A proposal of interest rate *dampener* for Solvency II Framework introducing a three factors mean reversion model

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- Solvability II : Main Objectives and Consequences
- The EIOPA's Standard Formula Approach
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  - Historical Behavior
- Introducing an Interest Rate Dampener
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- The Solvency II Directive is an EU directive that both reforms and harmonizes the EU insurance regulation
- Each Insurance undertaking has to hold sufficient capital to face the 0.5% probability worst case annual losses
- This main objective leads to three sub-objectives.
  - The first sub-objective is setting a process of VaR determination which, according to Christoffersen (1999), has at least the same importance as the results themselves
  - The second is quantifying and identifying each risk factor, and consequently introducing quantitative analysis in management
  - The third, is giving more transparency to equity holders and regulatory
  - The last, named Own Risk and Solvency Assessment (ORSA) is to integrate the risk measure in the decision process. The obligation of making provision for risk leads to think about capitalistic intensity
- Considering that risk has to be reserved (concept of risk margin, cost of options and warranties) this raises the issue of uncertainty about the risk itself (see Hugonnier (2012))

Let consider a simple insurance portfolio modeled as a series of zeros coupon both on liabilities and assets. These Zero Coupons are traditionally valuated as follow

$$P(T, r_T, s_{rtg}) = \frac{1}{(1 + r_T + s_{rtg})^T} \tag{1}$$

Where r_T is the risk free interest rate for the maturity T and s_{rtg} the spread for the rating rtg (s_{rtg} is equal to zero for liabilities).

Let's assume

$$\Delta NAV_{r_T} = P(T, r_T, s_{rtg}) - P(T, VaR_{r_T}, s_{rtg}) \tag{2}$$

$$\Delta NAV_{s_{rtg}} = P(T, r_T, s_{rtg}) - P(T, r_T, VaR_{s_{rtg}}) \tag{3}$$

Considering the assumption that r_T and s_{rtg} are distributed according to a Gaussian copula, $VaR_{99.5\%}$ of $P(T, r_T, s_{rtg}))$ can be written as follows :

$$VaR_{99.5\%}(P(T, r_T, s_{rtg})) = P(T, r_T, s_{rtg}) + \sqrt{\Delta NAV^2_{r_t} + \Delta NAV^2_{s_{rtg}} + 2\rho_{rs}\Delta NAV_{r_T}\Delta NAV_{s_{rtg}}} \tag{4}$$

With ρ_{rs} is defined as Pearson correlation between r_T and s_{rtg} for extreme values

- For each risk factor the $VaR_{99.5\%}$ has to be assessed and this can be done by various methods :
  - Time series theory
  - Market consistent diffusion process
  - Bootstrap
  - **Historical VaR based on overlapped one year variations (EIOPA chosen method)**
- The interest rate risk factor is assessed for each maturity on the overlapped one year variation of government zero coupon term structure calibration paper (CEIOPS 2010a)
- The spread risk factor is assessed for each rating on the overlapped one year variation of the Bank Of America Merrill Lynch Option adjusted spread Index

	EIOPA 1997 to 2009		1999 to 2009		1999 to 2013	
T	Down	Up	Down	Up	Down	Up
1	-75%	70%	-66%	76%	-78%	78%
2	-65%	70%	-58%	73%	-72%	77%
3	-56%	64%	-49%	66%	-67%	67%
4	-50%	59%	-43%	60%	-61%	59%
5	-46%	55%	-39%	55%	-57%	54%
6	-42%	52%	-35%	52%	-53%	50%
7	-39%	49%	-33%	48%	-50%	47%
8	-36%	47%	-31%	47%	-47%	46%
9	-33%	44%	-29%	45%	-45%	44%
10	-31%	42%	-27%	44%	-44%	43%
15	-27%	33%	-24%	38%	-41%	38%
20	-29%	26%	-24%	34%	-42%	33%
30	-30%	25%	-26%	30%	-42%	28%

Rating	1999 to 2009
AAA	90bps
AA	110bps
A	140bps
BBB	250bps
BB	450bps
B or lower	750bps
Unrated or lower	300bps

Figure : VaR of each factors (Historic VaR assessment)

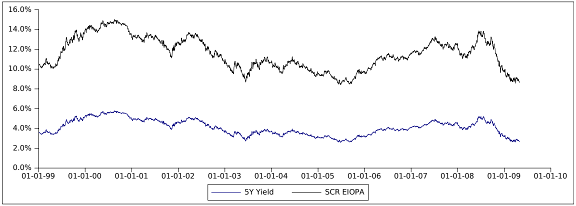

Figure : Historical Behavior of the 5Y AAA ZC Bonds SCR

- The evolution shows some inconsistencies in the EIOPA risk assessment : The capital needed to cover a raise of interest rate increase when the interest rates are higher (same logical for the capital needed to cover a decrease of interest rate)
- This behavior leads to a significant and useless pro-cyclical effect : Capital requirement increases after loss
- This is incompatible with ORSA : Risk measures have to be coherent in order to integrate the decision process
- Inconsistencies of the standard formula impose an internal model and generate a competitive advantage for large companies
- To reinstate the standard formula has. A trade-off between accuracy, simplicity and a recognition of reasonable counter-cyclicity has to be found

As we previously mentioned, our goal is to set up a model that

- Is not over-sophisticated but apprehends the main behavior of the yield curve.
- Gives priority to the possibility of giving economic justifications as well as allows an analytical expression of the quantile. The last point aims to avoid the use of Monte Carlo simulations

This model is based on the Nelson Siegel regression in which the yield curve is expressed by the following regression (see Nelson and Siegel (1987))

$$R(t,u) = l_t - s_t\phi(\frac{u}{\tau}) + c_t\psi(\frac{u}{\tau}) \tag{5}$$

Where $\phi(x) = \frac{1-e^{-x}}{x}$, $\psi(x) = \phi(x) - e^{-x}$, t is the date, u is the maturity of the interest rate. l_t is the level, s_t the slope and c_t the curvature.

The choice of these risk factors is confirmed from the principal component analysis on the variation of the yield curve (cumulated inertia of the first three Eigen vectors is about 95%)

The estimation of l_t, s_t, c_t and τ is traditionally made by least-square regression, (see Nelson 1987). Nevertheless this method shows instability (see Gili and al. (2010)). This makes the estimation of diffusion parameters difficult

Consequently we choose the Diebold and al. (2006) method where l_t, s_t and c_t are estimated by solving the following system for each date :

$$\begin{cases} R(t,0.25) = l_t - \phi(\frac{0.25}{\tau})s_t + \psi(\frac{0.25}{\tau})c_t \\ R(t,10) \quad = l_t - \phi(\frac{10}{\tau})s_t + \psi(\frac{10}{\tau})c_t \\ R(t,30) \quad = l_t - \phi(\frac{30}{\tau})s_t + \psi(\frac{30}{\tau})c_t \end{cases} \tag{6}$$

We have chosen

- τ in order to minimizes the correlation between risk factors ($\tau = 2.6$)
- The maturities triplet (0.25Y, 10Y, 30Y) in order to maximize the R^2 between model and observed curve

We then come to set stochastic differential equations. These equations are chosen for the small number of parameters, for the possibility of providing an economic interpretation, but especially for the existence of an analytical expression of the quantile

$$
\begin{aligned}
dl_t &= k_l(\theta_l - l_t)dt + \sigma_l dW_t^l && (7)\\
ds_t &= k_s(\theta_s - s_t)dt + \sigma_s dW_t^s && (8)\\
dc_t &= k_c(\theta_c - c_t)dt + \sigma_c dW_t^c && (9)
\end{aligned}
$$

Stochastic differential equations on l_t, s_t and c_t are integrated by applying the Ito Lemma. We then deduce the law of each risk factors :

$$
\begin{aligned}
l_T|F_{t_0} &\sim N(l_0 e^{-k_l T} + \theta_l(1 - e^{-k_l T}) \ , \ \sigma_l \sqrt{\frac{1 - e^{-2k_l T}}{2k_l}}) && (10)\\
s_T|F_{t_0} &\sim N(s_0 e^{-k_s T} + \theta_s(1 - e^{-k_s T}) \ , \ \sigma_s \sqrt{\frac{1 - e^{-2k_s T}}{2k_s}}) && (11)\\
c_T|F_{t_0} &\sim N(c_0 e^{-k_c T} + \theta_c(1 - e^{-k_c T}) \ , \ \sigma_c \sqrt{\frac{1 - e^{-2k_c T}}{2k_c}}) && (12)
\end{aligned}
$$

where T is the horizon of risk and equals to one year in Solvency II context.

We are able to obtain an analytical expression of the quantile. To clarify all the following equations, let we have

$$\alpha_l(T) = l_0 e^{-k_l T} + \theta_l(1 - e^{-k_l T}) \tag{13}$$

$$\beta_l(T) = \sigma_l \sqrt{\frac{1 - e^{-2k_l T}}{2k_l}} \tag{14}$$

$$\alpha_s(T) = s_0 e^{-k_s T} + \theta_s(1 - e^{-k_s T}) \tag{15}$$

$$\beta_s(T) = \sigma_s \sqrt{\frac{1 - e^{-2k_s T}}{2k_s}} \tag{16}$$

$$\alpha_c(T) = c_0 e^{-k_c T} + \theta_c(1 - e^{-k_c T}) \tag{17}$$

$$\beta_c(T) = \sigma_c \sqrt{\frac{1 - e^{-2k_c T}}{2k_c}} \tag{18}$$

Therefore l_T, s_T and c_T are expressed as follow

$$l_T | F_{t_0} \sim N(\alpha_l \ , \ \beta_l) \tag{19}$$

$$s_T | F_{t_0} \sim N(\alpha_s \ , \ \beta_s) \tag{20}$$

$$c_T | F_{t_0} \sim N(\alpha_c \ , \ \beta_c) \tag{21}$$

We deduce from (5),(19),(20) and (21) the distribution for $R(T,u)|F_{t_0}$:

$$R(T,u)|F_{t_0} \sim N(\alpha_R \ , \ \beta_R) \tag{22}$$

With

$$\begin{aligned}
\alpha_R &= \alpha_I + \phi(\frac{u}{T})\alpha_s + \psi(\frac{u}{T})\alpha_c \\
\beta_R^2 &= \beta_I^2 + \ \phi^2(\frac{u}{T})\beta_s^2 + \ \psi^2(\frac{u}{T})\beta_c^2 \\
&\quad + 2\ \phi(\frac{u}{T})\ \beta_I\ \beta_s\ \rho_{dW^I,dW^s} \\
&\quad + 2\ \psi(\frac{u}{T})\ \beta_I\ \beta_c\ \rho_{dW^I,dW^c} \\
&\quad + 2\ \phi(\frac{u}{T})\ \psi(\frac{u}{T})\ \beta_c\ \beta_s\ \rho_{dW^c,dW^s}
\end{aligned}$$

Therefore we have an analytical expression for the Solvency II percentiles and α_R is the proposed interest rate dampener

$$VaR_{99.5\%}(R(T,u)|F_{t_0}) = \beta_R U(99.5\%) + \alpha_R \tag{23}$$

$$VaR_{0.5\%}(R(T,u)|F_{t_0}) = \beta_R U(0.5\%) + \alpha_R \tag{24}$$

With $U(99.5\%) = 2.576$ and by symmetry $U(0.5\%) = -2.576$

According to Gillespie (1996) the exact discretization of an Ornstein-Uhlenbeck process is :

$$X_{t+\Delta_t} = e^{-k_X \Delta_t} X_t + (1 - e^{-k_X \Delta_t})\theta_X + \sigma_X \sqrt{\frac{1 - e^{-2k_X \Delta_t}}{2k_X}} dW_t^X \tag{25}$$

For this calibration we are looking for the linear relation which minimizes the residuals squares

$$X_{t+\Delta_t} = c_0^X + c_1^X X_t + \varepsilon_t^X \tag{26}$$

By identification we have:

$$k_X = -\frac{ln(c_1^X)}{\Delta_t} \tag{27}$$

$$\theta_X = \frac{c_0^X}{1 - e^{-k_X \Delta_t}} \tag{28}$$

$$\sigma_X = \sqrt{\frac{2k_X Var(\varepsilon_t^X)}{1 - e^{-2k_X \Delta_t}}} \tag{29}$$

Between January 4th 1999 and May 1st 2009 we estimate the risk factors l_t, s_t and c_t on a daily frequency basis excluding market closed days. Then we use the calibration method previously mentioned and we obtain the calibration below [1]:

$$
\begin{array}{lll}
k_l = 51.04\% & k_s = 29.76\% & k_c = 185.96\% \\
\theta_l = 4.947\% & \theta_s = 2.269\% & \theta_c = -0.208\% \\
\sigma_l = 0.796\% & \sigma_s = 0.921\% & \sigma_c = 1.953\%
\end{array}
$$

Finally we deflate the deterministic drift from l_t, s_t and c_t variations and we obtain the correlation between the three brownian motions

$$
\begin{aligned}
\rho_{dW^l,dW^s} &= 36.81\% \\
\rho_{dW^l,dW^c} &= -3.34\% \\
\rho_{dW^c,dW^s} &= -1.77\%
\end{aligned}
$$

[1] Data and source code are available upon request

- Our extended approach reveals dependence between the interest rate curve and the stress value
- To compare the stress with the EIOPA standard formula we define a stationary state where stresses are not time dependent
- In this state l_t, s_t and c_t are equal to their long term average $(\theta_l, \theta_s, \theta_c)$. A stationary curve is then defined with its associated stress
- We consider it as comparable to the EIOPA standard stress scenario on interest rates. However, in this state, the model stress does not exactly match with the EIOPA stress

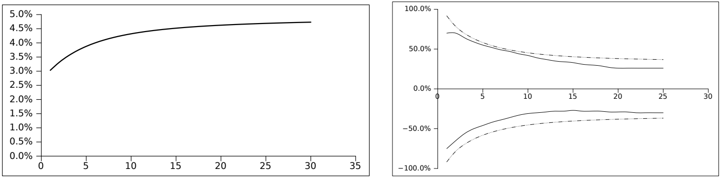

Figure : Stationary Yield Curve and Tunnel of doubt

- The mismatch between the two methods comes from the parameter's assessment methodology
  - EIOPA estimates directly the annual VaR on the series of overlapped annual returns on a daily basis
  - The proposed model estimates a daily volatility, then annualizes it with consideration of one order auto-correlation. The annual VaR is then deduced under a normal assumption by integration of the brownian process

- In order to quantify the bias of each method we simulate 10^4 Ornstein Uhlenbeck processes over 10 years (parameters are those of the I_t process), the annual VaR is consequently perfectly known. We then apply the two methodologies to assess the VaR
  - The average of the EIOPA's method VaR assessment is 35.31% with a standard deviation of 5.52%
  - The average of the auto regression method is 41.37% with a standard deviation of 0.59%

- This has to be compared to the theoretical VaR of the simulated process which is 41.29%. These results make appear that our method is less biased than EIOPA's and consequently the EIOPA risk framework could have a lack of prudency

- This lack of prudency comes from the overlapping method. Indeed this method considers only realized path in opposition to the auto regression method which allows consideration of unrealized path
- We notice a convergence between two methods when we extend significantly (over 100Years) the calibration window.
- Despite its bias we don't want to derive from the standard stress and we apply overlapping method to re-assess the volatility only for the level risk factor (l_t). We found 0.503% for the volatility instead of 0.796%. This leads to a VaR of 25% for the 30 year and fit the EIOPA's assessment (26%)
- The funnels of doubt of the model fit consequently the EIOPA's one

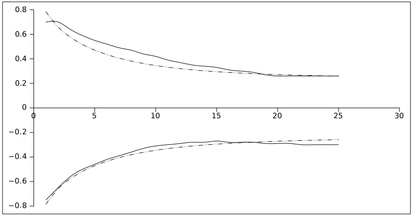

Figure : Convergence of EIOPA funnel of doubt model (plain lines) against model funnel of doubt (dotted lines) with EIOPA adjusted calibration

- With the proposed framework the up-stress is negatively correlated with interest rate levels and down-stress is positively correlated with interest rate levels
- As a proof of concept, we consider a simple asset modeled as a AAA 5Y Zero coupon, exposed to the upward stress and we compute its SCR with the EIOPA method and the proposed mean-reversion model between January 4th 1999 and May 1st 2012

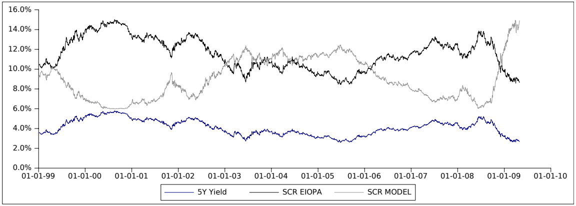

Figure : SCR Evolution of a 5Y AAA Asset (Modeled by a Zero Coupon) exposed to the increase of interest rate

- We consider two fictive insurance companies one life, one non-life and make an historical simulation from 1999 to 2013 of the solvency ratio of these companies
- Characteristics of the non-life company (exposed to an increase of interest rate) :

Settlement Date (t_0)	January 4th 1999
Asset Portfolio Value No discount factor (GA)	123.8MEUR
Best estimate of Liabilities No discount factor (GL)	103.2MEUR
Asset Portfolio Value (A)	100.0MEUR
Best estimate of Liabilities (L)	100.0MEUR
Risky Asset duration (u_a)	5Years
Risky Asset mean rating (rtg)	A
Liabilities duration (u_l)	1Year
Solvency Capital Requirement at t_0 (SCR)	9.3MEUR
Solvency Ratio (SR)	3.0
Own Funds (OF)	28.0MEUR

Table : Characteristics of the non-life company

For this company $\forall t$:

$$
\begin{aligned}
A(t) &= \frac{GA}{(1+R(t,u_A))^{u_A}} && (30)\\
L(t) &= \frac{GL}{(1+R(t,u_L))^{u_L}} && (31)\\
VaR_r(t) &= A(t)S_{eup}^{u_A}(t)R(t,u_A)u_A - L(t)S_{eup}^{u_L}(t)R(t,u_L)u_L && (32)\\
VaR_{spr}(t) &= A(t)u_A VaR_{spr}(rtg) && (33)\\
SCR(t) &= \sqrt{VaR_r(t)^2 + VaR_{spr}(t)^2 + 2\rho_{rs} VaR_r(t) VaR_{spr}(t)} && (34)\\
SR(t) &= OF(t)/SCR(t) && (35)
\end{aligned}
$$

Where $\rho_{rs} = 0.5$ when we consider a down stress and $\rho_{rs} = 0.0$ in the up stress We consider that Own Funds (OF) are determined recursively as follows

$$
\begin{aligned}
OF(t_0) &= 2SCR(t_o) && (36)\\
OF(t) &= OF(t-\Delta_t) + (A(t) - A(t-\Delta_t)) - (L(t) - L(t-\Delta_t)) && (37)
\end{aligned}
$$

We make an historical simulation of the solvency ratio between this date and May 1st 2012

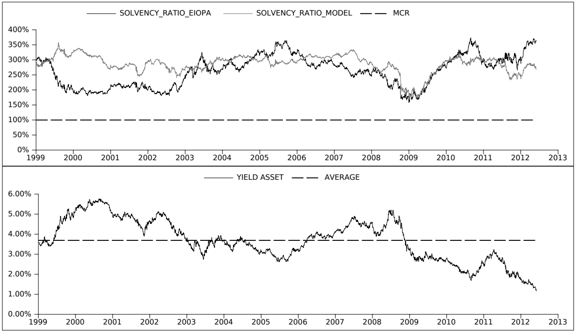

Figure : Historical Simulation of the Solvency ratio for a non-life company

	EIOPA Method	Proposed method
Average	263%	287%
Min	159%	172%
Max	374%	357%
Maximum Drawdown	215%	185%
Volatility	51%	30%

Table : Statistics about Solvency ratio

- The solvency ratio computed with the proposed model is higher than the ratio computed with the EIOPA's standard formula in high interest rates period, and lower in low interest rates period
- The proposed framework smoothes the switching between bad and good market environment conditions.
- This improvement also decreases the volatility of the solvency ratio, 51% with the EIOPA method versus 30% with the proposed framework and decreases the sensitivity of the solvency ratio to the yield curve
- As consequence it leads also to a simplification of the management of Own Funds and hedging strategy by decreasing the probability of a SCR higher than available Own funds (ratio below 100%)
- By introducing solvency ratio inertia the framework helps insurance undertakings to survive in temporary difficult market conditions
- Nevertheless the proposed framework has no effect on the spread variation. We can observe in the 2009 spread crisis that the solvency ratio drops because of increase of spread whatever the used risk framework
- Introduce a spread dampener in the spirit of the Matching Adjustment could consequently complete the proposed interest rate dampener

The same exercise with similar results can be made on a life insurance company, where liabilities are longer than assets and consequently exposed to a decrease of interest rate:

Settlement Date (t_0)	January 04th 1999
Asset Portfolio Value No discount(GA)	145.3MEUR
Best estimate of Liabilities No discount(GL)	251.3MEUR
Asset Portfolio Value (A)	100.0MEUR
Best estimate of Liabilities (L)	100.0MEUR
Risky Asset duration (u_A)	8Years
Risky Asset mean rating (rtg)	A
Liabilities duration (u_L)	20Year
Solvency Capital Requirement at t_0 (SCR)	25.1MEUR
Solvency Ratio (SR)	3.0
Own Funds (OF)	75.2MEUR

Table : Characteristics of the life company

We are keep the same management rules as previously and all formula are the same except for the $VaR_r(t)$ (we consider a decrease of interest rate) for which we have :

$$VaR_r(t) = L(t)S^{u_L}_{e_{dwn}}(t)R(t,u_A)u_L - A(t)S^{u_A}_{e_{dwn}}(t)R(t,u_L)u_A \tag{38}$$

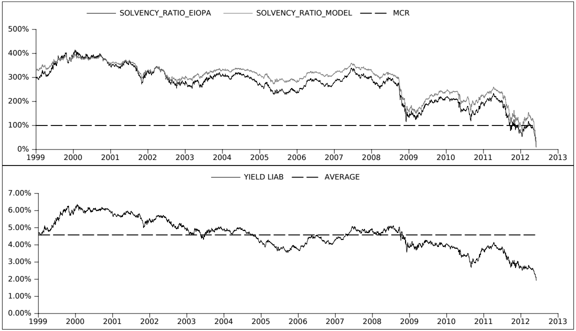

Figure : Historical Simulation of the Solvency ratio for a life company

	EIOPA Method	Proposed method
Average	268%	292%
Min	10%	14%
Max	415%	397%
Maximum Drawdown	405%	384%
Volatility	78%	69%

Table : Statistics about Solvency ratio

- Interest rates are an important source of risk exposure for insurance undertakings caused by a usual duration mismatch
- It appears that the standard risk approach is biased and results in an increase of the SCR requirement in a wrong timing
- This implies a dangerous pro-cyclical effect and misleads the meaning of a management driven by the standard risk formula
- This is a strong incentive to costly internal models development to the insurance industry in order to ensure meaningful decisions
- Our proposal, which is in line with the spirit of the standard approach, introduces simply but approved stochastic modifications, and corrects this bias of the standard formula for non-life and life insurance companies
- The proposed framework can also be improved. The dampener mechanism can be apply to other risk factors and the modeling could integrate the state of the art of stochastic processes theory

- Slides are based on the article :

  Alexandre Le Maistre and Frédéric Planchet. A proposal of interest rate dampener for Solvency II Framework introducing a three factors mean reversion model. Les cahiers de recherche de L'ISFA,n° 2013-2

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