CREDIBILITY FOR TREATY REINSURANCE EXCESS PRICING

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ABSTRACT:

Treaty reinsurance excess pricing ideally consists of both an exposure rating and an experience rating. The problem is how to put them together to reach a final rate.

This paper uses Hans Bühlmann's 1967 least squares credibility formulation for computing the final rate. We extend Erwin Straub's 1971 excess credibility model by considering uncertainty in the excess claims probability in addition to the uncertainty in the ground-up claim count expectation. We tie together excess credibilities for various attachment points into a consistent model utilizing a gamma/Poisson model for the ground-up number of claims. We discuss the a priori information available for excess exposure rating in the US casualty market and its problems. Likewise, we discuss the problems inherent in the normal reinsurance excess experience rating methodology. We discuss the question of subjectivity with regard to the information available in various actual pricing situations, and present a questionaire designed to elicit and codify an underwriter's judgement leading to an appropriate credibility structure. This paper is written for reinsurance practitioners.

1. INTRODUCTION

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This paper uses Hans Bühlmann's 1967 least squares credibility formulation for computing the final rate. We extend Erwin Straub's 1971 excess credibility model by considering uncertainty in the excess claims probability in addition to the uncertainty in the ground-up claim count expectation. We tie together excess credibilities for various attachment points into a consistent model utilizing a gamma/Poisson model for the ground-up number of claims. We discuss the a priori information available for excess exposure rating in the US casualty market and its problems. Likewise, we discuss the problems inherent in the normal reinsurance excess experience rating methodology. We discuss the question of subjectivity with regard to the information available in various actual pricing situations, and present a questionaire designed to elicit and codify an underwriter's judgement leading to an appropriate credibility structure. This paper is written for reinsurance practitioners.

2. STATEMENT OF THE PROBLEM AND PRELIMINARIES

Before getting to the mathematics, we will briefly discuss the problem, desirable characteristics of a solution, information availability, and various other preliminaries.

2.1 Treaty reinsurance excess exposure and experience rating

When pricing reinsurance excess coverage, two main methods are available to the actuary/underwriter. In each method, an estimate is made of the future reinsurance excess loss cost, which is then divided by an estimate of the future subject premium (primary premium for the underlying exposure which is subject to the reinsurance coverage) to obtain a loss cost rate. This loss cost rate is adjusted for expense and profit loadings to obtain the final flat rate, which may be adjusted further by retrospective rating.

One pricing method, generally called exposure rating, combines information on the reinsured's exposure by category of business and layer with the reinsurer's a priori loss estimates for such categories and layers. Usually the reinsurer's a priori loss estimates are based upon either a segment of their book of business or upon their interpretation of rating bureau statistics which combine the experience of many primary companies. This is analogous to the manual rating of primary business.

A second pricing method, generally called experience rating, relies upon an analysis of the history of the subject exposure, premiums and losses over the last several years and attempts to estimate expected losses or an expected loss cost rate for the future coverage year. This is analogous to the loss rating of primary business.

At a minimum, both exposure rating and experience rating provide estimates of expected premiums and losses or of the expected loss cost rate. A more sophisticated analysis will include separate estimates of the first and second moments of the claim counts and claim amounts. In the most sophisticated analyses, claim severity curves and aggregate loss distributions will be estimated.

If both exposure and experience rates have been successfully estimated and they differ, the actuary/underwriter is then faced with the question of which to believe. In some instances, one method is clearly superior to the other and the answer is obvious. For example, if the subject book of business has changed dramatically over the last several years, the experience rate may be meaningless. On the other hand, if the particular future subject exposure is very different from the general type of exposure used in the calculation of the exposure rate, then this rate may not be accurate for the particular case. Most situations, however, lie between these two extremes; thus a technical credibility procedure is desirable.

In its simplest form the basic credibility question is: how much weight should be given to an individual ceding company's experience? While much has been published on the topic of primary credibility, little has been written on the topic of excess credibility. We are extending the work of Erwin Straub discussed in his 1971 ASTIN paper.

2.2 Desirable characteristics for an excess credibility model

Our goal in this paper is to present a simple excess credibility actuarial model with the following characteristics:

1. It provides a credibility estimate of the expected claim count.

2. It produces consistent answers going from one excess attachment point to another.

3. It allows subjective reflection of the "goodness" of the prior loss cost estimates relative to the experience loss cost estimates.

4. It is simple and easy to explain to actuaries, underwriters and cedants, can be generally accepted by them, and is also supported by enough actuarial literature and common sense.

The need for property 3 may be better understood after reading about the problems with our a priori rates to be discussed in Section 2.3 and the problems with our experience rates to be discussed in Section 2.4. The need for property 4 is obvious to any practitioner.

Further work is necessary to incorporate claim severity into the model presented here and to perhaps replace some of the subjective judgment with a more sophisticated model. We hope that some readers may find this interesting to pursue.

2.3 *A priori* information for excess exposure rating

In the United States, we are fortunate to have available the huge databases of Insurance Services Office (ISO) and National Council on Compensation Insurance (NCCI). Rating information derived from these databases is widely used for pricing reinsurance excess coverage.

We will very briefly describe the ISO database and the rating information derived from it. ISO collects premium and loss information on an individual transaction basis from the majority of US insurers. The following table displays annual claim counts reported to ISO for several important categories of casualty business.

(2.3.1) ISO Individual claims database for certain casualty lines

Category	Approximate annual claim count
Commercial automobile liability	700,000
Premises/operations liability	300,000
Products liability	35,000
Medical malpractice liability	6,000

ISO actuaries annually review and publish both primary and excess pricing information. In particular, the published excess pricing information for most casualty lines includes Pareto parameters for curves fit to inflation-trended and developed (to settlement value) individual claims data.

A few problems with the ISO casualty claims database, claims severity curves, and therefore increased limits factors, should be mentioned here:

(2.3.2) Problems with ISO increased limits factors

1. The ISO databases include only claims occurring on primary policies; claims on excess and umbrella policies are not included.

Many U.S. actuaries, therefore, believe that while the ISO claim severity curves are very reliable up to $500,000 or $1,000,000, there is greater uncertainty at higher limits where most of the coverage sold is via either excess or umbrella policies.

2. ISO publishes maximum likelihood estimates of the Pareto parameters, but does not yet estimate any measure of the variability of these estimates.

In addition to parameter uncertainty arising from the MLE procedure, there is uncertainty arising from the subjective judgement used in selecting claims inflation trends, individual claims development (to settlement values), and truncation points. Also there are some problems with data quality.

3. There is a debate among U.S. actuaries as to whether the Pareto model is too severe for higher limits.

Although a lower-truncated 2-parameter Pareto model describes the claims data fairly well up to $1,000,000, there is some thought that its tail may be too severe above this point. Of course, it may instead be too low.

4. The size of the ISO database varies widely by line-of-business, as seen in Table (2.3.1).

For some specialty lines-of-business, the particular reinsured under consideration may have a larger database than ISO.

5. The ISO increased limits factors do not include a charge for allocated loss adjustment expense which may be shared by the excess reinsurer.

Most reinsurance excess contracts cover a share of the allocated loss adjustment expense on excess claims, or add the allocated to the indemnity loss on each claim before the application of the reinsurance attachment point. Thus ISO increased limits factors must be modified to give a correct excess exposure rate.

We have not yet investigated the ISO property claims database to see whether or not it may be useful for reinsurance excess pricing.

NCCI also captures individual claims data to estimate excess loss factors (ELF) which are useful for excess pricing. These factors are calculated for use in retrospective rating plans and vary by state and by workers compensation hazard group. Many of the problems discussed with respect to ISO severity curves and increased limits factors are also relevant for the NCCI claim severity curves and ELFs.

Despite these problems, the ISO and NCCI information is the best generally available. As long as we are aware of their inherent problems, we can use it for pricing with appropriate adjustments.

Our exposure rating methodology is based on this ISO and NCCI information, together with general industry rate-level information by line and a methodology for predicting individual company results. Our general approach is to utilize this information together with judgement to specify our prior distributions. Then the submitted experience data for a particular pricing situation is used to modify these priors.

2.4 Excess experience rating

Except where the primary company is writing a new type of business, a competent reinsurer will require historical premium and loss information for the business to be ceded. At a minimum, this historical information will include total subject premium, total ceded premium, and total ceded losses for many years prior. Sometimes more refined experience rating information is available, such as premium and loss information by type of business, loss development information, premium and loss trend and rate level information, data on deductible and limits shifts, etc. For excess reinsurance, it is desirable to have detailed information on each large loss as of annual evaluations.

The excess experience rating procedure revalues all premium and loss information to future coverage level, adjusts for differences in exposure by year and estimates expected losses for the coverage period to be priced. Many reinsurers rely heavily on experience rating. Especially outside the US, without the support of the large databases of ISO and NCCI, a reinsurer has little choice but to base their excess pricing upon experience rating.

It is important to understand that, even under the best of circumstances when the reinsurer is supplied with an impressive array of good data, the excess experience rate contains much uncertainty. Some of the major areas of uncertainty are listed below:

(2.4.1) Sources of uncertainty in the excess experience rate:

1. IBNR claim count
2. individual claim development
3. loss inflation trend - count and severity
4. exposure trend and actual rate level changes
5. changes in the mix of business
6. changes in the policy limits profile

You may be able to think of others.

2.5 A comparison of uncertainties

As discussed in the last two sections, both exposure rating and experience rating contain many areas of uncertainty. It is important to realize that individual submissions vary dramatically with respect to which method contains more uncertainty. Any complete credibility model must account for these uncertainties. The best way to do so is to codify the subjective judgement of the actuary/underwriter. As will be seen in Section 7, our model is built around a questionaire which structures the underwriter's judgement and thus leads to a coherent structure for the credibility weights.

2.6 Mathematical background and notation

The real item of interest when pricing a reinsurance excess cover is the random variable for the ceded aggregate loss. We would like to know its cumulative distribution function (cdf). Using standard risk theoretic models, this cdf is determined from the cdfs for the excess claim counts and excess claim severities. However, in this paper we restrict ourselves to consideration of claim counts, both ground-up and excess. And we will assume that all claims values are i.i.d. We will use the following notation.

(2.6.1) Notation:

N	random variable for number of ground-up claims	
d	excess attachment point	
$N(d)$	random variable for number of claims excess of attachment point d	
X	random variable for the ground-up amount of any given claim, with parameter μ	
$q(d)$	$\text{Prob}[X > d]$	probability that any given claim will exceed d
F_Y	cumulative distribution function of the rv Y	
f_Y	density or probability function of the rv Y	
$E[Y]$	expected value of the rv Y	

Var[Y]	variance of the rv Y	
CV[Y]	$\{Var[Y]\}^{1/2}/E[Y]$	coefficient of variation of the rv Y
P(Θ)	Poisson distribution with parameter Θ	
G(A)	gamma function of A (for integers, G(A)=(A-1)!)	
G(A,B)	gamma distribution with parameter (A,B)	
NB(k,p)	negative binomial distribution with parameter (k,p)	
Z	credibility value	
k	credibility constant, as in Z = m/(m+k)	

3. THE PRIMARY PRICING SITUATION

We start by reviewing the well known results for the gamma/Poisson (negative binomial) model for ground-up claim count. The model presented in this section was developed by (among others) Bailey (1950), Dropkin (1959), Mayerson (1964), and Bühlmann (1967).

3.1 The gamma/Poisson claim count model

Assume that we can describe the given underlying exposure by a parameter Θ such that N, given Θ, has a Poisson cdf:

(3.1.1) $N|\Theta \sim P(\Theta)$ (Poisson)

Then, the following results are well known:

(3.1.2) $f_N(n|\Theta) = \Theta^n \exp(-\Theta)/n!$ $n = 0, 1, 2, 3, \ldots.$

(3.1.3) $E[N|\Theta] = \Theta$

(3.1.4) $Var[N|\Theta] = \Theta$

(3.1.5) $CV[N|\Theta] = \Theta^{-1/2}$

Assume that Θ has a structure function given by a gamma distribution:

(3.1.6) $\Theta \sim G(A,B) \quad A > 0,\ B > 0$ (gamma)

with $f(\Theta) = B*\{\exp(-B\Theta)\}*(B\Theta)^{A-1}/G(A) \quad 0 \le \Theta < \infty$

Then the following facts are well known (see Hossack, Pollard and Zehnwirth (1983), p.86f):

(3.1.7) $E[\Theta] = A/B$

(3.1.8) $Var[\Theta] = A/B^2$

(3.1.9) $CV[\Theta] = A^{-1/2}$

Note that (3.1.9) says that the coefficient of variation of Θ depends only upon the gamma parameter component A. Thus A alone determines the relative dispersion of the distribution of Θ: the smaller A is, the more dispersed is the distribution of Θ. Thus the parameter component A is the key for expressing our relative *a priori* belief in the goodness of the primary rates for the particular excess rating situation.

3.2 The negative binomial claim count model

The probability function for the negative binomial distribution with parameter (k,p) is given by (ibid, p.96f):

(3.2.1) $f(x) = \{G(k+x)/x!G(k)\}p^k(1-p)^x$

for $x = 0, 1, 2, 3, \ldots$

with mean $= k(1-p)/p$ and variance $= (k(1-p)/p)(1/p)$

It has been proven (ibid, p.98) that if $N|\Theta$ is Poisson-distributed with parameter Θ, and Θ is gamma distributed with parameter (A,B), then the unconditional distribution of N is negative binomial with parameter (A, B/(1+B)):

(3.2.2) $N \sim NB(A, B/(1+B))$ (negative binomial)

It then follows that:

(3.2.3) $f_N(n) = \{G(A+n)/n!G(A)\}p^A(1-p)^n$

for $n = 0, 1, 2, 3, \ldots$

with $p = B/(1+B)$

(3.2.4) $E[N] = A/B$

(3.2.5) $Var[N] = (A/B)*((1+B)/B)$

(3.2.6) $CV[N] = \{(1+B)/A\}^{1/2}$

3.3 Historical justification for a negative binomial claim count model

The first use of the negative binomial for the distribution of claim counts that we know of was in a series of 1959-62 papers by Dropkin (1959), Harwayne (1959), Hewitt (1960), and Simon (1960 and 1962) in the Proceedings of the Casualty Actuarial Society. Although some of Dropkin's arguments in favor of the negative binomial were flawed, it is still viewed by many as a relatively sophisticated model of claim count (see the discussions of Dropkin's paper in the 1987 PCAS).

3.4 A primary claim count exposure rate

Assume that the exposure (manual) rating process determines an estimate (a,b) of the parameter (A,B). This determines our prior distribution of Θ, recasting formulas (3.1.6) through (3.1.9):

(3.4.1) $\Theta \sim G(a,b)$

(3.4.2) $E[\Theta] = E[N] = a/b$

(3.4.3) $Var[\Theta] = a/b^2$

(3.4.4) $CV[\Theta] = a^{-1/2}$

(3.4.5) $CV[N] = \{(1+b)/a\}^{1/2}$

As discussed in Section 2.3, our exposure rates are based upon ISO and NCCI data, together with general industry information. The rating bureaus publish expected values, but do not generally estimate variances. So, for each particular case, their information provides us with an estimate of E[N], thus of the quotient a/b. But we cannot get estimates for a and b. We will deal with this problem later in Section 6.

3.5 A primary claim count experience rate

Consider the experience rating process as producing a sample {n(1),...,n(m)} of claim counts over m years (adjusted by IBNR and adjusted to current exposure level). The experience rate can be considered to be an estimate of the expectation of N:

(3.5.1) $E[N|\{n(i)\}]^{est} = S/m$

where $S = \Sigma_i\{n(i)\}$

3.6 A primary claim count posterior Bayes rate

Given the sample {n(i)}, it has been shown (see Bailey (1950), Mayerson (1964) and Herzog (1984)) that if the prior distribution of the Poisson parameter is gamma, then the posterior Bayes distribution is also gamma with the following parameter change:

(3.6.1) $a \rightarrow a + S$ and $b \rightarrow b + m$

Thus we have the following:

(3.6.2) $\Theta|(a,b,S,m) \sim G(a+S,b+m)$

(3.6.3) $E[\Theta|a,b,S,m] = (a+S)/(b+m)$

(3.6.4) $CV[\Theta|a,b,S,m] = (a+S)^{-1/2}$

(3.6.5) $E[N|a,b,S,m] = (a+S)/(b+m)$

(3.6.6) $CV[N|a,b,S,m] = \{(1+b+m)/(a+S)\}^{1/2}$

3.7 A primary claim count credibility rate

As a result of our previous discussion, we can now state:

THEOREM 3.7.1 For the gamma/Poisson model described above,

(a) the least squares credibility weight Z to attach to the primary experience {n(i)} is given by:

$$Z = m/(m+b)$$

(b) and this credibility estimate is exact Bayesian.

Proof: (a) The general least squares credibility weight Z to attach to the primary experience {n(i)} is given by (see Bühlmann (1967), p.199f):

$$Z = m/(m+k)$$

$$\text{where} \quad k = E_\Theta[Var[N|\Theta]]/Var_\Theta[E[N|\Theta]]$$

and where the outer expectations are with respect to the gamma structure function for Θ.

For the gamma/Poisson model with prior parameter (a,b):

$E_\Theta[Var[N|\Theta]] = E[\Theta] = a/b$ by (3.1.4) and (3.4.2)

$Var_\Theta[E[N|\Theta]] = Var[\Theta] = a/b^2$ by (3.1.3) and (3.4.3)

So $k = b$ and $Z = m/(m+b)$.

(b) Now rewrite E[N|a,b,S,m] as:

$$\begin{aligned} E[N|a,b,S,m] &= (a+S)/(b+m) \\ &= \{m/(b+m)\}*(S/m) + \{b/(b+m)\}*(a/b) \\ &= Z*(S/m) + (1-Z)*E[N] \end{aligned}$$

So the credibility estimate is exact Bayesian for the gamma/Poisson model.

The main thrust of this paper is to extend these results to the excess layer as best as possible.

4. THE EXCESS PRICING SITUATION WHEN THE EXCESS PROBABILITY q(d) IS KNOWN

We want to consider credibility for N(d), the number of claims excess of attachment point d. When q(d), the probability that any particular claim is excess, is known, we will obtain a result similar to that above. The first demonstration of this that we know of was by Erwin Straub (1971).

4.1 An excess claim count model

Let μ be the parameter for X, the claim size random variable. Assume that we have an unspecified structure function for μ. Assume that μ and Θ are independent. And define $q(d|\mu)$ by:

(4.1.1) $q(d|\mu) = \text{Prob}[X > d|\mu]$

Let $N(d|\Theta,\mu)$ denote the random variable N(d), given Θ and μ.

<u>Lemma 4.1.2</u> If $N|\Theta$ is Poisson-distributed and $q(d|\mu)$ is known, then $N(d|\Theta,\mu)$ has a Poisson distribution:

$$N(d|\Theta,\mu) \sim P(q(d|\mu)*\Theta)$$

<u>Proof</u>: The probability that N(d) = n, given values Θ and μ, can be written

$$\text{Prob}(N(d)=n|\Theta,\mu) = \Sigma_{k=0}^{\infty}\{\text{Prob}(N=k|\Theta)\text{Prob}(N(d)=n|\mu,N=k)\}$$

$$= \Sigma_{k=n}^{\infty}\{(\Theta^k e^{-\Theta}/k!)(k!/((k-n)!n!))(1-q(d|\mu))^{k-n}q(d|\mu)^n\}$$

$$= \{q(d|\mu)^n e^{-\Theta}/n!\}\ \Sigma_{k=n}^{\infty}\{\Theta^k(1-q(d|\mu))^{k-n}/(k-n)!)\}$$

$$= (q(d|\mu)^n e^{-\Theta}\Theta^n/n!)\ \Sigma_{k=n}^{\infty}\{(\Theta*[1-q(d|\mu)])^{k-n}/(k-n)!\}$$

$$= \{q(d|\mu)^n e^{-\Theta}\Theta^n/n!\}\exp\{\Theta*(1-q(d|\mu))\}$$

$= (\{\Theta * q(d|\mu)\}^n \exp\{-\Theta * q(d|\mu)\})/n!$

So $N(d|\Theta,\mu) \sim P(q(d|\mu)*\Theta)$.

Define $\Theta(d) = q(d)*\Theta$ and $\Theta(d|\mu) = q(d|\mu)*\Theta$. Then:

Lemma 4.1.3 If Θ is gamma-distributed with parameter (A,B) and $q(d|\mu)$ is known, then $\Theta(d|\mu)$ has a gamma distribution:

$$\Theta(d|\mu) \sim G(A, B/q(d|\mu))$$

Proof: $\mathrm{Prob}(q(d|\mu)*\Theta \le x)$

$= \mathrm{Prob}(\Theta \le x/q(d|\mu))$

$= \int_0^{x/q(d|\mu)} \{B^A \Theta^{A-1} e^{-B\Theta}/G(A)\} d\Theta$

Let $\Theta' = \Theta(d|\mu) = q(d|\mu)*\Theta$

Then $\Theta = \Theta'/q(d|\mu)$ and $d\Theta = (1/q(d|\mu))d\Theta'$

So our integral above becomes:

$= \int_0^x \{B^A q(d|\mu)^{1-A} \Theta'^{A-1} \exp\{-B\Theta'/q(d|\mu)\}/G(A)\} * q(d|\mu)^{-1} d\Theta'$

$= \int_0^x \{[B/q(d|\mu)]^A \Theta'^{A-1} \exp\{-B\Theta'/q(d|\mu)\}/G(A)\} d\Theta'$

So $\Theta(d|\mu) \sim G(A, B/q(d|\mu))$.

Thus we have the following:

Corollary 4.1.4 (a) $E[\Theta(d)|\mu] = q(d|\mu)*A/B = q(d|\mu)*E[\Theta]$

(b) $CV[\Theta(d)|\mu)] = A^{-1/2} = CV[\Theta]$

Also, in this case it is clear that:

Corollary 4.1.5 $N(d)|\mu$ has a negative binomial distribution:

$$N(d)|\mu \sim NB(A, B/\{q(d|\mu)+B\})$$

4.2 An excess claim count exposure rate

Assume that the primary exposure rating process determines an estimate (a,b) of the parameter (A,B) as before. Also assume that it determines an estimate u of μ. As in lemmas (4.1.2) and (4.1.3), once again:

Corollary 4.2.1 $N(d|\Theta,u)$ has a Poisson distribution:

$$N(d|\Theta,u) \sim P(q(d|u)*\Theta)$$

Corollary 4.2.2 $\Theta(d|u)$ has a gamma distribution:

$$\Theta(d|u) \sim G(a,b/q(d|u))$$

Thus we have:

Corollary 4.2.3 (a) $E[\Theta(d|u)] = q(d|u)*a/b = q(d|u)*E[\Theta]$

(b) $CV[\Theta(d|u)] = a^{-1/2} = CV[\Theta]$

And we also have the following:

Corollary 4.2.4 $N(d)|u$ has a negative binomial distribution:

$$N(d)|u \sim NB(a,p(d|u)) \quad \text{where } p(d|u) = b/\{q(d|u) + b\}$$

Thus we also have:

Corollary 4.2.5 (a) $E[N(d)|u] = q(d|u)*a/b = q(d|u)*E[N]$

(b) $CV[N(d)|u] > CV[N]$ if $d > 0$

Proof: (a) is trivial

(b) $CV[N(d)|u] = \{[1 + b/q(d|u)]/a\}^{1/2}$ by (3.4.5)

$= \{[1 + b/q(d|u)]/(1 + b)\}^{1/2} * CV[N]$

$> CV[N]$ if $d > 0$

Thus the uncertainties discussed earlier in Section 2.3 regarding *a priori* exposure rates increase excess of d. The question then becomes, how fast is the uncertainty in the experience rate increasing excess of d?

4.3 An excess claim count experience rate

Consider the excess experience rating process as producing another sample {n(1),...,n(m)} of claim counts in excess of attachment point d over m years (adjusted by IBNR and adjusted to current exposure level). The experience rate can again be considered to be an estimate of the expectation of N(d):

(4.3.1) $E[N(d)|S(d),m]^{est} = S(d)/m$

where $S(d) = \Sigma_i\{n(i)\}$

4.4 An excess claim count posterior Bayes rate

Given the excess claim count sample {n(i)} and given the *a priori* value u for parameter μ, the excess posterior Bayes gamma parameter is given by:

(4.4.1) $a \rightarrow a + S(d)$ and $b/q(d|u) \rightarrow b/q(d|u) + m$

Thus we have the following:

(4.4.2) $\Theta(d)|(a,b,u,S(d),m) \sim G(a+S(d),b/q(d|u)+m)$

(4.4.3) $E[\Theta(d)|a,b,u,S(d),m] = (a+S(d))/(b/q(d|u)+m)$

(4.4.4) $CV[\Theta(d)|a,b,u,S(d),m] = (a+S(d))^{-1/2}$

(4.4.5) $E[N(d)|a,b,u,S(d),m] = (a+S(d))/(b/q(d|u)+m)$

(4.4.6) $CV[N|a,b,u,S(d),m] = \{(1+b/q(d|u)+m)/(a+S(d))\}^{1/2}$

4.5 An excess claim count credibility rate

As a result of the preceding discussion, we can now state:

<u>**THEOREM 4.5.1**</u> For the excess gamma/Poisson model with known excess probability q(d|u), given claim severity parameter u, as described above:

(a) the least squares credibility weight Z(d|u) to attach to the excess experience {n(i)} is given by:

$$Z(d|u) = m/\{m + b/q(d|u)\}$$

(b) and this credibility estimate is exact Bayesian.

<u>Proof</u>: (a) The general least squares credibility weight Z(d|u) to attach to the excess experience {n(i)} is given by (see Bühlmann (1967), p.199f):

$$Z(d|u) = m/(m+k)$$

where $k = E_\Theta[Var[N(d)|\Theta,u]]/Var_\Theta[E[N(d)|\Theta,u]]$

Since $N(d)|u,\Theta = N(d|\Theta,u)$ is Poisson with parameter $q(d|u)*\Theta$ by lemma (4.1.2)).

$$E_\Theta[Var[N(d)|\Theta,u]] = E_\Theta[q(d|u)*\Theta]$$

$$= q(d|u)*E[\Theta] = q(d|u)*a/b$$

$$Var_\Theta[E[N(d)|\Theta,u]] = Var_\Theta[q(d|u)*\Theta]$$

$$= q(d|u)^2 Var[\Theta] = q(d|u)^2 a/b^2$$

So $k = b/q(d|u)$ and $Z(d|u) = m/\{m + b/q(d|u)\}$.

(b) The credibility estimate is exact Bayesian according to Theorem (3.7.1), since $N(d|\Theta,u)$ is Poisson-distributed by Corollary (4.2.1), and $\Theta(d|u)$ is gamma-distributed by Corollary (4.2.2).

Note that Theorem (4.5.1) translates the primary gammma/Poisson model exactly into the excess case, changing only one parameter component, as long as the excess claims probability is known.

We also have the following result.

THEOREM 4.5.2 If $d < d'$ and the claim severity cdf determining $q(d|u)$ is strictly monotonic, then $Z(d|u) > Z(d'|u)$.

Proof:

$$\begin{aligned} Z(d|u) &= m/\{m + b/q(d|u)\} \\ &> m/\{m + b/q(d'|u)\} \text{ since } q(d|u) > q(d'|u) \\ &= Z(d'|u) \end{aligned}$$

Note that as a corollary, $Z(d|u) < Z$ if $d > 0$.

Intuitively, the above result is true because with known u, the increase in uncertainty as we go to a higher excess layer is greater for the experience rate than for the exposure rate. Both Theorems (4.5.1) and (4.5.2) are only true, however, for the case where the value u of μ is known. In the next section, we extend some of these results to the case where the value u of μ is not known.

5. THE EXCESS PRICING SITUATION WHEN THE EXCESS PROBABILITY q(d) IS UNKNOWN (NONDETERMINISTIC)

What can we say when the value u of μ is unknown? We assume that a structure function for μ is given. This structure function is based upon our a priori data, together with our belief in the relative goodness of the a priori claims severity distribution for our particular excess rating situation. In Section 2.3, we have already discussed the problems with our a priori estimates based upon ISO and NCCI data, together with general industry information and individual company rate-level estimates. Later, in Section 6, we will discuss the relative accuracy of the translation of the a priori model to particular excess rating situations. But, we need not precisely specify a structure function for μ in order to proceed.

5.1 An Excess Claim Count Model

Since the parameter μ now has a structure function, it can be considered to be a random variable. Since q(d) is a function of μ, it can also be considered to be a random variable. And of course $\Theta(d) = q(d)*\Theta$ is a random variable. Since we can consider Θ and μ to be independent, then Θ and q(d) are also independent. Thus the joint structure function for Θ and μ is the product of the individual structure functions, and products of functions of Θ and μ are separable, as follows for the case of the joint expectation:

$$(5.1.1) \quad E_{\Theta,\mu}[\Theta(d)] = E_{\mu}[q(d)]*E_{\Theta}[\Theta] = E_{\mu}[q(d)]*(a/b)$$

To write $Var_{\Theta,\mu}[\Theta(d)]$, we need a lemma.

Lemma 5.1.2 If Y and Z are any two independent random variables, then:

$$Var[Y*Z] = E[Y]^2*Var[Z] + E[Z]^2*Var[Y] + Var[Y]*Var[Z]$$

Proof: $Var[Y*Z] = E[Y^2*Z^2] - E[Y*Z]^2$

$= E[Y^2]*E[Z^2] - E[Y]^2*E[Z]^2$

$= \{E[Y]^2+ Var[Y]\}*\{E[Z]^2+ Var[Z]\} - E[Y]^2*E[Z]^2$

$= E[Y]^2*Var[Z] + E[Z]^2*Var[Y] + Var[Y]*Var[Z].$

Now we can write:

Corollary 5.1.3 $Var_{\Theta,\mu}[\Theta(d)] = (A/B^2)*\{E[q(d)]^2+ (A+1)*Var[q(d)]\}$

Proof: $Var_{\Theta,\mu}[\Theta(d)]$

$= E[\Theta]^2*Var[q(d)] + E[q(d)]^2*Var[\Theta] + Var[\Theta]*Var[q(d)]$

$= (A/B)^2*Var[q(d)] + E[q(d)]^2*(A/B^2) + (A/B^2)*Var[q(d)]$

$= (A/B^2)*\{E[q(d)]^2 + (A+1)*Var[q(d)]\}$

We now have all we need to know about the structure function of μ to be able to write the excess credibility.

5.2 An excess claim count credibility model

Assume that we have estimated excess exposure and experience rates as in Sections 4.2 and 4.3. In the case that μ is nondeterministic, the familiar gamma/Poisson model does not translate to the excess case. Thus we no longer have a situation where the least squares credibility rate is exact Bayesian.

However, we still obtain the following intuitively satisfying result.

THEOREM 5.2.1 If the primary claim count is described by a gamma/Poisson model G(a,b) as described above, then the least squares credibility weight Z(d) to attach to the excess experience {n(i)} is given by:

$$Z(d) = m/\{m+k(d)\}$$

$$\text{where} \quad k(d) = b/\{E[q(d)]*(1 + (a+1)*CV[q(d)]^2)\}$$

Proof: The general least squares credibility weight Z(d) to attach to the excess experience {n(i)} is given by:

$$Z(d) = m/\{m+k(d)\}$$

$$\text{where} \quad k(d) = E_{\Theta,\mu}[Var[N(d)|\mu,\Theta]]/Var_{\Theta,\mu}[E[N(d)|\mu,\Theta]]$$

and the outer expectations are with respect to the joint structure function for Θ and μ:

$$E_{\Theta,\mu}[Var[N(d)|\mu,\Theta]] = E_{\Theta,\mu}[q(d)*\Theta] \quad \text{by lemma (4.1.2)}$$

$$= E[q(d)]*E[\Theta] = E[q(d)]*a/b$$

$$Var_{\Theta,\mu}[E[N(d)|\mu,\Theta]] = Var_{\Theta,\mu}[q(d)*\Theta] \quad \text{by lemma (4.1.2)}$$

$$= Var_{\Theta,\mu}[\Theta(d)]$$

$$= (a/b^2)*\{E[q(d)]^2 + (a+1)*Var[q(d)]\}$$

by Corollary (5.1.3)

$$= (a/b^2)*E[q(d)]^2*\{1 + (a+1)*CV[q(d)]^2\}$$

$$\text{So } k(d) = b/\{E[q(d)]*(1 + (a+1)*CV[q(d)]^2)\}$$

Theorem (4.5.1a) is a special case of Theorem (5.2.1), since with known value u of μ, we have CV[q(d)] = 0 and E[q(d)] = q(d|u).

Once μ is not known, the extension of Theorem (4.5.2) may not be true. As seen in the result below, it depends on whether E[q(d)] decreases fast enough compared to the growth of CV[q(d)].

THEOREM 5.2.2 If d < d' and the cdf of q(d) is strictly monotonic, then:

(i) Z(d) > Z(d')

if and only if

(ii) $E[q(d)]*\{1+(a+1)*CV[q(d)]^2\} > E[q(d')]*\{1+(a+1)*CV[q(d')]^2\}$

Proof: Z(d) > Z(d')

iff $m/\{m + k(d)\} > m/\{m + k(d')\}$

iff k(d) < k(d')

if and only if

$E[q(d)]*\{1+(a+1)*CV[q(d)]^2\} > E[q(d')]*\{1+(a+1)*CV[q(d')]^2\}$

Theorem 4.5.2 is a special case of the Theorem 5.2.2, since with a value u for μ known, then CV[q(d)] = CV[q(d')] = 0, E[q(d)] = q(d|u) and E[q(d')] = q(d'|u).

If d < d' and the claim severity cdf $F_x(x|\mu)$ is strictly monotonic, then E[q(d)] will always be greater than E[q(d')]. The problem is that CV[q(d')] will also be greater than CV[q(d)]. This gets to the heart of the matter. It says that the credibility of the experience rate decreases as we move to a higher retention unless the uncertainty in the a priori estimate of the excess probability increases faster.

The next result compares Z(d) with Z(d|u). It only makes sense to compare these when E[q(d)] = q(d|u). We then have the following theorem.

<u>**THEOREM 5.2.3**</u> If u is specified so that E[q(d)] = q(d|u), then Z(d) > Z(d|u).

<u>Proof</u>: Z(d) > Z(d|u) if and only if k(d) < k(d|u)

iff $E[q(d)]*\{1+(a+1)*CV[q(d)]^2\} > q(d|u)$.

Since E[q(d)] = q(d|u), this last inequality is true.

This matches our intuition, which tells us that when we allow uncertainty as to the value of the <u>a priori</u> excess probability q(d), the experience rate should gain credibility.

We also have the following results.

<u>**THEOREM 5.2.4**</u> (a) E[N(d)] = E[q(d)]*E[N]

(b) $Var[N(d)] > E_{\Theta,\mu}[Var[N(d)]|\Theta,\mu]$

(c) $CV[N(d)]^2 = \{1 + k(d)^{-1}\}/E[N(d)]$

<u>Proof:</u> (a) $E[N(d)] = E_{\Theta,\mu}[N(d)]$

$= E_{\Theta,\mu}[q(d)*N\} = E[q(d)]*E[N]$

(b) Trivially, $Var[N(d)] = Var_{\Theta,\mu}[N(d)]$

$= E_{\Theta,\mu}[Var[N(d)|\Theta,\mu]] + Var_{\Theta,\mu}[E[N(d)|\Theta,\mu]]$

$> E_{\Theta,\mu}[Var[N(d)|\Theta,\mu]]$

(c) $Var[N(d)] = E_{\Theta,\mu}[Var[N(d)|\Theta,\mu]] + Var_{\Theta,\mu}[E[N(d)|\Theta,\mu]]$

$= \{(a/b)*E[q(d)]\} + (a/b^2)*E[q(d)]^2*\{1 + (a+1)*CV[q(d)]^2\}$

$= \{(a/b)*E[q(d)]\}*\{1 + (E[q(d)]/b)*(1 + (a+1)*CV[q(d)]^2)\}$

$= E[N(d)]*\{1 + k(d)^{-1}\}$

Thus $CV[N(d)]^2 = Var[N(d)]/E[N(d)]^2 = \{1 + k(d)^{-1}\}/E[N(d)]$

We will attach some numerical meaning to these results in the next section.

6. PARAMETERIZING THE MODEL: TECHNICAL PRELIMINARIES

To put the excess credibility model into practice, we must specify values for the parameters for specific rating situations. Let us first investigate how the credibility results vary as we vary the parameters. The attached Exhibits 1 through 5 display various credibility answers for various values of the parameters, and Exhibit 6 displays values of CV[q(d)] based upon various sample sizes for claim severity distribution.

6.1 Exhibits 1 to 5

Looking specifically at Exhibits 1 through 5, we have the following:

1. Input:

a. parameter A = 100, 300, or 500

b. parameter m = 5

c. E[N] = (10, 50, 100, 500, 1000)

d. E[q(d)] = (.1, .01, .001)

e. CV[q(d)] = (.15, .3, .4) or (.25, .4, .6)

2. Output:

a. CV[Θ] by formula (3.4.4)

b. b by formula (3.4.2)

c. CV[N] by formula (3.4.5)

d. Z by Theorem (3.7.1)

e. E[N(d)] by Theorem (5.2.4)

f. CV[N(d)] by Theorem (5.2.4)

g. k(d) by Theorem (5.2.1)

h. Z(d) by Theorem (5.2.1)

6.2 Selection of the parameter component A

The value of the first gamma parameter component A determines the degree of belief we have that the a priori primary rates may be applicable to the particular exposure underlying the reinsurance coverage. Remember that CV[Θ] = $A^{-1/2}$; so the larger the value of A, the greater confidence we have in the a priori rate. Given a value a for A, then the value b of the parameter component B is determined in each case by specifying E[N]. As we considered the value of the primary Z = m/(m+b) on Exhibits 1 through 5 arising from various values of A, we came to the belief that the value of A should lie in the range [100, 500], with the particular value selected according to the given situation. Exhibits 1 through 5 display Z and Z(d) values for A = 100, 300 and 500.

6.3 Selection of CV[q(d)]

Exhibit 6 displays values of CV[q(d)] estimated for various sample sizes for various values of E[q(d)] with respect to a Pareto distribution with parameter (B, Q) = (10,000, 1.1) for U.S. general liability premises/operations exposure using the ISO parameterization. The complement of the distribution function is given by:

(6.3.1) $q(d|(B,Q)) = \{B/(B+d)\}^Q$

The estimates of CV[q(d)] are obtained from the information matrix for the distribution function.

The values of CV[q(d)] used on Exhibits 1 through 5 are selected to reflect sample error in the maximum likelihood estimates of the cdf parameter (B,Q) and also to take into account our subjective beliefs regarding the accuracy of the *a priori* estimates of the parameter (B,Q) considering all the various uncertainties discussed earlier in Section 2.3 and the possible applicability of each cdf for our particular cedant's exposure for the rating year.

7. PARAMETERIZING THE MODEL: THE UNDERWRITER QUESTIONNAIRE

In order to determine the key selected credibility parameter (A, CV[q(d)]) for any given rating situation, we designed a questionaire to elicit and codify the underwriter's judgement as to the relative goodness of the exposure and experience rates. Note that we assume that both rates make sense, and it is only a matter of calculating technical weights to combine them into a final rate. In the case that the underwriter does not have confidence in one or both of the rates, we ask them not to use the credibility formula, but instead to obtain enough information to estimate good technical rates, or at least discard the rate which makes no sense at all.

The underwriter questionaire lists questions regarding the information upon which the exposure rate and the experience rate are based. The underwriter answers each question, and points are assigned to each answer as shown below. The point values are determined by the relative importance of the information discussed in each question. The total points range from -70 up to +43.

A score of +43 means the exposure rating information is as good as possible and the experience rating information is as weak as possible, but yet yielding a not-ridiculous, possibly-useable experience rate. A score of -70 is just the opposite. We will later discuss how the total point score for a particular rating situation determines the values of A and the simplified equation determining CV[q(d)] as a function of E[q(d)].

7.1 The reinsurance underwriter questionaire: exposure rating

Listed here are six questions regarding the information upon which the exposure rate is based.

1. The reinsurance attachment point lies in which interval?
    a. less than $250,000 (0 points)
    b. between $250,000 and $500,000 (-3 points)
    c. above $500,000 (-5 points)

The higher the attachment point the less confidence we should have in the exposure rate, because of the first problem discussed under heading (2.3.2).

2. From where was the policy limits distribution obtained?

   a. from a cedant report (0 points)

   b. from a sample (-2 points)

   c. from judgement (-10 points)

   d. from an industry default (-20 points)

The most accurate source for the policy limits distributions by line used in exposure rating is a comprehensive, careful report from the company. Next accurate is a sample taken of the company's policies by line. If neither of these are available, the underwriter may know that this company is very similar to another company or a type of company for which accurate policy limits information is available; thus the underwriter may be able to judgementally specify fairly accurate distributions. Least accurate are the all-industry default distributions from ISO or other such sources.

3. What percentage of excess claims do you expect to arise from multi-limit exposure, e.g., clash, stacking of limits, etc.?

   a. less than 10% (0 points)

   b. unknown (-5 points)

   c. more than 10% (-10 points)

The greater the potential excess exposure from limits stacking or the clash of various coverages or policies being involved in a single loss occurrence, the less accurate is the exposure rate.

This is so because the claim severity cdf's determining the excess exposure rate are based upon individual coverage claims, and thus neither they nor their derived increased limits factors account for multiple coverage or policy claims per occurrence.

4. How much excess loss potential do you think there may be arising from lines of business not separately rated in the exposure rating system? Also factor in how different you think the loss cost rates may be for these unrated lines.

    a. minor (0 points)

    b. major (-10 points)

    c. if the unrated lines includes heavy umbrella exposure (-20 points)

There are almost always miscellaneous exposures covered by a reinsurance excess contract which cannot be accurately rated because there are no accurate claim severity cdf's. Usually these miscellaneous exposures are each grouped with their most similar main exposure for rating purposes, and thus get the same excess rate as the main exposure type. The question here asks the underwriter for a very subjective judgement regarding the degree to which the miscellaneous exposures may not be accurately rated. It is clear that if heavy umbrella coverage is a major part of the not-explicitly-rated exposure, then the excess rate will be less accurate.

5. What level of confidence do you have in the technical predictions of the cedant's loss ratios for next year?

a. extremely confident (0 points)

b. very confident (-5 points)

c. fairly confident (-10 points)

The predictions of the company's primary loss ratios by line for the rating coverage period is absolutely crucial for exposure rating, since the excess rates key off the predictions of primary loss costs. The underwriter must have an opinion regarding these predictions. Note that we are assuming here that an underwriter won't attempt to exposure rate unless they can accurately predict the cedant's loss ratio, usually in relation to the industry, for next year.

6. From where were the distributions of the business within line to subline, and to hazard group for workers' compensation obtained?

a. from a cedant report (0 points)

b. from a mixture of sources (-2 points)

c. from industry defaults (-5 points)

The distribution of the underlying exposure by subline within line and by workers compensation hazard group is important for determining the excess exposure potential. Again, the best information source is a report from the company, with general industry defaults being least informative.

7.2 The reinsurance underwriter questionaire: experience rating

Listed here are six questions regarding the information upon which the experience rate is based.

1. How stable by year is the distribution of the cedant's business by line?
    - a. very stable (0 points)
    - b. unknown (3 points)
    - c. very unstable (5 points)

The excess claims in the rating period arise from the particular exposure covered during that period. If the underlying exposure is stable from year to year, then the historical experience is relevant to the future coverage period being priced. If the underlying exposure is not stable, then there is a question of how relevant the historical experience may be for rating future coverage.

2. How stable by year is the distribution of the cedant's policy limits?
    - a. very stable (0 points)
    - b. unknown (5 points)
    - c. very unstable (10 points)

Likewise, the historical excess claims arise within the context of the historical policy limits sold by the company. If these are very different from the policy limits which may be sold next year, or if it is very uncertain exactly what next year's distribution may be, then the historical experience is of questionable relevance.

3. What is the source of the rate changes and rate deviations by year?

   a. from a cedant report (0 points)

   b. from judgement or a mixture (5 points)

   c. from industry defaults (10 points)

Within the experience rating procedure, the historical subject premium by year is adjusted to future coverage level. The more accurate the information on exposure trend and historical rate changes, the more accurate will be the experience rate.

4. How stable is the cedant's excess loss development?

   a. very stable (0 points)

   b. unknown (5 points)

   c. very unstable (10 points)

Historical excess claims development is studied in the experience rating procedure, and may be used directly with perhaps some judgement modification. If it is very chaotic, then any ultimate loss predictions based in part upon the historical excess claims development is questionable.

5. Do the cedant's excess loss development factors lie between the lower and upper bounds given for this type of business?

a. yes (0 points)

b. no (5 points)

Our reinsurance excess experience rating procedure incorporates a comparison of the company's historical excess claims loss development lags, (percentage of loss reported each point in time) with "book" lags for similar excess exposure. If a company's development lags differ significantly from our more general information for similar excess exposure, we have less confidence in the company's loss development indications, or less confidence in our ability to accurately categorize and understand their excess exposure.

6. Is allocated loss adjustment expense covered on a pro rata basis?

a. yes (0 points)

b. no (3 points)

If allocated loss adjustment expense is covered pro rata with respect to indemnity loss, then the historical individual claims data may not clearly separate the allocated expense from the indemnity loss per excess claim. Thus we may not be able to accurately apply excess attachment points and limits.

7.3 The parameters determined by the underwriter questionaire

The total point score for a particular rating situation determines the value of gamma parameter component A and the multiplicative coefficient ß used in the simplified equation giving CV[q(d)] in terms of E[q(d)]. The simplified formula for CV[q(d)] we use is as follows:

(7.3.1) Simplified formula for CV[q(d)]

$$CV[q(d)] = \beta * \{-\ln(E[q(d)])\}^{0.76536}$$

We determined that this formula is a reasonably good approximation to the columns displayed in Exhibit 6.

Now we can write the table giving A and ß based upon the total questionaire total point score.

(7.3.2) Parameters from underwriter questionaire

Point range	Greater *a priori* belief in	A	ß
[12,43]	exposure	500	0.01878
[-13,11]	neutral	300	0.04200
[-70,-14]	experience	100	0.09391

You can see that these parameters are somewhat arbitrary. However they are based upon the reasonableness of the results displayed in Exhibits 1 through 5. And they yield reasonable-looking credibility values, as discussed in the next section.

7.4 The final credibility tables

Exhibits 7 through 10 display values of Z(d) corresponding to various values of E[N(d)] and E[N] (remember, E[q(d)] = E[N(d)]/E[N]). The values also depend upon the number of years in the experience rating period _m_ and upon the underwriter's degree of _a priori_ confidence in the information for either exposure or experience rate.

We hope the reader recognizes these tables as being intuitively reasonable. What is surprising is the great degree of credibility given to the excess experience for each of the cases. No reasonable person will henceforth be able to mention a full credibility standard for excess rating anywhere near the actuarial historical sacred relic of 1084 (or is it 1082?) claims.

8. SUMMARY

We have shown how a practical credibility model may be designed for excess-of-loss reinsurance. And we have discussed various uncertainties in both the exposure and experience rates and an intuitively reasonable range for the model parameters. Finally, we have discussed an underwriter questionaire to elicit and codify the underwriter's subjective judgement regarding the information underlying each rate, so that this important information can be incorporated into the credibility rate.

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GAMMA/POISSON CLAIM COUNT MODEL FOR CREDIBILITY

BASIC: θ ~GAMMA(a,b) N|θ ~ POISSON(θ) Z = m/(m+b)

SCENARIO:	a =	100		m =	5
	CV[θ] =	0.100			
E[N]	10	50	100	500	1000
b	10.000	2.000	1.000	0.200	0.100
CV[N]	0.332	0.173	0.141	0.110	0.105
Z	0.333	0.714	0.833	0.962	0.980

EXCESS OF d: q(d) = Prob[X>d], where X = claim size rv
Z(d) = m/(m+k(d))

SCENARIO:	E[q(d)] =	0.1		CV[q(d)] =	0.15
E[N(d)]	1.000	5.000	10.000	50.000	100.000
CV[N(d)]	1.016	0.482	0.364	0.230	0.207
k(d)	30.558	6.112	3.056	0.611	0.306
Z(d)	0.141	0.450	0.621	0.891	0.942

SCENARIO:	E[q(d)] =	0.01		CV[q(d)] =	0.3
E[N(d)]	0.100	0.500	1.000	5.000	10.000
CV[N(d)]	3.178	1.449	1.049	0.549	0.448
k(d)	99.108	19.822	9.911	1.982	0.991
Z(d)	0.048	0.201	0.335	0.716	0.835

SCENARIO:	E[q(d)] =	0.001		CV[q(d)] =	0.4
E[N(d)]	0.010	0.050	0.100	0.500	1.000
CV[N(d)]	10.009	4.491	3.189	1.474	1.082
k(d)	582.751	116.550	58.275	11.655	5.828
Z(d)	0.009	0.041	0.079	0.300	0.462

GAMMA/POISSON CLAIM COUNT MODEL FOR CREDIBILITY

BASIC: θ ~GAMMA(a,b)   N|θ ~ POISSON(θ)   Z = m/(m+b)

SCENARIO:	a =	100		m =	5
	CV[θ] =	0.100			
E[N]	10	50	100	500	1000
b	10.000	2.000	1.000	0.200	0.100
CV[N]	0.332	0.173	0.141	0.110	0.105
Z	0.333	0.714	0.833	0.962	0.980

EXCESS OF d: q(d) = Prob[X>d], where X = claim size rv
Z(d) = m/(m+k(d))

SCENARIO:	E[q(d)] =	0.1		CV[q(d)] =	0.25
E[N(d)]	1.000	5.000	10.000	50.000	100.000
CV[N(d)]	1.036	0.523	0.416	0.305	0.288
k(d)	13.675	2.735	1.368	0.274	0.137
Z(d)	0.268	0.646	0.785	0.948	0.973

SCENARIO:	E[q(d)] =	0.01		CV[q(d)] =	0.4
E[N(d)]	0.100	0.500	1.000	5.000	10.000
CV[N(d)]	3.189	1.474	1.082	0.610	0.521
k(d)	58.275	11.655	5.828	1.166	0.583
Z(d)	0.079	0.300	0.462	0.811	0.896

SCENARIO:	E[q(d)] =	0.001		CV[q(d)] =	0.6
E[N(d)]	0.010	0.050	0.100	0.500	1.000
CV[N(d)]	10.019	4.514	3.221	1.541	1.172
k(d)	267.666	53.533	26.767	5.353	2.677
Z(d)	0.018	0.085	0.157	0.483	0.651

GAMMA/POISSON CLAIM COUNT MODEL FOR CREDIBILITY

BASIC: θ ~GAMMA(a,b)   N|θ ~ POISSON(θ)   Z = m/(m+b)

SCENARIO:	a =	100		m =	5
	CV[θ] =	0.100			
E[N]	10	50	100	500	1000
b	10.000	2.000	1.000	0.200	0.100
CV[N]	0.332	0.173	0.141	0.110	0.105
Z	0.333	0.714	0.833	0.962	0.980

EXCESS OF d: q(d) = Prob[X>d], where X = claim size rv
Z(d) = m/(m+k(d))

SCENARIO:	E[q(d)] =	0.1		CV[q(d)] =	0.25
E[N(d)]	1.000	5.000	10.000	50.000	100.000
CV[N(d)]	1.036	0.523	0.416	0.305	0.288
k(d)	13.675	2.735	1.368	0.274	0.137
Z(d)	0.268	0.646	0.785	0.948	0.973

SCENARIO:	E[q(d)] =	0.01		CV[q(d)] =	0.5
E[N(d)]	0.100	0.500	1.000	5.000	10.000
CV[N(d)]	3.204	1.504	1.124	0.680	0.602
k(d)	38.095	7.619	3.810	0.762	0.381
Z(d)	0.116	0.396	0.568	0.868	0.929

SCENARIO:	E[q(d)] =	0.001		CV[q(d)] =	0.75
E[N(d)]	0.010	0.050	0.100	0.500	1.000
CV[N(d)]	10.029	4.536	3.252	1.606	1.256
k(d)	172.973	34.595	17.297	3.459	1.730
Z(d)	0.028	0.126	0.224	0.591	0.743

GAMMA/POISSON CLAIM COUNT MODEL FOR CREDIBILITY

BASIC: θ ~GAMMA(a,b) N|θ ~ POISSON(θ) Z = m/(m+b)

SCENARIO:	a =	300		m =	5
	CV[θ] =	0.058			
E[N]	10	50	100	500	1000
b	30.000	6.000	3.000	0.600	0.300
CV[N]	0.321	0.153	0.115	0.073	0.066
Z	0.143	0.455	0.625	0.893	0.943

EXCESS OF d: q(d) = Prob[X>d], where X = claim size rv
Z(d) = m/(m+k(d))

SCENARIO:	E[q(d)] =	0.1		CV[q(d)] =	0.15
E[N(d)]	1.000	5.000	10.000	50.000	100.000
CV[N(d)]	1.013	0.475	0.355	0.214	0.189
k(d)	38.598	7.720	3.860	0.772	0.386
Z(d)	0.115	0.393	0.564	0.866	0.928

SCENARIO:	E[q(d)] =	0.01		CV[q(d)] =	0.3
E[N(d)]	0.100	0.500	1.000	5.000	10.000
CV[N(d)]	3.177	1.447	1.046	0.542	0.440
k(d)	106.800	21.360	10.680	2.136	1.068
Z(d)	0.045	0.190	0.319	0.701	0.824

SCENARIO:	E[q(d)] =	0.001		CV[q(d)] =	0.4
E[N(d)]	0.010	0.050	0.100	0.500	1.000
CV[N(d)]	10.008	4.490	3.188	1.471	1.079
k(d)	610.252	122.050	61.025	12.205	6.103
Z(d)	0.008	0.039	0.076	0.291	0.450

EXHIBIT 5

GAMMA/POISSON CLAIM COUNT MODEL FOR CREDIBILITY

BASIC: Θ ~GAMMA(a,b)   N|Θ ~ POISSON(Θ)   Z = m/(m+b)

SCENARIO:	a =	500		m =	5
	CV[Θ] =	0.045			
E[N]	10	50	100	500	1000
b	50.000	10.000	5.000	1.000	0.500
CV[N]	0.319	0.148	0.110	0.063	0.055
Z	0.091	0.333	0.500	0.833	0.909

EXCESS OF d: q(d) = Prob[X>d], where X = claim size rv
Z(d) = m/(m+k(d))

SCENARIO:	E[q(d)] =	0.1		CV[q(d)] =	0.15
E[N(d)]	1.000	5.000	10.000	50.000	100.000
CV[N(d)]	1.012	0.474	0.353	0.211	0.186
k(d)	40.741	8.148	4.074	0.815	0.407
Z(d)	0.109	0.380	0.551	0.860	0.925

SCENARIO:	E[q(d)] =	0.01		CV[q(d)] =	0.3
E[N(d)]	0.100	0.500	1.000	5.000	10.000
CV[N(d)]	3.177	1.446	1.045	0.541	0.438
k(d)	108.483	21.697	10.848	2.170	1.085
Z(d)	0.044	0.187	0.315	0.697	0.822

SCENARIO:	E[q(d)] =	0.001		CV[q(d)] =	0.4
E[N(d)]	0.010	0.050	0.100	0.500	1.000
CV[N(d)]	10.008	4.490	3.188	1.470	1.078
k(d)	616.067	123.213	61.607	12.321	6.161
Z(d)	0.008	0.039	0.075	0.289	0.448

EXCESS PROBABILITIES FOR PARETO CDF

PARETO (B,Q) CDF:
F(X)= 1-(B/(B+X))^Q

PARAMETER B= 10,000
Q= 1.1

CV[q(d)] FOR VARIOUS VALUES OF E[q(d)]
AND FOR VARIOUS SAMPLE SIZES.

		sample size:				
		1	10	100	1,000	10,000
E[q(d)]	d					
0.2	33,194	6.166	1.950	0.617	0.195	0.062
0.1	71,113	7.991	2.527	0.799	0.253	0.080
0.05	142,319	9.633	3.046	0.963	0.305	0.096
0.01	647,933	13.155	4.160	1.316	0.416	0.132
0.001	5,326,699	18.004	5.694	1.800	0.569	0.180
0.0001	43,277,613	22.826	7.218	2.283	0.722	0.228
0.000001	2,848,025,868	32.475	10.270	3.248	1.027	0.325

EXCESS EXPOSURE AND EXPERIENCE RATE CREDIBILITY

A. Number of years used in rating period = 5
B. Expected number of claims ground-up = 5000

(1) Expected # claims excess	(2) Credibility of excess experience rate based upon a priori confidence in exposure	(3) neutral	(4) experience
1	5.4%	20.1%	55.2%
2	9.3%	30.8%	68.5%
3	12.6%	38.3%	75.1%
4	15.5%	43.9%	79.2%
5	18.0%	48.3%	81.9%
6	20.4%	52.0%	84.0%
7	22.5%	55.0%	85.5%
8	24.5%	57.6%	86.8%
9	26.3%	59.3%	87.8%
10	28.0%	61.3%	88.6%
11	29.6%	63.5%	89.4%
12	31.1%	65.1%	90.0%
13	32.5%	66.5%	90.5%
14	33.9%	67.7%	91.0%
15	35.1%	68.9%	91.4%
16	36.3%	69.9%	91.8%
17	37.5%	70.9%	92.1%
18	38.6%	71.8%	92.4%
19	39.6%	72.6%	92.7%
20	40.6%	73.4%	92.9%
21	41.5%	74.1%	93.2%
22	42.5%	74.7%	93.4%
23	43.3%	75.4%	93.6%
24	44.2%	76.0%	93.8%
25	45.0%	76.5%	94.0%

EXCESS EXPOSURE AND EXPERIENCE RATE CREDIBILITY

A. Number of years used in rating period = 7
B. Expected number of claims ground-up = 5000

(1)	(2)	(3)	(4)
Expected # claims excess	Credibility of excess experience rate based upon a priori confidence in exposure	neutral	experience
1	7.4%	26.0%	63.3%
2	12.5%	38.4%	75.3%
3	16.8%	46.5%	80.9%
4	20.4%	52.3%	84.2%
5	23.6%	56.7%	86.4%
6	26.4%	60.2%	88.0%
7	28.9%	63.1%	89.2%
8	31.2%	65.5%	90.2%
9	33.4%	67.6%	91.0%
10	35.3%	69.4%	91.6%
11	37.1%	70.9%	92.2%
12	38.8%	72.3%	92.6%
13	40.3%	73.5%	93.0%
14	41.8%	74.6%	93.4%
15	43.1%	75.6%	93.7%
16	44.4%	76.5%	94.0%
17	45.6%	77.3%	94.2%
18	46.8%	78.1%	94.5%
19	47.9%	78.8%	94.7%
20	48.9%	79.4%	94.9%
21	49.9%	80.0%	95.0%
22	50.8%	80.6%	95.2%
23	51.7%	81.1%	95.3%
24	52.5%	81.6%	95.5%
25	53.4%	82.0%	95.6%

EXCESS EXPOSURE AND EXPERIENCE RATE CREDIBILITY

A. Number of years used in rating period = 5
B. Expected number of claims ground-up = 1000

(1)	(2)	(3)	(4)
Expected # claims excess	Credibility of excess experience rate based upon a priori confidence in exposure	neutral	experience
1	4.2%	15.8%	47.6%
2	7.2%	24.4%	60.9%
3	9.8%	30.7%	68.0%
4	12.0%	35.5%	72.5%
5	14.0%	39.4%	75.7%
6	15.9%	42.7%	78.0%
7	17.6%	45.5%	79.9%
8	19.2%	48.0%	81.4%
9	20.7%	50.1%	82.6%
10	22.1%	52.0%	83.6%
11	23.4%	53.7%	84.5%
12	24.6%	55.2%	85.3%
13	25.8%	56.6%	86.0%
14	26.9%	57.9%	86.6%
15	28.0%	59.1%	87.1%
16	29.0%	60.2%	87.6%
17	30.0%	61.2%	88.0%
18	30.9%	62.1%	88.4%
19	31.8%	63.0%	88.8%
20	32.7%	63.8%	89.1%
21	33.5%	64.5%	89.4%
22	34.3%	65.3%	89.7%
23	35.1%	65.9%	90.0%
24	35.8%	66.6%	90.2%
25	36.5%	67.2%	90.4%

EXCESS EXPOSURE AND EXPERIENCE RATE CREDIBILITY

A. Number of years used in rating period = 7

B. Expected number of claims ground-up = 1000

(1) Expected # claims excess	(2) Credibility of excess experience rate based upon a priori confidence in exposure	(3) neutral	(4) experience
1	5.8%	20.8%	56.0%
2	9.8%	31.2%	68.6%
3	13.2%	38.2%	74.8%
4	16.1%	43.5%	78.7%
5	18.6%	47.7%	81.3%
6	20.9%	51.1%	83.3%
7	23.0%	53.9%	84.7%
8	24.9%	56.3%	85.9%
9	26.7%	58.4%	86.9%
10	28.4%	60.3%	87.7%
11	29.9%	61.9%	88.4%
12	31.4%	63.3%	89.0%
13	32.7%	64.6%	89.6%
14	34.0%	65.8%	90.0%
15	35.2%	66.9%	90.4%
16	36.4%	67.9%	90.8%
17	37.5%	68.8%	91.1%
18	38.5%	69.6%	91.4%
19	39.5%	70.4%	91.7%
20	40.4%	71.1%	92.0%
21	41.4%	71.8%	92.2%
22	42.2%	72.5%	92.4%
23	43.1%	73.1%	92.6%
24	43.9%	73.6%	92.8%
25	44.6%	74.1%	93.0%