Influence of Economic Factors on the Credit Rating Transitions and Defaults of Credit Insurance Business

Anisa Caja*[1], Quentin Guibert†[1], and Frédéric Planchet‡[1,2]

[1] *ISFA, Laboratoire SAF, Université Claude Bernard Lyon 1*§

[2] *Prim'Act*¶

July 20, 2015

Abstract

This paper presents a model for the determination and forecast of the number of defaults and credit changes by estimating a reduced-form ordered regression model with a large data set from a credit insurance portfolio. Similarly to banks with their classical credit risk management techniques, credit insurers measure the credit quality of buyers with rating transition matrices depending on the economical environment. Our approach consists in modeling stochastic transition matrices for homogeneous groups of firms depending on macroeconomic risk factors. One of the main features of this business is the close monitoring of covered firms and the insurer's ability to cancel or reduce guarantees when the risk changes. As our primary goal is a risk management analysis, we try to account for this leeway and study how this helps mitigate risks in case of shocks. This specification is particularly useful as an input for the *Own Risk Solvency Assessment* (ORSA) since it illustrates the kind of management actions that can be implemented by an insurer when the credit environment is stressed.

Keywords. Credit insurance; Rating transition matrix; Cumulative link model; Doubly stochastic assumption; Macroeconomic model; VECM; Stress test.
Classification JEL. G22; C32; G33.

*Email: ani_caja@yahoo.com.
†Email: q.guibert@hotmail.fr.
‡Email frederic.planchet@univ-lyon1.fr.
§Université de Lyon, Université Claude-Bernard Lyon 1, Laboratoire de Sciences Actuarielle et Financière, Institut de Science Financière et d'Assurances, 50 Avenue Tony Garnier, F-69007 Lyon, France.
¶Prim'Act, 42 Avenue de la Grande Armée, 75017 Paris, France.

Contents

.

1 Introduction

Credit insurance is concerned by business and credit cycle fluctuations, but in an unusual way. Commercial businesses need a credit insurer if they think their clients, called buyers, might not be able to pay their invoices within a pre-defined time (protracted default), or are likely to become insolvent shortly (solvency default). They subscribe an insurance policy which guarantees them that a part of the whole invoice amount will be reimbursed by the insurer in case the client couldn't pay. Credit insurers have an important leeway to manage this credit risk within the insured portfolio and are able to limit or cancel the offered guarantees at any time, depending on the change in the credit quality of the insured firm, thus eventually on the evolutions of the business cycle and the macroeconomic environment (Caja and Planchet, 2014). For that purpose, insurers need to pinpoint those firms the quality of which deteriorates and which have a greater risk of default. Practically, they deploy internal credit rating systems for risk management in a way that is similar to that of banks for the Basel requirements. These systems compute ratings for all the buyers and give an accurate view of the portfolio risk on a monthly basis.

In this paper, we develop a reduced-form model measuring over time the effects of macroeconomic risk factors on the credit quality of a portfolio of buyers and we analyze the response in term of risk management. Our analysis is built on an historical database of credit ratings provided by a French credit insurer. The academic literature regarding risk management of credit insurance is extremely scarce and the only available papers (Passalacqua, 2006, 2007) discuss the pricing and the forecast of the credit loss distribution without analyzing the dynamics of credit ratings. To the best of our knowledge, we are the first to model the linkage between macroeconomic risk factors and the credit insurance activity. To our knowledge, practitioners use structural models based on unobservable credit risk factors, e.g. the Merton's model (Merton, 1974) or the KMV model. They generally consider heterogeneity across portfolio with specific balance sheet indicators and specify a simple default correlation for buyer's asset values. Recently, Caja and Planchet (2014) extend this type of model and seek to capture good and bad credit cycles with the help of two constant migration matrices calibrated on two different periods with presupposed different regimes. However, such models do not fit adequately the observed data and remain very limited.

We propose a discrete version of a doubly-stochastic Markov chain model for default rate and migration matrix dynamics, also called stochastic migration matrices (Gagliardini and Gouriéroux, 2005), driven by macroeconomic variables. This type of specification, with a doubly-stochastic independence assumption, is used in many banking credit risk models (see e.g. Duffie *et al.*, 2007) for default rate dynamics. Our current approach allows to compute upgrades, downgrades and defaults combining idiosyncratic and systematic risks, and to detect potential vulnerabilities coming from these factors. Although it could be possible to calibrate constant transition matrices, our dynamic model is attractive for practitioners who forecast the multi-period distribution of the number of defaults since it allows to activate the insurer's management actions depending on the change in the buyer' creditworthiness. The approach also helps insurers for stress testing and enables them to take appropriate decisions relative to the changes in the portfolio's credit quality. With the recent Solvency II project (European Parliament and Council of the European Union, 2009), insurers are indeed encouraged to assess and manage adequately their own risks. In particular, the Own Risk Solvency Assessment (ORSA)[1] introduces a framework to measure the specific

[1] Consistency with the Article 45 of the Solvency II Directive, the ORSA is a fundamental part of the internal management system for an insurer, taking into account its specific risk profile, its risk tolerance ant its strategy. This system allows to comply continuously with the Solvency II requirements and to monitor the relevance of the capital calculation model (Guibert *et al.*, 2014). Stress testing is typically a key component of the ORSA as it allows an insurer to explore adverse scenarios in order to investigate their impact and the potential management actions that could be taken. This procedure should be based on historical data if possible and properly justified. Insurers need to ensure that any such scenario is fully thought about and is internally consistent. Boards and regulators are likely to challenge insurers to demonstrate that the scenarios are comprehensive and have covered all of the insurer's material risks.

risk profile of the insurer. In addition, our model gives estimations which can be used for comparison with the solvency capital results given the standard formula or an internal model.

With the Basel II Accord, issued in 2004, the banking literature dedicated to credit risk assessment for corporate and sovereign bonds has considerably increased and it has especially investigated the linkage with macroeconomic variables both for default rates and migration matrices. Since the empirical works of Nickell *et al.* (2000) and Bangia *et al.* (2002) which have demonstrated the link between credit rating movements and business cycle (as characterized by the NBER index, the GDP growth rates, the issuer's industry and country), numerous studies have focused on the dependence between the credit quality and the business, financial and economic environments (see e.g. Hu *et al.*, 2002; Chava and Jarrow, 2004; Couderc and Renault, 2005; Duffie *et al.*, 2007; Koopman *et al.*, 2009, 2011; Figlewski *et al.*, 2012; Fei *et al.*, 2012). A recurrent result in this literature is that defaults and downgrade probabilities increase during a recession period. In these models, the macroeconomic factors display the state of the economy and are considered as observable systematic risk factors. More recently in this literature, a lot of approaches in the corporate sector consider latent factors for the sake of efficiency. Among these approaches, Koopman *et al.* (2008) and Koopman *et al.* (2009) have introduced a continuous time model taking into account both observable and unobervable parts of the credit cycle for transition intensities. In parallel, Duffie *et al.* (2009) have focused on individual default intensities with a dynamic frailty covariate and individual observable covariates. This aims to give better individual prediction of the latent part beyond the economical and firm-specific variables. Chava *et al.* (2011) provides a more complete joint model for default rates and recovery rates where several frailties take into account an unobserved effect related to an industry-specific distress. Wendin and McNeil (2006), McNeil and Wendin (2007) and Stefanescu *et al.* (2009) have also developed models capturing the unobservable part of the credit cycle at the firm level and infer their models with Bayesian techniques. These models are built in discrete time and allow credible intervals for transition probabilities. Azizpour *et al.* (2014) discuss the different sources of corporate defaults including contagion effects. Creal *et al.* (2014) propose also a flexible mixed approach modeling both factors and frailty dynamics for the loss distribution forecasting.

These examples of current econometric models allow to account for complex effects (industrial contagion, unobserved characteristics, frailty, various macroeconomics factors) analyzed over a long period with large corporate data set (major banks or credit ratings agencies). However, the credit insurance portfolios are diversified, mixing small, medium[2] and large firms from different industries, and their features are little known. For this reason, a sophisticated model is not necessarily desirable at a first step as the available data is not equivalent to other more standardized sectors. Several points are singular. First, insurance coverage distinguishes two different types of default events: protracted and insolvency defaults. The first type is not necessarily a terminal state but still triggers insurer's payments. It should be analyzed specifically as this state has a momentum effect. Such analyses are rarely done in the credit literature, as noted by Malik and Thomas (2012). Otherwise, we remark that the selection effect induced by contracts cancellation by insurers is clearly non neutral and aims to reduce the credit risk within the portfolio. After this selection effect, the observed default rates seem to be less sensible to the macroeconomic variables, even during the 2008-2009 crisis. On the contrary, changes of environmental variables have a clear effect on the transition probabilities from one rating to another.

Our empirical results are based on a migration model estimated with a cumulative link specification[3], which is more appropriate to describe the rating process characteristics than a simple discrete multinomial

[2]On another data set given by a French credit insurer, Dietsch and Petey (2002) and Dietsch and Petey (2004) show for example that small and medium firm' correlations to default are significantly different to those assumed in the Basel II formulas based on large firms.

[3]Also know as ordinal regression model (McCullagh, 1980; Agresti, 2002).

model, or a continuous model with constant transition intensity over a fixed period. Since a majority of the buyers in our data is not publicly quoted, it is not possible to use a stock-price based model. The selected specification also permits to increase convergence with the internal rating system of insurers. In terms of implementation, an advantage of our approach is that it is based on observable factors. Thus, inference requires simple maximum likelihood techniques, as the rating processes do not predict the systematic factors. This allows to decouple inference of the factors model and the transition probabilities model. By contrast, this estimation methodology is relatively straightforward compared to the Bayesian techniques or the state space methods used to deal with the computation of likelihood with latent dynamic factors. Additionally, these two models could be specified independently. The transition probabilities in our model can not be performed with analytic formula and require Monte-Carlo simulation techniques.

Our factors model entails a simple vector autoregressive (VAR) econometric model predicting the unobservable credit score used in the transition probabilities model. In contrast with Pesaran *et al.* (2004) and Pesaran *et al.* (2006)[4] who used global vector autoregressive macroeconomic models in the international framework, we limited our approach to a few variables which are the most statistically significant for our local portfolio, as the study of global (domestic and foreign) macroeconomic equilibrium is beyond the scope of this paper. However, our model includes a vector error-correcting model (VECM) to consider simple long run interactions between these factors. A desirable feature of this approach for risk management is that we can analyze how a shock on these factors affects the default probability.

This paper is structured as follows. In Section 2, we recall the main characteristics of credit insurance. Section 3 exposes the model for an individual credit rating process. We specify both the VAR time-series models for systematic factors and the link with transition and default probabilities along with the estimation process. In Section 4, we describe the data set used in our numerical application. The empirical results for the transition probabilities model and the factors model are discussed in Section 5. We then analyze the effect for risk management in Section 6. Finally, Section 7 concludes the paper and presents some potential improvements.

2 Credit insurance contracts

This section gives a brief overview about the characteristics of credit insurance contracts and explains the importance of migration matrices for risk management. In the credit insurance business, the contractual relation involves two parties, namely a credit insurer and a policyholder. A third party,the policyholder's client, intervenes in this relation as the source of the risk. This third party is called the buyer. The reason why credit insurance contracts exist is because companies (policyholders) buy such contracts to protect themselves in case of default of one of their clients (the buyers). Figure 1 represents this insurance scheme.

[Figure 1 about here.]

Except for small policyholders, a standard insurance contract only covers one buyer and a policyholder can subscript many policies associated to different buyers. It is therefore possible than a certain buyer is a party in many policies without necessarily being informed. If a buyer defaults during the coverage period and the policyholder declares the corresponding claim, the credit insurer must indemnify the policyholder, after taking into account the different policy parameters. Those parameters make it possible for the credit insurer to mitigate the effects of a default. Hence, an effective measurement of credit risk in a portfolio involves two quantities: the amount of financial loss in the event of a buyer's default and the probabilities of default for different buyers.

[4]Such models are typical examples of stress testing approach developed by central banks and national supervisory authorities (Foglia, 2008).

2.1 Default events

Two main events are labeled as *default event*: 1) the legal insolvency or court insolvency of the buyer, and 2) the non-payment of receivables by the buyer within the contractual period, called the protracted default[5]. The frequency of these events is measured by default probabilities which depend on the rating class, the industrial sector and the area. Hence, credit insurers have developed internal rating systems based on buyer's individual features which give a picture of the trustworthiness of each buyer at each assessment time for risk management purpose.

2.2 Loss in case of default

An important feature of the credit insurance business is the ability for a credit insurer to mitigate the loss in case of default and also during the coverage period. Indeed, the credit insurer does not reimburse the entire amount of the outstanding debt to the policyholder if a buyer defaults, since preventive measures can be taken, particularly after the contract is signed.

The first measure is the ability to reduce the policy's limit, i.e. the credit limit granted to the policyholder with regard to the defaulting buyer, at any time, depending on the creditworthiness of the buyer. Indeed, the insurer has the right to immediately restrict or cancel the contract if the risk deteriorates. This decision concerns all subsequent invoices, and the default of the buyer has thus a reduced consequence on the insurer. This is contractually well founded as the insurance premium is initially calculated based on a certain exposure amount and at fixed risk level. In the best possible case for the insurer, the contract is canceled in good time before the default of the buyer and the credit insurer is free of any legal obligation. If the perceived risk is lower, the default can occur before the cancellation and then the insurer has to pay only a proportion of the claim. Note also that in many cases the insurer terminates the policy soon after the occurrence of a default and it is possible that a buyer be reinstated some time after the cancellation. This contractual leeway justifies the need to develop reliable monitoring tools to anticipate default events.

In addition to this, when the credit insurer decreases the granted limit, the policyholder is informed and could rationally lower the trading activity with this particular buyer. There are two reasons which lead this behavior. The first reason is that if the insurer decreases the limit, the policyholder interprets this measure as a warning saying that the buyer is less capable of paying off the debt, so the likelihood it never reimburses the invoices increase. The second reason is that the amount the policyholder loses in case of default of its buyer is greater when the granted limit decreases.

The second means for an insurer to limit its loss is generated by the contract clauses which define uninsured percentages, deductibles and maximum liability amounts, i.e. the maximum total amount of indemnification. There also exists an annual aggregate amount which is deductible, in case of default: if the sum is smaller than the deductible nothing is paid, otherwise the amount exceeding the franchise is paid. After the indemnification, the insurer is generally able to recollect part of the claims. Ultimately, the credit insurer may choose to share the risk with a reinsurer.

3 Migration model with macro factors

This section describes our discrete time stochastic model for dependent grade history of a large number of buyers in a credit insurance portfolio based on a factors model. Our modeling approach includes two

[5] The payment period lasts longer than it was initially agreed on by the policyholder and its client. In France, the payment period is 90 days, except for rare exceptions.

steps: the linkage between credit risk and macro factors, and the macro factors dynamics. Then, we detail the estimation method and the forecasting approach.

3.1 Econometric model

Let $T \in \mathbb{N}$ be a fixed time horizon for a discrete time setting with $t \in \mathcal{T} = \{0, 1, \ldots, T\}$. Let $(\Omega, \mathcal{F}, \mathbb{P})$ denote a probability space and $\mathbb{F} = (\mathcal{F}_t)_{t \in \mathcal{T}}$ a general informative filtration satisfying the usual conditions of completeness and right-continuity.

We consider a portfolio of K buyers observed over $\mathcal{T}$ and their respective rating history chains $(X_k(t))_{t \in \mathcal{T}}$, $k = 1, \ldots, K$. The individual rating history chains are considered $\mathbb{F}$-adapted and take values in a finite state space $\mathcal{R} = \{1, \ldots, R\}$ that is a set of rating classes of decreasing creditworthiness used by an insurer. The higher grade R corresponds to the insolvency default, i.e. an absorbing state, and the grade $R - 1$ correspond to the protracted default.

We divide the portfolio of buyers into a set $\mathcal{H} = \{1, \ldots, H\}$ of subgroups assumed to be homogeneous. Each subgroup can refer to a combination of discrete buyer-specific covariates such as industry sector, business area or size. This set of cross section is assumed to be constant over the period but time-covariates can be easily handled (Wendin, 2006; Duffie *et al.*, 2007). Let $(\boldsymbol{Z}(t))_{t \in \mathcal{T}}$ be a discrete time chain comprising a vector of d-macroeconomic and financial risk factors that we call macro factors for the sake of simplicity. We consider that the general information set is entirely defined by the full information about the credit and macroeconomic environments.

Assumption 1. *Denoting by $\mathbb{G} = (\mathcal{G}_t, t \in \mathcal{T})$ the filtration comprising both financial and macroeconomic information, i.e. information generated by $\boldsymbol{Z}$, and the natural filtration $\mathbb{F}^{X_k}$ related to X_k for all $t \in \mathcal{T}$, we assume that the general information set $\mathcal{F}_t$ at time $t \in \mathcal{T}$ is defined by $\mathcal{F}_t = \mathcal{G}_t \vee \mathcal{F}_t^X \vee \sigma(\mathcal{H})$ where $\mathcal{F}_t^X = \vee_{k=1}^K \mathcal{F}_t^{X_k}$.*

For the purpose of credit management, future rating prediction is the cardinal at the more granular level. Thus, the model should take into account the heterogeneity within each grade in order to predict adequately the individual rating chain. Following the classic approach taken by rating agencies, we consider that the heterogeneity across buyers is entirely modeled with the initial subgroups $1, \ldots, H$ conditionally to macro factors. In practice, the homogeneous assumption may be critical since it is tricky to include all significant information beyond rating classes and observed covariates. As a consequence, only a limited number of dimensions can be considered and we acknowledge that our model partially addresses the heterogeneity phenomena. The rest of the model is formalized as follows.

Assumption 2. *We assume for the k-th buyer belonging to the subgroup $h \in \mathcal{H}$ that the discrete time process X_k follows a doubly-stochastic Markov chain driven by the factors $\boldsymbol{Z}$. The chain is characterized by a stochastic migration or transition matrix $\boldsymbol{Q}_h(t) = (q_{ij,h}(t) = q_{ij,h}(\boldsymbol{Z}(t)))_{i,j \in \mathcal{R}}$[6] for all $t \in \mathcal{T}$. We assume also that all buyers in the same cross-section, $(X_k)_{k \in h}$, are independent conditionally on $\boldsymbol{Z}$.*

Hence, the process X_k is a Markov chain given its initial subgroup h, the previous rating and the current state of the environment. Thus, we have

$$q_{ij,h}(t) = \mathbb{P}(X_k(t+1) = j \mid X_k(t) = i, \boldsymbol{Z}(t), k \in h) \ . \tag{3.1}$$

As the stochastic transition matrix over one period is a function of the factor $\boldsymbol{Z}$, the Markov assumption is easily verified if the factor is a Markov process itself. This allows to define the process in discrete

[6] $q_{ij,k}(t)$ corresponds to the transition probability from the rating class i to j between t and $t+1$.

time but this framework could be easily formalized equivalently in continuous time. We now introduce the counting variables

$$\boldsymbol{N}(t) = (\boldsymbol{N}_1(t), \ldots, \boldsymbol{N}_H(t))_{t\in\mathcal{T}} \text{ and for } h \in \mathcal{H},\ \boldsymbol{N}_h(t) = (N_{ij,h}(t))_{i,j\in\mathcal{R}}, \tag{3.2}$$

where $N_{ij,h}(t)$ represents the number of transitions from i to j observed for the cross-section h between time t and $t+1$.

Assumption 2 implies that the migration counts are correlated through the dependence between the migration matrices related to each buyer' homogeneous group. This specification is especially useful for the forecast of a large credit portfolio including dependent migrations *via* a common random factor. We emphasize also that the migration counts do not cause the process $\boldsymbol{Z}$. In addition, as the factors traduce the general state of the economy, this introduces the time-heterogeneity of transition probabilities. By denoting $Y_{i,h}(t)$, the number of exposures of the h-th subgroup characterized by rating i at the time t, our specification induces

$$\boldsymbol{N}_{i,h}(t) = (\boldsymbol{N}_{ij,h}(t))_{j\in\mathcal{R}} \sim \text{Multinomial}\left(Y_{i,h}(t), (q_{ij,h}(t))_{j\in\mathcal{R}}\right). \tag{3.3}$$

The count $Y_{i,h}(t)$ is observed and should be reassessed at the beginning of each period.

Next, we consider the econometric model in discrete time for conditional transition matrices (3.1). As ratings are naturally ordered, we adopt a cumulative link model, also called ordinal regression model (McCullagh, 1980; Agresti, 2002) based on observable factors. Such a specification is quite classical in the credit literature (see e.g. Bangia *et al.*, 2002; Feng *et al.*, 2008). Compared to a continuous model similar to a simple multinomial model, our discrete specification is more flexible and account the natural order between ratings. We assume that the conditional cumulative probabilities satisfy, for buyer k belonging to subgroup h

$$\mathbb{P}(X_k(t+1) \leq j \mid X_k(t) = i, \boldsymbol{Z}(t)) = g\left(\mu_{ij,h} + \boldsymbol{\theta}_{ij,h}^\top \widetilde{\boldsymbol{Z}}(t)\right),\ i,j \in \mathcal{R}, \tag{3.4}$$

where $(\mu_{ij,h})_{j\in\mathcal{R}}$ represents a sequence of unobserved threshold values specific to each credit class, $\boldsymbol{\theta} = (\boldsymbol{\theta}_{ij,h})_{i,j\in\mathcal{R}}$ is the vector of parameters of size d representing the sensitivity to each factor and $g : \mathbb{R} \to [0,1]$ is a link function. Common choices for g are probit, logit or log-log links. We use the notation $\widetilde{\boldsymbol{Z}}$ instead of $\boldsymbol{Z}$ since the migration model may depend only on a subset of all the macroeconomic factors required to model the economic and financial environment. This notation can also be used to code for lagged versions of the contemporaneous factors $\boldsymbol{Z}$. One can remark that we do not adopt an *a priori* parallelism assumption, for example, a proportional odds model, where the parameter $\boldsymbol{\theta}$ would be equal for all transitions, because factors may have different effects on each transition. This allows to take into account the heterogeneity across transitions and subgroups. Notice also that we can constraint this specification by considering the subgroup as a dummy variable to have a more parsimonious model.

From Equation (3.4), we have easily the conditional transition probabilities for all $i \in \mathcal{R}$ and $h \in \mathcal{H}$

$$\begin{cases} q_{i,1,h}(t) = g\left(\mu_{i,1,h} + \boldsymbol{\theta}_{i,1,h}^{\top}\widetilde{\boldsymbol{Z}}(t)\right) \\ \vdots \\ q_{ij,h}(t) = g\left(\mu_{ij,h} + \boldsymbol{\theta}_{ij,h}^{\top}\widetilde{\boldsymbol{Z}}(t)\right) - g\left(\mu_{i,j-1,h} + \boldsymbol{\theta}_{i,j-1,h}^{\top}\widetilde{\boldsymbol{Z}}(t)\right) \\ \vdots \\ q_{iR,h}(t) = 1 - g\left(\mu_{i,R-1,h} + \boldsymbol{\theta}_{i,R-1,h}^{\top}\widetilde{\boldsymbol{Z}}(t)\right) \end{cases} . \tag{3.5}$$

As cardinal of $\mathcal{R}$ is R, note that only $R-1$ thresholds are required for the model. The state of terminal default ("insolvency") is characterized by the threshold $\mu_{i,R-1,h}$. Remark also that the term $\mu_{ij,h} + \boldsymbol{\theta}_{ij,h}^{\top}\widetilde{\boldsymbol{Z}}$ in (3.4) do as not contain idiosyncratic risk as it is included afterwards in (3.3).

This econometric framework is quite standard in the credit literature and could be linked with models based on Merton's methodology (Merton, 1974), used by practitioners. This gives economic interpretation of the factors. More precisely, the rating process could be projected considering $(\varepsilon_1(t), \ldots, \varepsilon_K(t))$, K i.i.d. random variables with distribution function g. Indeed, the k-th buyer in subgroup h, rated i at time t, makes a transition to j such as

$$X_k(t+1) = j \Longleftrightarrow \varepsilon_k(t) \in \left]\mu_{i,j-1,h} + \boldsymbol{\theta}_{i,j-1,h}^{\top}\widetilde{\boldsymbol{Z}}(t), \mu_{ij,h} + \boldsymbol{\theta}_{ij,h}^{\top}\widetilde{\boldsymbol{Z}}(t)\right],$$

where $\varepsilon_k(t)$ represents the idiosyncratic risk whereas the term $\boldsymbol{\theta}_{ij,h}^{\top}\widetilde{\boldsymbol{Z}}(t)$ corresponds to the systematic effect.

3.2 Dynamic model for macro factors

In this section, we specify the model for the time-series covariates $\boldsymbol{Z}$, as we are interested in forecasting the loss-distribution over multiple periods. As we assume that risk-specific information is captured by the rating's past history and the initial cross-sections, we specify a model for the macroeconomic factors and financial factors with a general VAR(m) representation with $m \geq 1$. This framework is used in econometrics to model the relationships between variables and is relatively common in the macroeconomic field. Such models in the context of credit risk are previously applied by Pesaran *et al.* (2004), Pesaran *et al.* (2006), Duffie *et al.* (2007) and more generally by central banks and authorities interested in financial stability (Foglia, 2008).

As a large part of the buyers portfolio comprises small and medium companies, it seems reasonable to assume that the buyer rating histories $\boldsymbol{Y}$ do not predict the macroeconomic factors. Then, we assume the macroeconomic factors are given at time t by

$$\boldsymbol{Z}(t) = \sum_{i=1}^{m} \boldsymbol{A}_i \boldsymbol{Z}(t-i) + \boldsymbol{b}_0 + \boldsymbol{b}_1 t + \boldsymbol{\zeta}(t), \tag{3.6}$$

where $\boldsymbol{b}_0$, $\boldsymbol{b}_1$ are $d \times 1$ vectors of unknown parameters, $\boldsymbol{A}_i$, $i = 1, \ldots, m$, are $d \times d$ time-invariant matrices of unknown parameters and $\boldsymbol{\zeta}(t)$ is a i.i.d. standard normal vector of dimension $(d \times 1)$ with $\boldsymbol{\Sigma}$ the variance-covariance matrix.

In general, the macroeconomic factors are not stationnary[7] but are cointegrated[8]. In other word, they have a common stochastic trend and the series should be modeled together considering the relationships between them. For convenience, we assume hereafter that the factors are I(1), that is integrated with order 1. As commonly done when dealing with cointegration relations, we rewrite Equation (3.6) under its VECM reduced form[9]

$$\Delta \boldsymbol{Z}(t) = \boldsymbol{\Pi} \boldsymbol{Z}(t-1) + \sum_{i=1}^{m-1} \boldsymbol{\Psi}_i \Delta \boldsymbol{Z}(t-i) + \boldsymbol{b}_0 + \boldsymbol{b}_1 t + \boldsymbol{\zeta}(t), \tag{3.7}$$

where $\boldsymbol{\Pi} = -\left(I_d - \sum_{i=1}^{m} \boldsymbol{A}_i\right)$ and $\boldsymbol{\Psi}_i = -\sum_{j=i+1}^{m} \boldsymbol{A}_j$, $i = 1, \ldots, m-1$. The matrix $\boldsymbol{\Pi}$ is the so-called equilibrium matrix and its rank represents the number of cointegration equations of the system or the number of long run relationships. This matrix is expressed as $\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}^\top$ where $\boldsymbol{\alpha}$ is called the loading matrix and $\boldsymbol{\beta}$ is the cointegration matrix. This second term corresponds to the long-run coefficients. As $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are not unique, the classical strategy adopted in econometric consists in restricting $\boldsymbol{\beta}$ coefficients[10] consistently with some long-run steady-state relationships based on macroeconomic theory, see e.g. Garratt *et al.* (2003). The construction of such an econometric model theoretically requires considering both domestic and foreign variables.

3.3 Maximum likelihood estimation

The analysis of this article is based on panel data in which the individual rating history of buyer k, $k = 1, \ldots, K$, is observed over time intervals. Hence, the rating of a buyer is only known for a finite number of dates. We introduce the process $(C_k(t))_{t \in \mathcal{T}}$ which indicates with 1 if the rating of the k-th buyer is observed for each assessment time and 0 otherwise. There is a loss of information due to this process and the information available for all the observed buyers is now denoted by observable filtration $\widetilde{\mathcal{F}_t}^{\boldsymbol{X}}$. For the sake of simplicity, we make the following assumption and we discuss its validity for our data set in Section 4.

Assumption 3. *We assume hereafter that the macroeconomic covariates are not subject to missingness and the censoring process* $(C_k(t))_{t \in \mathcal{T}}$ *is not informative.*

As the covariates' processes are not predicted by individual rating processes, we denote for convenience $f(\cdot \mid (\boldsymbol{Z}(t-1), \ldots, \boldsymbol{Z}(t-m)), \boldsymbol{\Gamma})$ the conditional joint distribution of $\boldsymbol{Z}(t)$ where $\boldsymbol{\Gamma}$ is the vector of parameters to be estimated and characterizing the covariates time-series model, see Section 3.2. Under the doubly stochastic framework (Assumption 2) and Assumption 3, inference for $(\boldsymbol{\mu}, \boldsymbol{\theta}, \boldsymbol{\Gamma})$ can be based only on the observed data. The likelihood function, conditionally upon the initial state, is the product of separate terms involving $\boldsymbol{\theta}$ and $\boldsymbol{\gamma}$ similarly to Duffie *et al.* (2007, Section 2.2)

$$\mathcal{L}\left(\boldsymbol{\mu}, \boldsymbol{\theta}, \boldsymbol{\Gamma}; \mathcal{G}_T \vee \widetilde{\mathcal{F}_T}^{\boldsymbol{X}}\right) = \mathcal{L}(\boldsymbol{\Gamma}; \mathcal{G}_T) \times \mathcal{L}\left(\boldsymbol{\mu}, \boldsymbol{\theta}; \mathcal{G}_T \vee \widetilde{\mathcal{F}_T}^{\boldsymbol{X}}\right), \tag{3.8}$$

where

$$\mathcal{L}(\boldsymbol{\Gamma}; \mathcal{G}_T) = \prod_{t \geq 1} f(\boldsymbol{Z}(t) \mid (\boldsymbol{Z}(t-1), \ldots, \boldsymbol{Z}(t-m)), \boldsymbol{\Gamma}), \tag{3.9}$$

[7] The process is stationnary if the polynomial $\det\left(I_d - \sum_{i=1}^{m} \boldsymbol{A}_i \boldsymbol{x}^i\right)$ has no roots in and on the complex unit circle, see Lütkepohl and Krätzig (2004, Chapter 3).
[8] There exist linear combinaisons of them that are stationary.
[9] Δ represents the differenting operator.
[10] $\boldsymbol{\alpha}$ coefficients can also be restricted to 0. That allows testing weak exogeneity of a cointegration equation.

and

$$\mathcal{L}\left(\boldsymbol{\mu},\boldsymbol{\theta};\mathcal{G}_T \vee \widetilde{\mathcal{F}_T}^{\boldsymbol{X}}\right)=\prod_{t\geq 1}\prod_{h=1}^{H}\prod_{i,j\in\mathcal{R}} q_{ij,h}\left(t\right)^{n_{ij,h}(t)}, \tag{3.10}$$

where $n_{ij,h}\left(t\right)=\sum_{k\in h}\mathbb{1}_{\{X_k(t)=j,X_k(t-1)=i,C_k(t)=C_k(t-1)=1\}}$ is the number of observed transition from i to j between t et $t+1$. This corresponds to the multinomial ditribution identified in Equation (3.3) where the exposure can be written $y_{i,h}\left(t\right)=\sum_{k\in h}\mathbb{1}_{\{X_k(t)=i,C_k(t)=1\}}$. Remark that the Equation (3.10) is separable and allows for different specifications of the linear regressors $\mu_{ij,h}+\boldsymbol{\theta}_{ij,h}^{\top}\widetilde{\boldsymbol{Z}}$ (selected covariates, restrictions of the coefficients ...). We also see that each transition function from state i and group h can be estimated separately similarly to the parametric transition intensity for competing risk data.

When the factors are latent,the statistical inference process may be complex depending on the dynamics specification of these factors (see e.g. Gagliardini and Gouriéroux, 2005). A desirable feature of our approach is that we only need a standard maximum likelihood technique to infer the parameters since all factors are observed. Indeed, the overall maximum likelihood problem can be decomposed into two separate optimization problems as the log-likelihood of (3.8) is the sum of the logarithm of (3.9) and (3.10). This decomposition allows for the separate estimation of the parameters $\boldsymbol{\Gamma}$ related to the time-series models of factors (see Section 3.2) and the parameters $(\boldsymbol{\mu},\boldsymbol{\theta})$ characterizing the transition model (see Section 3.1). For Equation (3.9), the estimation of a classic VAR model (3.6) is led with maximum likelihood techniques, see for instance Lütkepohl and Krätzig (2004, Section 3.3.1) or more generally Juselius (2006). As the economic variables are not $I\left(0\right)$, we then used the so-called Johansen procedure to estimate its VECM form (Johansen, 1991). Regarding Equation (3.10), estimates $\left(\widehat{\boldsymbol{\mu}},\widehat{\boldsymbol{\theta}}\right)$ are given with a simple maximum likelihood estimation (Agresti, 2002).

3.4 Forecasting

For risk management purposes, the insurer is interested in forecasting the future risk profile of the portfolio of buyers over a fixed number of periods. Assuming the risk manager is able to mitigate the portfolio by reducing the riskiest exposures, our specification with observable variables is particularly relevant and permits to build strategies to react to a change in the economic environment. However, we have to predict beforehand the future values of the exogenous factors. We first give the expected transition probabilities and explain how to simulate the future rating distribution.

3.4.1 Forecasting the portfolio of buyers

As the factors are assumed to be observable, we use the VAR model (3.6) from the parameters[11] to forecast the value of the factors at time $t+s$ with $s\geq 1$

$$\boldsymbol{Z}\left(t+s\right)=\sum_{i=1}^{m}\widehat{\boldsymbol{A}}_i\boldsymbol{Z}\left(t+s-i\right)+\widehat{\boldsymbol{b}}_0+\widehat{\boldsymbol{b}}_1 t+\boldsymbol{\zeta}\left(t,t+s\right), \tag{3.11}$$

where $\boldsymbol{\zeta}\left(t,t+s\right)$ captures the forecast error and follows a multivariate normal distribution with variance-covariance matrix (Lütkepohl and Krätzig, 2004)

$$\boldsymbol{\Sigma}\left(s\right)=\sum_{i=1}^{s-1}\boldsymbol{\Upsilon}_i\boldsymbol{\Sigma}\boldsymbol{\Upsilon}_i^{\top},$$

[11] the VECM model can be rewritten in VAR form.

with $\boldsymbol{\Upsilon}_i = \sum_j^i \boldsymbol{\Upsilon}_{i-j} \widehat{\boldsymbol{A}}_j$.

Knowing future values of $\boldsymbol{Z}$, we can apply selection operations to deduce $\widetilde{\boldsymbol{Z}}$. Then, the transition matrix between times t and $t+s$ verifies the following proposition for a buyer belonging to the subgroup h at time t

$$\boldsymbol{Q}_h(t, t+h) \mid \widetilde{\boldsymbol{Z}}(t), \ldots, \widetilde{\boldsymbol{Z}}(t+s) = \prod_{u=0}^{s} \boldsymbol{Q}_h\left(\widetilde{\boldsymbol{Z}}(t+u)\right), \tag{3.12}$$

In practice, Equation (3.12) is computed easily by Monte-Carlo simulations. Idiosyncratic risk can be also considered by drawing multinomial realizations of (3.3).

3.4.2 Stress scenarios

A main interest of our approach is to derive the future portfolio distribution under changes in different macroeconomic factors. For that, we use a typical approach similar to the macro stress testing defined by supervisory authorities (Foglia, 2008). We then examine impulse response functions to an isolated shock following the generalized approach proposed by Koop *et al.* (1996) and implemented e.g. by Pesaran *et al.* (2006) and Dees *et al.* (2007). This approach is particularly useful for risk management purposes as the shock is unanticipated and has effect on the non-primarily stressed factors.

Assuming that we have a shock for factor f of one standard error (obtained with the historical variance-covariance matrix) over one period $\zeta_f(t+1) = \sqrt{\sigma_{ff}}$, we obtain that

$$\boldsymbol{\zeta}_{-f}(t+1) \mid \zeta_f(t+1) = \sqrt{\sigma_{ff}} \sim IIN\left(\frac{1}{\sqrt{\sigma_{ff}}}\boldsymbol{\rho}, \Sigma_{-f} - \frac{1}{\sigma_{ff}}\boldsymbol{\rho}\boldsymbol{\rho}^\top\right),$$

where $\boldsymbol{\rho}$ is the f-th column of $\boldsymbol{\Sigma}$ without the row f, Σ_{-f} is the variance-covariance matrix of $\boldsymbol{\zeta}_{-f}(t)$ which is the standard normal vector $\boldsymbol{\zeta}(t)$ without the f-th component. The effect of a such shock can be easily measured by Monte-Carlo simulation.

4 Data description

This section describes our credit insurance data set and the observed macroeconomic and financial variables used to model the credit cycle.

4.1 Credit ratings data

Contrary to what is commonly observed in the credit literature, we do not use credit-rating agencies' data but rather we analyze a large data set that we extracted from the internal credit rating system of a French insurer. This data set contains individual rating histories of buyers observed under a discrete observation scheme from March 2004 to September 2012 and comprises both small, medium and large firms from different countries and sectors. The ratings are determined at each date on the basis of a quantitative score depending upon individual characteristics, along with the declaration times for default (insolvency and protracted). Note that the rating can be sometimes adjusted manually by the experts in charge of the rating system if the observed characteristics do not provide sufficient information.

For this case study, we select a subset of 1,604,533 French buyers[12] from a lot of industrial sectors over the observation period which have been grouped into $H = 5$ industrial groups: (1) Agriculture,

[12] Data from different countries should be inferred separately since the default definition may be different. For French data, the definition of protracted default is generally 90 days.

(2) Finance and Real estate, (3) Finished product, (4) Raw product and (5) Service and Trades. These groups are built by the insurer from NACE codes[13] and are used in daily management, but for reasons of confidentiality the codes and the names of the firms are not identifiable. Summary statistics is given in Table 1.

[Table 1 about here.]

The presence of an industrial sector indicator is useful as the existence of industrial effects is proven in the credit literature, see Nickell *et al.* (2000), Bangia *et al.* (2002) or more recently Xing *et al.* (2012). As the industrial sector is the finest individual information at our disposal, we assume that buyers in the same rating class and in the same sector define an homogeneous subgroup. This assumption is classical in the credit literature but we acknowledge that this segmentation may contain residual heterogeneity. In particular, we think that the probability of protracted defaults depends not only on the buyer' features but also on the business relationship with the policyholders. In fact, it would seem natural that a buyer would prefer to favor a good client in case of default. Unfortunately, we did not have access to this information, yet our model could be easily adapted to account for the supplier customer relationship e.g. through a covariate.

4.1.1 State definition and censoring

The internal rating system contains initially 10 grades of risk that we group into 5 classes, denoted "1", "2", ..., "5". To this end, we eliminate the classes that have a very small number of rating transitions. The class "1" is for the highest rating category and "5" alternatively the lowest. During their rating history, buyers can also experiment the protracted default state P or the insolvency state I. They are noted C when the insurer closes the contract. Note that I and C are considered as absorbing states whereas transitions are possible from the state P. In fact, we consider reinstatement of a contract after cancellation as a new policy. Then, we observe 42 possible transitions. The insurer indicated that there were no important changes in the credit rating system during the selected period, expect in January 2007 where the insurer largely reclassified buyers in classes 1 and 2 to class 5. In the rest of the paper, we handle this particular effect with a mean-shift dummy variable separating the periods of both sides of this date.

A buyer's rating history may be censored for several reasons. First, the rating can be unobserved due to the withdrawal or the cancellation of the contract. However, the insurer sometimes continues to monitor the non-rated buyers in existing contracts and we then observe the default events occurred in this case. Similarly, the insurer may sometimes fill the defaults that occurred after the cancellation of the contract, as parts of the claims are covered by the contract. However, in other cases, we have no available information on the buyer's state after the cancellation. In order to limit the potential bias due to these actions on default rates, we have dropped the non-rating transitions, as they are commonly consider as non-informative in the credit literature, and the entries into state C, and we kept the last known rating when a buyer experienced one of this two states less than 1 year before defaulting. For the other cases, we have chosen to model jointly the observed rating transitions and the cancellation as an additional transition.

Two reasons motivated our choice to consider the cancellation (not explicitly followed by default) as a particular event. First, realistic modeling of credit loss distribution requires considering the cancellation ability as a legitimate management action. Hence, it is interesting to investigate how this ability affects

[13] European industry standard classification system used in the European Community (http://ec.europa.eu/competition/mergers/cases/index/nace_all.html).

credit losses and is sensible to the economic environment. Secondly, as the full rating process is censored by cancellations, we would have to analyze how Assumption 3 is verified for this type of censoring if we want to estimate the model on the complete likelihood. For most cases, the fixed term of the contract is reached and the cancellation can be reasonably consider such as missing completely at random. In this case, the cancellation could be simply ignored when the amount of data is sufficient. However, the insurer is also able to mitigate credit risk by terminating an insurance contract especially in stress situation. As a result, the cancellation is not at random (MNAR) and then inference for the complete model would become tricky as cancellation induces non identifiable selection bias due to unobservables. Figure 2 displays the cancellation rates over the period and in particular during the year 2009.

[Figure 2 about here.]

For all the period, the cancellation rates do not exceed 10%, except during the first quarter of 2009 where the buyers with bad rating (Class 5) are amply removed from the portfolio. We feel it could be considered as a signal of downgraded quality and in this case the default events would be not at random. As seen below, this action is applied conservatively and reduces the observed default rates. As we observe this behavior only on a short period of time (only 2 or 3 dates) and due to the lack of information after cancellation, we are not able to gauge the magnitude of the selection bias induced. Consequently, specifyong a model with selection bias functional or an imputation method seems not possible with our current data set. This interesting[14] question is left for future research.

An other source of missing information lies in the observation scheme in discrete time which does not accounts for the possible transitions between two assessment dates. However, this source of censoring only induces a small bias, because the probability that one firm undergoes more than one transition over one month is extremely low. Finally, there is on average around $660,000$ buyers in each quarter and at least about $515,000$ buyers when accounting for cancellations, withdrawals and deletion due to insolvency defaults.

We give additional details on our data set as follows. The number of exposures increases over time, except in 2009, and Figure 3 describes the distribution of this sample according to rating classes. A similar evolution is observed for each sector. The data set contains mainly medium and high risk firms. We noted than the proportion of bad risk increases during the period 2004-2008, when the economic environment is favorable and vice versa. This effect results from the underwriting policy of the insurer. We emphasize that the effects of the crisis since 2009 are clear for bad grades in terms of exposure.

[Figure 3 about here.]

The overall number of direct transitions is presented in Table 2. The number of closing events is naturally important and a large part results from the normal termination of the policy. For classes 2 to 5, most transitions take place to adjacent rating classes but this result does not hold for class 1 and buyers who experienced protracted default. Large up-grades are very rare and down-grades are clearly more frequent.

[Table 2 about here.]

Due to the frequency of the rating process, we could estimate transition matrices on a monthly basis. Since transitions between some states are very rare, such estimation may induce relatively poor quality of fit. Thus, we consider the three-months migration counts which are more robust and also

[14] To the best of our knowledge, a very little attention has been given to infer transition models when missing data are MNAR (see e.g. Chen *et al.*, 2011).

more correlated to changes in the economy. In order to limit the bias (Shumway, 2001) induced from a discrete model compared to a continuous model, we adjust the number of transitions for each quarter such that $n_{ij,h}(t+3) := y_{i,h}(t)\,\widehat{q}_{ij,h}(t,t+3)$ where $\widehat{q}_{ij,h}(t,t+3)$ is the product-limit estimator of the transition matrix for non-homogeneous Markov chains following Lando and Skødeberg (2002) between t and $t+3$ months. This corresponds to the non-parametric Aalen-Johansen estimators (Andersen *et al.*, 1993) for three months periods that we have computed to our discrete time data assuming that monthly observations can be approximated as continuous time observations[15]. This adjustment seems reasonable as we have a large data set, and allows to consider intermediate transitions.

4.1.2 Transition matrices and the order of transition from the default state

In this subsection, we are interested in the transition matrix which displays the rating movements during a reference period, taken as three-months. At first, we estimate for illustration the average transition matrices over a three months horizon for all the sectors with a duration approach[16] under the assumption of time homogeneity (Lando and Skødeberg, 2002). This gives a more efficient estimate than performing quarterly transition matrices with a cohort approach. The results are given in Table 3. As the default I and the cancellation are absorbing states, the rows for states C and I are discarded since they have only zeros except for the last elements which are equal to one.

[Table 3 about here.]

As expected, buyers in the riskiest class 5 have higher default and cancellation probabilities. The latter probabilities decrease when the credit quality decreases until the class 3. For protracted default and cancellation transitions, this trend is less clear for rating classes $1-3$ but we remark surprisingly that the cancellation probabilities are quite important for class 1. Another interesting result is that the probability to move from state 2 to protracted default is higher than the probability to move from 3 to P. This result seems to be explained by the cancellation action which are rarer for class 2. The examination of Table 3 reveals that the default rates for the Agriculture sector differs significantly from the other industries.

The transition probabilities from state P are quite important and are not intuitive. This results from the Markov assumption of order 1 which is not verified for these transitions since a buyer who has defaulted temporarily is more likely to return around his previous rating. Hence, we analyze the relevance of a second order chain for the transitions from this state. The order one Markov assumption is rarely challenged in the credit literature, except for data sets with particular features (see e.g. Malik and Thomas, 2012), and the current credit score used in the credit rating system is considered sufficiently statistic to predict the next state. Table 4 presents the results where the transition rates are estimated in a way similar to Table 3. In order to keep the length of the table manageable, we ignore transitions from states $1-5$ as transitions rates are very close[17] to those reported in Table 3.

[Table 4 about here.]

The study of Table 4 shows a strong propensity to return to the states that precede the default, except in case of consecutive default payments. This confirms that the order one Markov assumption

[15] This approximation suggests that we could use continuous time model based on intensity specification (see e.g. Cox, 1972). Leow and Crook (2014) have used this approximation for intensity model for credit card loans. However, a discrete time Markov model is equivalent to a series of multinomial models and we prefer to fit our data from techniques for multinomial regression for efficiency reasons as noted by Jackson (2011).

[16] As data is observed on a monthly basis, the generator matrix is computed assuming only one transition can occur during one month for one buyer.

[17] Introducing the second order assumption for state P has low impact on the other states since the exposition in state P is negligible compared to the other states, as displayed in Figure 3.

does not hold for state P. We remark that the probabilities to move around the diagonal elements, i.e. transitions of type $(x, P) \to x - 1$ or $(x, P) \to x + 1$, increase after protracted default events in most cases. In particular, this increase is more apparent for transitions where the last observed rating is 4 or 5. Interestingly, we also note that protracted default has a positive effect on cancellation probabilities, successive protracted default payments or insolvency. For example, being in state 5, and after a stay in state P, buyers in the Service/Trade sector have a probability over 9 times higher to become insolvent. Conversely, large upgrades and downgrades are less frequent. Downgrading events after protracted default seem natural but the opposite movement is more complex to understand. We suppose that for this last case, the explanation could be given by observing higher order models. Yet, such analysis is tricky for reason of robustness due to the low number of observed transitions with orders higher than two.

For the rest of the paper, we consider that state P follows the second order Markov assumption but due to the low number of transitions from this state, we consider them to be time-constant.

4.2 Covariates description

The study of the relation between the credit risk of corporate bonds and the general macroeconomic variables is an active area of research, see for example (see e.g. Couderc and Renault, 2005; Duffie *et al.*, 2007; Koopman *et al.*, 2009, 2011; Figlewski *et al.*, 2012; Azizpour *et al.*, 2014; Creal *et al.*, 2014). However, the effect of observed macrovariables on transition matrix for credit insurance portfolios has never been analyzed, to the best of our knowledge. Interestingly, our data set contains the period around the financial crisis in 2008-2009 and we observe how credit managers have adjusted their portfolio during the economic slowdown. The question of which variables affect truly the transition is a complex investigation.

We examine the effect of 8 factors that we grouped into 3 categories (General macroeconomic conditions, direction of the economy and financial market conditions) in the same spirit that Figlewski *et al.* (2012) but adapted to the French area. Our choice is also geared toward variables which are meaningful in a stress test approach for an insurer. We considered the following explanatory variables.

- *General macroeconomic conditions (quarterly growth rate of the French unemployment, quarterly growth rate of the French consumer price index, quarterly growth rate of the French enterprises birth rate):* The unemployment growth is considered here as an indicator of the health of the economy. We expect that an increase in unemployment has a negative impact on the credit portfolio. The consumer price index (CPI) is a central economic indicator but the effects of an increase in inflation is not clear and may depend on the economic sectors to which the buyers belongs and on how buyers trade with foreign countries. The enterprises birth rate may have significant effect on the business demography by introducing new firms in the population. In France, Dolignon (2011) remarks that the boost in business creation induced in year by French laws (*lois Dutreil*) introduced new firms which are potentially more unstable.

- *Direction of the economy (quarterly growth rate of the French real gross domestic production, quarterly growth rate of the French industrial production):* The real GDP is a macroeconomic factor commonly used in the credit literature to represent the current state in the economic cycle. When the economy grows more rapidly, the credit situation of buyers should be improve and vice versa. As it is a quite general factor, GDP should be associated with an aggregated proxy to account for the profits or losses of any kind of buyer. We also consider the industrial production as a possibly more precise indicator as it excludes the contribution of non industrial sectors (government, non-corporate business ...) to economic growth.

- *Financial market conditions (the 10-year French Government Debt rates, the trailing 1-year return of the CAC40 index, the trailing 1-year volatility of the CAC40 index):* The interest rates play a strong role in the funding of the buyers and we expect an increase in interest rates to have a negative effect on the buyers' credit situation. For larger companies, the stock market returns are an indicator of the health of the corporate sector. Intuitively, we expect that a rise of the volatility index or a sudden drop of the financial market would have a negative impact on the credit situation of a buyer.

These variables are listed in Table 5 with their sources and the transformations applied to include them in our migration model.

[Table 5 about here.]

All these variables are observed at least quarterly, at the same date than the rating changes. These variables attempts to capture the state of economy, yet as it is not possible to identify the buyers' exposures to the foreign economy, our analysis is limited to the French environment.

5 Empirical results

In this section, we estimate our migration model with different specifications and discuss the in-sample estimation results. Then, we analyze the out-of-sample forecasting results for credit risk. We conclude this section by calibrating a simple VAR model for macro factors. In the following, the computations are carried out with the software `R` (R Core Team, 2015).

5.1 Estimation of the migration model

We estimate[18] the model described in subsection 3.1 using the data and the covariates presented above. In Equation (3.4), we decide to employ throughout this application the logit link[19] function $x \mapsto (1+\exp\{-x\})^{-1}$. The cumulative logit model is suitable to the migration dynamics but gets trickier when adding the cancellation feature, as this state is not naturally ordered with others. Malik and Thomas (2012) have faced the same problem and they developed a two-stage approach where the rating process and the default events are modeled conditionally to the non-closing event. They proposed to estimate the closing probability separately with for instance a simple logit model. However, this specification implicitly assumes independence between the closing events and the rating transitions, which does not seem to be the case. Because of the conservative nature of the cancellation action, we decide to force this state in our ordered logit model and to position it as the first state preceding downgrades[20] or, for state 5, just before the default state.

We first discuss different model specifications with regard to the term $\boldsymbol{\theta}_{ij,h}^{\top}\widetilde{\boldsymbol{Z}}(t)$ in Equation (3.4). The estimation is performed separately for each initial rating class because of the additive form of the log-likelihood function. Due to the size of Agriculture and Finance/Real estate sectors, we choose to consider the information on the sector as a specific covariates. As transitions from states 1 and 2 to default and from state 5 to state 1 are quite limited, these transitions are assumed to be constant (closed to zero) and therefore not modeled. Table 6 shows the results with different specifications (*General macroeconomic conditions* (M1), *Direction of the economy* (M2), *Financial market conditions* (M3), *All variables* (M4)) where the loading factors for macro factors are the same for all transitions from any initial state (parallel

[18] For numerical estimation, we use the `R` package `VGAM` (Yee, 2010).
[19] The results given with probit link are very close.
[20] We have tested other combinations with not significant difference.

assumption), i.e. $\boldsymbol{\theta}_{ij} = \boldsymbol{\theta}_i$ except for the intercepts and the sector coefficients. This assumption is relatively common for migration models. To keep the table manageable, we have omitted the estimated threshold coefficients μ_{ij} and the sector coefficients.

[Table 6 about here.]

Models M1 and M3 globally have the lowest performance with 2 and 3 variables and the model M4 is clearly the better although it depends upon 8 variables. The estimated results for the four models give an insight of the effects of macro factors on the transition probabilities. In general, these effects are highly significant. In this framework, a positive (resp. negative) coefficient with an increase of the relative macro factor have positive (resp. negative) effect on the creditworthiness of the buyer. However, the factors are highly correlated and the estimated coefficients may have unexpected sign due to the combination between covariates. For example, this explains the opposite signs of estimated coefficients related to the growth rate of the real GDP and the industrial production for M2. Interestingly, it seems that an higher enterprises births rates has negative effect probably because it introduces new buyers who are more unstable. The progressive decrease in the interest rates seems to have a positive effect over the period.

Moreover, we note these four models appear to have relatively poor fit because all the transitions from a particular state have different shapes which can not be handled with the parallel assumption. This contributes to the lack of meaning of the coefficients. In addition, we do not observe a symmetrical effect for upgrades, downgrades and defaults. We therefore decide to release the parallel assumption. Such a specification is chosen for example by Figlewski *et al.* (2012) and is more laborious to work out as the number of parameters is fairly important. Table 7 compares the quality of the previously estimated models, but without the parallel assumption (M1* to M4*) model M0 used as a benchmark and which does not contain any macro factor. As our variables are highly correlated, we conduct a forward-backward selection analysis based on the log-likelihood and the Akaike information criterion (AIC) and then select the best models with only three covariates to avoid over-fitting the model. The resulting parsimonious model M5* is specified with the growth rate of the real GDP, the volatility of the CAC40 and the level of the 10-years interest rate. In practice, there is not a unique best choice for the set of selected variables and alternative choices could also give good performances in terms of fit. We have examined many competing specifications with or without lags as well as their goodness of fit and some of them could have also been selected.

[Table 7 about here.]

The models are compared through the AIC, BIC, log-likelihood, Mc Fadden R^2 and Mc Fadden adjusted R^2. The quality of models M1*, M2* and M3* is significantly better than to their equivalent with the parallel assumption. Note also that the performance of model M4 is quite good compared to M1*-M3*. As expected, model M5* gives better fit[21] than these previous models but model M4* seems to be superior to M5*. However, model M4* is more parameterized and there is a risk of overfitting. For this reason, we choose to present in the following the estimated results for model M5*. Notice, that M2* gives also good performance for transition from states 4 and 5.

Tables 8 and 9 present the estimated threshold coefficients, the estimated coefficients characterizing the industrial effect relative to Service/Trade sector, the estimated coefficients related to the macro factors and their related estimated asymptotic standard deviations for model M5*. Notice that for transitions

[21] We have also tested an altervative model for M5* remplacing the industrial production growth by the real GDP growth. The results are close but slightly worst indicating, notably for states 4 and 5, than transition probabilities are rather driven by the industrial production growth than the real GDP growth.

from the rating classes 1 and 2, thresholds to the default class are not considered due to the very limited number of occurrences of these events in our data,and the same goes for upgrades from 5 to 1.

[Table 8 about here.]

[Table 9 about here.]

The threshold coefficients are clearly significant and give an insight on the transition probabilities without the factor's effect. The sector' coefficients are almost all statically significant, meaning that there is a large heterogeneity among buyers facing the same transition, which we capture partially with industrial information. Remember that we have considered sectors as a covariate because the results obtained analyzing each sector separately have a lower quality and the estimates are not statistically significant for Agriculture, a sector with include fewer observed transitions.

For each initial state, we finally choose three factors as a maximum and select the set of variables that provides the best fit. Virtually all the loaders are highly significant indicating that both upgrade and downgrade movements are sensitive to changes in economic environment. Except for buyers in state 1, the sign and magnitude of the coefficients indicate, as expected, that the growth of the CAC40 volatility indicator has a negative effect on the creditworthiness. The results are more ambiguous for the growth rate of the industrial production as a decrease has a negative effect only for the best buyers. Indeed, we observe that the worst rated buyers are impacted in the opposite direction and this suggests a possible selection effect through cancellations consisting in eliminating the riskiest buyers. The general decrease in the 10Y-French government bond rate over the period has a broadly positive effect.

The linkage between credit risk and the environmental variables is more clearly visible with the display of the estimated transitions. Let us focus on upgrade and downgrade probabilities. The time series of upgrade and downgrade probabilities are displayed in Figure 4 for the Services/Trade sector and for each rating classes. Transition probabilities for the other sectors can be obtained by a parallel shift.

[Figure 4 about here.]

As one can expect, upgrade probabilities increase with decreasing rating category. Alternatively, downgrade probabilities decrease with decreasing rating category. The probabilities located on the upper and lower diagonals of the migration matrices are those which are the highest changing. We remark that the economic environment has effects both on good and bad credit rating categories. The 2008-2009 financial and economic crisis globally has strong effects on the creditworthiness of buyers. As expected the downgrade probabilities are increasing sharply but only for the upper-rated buyers because the riskiest buyers are excluded from the portfolio by credit managers. As a results, the downgrade probabilities from classes 3 and 4 conditionally to the non-closing event decrease significantly during the crisis. In contrast, we observe a significant increase of the upgrade probabilities for the worst classes just after this period due to this selection effect.

Figure 5 presents the default probabilities obtained with the cumulative logit model and the pointwise 95% confidence interval, associated with parameter uncertainty of $(\boldsymbol{\mu}, \boldsymbol{\theta})$ and obtained with the "Delta method". Due to the lack of observations, we have only plotted insolvency default probabilities time-series for the three most populous sectors, i.e. Finished Product, Raw Product and Services/Trade. For comparison, we display on Figure 5 with points the non-parametric Aalen-Johansen estimator, computed as described in Section 4.

[Figure 5 about here.]

This constitutes an approximation of the crude default rates. For protracted defaults, the plots show a break at the beginning of 2007 due to the structural change in data as detailed before. We globally observe a small decrease for rating transitions over the period for all sectors although this fall is less steep for Agriculture and Finance/Real-estate. After the beginning of 2007, the estimated protracted default probabilities appear relatively stable for bad-rated buyers and we do not observe the effects of the the 2008-2009 crisis confirming that the closing operations at this date have limited the potential rise in default probabilities[22]. Regarding the insolvency default rates, a spike at this date is notable for default transitions from classes 3 and 5 but is clearly smoother than one could have expected.

Finally, Figure 6 presents the cancellation probabilities obtained with the same method as Figure 5.

[Figure 6 about here.]

The cancellation probabilities are clearly sensitive to a difficult economic environment and we observe the effects of the shock occurring during the financial crisis for the riskiest buyers and for all sectors. Remark that our model has some difficulty to fit transitions from state 4 due to some outliers in the Agriculture and Financial sectors, and the model overestimates cancellation rates by a few percent. Note also that the model misses some peaks which would require a more advanced model for example with frailties.

5.2 Out-of-sample results for the migration model

In this subsection, we perform out-of-sample tests in order to assess the forecasting performance of the model. Following a recursive approach, we perform the out-of-sample forecasting of default and transition probabilities (conditionally upon the non-closing event) for each quarter $[t, t+1]$ with the parameters of the cumulative logit model estimated on the window $[2004:1, t]$. Such an approach is used extensively in the credit literature (see e.g. Frydman and Schuermann, 2008; Koopman *et al.*, 2008; Stefanescu *et al.*, 2009; Chava *et al.*, 2011; Fei *et al.*, 2012). The out-of-sample period is defined as Sept. 2009 to June. 2012. In order to forecast the one quarter default probabilities, it is required to account for the macroeconomic environment at the end of the window and the current exposure of the portfolio in terms of rating and industrial sector. As these information are known at the beginning of each forecasting period, the default probabilities are easily deduced from Equations (3.4) and (3.5).

For risk management purposes, it is quite important to generate as precisely as possible the number of rating changes in the portfolio of buyers as well as the count of defaults. Thus, we measure the forecasting accuracy of the model by focusing on the forecasting errors $\widehat{q}_{ij,h} - \bar{q}_{ij,h}$ for all buyers, where $\widehat{q}_{ij,h}$ refers to the estimated transition probabilities (including default) and $\bar{q}_{ij,h}$ is obtained with the non-parametric Aalen-Johansen estimators. We consider for each date t between Sept. 2009 and June. 2012 the mean absolute error (MAE) and the mean squared error statistic (RMSE) for upgrades, downgrades, defaults and cancellations.

This analysis focuses on the results of models M4, M4* and M5* as these are the best models identified in the in-sample analysis, with and without the parallel assumption. Table 10 gives the forecasting errors by quarter for these models and the results with model M0 as a benchmark. This table compares the accuracy of the models for upgrades, downgrades and defaults.

[Table 10 about here.]

In general, the model give predictions that are quite close to those obtained with the non-parametric estimators. Note also than the transition probabilities are quite stable during the out-of-sample period

[22] A small rise occured at the end of 2010 is visible for Raw Product sector ans is not encountered by the model.

and relatively close to the average probabilities over the period. This explains why the error for M0 is quite smaller than what one could expect. For the upgrade probabilities, models M4 and M4* have equivalent performance and M5* outperforms them although the gain is relatively small in particular from the third quarter of 2010 on ward. Models M4, M4* and M5* have poor fit for the first date of the out-of-sample period. For the downgrades, the errors of model M5* are more often better than for models M4 and M4*. As noted with the in-sample results, the default probabilities seem to be less sensitive to the economic and financial environment. Hence, our forecasting results are almost identical, except for model M4. In addition, M5* is also the best model for cancellation probabilities, although the errors for this transition are higher than for the others. Recall that this gap is also observed for the in-sample estimation results.

Finally, similarly to Koopman *et al.* (2008), we deploy the Jafry-Schuerman metric (Jafry and Schuermann, 2004) in order to compare the forecasts of migration matrices over one quarter, defined as the average of the singular values of the migration matrix minus the unit matrix

$$SVD\left(t\right)=\frac{1}{R}\sum_{i=1}^{R}\sqrt{\lambda_i\left(\left(\widetilde{\boldsymbol{Q}}_k\left(t\right)\right)^{\top}\widetilde{\boldsymbol{Q}}_k\left(t\right)\right)},$$

with $\widetilde{\boldsymbol{Q}}_k\left(t\right)=\boldsymbol{Q}_k\left(t\right)-\boldsymbol{I}$, $\boldsymbol{I}$ the identity matrix and $\lambda_i\left(\boldsymbol{Q}\right)$ the function returning the i-th eigenvalue of $\boldsymbol{Q}$. The results are reported on Figure 7.

[Figure 7 about here.]

The forecasting results do not change substantially between models M4, M4* and M5* from the beginning of 2010 onward. All four models capture partially the dynamics of the crude rates realization and e.g. misses the fall at the end of 2010. A possible way to increase the performance of our models could be to add frailty factors (see e.g. Koopman *et al.*, 2008; Duffie *et al.*, 2009; Koopman *et al.*, 2011). This analysis is left for future research.

5.3 Estimation of the macro factors model

In this section, we perform estimations of the model for macroeconomic factors given by the general Equation (3.7)[23].

As the macroeconomic time-series model and the migration model can be estimated separately, the data that we have used for the factors model covers the period from September 1992 to September 2012 on a quarterly basis (81 quarterly observations). However, the enterprise birth rate is not observed over the full period. Consequently, we ignore this variable in the factors model and assume this variable remains constant for forecasting purposes.

Before estimating the model, we start our empirical investigation with a test for the first order integration hypothesis (noted I(1)) of univariate time series $\left(UR\left(t\right),CPI\left(t\right),GDP\left(t\right),IP\left(t\right),r\left(t\right),CAC\left(t\right),\sigma\left(t\right)\right)$. The Augmented Dickey-Fuller (ADF) statistics have been computed to conduct unit root tests. We have considered the possibility that the series contain deterministic terms (constant and trend). The adequate number of lagged differences for the series in the ADF regression is the value suggested by the AIC criterion with a maximum lag order of 10. The results of the ADF tests are reported in Table 11 for the levels and the first differences. This tables also gives the results for the second differences of the CPI.

[Table 11 about here.]

[23] The results are obtained using the packages `urca` and `vars` developed under the `R` software, see Pfaff (2008) and the related references.

Given the results of the unit root tests, it seems reasonable to consider that the variables $(UR(t), \Delta CPI(t), GDP(t), IP(t), r(t), CAC(t), \sigma(t))$ are at least I(1) and this enables us to search a VECM formulation. We first select the number of lags based on the AIC criterion without exceeding a lag order of 5 due to the length of our data set. We obtain a lag order of 2 for the underlying VAR model with a deterministic constant and a trend. Some univariate and multivariate diagnostics tests are conducted on the residuals (normality, homoskedasticity, serial correlation) which are globally satisfying. However, the residuals analysis indicates that there is some autocorrelation in the log inflation index as well as in the log unemployment rate. This problem may be solved by considering a VARMA specification but we consider here a simple VAR representation to be acceptable for our application. In addition, the normality assumption is violated for industrial production, stocks returns and volatilities which is not rely surprising given the presence of outliers during crisis periods.

Secondly, cointegration analysis is achieved on this set of variables to specify the factors model. We determinate the number of cointegration relationships r with the Johansen's trace statistic and maximum eigenvalue statistic and report the results in Table 12.

[Table 12 about here.]

The Johansen cointegration tests suggest that $r = 3$ at 5% level for the maximum eigenvalue statistics and $r = 4$ at 1% level for the trace statistics. Notice that three cointegrations are not founded with the trace statistics. As this last statistic is more robust when the normality assumption does not hold, we refer to it and we thus consider $r = 4$ cointegration relationships.

We can now estimate the cointegrated VECM model based on an underlying VAR(2) specification. Most of general macroeconometric models (see e.g. Pesaran *et al.*, 2006) consider long-run steady-state relationships to impose restrictions. Such equilibrium functions are beyond the scope of this paper and we only apply the Johansen's normalization (Johansen, 1995). The results obtained from the maximum likelihood estimation are given in Table 13.

[Table 13 about here.]

As we do not impose specific restrictions on $\widehat{\boldsymbol{\beta}}$, we should not expect the coefficient to be particularly interpretable. Some results are nonetheless interesting. The first relation describes an GDP equation with negative effects of interest rates and the stock volatility. The sign of the inflation rate is positive but the coefficient is very small. On the second relation, the industrial production is affected negatively by interest rates and positively by inflation rates. The unemployment rate depends to the same factors, yet with opposite coefficients, and the sign of the stock volatility coefficient seems to be consistent. In the last relation, the stock index is linked with strong negative effects to interest and inflation rates. The relation with the stock volatility seems also to be realistic.

6 Application for risk management

In this section, we consider implications for risk management. The migration model and the factors model can be used jointly to forecast the number of future defaults and the changes in the credit quality of the portfolio. The factor model is used as an economic scenario generator in a first step and the migration model processes the link between the credit quality and the macroeconomic and financial environment in the same spirit than Pesaran *et al.* (2006). As we do not estimate a model for the loss in case of default, we are not able to combine this information with the rating transitions and the default events to compute the loss distribution. However, it can be easily done practically if a credit manager adds the expected loss in case of default. Our model could be used as a component of an internal model, or in component with the various stress-testing analysis for the ORSA. However, the users should be aware that the factors model generates simulations with Gaussian laws which is not appropriate to study extremes scenarios. This can be improved considering frailty factors, e.g. Creal *et al.* (2014) for recent application.

For our application, we consider a fictitious buyer portfolio containing 100,000 buyers with the same exposure as our sample at the end of the estimation period, i.e. in September 2012, and we forecast the distribution of the cumulative number of defaults for a fixed time horizon T in run-off with a quarterly step. Initially, we observe 943 buyers in class 1, 3,207 buyers in class 2, 25,855 buyers in class 3, 49,299 buyers in class 4, 20,600 buyers in class 5 and 96 buyers who have already experienced protracted default.

With the estimated factors model and the migration model M5*, we simulate the future transition matrices including the cancellation rates and compute the rating composition of the buyer portfolio via the multinomial distribution, as described in Subsection 3.4. The factors are simulated with 100,000 replications using Gaussian innovations. Although model M5* directly depends on only three factors, the effect of other factors is taken into account through the relationships estimated with the VECM model. For each drawing, we remove buyers who become insolvent or are canceled. In case of protracted default, the buyer' life time is simulated with the constant second order transition matrix given in Table 4. Note finally that the non-modeled transitions from states 1 and 2 to default and from state 5 to state 1 are assumed to be constant and are given in Table 3.

Figure 8 gives the number of defaults and cancellation distributions for different time horizons $T = 2$ quarters, 4 quarters, 8 quarters and 12 quarters for models M5* and M0 as a benchmark. Remark that model M0 is only affected by poolable risk.

[Figure 8 about here.]

The difference between models M0 and M5* for cancellation is clearly visible from the second quarter onward as M5* is more uncertain. Conversely, distributions of the number of defaults remain relatively close across models and we observe convergence of the results after 12 quarters. As noted before, the default rates of our sample after cancellation are very insensitive to the changes in the environment and thus the number of default mainly evolves due to the changes in credit quality. This management action allows to virtually immunize the buyer portfolio against systematic risks, assuming the conservative management policy in terms of cancellation stays the same.

We study the impact of several specific shocks occurring over the first quarter in addition to the initial distributions obtained with random shocks. The following stress events are considered:

- a -2.58σ[24] shock to the log real GDP $(GDP\,(t))$,
- a -2.58σ shock to the log Industrial production $(IP\,(t))$,
- a $+2.58\sigma$ shock to the 10-years government rate $(r\,(t))$,
- a $+2.58\sigma$ shock to the CAC40 volatility $(\sigma\,(t))$.

The results of these shock are displayed in Figure 9. For comparison, the figure also presents the distribution of the number of defaults and cancellations obtained with models M0 and M5* without shock.

[Figure 9 about here.]

[24] A 2.58 $\times$ standard deviation shock is approximately equivalent to a 99.5% quantile of a Gaussian law which is the level of confidence used in the Solvency II framework.

The stress event on the log industrial production has an immediate impact on the number of cancellations. This confirms the analysis of the factors presented in Tables 8 and 9 as the main determinant and is accentuated by the deterioration in the credit quality especially for the riskiest classes. Because of the strong correlation between GDP growth and industrial production, the response to a shock on the log GDP is close to the previous one. The effects of these shocks fade away within 2 and 3 years. The increase in the volatility of the CAC40 is more moderate and the impacts of this shock vanish after 2 years. Conversely, the number of cancellation is not immediately impacted by a rise in interest rates but the response comes later with the deterioration of the credit quality and through a second order effect with the responses of others macro factors.

As expected, the impact of shocks in terms of defaults is relatively limited as they ahve largely absorbed by the cancellations. While these shocks impulse a decrease in the credit quality, there is, a decline in the number of defaults. Except for the upward shock on rates, the variance is also not affected by the shocks.

Finally, we analyze how the tail of the distributions evolves. For each scenario, we compute the empirical mean, the value-at-risk (VaR) at 99.5% and the conditional tail expectation (CTE) for the same level and report the results in Table 14.

[Table 14 about here.]

As we have already seen, shocks on the log real GDP and on the log of industrial production have the greatest effect on cancellations. Yet, in these situations the gap between the mean and the VaR decreases indicating that the propensity to cancel contracts is higher and less uncertain. As we do not consider future business, more cancellations at the beginning of the projection leads to a lower risk exposure at the term of the projection. Then, fewer cancellations are observed at the end if a shock occurs. We can verify with this results that the cancellation policy permits to clearly reduce the risk even at the distribution tail. The most severe effect on defaults (a rise of roughly 5% for the VaR and the CTE) is observed after 4 quarters in response to the increase in interest rate as this shock does not trigger management actions. A similar result is seen for the CAC40 volatility but with smaller magnitude. Recall also that the effects of the shocks are rapidly absorbed and no significant impact is notable after 2 years.

On the whole, our results on the current portfolio of buyers[25] illustrate the strength of the ability to close the riskiest contracts, as defined with the credit classes, in a stressed economic environment.

7 Conclusion

Modeling and managing the risks underlying the credit insurance business have been rarely described in the literature. Yet, similarly to more classical financial institutions, these insurance companies have to manage credit risk, which is exposed to financial and economic fluctuations. In this paper, we have introduced an econometric framework which links dynamically the credit risk and the systematic macroeconomic and financial risk factors. It allows to retrofit the credit quality of a portfolio of buyers when the environment changes. Our modeling approach is quite simple and uses an ordered regression model based on observable factors, and a VAR model as an economic engine for macro variables.

Such a framework is useful for credit insurance risk managers as it gives the estimated impacts of economic shocks on the credit situation of the portfolio. As this model permits multi-periods forecasting of the risk profile, credit insurers could use it as an input for their ORSA exercises or as a brick to develop an internal model valuing the Solvency Capital Requirement under the Solvency II framework.

[25] The results goes in the same direction even when the portfolio at the beginning of the forecasting exercise has a better credit quality.

Contrary to the traditional buy and sell management strategy implemented for a bonds portfolio, a credit insurer has much leeway to pilot its credit risk by reducing its exposure to one contract or by directly closing it. After estimating the model and checking its robustness, we analyze more specifically the effect of this last management action combined with changes in credit quality. Empirically, we remark that this induces significant selection bias on our data since this action aims at controlling the number of defaults. Indeed, by removing a large part of the riskiest contracts from the portfolio, we observe that this technique clearly limited the effects of the 2008-2009 financial crisis and is very effective on our sample by reacting quickly to the observed economic shocks. By replicating the observed cancellation' behavior, our forecasting analysis indicates that this allows to control for the number of defaults and the deterioration of the credit quality after such shocks. However, it is really difficult to gauge the real effects of these shocks on the closed contracts as this policy seems to be cautious and removes contracts that will experience defaults as well as others that will not. As a consequence, we do not measure how this action protects the insurer' future incomes. Another interesting question for future research is to forecast jointly the number of defaults with the underlying exposure.

Acknowledgments

The authors gratefully thank Pierre-Emmanuel Darpeix for his helpful comments which greatly improved the quality of the first draft of this paper. The authors are also thankful to Cécile Welter-Nicol, Stéphane Loisel and participants in an ACPR internal seminar for their suggestions.

References

Agresti, A. (2002). *Categorical Data Analysis*. Number Wiley Series in Probability and Statistics. John Wiley & Sons, Inc., 2 edition.

Andersen, P. K., Borgan, r., Gill, R. D., and Keiding, N. (1993). *Statistical Models Based on Counting Processes*. Springer Series in Statistics. Springer-Verlag New York Inc.

Azizpour, S., Giesecke, K., and Schwenkler, G. (2014). Exploring the Sources of Default Clustering. Technical report.

Bangia, A., Diebold, F. X., Kronimus, A., Schagen, C., and Schuermann, T. (2002). Ratings migration and the business cycle, with application to credit portfolio stress testing. *Journal of Banking & Finance*, **26**(2-3), 445–474.

Caja, A. and Planchet, F. (2014). Modelling cycle dependence in credit insurance. *Risks 2*, **1**, 74–88.

Chava, S. and Jarrow, R. A. (2004). Bankruptcy Prediction with Industry Effects. *Review of Finance*, **8**(4), 537–569.

Chava, S., Stefanescu, C., and Turnbull, S. (2011). Modeling the Loss Distribution. *Management Science*, **57**(7), 1267–1287.

Chen, B., Yi, G. Y., and Cook, R. J. (2011). Progressive multi-state models for informatively incomplete longitudinal data. *Journal of Statistical Planning and Inference*, **141**(1), 80–93.

Couderc, F. and Renault, O. (2005). Time-to-default: Life cycle, global and industry cycle impacts. Research Paper No. 142, University of Geneva.

Cox, D. R. (1972). Regression Models and Life-Tables. *Journal of the Royal Statistical Society. Series B (Methodological)*, **34**(2), 187–220.

Creal, D., Schwaab, B., Koopman, S. J., and Lucas, A. (2014). Observation-Driven Mixed-Measurement Dynamic Factor Models with an Application to Credit Risk. *Review of Economics and Statistics*, **96**(5), 898–915.

Dees, S., Mauro, F. d., Pesaran, M. H., and Smith, L. V. (2007). Exploring the international linkages of the euro area: a global VAR analysis. *Journal of Applied Econometrics*, **22**(1), 1–38.

Dietsch, M. and Petey, J. (2002). The credit risk in SME loans portfolios: Modeling issues, pricing, and capital requirements. *Journal of Banking & Finance*, **26**(2–3), 303–322.

Dietsch, M. and Petey, J. (2004). Should SME exposures be treated as retail or corporate exposures? A comparative analysis of default probabilities and asset correlations in French and German SMEs. *Journal of Banking & Finance*, **28**(4), 773–788.

Dolignon, C. (2011). Facteurs explicatifs des évolutions récentes des défaillances d'entreprises : une analyse économétrique. *Économie & prévision*, **197**(1), 161–167.

Duffie, D., Saita, L., and Wang, K. (2007). Multi-period corporate default prediction with stochastic covariates. *Journal of Financial Economics*, **83**(3), 635–665.

Duffie, D., Eckner, A., Horel, G., and Saita, L. (2009). Frailty Correlated Default. *The Journal of Finance*, **64**(5), 2089–2123.

European Parliament and Council of the European Union (2009). Directive 2009/138/CE of 25 november 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II). OJ L. 335/1.

Fei, F., Fuertes, A.-M., and Kalotychou, E. (2012). Credit Rating Migration Risk and Business Cycles. *Journal of Business Finance & Accounting*, **39**(1-2), 229–263.

Feng, D., Gouriéroux, C., and Jasiak, J. (2008). The ordered qualitative model for credit rating transitions. *Journal of Empirical Finance*, **15**(1), 111–130.

Figlewski, S., Frydman, H., and Liang, W. (2012). Modeling the effect of macroeconomic factors on corporate default and credit rating transitions. *International Review of Economics & Finance*, **21**(1), 87–105.

Foglia, A. (2008). Stress Testing Credit Risk: A Survey of Authorities' Approaches. Bank of Italy Occasional Paper No. 37, Bank of Italy.

Frydman, H. and Schuermann, T. (2008). Credit rating dynamics and Markov mixture models. *Journal of Banking & Finance*, **32**(6), 1062–1075.

Gagliardini, P. and Gouriéroux, C. (2005). Stochastic Migration Models with Application to Corporate Risk. *Journal of Financial Econometrics*, **3**(2), 188–226.

Garratt, A., Lee, K., Pesaran, M. H., and Shin, Y. (2003). A Long run structural macroeconometric model of the UK. *The Economic Journal*, **113**(487), 412–455.

Guibert, Q., Juillard, M., Nteukam Teuguia, O., and Planchet, F. (2014). *Solvabilité prospective en assurance: Méthodes quantitatives pour l'ORSA*. Assurance Audit Actuariat. Economica, Paris.

Hu, Y.-T., Kiesel, R., and Perraudin, W. (2002). The estimation of transition matrices for sovereign credit ratings. *Journal of Banking & Finance*, **26**(7), 1383–1406.

Jackson, C. H. (2011). Multi-State Models for Panel Data: The msm Package for R. *Journal of Statistical Software*, **38**(8).

Jafry, Y. and Schuermann, T. (2004). Measurement, estimation and comparison of credit migration matrices. *Journal of Banking & Finance*, **28**(11), 2603–2639.

Johansen, S. (1991). Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models. *Econometrica*, **59**(6), 1551.

Johansen, S. (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford University Press.

Juselius, K. (2006). *The Cointegrated VAR Model: Methodology and Applications*. Oxford University Press.

Koop, G., Pesaran, M. H., and Potter, S. M. (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics*, **74**(1), 119–147.

Koopman, S. J., Lucas, A., and Monteiro, A. (2008). The multi-state latent factor intensity model for credit rating transitions. *Journal of Econometrics*, **142**(1), 399–424.

Koopman, S. J., Kräussl, R., Lucas, A., and Monteiro, A. (2009). Credit cycles and macro fundamentals. *Journal of Empirical Finance*, **16**(1), 42–54.

Koopman, S. J., Lucas, A., and Schwaab, B. (2011). Modeling frailty-correlated defaults using many macroeconomic covariates. *Journal of Econometrics*, **162**(2), 312–325.

Lando, D. and Skødeberg, T. M. (2002). Analyzing rating transitions and rating drift with continuous observations. *Journal of Banking & Finance*, **26**(2–3), 423–444.

Leow, M. and Crook, J. (2014). Intensity models and transition probabilities for credit card loan delinquencies. *European Journal of Operational Research*, **236**(2), 685–694.

Lütkepohl, H. and Krätzig, M. (2004). *Applied time series econometrics*. Cambridge University Press, Cambridge, UK; New York.

Malik, M. and Thomas, L. C. (2012). Transition matrix models of consumer credit ratings. *International Journal of Forecasting*, **28**(1), 261–272.

McCullagh, P. (1980). Models for Ordinal Data. *Journal of the Royal Statistical Society. Series B (Methodological)*, **42**(2), 109–142.

McNeil, A. J. and Wendin, J. P. (2007). Bayesian inference for generalized linear mixed models of portfolio credit risk. *Journal of Empirical Finance*, **14**(2), 131–149.

Merton, R. C. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *The Journal of Finance*, **29**(2), 449.

Nickell, P., Perraudin, W., and Varotto, S. (2000). Stability of rating transitions. *Journal of Banking & Finance*, **24**(1-2), 203–227.

Passalacqua, L. (2006). A pricing model for credit insurance. *Giornale dell'Istituto Italiano degli Attuari*, **LXIX**, 87–123.

Passalacqua, L. (2007). Measuring Effect of Excess-of-Loss Reinsurance on Credit Insurance Risk Capital. *Giornale dell'Istituto Italiano degli Attuari*, **LXX**, 81–102.

Pesaran, M. H., Schuermann, T., and Weiner, S. M. (2004). Modeling Regional Interdependencies Using a Global Error-Correcting Macroeconometric Model. *Journal of Business & Economic Statistics*, **22**(2), 129–162.

Pesaran, M. H., Schuermann, T., Treutler, B.-J., and Weiner, S. M. (2006). Macroeconomic Dynamics and Credit Risk: A Global Perspective. *Journal of Money, Credit and Banking*, **38**(5), 1211–1261.

Pfaff, B. (2008). VAR, SVAR and SVEC Models: Implementation Within R Package vars. *Journal of Statistical Software*, **27**(4), 1–32.

R Core Team (2015). *R: A Language and Environment for Statistical Computing*. Vienna, Austria.

Shumway, T. (2001). Forecasting Bankruptcy More Accurately: A Simple Hazard Model. *The Journal of Business*, **74**(1), 101–124.

Stefanescu, C., Tunaru, R., and Turnbull, S. (2009). The credit rating process and estimation of transition probabilities: A Bayesian approach. *Journal of Empirical Finance*, **16**(2), 216–234.

Wendin, J. (2006). *Bayesian methods in portfolio credit risk management*. Ph.D. thesis, Swiss Federal Institute of Technology Zürich, Zürich.

Wendin, J. and McNeil, A. J. (2006). Dependent credit migrations. *Journal of Credit Risk*, **2**(3), 87–114.

Xing, H., Sun, N., and Chen, Y. (2012). Credit rating dynamics in the presence of unknown structural breaks. *Journal of Banking & Finance*, **36**(1), 78–89.

Yee, T. W. (2010). The VGAM Package for Categorical Data Analysis. *Journal of Statistical Software*, **32**(10), 1–34.

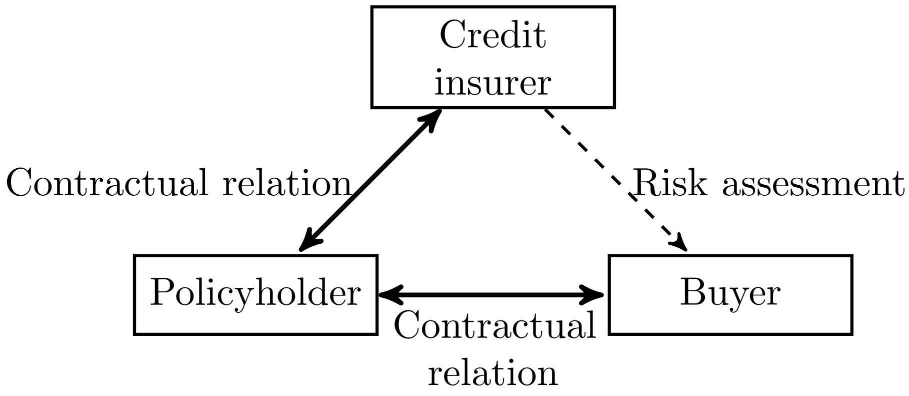


Figure 1: Triangle relationship in credit insurance.

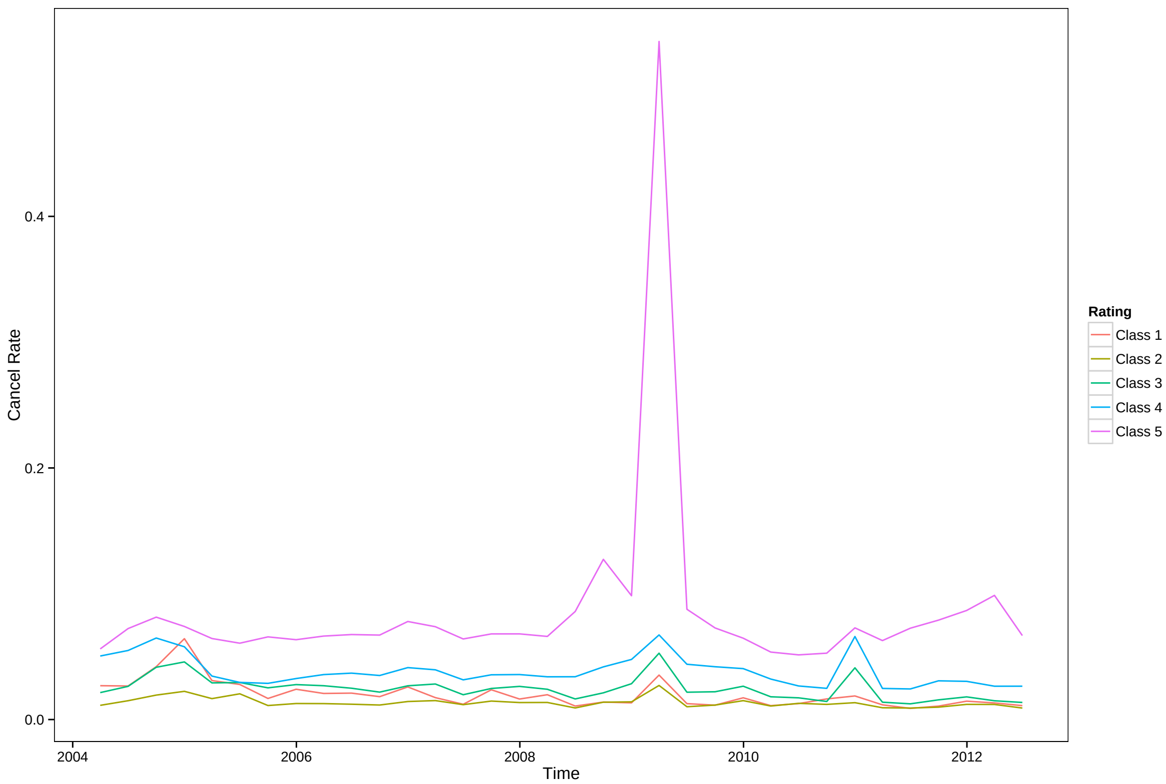


Figure 2: The cancellation rates for each rating classes (Class 1 to 5).

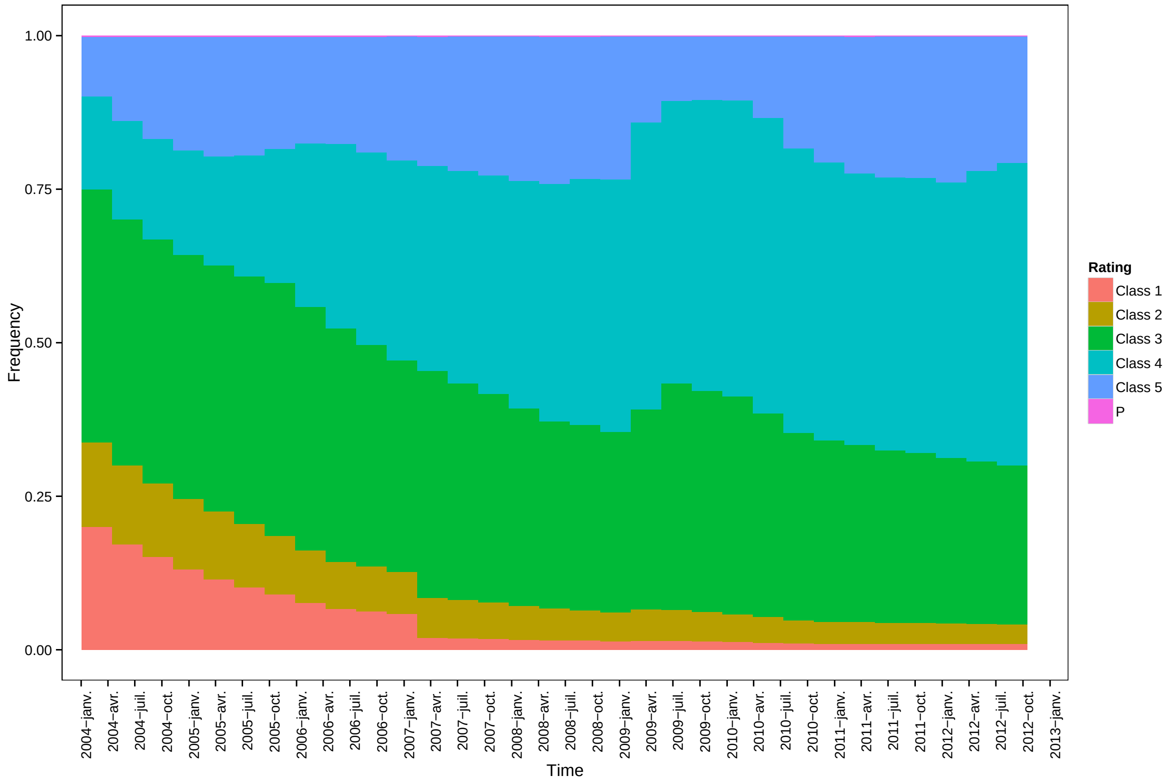


Figure 3: The total exposure broken down by rating over the observation period.

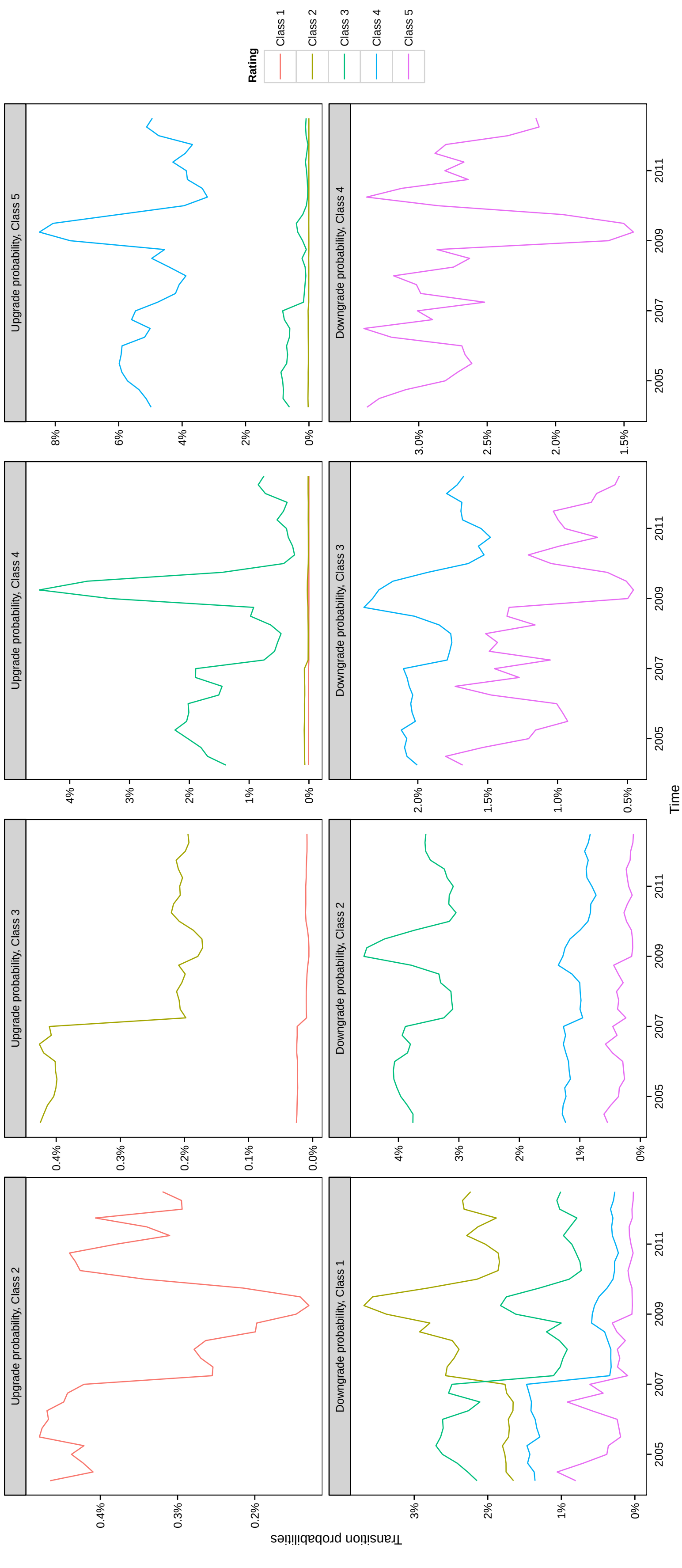
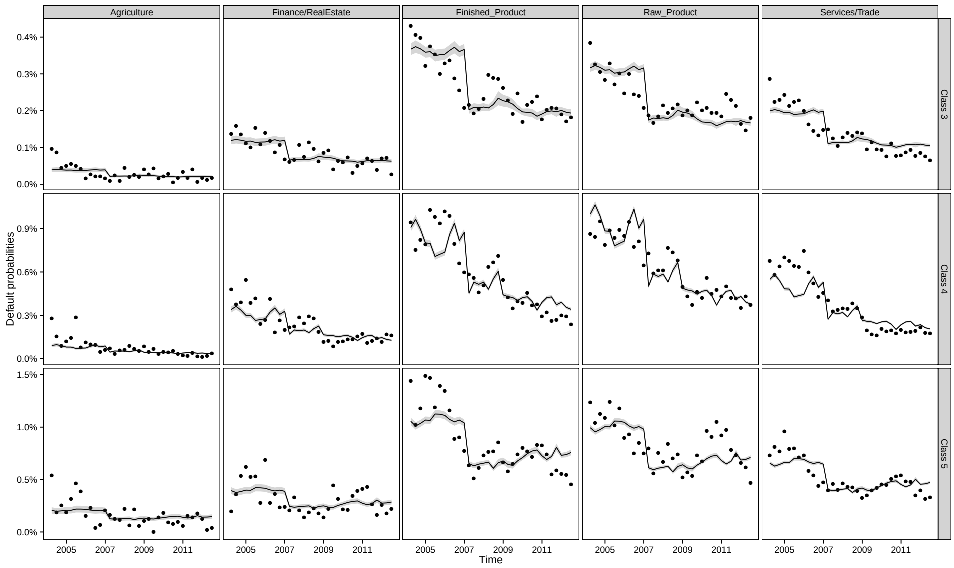
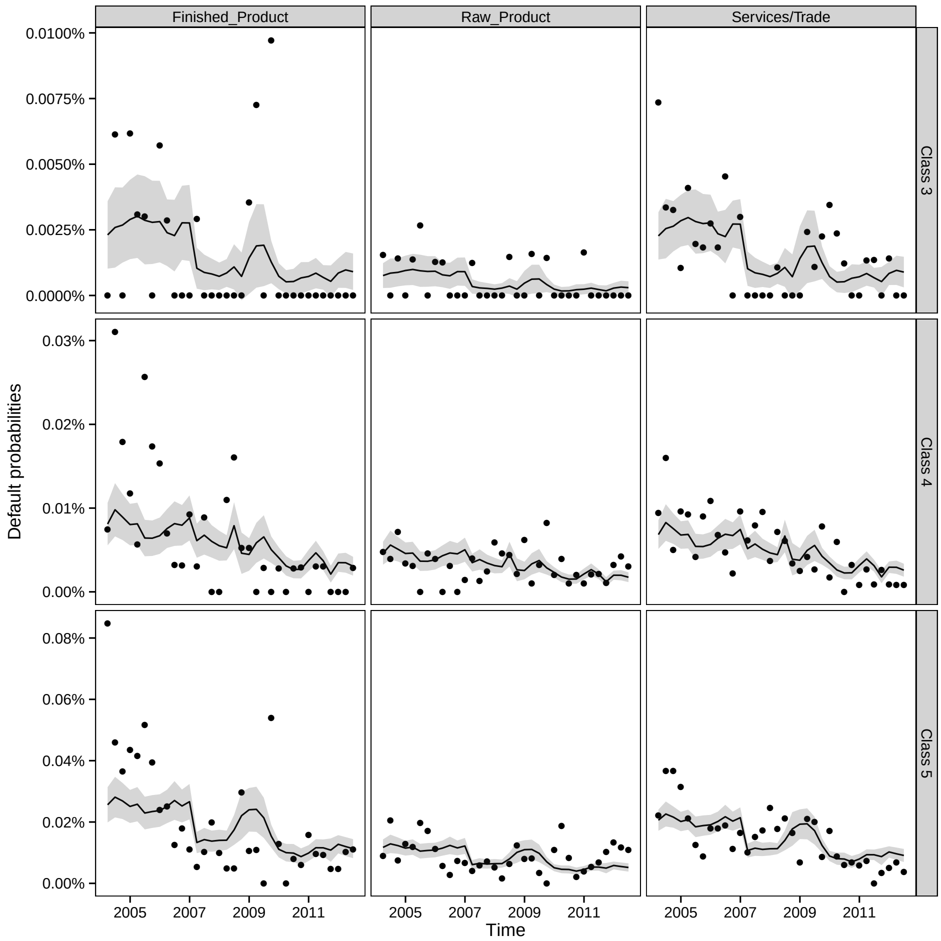


Figure 4: Quarterly estimated transition probabilities for the Services/Trade sector. The first row presents the upgrade probabilities from one rating class (Class 2-5) to another. The second row presents the downgrade probabilities from one rating class (Class 1-4) to another.



(a) Protracted default.



(b) Insolvency default.

Figure 5: Quarterly estimated (a) protracted default and (b) insolvency default probabilities from rating classes 3-5 and the pointwise 95% confidence interval associated with parameter uncertainty. The protracted default rates are displaying for all sectors and the insolvency default rates are ignored for Agriculture and Finance/Real estate sectors due to lack of observations. The points corresponds to the crude default probabilities estimated with the non-parametric Aalen-Johansen estimators.

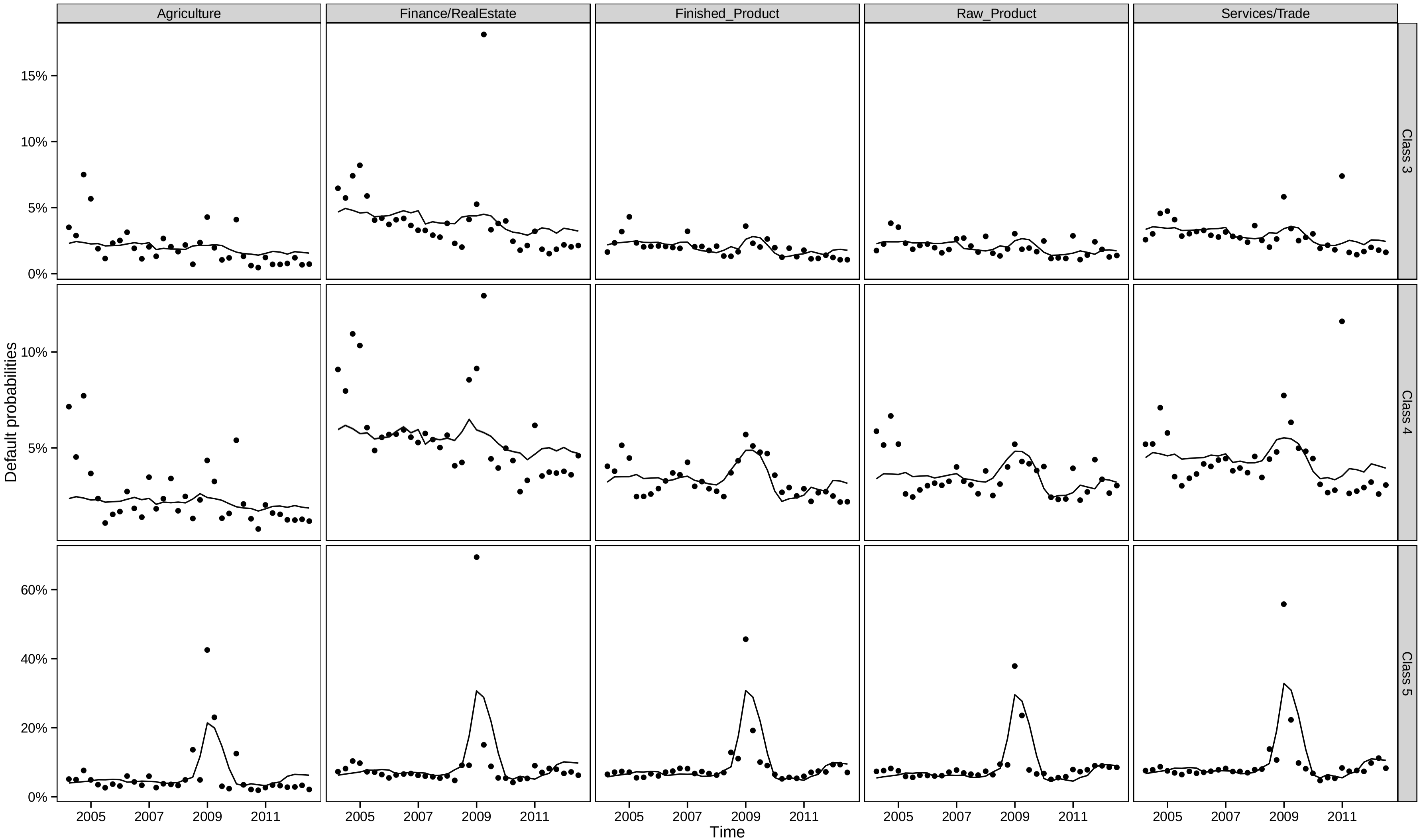


Figure 6: Quarterly estimated cancellation probabilities for all sectors and the pointwise 95% confidence interval associated with parameter uncertainty. The points corresponds to the crude default probabilities estimated with the non-parametric Aalen-Johansen estimators.

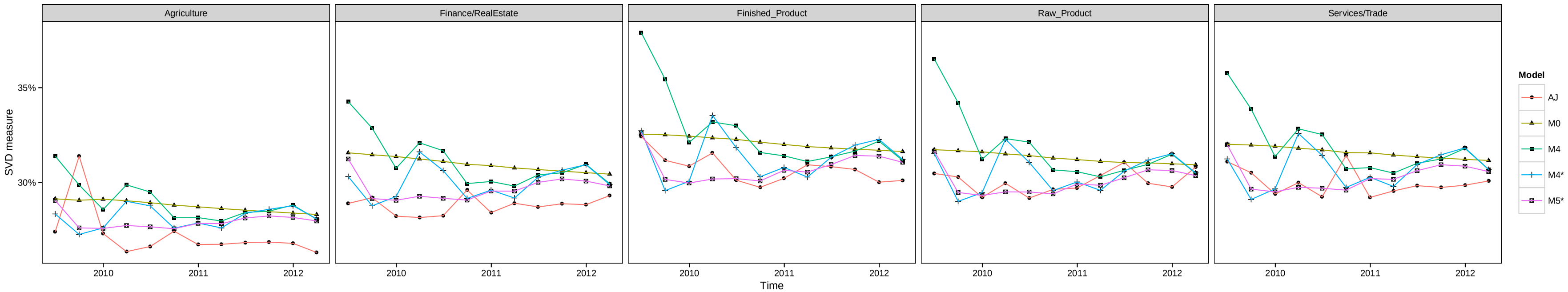


Figure 7: SVD measure for quarterly forecast default probabilities for all the sectors. SVD for the out-of-sample probabilities are forecast from models M0, M4, M4* and M5. The accuracy of the models is measured by comparing the estimated results with the SVD measure for probabilities computed with Aalen-Johansen estimators.

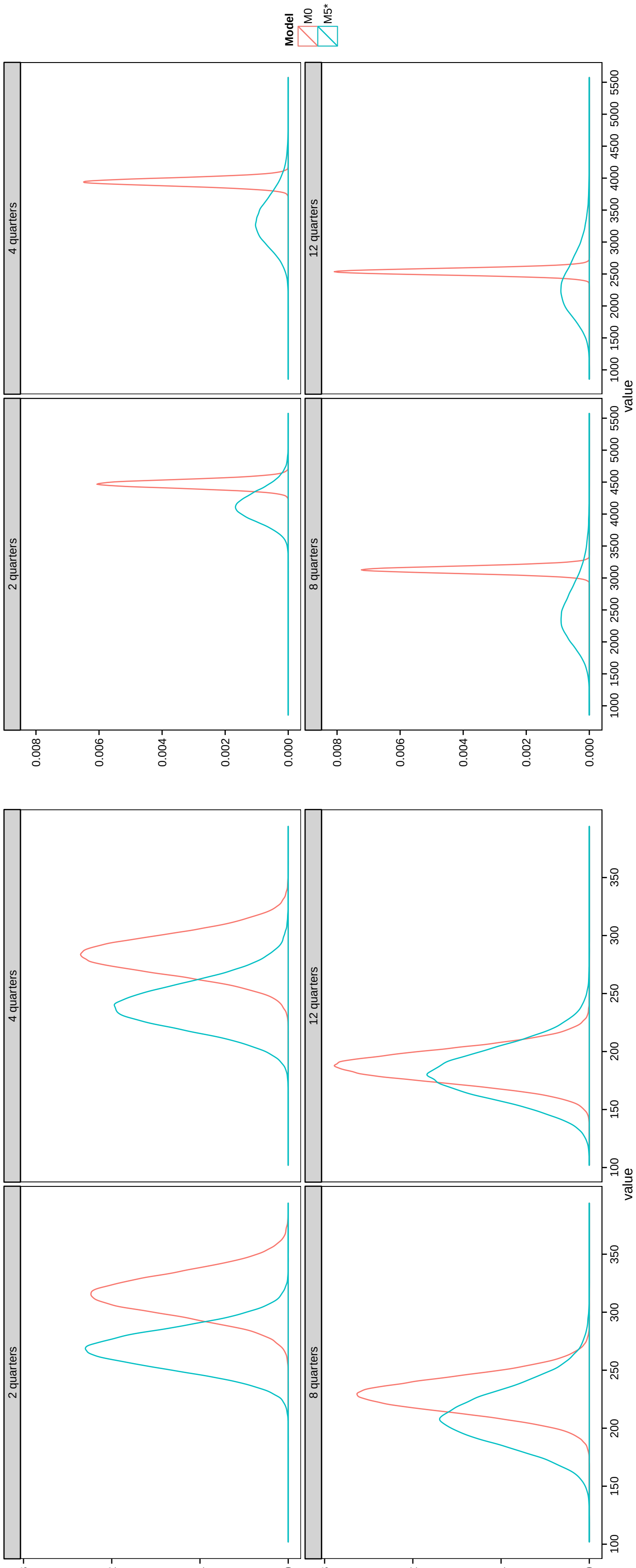


Figure 8: Number of defaults and cancellations obtained with models M0 and M5*. The plots display the distribution of the number of defaults (left) and cancellations (right) for different horizons.

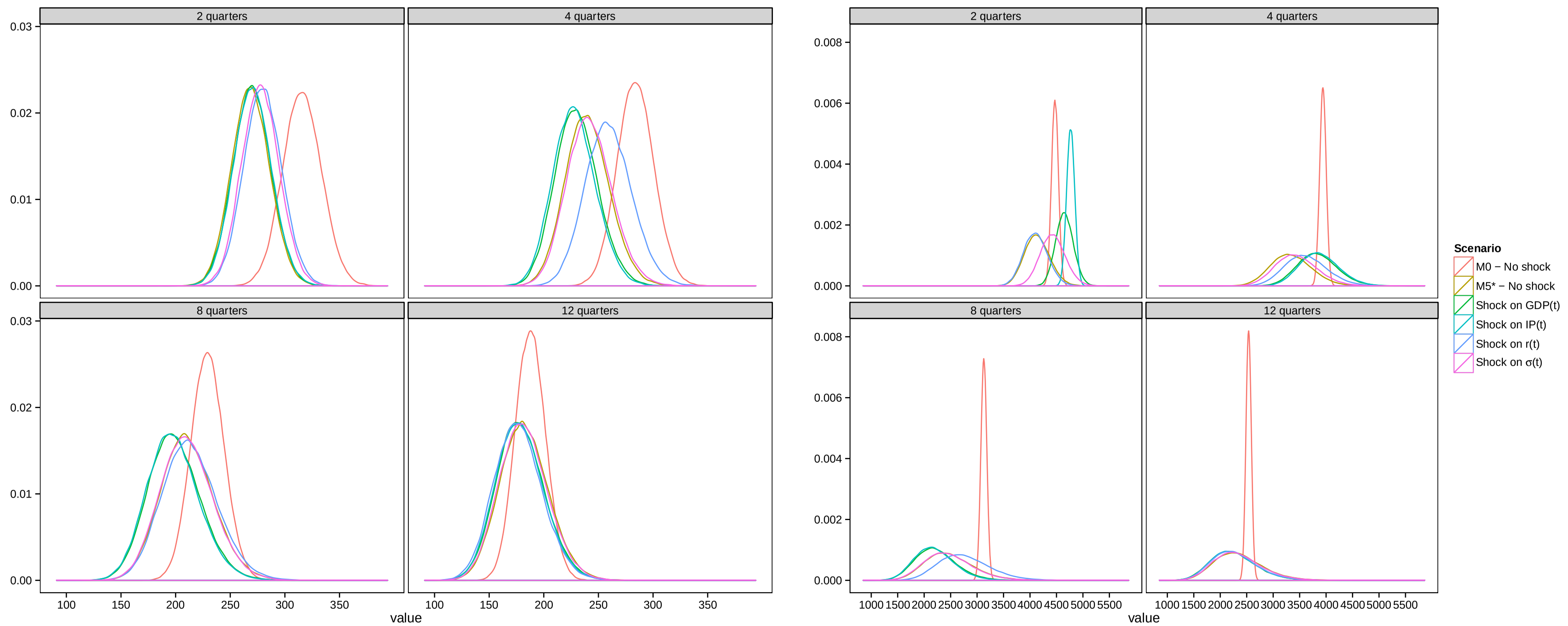


Figure 9: Number of defaults (left) and cancellations (right) predicted by M5* for different horizons under several stressed situations. Models M0 and M5* without shock are also plotted for comparison.

Table 1: Summary statistics on the selected insurance sample.

Sector	Name	Number of buyers
1	Agriculture	93,697
2	Finance/Real estate	148,582
3	Finished Product	187,857
4	Raw Product	471,270
5	Services/Trade	703,127

Note: This table presents the number of total observed buyers over the period for each sector.

Table 2: Number of direct transitions observed.

Initial state	Total Exposure	Transition state							
		1	2	3	4	5	P	C	I
1	2,802,022	–	18,221	47,147	10,676	4,186	2,658	26,537	7
2	4,211,672	5,928	–	55,901	16,511	4,941	3,461	20,576	5
3	22,998,099	1,279	25,536	–	158,009	82,947	12,941	195,022	79
4	24,890,613	198	2,325	112,622	–	229,832	33,484	312,732	287
5	13,035,742	74	451	13,603	246,523	–	28,452	427,807	547
P	105,801	2,564	3,324	12,297	32,212	27,904	–	13,509	242

Note: This table contains the total number of direct transitions observed during the period. The total exposure corresponds to the sum of buyers observed at the beginning of each month per rating. The state are ordered from class 1 to 5 and we count transitions to state P (protracted default), C (cancellation) and I (insolvency default).

Table 3: Average quarterly transition probabilities.

Initial state	Transition state							
	1	2	3	4	5	P	C	I
Agriculture								
1	94.266	0.548	1.660	0.563	0.229	0.007	2.727	0
2	0.096	95.370	1.702	0.858	0.316	0.018	1.640	0
3	0.003	0.032	97.197	0.509	0.313	0.009	1.937	0
4	0.001	0.004	0.200	97.115	0.585	0.016	2.079	0
5	0.002	0.005	0.277	1.942	91.507	0.045	6.221	0.001
P	0.981	1.764	19.456	37.098	17.667	5.691	17.344	0
Finance/Real estate								
1	94.439	0.900	0.830	0.390	0.129	0.037	3.276	0
2	0.398	93.451	1.792	0.764	0.181	0.044	3.368	0
3	0.031	0.222	93.972	1.085	0.606	0.029	4.054	0
4	0.012	0.054	0.781	92.714	1.062	0.063	5.313	0.001
5	0.015	0.034	0.364	3.363	85.893	0.090	10.231	0.009
P	4.467	4.323	15.994	35.046	17.282	6.107	16.751	0.030
Finished Product								
1	90.335	3.187	2.436	1.288	0.566	0.135	2.050	0.002
2	0.566	91.626	4.779	1.399	0.473	0.111	1.044	0.002
3	0.035	0.677	92.561	3.148	1.410	0.100	2.065	0.003
4	0.014	0.069	2.250	90.697	3.359	0.201	3.402	0.008
5	0.018	0.040	0.508	6.690	83.694	0.302	8.726	0.022
P	2.565	4.288	13.494	29.641	25.061	8.709	15.635	0.606
Raw Product								
1	90.383	1.812	2.989	1.559	0.619	0.166	2.470	0.001
2	0.340	92.591	3.997	1.293	0.452	0.122	1.205	0.001
3	0.016	0.303	94.249	2.223	1.048	0.083	2.076	0.001
4	0.012	0.033	1.411	91.939	2.931	0.203	3.468	0.004
5	0.015	0.024	0.340	5.668	84.792	0.272	8.880	0.009
P	2.456	2.897	11.447	32.997	25.588	7.540	16.875	0.201
Services/Trade								
1	90.989	1.827	2.121	1.235	0.526	0.078	3.223	0.001
2	0.360	92.814	3.631	1.151	0.376	0.066	1.602	0.001
3	0.017	0.283	93.619	1.912	1.123	0.049	2.994	0.002
4	0.007	0.035	1.227	91.672	2.618	0.109	4.328	0.005
5	0.009	0.021	0.370	4.727	84.697	0.164	9.997	0.015
P	2.165	3.095	12.447	31.233	27.557	6.585	16.771	0.148

Note: This table gives the average transition matrices over one quarter estimated (in %) from the duration approach with homogeneous assumption. The states are ordered from class 1 to 5 and we count transitions to state P (protracted default), C (cancellation) and I (insolvency default). The rows from states C and I are discarded.

Table 4: Second order quarterly average transition probabilities from state P.

(Previous state,Initial state)	Transition state							
	1	2	3	4	5	P	C	I
Agriculture								
(1,P)	81.978	0.341	1.041	9.651	0.181	4.984	1.823	0
(2,P)	0.059	84.413	1.042	0.518	0.191	6.963	6.814	0
(3,P)	0.002	0.020	84.121	3.691	2.876	5.077	4.215	0
(4,P)	0	0.003	1.138	84.125	2.952	5.546	6.236	0
(5,P)	0	0.002	0.153	11.797	69.650	5.665	12.733	0
(P,P)	0.003	3.743	7.696	23.137	29.249	15.189	20.984	0
Finance/Real estate								
(1,P)	84.728	1.119	2.717	1.883	0.608	5.559	3.387	0
(2,P)	0.249	83.225	5.084	2.184	0.665	5.682	2.912	0
(3,P)	0.018	0.301	79.396	5.176	2.442	5.170	7.497	0
(4,P)	0.004	0.110	2.184	75.607	4.316	5.830	11.949	0.001
(5,P)	0.003	0.016	0.368	7.136	66.827	5.850	19.795	0.005
(P,P)	3.816	1.939	6.599	35.014	19.383	13.779	18.810	0.660
Finished Product								
(1,P)	82.219	3.668	3.598	2.107	0.882	5.379	2.147	0.001
(2,P)	0.998	78.556	8.700	2.851	1.190	6.082	1.386	0.236
(3,P)	0.021	1.334	75.882	7.493	4.114	6.615	4.383	0.157
(4,P)	0.002	0.070	2.659	70.815	8.045	7.851	10.296	0.261
(5,P)	0.001	0.010	0.450	8.010	63.720	8.088	19.202	0.518
(P,P)	0.452	1.278	5.678	25.035	26.664	17.846	20.555	2.492
Raw Product								
(1,P)	83.663	2.730	2.794	2.033	0.609	6.036	2.133	0.001
(2,P)	1.032	81.416	6.804	2.257	1.224	5.787	1.480	0
(3,P)	0.062	0.970	77.246	6.624	4.081	6.210	4.711	0.096
(4,P)	0.007	0.021	1.825	72.863	7.661	7.008	10.496	0.118
(5,P)	0.001	0.013	0.379	7.978	63.562	6.825	21.034	0.209
(P,P)	1.435	0.959	5.837	28.722	23.296	16.577	22.318	0.857
Services/Trade								
(1,P)	81.825	3.506	3.422	1.595	0.970	5.587	3.093	0.001
(2,P)	0.875	82.230	5.451	2.817	1.128	5.435	2.065	0
(3,P)	0.011	0.770	79.570	5.820	3.327	5.798	4.680	0.024
(4,P)	0.002	0.028	1.770	75.483	7.202	6.198	9.231	0.088
(5,P)	0.001	0.007	0.352	6.663	67.675	6.196	18.965	0.142
(P,P)	1.181	1.354	6.539	27.160	27.417	14.675	20.601	1.075

Note: This table gives the second order average transition matrices over one quarter estimated (in %) from the duration approach with homogeneous assumption. The states are ordered from class 1 to 5 and we count transitions to state P (protracted default), C (cancellation) and I (insolvency default). The rows from states C and I are discarded.

Table 5: Macroeconomic and financial covariates used for the migration model.

Variable	Definition	Source	Mean	Std. Dev.
		General macroeconomic conditions		
$UR(t) - UR(t-4)$	The annual change in log unemployment rate $UR(t)$ at the end of quarter t.	Harmonized unemployment rate of all business for France. FRED database at the Federal Reserve Bank of St. Louis.	1.14%	8.96%
$CPI(t) - CPI(t-4)$	The annual change in log consumer price index $CPI(t)$ at the end of quarter t.	Consumer Price Index of all items in France. FRED database at the Federal Reserve Bank of St. Louis.	1.71%	0.82%
$EB(t) - EB(t-4)$	The annual change in log number of business creation $EB(t)$ at the end of quarter t.	Number of business creation all items in France. INSEE, business demography.	0.52%	11.34%
		Direction of the economy		
$GDP(t) - GDP(t-4)$	The annual change in log real growth domestic product $GDP(t)$ at the end of quarter t.	Growth Domestic Product in France. FRED database at the Federal Reserve Bank of St. Louis.	1.18%	1.85%
$IP(t) - IP(t-4)$	The annual change in log industrial production $IP(t)$ at the end of quarter t.	Production of Total Industry in France. FRED database at the Federal Reserve Bank of St. Louis.	-1.04%	6.12%
		Financial market conditions		
$r(t) = 0.25ln(1 + R(t))$	The quarterly transformation of the 10-year French government bond rate on annual basis $R(t)$ at the end of quarter t.	Banque de France.	0.90%	0.14%
$\rho(t) = CAC(t) - CAC(t-4)$	The annual change in log CAC40 index $CAC(t)$ at the end of quarter t.	Yahoo!Finance	-0.01%	21.62%
$\sigma(t)$	The CAC40 quarterly volatility of the CAC40 index $CAC(t)$ at the end of quarter t. Volatility is estimated as the annualized (rounded to 260 trading days) standard deviation of daily returns within the quarter (rounded to 90 trading days).	Yahoo!Finance	21.94%	10.60%

Note: This table describes the covariates used for the different migration model specifications used in this paper. Summary statistics are calculated over the period 2004:1-2012:3.

Table 6: Estimated parameters for different migration models with parallel assumption.

	Initial state									
	1		2		3		4		5	
	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2
Unemployment	−1.25***		−1.17***		0.19***		−0.25***		−5.38***	
	(0.07)		(0.06)		(0.03)		(0.03)		(0.03)	
Inflation	−3.46**		2.92***		15.16***		1.48***		14.83***	
	(1.01)		(0.64)		(0.28)		(0.23)		(0.22)	
Entreprises births	−0.07		−0.30***		−0.59***		−1.29***		−3.53***	
	(0.07)		(0.06)		(0.03)		(0.03)		(0.03)	
Real GDP		17.17***		7.48***		−10.37***		4.72***		−3.37***
		(1.24)		(0.90)		(0.40)		(0.32)		(0.37)
Industrial production		−2.49***		−0.40		3.81***		−3.12***		6.60***
		(0.36)		(0.25)		(0.11)		(0.09)		(0.10)
AIC	16569.76	16364.82	13840.93	13819.14	69009.62	69945.37	175787.64	174640.69	220846.62	196586.05
BIC	22113.76	21740.82	19384.93	19195.14	76569.62	77337.37	183347.64	182032.69	227398.62	202970.05
Log Likelihood	-8251.88	-8150.41	-6887.47	-6877.57	-34459.81	-34928.68	-87848.82	-87276.35	-110384.31	-98255.02
	M3	M4	M3	M4	M3	M4	M3	M4	M3	M4
Unemployment rate		−1.65***		−0.46***		−0.05		−1.90***		−1.99***
		(0.10)		(0.08)		(0.04)		(0.04)		(0.04)
Inflation		8.73***		−2.63**		18.91***		16.26***		20.74***
		(1.32)		(0.95)		(0.41)		(0.32)		(0.35)
Enterprises births		−1.63***		0.09		−0.30***		−1.43***		−4.07***
		(0.10)		(0.08)		(0.04)		(0.03)		(0.03)
Real GDP		19.01***		8.48***		0.61		8.33***		9.81***
		(1.58)		(1.16)		(0.51)		(0.41)		(0.45)
Industrial production		−5.45***		0.41		−2.08***		−8.77***		1.14***
		(0.52)		(0.36)		(0.16)		(0.13)		(0.14)
10-years interest rate	−35.54***	−26.56***	−18.09***	−44.67***	−63.99***	−80.38***	−43.03***	−93.02***	23.37***	64.45***
	(3.70)	(5.12)	(2.69)	(3.50)	(1.19)	(1.64)	(0.95)	(1.46)	(0.92)	(1.41)
CAC40 Return	0.62***	1.51***	0.15***	−0.68***	0.16***	0.57***	−0.73***	1.01***	1.69***	1.09***
	(0.04)	(0.08)	(0.03)	(0.06)	(0.01)	(0.03)	(0.01)	(0.02)	(0.01)	(0.02)
CAC40 Volatility	−0.97***	2.16***	−0.44***	−0.41***	−0.23***	−0.31***	−0.99***	−0.51***	−0.32***	0.30***
	(0.12)	(0.10)	(0.06)	(0.06)	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)
AIC	16140.48	15264.81	14257.66	13331.84	68364.90	64948.56	174533.85	165414.31	204793.23	177417.80
BIC	21684.48	21648.81	19801.66	19715.84	75924.90	73348.56	182093.85	173814.31	211345.23	184809.80
Log Likelihood	-8037.24	-7594.41	-7095.83	-6627.92	-34137.45	-32424.28	-87221.92	-82657.16	-102357.62	-88664.90

Note: This table gives the estimated parameters and their standard errors in parentheses for the four migration models (cumulative logit model) with parallel assumption: *General macroeconomic conditions* (M1), *Direction of the economy* (M2), *Financial market conditions* (M3), *All variables* (M4). Each model can be compared with their log-likelihood, Akaike information criterion (AIC) and Bayesian information criterion (BIC). Standard errors are computed using the inverse Hessian matrix of the maximized log-likelihood. Threshold estimated coefficients μ_{ij} and the sector estimated coefficients are omitted. Transitions from states 1 and 2 to default and from state 5 to state 1 are assumed to be constant. Significance with the Wald-test: $^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$.

Table 7: Information criteria and performance for different migration models.

Model	Initial state 1	2	3	4	5
M0					
AIC	16900.29	14547.47	72012.62	182711.31	270704.55
BIC	21940.29	19587.47	79068.62	189767.31	276752.55
Log Likelihood	-8420.14	-7243.74	-35964.31	-91313.65	-135316.28
McFadden R^2	0.53	0.40	0.47	0.29	0.14
McFadden adjusted R^2	0.48	0.29	0.45	0.28	0.13
M1*					
AIC	15417.54	12875.74	60663.97	126999.99	157815.43
BIC	22977.54	20435.74	71247.97	137583.99	166887.43
Log Likelihood	-7663.77	-6392.87	-30268.99	-63437.00	-78853.71
McFadden R^2	0.59	0.46	0.56	0.52	0.51
McFadden adjusted R^2	0.53	0.36	0.54	0.51	0.50
M2*					
AIC	16081.48	13569.05	64006.56	107196.71	98392.70
BIC	22801.48	20289.05	73414.56	116604.71	106456.70
Log Likelihood	-8000.74	-6744.53	-31947.28	-53542.35	-49148.35
McFadden R^2	0.56	0.55	0.53	0.60	0.70
McFadden adjusted R^2	0.51	0.46	0.51	0.59	0.69
M3*					
AIC	14414.26	13514.79	62888.78	126951.08	125985.13
BIC	21974.26	21074.79	73472.78	137535.08	135057.13
Log Likelihood	-7162.13	-6712.39	-31381.39	-63412.54	-62938.57
McFadden R^2	0.63	0.51	0.54	0.52	0.61
McFadden adjusted R^2	0.57	0.43	0.53	0.51	0.60
M4*					
AIC	12592.34	11190.12	50472.16	85440.00	70664.23
BIC	24352.34	22950.12	66936.16	101904.00	84776.23
Log Likelihood	-6226.17	-5525.06	-25138.08	-42622.00	-35248.12
McFadden R^2	0.69	0.65	0.64	0.68	0.79
McFadden adjusted R^2	0.64	0.56	0.63	0.67	0.78
M5*					
AIC	14764.97	12996.75	59513.61	102565.07	94791.34
BIC	22324.97	20556.75	70097.61	113149.07	103863.34
Log Likelihood	-7337.49	-6453.38	-29693.81	-51219.54	-47341.67
McFadden R^2	0.61	0.54	0.57	0.61	0.71
McFadden adjusted R^2	0.55	0.45	0.55	0.61	0.70

Note: This table gives the log-likelihood, Akaike information criterion (AIC), Bayesian information criterion (BIC), McFadden R^2 and McFadden adjusted R^2 for the cumulative logit model without the parallel assumption for different specifications (M0, M1* - M5*). Transitions from states 1 and 2 to default and from state 5 to state 1 are assumed to be constant.

Table 8: Estimated parameters for the migration model M5* (Initial states 1 and 2).

Initial state	Transition state				
1	$< C$	< 2	< 3	< 4	< 5
Threshold	3.22***	3.60***	4.71***	7.20***	12.74***
	(0.04)	(0.05)	(0.06)	(0.10)	(0.22)
Agriculture	0.50***	0.66***	0.49***	0.77***	0.82***
	(0.02)	(0.03)	(0.03)	(0.05)	(0.09)
Finance/Real estate	0.48***	0.94***	1.01***	1.15***	1.30***
	(0.01)	(0.02)	(0.02)	(0.04)	(0.08)
Finished Product	−0.08***	−0.29***	−0.13***	−0.08**	−0.11*
	(0.01)	(0.01)	(0.02)	(0.02)	(0.04)
Raw Product	−0.04***	−0.19***	−0.25***	−0.14***	−0.06
	(0.01)	(0.01)	(0.01)	(0.02)	(0.03)
CAC40 Volatility	1.07***	0.55***	0.43**	−1.07***	−3.35***
	(0.09)	(0.10)	(0.13)	(0.22)	(0.43)
Industrial production	3.31***	3.05***	3.19***	1.39**	−5.57***
	(0.14)	(0.16)	(0.23)	(0.44)	(1.09)
10-years interest rate	−70.04***	−53.39***	−62.70***	−162.74***	−528.87***
	(3.90)	(4.52)	(5.76)	(9.82)	(20.79)
2	< 2	$< C$	< 3	< 4	< 5
Threshold	−5.29***	2.96***	3.21***	5.46***	8.87***
	(0.12)	(0.03)	(0.03)	(0.06)	(0.14)
Agriculture	−1.27***	0.42***	0.60***	0.23***	0.05
	(0.18)	(0.03)	(0.03)	(0.05)	(0.10)
Finance/Real estate	0.13**	0.09***	0.65***	0.46***	0.69***
	(0.05)	(0.01)	(0.02)	(0.03)	(0.07)
Finished Product	0.46***	−0.14***	−0.27***	−0.19***	−0.22***
	(0.03)	(0.01)	(0.01)	(0.02)	(0.04)
Raw Product	−0.06	−0.02*	−0.10***	−0.10***	−0.14***
	(0.04)	(0.01)	(0.01)	(0.02)	(0.03)
CAC40 Volatility	0.75**	−0.11*	−0.16**	−0.67***	−2.09***
	(0.23)	(0.05)	(0.06)	(0.10)	(0.23)
Industrial production	4.80***	1.49***	1.46***	0.64***	−3.92***
	(0.39)	(0.08)	(0.08)	(0.17)	(0.43)
10-years interest rate	−79.65***	−17.09***	−12.27***	−94.46***	−250.81***
	(11.17)	(2.79)	(3.08)	(5.89)	(12.81)

Note: This table provides the estimated threshold parameters μ_{ij}, the estimated parameters for sectorial dummies and macro factors related to transitions from states 1 and 2 obtained with the cumulative logit model M5* without the parallel assumption. Standard errors are computed using the inverse Hessian matrix of the maximized log-likelihood and are in parentheses. Transitions from states 1 and 2 to default are assumed to be constant. Significance with the Wald-test: $^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$.

Table 9: Estimated parameters for the migration model M5* (Initial states 3, 4 and 5).

Initial state			Transition state				
3	< 2	< 3	$< C$	< 4	< 5	$< P$	$< I$
Threshold	−9.33***	−6.32***	3.49***	4.44***	6.42***	7.05***	11.30***
	(0.25)	(0.05)	(0.01)	(0.02)	(0.03)	(0.07)	(1.05)
Agriculture	−1.69***	−2.12***	0.81***	1.33***	1.31***	1.64***	16.28
	(0.26)	(0.07)	(0.01)	(0.01)	(0.02)	(0.08)	(697.65)
Finance/Real estate	0.68***	−0.15***	0.05***	0.57***	0.60***	0.52***	2.25*
	(0.09)	(0.03)	(0.01)	(0.01)	(0.02)	(0.05)	(1.01)
Finished Product	0.72***	0.87***	−0.11***	−0.43***	−0.28***	−0.61***	−0.02
	(0.07)	(0.02)	(0.00)	(0.01)	(0.01)	(0.02)	(0.28)
Raw Product	−0.21**	0.06***	0.12***	−0.09***	0.00	−0.46***	1.10**
	(0.08)	(0.02)	(0.00)	(0.00)	(0.01)	(0.02)	(0.32)
CAC40 Volatility	0.12	0.39***	−0.14***	−0.58***	−1.32***	−0.16	2.27
	(0.49)	(0.10)	(0.02)	(0.03)	(0.05)	(0.13)	(2.04)
Industrial production	2.67**	1.16***	0.93***	−0.23***	−3.89***	0.51**	5.56*
	(0.77)	(0.15)	(0.03)	(0.04)	(0.08)	(0.19)	(2.60)
10-years interest rate	4.61	8.51	−66.66***	−86.33***	−176.31***	−22.23**	−14.00
	(24.08)	(5.23)	(1.23)	(1.58)	(2.62)	(7.09)	(103.19)
4	< 2	< 3	< 4	$< C$	< 5	$< P$	$< I$
Threshold	−12.13***	−8.94***	−4.54***	3.13***	4.34***	6.97***	11.44***
	(0.68)	(0.18)	(0.03)	(0.01)	(0.02)	(0.04)	(0.49)
Agriculture	−1.52*	−2.05***	−1.81***	1.01***	1.54***	1.81***	3.16**
	(0.69)	(0.25)	(0.03)	(0.01)	(0.02)	(0.06)	(1.03)
Finance/Real estate	0.97***	0.56***	−0.39***	0.10***	0.83***	0.49***	2.16***
	(0.21)	(0.07)	(0.02)	(0.01)	(0.01)	(0.03)	(0.59)
Finished Product	0.27	0.47***	0.58***	0.01	−0.28***	−0.50***	−0.17
	(0.20)	(0.05)	(0.01)	(0.00)	(0.01)	(0.02)	(0.16)
Raw Product	−0.50*	−0.33***	0.12***	0.05***	−0.17***	−0.60***	0.39**
	(0.20)	(0.05)	(0.01)	(0.00)	(0.00)	(0.01)	(0.14)
CAC40 Volatility	−1.34	−0.79*	−2.71***	−0.47***	−1.16***	−0.58***	1.53
	(1.32)	(0.33)	(0.04)	(0.02)	(0.03)	(0.07)	(0.83)
Industrial production	1.52	−4.10***	−12.31***	−0.09**	−3.48***	−0.24*	2.48*
	(1.97)	(0.44)	(0.05)	(0.03)	(0.04)	(0.11)	(1.13)
10-years interest rate	126.27*	31.76	11.11***	−48.52***	−60.78***	−100.42***	−176.56***
	(63.99)	(18.21)	(2.85)	(1.02)	(1.48)	(4.20)	(45.85)
5	< 3	< 4	< 5	$< C$	$< P$	$< I$	
Threshold	−11.56***	−7.15***	−2.65***	2.07***	5.11***	9.94***	
	(0.47)	(0.09)	(0.02)	(0.01)	(0.05)	(0.35)	
Agriculture	−1.03	−0.17**	−0.85***	0.60***	1.19***	2.71**	
	(0.52)	(0.06)	(0.02)	(0.01)	(0.08)	(0.97)	
Finance/Real estate	0.94***	0.18***	−0.32***	0.11***	0.49***	0.01	
	(0.16)	(0.04)	(0.01)	(0.01)	(0.04)	(0.20)	
Finished Product	0.34**	0.25***	0.36***	0.08***	−0.47***	−0.22	
	(0.13)	(0.02)	(0.01)	(0.01)	(0.02)	(0.12)	
Raw Product	−0.19	−0.08***	0.15***	0.14***	−0.40***	0.56***	
	(0.12)	(0.02)	(0.00)	(0.00)	(0.01)	(0.11)	
CAC40 Volatility	−2.56**	−4.13***	−1.31***	−0.40***	−0.23**	−0.18	
	(0.92)	(0.17)	(0.03)	(0.02)	(0.09)	(0.58)	
Industrial production	−2.19	−8.76***	−4.54***	8.05***	−0.47***	3.40***	
	(1.44)	(0.23)	(0.04)	(0.03)	(0.13)	(0.89)	
10-years interest rate	231.79***	142.29***	−10.09***	39.90***	39.83***	−78.45*	
	(43.49)	(8.14)	(1.56)	(1.20)	(4.51)	(34.18)	

Note: This table provides the estimated threshold parameters μ_{ij}, the estimated parameters for sectorial dummies and macro factors related to transitions from states 3, 4 and 5 obtained with the cumulative logit model M5* without the parallel assumption. Standard errors are computed using the inverse Hessian matrix of the maximized log-likelihood and are in parentheses. Transitions from state 5 to state 1 are assumed to be constant. Significance with the Wald-test: $^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$.

Table 10: Out-of-sample results.

	Upgrades		Downgrades		Defaults		Cancellations		Upgrades		Downgrades		Defaults		Cancellations	
Date	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE
M0									M4*							
2009:3	9.45e-02	2.81e-01	4.33e-03	7.14e-03	2.94e-03	3.60e-03	2.52e-02	4.56e-02	1.49e-02	3.49e-02	5.00e-03	8.20e-03	4.04e-04	6.46e-04	1.85e-02	3.02e-02
2009:4	9.13e-02	2.70e-01	3.32e-03	4.78e-03	2.96e-03	3.63e-03	2.23e-02	3.77e-02	4.57e-03	1.07e-02	4.82e-03	7.13e-03	4.13e-04	8.35e-04	2.38e-02	3.80e-02
2010:1	9.86e-02	2.92e-01	3.81e-03	6.75e-03	2.93e-03	3.61e-03	3.03e-02	5.06e-02	1.61e-03	3.09e-03	4.74e-03	7.41e-03	5.37e-04	1.32e-03	1.98e-02	2.10e-02
2010:2	9.80e-02	2.91e-01	4.61e-03	7.77e-03	2.94e-03	3.63e-03	2.92e-02	4.83e-02	1.30e-02	2.87e-02	5.69e-03	8.94e-03	5.67e-04	1.08e-03	3.14e-02	3.61e-02
2010:3	9.84e-02	2.92e-01	4.41e-03	6.23e-03	2.93e-03	3.64e-03	2.79e-02	4.71e-02	3.11e-03	5.61e-03	4.64e-03	7.62e-03	5.55e-04	1.27e-03	2.18e-02	2.79e-02
2010:4	9.66e-02	2.86e-01	4.81e-03	6.56e-03	2.94e-03	3.66e-03	2.90e-02	4.86e-02	2.53e-03	5.60e-03	3.70e-03	4.83e-03	3.66e-04	7.26e-04	1.71e-02	2.67e-02
2011:1	9.52e-02	2.83e-01	3.55e-03	5.11e-03	2.94e-03	3.67e-03	2.49e-02	3.86e-02	1.41e-03	2.93e-03	3.27e-03	4.88e-03	4.08e-04	8.41e-04	1.20e-02	1.35e-02
2011:2	9.45e-02	2.78e-01	3.57e-03	5.09e-03	2.95e-03	3.70e-03	2.12e-02	3.23e-02	2.48e-03	4.81e-03	3.38e-03	5.28e-03	3.37e-04	6.66e-04	9.70e-03	1.23e-02
2011:3	9.56e-02	2.83e-01	2.96e-03	4.16e-03	2.93e-03	3.68e-03	2.29e-02	3.61e-02	1.86e-03	3.26e-03	4.30e-03	7.05e-03	8.37e-04	1.64e-03	1.86e-02	3.07e-02
2011:4	9.27e-02	2.76e-01	5.71e-03	7.55e-03	2.88e-03	3.63e-03	1.65e-02	2.81e-02	2.31e-03	4.90e-03	4.96e-03	7.17e-03	5.59e-04	1.07e-03	1.50e-02	2.40e-02
2012:1	9.29e-02	2.76e-01	4.56e-03	6.14e-03	2.84e-03	3.58e-03	1.86e-02	2.98e-02	3.32e-03	7.67e-03	4.36e-03	6.27e-03	6.34e-04	1.34e-03	2.10e-02	2.71e-02
2012:2	9.45e-02	2.80e-01	2.84e-03	3.85e-03	2.81e-03	3.54e-03	1.77e-02	2.79e-02	2.06e-03	4.92e-03	3.03e-03	4.13e-03	5.86e-04	1.23e-03	1.17e-02	1.80e-02
M4									M5*							
2009:3	4.40e-02	1.19e-01	5.30e-03	8.35e-03	9.48e-04	2.09e-03	1.76e-02	2.82e-02	4.53e-03	1.18e-02	4.36e-03	6.91e-03	3.88e-04	6.67e-04	1.95e-02	3.30e-02
2009:4	2.21e-02	6.53e-02	4.79e-03	7.45e-03	1.47e-03	2.78e-03	2.43e-02	3.60e-02	1.94e-03	4.93e-03	3.66e-03	5.76e-03	7.96e-04	1.37e-03	1.55e-02	2.37e-02
2010:1	7.60e-03	2.10e-02	3.65e-03	5.90e-03	8.07e-04	1.71e-03	1.56e-02	1.86e-02	4.09e-03	1.06e-02	5.29e-03	8.65e-03	6.53e-04	1.21e-03	6.13e-03	8.09e-03
2010:2	2.26e-03	5.90e-03	6.97e-03	1.10e-02	5.26e-04	1.19e-03	4.00e-02	7.41e-02	3.15e-03	8.27e-03	5.98e-03	9.33e-03	5.78e-04	1.20e-03	9.60e-03	1.23e-02
2010:3	2.11e-03	4.36e-03	6.49e-03	1.07e-02	3.90e-04	7.29e-04	3.05e-02	4.87e-02	3.04e-03	8.48e-03	3.71e-03	5.90e-03	3.54e-04	7.92e-04	6.62e-03	8.72e-03
2010:4	8.32e-03	2.12e-02	4.52e-03	6.25e-03	7.05e-04	1.71e-03	1.31e-02	2.21e-02	2.30e-03	6.26e-03	3.27e-03	4.74e-03	2.82e-04	5.17e-04	1.58e-02	2.48e-02
2011:1	4.06e-03	9.38e-03	3.12e-03	4.44e-03	6.53e-04	1.45e-03	9.31e-03	1.12e-02	9.07e-04	1.85e-03	2.69e-03	3.84e-03	3.19e-04	6.95e-04	9.40e-03	1.09e-02
2011:2	3.58e-03	6.75e-03	2.74e-03	4.32e-03	4.41e-04	9.39e-04	7.59e-03	9.62e-03	3.26e-03	6.22e-03	2.43e-03	3.77e-03	3.46e-04	7.62e-04	7.89e-03	9.98e-03
2011:3	2.47e-03	4.46e-03	3.62e-03	6.28e-03	4.28e-04	9.20e-04	1.97e-02	3.43e-02	1.62e-03	2.98e-03	3.54e-03	5.87e-03	6.58e-04	1.42e-03	1.40e-02	2.03e-02
2011:4	3.12e-03	7.22e-03	5.08e-03	6.73e-03	3.09e-04	6.75e-04	1.39e-02	2.39e-02	1.40e-03	2.90e-03	5.11e-03	7.51e-03	3.18e-04	6.28e-04	9.62e-03	1.48e-02
2012:1	2.81e-03	7.04e-03	5.16e-03	7.08e-03	6.02e-04	1.22e-03	1.92e-02	2.97e-02	1.81e-03	3.48e-03	3.57e-03	5.33e-03	4.22e-04	8.07e-04	9.45e-03	1.32e-02
2012:2	2.90e-03	6.31e-03	2.96e-03	4.14e-03	3.23e-04	6.80e-04	1.22e-02	1.97e-02	2.28e-03	5.07e-03	2.67e-03	4.00e-03	4.53e-04	9.76e-04	1.08e-02	1.58e-02

Note: This table gives the out-of-sample performance for models M0, M4, M4* and M5. The performance is measured by the Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) during the period $[2009:3, 2012:2]$ for upgrades, downgrades, defaults and cancellations.

Table 11: Results of ADF tests for macroeconomic factors.

Factor	Deterministic terms	Lags	Test value	Critical values 1%	5%	10%
$UR\,(t)$	constant, trend	5	-0.44	-4.04	-3.45	-3.15
$\Delta UR\,(t)$	constant	4	-4.09	-3.51	-2.89	-2.58
$IPC\,(t)$	constant, trend	9	-2.18	-4.04	-3.45	-3.15
$\Delta IPC\,(t)$	constant	8	-2.181	-3.51	-2.89	-2.58
$\Delta^2 IPC\,(t)$	constant	7	-5.34	-3.51	-2.89	-2.58
$GDP\,(t)$	constant, trend	3	-1.19	-4.04	-3.45	-3.15
$\Delta GDP\,(t)$	constant	2	-3.51	-3.51	-2.89	-2.58
$IP\,(t)$	constant, trend	2	-1.97	-4.04	-3.45	-3.15
$\Delta IP\,(t)$	constant	1	-4.78	-3.51	-2.89	-2.58
$r\,(t)$	constant, trend	1	-3.71	-4.04	-3.45	-3.15
$\Delta r\,(t)$	constant	1	-6.08	-3.51	-2.89	-2.58
$CAC\,(t)$	constant, trend	1	-1.62	-4.04	-3.45	-3.15
$\Delta CAC\,(t)$	constant	1	-5.74	-3.51	-2.89	-2.58
$\sigma\,(t)$	constant, trend	1	-4.37	-4.04	-3.45	-3.15
$\Delta\sigma\,(t)$	constant	8	-3.98	-3.51	-2.89	-2.58

Note: The Augmented Dickey Fuller (ADF) tests are calculated on univariate AR(p) model with order p based on AIC order selection.

Table 12: Johansen cointegration tests

Trace statistic			Critical values		
$\mathcal{H}_0$	$\mathcal{H}_1$	Test statistic	10%	5%	1%
$r = 0$	$r \geq 1$	220.18	141.01	146.76	158.49
$r \leq 1$	$r \geq 2$	155.77	110.42	114.90	124.75
$r \leq 2$	$r \geq 3$	107.24	83.20	87.31	96.58
$r \leq 3$	$r \geq 4$	65.33	59.14	62.99	70.05
$r \leq 4$	$r \geq 5$	34.73	39.06	42.44	48.45
Maximum eigenvalue statistic			Critical values		
$\mathcal{H}_0$	$\mathcal{H}_1$	Test statistic	10%	5%	1%
$r = 0$	$r \geq 1$	64.41	46.32	49.42	54.71
$r \leq 1$	$r \geq 2$	48.54	40.91	43.97	49.5
$r \leq 2$	$r \geq 3$	41.90	34.75	37.52	42.36
$r \leq 3$	$r \geq 4$	30.60	29.12	31.46	36.65
$r \leq 4$	$r \geq 5$	18.00	23.11	25.54	30.34

Note: This table presents the Johansen cointegration tests with trace statistic and maximum eigenvalue statistic for VAR(2) model with unrestricted constant and restricted trend.

Table 13: VECM parameters.

Long-run parameters $\widehat{\boldsymbol{\beta}}$	β_1	β_2	β_3	β_4			
$GDP\,(t-1)$	1.000	0.000	0.000	0.000			
	–	–	–	–			
$IP\,(t-1)$	0.000	1.000	0.000	0.000			
	–	–	–	–			
$UR\,(t-1)$	0.000	0.000	1.000	0.000			
	–	–	–	–			
$CAC\,(t-1)$	0.000	0.000	0.000	1.000			
	–	–	–	–			
$\Delta CPI\,(t-1)$	−0.840	−32.150	−9.696	207.418			
	(0.065)	(0.110)	(0.002)	(0.000)			
$r\,(t-1)$	21.612	16.199	−133.544	409.110			
	(1.837)	(3.120)	(0.060)	(0.000)			
$\sigma\,(t-1)$	0.327	−0.055	−1.813	4.911			
	(0.291)	(0.494)	(0.009)	(0.000)			
Trend	−0.001	0.002	−0.015	0.050			
	(0.229)	(0.388)	(0.007)	(0.000)			
Loading parameters $\widehat{\boldsymbol{\alpha}}$	$\Delta GDP\,(t)$	$\Delta IP\,(t)$	$\Delta UR\,(t)$	$\Delta CAC\,(t)$	$\Delta^2 CPI\,(t)$	$\Delta r\,(t)$	$\Delta\sigma\,(t)$
α_1	0.148	0.428	−1.012	3.841	−0.075	0.023	−5.455
	(0.067)	(0.240)	(0.319)	(1.937)	(0.066)	(0.015)	(1.073)
α_2	−0.030	−0.125	0.042	−0.744	0.019	−0.007	1.262
	(0.013)	(0.047)	(0.062)	(0.377)	(0.013)	(0.003)	(0.209)
α_3	0.018	0.028	−0.187	0.349	−0.028	0.001	−0.329
	(0.009)	(0.033)	(0.044)	(0.267)	(0.009)	(0.002)	(0.148)
α_4	−0.004	−0.016	−0.010	−0.109	−0.005	−0.001	0.149
	(0.002)	(0.006)	(0.007)	(0.045)	(0.002)	(0.000)	(0.025)
$\widehat{\boldsymbol{\Psi}}$-parameters $\widehat{\boldsymbol{\alpha}}$	$\Delta GDP\,(t)$	$\Delta IP\,(t)$	$\Delta UR\,(t)$	$\Delta CAC\,(t)$	$\Delta^2 CPI\,(t)$	$\Delta r\,(t)$	$\Delta\sigma\,(t)$
Constant	−1.733	−4.762	13.233	−45.048	0.973	−0.256	63.173
	(0.829)	(2.955)	(3.927)	(23.853)	(0.810)	(0.180)	(13.210)
$\Delta GDP\,(t-1)$	−0.167	0.497	0.576	−4.575	0.278	−0.011	4.357
	(0.217)	(0.774)	(1.029)	(6.248)	(0.212)	(0.047)	(3.460)
$\Delta IP\,(t-1)$	0.123	0.410	−0.638	3.048	0.038	0.009	−2.098
	(0.062)	(0.222)	(0.295)	(1.794)	(0.061)	(0.014)	(0.994)
$\Delta UR\,(t-1)$	−0.038	−0.030	0.122	0.115	−0.006	−0.009	0.139
	(0.028)	(0.100)	(0.133)	(0.807)	(0.027)	(0.006)	(0.447)
$\Delta CAC\,(t-1)$	0.006	0.013	0.019	−0.020	0.008	−0.001	−0.280
	(0.005)	(0.016)	(0.022)	(0.131)	(0.004)	(0.001)	(0.072)
$\Delta^2 CPI\,(t-1)$	0.175	0.201	0.015	6.783	0.234	0.092	−1.197
	(0.133)	(0.475)	(0.631)	(3.832)	(0.130)	(0.029)	(2.122)
$\Delta r\,(t-1)$	0.349	−0.597	−3.375	4.496	−0.145	0.393	−4.497
	(0.548)	(1.953)	(2.596)	(15.765)	(0.535)	(0.119)	(8.730)
$\Delta\sigma\,(t-1)$	0.007	0.019	−0.031	0.008	0.019	0.002	−0.130
	(0.007)	(0.026)	(0.034)	(0.209)	(0.007)	(0.002)	(0.116)

Note: This table gives the VECM parameters with the standard errors in parentheses: the loading parameter $\boldsymbol{\alpha}$, long-run parameters $\boldsymbol{\beta}$, the unrestricted constant, the restricted trend and the $\boldsymbol{\Psi}$-matrix of parameters.

Table 14: Mean, VaR and CTE for different shocks and time horizons.

	T=2 quarters			T=4 quarters			T=8 quarters			T=12 quarters		
	Mean	VaR 99.5%	CTE 99.5%	Mean	VaR 99.5%	CTE 99.5%	Mean	VaR 99.5%	CTE 99.5%	Mean	VaR 99.5%	CTE 99.5%
Number of defaults												
M0	316	362	369	284	329	335	229	269	275	188	224	229
M5*	269	315	321	240	298	307	211	280	290	182	244	253
M5* - Shock on $GDP\,(t)$	270	316	322	230	285	293	199	268	278	179	241	250
M5* - Shock on $IP\,(t)$	270	317	323	228	282	290	198	266	277	179	241	250
M5* - Shock on $r\,(t)$	280	325	332	259	318	327	213	285	296	177	240	249
M5* - Shock on $\sigma\,(t)$	278	324	331	242	300	309	210	279	289	182	244	254
Number of cancellations												
M0	4,472	4,641	4,661	3,941	4,100	4,120	3,128	3,270	3,287	2,537	2,666	2,683
M5*	4,136	4,806	4,905	3,347	4,459	4,615	2,480	3,919	4,136	2,354	3,808	4,051
M5* - Shock on $GDP\,(t)$	4,652	5,098	5,164	3,838	4,898	5,042	2,226	3,415	3,609	2,262	3,631	3,861
M5* - Shock on $IP\,(t)$	4,768	4,964	4,989	3,876	4,930	5,090	2,197	3,335	3,515	2,264	3,614	3,827
M5* - Shock on $r\,(t)$	4,108	4,737	4,824	3,622	4,758	4,915	2,765	4,254	4,513	2,278	3,649	3,857
M5* - Shock on $\sigma\,(t)$	4,430	5,069	5,154	3,441	4,553	4,711	2,467	3,884	4,103	2,336	3,757	3,992

Note: This table reports the empirical mean, the value-at-risk (VaR) and the conditional tail expectation (CTE) related to the simulated distribution (100,000 replications) of the numbers of defaults and cancellations observed at times $T = 2$ quarters, 4 quarters, 8 quarters and 12 quarters. The results are separated by scenarios: random shocks for the model M0 and M5*, a -2.58 one standard deviation shock to the log real GDP, a -2.58σ shock to the log industrial production, a $+2.58\sigma$ shock to the 10-years government bond rate and a $+2.58\sigma$ shock to the CAC40 volatility.