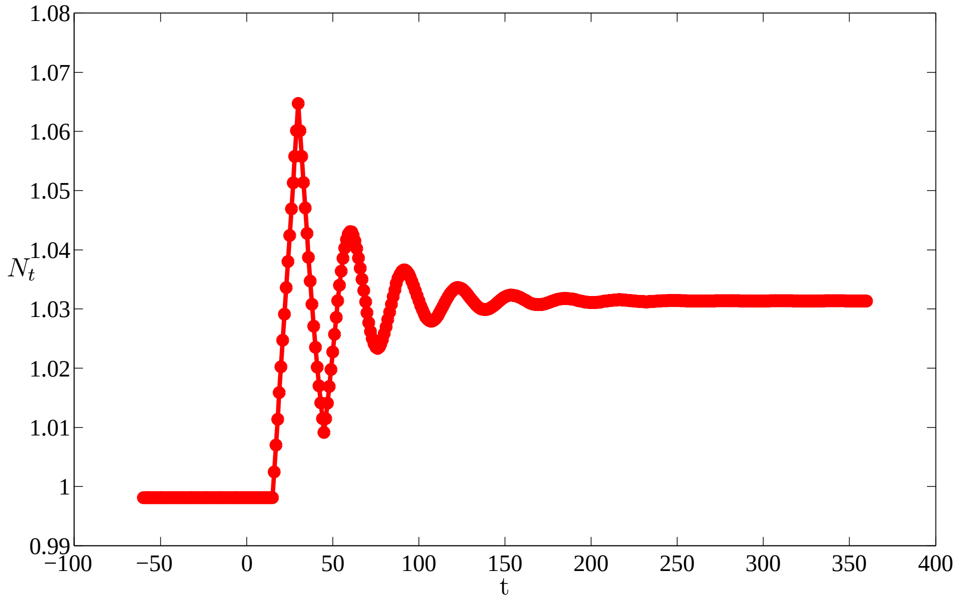
at the case where cohort 0 was suddenly larger I now look at the case where cohort 0 suddenly increases its reproductive behavior and $\psi_0 = 2$, while $\psi_t = 1$ for $t \neq 0$.

`Insert Figure 5.1 about here`

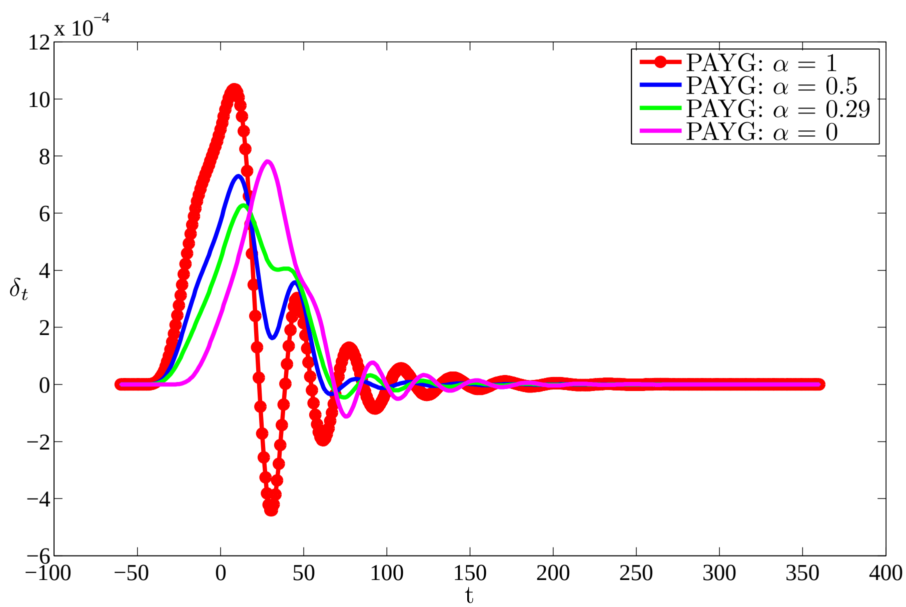
In Figure 5.1a I report the pattern of N_t that follows from this shock to the reproductive behavior of only one generation. It leaves a permanent impact one the cohort sizes (and the total population size) since the larger children generations will themselves have more children than in the reference case etc. If one is guided by the reward-principle of fairness then one would like to have a policy that primarily remunerates generation 0 since it is the only generation with a higher reproduction rate and thus more children than its own size. By looking at Figure 5.1b one sees that none of the different variants of the PAYG system fulfill this property. In all cases a large number of cohorts gets higher or lower IRRs than normal, meaning that there are "free-riders" and "innocent payers". The generation with the largest IRR is generation 8 (for $\alpha = 1$), 11 (for $\alpha = 0.5$), 14 (for $\alpha = 0.29$) and 28 (for $\alpha = 0$). In all cases except the DB system the IRR of generation 0 is close to this maximum value. For all of these cases there is some degree of "collateral damage", i.e. a situation where cohorts are punished or rewarded for the behavior of cohort 0 and it seems to be most pronounced for the DC policy. On the whole, the reward principle of fairness does not lead to a clear choice among the policies up to values of about $\alpha = 0.25$. On the other hand, one can say that the case for a defined benefit system is weak when looked at under this perspective of intergenerational fairness.[10]

All in all, the use of the DC variant of the PAYG system is supported by these considerations or — to put it the other way round — the use of a DB variant is strongly rejected. One has to remember, however, that the sustainability factor (2.8) stipulates one pension level q_t at each point in time that is the same for all pensioners and does not distinguish between different generations or different members of a specific generation. It might be possible to adapt the sustainability factor to capture inter- and intragenerational differences in reproductive behavior but this is not straightforward. Despite these difficulties the reward-based principles of fairness and the fluctuations in cohort sizes have lead a number of economists and laypersons to plead vehemently to take the reproductive behavior into account when determining the size of pension benefits. One of the most

[10] Similar results have emerged in simulations where I have regressed the IRR of different generations on the size of the potential children generations (that have been calculated using again the triangular fertility distribution). The correlations have appeared to be weak, although one gets again the result that the coefficients are largest (and most significant) for the case of a DC system.



(a) The Development of Cohort Sizes for a One-Time-Shock in the Triangular Fertility Distribution



(b) The Pattern of the IRR for a One-Time-Shock in the Triangular Fertility Distribution

Figure 5.1: The IRR for a PAYG pension systems and four different values of α when life expectancy is $Y = 60$, workers retire at age $X = 45$ and $\hat{\tau} = 1/4$, $\hat{q} = 3/4$ and there is a one-time-change for generation 0 in the triangular fertility distribution.

ardent supporters of this proposal is the German economist Hans Werner Sinn: "In order to be able to consume in old age and enjoy a decent retirement life, a working generation has to save or to raise children who will later be able to pay them a pension. Or, to put it more bluntly, the working generation has to invest in real or in human capital. If it does not invest in either real or in human capital it will have to starve because nothing breeds nothing" (Sinn 2000, 24). On an individual level this would then amount to link the contributions and benefits to a person's number of children. "Instead of placing collective responsibility on a whole generation, the necessary pension cuts and the compensating new savings plan should be concentrated on the childless. Whoever has not raised children can be expected to take a pension cut of one half"(Sinn 2003, 1). This proposal is regarded as justified from the viewpoints of both the causation and the ability principle. The childless have caused the decline in cohort sizes and should bear the consequences while at the same time being in a position to be able to afford this since they do not face the costs of child rearing.[11]

This view is, however, challenged on various grounds. Besides ideological reservations that are related to disreputable historical precursors of such proposals[12] it is often argued that it is hard to tell which persons are exactly to be held responsible for the size of a succeeding cohort. In modern societies it is not true that the natural parents are the only — or sometimes not even the main — sponsors of their offsprings. The welfare and tax systems know many channels by which the society as a whole shoulders part of the costs for the upbringing and the education of younger generations.[13] More generally the reproductive behavior is heavily influenced by the legal system and by common rules and norms. This system of incentives and disincentives is, however, shaped by the society at large (via electoral and political behavior and via general social activities) such that every individual and every generation bears at least some responsibility for the size of the succeeding generations. As a baseline one should probably keep the arguments for a "child pension" (together with the counterarguments) in mind when designing a PAYG pension system, since some reflection of the individual reproductive behavior seem justified

[11] Wilke (2008) analyzes the effects of an introduction of a NDC system in Germany instead of the existing earnings-points system. She uses a measure for intergenerational distribution (the ratio of total benefits to total contributions) that is similar to the IRR and comes to the conclusion that the such a transition "implies a redistribution mainly from the early 1940 cohorts to the baby-boom generation. Given the fact that the baby-boom generation partly induced the demographic challenges the system now has to cope with, this cannot be a political objective" (Wilke 2008, p. 28).

[12] Cf. Barbier (2003).

[13] For Germany, e.g., it was calculated that the public sector pays about 40% of all related costs (Werding 1999).

according to leading concepts of fairness.[14]

In closing this section I want to emphasize an interesting implication. As shown in chapter 3 a demographic shock has also a discernible impact on the intergenerational distribution in a funded system. Depending on the availability of assets and the investment rules the intergenerational pattern might be identical to the one of a DC-PAYG system (see section 3.4) or it might show smoother or wilder fluctuations. All cases discussed in chapter 3, however, have shown a *qualitatively* similar intergenerational pattern of the IRR. This means, in other words, that the reproductive behavior of one cohort (or of one individual as a matter of fact) has an impact on the rate of return of all other cohorts (or individuals). This is a clear example of a "pecuniary externality" that has, so far, not been explicitly noted. It might not have a detrimental effect on the efficiency of the resulting allocation but it raises important questions about its intergenerational fairness.

5.4 Intergenerational fairness and increases in life expectancy

Increases in life expectancy raise a number of problems for welfare economics and questions for intergenerational fairness. In fact, these are partly related to the question of how to treat the welfare of yet unborn generations (cf. Cowen & Parfit 1992). Without going into the details of this issue, I just want to mention in the following some interesting implications of the assumption of increasing life expectancy. To anticipate the results I will show that a constant process of aging will lead to an increase in the IRR for all generations if the PAYG systems uses a "life expectancy factor" as specified in chapter 2. Under the assumption that the elements that contribute to the increases in life expectancy are either directly influenced by the individuals (personal health care etc.) or can be treated as "quasi-exogenous" (medical progress etc.) these equal gains for all generations might be deemed as intergenerationally fair. An adjustment to increases in life expectancy that is just based on increases in the contribution rate, on the other hand, might lead to a highly unequal treatment of different generations.

In order to show this I start with the situation where increases in life expectancy are accompanied by a constant or constantly increasing cohort size (cf. section 2.5). The use of a life expectancy factor according to the formula in equation (2.15) is associated

[14] In fact, most pension systems include an allowance for non-contributory years that are due to childcare and that increase the pension benefit.

with a situation where both the contribution rate and the pension level are constant over time ($\tau(t) = \hat{\tau}$, $q(t) = \hat{q}$, $\forall t$). The continuous time equivalent to the definition of the internal rate of return in (3.16) is given by $\hat{\tau} \int_0^{X(t)} e^{(g-\delta(t))x} dx = \hat{q} \int_{X(t)}^{Y(t)} e^{(g-\delta(t))x} dx$. Using the linearization $e^{(g-\delta(t))x} \approx 1 + (g - \delta(t))x$ one can then derive an explicit solution for the IRR:

$$\delta(t) = g + 2\frac{Y(t) - X(t)(1+\hat{z})}{[Y(t)]^2 - [X(t)]^2(1+\hat{z})}. \tag{5.1}$$

For the calculation of the IRR we thus need an expression for the retirement age of the generation born in period t, i.e. $X(t)$. This variable is implicitly defined by $E(t+X(t)) = X(t)$.[15] Using (2.15) it comes out as:

$$X(t) \approx \frac{Y(t)}{1+\hat{z}(1+\gamma)}\left[1 - \frac{n\hat{z}(1+\hat{z})(1+\gamma)Y(t)}{2\left[(1+\hat{z}(1+\gamma))^2 + n\hat{z}\gamma Y(t)\right]}\right], \tag{5.2}$$

where for $n = 0$ the expression is exact. One can now use this solution for $X(t)$ in (5.1) to derive an approximation for the internal rate of return of the PAYG pension system. This leads to a surprisingly simple expression for the IRR as specified below.[16]

$$\delta(t) \approx g + n + \gamma\frac{2}{Y(t)} \tag{5.3}$$

Equation (5.3) contains a number of interesting results. First, the IRR increases with the growth rate of wages g. This is obvious and follows from the fact that increasing wages will also raise future pension benefits.

Second, the IRR increases in n ($\frac{\partial\delta(t)}{\partial n} > 0$). Population growth makes a PAYG system more attractive, since this increases the internal return of the system. In fact, for constant life expectancy ($\gamma = 0$) equation (5.3) reduces to $\delta(t) = g + n$, i.e. the internal rate of return of the PAYG system is simply given by the growth rate of the wage sum. This is the classic "biological interest rate" associated with population growth for which Samuelson (1958) has given the following "common-sense explanation": "In a growing population [...] retired men are outnumbered by workers more than in the ratio of the work span to the retirement span. With more workers to support them, the aged live better than in the stationary state—the excess being positive interest on their savings" (Samuelson 1958, p.

[15] In order to see this note that generation t retires in period $t + X(t)$. In this period the actual, observed retirement age is thus given by $E(t + X(t))$. It thus holds that: $E(t + X(t)) = X(t)$.

[16] This uses a number of approximations and the expression is also only valid in an approximate sense. Numerical calculations have shown, however, that the pattern of the exact results is completely parallel to the approximated solution.

473).

Third, equation (5.3) implies that the IRR also increases in γ ($\frac{\partial \delta(t)}{\partial \gamma} > 0$). An increasing life expectancy thus also gives rise to an additional, less well-known "biological interest rate", similar to the one associated with a growing population. The intuition behind this result can be grasped most easily for the case where $n = 0$. The determination of the retirement age (2.15) implies that every cohort works longer than the preceding cohorts but not by the full increase in life expectancy. In fact, the generation-specific retirement age is given by $X(t) = \frac{Y(t)}{1+\hat{z}(1+\gamma)}$. This means that for each generation the ratio of years in pension to working years ($\frac{Y(t)-X(t)}{X(t)} = (1+\gamma)\hat{z}$) is higher than the dependency ratio which is observed in the year of its birth ($\frac{F(t)-E(t)}{E(t)} = \hat{z}$). Every generation can spend a larger fraction of its lifetime in pension without having to pay higher contribution rates or receiving lower pension levels. This situation is compatible with a constantly balanced budget since the extra time in pension for generation t is paid by the succeeding cohorts who retire later and thus increase the contribution base. The "gratification" for their longer working life is again a longer pension period which is in turn paid by the later retirement of their succeeding cohorts etc. The logic behind the biological rate of interest due to increases in life expectancy is thus completely parallel to the biological rate of interest due to population growth. Again, the "aged live better than in the stationary state" because a constantly postponed retirement age leads to a constantly increasing number of workers that finance the longer pension years.

Given that life expectancy increases in a constant way it could be regarded as "fair" that all generations get some benefit from this process. This is even more so if one ascribes the responsibility for the increases in life expectancy to a combination of personal circumspection (health-conscious life-style etc.) and medical progress that is due to research and development that is viewed as an "intergenerational public good". In this respect it might only be considered as problematic that the IRR is not completely identical for different generations but rather decreases over time. This property, however, is only due to the use the internal rate of return as the specific measure of intergenerational distribution. The use of other measures (e.g. the present value ratio, see Geanakoplos et al. (1999)) might imply an equal treatment across generations.

This can be illustrated by an example. Say $n = g = 0$, $\gamma = 0.5$, $\hat{\tau} = 1/4$ and $\hat{q} = 3/4$. For one generation A it holds that $Y(A) = 60$ and then $X(A) = 40$ according to the formula (5.2). For a later-born generation B, on the other hand, one has that

that $Y(B) = 66$ and $X(B) = 44$.[17] The IRR of the two generations is approximately given by $\delta(A) = 0.0167$ and $\delta(B) = 0.0152$. The younger generation thus receives a lower IRR which could be viewed as unfair. But this outcome is partly due to specific properties of this measure. One could, e.g., also use the present value ratio (PVR) which is defined as the ratio of the present value of total benefits to the present value of total contributions. Under the assumption that the discount rate (used to calculate the present value) is equal to the growth rate of wages then we can make the following calculations in order to derive the PVR. Generation A pays a total of $40 \cdot \hat{\tau}$ "contribution units" (relative to the wage) and receives $20 \cdot \hat{q}$ "pension level units" (relative to the wage). For generation B the two magnitudes are $44 \cdot \hat{\tau}$ and $22 \cdot \hat{q}$, respectively. So the PVR of the two generations is the same and given by: $1 + \gamma$.[18] The presence of the second "biological interest rate" is thus even more clearly manifested when using the alternative measure for intergenerational distribution. The reason why the IRR decreases over time is the fact that younger generations retire later (according to (5.2)) and thus get their gratification (the pension benefits) later in life. This means that the discounted value of the sum of the "pension level units" is smaller and this is reflected in the lower δ_t. This outcome is therefore partly an "artifact" of the role of discounting.[19]

It is possible to doubt whether the assumption about a constantly increasing life expectancy (cf. (2.11)) is reasonable. Taken literally it seems inconceivable to assume that life expectancy increases without bound. On the other hand, even though one would not believe that the increase in life expectancy can go on forever, the development over the last decades and the forecast over the next 50 years is nevertheless best described by the assumption of a constant linear increase. In fact, a similar controversy has already been raised by the publication of Samuelson's original article where the population growth rate has been the focus of the discussion. In particular, Lerner has criticized the assumption of a constantly growing population as a "mirage" and a "chain letter swindle", arguing that the biological interest rate will collapse once the growth will come to an end (Lerner 1959, p. 523f.). In order to deal with this issue one has to use alternative assumptions about the demographic development that involve discontinuous changes in both population size and life expectancy. I leave this for future research.

So far, I have only discussed the use of the "life expectancy factor" ((5.2)) as a reaction

[17] So the first generation might be born in $t = 0$ and the second in $t = 12$.

[18] This follows since the PVR can be written as $\frac{(Y(t)-X(t))\hat{q}}{X(t)\hat{\tau}} = (1+\gamma)\hat{z}\frac{1}{\hat{z}} = 1 + \gamma$.

[19] The question of what is the correct individual or social discount rate has a long standing in welfare economics. See, e.g., Cowen & Parfit (1992) or Queisser & Whitehouse (2006).

to the increase in life expectancy. This proposal is, however, not very popular in the public arena where one can frequently hear the argument that present and future increases in life expectancy should be paid for by present and future productivity growth while leaving pension benefits and the retirement age unchanged.[20] At first sight, this sounds like a reasonable suggestions where no generation is encumbered with unacceptable burdens. On second sight, however, this argument involves a number of questionable implications. First, if the pension payment is expressed in relation to the wage rate (as is the case in the sustainability factor (2.8)) then this proposal amounts to a continuous increase in the contribution rate. In fact it means that the increase in the dependency ratio z_t should just be countered by increases in τ_t (i.e. $\alpha = 0$). Evidently, if the process of aging continues then at some point the contribution rate will hit the 100% upper limit when at the latest (but probably much earlier than that) this way of financing the increases in life expectancy by the gains in productivity will have to stop. This means, however, that there will be a strong discontinuity in the development of the IRRs between generations. The earlier cohorts are the clear winners since they get the same pension level $\hat{q}$ for a longer period of time $Y(t) - X$. On the other hand the generations after the increases in the contribution rate stopped will be the losers since they will have to bear the brunt of adjustment (however this might look like).

If the pension is fixed in absolute terms (say $P_s = P_t = \hat{q}W_t, \forall s \geq t$) then it might be possible to stabilize the contribution rate and the absolute pension payments if the real growth rate of the economy is large enough. This DC policy, however, will lead to a permanent deterioration in the *relative* pension level q_t which might sooner or later run into conflict with the objective of poverty prevention. This pattern might again be regarded as unfair by some generations and furthermore it is unlikely that it is a politically stable constellation.

5.5 Conclusion

In this chapter I have presented various principles of fairness (exogenous rights, compensation, reward and fitness) and I have discussed how they might be extended to the context of intergenerational fairness. To this end I have again used the example of the two-sided demographic challenge and I have shown how different principles of fairness suggest different policy measures if cohort sizes fluctuate or if life expectancy increases in

[20] In the scientific literature a similar argument can be found in Bütler & Kirchsteiger (2000).

a linear fashion. The reflection of this chapter should only be understood as a first step in this direction. The often heated public discussions about pension reforms and various "folk definitions" of fairness would in any case benefit from a more thoughtful use of the different concepts of intergenerational justice. For this reason alone it were desirable if research on intergenerational fairness and its application to real-world problems of pension policy would be intensified in the future years.

5.6 Related literature

A careful exposition of the different principles of fairness and how they are treated in the microeconomic and social choice literature is Moulin (2003). A somewhat older book with a similar focus is Young (1994). A general account on the issue of justice is Sandel (2009) and a recent book with a more economic leaning is Sen (2009). The classic modern treatment of justice is Rawls (1971) which contains, however, only a few pages on the topic of intergenerational fairness. A paper that contains results of a survey about people's opinion on fairness is Boeri et al. (2001).

Chapter 6

How to Design a Reasonable PAYG System?—Grand Principles and Smaller Details

6.1 Introduction

Economist are often reproached for using hypothetical concepts that are derived in an artificial world and based on irrelevant and fantastic institutions (cf. Caballero 2010). Without doubt it would be wrong to dismiss all simplifications since they are inevitable in order to isolate, study and explain important regularities and mechanisms of action. Sometimes, however, the simplifications are misleading since they exclude crucial elements or direct the view to side shows while failing to focus on the main relations. My decision to use in most parts of this monograph a multiperiod model instead of the more common two-period set-up for the generational structure was exactly motivated by the desire to work within a "realistic" demographic set-up in order to investigate the impact of shocks and the functioning of possible policy reactions. On the other hand, it was necessary to simplify other parts of the model, both concerning the range of strategies and decision variables of households and concerning the detailedness of the environment in which they are situated. The same is also true for the adjustment policies that I have presented in chapter 2. Both are rather stylized creations but they have been sufficient for the purpose of proving that it is possible to make a PAYG system resilient whilst the two-sided demographic challenge.

In this chapter I want to deal explicitly with the question of how to design a functioning and acceptable PAYG system for real world application. This is important since it is a rather futile exercise to conceive a highly sophisticated construct that works fine in the sheltered realm of the theoretical model but fails in practice due to a lack of data, unforeseen interrelations, unintended consequences or the sheer refusal of the insured population. A popular form of PAYG systems that has gotten much press in recent years is the so-called notional defined contribution (NDC) system (see Palmer 2000, Palmer 2006). It has been conceived in Italy and Sweden and is being adopted in an increasing number of countries. Due to its success I use it therefore as my benchmark model of existing PAYG systems. Before describing its main logic and its working principles I want to start with a few remarks.

First, the two adjustment factors of chapter 2 do not look so complicated or ethereal as to not be incorporable into a real-world PAYG system. In fact, as discussed in section 2.3, the German system has introduced a sustainability factor in 2003 that is closely related to the abstract factors defined by (2.7) and (2.8).[1] The latter equation states

[1] Of course the adjustment factors as they are legislated in Germany deviate from the formulas in various aspects. E.g., they include time lags and they have some bells and whistles attached but the

how to determine the pension level q_t which is the same for all generations of pensioners and for all members of each generation in a certain year t. In other words, I have looked in chapters 2 and 3 at a representative member of each generation since my focus was on intergenerational rather than on intragenerational questions. In order to account for the latter dimension one has to specify how the pension systems treats members of one generation that differ with respect to important characteristics, e.g. with respect to their income. In the German system, e.g., this is done via the recording of earnings points that reflect the relative income of every insured individual in each year. The pension payment is then the product of the pension level q_t (which is the same for all pensioners) and the overall sum of earnings-points (that differs across individuals and corresponds to lifetime income). Details of the German system can be found in Börsch-Supan et al. (2003) and Knell (2005). On the other hand, the life expectancy factor, as stated in (2.13), does not seem to be in use in any existing pension system. This, however, might be mainly due to the general unpopularity of a longer working life and less due to its basic structure. In fact, the life expectancy factor is also a quite intuitive concept and I see no reason why it could not be used in actual pension systems in order to calculate the "normal retirement age" for every cohort t. It would still be possible to retire earlier or later but this normal age would be the benchmark for which the "normal replacement ratio" will be paid out.

As a second preliminary remark I want to note that it would be certainly naïve to think that there exists only one design for a workable PAYG system and that all countries should sooner or later converge on exactly the same system. Even the World Bank has stressed in its 2005 assessment of it pension strategy that it is important to respect the specific historic situations and peculiarities of every country and that its own suggestion should only be regarded as "a benchmark, not a blueprint". This is also strongly emphasized by Barr and Diamond: "A country's pension arrangements reflect the relative weights attached to its various objectives and the pattern of constraints it faces. [...] Countries have successfully implemented pension systems using very different mixes of structures" (Barr & Diamond 2009, p. 148f.). Pension systems are often children of their time and their local surroundings and they are sometimes deeply embedded into the specific social, economic (and sometimes even cultural) texture of a nation. They are often in use for a long time and grown-ups start to learn and understand their workings like the usage of their mother tongue. In such a situation it does not make much sense to introduce (or rather to superimpose) a new, grandiose new structure. Even if this novel system is

basic structure is identical. See Knell (2010*a*) for a more detailed discussion of the similarities and differences.

superior to the old one it will not be very successful if it is neither understood not accepted by the " indigenous" population. It is better to take the existing system as the point of departure and adapt its structure, strengthen sensible element, abolish weaknesses and add specific features in order to reform it in the right direction.

My third and last remark, is related to the last point. It can be shown that many of the "best practices" or the most reasonable design features of successful, accepted and fair pension systems are in fact rather similar.[2] Many roads lead to Rome (or Stockholm, in this case), as has been argued and demonstrated by a number of authors. There are two systems that lend themselves as benchmark models of PAYG schemes that have been implemented in real world economies: the (Swedish) notional defined contribution (NDC) system and the (German) earnings-points system. I will focus in this chapter on the NDC system since it is more frequently discussed in the literature as a benchmark and it is also steadily exported to other countries. In the next section I will give a brief overview over its main features while in section 6.3 I will focus on some crucial design features of NDC systems and show how its sustainability and intergenerational distribution depend on seemingly trifle details like the notional interest rate, the concept of life expectancy used for annuitization and the determination of deductions (supplements) for early (late) retirement. I concentrate on these issues because they are key elements of NDC systems and also because some some of them have so far gone unnoticed in the related literature. In section 6.4 I will conclude this chapter and the monograph by comparing NDC systems with "quasi-NDC" systems (like the German earnings-point scheme).

6.2 The basic structure of NDC systems

In the following I want to give a short overview of the main characteristics of NDC systems with a focus on the version implemented in Sweden. There is a fixed contribution rate to the NDC pension system, which in Sweden amounts to 16%.[3] Contributions paid into the personal notional account are revalued at the notional interest rate, which in Sweden equals the growth rate of average earnings. There exists an "automatic balancing mecha-

[2]It would be interesting to investigate to what extent the acceptance of a pension system has to do with the fact that the insured population regards it as intergenerationally and intrageneretionally fair.

[3]In fact, in Sweden the total contribution rate on earnings amounts to 18.5% (split evenly between employer and employee), but 2.5% is channeled into a mandatory funded pillar. The structure of the funded pillar must also be carefully designed. This involves issues like the size of the contributions, the number of available investment vehicles, the existence of a clearing house etc. I will not deal with these aspects here. Details on the regulation of the second pillar in Sweden can be found in Sundén (2004).

nism" that lowers the notional interest rate when the financial balance of the system looks unsatisfactory (cf. Settergren 2001). At the time of retirement, the capital accumulated in the notional account is converted into an annuity. In the most straightforward version of this model, the notional capital is simply divided by life expectancy, which is why increased longevity automatically translates into reduced pension benefits. In Sweden this annuitization is not so straightforward, since a certain amount of the expected increases of the pension capital during retirement is "front-loaded" (see Appendix 1 in Palmer 2000). In Sweden this front-loaded part amounts to 1.6% and existing pensions are then only adjusted with the difference between the actual growth rate and this stipulated growth rate of 1.6%. All of this is summarized by a central category in the Swedish system called the "G-factor". Individuals are allowed to retire earlier or later than the normal retirement age (above some minimum eligible age) but this will of course change their pension payments.

The main features of a NDC system can thus be summarized as follows:

- Fixed contribution rate.
- Life-time assessment period.
- Close relation between contributions and benefits.
- Past contributions are revalued with an appropriate notional interest rate.
- At retirement the notional capital is transformed into annual pension payments by taking the development of life expectancy into account.
- Flexibility in the time of retirement is allowed with actuarial deductions and supplements.
- Individual accounts and annual statements increase the transparency of the system.
- Despite the framing in the language of a "defined contribution" system, there is no funding involved and all "investments" and "accounts" are purely "fictitious" (or "notional").

This verbal description can also be translated into a formal model. This is done in the following where I use the same notation as in chapters 2 and 3. The fixed contribution rate is again denoted by $\hat{\tau}$ and the size of cohort t by N_t. The maximum age that can be reached by cohort t is denoted by Y_t and I again assume that all members of a generation actually reach this age. The wage level earned during a working period t is denoted by W_t and the (possibly cohort-specific) retirement age is given by X_t. I assume that both Y_t and X_t are non-decreasing over time. As in section 2.4 I use two additional variables. E_t denotes the number of cohorts that work in period t while F_t stands for the highest age observed in this period. The size of the labor force and the retirement pool are again given by equations (2.1) and (2.2).

The notional capital $NC_{X_t+1,t+X_t}$, accumulated by generation t over their working periods and valued at the beginning of the year when its members receive the first pension installment (i.e., in period $t+X_t$) is given by:

$$NC_{X_t+1,t+X_t} = \sum_{a=1}^{X_t} \hat{\tau} W_{t+a-1} NR_{t+a-1}^{t+X_t}, \tag{6.1}$$

where $NR_{t+a-1}^{t+X_t}$ is the (cumulative) notional interest rate that is used to revalue contributions paid in period $t+a-1$ to the year of the first pension payment $t+X_t$. I will discuss some possibilities how to determine the notional interest rate below.

The first pension received by generation t in period $t+X_t$ is given by:

$$P_{X_t+1,t+X_t} = \frac{NC_{X_t+1,t+X_t}}{G_{X_t+1,t+X_t}}, \tag{6.2}$$

where the notional capital for generation t is transformed into an annuity via the "G-factor" $G_{X_t+1,t+X_t}$. Existing pensions are adjusted according to:

$$P_{a,t+a-1} = P_{X_t+1,t+X_t} AF_{t+X_t}^{t+a-1}, \text{ for } X_t+1 \leq a < Y_t, \tag{6.3}$$

where $AF_{t+X_t}^{t+a-1}$ is the (cumulative) adjustment factor that stipulates how the first pension $P_{X_t+1,t+X_t}$ received by generation t in period $t+X_t$ is adjusted to give the pension payment in period $t+a-1$.[4]

In general the G-factor depends on two elements: (i) The concept of life expectancy used for its calculation and (ii) the inclusion or exclusion of a "wage norm" (i.e. a front-loading component). In order to keep the analysis simple I will abstract from wage norms in the following. The concept of life expectancy used to calculate the annuity for

[4] I define $AF_{t+X_t}^{t+X_t} \equiv 1$. Therefore formula (6.3) is also valid for the initial pension where $a = X_t + 1$.

generation t is denoted by LE_t. The G-factor for generation t can then be written as:

$$G_{X_t+1,t+X_t} = LE_t - X_t. \tag{6.4}$$

If life expectancy forecasts are used to estimate life expectancy then $LE_t = Y_t$ while the use of historic (cross-section) life tables will result in $LE_t = F_{t+X_t}$. In the first case the G-factor is simply the number of years a pension is received.

Three crucial factors that have to be defined at the outset for a functioning NDC system are thus the notional interest rate, the adjustment factor and the concept of life expectancy that is used in the G-factor. These choices are important as will be shown in the following section.

Before turning to this I have to talk about the budget of the NDC system. The income and the expenditures of the system in a certain period t are given by:

$$IN_t = \sum_{a=1}^{E_t} \hat{\tau} W_t N_{t-a+1} \tag{6.5}$$

$$EX_t = \sum_{a=E_t+1}^{F_t} P_{a,t} N_{t-a+1} \tag{6.6}$$

The total deficit in a certain period is denoted by

$$D_t = EX_t - IN_t. \tag{6.7}$$

There are two interesting concepts of a "balanced budget" in this context. The first is the concept of "balanced in every period" or $D_t = 0, \forall t$. In chapter 2 I have discussed adjustment policies (the sustainability factors (2.7) and (2.8) and the life expectancy factors (2.13) and (2.15)) that imply a constant fulfillment of this condition.

The second definition of a balance refers to the intertemporal budget constraint and requires that the budget is "balanced in present value terms" or: $\sum_{s=t}^{\infty} D_s \frac{1}{\prod_{j=t}^{s} \xi_j} = 0$, where ξ_j is the discount factor that is used to calculate the present value and $\xi_t \equiv 1$. There exists a long discussion on what is the correct intergenerational discount factor and in the following I will mostly refer to the case where the actual growth rate of wages $(1+g_j)$ is used as the discount factor. This has the convenient property that the long-term solidity of the budget does not depend on the temporal sequence of deficits and surpluses.

For the graphical representation I will use the deficit ratio d_t—defined as the percentage of the deficit in relation to the income of the pension insurance IN_t, i.e. $d_t = \frac{D_t}{IN_t} = \frac{EX_t}{IN_t} - 1$. This allows for a better quantitative interpretability of the results.

6.3 Crucial choices in NDC systems and the reaction to shocks

A reasonable NDC system must have a clear and coherent structure. The design involves the choice of some crucial parameters which have an effect on the long-run performance of the entire system. It is important to carefully consider these choices, even if (for some elements) there might not exist a clear "optimal design". In the following I will deal with a number of these crucial factors, in particular with the notional interest rate, the adjustment factor, the choice of life expectancy and the determination of supplements and deductions.

I will mainly study these issues one by one, i.e. I will look at the interaction of one design feature and one (demographic) shock while holding all other variables constant. This is done in order to understand the main mechanisms involved and to clearly separate them. One should keep in mind, however, that different demographic and macroeconomic developments often interact and that there are important complementarities. This has already been a topic of section 2.5 where it was shown that in the presence of constant trends in both population growth and aging, the life expectancy factor has to be adapted ((2.15) instead of (2.13)) in order to be compatible with a balanced budget. These interaction effects might cumulate and do not cancel out in the aggregate. Nevertheless, it is also useful to look at the different elements in isolation since one can expect that if the variability of a certain factor leads to budgetary problems in this simple one-by-one setting then it will also cause instability in a situation where the ceteris paribus assumption is lifted. In a next step it would of course be interesting to study the joint effect in a calibrated simulation exercise. This goes, however, beyond the scope of this chapter.

6.3.1 The choice of the notional interest rate

There are two possible methods of indexation that are often discussed in the literature and that are used in real-world pension systems: an indexation with the growth rate of average wages and one with the growth rate of the wage sum. Indexation with the growth rate of average wages means that:

$$NR_{t+a-1}^{t+X_t} = \frac{W_{t+X_t}}{W_{t+a-1}} \text{ and } AF_{t+X_t}^{t+a-1} = \frac{W_{t+a-1}}{W_{t+X_t}}, \tag{6.8}$$

while indexation with the growth rate of the wage sum means that:

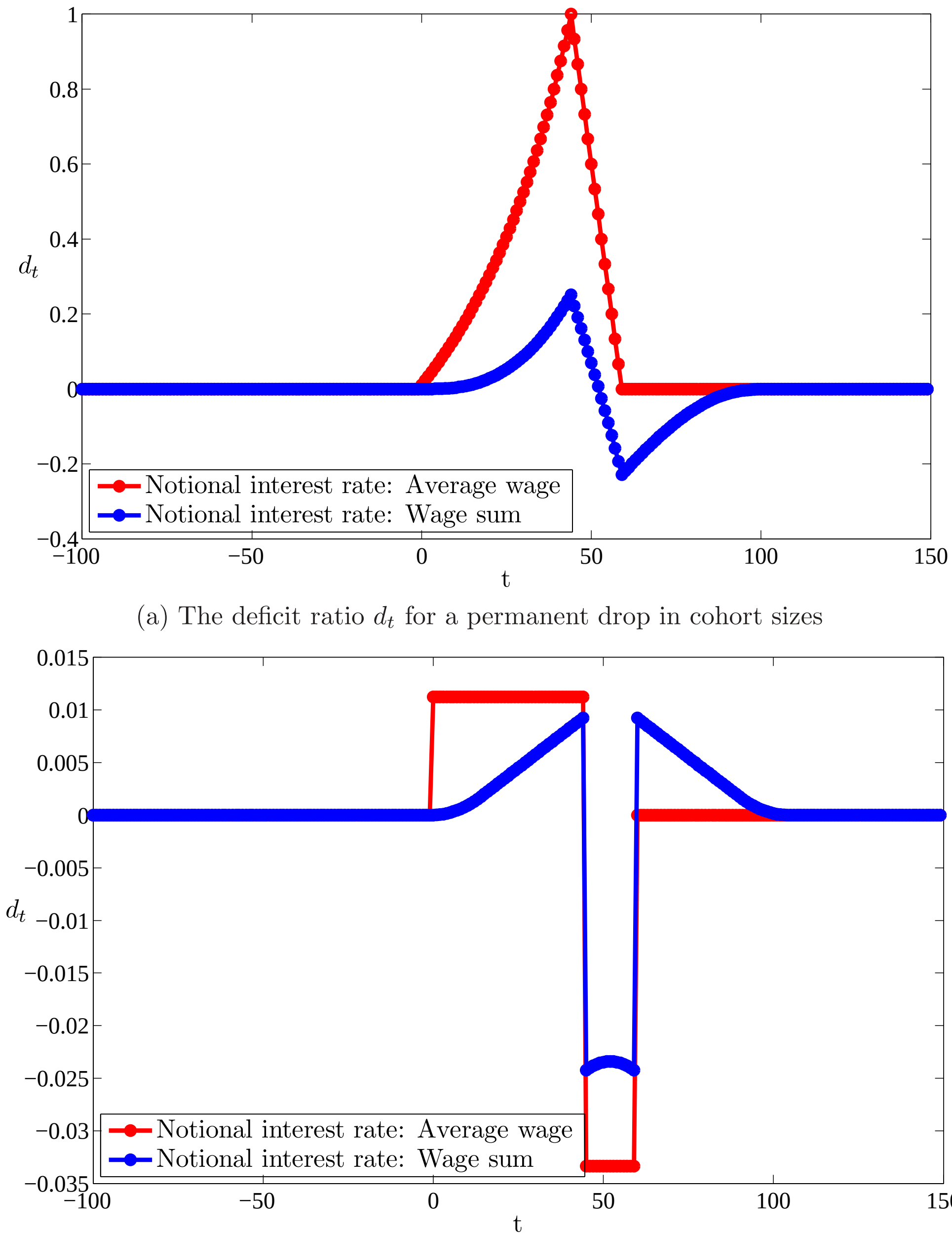
$$NR_{t+a-1}^{t+X_t} = \frac{W_{t+X_t}}{W_{t+a-1}}\frac{L_{t+X_t}}{L_{t+a-1}} \text{ and } AF_{t+X_t}^{t+a-1} = \frac{W_{t+a-1}}{W_{t+X_t}}\frac{L_{t+a-1}}{L_{t+X_t}}. \tag{6.9}$$

Note that the cumulative rates can also be expressed in the more conventional form as the product of annual rates, e.g.: $NR_{t+a-1}^{t+X_t} = \prod_{s=a}^{X_t} nr_{t+s}$, where nr_{t+s} is the annual notional interest rate (factor).

How will the choice of the notional interest rate and the adjustment factor will affect the reaction of a NDC system to a demographic shock? In order to answer this question I want to take up the approach of chapter 3 and focus again on discretionary changes in the cohort size. In particular, I look at the case where life expectancy and the retirement age are constant, i.e. $Y_t = Y$ and $X_t = X$. From this it follows that $LE_t = Y$ and that the G-factor (6.4) is given by $Y - X$. For the examples I use the same benchmark values as in chapter 3: $Y = 60$, $X = 45$ and $\hat{\tau} = 1/4$. Wages are assumed to grow at an arbitrary rate g_t. The following numerical examples will illustrate that as a reaction to a change in the cohort sizes indexation with the wage-sum will lead at least to a situation where the budget of the pension system is balanced over time, while indexation with the average wage will lead to a budget that is permanently in surplus or in deficit.

`Insert Figure 6.1 about here`

I start with the situation where there is a permanent change in the cohort size. At time $t = 0$ it drops from $N_t = \hat{N}$ to $N_t = \frac{1}{2}\hat{N}$ and it stays there forever. In Figure 6.1a I illustrate the consequences of this drop on the deficit-ration d_t for both indexation methods. One observes that with average-wage-indexation there is no automatic "rebound-effect" and the drop in cohort size leads to a number of periods where—due to the decrease in the number of contributors—the pension system runs a deficit ($D_t > 0$). The deficit is largest in the period where *all* working cohorts are small and *all* retired cohorts are large (i.e. in period $t = 44$). After this peak the deficit shrinks again because from then on more and more small cohorts are also part of the retired population which alleviates the budgetary pressure. The deficit remains positive, however, until the new demographic steady state is reached, which happens only after the last large cohort has died, i.e. in



(a) The deficit ratio d_t for a permanent drop in cohort sizes

(b) The deficit ratio d_t for a one-time drop in cohort sizes

Figure 6.1: The diagrams illustrate the development of the deficit ratio d_t for two assumptions about the development of cohort sizes. In panel (a) there is a permanent drop in cohort sizes at time $t = 0$, i.e. $N_t = \hat{N}$ for $t < 0$, $N_t = \frac{1}{2}\hat{N}$ for $t \geq 0$. In panel (b) there is a one-time drop in cohort sizes, i.e. $N_t = \hat{N}$, $\forall t \neq 0$ and $N_0 = \frac{1}{2}\hat{N}$. Life expectancy is $Y = 60$, workers retire at age $X = 45$, $\hat{\tau} = 1/4$ and the notional interest rate is either the growth rate of average wages or the growth rate of the wage sum.

period $t = 59$. The sixty loss-making periods are not counterbalanced by even a single period where the pension system runs a surplus. The demographic shift will thus cause a situation where the budget is neither balanced in every period nor balanced in present value terms.

This is different for wage-sum-indexation where the increase in the deficit ratio until period $t = 44$ is less pronounced than for the alternative indexation method and where the period of the deficits is followed by a period of surpluses. In fact the latter period is the *exact mirror image* of the first one. This has the consequence that the average deficit ratio under wage-sum-indexation and the given demographic shift is zero, i.e. $\sum_{t=0}^{\infty} d_t = 0$. The reason for this different behavior is that the drop in cohort size from period $t = 0$ on does not only change the relative size of contributors to pensioners (as for wage-indexation) but has also two additional effects. First the smaller size of L_t decreases the adjustment factor such that from period $t = 0$ to $t = 44$ the total expenditures for pensions are lower than in the other indexation regime. In period $t = 44$ the last large cohort retires and thus L_t is from now on again constant, i.e. for $t \geq 44$ the notional interest rate and the adjustment factor are again identical for both indexation regimes. But there exists a second additional effect that leads to a diverging behavior for the two methods of indexation even after $t = 44$. All contributions that are paid from period $t = 0$ to period $t = 44$ are revalued with a lower notional interest rate than under wage-indexation which lowers the pension claims of all pensions that are based on *any* contribution that falls into this time span.[5] I want to note, however, that even though the average deficit *ratio* is zero in this case this is not true for the average absolute deficit. It can be shown that only if one uses the growth rate of the wage-sum as the discount factor ξ_j one gets that $\sum_{t=0}^{\infty} D_t \frac{1}{\prod_{j=0}^{t} \xi_j} = 0$. Even if this is not fulfilled it is save to say that this deviation from present-value balancing will be rather minor when compared to wage-indexation.

In Figure 6.1b I report the behavior of the deficit ratio for the case of a "one-period-change" in cohort size, which is exactly the same case that has been used extensively in chapter 3. In particular, $N_t = \hat{N}$, $\forall t \neq 0$ and $N_0 = \frac{1}{2}\hat{N}$. For wage-indexation one sees that in every period where this smaller cohort works (i.e. periods $t = 0$ to $t = 44$) there is a smaller total working population, a smaller sum of total revenues and thus a deficit. This changes for the periods where the cohort is in retirement (from $t = 45$ to

[5] Many cohorts are affected by this lower-than-normal notional interest rates. The first one is the cohort that retires in period $t = 1$ and has one year of lower revaluation (namely its last working period in $t = 0$). The last cohort that is affected is the one that entered the labor market in the last year of the subnormal revaluation, i.e. in period $t = 43$. This cohort dies in period $t = 102$ and so from $t = 103$ on the deficit ratio is again given by $d_t = 0$.

$t = 60$), where the total number of pensioners and thus the total amount of expenditures is lower than normal. Nevertheless these two effects do not balance in this case and we get that $\sum_{t=0}^{\infty} d_t > 0$. For the case of wage-sum-indexation we have again the two additional effects via the adjustment factor and the notional interest rate that dampen the increase in the deficit ratio until period $t = 44$. Now, however, we have a situation where the notional interest rate was for one period larger than normal, i.e. in period $t = 45$ where the small cohort has retired. This increases the pension claim for all cohorts that have been working at that time and this is reflected by the positive deficit ratios from $t = 60$ to $t = 103$. Again one has, however, that $\sum_{t=0}^{\infty} d_t = 0$.

It can also be shown that in the case of constant population growth (cf. (2.10)) indexation with the wage sum leads to a deficit ratio of zero while indexation with average wages leads to a situation where the budget is permanently in surplus or in deficit.

Summing up, the choice of the indexation method has an important and non-negligible impact on the financial stability of PAYG pension systems. Indexation with the growth rate of the wage-sum is more or less compatible with a balanced budget, although only over time and only under specific assumptions about the discount (or market interest) rate. It is remarkable to note, however, that despite the fact that this result is well-known for quite some time (cf. Lindbeck & Persson 2003, 86f.) one can still frequently find the claim that wage-sum-indexation is associated with a *constantly* balanced budget. This confusion probably follows from the erroneous generalization of the standard two-period model. In the language of the present model this benchmark case can be described by $X = 1$ and $Y = 2$. One can insert these values into the main equations of the models to observe that for the two-period model wage-sum-indexation leads to a permanently balanced budget but that this is not true in general. This is also stressed by Valdés-Prieto (2000).

As far as the empirical side is concerned one can find different choices of the notional interest rate that range form the growth rate of GDP (Italy) to per-capita wage growth (Sweden) to the growth rate of the covered wage bill (Poland and Latvia). In Sweden there exists an additional balancing mechanism that reduces the notional interest rate when the relation of (implicit) assets and (implicit) liabilities of the system starts to diverge. In any case a NDC system needs a sizable reserve fund in order to deal with the fluctuation of the budget over time. I want to stress the fact that the defined contribution version of the sustainability factor (cf. (2.7) and (2.8) with $\alpha = 1$) is by construction balanced in every instant and thus the required reserve fund is tiny.

6.3.2 The choice of life expectancy to calculate the G-factor

The second important demographic development that has put considerable pressure on PAYG pension systems is the increase in life expectancy. It is often claimed that a NDC system is particularly qualified to deal with such a situation since the rising life expectancy is taken into consideration when the notional capital is annuitized, where a higher life expectancy will simply lower the annual pension benefits and might thus also induce the individual to work longer in order to compensate for this decline. At the same time, however, it has also been established that the specific way Sweden has chosen for the determination of this annuity is not compatible with a balanced budget since only historic values are used. In the following I want to show that the sole use of either historic or forecasted values might lead to a situation where the budget of the pension system is not even balanced in present value terms.

In order to be able to focus on the pure influence of life expectancy developments I will again deal with a situation where every other variable besides life expectancy (and wage growth) is constant. In particular, $N_t = \hat{N}$ and $X_t = X, \forall t$. On the other hand, there is a one-time increase in life expectancy. While all generations that were born before $t = 0$ reach a maximum age of $Y_t = 60$, all generations born after that time will reach an age of $Y_t = 65$.

`Insert Figure 6.2 about here`

In Figure 6.2 I illustrate the development of the deficit ratio for three cases that differ in the methods that are used to regulate the cohort-specific remaining life expectancy LE_t and thus determines the crucial G-factor. Note first, that for a long time there is no effect on the deficit since generation 0 will first work for 45 periods and thus until period $t = 45$ there is no change in pension payments and total expenditures. Now start with the case where historic data are used to determine the remaining life expectancy, i.e. $LE_t = F_{t+X_t}$. Until period $t = 59$ there is no effect on the deficit since the oldest cohorts in each of these periods are still only 60 years old (i.e. $F_t = 60$ for $t = 0, ..., 59$). Then, however, the longer life expectancy of generation 0 manifests itself and in period $t = 60$ it is still alive at the age of 61. For the cohorts that retire in this period the annuity will thus be based on the value of $F_{60} = 61$. This period-specific maximum age will increase in annual steps until period $t = 64$ when generation 0 will die at the age of 65. For each of these periods the awarded pensions will be based on the historic life expectancies that are too low given the actual cohort-specific life expectancy. Thus from period $t = 60$ onwards the pension

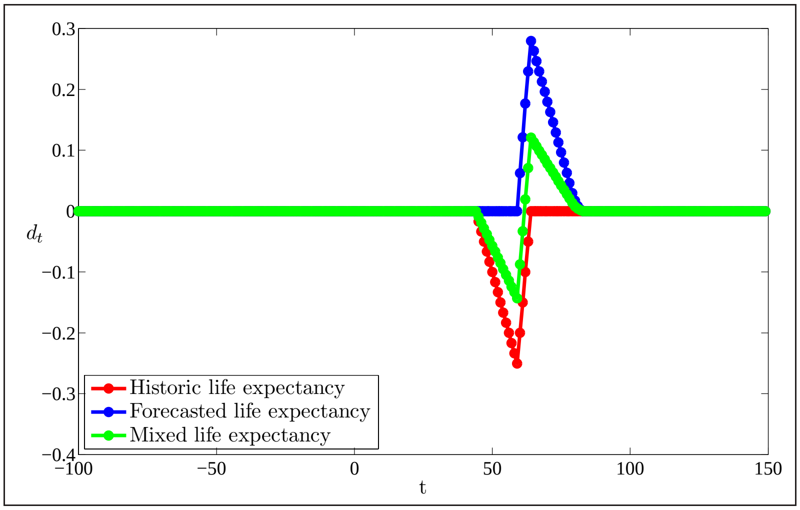


Figure 6.2: The diagrams illustrate the development of the deficit ratio d_t for a one-time increase in life expectancy. In particular, $Y_t = 60$ for $t < 0$ and $Y_t = 65$ for $t \geq 0$. The concept of life expectancy LE_t in order to calculate the G-factor is either based on historic data, forecasted data or a mix of both with an equal weight $\varsigma = 1/2$. Cohort size is constant and the rest of the parameters is like in Figure 6.1.

system will run a deficit that goes on until the period when the last generation that was awarded a "wrongly calculated" pension dies (i.e. in period $t = 83$).

The reverse picture emerges if forecasted data—i.e. the real cohort-specific values—are used to determine the remaining life expectancy of new pensioners, i.e. $LE_t = Y_t$. In this case the initial pension of generation 0 that retires in period $t = 45$ is already calculated on the basis of these 20 years remaining life span. But this longer life expectancy of generation 0 has no immediate financial consequences for the budget in periods $t = 45$ to $t = 59$ and so the pension system will run a surplus ($d_t < 0$) until the new demographic steady state is reached in $t = 64$.

The third line in 6.2 uses a mixture of the two life expectancy concepts in order to calculate the annuity value for the initial pension. In particular I assume that $LE_t = \frac{1}{2}\left(Y_t + F_{t+X_t}\right)$. This leads to smaller fluctuations in the deficit ratio and to a total deficit that is almost balanced in present value terms ($\sum_{t=0}^{\infty} d_t = -0.087$).

The one-time jump in life expectancy that underlies the example in panel (a) is of course fairly unrealistic. Most empirical studies have found a rather constant linear increase in life expectancy, as discussed in chapter 2. One can also use assumption (2.11) and the model set in continuous time (i.e $Y(t) = Y(0) + \gamma \cdot t$), to see the effect on the permanent budget balance under various assumptions about the determination of $LE(t)$.

In particular, assume that $LE(t) = \varsigma Y(t) + (1-\varsigma)F(t+X(t))$. Similar to the example for the constant change in cohort size one also gets for a *constant* change in life expectancy that the deficit ratio is stable over time. What is more, it can be shown that it is in this case given by:

$$\hat{d} = -\frac{(1-\gamma)\ln(1-\gamma)}{\gamma\,[1-\gamma(1-\varsigma)]} - 1. \tag{6.10}$$

So for $\gamma = 0.2$ we get for $\varsigma = 0$ (only historic data) that $\hat{d} = 0.116$, for $\varsigma = 1$ (only forecasted data) that $\hat{d} = -0.107$ and for $\varsigma = 0.5$ (mixed case) that $\hat{d} = -0.008$. One can also calculate the value for ς that will lead to a balanced steady state budget. This depends on γ and comes out as: $\varsigma^* = -\frac{(1-\gamma)}{\gamma^2}\,[\gamma + \ln(1-\gamma)]$. For $\gamma = 0.2$ one gets that $\varsigma^* = 0.463$. The intuition for these results is parallel to the one that has been mentioned in connection with the example of a one-time jump in life expectancy.

The punchline of these consideration is surprising and has, to the best of my knowledge, not been stated clearly before. Both methods of calculating the remaining life expectancy LE_t for annuitization at the time of retirement (the use of historic life tables and the use of forecasted data) might lead to an unbalanced budget, even in present value terms. The first method is too "generous" causing persistent deficits while the second method is too "harsh" leading to considerable ongoing surpluses. It might thus be recommendable to use some mixture of the two basic concepts to determine the value of the annuity (or to regulate the G-factor).

Related to this, one can sometimes hear the suggestion that the G-factor should be recalculated every year and adjusted for modifications in life expectancy forecasts. "The generic NDC annuity embodies [...] cohort life expectancy at the time the annuity is claimed" (Palmer 2006).[6] This suggests that the annuit should be based on accurate forecasts of Y_t. The examples of this section, however, have shown clearly that such a strategy is—so to say—overambitious and will necessarily lead to surpluses. It is better to use some mixture of historic and forecasted values to arrive at a more balanced approach.[7]

As far as the actual situation is concerned, most countries use a backward-looking concept, Latvia uses a forward-looking definition and none uses a mixed approach.

[6] Barr and Diamond have a different opinion: "A process of automatic adjustment that relies heavily on projected mortality rates could easily become politicized. Thus a system may function better if it adjusts benefits on the basis of realized mortality information" (Barr & Diamond 2009, p. 89).

[7] Palmer & Franco (2009), however, argue that the use of backward-looking life expectancy gives some safety margin for the sustainability of the system.

6.3.3 The choice of reductions and supplements for early or late retirement

There exists a heated debate about the right values for changes to the pension payment if individuals retire earlier or later than the "normal retirement age". On the one hand, the solution to this problem looks fairly easy since everything is already stipulated by the NDC system with the G-factor. Assume again a steady state with $N_t = \hat{N}, \forall t$ and constant life expectancy $Y_t = Y$. Furthermore assume that all individuals of one cohort retire at age X while there is only one individual i that retires at $X_t^i = X + 1$. The notional capital for this person is given by $\hat{\tau}(X + 1)$ and the first pension by $\frac{\hat{\tau}(X+1)}{Y-X-1}$. The pension of a "normal member" of the generation is in contrast given by $\frac{\hat{\tau}X}{Y-X}$. Using the standard calibration this means that the first pension installment of individual i is larger by 9.5% (pension level $q^i = 0.82$ instead of $q = 0.75$). If the individual retires at $X^i = X - 1$ then the first installment is lower by 8.3% (pension level $q^i = 0.69$ instead of $q = 0.75$). So there is a slight asymmetry in these values and, interestingly, this is also reflected in the empirically observed values (for both NDC and non-NDC systems): "The average reduction in benefits for early retirement is 5.1% a year, while the average increase for late retirement is 6.2%" (Whitehouse & Zaidi 2008, p. 9).

In this thought experiment I have assumed, however, that only one single individual has a retirement age that differs from the rest of the cohort. The effect on the total budget will in this case be negligible. This is even more so if one would assume that individual retirement ages fluctuate randomly around the legally stipulated normal age X. This seems to be the implicit assumption that is typically used to determine the "actuarially fair" reductions and supplements of the system (and that emerge automatically in NDC systems with the use of the G-factor). The problem with this approach, however, is that the system only remains stable if the *average* retirement age remains constant over time. If the retirement changes then this will also lead to a permanent budgetary surplus or deficit. Also this mechanisms does not seem to be part of the common knowledge on NDC design. It is illustrated in Figure 6.3 where the retirement age jumps at time $t = 0$ from $X = 45$ to $X = 50$ and remains there forever. One can think of this as the (rather unrealistic) case where in year 0 the retirement age is made flexible and where all insured people suddenly start to retire five years later.

`Insert Figure 6.3 about here`

If the notional interest rate is equal to the average growth rate of wages then the

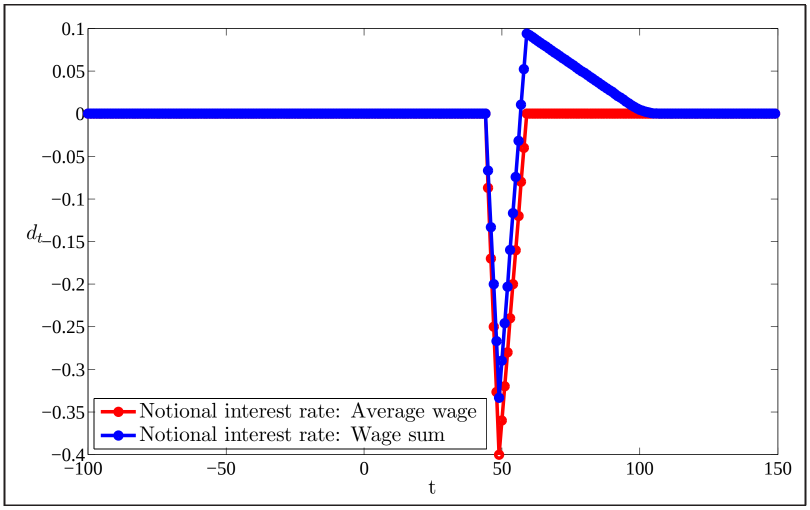


Figure 6.3: The diagram illustrates the development of the deficit ratio d_t for a one-time increase in the retirement age. In particular, $X_t = 45$ for $t < 0$ and $X_t = 50$ for $t \geq 0$. Cohort size is constant and the rest of the parameters is like in Figure 6.1. The notional interest rate is either the growth rate of average wages or the growth rate of the wage sum.

increase in the retirement age leads to a permanent surplus in the system. The first surprisingly good years, when the first cohort start to work longer give a boost to the system that is never followed by a year where expenditures exceed income. For the (maybe more relevant) case where people retire at the earliest possible moment, the reverse is true and the system will run a permanent deficit. The reason is that the drop in the retirement age leads to a windfall loss of the system that is comparable to the consequences at the start of a PAYG scheme where some cohorts get a benefit without fully contributing to the system. One could of course take care of these effects by adjusting the actuarially fair reductions and supplements that could, e.g., be conditioned on the difference between the own retirement age and the "typical" (or last period's) retirement age.

Alternatively, however, one can also set the notional interest rate equal to the growth rate of the wage sum. As illustrated in Figure 6.3 this leads to a situation where the deficit ratio is balanced over time. The reason is that the labor force increases in the first years when the cohorts start to work longer and this also increases the growth rate of the wage sum which leads to higher pension payments for these generations.

The topic of a flexible retirement age involves a number of additional issues beyond the determination of the appropriate level of reductions and supplements. One question, e.g., is whether individuals should be allowed to retire at any age they please or whether the possibility to enter the pension period should be conditioned on reaching a certain minimum pension age ("earliest eligibility age") or having acquired a pension payment that is above a certain minimum level? If the choice of the retirement age is left completely unrestricted this might cause problems related to myopic behavior, moral hazard and the danger of an increase of old-age poverty. This is also an important issue for non-NDC countries. It might make sense to define a "corridor pension" (as has been done in some countries) where there is an interval between which individuals are allowed to retire (possible if they exceed a specific minimum pension payment) and where the boundaries of this interval should be adjusted to increases in life expectancy (probably according to a factor similar to (2.13)).[8]

In any case, the examples of this section emphasize that the backside to a larger degree of flexibility in retirement are possibly larger year-to-year fluctuations in the budget. The size of the reserve fund (which is inevitable in a NDC system) thus has to be upscaled if flexibility is increased.

6.3.4 Additional issues

So far I have focused on three of the most important design features of NDC systems. There are, however, a number of additional elements that are also quite significant and that deserve a thorough analysis.

Indexation of Pension Benefits

The first of these issues is of particular importance: how should pensions be adjusted after the first pension is calculated? In section 6.3.1 I have simply assumed that the adjustment factor of pensions is identical to the notional interest rate. This is one possibility that assures that the growth rate of pensions and (total) wages is identical. There exists, however, a number of alternative possibilities to adjust pensions. The Swedish system, e.g., uses a "wage norm" to front-load some of the expected present value of the total pension capital. In particular, it is assumed that real wages grow at a rate of 1.6%. If this is in fact the case than this will also be the notional interest rate and the initial pension

[8] In the Swedish system, the minimum pensionable age is not adjusted to increases in life expectancy which might create problems since "a person whose personal discount rate exceeds the rate of actuarial adjustment of the pension will retire as soon as possible, creating potential pensioner poverty. Thus consideration needs to be given to adjusting the minimum pensionable age as well" (Barr & Diamond 2009, p. 88). They also note, however, that "The case for an explicit rule automatically adjusting the earliest eligibility age is weaker than the case for periodic reevaluation, since the normative analysis of the choice depends on much more than just life expectancy" (Barr & Diamond 2009, p. 91).

is not adjusted anymore. If this is not the case, then the adjustment will be with the difference between the notional interest rate and the wage norm of 1.6%.

The choice of the right adjustment of existing pensions is difficult and also present in non-NDC systems. Many of these countries, e.g., adjust pensions solely with the rate of inflation. This keeps the purchasing power of pensions constant while leading to a steady deterioration in the relative standing of pensioners vis-à-vis workers (cf. chapter 4). "A key policy question is, therefore, whether the benchmark for assessing purchasing-power risk should be the *cost* of living (that rise in line with price inflation) or *standards* of living (which increase in line with nominal earnings or household incomes)" (Whitehouse 2009, p. 25).[9] Front-loading thus entails the risk of increasing the poverty threat of pensioners, especially if they reach an old age. This was noted in a recent discussion of various NDC systems where the front-loading of the Swedish model was viewed rather skeptical.

> "As it turns out the Italian-Swedish model of front-loading benefits with a lifetime real rate of economic return is not as good an idea as it might first appear to be. For a given sum of capital at retirement, this way of calculating the annuity distributes money from years as an old pensioner to years as a young pensioner. [...] Formally in the case in Italy and de facto in the case in Sweden, benefits are indexed to price dynamics. [...] The issue is particularly acute for relatively small pensions. In the long run, if indexed only to prices, these pensions may provide income on the borderline of poverty" (Palmer & Franco 2009, p. 29f.).

On the other hand, however, front-loading looks "fairer" to individuals with a lower life expectancy since they benefit from the fact that they get the control over a larger part of their pension capital earlier in life. Since life expectancy and income are positively correlated this is not a minor issue since front-loading involves a strong regressive element. "It is well established that people with lower incomes have a shorter life expectancy. [...] This means that more generous indexation procedures – to earnings rather than prices, for example – redistribute from poor to rich." (Whitehouse 2009, p. 27). [10] One solution to this dilemma would of course be to use different life tables for different sociodemographic groups when calculating the first pension. This, however, is also not straightforward due

[9] It might be interesting to note that the OECD shows some mild preference for indexation with prices (i.e. some front-loading). "Price indexation of pensions in payment provides more flexibility for individuals to follow their preferred consumption path during retirement. [...] The paper argued that price indexation of pensions in payment is appropriate for most OECD countries for distributional reasons and because of the greater flexibility it offers for pensioners" (Whitehouse 2009, p. 27, 35).

[10] See also Barr & Diamond (2009, p. 82).

to lack of data and also a large degree of arbitrariness in this differentiation which might make it unworkable form a political point of view.

As a last element in these considerations it should also be beard in mind that the popular adjustment with the rate of inflation is often a “second-best solution” to involve already retired people in the financing of a reform. Often the most important piece of information is the first pension payment and cuts in this number are highly politicized while changes in the adjustment mechanisms typically cause much less upheaval.

Seniority wages and pensions

So far I have abstracted from the issues of seniority wages. As shown in chapter 2 it is rather straightforward in this case to design a life expectancy factor that keeps the budget of the pension system in balance by slowly increasing the retirement age. The mechanisms of the NDC system are somewhat different and they involve (as shown above) some sophisticated difficulties but if designed in the right way both of them will also lead to a balanced budget over time. Both of these suggested schemes will, however, run into problems if there is a considerable degree of seniority in wage payments. First, the steepness of the seniority wage curve will depend on the originally assumed retirement age and firms might be unwilling to employ older workers whose wage level will exceed their productivity for the additional years (“The labour-market opportunities for older people can often be limited by age discrimination and other barriers, such as pay schedules that link earnings strongly to seniority (thereby making older workers expensive to hire or retain)” (OECD 2009, p. 71)). Furthermore, it can be shown that a one-time-shift in the seniority profile will also lead to a permanent deficit or surplus of the system just as it has been the case for a one-time-jump in the retirement age (and indexation with average wages).

Important system-inherent exceptions

Each PAYG pension system entails some elements that go beyond the mere objective of individual consumption smoothing. Pension systems are also a tool to insure individuals against life-time risks, to prevent poverty and also (sometimes) to foster some generally agreed upon social goals. The system has to stipulate how to deal with the time of unemployment, the case of disability and survivor pensions. A crucial issue is of course the valuation of non-contributory periods due to childcare. As noted in chapter 5 there are good reasons to argue that the number of children should be taken into account when calculating the pension. This might be done via an explicit "child pension" as, e.g., suggested by Sinn (2000) or via a generous granting of pension capital (or earnings-points in non-NDC systems) when the insured are absent from the labor market due to childcare.

6.4 Comparison of NDC and "quasi-NDC" systems and concluding remarks

Overall, a NDC structure can be regarded as a useful framework to design PAYG pension systems in a sustainable, transparent and intergenerationally balanced way. They put special emphasis on actuarial elements and they are particularly appropriate for the purpose of consumption-smoothing of a typical (full-career) worker. In order to target the other objectives of pension systems (poverty relief, redistribution) NDC systems have to be amended by elements like minimum pensions, payments for non-contributory periods etc. As shown in this chapter, there also exist a number of design features (notional interest rate, life expectancy for annuitization, reductions and supplements etc.) that have to be set in an appropriate way in order to keep the system sustainable, generally accepted and crisis-proof.

At the same time it is important to note that a reasonably designed NDC system might be a sufficient condition for a viable PAYG scheme but not a necessary one. As noted in the beginning of this chapter, there exist a number of different PAYG systems that are also manageable and sustainable frameworks. The stylized structure that has been presented in chapter 2 that uses a sustainability factor and a life expectancy factor is, e.g., perfectly practicable. In fact, this structure would be similar to the German earnings-points system with the exception that the latter so far does not involve an automatic adjustment to increases in life expectancy but only a discretionary (and highly politicized) process of changing it.

In Knell (2005) I compare in detail the NDC system (as used, e.g., in Sweden, Italy, Poland) with an earnings-point system (as used, e.g., in Germany and France) and a "notional defined benefit" system (as used in Austria). These alternative systems are in many respects similar to NDC systems. They also have a lifelong assessment period, a strong link between contributions and benefits and a wage-oriented notional interest rate.[11] In normal weather (a demographic "steady state") NDC and these "quasi-NDC"

[11] "NDC plans are very close to some existing DB plans, particularly those based on a person's lifetime earnings history, differing primarily in vocabularies used to describe them" (Barr & Diamond 2009, p.

systems are thus basically identical. Only in stormy sea the differences stand out, not the least because the incentive structures might be different in the two systems despite identical steady state results.

The main difference of these "quasi-NDC" systems to NDC systems is that developments of life expectancy are not explicitly taken into account (via annuitization at the moment of retirement). In Germany, e.g., the influence is only indirect in as far as an increase in life expectancy will also change the dependency ratio and thus—via the sustainability factor—also the pension level. The Austrian system, on the other hand, would be similar to a NDC system if the 80% target in the 45/65/80 formula would be indexed to life expectancy.[12] Furthermore, even if life expectancy were in one way or another included in the quasi-NDC systems, they would still differ from classic NDC system in their reaction to demographic shocks. As I have discussed earlier, NDC systems often imply a balanced budget only over the medium run while the German earnings-point system is associated with a budget that is balanced in every period. Pari passu, one can expect that the intergenerational distribution of the adjustment burden will also be different in NDC and quasi-NDC systems. As far as the sustainability factor is concerned, I have discussed some issues of intergenerational fairness in chapter 5. For reasons of brevity I have not repeated these analyses in this chapter for the NDC system and this remains a promising and important topic for future research. Finally, there are also differences in transparency and simplicity between the systems, maybe even a trade-off between simplicity and optimality. The biggest advantage of NDC systems is probably that they are easy to understand, they involve few parameters and they offer a clear framework to think about pensions and individual decisions. They also appear to be better compatible with the "meta adjustment principle" which says that adjustment factors should themselves be rather easy to adjust when the necessity comes about. Alternative systems will either tend to mimic DC systems or remain somewhat opaque and manipulable. A NDC structure could probably also function as a useful device for the development of a coordinated Pan-European pension system with better cross-country portability (cf. Holzmann 2004). For the reasons mentioned above it does not make much sense, however, to intervene here

34).

[12] Austria has since 2005 a NDB system (non-financial defined benefit; or better: LIKU [leistungsorientiertes Individualkonto auf Umlagebasis]). The system works exactly like a earnings-point system plus individual accounts. The Austrian sustainability factor differs from the German counterpart in that (i) it responds only to deviations from forecasts, (ii) it does not provide for an automatic adjustment (a process rather than a factor), (iii) it does not specify in detail how the five crucial pension system parameters are to be adjusted ("with an equal distribution on the parameters".) See Knell (2005).

in a strict or dirigistic manner. Most likely a Pan-European system will emerge without explicit centralization although maybe with the support of some institution (like the EPC) that coordinates these efforts (via international advice and sharing of expertise; prevention of design flaws and costly lock-in effects; support for national experts and reform champions etc.).

Even after having taken all these arguments in favor of a NDC system into account one should still keep in mind that there are “many different ways of achieving the various objectives of pension systems. The diversity of (more or less) well-run systems is considerable. This [...] is as it should be” (Barr & Diamond 2009, p. 165). This is the closest I can get to answer the demand of the *Hannes Androsch Prize* to provide a proposal “for an alternative design, which would optimize the magnitude and stability of pensions over time, and confront the double challenge of demographic developments and financial market risk”. The preceding pages have made a long, investigative journey through various fields in order to answer this question in a comprehensive manner. The search for a completely novel, so far unthought and untried alternative pension systems has turned out to be elusive. The different considerations that have looked at the problem from various angles suggest, however, that a pension scheme with a long pedigree—the PAYG system—looks like an attractive candidate to fulfill the desiderata to a satisfactory extent. I have argued that a PAYG scheme can be designed in a way that the system is robust to demographic shocks (chapter 2), that it is likely to react to demographic (chapter 3) and financial market (chapter 4) shocks in a way that is more favorable than the one of a funded system and that it can be be calibrated in a fashion such as to accord to various principles of intergenerational fairness (chapter 5). The exact design of the PAYG system (chapter 6), its size, redistributive elements and the eventual mixture with a (small) funded pillar will depend on particular circumstances and national preferences. This flexibility is, however, a further advantage of the PAYG system which is capable to adapt and to evolve in a permanently changing environment.

6.5 Related literature

A detailed description of NDC systems can be found in Palmer (2000), Disney (1999), Börsch-Supan (2003), Wilke (2008) and in the book edited by Holzmann & Palmer (2006). In-depth and partly critical discussions NDC system can be found in Börsch-Supan (2004) and Williamson & Williams (2003).

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