Solvency assessment – a pragmatic approach

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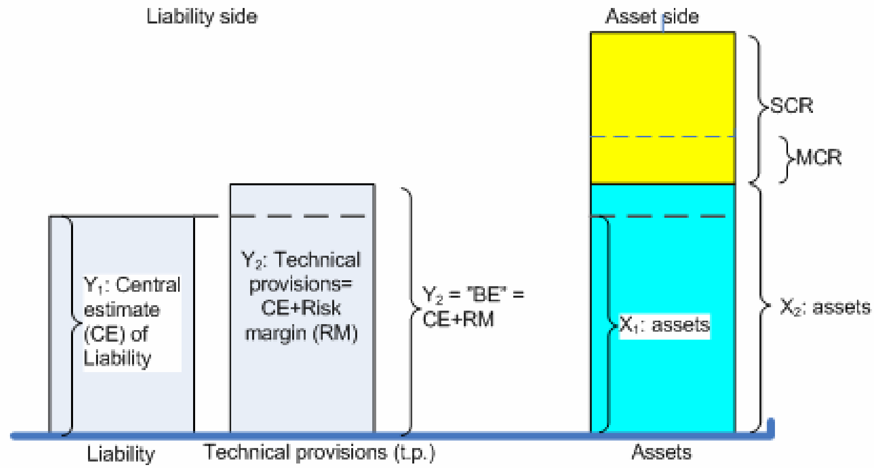
NO, I INSIST!
cara

SOLVENCY ASSESSMENT – Supervisory process

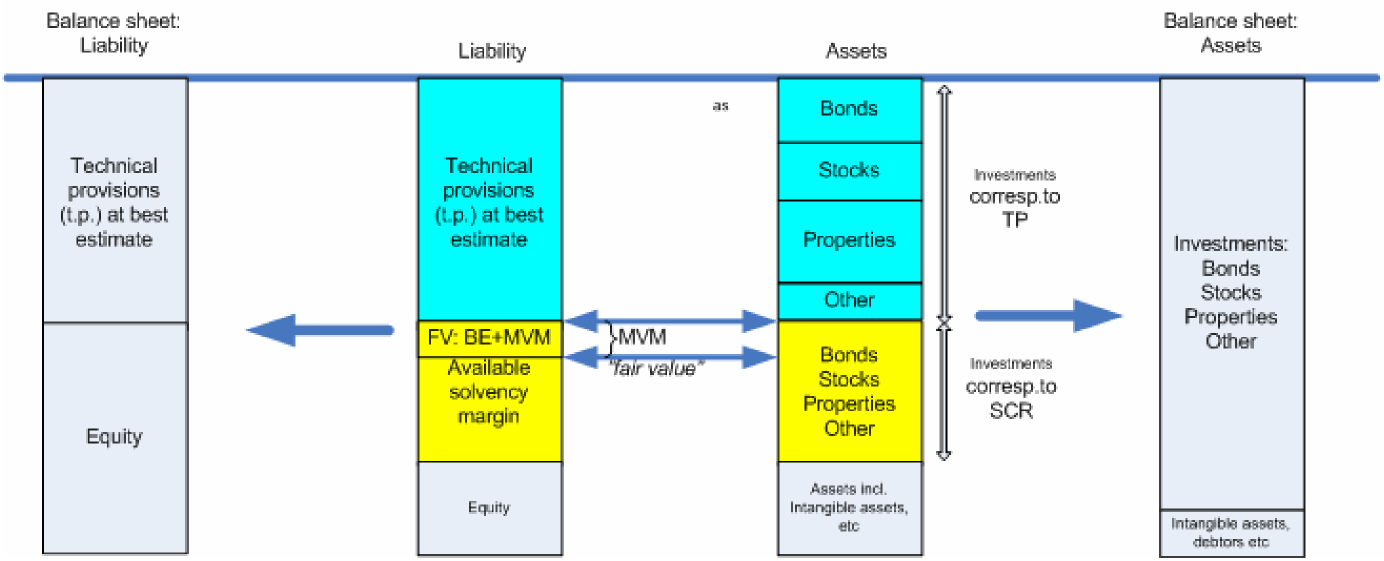


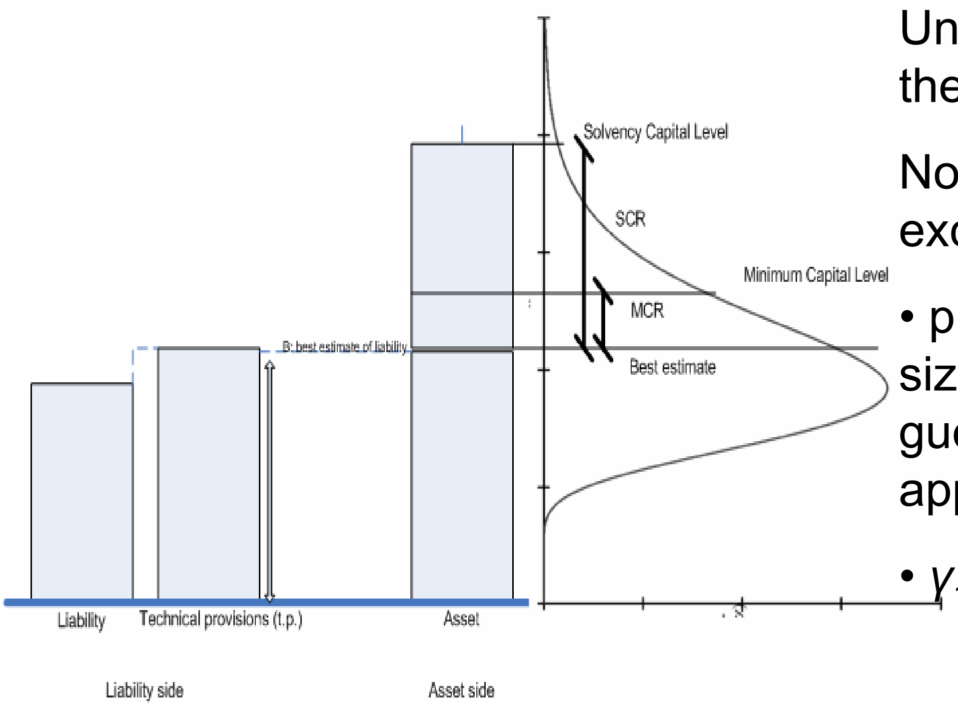
Risk margin, RM = $f_1(Y_1)$ non-life
Risk margin, RM = $k(y_1,x_1) = f_1(y_1) + f_2(X_1)$ life

Market value Margin, MVM = $g(Y_2,X_2)$

SCR = $h(Y_2,X_2)$

ACCOUNTING ASSESSMENT – Financial reporting



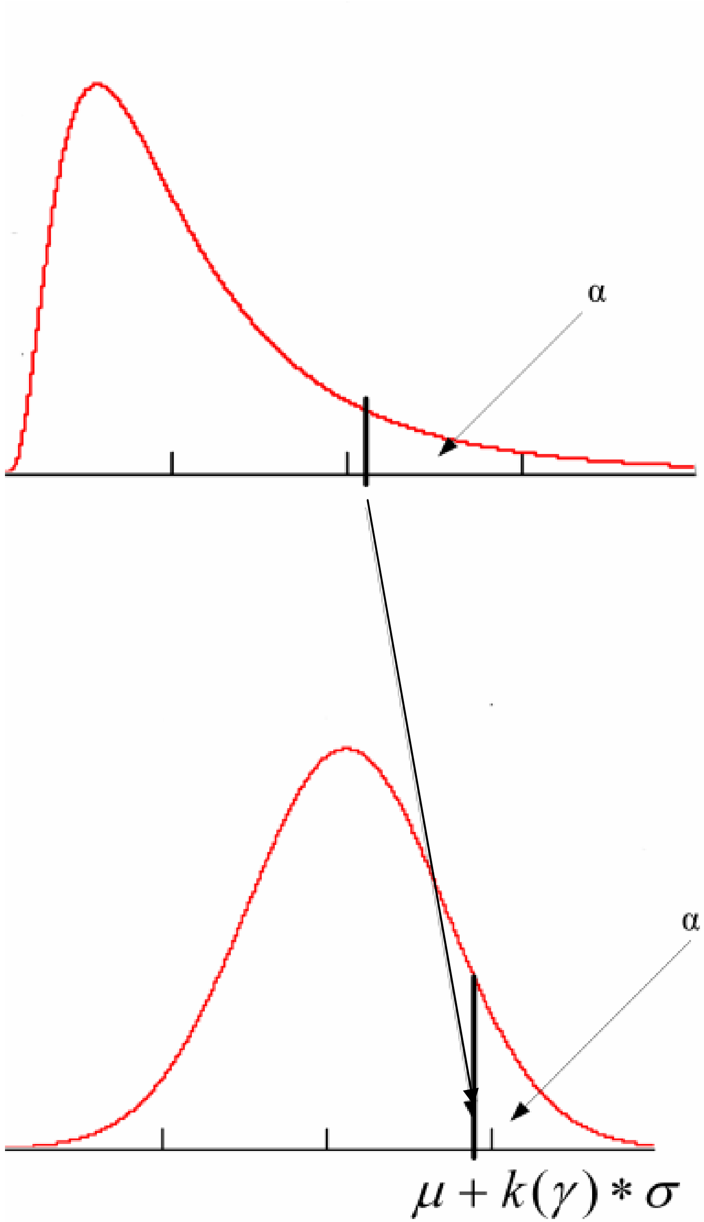


Unknown, but surely skewed to the right.

No distributional assumptions, except the skewness ➔ γ_1

- probably easier to guess the size of the skewness than to guess what distribution is most appropriate!

- $\gamma_1 > 0$



1st pragmatic approach:

use NP-approximation

NP-approximation of VaR in the skew, unknown, distribution transforms it to a standard normal distribution:

$\mu + k(\gamma_1) * \sigma$,

where k(•) depends on the percentile and the skewness!

- $k(\gamma_1)=k_{1-\alpha} + \gamma_1 (k^2_{1-\alpha} -1)/6$

$1-\alpha$	$k_{1-\alpha}$	$k(\gamma_1)$
0.990	2.33	$2.33+0.74\gamma_1$
0.995	2.58	$2.58+0.94\gamma_1$
0.999	3.09	$3.09+1.43\gamma_1$

$1-\alpha=0.995$ and $\gamma_1 =1$: $k(\bullet) = 3.52$

IAA-baseline as start

- General structure from linear correlation structure:

  for example, 4 risk categories, and let

  $$C_i = k\sigma_i$$

  then the total risk is

$$C = \sqrt{C_1^2 + C_2^2 + C_3^2 + C_4^2 + 2\rho_{12}C_1C_2 + 2\rho_{13}C_1C_3 + 2\rho_{14}C_1C_4 + 2\rho_{23}C_2C_3 + 2\rho_{24}C_2C_4 + 2\rho_{34}C_3C_4}$$

	1	2	3	4
1	1	?	?	?
2		1	?	?
3			1	?
4				1

Assume that risk category 4 is fully correlated with the other three

	1	2	3	4
1	1	?	?	**1**
2		1	?	**1**
3			1	**1**
4				1

Linear correlation implies the exact correlation structure ➔

	1	2	3	4
1	1	**1**	**1**	1
2		1	**1**	1
3			1	1
4				1

- if the risks are fully correlated then

$$\Rightarrow C = C_1 + C_2 + C_3 + C_4$$

- but this is not always what we believe in!

- Note the following:

$$\rho_{12} = 0 \Rightarrow \sigma_{12}^2 = \sigma_1^2 + \sigma_2^2 \Rightarrow \sigma_{12} = \sqrt{\sigma_1^2 + \sigma_2^2}$$

$$\rho_{12} = 1 \Rightarrow \sigma_{12} = \sigma_1 + \sigma_2$$

	1	2	3	4
1	1	?	?	?
2		1	?	?
3			1	?
4				1

Once again, assume that risk category 4 is fully correlated with the other three

	1	2	3	4
1	1	?	?	**1**
2		1	?	**1**
3			1	**1**
4				1

Benchmark approach:

non-linear relationships, for example like this ➔

	1	2	3	4
1	1	**0**	**1**	1
2		1	**0**	1
3			1	1
4				1

Look at the Müller report (1997, p. 55) describing the NAIC-RBC system:

Once all RBC values of the individual categories have been calculated they are combined into the total RBC. For this the individual values are, however, not simply added up but compensation is made because not all risks will cause losses simultaneously. If it is assumed that both asset risk and interest rate risk (C1 and C3) are completely correlated and the technical risk (C2) is not related to either of them and in addition that the business risk (C4) is completely correlated with the other three risks this will result in a total RBC in life insurance (RBCLV) as follows:

$$RBC_{LV} := C_4 + \sqrt{C_2^2 + \left(C_1 + C_3\right)^2}$$

2nd pragmatic approach: use the benchmark modeling structure

We do as follows:

- The fourth risk is fully correlated with all the other three, i.e. $\rho_{4,(123)} = 1$

  giving us the following pragmatic structure:

$$C = C_4 + C_{(123)}$$

- consider now $C_{(123)}$: the second risk is uncorrelated with the two others, i.e.

$$C_2^2 + C_{(13)}^2$$

- The last risk is $C_{(13)}^2 = \left(C_1 + C_3\right)^2$ since they are assumed to be fully correlated.

- This gives us the RBC-structure above!

- Note that each main risk category can be thought of as consisting of different sub-risks;

  for example we can have the following

  structure for risk C_2 :

$$C_2^2 = C_{21}^2 + \left(C_{22} + C_{23} + C_{24}\right)^2 + 2\rho_{21,25} C_{21} C_{25} + C_{25}^2$$

- To be more pragmatic: even if we have started with

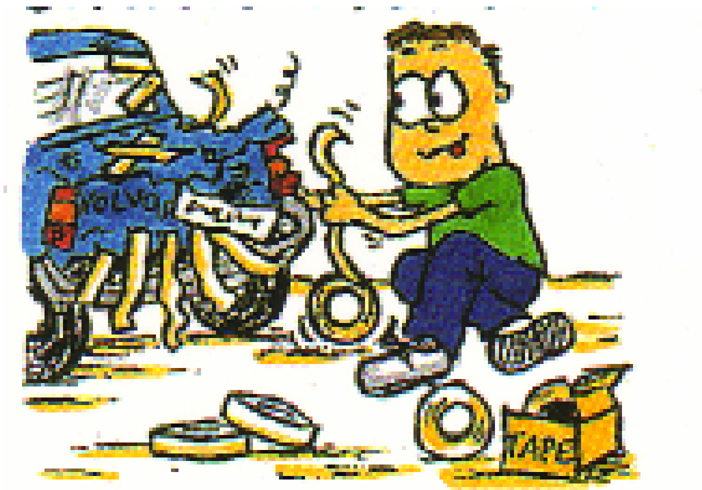
  $C_* = k(\gamma_1)\sigma_*$

  we may choose to let C_* be the result of a stress test (= the capital charge based on a stress test).

Summary

- Pragmatic approach 1:
  - – Assume skewness and not a d.f.
  - – Use NP approximation and hence a standard normal framework

- Pragmatic approach 2:
  - – Benchmark structure for risk categories



SWEDISH PRAGMATISM