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Elias KARAM

Measuring and Managing Operational Risk in the Insurance and Banking Sectors

Composition du jury

Rapporteurs: Pr. Didier FOLUS

Pr. Michel BÉRA

Examineurs: Mr. Adel N. SATEL

Pr. Toni SAYAH

Pr. Jean-Claude AUGROS

Pr. Frédéric PLANCHET (directeur de thèse)

Abstract

Measuring and Managing Operational Risk in the Insurance and Banking Sectors

Operational risk existed longer than we know, but its concept was not interpreted until after the year 1995. Though its application varied by institutions, Basel III for banks and Solvency II for insurance companies, the idea stays the same. Firms are interested in operational risk because exposure can be fatal. As so, and since Basel and Solvency frameworks set forth many calculation criteria, our interest in this thesis is first to combine the different measurement techniques for operational risk in financial companies, and we highlight more and more the consequences of estimation risk which is treated as a particular part of operational risk.

In the first part, we will present a full overview of operational risk, from the regulatory laws and regulations to the associated mathematical and actuarial concepts as well as a numerical application regarding the Advanced Measurement Approach, like Loss Distribution to calculate the capital requirement, then applying the Extreme Value Theory. This part answers to the qualitative and quantitative aspects of operational risk, in addition to some application. We will be interested as well, in the use of scenario analysis of expert opinion in conjunction with internal data to evaluate our exposure to events. We conclude this first part by setting a scaling technique based on (OLS) enabling us to normalize our external data to a local Lebanese Bank. Hence, setting the platform for the second part regarding the importance of estimation risk treated as a part of operational risk.

On the second part, we feature estimation risk by first measuring the error induced on the SCR by the estimation error of the parameters, to having an alternative yield curve estimation and finishing by calling attention to the reflections on assumptions of the calculation instead of focusing on the so called hypothesis "consistent with market values", would be more appropriate and effective than to complicate models and generate additional errors and instability. Chapters in this part illustrate the estimation risk in its different aspects which is a part of operational risk, highlighting as so the attention that should be given in treating our models.

Keywords: Operational Risk, Basel III, Solvency II, Loss Distribution Approach, Extreme Value Theory, Bayesian Statistics, VaR, Estimation Risk, ORSA.

Résumé

La Mesure et la Gestion du Risque Opérationnel dans l'Assurance et le secteur Bancaire

La notion du risque opérationnel a été interprétée dans les années 1995. Bien que son application varie selon les établissements, Bâle III pour les banques et Solvabilité II pour les compagnies d'assurance, l'idée reste la même. Les entreprises sont intéressées par le risque opérationnel, car l'exposition peut être grave. Notre intérêt dans cette thèse est de combiner les différentes techniques de mesure du risque opérationnel dans les secteurs financiers, et on s'intéresse plus particulièrement aux conséquences du risque d'estimation dans les modèles, qui est un risque opérationnel particulier.

Dans la première partie, nous allons présenter les concepts mathématiques et actuarielles associés ainsi qu'une application numérique en ce qui concerne l'approche de mesure avancée comme Loss Distribution pour calculer l'exigence en capital, puis en appliquant la théorie des valeurs extrêmes. Les chapitres dans cette partie illustrent les différentes méthodes qualitatives et quantitatives avec des applications directes sur le risque opérationnel. En plus, on se concentre sur le risque d'estimation illustré avec l'analyse des scénarios de l'opinion d'experts en conjonction avec des données de pertes internes pour évaluer notre exposition aux événements de gravité. Nous concluons cette première partie en définissant une technique de mise à l'échelle sur la base de (MCO) qui nous permet de normaliser nos données externes à une banque locale Libanaise.

Dans la deuxième partie, on donne de l'importance sur la mesure de l'erreur induite sur le SCR par l'erreur d'estimation des paramètres, on propose une méthode alternative pour estimer une courbe de taux et on termine par attirer l'attention sur les réflexions autour des hypothèses de calcul et ce que l'on convient de qualifier d'hypothèse "cohérente avec les valeurs de marché" serait bien plus pertinente et efficace que la complexification du modèle, source d'instabilité supplémentaire, ainsi mettre en évidence le risque d'estimation qui est lié au risque opérationnel et doit être accordé beaucoup plus d'attention dans nos modèles de travail.

Mots clés: Risque Opérationnel, Bâle III, Solvabilité II, Statistique Bayésienne, LDA, Théorie des valeurs extrêmes, VaR, Risque d'estimation, ORSA.

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Ultimately, I would like to express my gratitude for all those who made this thesis possible.

*"Focus on the journey, not the destination.
Joy is found not in finishing an activity
but in doing it"*
Greg Anderson

*To Salim,
To Gisèle,
To Chantal,
To Désirée,
To Noah.*

Contents

General Introduction	1
Choice of the study subject	1
Context of the study	2
Organization of the study	2
I Measuring and Managing Operational Risk in a Bank	5
Part I - Introduction	7
1 Laws and Regulations	9
1.1 Basic Indicator and Standardized Approach	9
1.1.1 Example of the BIA and SA Calculations	10
1.1.2 The Capital Requirement Under the Basic Indicator and Standard- ized Approach	11
1.2 Capital Requirement Review	12
1.2.1 The Basic Indicator Approach	14
1.2.2 The Standardized Approach	14
1.3 The Advanced Measurement Approach	16
1.3.1 The Loss Distribution Approach (LDA)	17
1.3.2 Internal Measurement Approach (IMA)	20
1.3.3 Scocard Approach (SCA)	21
1.3.4 Scenario-Based AMA (sbAMA)	22
1.4 Solvency II Quantification Methods	23
2 Quantitative Methodologies	27
2.1 Risk Measures	28
2.1.1 VaR calculation	28
2.1.2 Expected Shortfall	28
2.2 Illustration of the LDA method	29
2.2.1 Severity of Loss Distributions	29
2.2.2 Loss Frequency Distribution	32
2.2.3 Aggregate Loss Distribution	33
2.2.3.1 Panjer's recursive algorithm	34
2.2.3.2 Monte Carlo method	35

2.3	Treatment of Truncated Data	36
2.3.1	Estimating Parameters using MLE	38
2.3.2	Kolmogorov-Smirnov test adapted for left truncated Data	39
2.4	Introduction to Robust Statistics	40
2.4.1	Maximum Likelihood Estimation (MLE)	40
2.4.2	Influence Function	41
2.4.2.1	The Influence Function for MLE	42
2.4.2.2	The Influence function for MLE case of the truncated Severity Distributions	43
2.4.2.3	General Example for the LogNormal Distribution	44
2.5	Working with Extremes for Catastrophic Risks	45
2.5.1	Generalized Extreme Value Distribution: Basic Concepts	47
2.5.2	Block Maxima Method	48
2.5.3	Generalized Pareto Distribution	48
2.5.4	Excess Loss Distribution	49
2.5.5	The Peak Over Threshold	51
2.6	Application to a Legal Events Database	52
2.6.1	Some Probability Distributions	52
2.6.1.1	Exponential Distribution	53
2.6.1.2	Lognormal Distribution	53
2.6.1.3	Weibull Distribution	53
2.6.1.4	Pareto Distribution	54
2.6.1.5	Output Analysis	54
2.6.2	LDA and Application of Extreme Value Theory	56
2.6.2.1	Application to an internal Loss Data	57
2.6.2.2	Application of the Extreme Value theory	58
2.7	The Bayesian Approach: Internal Data, External Data and Expert Opinion	59
2.7.1	A simple Model	60
2.7.2	Illustration of the Bayesian Approach	61
3	Combining Internal Data with Scenario Analysis	65
3.1	Introduction	65
3.2	Bayesian techniques in combining two data sources: Conjugate prior	66
3.2.1	Modelling Frequency distribution: The Poisson Model	67
3.2.2	Modelling severity distribution: Lognormal $\mathcal{LN}(\mu, \sigma)$ distribution with unknown μ	68
3.2.3	Modelling frequency and severity distributions: Unknown Poisson(λ) parameter and Lognormal $\mathcal{LN}(\mu, \sigma)$ distribution with unknown μ	69
3.2.4	Sensitivity Analysis	70
3.3	Bayesian techniques in combining two data sources: MCMC-Metropolis Hastings algorithm	71
3.3.1	MCMC with the Poisson(λ) distribution	72
3.3.2	MCMC with Unknown Poisson(λ) parameter and Lognormal $\mathcal{LN}(\mu, \sigma)$ distribution with unknown μ	72

3.3.3	General case: MCMC with Unknown Poisson(λ) parameter and Lognormal $\mathcal{LN}(\mu, \sigma)$ distribution with unknown μ and σ	74
3.3.4	Confidence Interval calculation	75
3.4	Bayesian approach reviewed	77
3.4.1	Operational Risk Standardized Approach	78
3.4.2	Numerical Results for Expert opinion treated as SA	78
3.5	Conclusion	79
4	Operational Risk Management in a Bank	81
4.1	Introduction	81
4.2	Risk and Control Self Risk Assessment	82
4.2.1	Expressing Risk Tolerance	83
4.2.2	Setting the rating scales	83
4.2.3	Defining the fields of the RCSA table	84
4.3	Incident Reporting	85
4.3.1	Definition	85
4.3.2	Example of incidents	85
4.4	Key Risk Indicators	86
4.4.1	Definition	86
4.4.2	KRIs Process	87
4.4.2.1	Identification of Key Risk Indicators	87
4.4.2.2	Assessment of Key Risks Indicators	87
4.4.2.3	Setting the KRI Scale	88
4.4.2.4	Determination of Thresholds	88
4.4.3	Periodical KRI Reporting	88
4.4.4	Review of KRIs and their Thresholds	89
4.5	Incorporation of External Data	89
4.6	Scenario Analysis	90
4.6.1	Scenario Analysis Algorithm and Procedures	90
4.6.2	Scenario risk drivers	91
4.7	Insurance Covering Operational Risks	93
4.7.1	Insurance Policies and Coverage	93
4.7.1.1	Bankers Blanket Bond Insurance	93
4.7.1.2	Directors and Officers Liability Insurance	95
4.7.1.3	Political Violence Insurance	96
4.7.1.4	Electronic and Computer Crime Policy	97
4.7.1.5	Plastic Card Insurance Policy	98
4.7.2	Advantages of Insuring Operational Risks	99
4.7.3	Basel II views	99
4.7.4	Capital assessment under insurance on operational losses	100
4.7.4.1	Appropriate haircuts	101
4.7.4.2	Payment uncertainty	102
4.7.4.3	Counterparty risk	102
4.8	Scaling Severity for External Loss Data	103
4.8.1	Loss data and scenarios	103

4.8.2	Description of the External Data	104
4.8.3	Model specification	105
4.8.3.1	Theoretical scaling model	105
4.8.3.2	Description of the variables	105
4.8.3.3	Linear regression	107
4.8.3.4	Simple Regression Results	107
4.8.3.5	Normalization formula and resulted output	109
4.8.4	Data requirements for specifying frequency distributions	111
Part I - Conclusion		113
II Addressing Estimation Risk		115
5	Estimation Errors and SCR Calculation	117
5.1	Introduction	117
5.2	Model Presentation	118
5.2.1	Simulating Interest rates: The Vasicek model	118
5.2.2	Parameter Estimation	119
5.2.3	Calculation of the SCR	120
5.3	Estimation Risk	121
5.3.1	Case of one zero coupon bond	121
5.3.2	Estimation Risk of a Portfolio consisting of two Zero-Coupon bonds	122
5.3.3	Numerical Results	123
5.4	Conclusion	125
6	An Alternative Yield Curve Estimation	127
6.1	Introduction	127
6.2	Model Presentation	127
6.2.1	Curve Fitting: The Nelson-Siegel model	128
6.2.2	Estimation Risk	130
6.2.3	Numerical Results	131
6.3	Conclusion	132
7	Market Consistency and Economic Evaluation	135
7.1	Introduction	135
7.2	Market Consistency	136
7.2.1	Economic Evaluation	136
7.2.2	Balance Sheet Exposure	138
7.2.2.1	Restructuring the Yield Curve	140
7.2.2.2	Sensitivity to the age of the portfolio	142
7.2.2.3	Stress Test	144
7.3	Conclusion	147
General Conclusion		149

Bibliography	150
A Qualifying Criteria for Standardized Approach for Operational Risks	157
B Business Lines and Event types	159
C Detailed loss event type classification	161
D Moments calculation for the Generalized Pareto Distribution	163
E Derivation of MLE Estimators for LogNormal Parameters	165
F Simple Credibility Model	167
G Jeffreys prior distribution	168
H MCMC Metropolis-Hastings algorithm	169
H.1 Applying MCMC with Metropolis-Hastings algorithm for λ	169
H.2 Applying MCMC with Metropolis-Hastings algorithm for μ	169
H.3 Applying MCMC with Metropolis-Hastings algorithm for $\omega = (\mu, \sigma)$	170
I The Inverse of Fisher Information Matrix	171
J The Inverse of Fisher Information Matrix for Nelson-Siegel Maximum Likelihood Estimation Technique	173

General Introduction

Choice of the study subject

Operational Risk Management policies formalizes the Financial Institutions' approaches to operational risk management. It is meant at a minimum to comply with the qualifying qualitative criteria of the Basel Capital Accord (cf. BCBS [2005]), and Solvency Directive requirements (cf. CEIOPS [2009]), as well as local and cross-border regulatory requirements for defining, measuring and managing operational risk.

A set of fundamental behavioral, organizational and operational principles that embody the importance given by financial institutions in managing operational risks, help facilitate the development of a culture in which managing and mitigating operational risks is viewed as everybody's responsibility. A wide variety of definitions are used to describe operational risk of which the following is just a sample:

- All types of risk other than credit and market risk.
- The risk of loss due to human error or deficiencies in systems or controls.
- The risk that a firm's internal practices, policies and systems are not rigorous or sophisticated enough to cope with unexpected market conditions or human or technological errors.
- The risk of loss resulting from errors in the processing of transactions, breakdown in controls and errors or failures in system support.

Basel II Committee has defined operational risk as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events (cf. BCBS, Definition of Operational Risk [2001]). For example, an operational risk could be losses due to an IT failure; transactions errors; or external events like a flood, an earthquake or a fire such as the one at Crédit Lyonnais in May 1996 which resulted in extreme losses. Currently, the lack of operational risk loss data is a major issue on hand but once the data sources become available, a collection of methods will be progressively implemented.

Hence, since operational risk is a sort of a carryall including all types of risk other than the ones explicitly taken in other modules of risk like credit and market risk, it is essential for regulators to propose some modeling techniques for the different types of losses encountered. We have to mention as well, that regardless of the method chosen for the measurement of the capital requirement for this risk, financial institutions must prove that its measures are highly solid and reliable. Yet, not taking into consideration the complexity of some models and the estimation errors behind will create some additional instability and will have severe influences on our Solvency Capital Requirement. Based on operational risk definition, estimation risk is to be treated as a part of this risk. The present study fits within this frame of reference.

Context of the study

Sensitive to the need for suitable tools to develop operational risk, in 2001, the Basel Committee started a series of surveys and statistics regarding operational risks that most banks encounter. The idea was to develop and correct measurements and calculation methods. Additionally, the European Commission also started preparing for the Solvency II Directive, taking into consideration the operational risk for insurance and reinsurance companies.

As so, and since Basel and Solvency accords set forth many calculation criteria, our interest in this work, which was developed in parallel with my work on the PhD thesis at bank Audi Lebanon and at the research laboratory of ISFA, Claude Bernard University-Lyon, is to elaborate the different quantitative measurement techniques for operational risk in financial institutions, (particularly in Banks and Insurance companies). We are going to present the associated mathematical and actuarial concepts as well as a numerical application regarding the Advanced Measurement Approaches, and focus on the qualitative part of operational risk, mostly the potential for large, unexpected losses, either on a per event basis or within a set time period. Hence, pointing out the importance of both qualitative and quantitative measurements and highlighting the necessity of an operational risk framework. Furthermore, we direct our study work to a more specific type of operational risk which is the estimation risk. We explored such risk to some extent by the use of scenario analysis based on expert opinion in conjunction with internal data both used to evaluate our exposure to events. In addition, the study work includes some reflections regarding the measurement of the error induced on the SCR through the estimation error of the parameters. Hence, revealing the importance of calling attention to the reflections on assumptions of the calculations.

In practice, this work attempts to present the different modeling tools for assessing operational risk and more particularly, pointing up the consequences of estimation risk behind. This draws the attention to the conclusion that it would be more appropriate and effective in reality to privilege more simple and prudent models than to complicate things and generate additional errors and instability.

Organization of the study

The main objective of this thesis is to highlight the importance of estimation risk treated as a particular case of operational risk. The study is organized in two parts:

- The first part provides a full overview of operational risk in a bank and insurance company, from the regulatory laws and regulations to the associated mathematical and actuarial concepts.
- The second part is dedicated to the consequences of estimation risk in model development.

The first part is divided into four chapters: Laws and Regulations, Quantitative Methodologies, Combining Internal Data with Scenario Analysis and Operational Risk Management in a Bank. This part features the different perspectives of operational risk, that have risen to the point of holding a significant position in risk assessment, as seen by the fact that many banking failures in the last 20 years have demonstrated the serious dangers of operational risk events. This has incited the regulators of both the Basel Accords for banks and the Solvency Directive for insurance companies, to propose some quantification methods of operational risk to help financial institutions ensure its mitigation.

As so, many calculation criteria have been developed, ranging from the Basic, Standardized reaching the Advanced Measurement Approach. Through this part, operational risk has been defined as per the different theories and approaches presented for financial institutions. While the standardized approach is widely applied by banks and insurance companies, this study shows that applying more advanced approaches and theories such as Loss Distribution, Extreme Value Theory, or Bayesian updating techniques may present more robust analyses and framework to model this risk. Additionally, we focus on the Bayesian inference approach which offers a methodical concept that combines internal data with experts' opinions. Joining these two elements with precision is certainly one of the challenges in operational risk. We are interested in applying a robust Bayesian inference technique to estimate an operational risk capital requirement that best approaches the reality. In addition, we illustrate the importance of consistent scenario analysis in showing how the expert opinion coherence is a major factor for capital calculations, since it creates an estimation risk that highly influences capital requirement.

At the end, we emphasize the qualitative management of operational risk in a Lebanese bank (Bank Audi) summarized by: Risk and Control Self Assessment (RCSA), Incident reporting, Key Risk Indicators (KRIs) and the Incorporation of External Data. We are going as well to assess the use of insurance policies as an option to transfer a part of the risk to the insurance company. This will lead us to justify how insurance policies play an important role in decreasing the financial impact of operational losses and can therefore contribute to a better performance by covering a variety of potential operational losses. Furthermore, we point out the effects of the increased use of insurance against major operational risk factors, and incorporate these in the performance analyses. The Basel Committee recognizes this potential and has accordingly allowed a reduction in the required minimum capital for a particular risk category for any institution possessing an insurance policy. In the long run, this regulation would allow banks to replace operational risk with counterparty risk.

Moreover, since the use of external data is absolutely important to the implementation of an advanced method for calculating operational risk capital such as the LDA method, we scale the severity of external losses for integrating them with internal data; a similar approach was published by DAHEN & DIONNE [2008]. We resume this section by illustrating three examples of losses extracted from our external loss database to show the details of how the scaling was done and how losses were normalized.

The second part, starts by an overview of the analytical framework, we will be more interested in the consequences of estimation risk, which according to the first part,

is considered as a particular case of operational risk. This part is divided into three chapters: Estimation Errors and SCR Calculation, An Alternative Yield Curve Estimation and Market Consistency and Economic Evaluation.

In the first chapter of this part, we will intent to measure the error induced on the SCR by the estimation error of the parameters. We expand this analytical framework where an insurer must calculate a VaR to a confidence level of 99.5% on a distribution which we must estimate the parameters. This estimation might lead to important differences in the numerical results. To be able to illustrate this situation we took the a particular case of the only market risk for an asset consisting of a zero coupon bond, and we highlight the possible undervaluation of the Solvency Capital Requirement if a special attention is not given to the risk of parameter estimation.

In the second chapter, we present a new approach for fitting a yield curve, that leads to a much more robust assessment of risk. We will propose a new method of calibration by using Nelson-Siegel Maximum Likelihood Estimation technique (MLE) then we show that the estimation risk is low. Thus, eliminating the potential loss of accuracy from estimation error when calculating a Value-at-Risk.

At the end, the last chapter reveals the importance of calling attention to the reflections on assumptions of the calculations and since Solvency II has chosen a framework for the evaluation of technical provisions consistent with market values: the interest rate curve is to be used for discounting purposes. The implementation of this framework in practice leads to important volatility assessments that might have high consequences on balances. Based on this, we illustrate a simple proposal of correction, where we apply the moving average technique and try to reconstruct a Yield Curve in a way, to stabilize the volatility and smooth our curve.

The work carried out within the framework of this thesis, has been subject for the following publications ¹:

KARAM E. & PLANCHET F. [2012], *Operational Risks in Financial Sectors*, Advances in Decision Sciences, Vol. 2012, Article ID 385387, 57 pages. doi:10.1155/2012/385387

KARAM E. & PLANCHET F. [2013a], *Combining internal data with scenario analysis*, Cahiers de recherche de l'ISFA, 2013.3, (submitted to Advances in Decision Sciences)

KARAM E. & PLANCHET F. [2013b], *Estimation Errors and SCR Calculation*, Bulletin Français d'Actuariat, Vol. 13 n° 26, juin - décembre 2013, pp. 79-92, (paper presented at the AFIR Colloquium on June 25 2013)

¹cf. www.ressources-actuarielles.net

Part I

Measuring and Managing Operational Risk in a Bank

Part I - Introduction

Operational risk existed longer than we know, but its concept was not interpreted until after the year 1995 when one of the oldest banks in London, Barings bank, collapsed because of Nick Leeson, one of the traders, due to unauthorized speculations. A wide variety of definitions are used to describe operational risk. The Basel Committee, however, defined operational risk as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events (cf. BCBS, Definition of Operational Risk [2001b]). For example, an operational risk could be losses due to an IT failure; transactions errors; or external events like a flood, an earthquake or a fire such as the one at Crédit Lyonnais in May 1996 which resulted in extreme losses.

In 2001, the Basel Committee started a series of surveys and statistics regarding operational risks that most banks encounter. The idea was to develop and correct measurements and calculation methods. Additionally, the European Commission also started preparing for the new Solvency II Accord, taking into consideration the operational risk for insurance and reinsurance companies.

As so, and since Basel and Solvency accords set forth many quantitative and qualitative criteria, the main idea in this first part is to set the grounds of operational risk, we are going to present an overview of this risk, from the regulatory laws and regulations to the associated mathematical and actuarial concepts. The chapters in this part are related to the published article Operational risks in Financial Sectors, Advances in Decision Sciences (cf. KARAM & PLANCHET [2012]).

Chapter 1

Laws and Regulations

Basel II cites three ways of calculating the capital charges required in the first pillar of operational risk. The three methods, in increasing order of sophistication, are as follows:

- The Basic Indicator Approach (BIA)
- The Standardized Approach (SA)
- The Advanced Measurement Approach (AMA)

Regardless of the method chosen for the measurement of the capital requirement for operational risk, the bank must prove that its measures are highly solid and reliable. Each of the three approaches have specific calculation criteria and requirements, as explained in the following sections.

1.1 Basic Indicator and Standardized Approach

Banks using the BIA method have a minimum operational risk capital requirement equal to a fixed percentage of the average annual gross income over the past three years. Hence, the risk capital under the BIA approach for operational risk is given by:

$$K_{BIA} = \frac{\alpha}{Z} \sum_{i=1}^3 \max(GI^i, 0)$$

Where, $Z = \sum_{i=1}^3 I_{\{GI^i > 0\}}$, GI^i stands for gross income in year i , and $\alpha = 15\%$ is set by the Basel Committee. The results of the first two Quantitative Impact Studies (QIS) conducted during the creation of the Basel Accord showed that on average 15% of the annual gross income was an appropriate fraction to hold as the regulatory capital. Gross income is defined as the net interest income added to the net non-interest income. This figure should be gross of any provisions (unpaid interest), should exclude realized profits and losses from the sale of securities in the banking book, which is an accounting book that includes all securities that are not actively traded by the institution, and exclude extraordinary or irregular items. No specific criteria for the use of the Basic Indicator Approach are set out in the Accord.

The Standardized Approach

In the Standardized Approach, banks' activities are divided into 8 business lines (cf. BCBS [2005]): corporate finance, trading & sales, retail banking, commercial banking, payment & settlements, agency services, asset management, and retail brokerage. Within each business line, there is a specified general indicator that reflects the size of the banks' activities in that area. The capital charge for each business line is calculated by multiplying gross income by a factor β assigned to a particular business line.

Business line (j)	Beta factors(β_j)
$j = 1$, corporate finance	18%
$j = 2$, trading & sales	18%
$j = 3$, retail banking	12%
$j = 4$, commercial banking	15%
$j = 5$, payment & settlement	18%
$j = 6$, agency services	15%
$j = 7$, asset management	12%
$j = 8$, retail brokerage	12%

Table 1.1: Business lines and the Beta factors

As in the Basic Indicator Approach, the total capital charge is calculated as a three year average over all positive gross income (GI) as follows:

$$K_{SA} = \frac{\sum_{i=1}^3 \max(\sum_{j=1}^8 \beta_j GI^i, 0)}{3}$$

The second QIS issued by the Basel Committee, covering the same institutions surveyed in the first study, resulted in 12%, 15% and 18% as appropriate rates in calculating regulatory capital as a percentage of gross income. The general criteria for using the Standard Approach are given in the Appendix A.

Before tackling the third Basel approach (AMA), we give a simple example to illustrate the calculation for the first two approaches.

1.1.1 Example of the BIA and SA Calculations

In the table 1.2 above, we see the basic and standardized approach for the 8 business lines. The main difference between the BIA and the SA is that the former does not distinguish its income by business lines. As shown in the tables, we have the annual gross incomes related to year 3, year 2 and year 1. With the Basic Approach, we do not segregate the income by business lines, and therefore, we have a summation at the bottom. We see that three years ago, the bank had a gross income of around 132 million which then decreased to -2 million the following year, and finally rose to 71 million. Moreover, the

Basic Indicator Approach (BIA)				Standardized Approach (SA)				
<i>Gross Income (GI) per million of €</i>								
Business lines	t-3	t-2	t-1	Beta	t-3	t-2	t-1	
Corporate finance	20.00€	-14.00€	-1.00€	18%	3.60€	-2.52€	-0.18€	
Trading & Sales	19.00€	3.00€	18.00€	18%	3.42€	0.54€	3.24€	
Retail banking	14.00€	-15.00€	18.00€	12%	1.68€	-1.80€	2.16€	
Commercial banking	16.00€	10.00€	11.00€	15%	2.40€	1.50€	1.65€	
Payments & settlements	17.00€	-8.00€	10.00€	18%	3.06€	-1.44€	1.80€	
Agency services	18.00€	13.00€	13.00€	15%	2.70€	1.95€	1.95€	
Asset management	16.00€	4.00€	-4.00€	12%	1.92€	0.48€	-0.48€	
Retail brokerage	12.00€	5.00€	6.00€	12%	1.44€	0.60€	0.72€	
Bank	132.00€	-2.00€	71.00€		20.22€	-0.69€	10.86€	
Treat negatives	132.00€		71.00€		20.22€	0.00	10.86€	
<i>Average of the 3 years excluding negatives:</i>			101.50€					
<i>Alpha(α):</i>			15%					
Capital requirement under BIA				15.23€	Capital requirement under SA			10.36€

Table 1.2: Simple example related to the BIA and SA calculation criteria

Basic Indicator Approach doesn't take into consideration negative gross incomes. So, in treating the negatives, the -2 million was removed. To get our operational risk charge, we calculate the average gross income excluding negatives and we multiply it by an alpha factor of 15% set by the Basel Committee. We obtain a result of 15.23 million €.

Similarly to the BI Approach, the Standardized Approach has a Beta factor for each of the business lines as some are considered riskier in terms of operational risk than others. Hence, we have eight different factors ranging between 12 and 18 percent as determined by the Basel Committee. For this approach, we calculate a weighted average of the gross income using the business line betas. Any negative number over the past years is converted to zero before an average is taken over the three years. In this case, we end up with a capital charge of around 10.36 million €.

1.1.2 The Capital Requirement Under the Basic Indicator and Standardized Approach

As depicted in the previous example, the capital charge relating to the Standardized Approach was lower than that of the Basic Approach. This, however, is not always the case, thus causing some criticism and raising questions such as why would a bank use a more sophisticated approach when the simpler one would cost them less?

In this section, we show that the capital charge could vary between different approaches.

To start with, let $K_{BIA} = \alpha GI$ and $K_{SA} = \sum_{i=1}^8 \beta_i GI_i$,

where $\alpha = 15\%$, GI_i is the gross income related to the business line i , and GI is the total gross income.

Compiling these equations, we have:

$$K_{BIA} > K_{SA} \Leftrightarrow \alpha GI > \sum_{i=1}^8 \beta_i GI_i$$

and, consequently:

$$\alpha > \frac{\sum_{i=1}^8 \beta_i GI_i}{GI} \quad (1.1)$$

Therefore, the BIA produces a higher capital charge than the SA is under the condition that the alpha factor under the former is greater than the weighted average of the individual betas under the latter.

There is no guarantee that the condition will be satisfied, which means that moving from the BIA to the SA may or may not produce a lower capital charge (cf. Moosa [2008]).

1.2 Capital Requirement Review

Several Quantitative Impact Studies (QIS) have been conducted for a better understanding of operational risk significance on banks and the potential effects of the Basel II capital requirements. During 2001 and 2002, QIS 2, QIS 2.5 and QIS 3 were carried out by the committee using data gathered across many countries. Furthermore, to account for national impact, a joint decision of many participating countries resulted in the QIS 4 being undertaken. In 2005, to review the Basel II framework, BCBS implemented QIS 5.

Some of these quantitative impact studies have been accompanied by operational Loss Data Collection Exercises (LDCE). The first two exercises conducted by the Risk Management Group of BCBS on an international basis are referred to as the 2001 LDCE and 2002 LDCE. These were followed by the national 2004 LDCE in USA and the 2007 LDCE in Japan.

Detailed information on these analyses can be found on the BCBS web site: www.bis.org.

Before analyzing the quantitative approaches, let's take a look at the minimum regulatory capital formula and definition (cf. BCBS [2002]).

Total risk-weighted assets are determined by multiplying capital requirements for market risk and operational risk by 12.5, which is a scaling factor determined by the Basel Committee, and adding the resulting figures to the sum of risk-weighted assets for credit risk. The Basel II committee defines the minimum regulatory capital as 8% of the total risk-weighted assets, as shown in the formula below:

$$\frac{\text{Total regulatory capital}}{RWA_{Credit} + [MRC_{Market} + ORC_{Op}] * 12.5} \geq 8\%$$

$$\text{Minimum regulatory capital} = 8\%[RWA_{Credit} + (MRC_{Market} + ORC_{Opr}) * 12.5]$$

The Committee applies a scaling factor in order to broadly maintain the aggregate level of minimum capital requirements while also providing incentives to adopt the more advanced risk-sensitive approaches of the framework.

The Total Regulatory Capital has its own set of rules according to 3 tiers:

- The first tier, also called the core tier, is the core capital including equity capital and disclosed reserves.
- The second tier is the supplementary capital which includes items such as general loss reserves, undisclosed reserves, subordinated term debt, etc.
- The third tier covers market risk, commodities risk, and foreign currency risk.

The Risk Management Group (RMG) has taken 12% of the current minimum regulatory capital as its starting point for calculating the basic and standardized approach.

The Quantitative Impact Study (QIS) survey requested banks to provide information on their minimum regulatory capital broken down by risk type (credit, market, and operational risk) and by business line. Banks were also asked to exclude any insurance and non-banking activities from the figures. The survey covered the years 1998 to 2000.

Overall, more than 140 banks provided some information on the operational risk section of the QIS. These banks included 57 large, internationally active banks (called type 1 banks in the survey) and more than 80 smaller type 2 banks from 24 countries. The RMG used the data provided in the QIS to gain an understanding of the role of operational risk capital allocations in banks and their relationship to minimum regulatory capital for operational risk. These results are summarized in the table below 1.3:

	Median	Mean	Min	25th %	75th %	Max	N
Operational Risk Capital/Overall Economic Capital	0.150	0.149	0.009	0.086	0.197	0.351	41
Operational risk capital/Minimum Regulatory Capital	0.128	0.153	0.009	0.074	0.17	0.876	41

Table 1.3: Ratio of Operational Risk Economic Capital to Overall Economic Capital and to Minimum Regulatory Capital

The results suggest that on average, operational risk capital represents about 15 percent of overall economic capital, though there is some dispersion. Moreover, operational risk capital appears to represent a rather smaller share of minimum regulatory capital over 12% for the median.

These results suggest that a reasonable level of the overall operational risk capital charge would be about 12 percent of minimum regulatory capital. Therefore, a figure of 12% chosen by the Basel Committee for this purpose is not out of line with the proportion of internal capital allocated to operational risk for most banking institutions in the sample.

1.2.1 The Basic Indicator Approach

Under the BIA approach, regulatory capital for operational risk is calculated as a percentage α of a bank’s gross income. The data reported in the QIS concerning banks’ minimum regulatory capital and gross income were used to calculate individual alphas for each bank for each year from 1998 to 2000 to validate the 12% level of minimum regulatory capital (cf. BCBS [2001a]).

The calculation was:

$$\alpha_{j,t} = \frac{12\% * MRC_{j,t}}{GI_{j,t}}$$

Here, $MRC_{j,t}$, is the minimum regulatory capital for bank j in year t and $GI_{j,t}$ is the gross income for bank j in year t. Given these calculations, the results of the survey are reported in table 1.4 below:

Individual Observations	Median	Mean	WA	Std	WA Std	Min	Max	25th %	75th %	N
All Banks	0.190	0.221	0.186	0.135	0.120	0.019	0.831	0.137	0.246	355
Type 1 Banks	0.168	0.218	0.183	0.136	0.121	0.048	0.659	0.136	0.225	151
Type 2 Banks	0.205	0.224	0.220	0.134	0.111	0.019	0.831	0.139	0.253	204

Table 1.4: Analysis of QIS data: BI Approach (Based on 12% of Minimum Regulatory Capital)

Table 1.4 presents the distribution in two ways - the statistics of all banks together, and the statistics according to the two types of banks by size. The first three columns of the table contain the median, mean and the weighted average of the values of the alphas (using gross income to weight the individual alphas). The median values range between 17% and 20% with higher values for type 2 banks. The remaining columns of the table present information about the dispersion of alphas across banks.

These results suggest that an alpha range of 17% to 20% would produce regulatory capital figures approximately consistent with an overall capital standard of 12% of minimum regulatory capital. However, after testing the application of this alpha range, the Basel Committee decided to reduce the factor to 15% because an alpha of 17 to 20 percent resulted in an excessive level of capital for many banks.

1.2.2 The Standardized Approach

As seen previously, the minimum capital requirement for operational risk under the Standardised Approach is calculated by dividing a bank’s operations into eight business lines. For each business line, the capital requirement will be calculated according to a certain percentage of gross income attributed for that business line.

The QIS data concerning distribution of operational risk across business lines was used and, as with the Basic Approach, the baseline assumption was that the overall level of operational risk capital is at 12% of minimum regulatory capital. Then, the business line capital was divided by business line gross income to arrive at a bank-specific β for that business line, as shown in the following formula:

$$\beta_{j,i} = \frac{12\% * MRC_j * OpRiskShare_{j,i}}{GI_{j,i}}$$

Where, $\beta_{j,i}$ is the beta for bank j in business line i , MRC_j is the minimum regulatory capital for the bank, $OpRiskShare_{j,i}$ is the share of bank j 's operational risk economic capital allocated to business line i , and $GI_{j,i}$ is the gross income in business line i for bank j .

In the end, 30 banks reported data on both operational risk economic capital and gross income by business line, but only the banks that had reported activity in a particular business line were included in the line's beta calculation (i.e., if a bank had activities related to six of the eight business lines, then it was included in the analysis for those six business lines).

The results of this analysis are displayed in the table 1.5 below:

	Median	Mean	WA	Std	WA Std	Min	Max	25th %	75th %	N
Corporate Finance	0.131	0.236	0.12	0.249	0.089	0.035	0.905	0.063	0.361	19
Trading & Sales	0.171	0.241	0.202	0.183	0.129	0.023	0.775	0.123	0.391	26
Retail Banking	0.125	0.127	0.110	0.127	0.006	0.008	0.342	0.087	0.168	24
Commercial Banking	0.132	0.169	0.152	0.116	0.096	0.048	0.507	0.094	0.211	27
Payment & Settlement	0.208	0.203	0.185	0.128	0.068	0.003	0.447	0.1	0.248	15
Agency Services & Custody	0.174	0.232	0.183	0.218	0.154	0.056	0.901	0.098	0.217	14
Retail Brokerage	0.113	0.149	0.161	0.073	0.066	0.05	0.283	0.097	0.199	15
Asset Management	0.133	0.185	0.152	0.167	0.141	0.033	0.659	0.079	0.210	22

Table 1.5: Analysis of QIS data: the Standardized Approach (Based on 12% of Minimum Regulatory Capital)

The first three columns of the table 1.5 present the median, mean and weighted average values of the betas for each business line, and the rest of the columns present the dispersion across the sample used for the study. As with the Basic Approach, the mean values tend to be greater than the median and the weighted average values, thus reflecting the presence of some large individual Beta estimates in some of the business lines.

Additionally, the QIS ranked the betas according to the business lines with "1" representing the smallest beta and "8" the highest. The table 1.6 below depicts this ranking, and we see that Retail Banking tends to be ranked low while Trading & sales with Agency Services & Custody tend to be ranked high.

Table 1.6 shows us the disparity that exists of "typical" beta by business line in columns 4 to 9 and so, we want to find out whether this dispersion allows us to separate the different

	Median	Mean	Weighted Average
Corporate Finance	3	7	2
Trading & Sales	6	8	8
Retail Banking	2	1	1
Commercial Banking	4	3	7
Payment & Settlement	8	5	7
Agency Services & Custody	7	6	6
Retail Brokerage	1	2	5
Asset Management	5	4	3

Table 1.6: Size Ranking Across Three Measures of "Typical" Beta by Business Lines

beta values across business lines. Through statistical testing of the equality of the mean and the median, the results do not reject the null hypothesis that these figures are the same across the eight business lines.

These diffusions observed in the beta estimate could be reflected in the calibration difference of the internal economic capital measures of banks. Additionally, banks may also be applying differing definitions of the constitution of operational risk loss and gross income as these vary under different jurisdictions. Given additional statistics and data, the Basel Committee decided to estimate the beta factors between 12% to 18% for each of the different business lines.

1.3 The Advanced Measurement Approach

With the Advanced Measurement Approach (AMA), the regulatory capital is determined by a bank's own internal operational risk measurement system according to a number of quantitative and qualitative criteria set forth by the Basel Committee. However, the use of these approaches must be approved and verified by the national supervisor.

The AMA is based on the collection of loss data for each event type. Each bank is to measure the required capital based on its own loss data using the holding period and confidence interval determined by the regulators (1 year and 99.9%).

The capital charge calculated under the AMA is initially subjected to a floor set at 75% of that under the Standardized Approach, at least until the development of measurement methodologies is examined. In addition, the Basel II Committee decided to allow the use of insurance coverage to reduce the capital required for operational risk, but this allowance does not apply to the SA and the BIA.

A bank intending to use the AMA should demonstrate accuracy of the internal models within the Basel II risk cells, (eight business lines \times seven risk types, as shown in Appendix B and more details for the loss event type classification in Appendix C), rele-

vant to the bank and satisfy some criteria including:

- The use of the internal data, relevant external data, scenario analyses and factors reflecting the business environment and internal control systems;
- Scenario analyses of expert opinion;
- The risk measure used for capital charge should correspond to a 99.9% confidence level for a one-year holding period;
- Diversification benefits are allowed if dependence modelling is approved by a regulator;
- Capital reduction due to insurance is fixed at 20%.

The relative weight of each source and the combination of sources is decided by the banks themselves; Basel II does not provide a regulatory model.

The application of the AMA is, in principle, open to any proprietary model, but the methodologies have converged over the years and thus specific standards have emerged. As a result, most AMA models can now be classified into:

- Loss Distribution Approach (LDA)
- Internal Measurement Approach (IMA)
- Scenario-Based AMA (sbAMA)
- Scocard Approach (SCA)

1.3.1 The Loss Distribution Approach (LDA)

The Loss Distribution Approach (LDA) is a parametric technique primarily based on historic observed internal loss data (potentially enriched with external data). Established on concepts used in actuarial models, the LDA consists of separately estimating a frequency distribution for the occurrence of operational losses and a severity distribution for the economic impact of the individual losses. The implementation of this method can be summarized by the following steps:

- 1. Estimate the loss severity distribution
- 2. Estimate the loss frequency distribution
- 3. Calculate the capital requirement
- 4. Incorporate the experts' opinions

For each business line and risk category, we establish two distributions (cf. Dahan [2006]): one related to the frequency of the loss events for the time interval of one year (the loss frequency distribution), and the other related to the severity of the events (the loss severity distribution).

To establish these distributions, we look for mathematical models that best describe the two distributions according to the data and then we combine the two using Monte-Carlo simulation to obtain an aggregate loss distribution for each business line and risk type. Finally, by summing all the individual VaRs calculated at 99.9%, we obtain the capital required by Basel II.

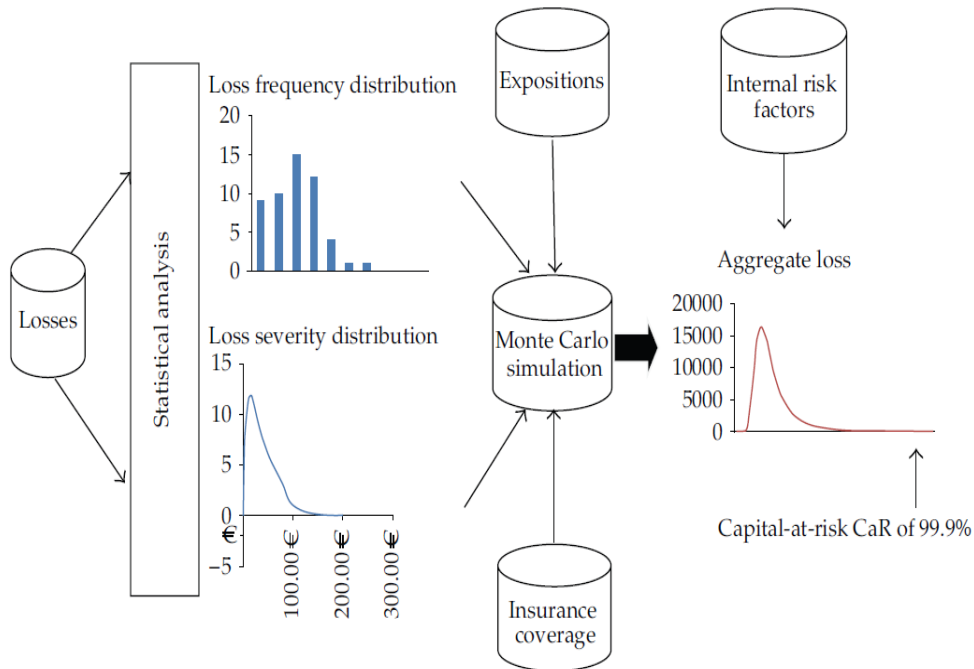


Figure 1.1: Illustration of the Loss Distribution Approach method (LDA) (cf. Maurer [2007])

We start with defining some technical aspects before demonstrating the LDA (cf. Maurer [2007]).

Definition 1

Value at Risk OpVaR: The capital charge is the 99.9% quantile of the aggregate loss distribution. So, with N as the random number of events, the total loss is

$$L = \sum_{i=0}^N \psi_i \text{ where } \psi_i \text{ is the } i^{\text{th}} \text{ loss amount. The capital charge would then be:}$$

$$P(L > OpVaR) = 0.1\%$$

Definition 2

OpVaR unexpected loss: This is the same as the Value at Risk OpVaR while

adding the expected and the unexpected loss. Here, the Capital charge would result in:

$$\mathbb{P}(L > UL + EL) = 0.1\%$$

Definition 3

OpVar beyond a threshold: The capital charge in this case would be a 99.9% quantile of the total loss distribution defined with a threshold H as

$$\mathbb{P}\left(\sum_{i=0}^N \psi_i \times \mathbb{I}\{\psi_i \geq H\} > OpVaR\right) = 0.1\%$$

The three previous methods are calculated using a Monte Carlo simulation.

For the LDA method which expresses the aggregate loss regarding each business line \times event type L_{ij} as the sum of individual losses, the distribution function of the aggregate loss, noted as \mathbf{F}_{ij} , would be a compound distribution (cf. Frachot *et al.* [2001]).

So, the Capital-at-Risk (CaR) for the business line i and event type j (shown in Appendix B) correspond to the α quantile of \mathbf{F}_{ij} as follows:

$$CaR_{ij}(\alpha) = \mathbf{F}_{ij}^{-1}(\alpha) = \inf\{x | \mathbf{F}_{ij}(x) \geq \alpha\}$$

And, as with the second definition explained previously, the CaR for the element ij is equal to the sum of the expected loss (EL) and the unexpected Loss (UL):

$$CaR_{ij}(\alpha) = EL_{ij} + UL_{ij}(\alpha) = \mathbf{F}_{ij}^{-1}(\alpha)$$

Finally, by summing all the the capital charges $CaR_{ij}(\alpha)$, we get the aggregate CaR across all business lines and event types:

$$CaR(\alpha) = \sum_{i=1}^I \sum_{j=1}^J CaR_{ij}(\alpha)$$

The Basel committee fixed an $\alpha = 99.9\%$ to obtain a realistic estimation of the capital required. However, the problem of correlation remains an issue here as it is unrealistic to assume that the losses are not correlated. For this purpose, Basel II authorized each bank to take correlation into consideration when calculating operational risk capital using its own internal measures. For more on operational risk and modelling dependence see GICAMOTTI *et al.* [2008], PETERS *et al.* [2009] and MITTNIC *et al.* [2013].

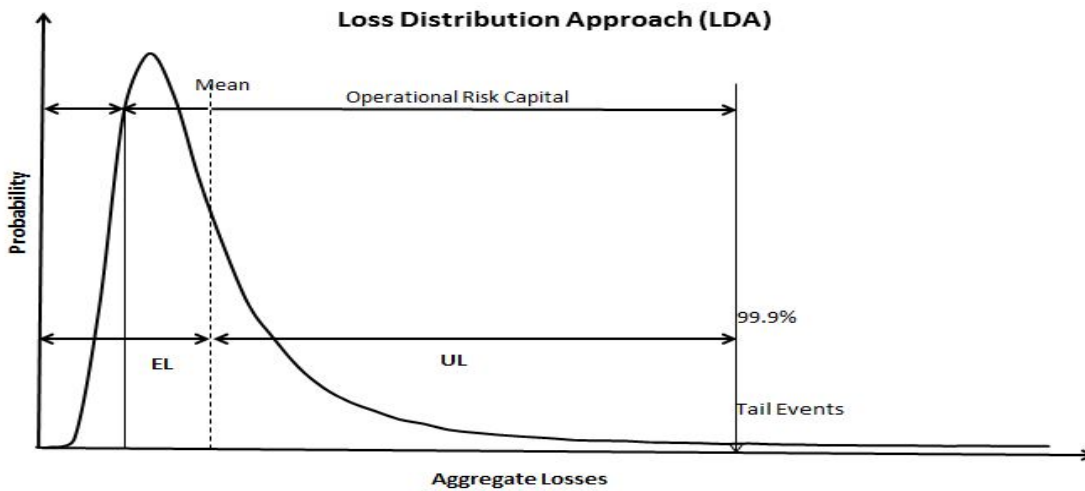


Figure 1.2: Operational Risk Capital-at-Risk (CaR) (cf. Maurer [2007])

1.3.2 Internal Measurement Approach (IMA)

The IMA method (cf. BCBS [2001b]), provides carefulness to individual banks on the use of internal loss data, while the method to calculate the required capital is uniformly set by supervisors. In implementing this approach, supervisors would impose quantitative and qualitative standards to ensure the integrity of the measurement approach, data quality, and the adequacy of the internal control environment.

Under the IM approach, capital charge for the operational risk of a bank would be determined using:

- A bank's activities are categorized into a number of business lines, and a broad set of operational loss types is defined and applied across business lines.
- Within each business line/event type combination, the supervisor specifies an exposure indicator (EI) which is a substitute for the amount of risk of each business line's operational risk exposure.
- In addition to the exposure indicator, for each business line/loss type combination, banks measure, based on their internal loss data, a parameter representing the probability of loss event (PE) as well as a parameter representing the loss given that event (LGE). The product of $EI * PE * LGE$ is used to calculate the Expected Loss (EL) for each business line/loss type combination.
- The supervisor supplies a factor γ for each business line/event type combination, which translates the expected loss (EL) into a capital charge. The overall capital charge for a particular bank is the simple sum of all the resulting products.

Let's reformulate all the points mentioned above; calculating the expected loss for each business line so that for a business line i and an event type j , the capital charge K is

defined as: $K_{ij} = EL_{ij} \times \gamma_{ij} \times RPI_{ij}$

Where EL represents the expected loss, γ is the scaling factor and RPI is the Risk Profile Index.

The Basel Committee on Banking Supervision proposes that the bank estimates the expected loss as follows:

$$EL_{ij} = EI_{ij} \times PE_{ij} \times LGE_{ij}$$

Where EI is the exposure indicator, PE is the probability of an operational risk event and LGE is the loss given event.

The committee proposes to use a risk profile index RPI as an adjustment factor to capture the difference of the loss distribution tail of the bank compared to that of the industry wide loss distribution.

The idea is to capture the leptokurtic properties of the bank loss distribution and then to transform the exogeneous factor γ into an internal scaling factor λ such that:

$$\begin{aligned} K_{ij} &= EL_{ij} \times \gamma_{ij} \times RPI_{ij} \\ &= EL_{ij} \times \lambda_{ij} \end{aligned}$$

By definition, the RPI of the industry loss distribution is one. If the bank loss distribution has a fatter tail than the industry loss distribution RPI would be larger than one. So two banks which have the same expected loss may have different capital charge because they do not have the same risk profile index.

1.3.3 Scorecard Approach (SCA)

The Scorecards approach¹ incorporates the use of a questionnaire which consists of a series of weighted, risk-based questions. The questions are designed to focus on the principal drivers and controls of operational risk across a broad range of applicable operational risk categories, which may vary across banks.

The questionnaire is designed to reflect the organization's unique operational risk profile by:

- Designing organization-specific questions that search for information about the level of risks and quality of controls.
- Calibrating possible responses through a range of "unacceptable" to "effective" to "leading practice".
- Applying customized question weightings and response scores aligned with the relative importance of individual risks to the organization. These can vary significantly between banks (due to business mix differences) and may also be customized along

¹http://www.fimarkets.com/pages/risque_operationnel.php

business lines within an organization. Note that scoring of response options will often not be linear.

The Basel Committee did not put any kind of mathematical equation regarding this method, but working with that method made banks propose a formula related which is:

$$K_{SCA} = EI_{ij} \times \omega_{ij} \times RS_{ij}$$

Where, EI is the exposure indicator, RS the risk score and ω the scale factor.

1.3.4 Scenario-Based AMA (sbAMA)

Risk is defined as the combination of severity and frequency of potential loss over a given time horizon, is linked to the evaluation of scenarios. Scenarios are potential future events. Their evaluation involves answering two fundamental questions: firstly, what is the potential frequency of a particular scenario occurring and secondly, what is its potential loss severity?

The scenario-based AMA² (or sbAMA) shares with LDA the idea of combining two dimensions (frequency and severity) to calculate the aggregate loss distribution used to obtain the OpVaR. Banks with their activities and their control environment, should build scenarios describing potential events of operational risks. Then experts are asked to give opinions on probability of occurrence (i.e., frequency) and potential economic impact should the events occur (i.e., severity); But Human judgment of probabilistic measures is often biased and a major challenge with this approach is to obtain sufficiently reliable estimates from experts. The relevant point in sbAMA is that information is only fed into a capital computation model if it is essential to the operational risk profile to answer the "what-if" questions in the scenario assessment. Furthermore the overall sbAMA process must be supported by a sound and structured organisational framework and by an adequate IT infrastructure. The sbAMA comprises six main steps, which are illustrated in the figure 1.3 below. Outcome from sbAMA shall be statistically compatible with that arising from LDA so as to enable a statistically combination technique. The most adequate technique to combine LDA and sbAMA is Bayesian inference, which requires experts to set the parameters of the loss distribution.

²<http://www.newyorkfed.org/newsevents/events/banking/2003/con0529d.pdf>

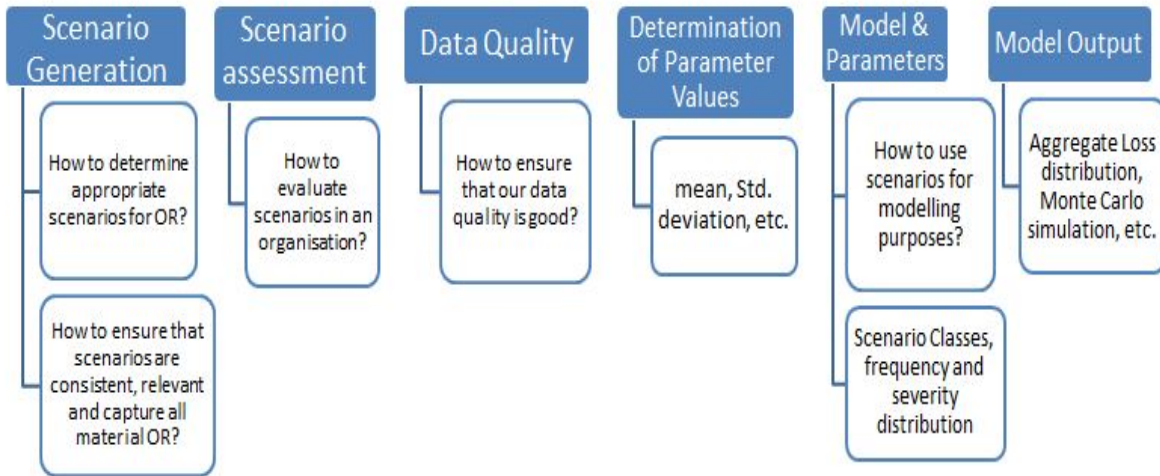


Figure 1.3: Overview of the sbAMA

1.4 Solvency II Quantification Methods

Solvency II imposes a capital charge for the operational risk that is calculated regarding the standard formula given by regulators or an internal model which is validated by the right authorities.

For the enterprises that have difficulties running an internal model for operational risk, the standard formula can be used for the calculation of this capital charge.

The European Insurance and Occupational Pensions Authority (EIOPA), previously known as the Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS), tests the standard formulas in markets through the use of surveys and questionnaires called Quantitative Impact Studies (QIS). The QIS allows the committee to adjust and develop the formulas in response to the observations and difficulties encountered by the enterprises.

Standard Formula Issued by QIS5

The Solvency Capital Requirement (SCR) concerns an organization’s ability to absorb significant losses through their own basic funds of an insurance or reinsurance policy. This ability is depicted by the company’s Value-at-Risk at a 99.5% confidence level over a one-year period and the objective is applied to each individual risk model to ensure that different modules of the standard formula are quantified in a consistent approach. Additionally, the correlation coefficients are set to reflect potential dependencies in the distributions’ tails. The breakdown of the SCR is shown in the figure 1.4 below:

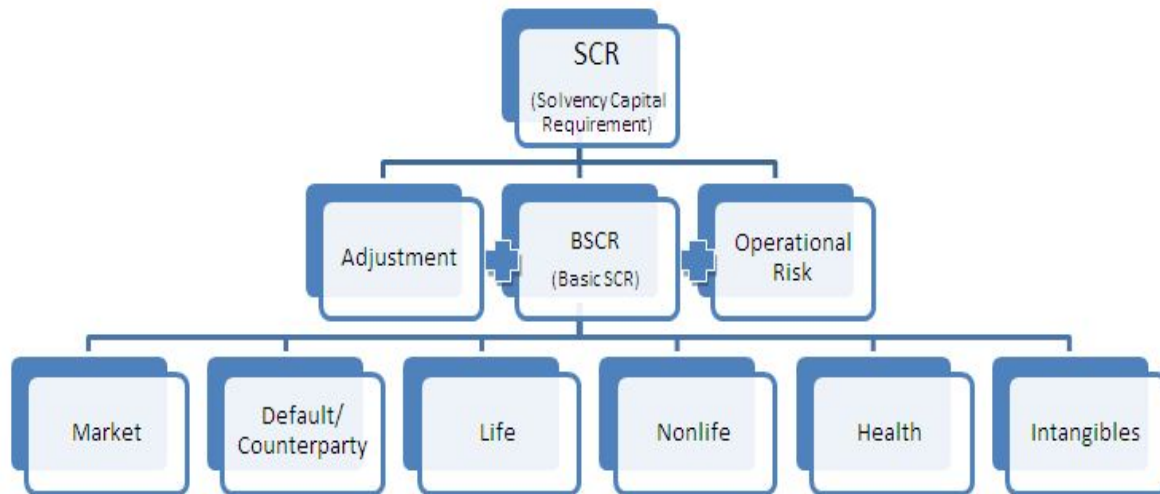


Figure 1.4: Solvency Capital Requirement (SCR)

With the calculation of the *BSCR*:

$$BSCR = \sqrt{\sum_{ij} Corr_{ij} \times SCR_i \times SCR_j} + SCR_{Intangibles}$$

Corr	Market	Default	Life	Health	Non-life
Market	1				
Default	0.25	1			
Life	0.25	0.25	1		
Health	0.25	0.25	0.25	1	
Non-life	0.25	0.5	0	0	1

Table 1.7: Correlation Matrix for the different risks

In relation to previous surveys, respondents suggested that:

- The operational risk charge should be calculated as a percentage of the BSCR or the SCR.
- The operational risk charge should be more sensitive to operational risk management.
- The operational risk charge should be based on entity-specific operational risk sources, the quality of the operational risk management process, and the internal control framework.

- Diversification benefits and risk mitigation techniques should be taken into consideration.

In view of the above, EIOPA has considered the following (cf. CEIOPS [2009]):

- The calibration of operational risk factors for the standard formula has been revised to be more consistent with the assessment obtained from internal models.
- A zero floor for all technical provisions has been explicitly introduced to avoid an undue reduction of the operational risk SCR.
- The Basic SCR is not a sufficiently reliable aggregate measure of the operational risk, and that a minimum level of granularity would be desirable in the design of the formula.

And so after additional analysis and reports, EIOPA recommends the final factors to be as follows:

TP life	0.45%
TP non life	3%
Premium life	4%
Premium non life	3%
UL factor	25%
BSCR cap life	30%
BSCR cap non life	30%

Table 1.8: QIS5 Factors

Before going into the formula let's define some notations (cf. CEIOPS [2010]):

- TP_{life} = Life insurance obligations. For the purpose of this calculation, technical provisions should not include the risk margin, should be without deduction of recoverables from reinsurance contracts and special purpose vehicles
- $TP_{non\ life}$ = Total non-life insurance obligations excluding obligations under non-life contracts which are similar to life obligations, including annuities. For the purpose of this calculation, technical provisions should not include the risk margin and should be without deduction of recoverables from reinsurance contracts and special purpose vehicles
- $TP_{life\ ul}$ = Life insurance obligations for life insurance obligations where the investment risk is borne by the policyholders. For the purpose of this calculation, technical provisions should not include the risk margin, should be without deduction of recoverables from reinsurance contracts and special purpose vehicle
- $pEarn_{life}$ = Earned premium during the 12 months prior to the previous 12 months for life insurance obligations, without deducting premium ceded to reinsurance

- $pEarn_{life\ ul}$ = Earned premium during the 12 months prior to the previous 12 months for life insurance obligations where the investment risk is borne by the policyholders, without deducting premium ceded to reinsurance
- $Earn_{life\ ul}$ = Earned premium during the previous 12 months for life insurance obligations where the investment risk is borne by the policyholders without deducting premium ceded to reinsurance
- $Earn_{life}$ = Earned premium during the previous 12 months for life insurance obligations, without deducting premium ceded to reinsurance
- $Earn_{non\ life}$ = Earned premium during the previous 12 months for non-life insurance obligations, without deducting premiums ceded to reinsurance
- Exp_{ul} = Amount of annual expenses incurred during the previous 12 months in respect life insurance where the investment risk is borne by the policyholders
- $BSCR$ = Basic SCR.

Finally the Standard formula resulted to be:

$$SCR_{op} = \min(0.3BSCR, Op_{all\ none\ ul}) + 0.25Exp_{ul}$$

Where, $Op_{all\ none\ ul} = \max(Op_{premiums}, Op_{provisions})$

$$Op_{premiums} = 0.04 * (Earn_{life} - Earn_{life\ ul}) + 0.03 * (Earn_{non\ life}) + \max(0, 0.04 * (Earn_{life} - 1.1pEarn_{life} - (Earn_{life\ ul} - 1.1pEarn_{life\ ul}))) + \max(0, 0.03 * (Earn_{non\ life} - 1.1pEarn_{non\ life}))$$

and:

$$Op_{provisions} = 0.0045 * \max(0, TP_{life} - TP_{life\ ul}) + 0.03 * \max(0, TP_{non\ life})$$

Chapter 2

Quantitative Methodologies

A wide variety of risks exist, thus necessitating their regrouping in order to categorize and evaluate their threats for the functioning of any given business. The concept of a risk matrix, coined by Richard Prouty (1960), allows us to highlight which risks can be modeled. Experts have used this matrix to classify various risks according to their average frequency and severity as seen in the figure 2.1 below:



Figure 2.1: Risk Matrix

There are in total four general categories of risk:

- Negligible risks: with low frequency and low severity, these risks are insignificant as they don't impact the firm very strongly.
- Marginal risks: with high frequency and low severity, though the losses aren't substantial individually, they can create a setback in aggregation. These risks are modeled by the Loss Distribution Approach (LDA) which we discussed earlier.
- Catastrophic risks: with low frequency and high severity, the losses are rare but have a strong negative impact on the firm and consequently, the reduction of these

risks is necessary for a business to continue its operations. Catastrophic risks are modeled using the Extreme Value Theory and Bayesian techniques.

- Impossible: with high frequency and high severity, the firm must ensure that these risks fall outside possible business operations to ensure financial health of the corporation.

Classifying the risks as per the matrix allows us to identify their severity and frequency and to model them independently by using different techniques and methods. We are going to see in the arriving sections the different theoretical implementation and application of different theories and models regarding Operational risk.

2.1 Risk Measures

Some of the most frequent questions concerning risk management in finance involve extreme quantile estimation. This corresponds to determining the value a given variable exceeds with a given (low) probability. A typical example of such a measure is the Value-at-Risk (VaR). Other less frequently used measures are the expected shortfall (ES) and the return level (cf. Gilli & Kellezi [2003]).

2.1.1 VaR calculation

A risk measure of the risk of loss on a specific portfolio of financial assets, VaR is the threshold value such that the probability that the mark-to-market loss on the portfolio over the given time horizon exceeds this value is the given probability level. VaR can then be defined as the q -th quantile of the distribution F :

$$VaR_q = F^{-1}(q)$$

Where F^{-1} is the quantile function which is defined as the inverse function of the distribution function F . For internal risk control purposes, most of the financial firms compute a 5% VaR over a one-day holding period.

2.1.2 Expected Shortfall

The expected shortfall is an alternative to VaR that is more sensitive to the shape of the loss distribution's tail. The expected shortfall at a $q\%$ level is the expected return on the portfolio in the worst $q\%$ of the cases:

$$ES_q = E(X \mid X > VaR_q)$$

2.2 Illustration of the LDA method

Even a cursory look at the operational risk literature reveals that measuring and modeling aggregate loss distributions are central to operational risk management. Since the daily business operations have considerable risk, quantification in terms of an aggregate loss distribution is an important objective. A number of approaches have been developed to calculate the aggregate loss distribution.

We begin this section by examining the severity distribution, the frequency distribution function and finally the aggregate loss distribution.

2.2.1 Severity of Loss Distributions

Fitting a probability distribution to data on the severity of loss arising from an operational risk event is an important task in any statistically based modeling of operational risk. The observed data to be modeled may either consist of actual values recorded by business line or may be the result of a simulation. In fitting a probability model to empirical data, the general approach is to first select a basic class of probability distributions and then find values for the distributional parameters that best match the observed data.

Following is an example of the Beta and Lognormal Distributions:

The standard Beta distribution is best used when the severity of loss is expressed as a proportion. Given a continuous random variable x , such that $0 \leq x \leq 1$, the probability density function of the standard beta distribution is given by $f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)}$

where

$$B(\alpha, \beta) = \int_0^1 u^{\alpha-1}(1-u)^{\beta-1} du, \quad \alpha > 0, \quad \beta > 0$$

The parameters α and β control the shape of the distribution.

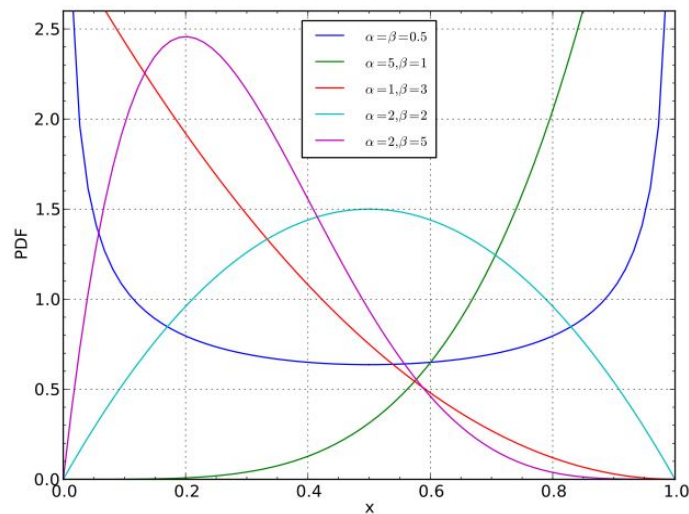


Figure 2.2: Loss severity of a Beta distribution

The mean of the beta distribution is given by

$$\text{Mean} = \frac{\alpha}{(\alpha + \beta)}$$

$$\text{and standard deviation} = \sqrt{\frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}}$$

In our example, we will be working with lognormal distributions (see Fig. 2.3). A log-normal distribution is a probability distribution of a random variable whose logarithm is normally distributed. So if X is a random variable with a normal distribution, then $Y = \exp(X)$ has a log-normal distribution. Likewise, if Y is Lognormally distributed, then $X = \log(Y)$ is normally distributed.

The probability density function of a log-normal distribution is:

$$f_X(x, \mu, \sigma) = \frac{1}{x\sigma} \sqrt{2\pi} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}$$

Where μ and σ are called the location and scale parameter, respectively. So, for a log-normally distributed variable X , $\mathbb{E}[X] = e^{-\frac{1}{2}\sigma^2}$ and $\text{Var}[X] = (e^{\sigma^2} - 1)e^{2\mu + \sigma^2}$

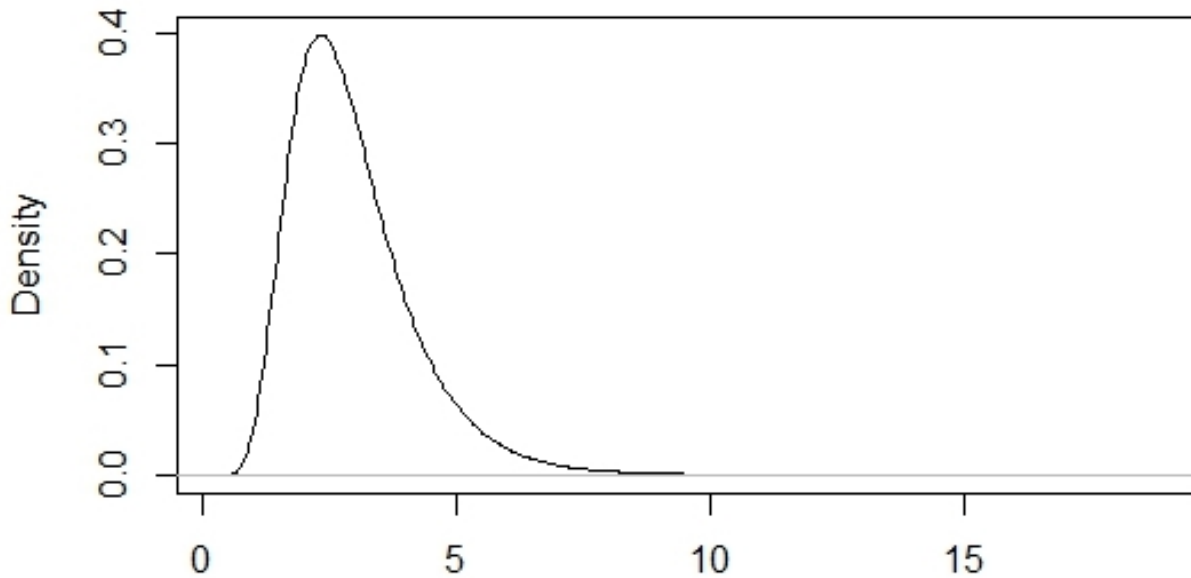


Figure 2.3: Loss severity of a Lognormal distribution

Statistical and Graphical Tests

There are numerous graphical and statistical tests for assessing the fit of a postulated severity of a loss probability model to empirical data. In this section, we focus on four of

the most general tests: Probability plots, Q-Q Plots, the Kolmogorov-Smirnov goodness of fit test, and the Anderson-Darling goodness of fit test. In discussing the statistic tests, we shall assume a sample of N observations on the severity of loss random variable X .

Furthermore, we will be testing:

- H_0 : Samples come from the postulated probability distribution, against
- H_1 : Samples do not come from the postulated probability distribution.

Probability Plot: A popular way of checking a model is by using Probability Plots¹. To do so, the data are plotted against a theoretical distribution in such a way that the points should form approximately a straight line. Departures from this straight line indicate departures from the specified distribution.

The probability plot is used to answer the following questions:

- Does a given distribution provide a good fit to the data?
- Which distribution best fits my data?
- What are the best estimates for the location and scale parameters of the chosen distribution?

Q-Q Plots: Quantile-Quantile Plots (Q-Q Plots)² are used to determine whether two samples come from the same distribution family. They are scatter plots of quantiles computed from each sample, with a line drawn between the first and third quartiles. If the data falls near the line, it is reasonable to assume that the two samples come from the same distribution. The method is quite robust, regardless of changes in the location and scale parameters of either distribution.

The Quantile-Quantile plots are used to answer the following questions:

- Do two data sets come from populations with a common distribution?
- Do two data sets have common location and scale parameters?
- Do two data sets have similar distributional shapes?
- Do two data sets have similar tail behavior?

Kolmogorov-Smirnov goodness of fit test: The Kolmogorov-Smirnov test statistic is the largest absolute deviation between the cumulative distribution function of the sample data and the cumulative probability distribution function of the postulated probability density function, over the range of the random variable:

$$T = \max |F_N(x) - F(x)|$$

¹<http://www.itl.nist.gov/div898/handbook/eda/section3/probplot.htm>

²<http://www.itl.nist.gov/div898/handbook/eda/section3/qqplot.htm>

over all x , where the cumulative distribution function of the sample data is $F_N(x)$, and $F(x)$ is the cumulative probability distribution function of the fitted distribution. The Kolmogorov-Smirnov test relies on the fact that the value of the sample cumulative density function is asymptotically normally distributed. Hence, the test is distribution free in the sense that the critical values do not depend on the specific probability distribution being tested.

Anderson-Darling goodness of fit test:

The Anderson-Darling test statistic is given by:

$$\hat{T} = -N - \frac{1}{N} \sum_{i=1}^N 2(i-1) \{ \ln F(\tilde{x}_i) + \ln [1 - F(\tilde{x}_{N+1-i})] \}$$

where \tilde{x}_i are the sample data ordered by size. This test is a modification of the Kolmogorov-Smirnov test which is more sensitive to deviations in the tails of the postulated probability distribution. This added sensitivity is achieved by making use of the specific postulated distribution in calculating critical values. Unfortunately, this extra sensitivity comes at the cost of having to calculate critical values for each postulated distribution.

2.2.2 Loss Frequency Distribution

The important issue for the frequency of loss modeling is a discrete random variable that represents the number of operational risk events observed. These events will occur with some probability p .

Many frequency distributions exist, such as the binomial, negative binomial, geometric, etc., but we are going to focus on the Poisson distribution in particular for our illustration. To do so, we start by explaining this distribution.

The probability density function of the Poisson distribution is given by

$$\mathbb{P}(X = k) = \frac{\exp^{-\lambda} \lambda^k}{k!}$$

where $k \geq 0$ and $\lambda > 0$ is the mean and $\sqrt{\lambda}$ is the standard deviation.

Estimation of the parameter can be carried out by maximum likelihood.

Much too often, a particular frequency of a loss distribution is chosen for no reason other than the risk managers familiarity of it. A wide number of alternative distributions are always available, each generating a different pattern of probabilities. It is important, therefore, that the probability distribution is chosen with appropriate attention to the degree to which it fits the empirical data. The choice as to which distribution to use can be based on either a visual inspection of the fitted distribution against the actual data or a formal statistical test such as the chi-squared goodness of fit test. For the chi-squared goodness of fit test, the null hypothesis is:

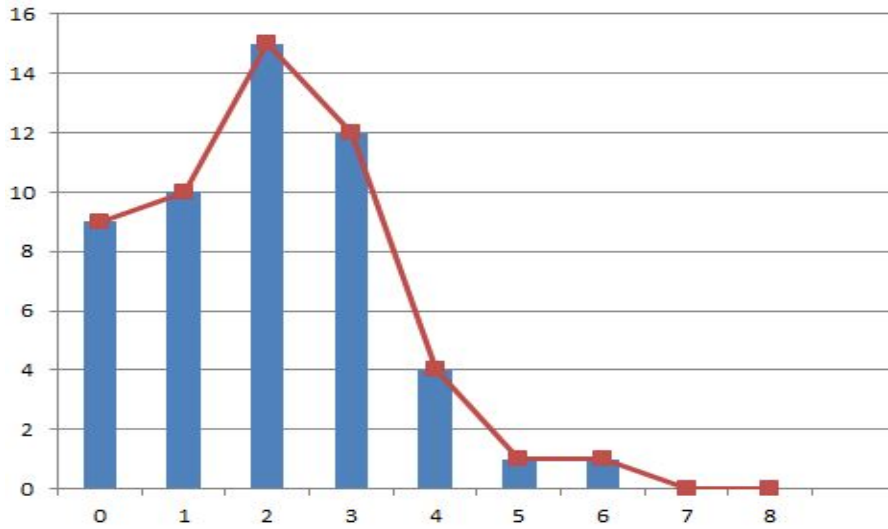


Figure 2.4: Loss Frequency Distribution

H_0 = The data follow a specified distribution

and,

H_1 = The data do not follow the specified distribution

The test statistic is calculated by dividing the data into n sets and is defined as:

$$\tilde{T} = \sum_{i=1}^n \frac{(E_i - O_i)^2}{E_i}$$

Where, E_i is the expected number of events determined by the frequency of loss probability distribution, O_i is the observed number of events and n is the number of categories.

The test statistic is a measure of how different the observed frequencies are from the expected frequencies. It has a chi-squared distribution with $n - (k - 1)$ degrees of freedom, where k is the number of parameters that need to be estimated.

2.2.3 Aggregate Loss Distribution

Even though in practice we may not have access to a historical sample of aggregate losses, it is possible to create sample values that represent aggregate operational risk losses given the severity and frequency of a loss probability model. In our example, we took the Poisson(2) and Lognormal(1.42,2.38) distributions as the frequency and severity distributions, respectively. Using the frequency and severity of loss data, we can simulate aggregate operational risk losses and then use these simulated losses for the calculation of the Operational risk capital charge.

The simplest way to obtain the aggregate loss distribution is to collect data on frequency

and severity of losses for a particular operational risk type and then fit frequency and severity of loss models to the data. The aggregate loss distribution then can be found by combining the distributions for severity and frequency of operational losses over a fixed period such as a year.

Let's try and explain this in a more theoretical way: Suppose N is a random variable representing the number of OR events between time t and $t + \delta$, (δ is usually taken as one year) with associated probability mass function $p(N)$ which is defined as the probability that exactly N losses are encountered during the time limit t and $t + \delta$. and let's define X as a random variable representing the amount of loss arising from a single type of OR event with associated severity of loss probability density function $f_X(x)$; Assuming the frequency of events N is independent of the severity of events, the total loss from the specific type of OR event between the time interval is:

$$S = X_1 + X_2 + \cdots + X_{N-1} + X_N$$

The probability distribution function of S is a compound probability distribution:

$$G(x) = \begin{cases} \sum_{i=1}^{\infty} p(i) \times F^{i*}(x) & \text{if } x > 0 \\ p(i) & \text{if } x = 0 \end{cases}$$

where $F(x)$ is the probability that the aggregate amount of i losses is x , $*$ is the convolution operator on the functions F and $F^{i*}(x)$ is the i -fold convolution of F with itself.

The problem is that for most distributions, $G(x)$ cannot be evaluated exactly and it must be evaluated numerically using methods such as *Panjer's recursive algorithm* or *Monte Carlo simulation*.

2.2.3.1 Panjer's recursive algorithm

If the frequency of loss probability mass function can be written in the form (cf. Mcneil *et al.* [2005] p. 480):

$$p(k) = p(k-1) \left(a + \frac{b}{k} \right) \quad k = 1, 2, \dots$$

where a and b are constants, Panjer's recursive algorithm can be used.

The recursion is given by

$$g(x) = p(1)f(x) + \int_0^x \left(a + b\frac{y}{x} \right) f(y)g(x-y)dy, \quad x > 0$$

where $g(x)$ is the probability density function of $G(x)$.

Usually, Poisson distribution, binomial distribution, negative binomial distribution, and geometric distribution satisfy the form. For example, if our severity of loss is the Poisson distribution seen above,

$$p(k) = \frac{\exp^{-\lambda} \lambda^k}{k!}$$

then $a = 0$ and $b = \lambda$.

A limitation of Panjer's algorithm is that only discrete probability distributions are valid. This shows that our severity of loss distribution, which is generally continuous, must be made discrete before it can be used. Another much larger drawback to the practical use of this method is that the calculation of convolutions is extremely long and it becomes impossible as the number of losses in the time interval under consideration becomes large.

2.2.3.2 Monte Carlo method

The Monte Carlo simulation is the simplest and often most direct approach. It involves the following steps (cf. Dahen [2006]):

- 1- Choose a severity of loss and frequency of loss probability model;
- 2- Generate n number of loss daily or weekly regarding the frequency of loss distribution
- 3- Generate n losses X_i , ($i = 1, \dots, n$) regarding the loss severity distribution;
- 4- Repeat steps 2 and 3 for $N = 365$ (for daily losses) or $N = 52$ (for weekly). Summing all the generated X_i to obtain S which is the annual loss;
- 5- Repeat the steps 2 to 4 many times (at least 5000) to obtain the annual aggregate loss distribution.
- 6- The VaR is calculated taking the 99.9th percentile of the aggregate loss distribution.

Now focusing on our example taking as Lognormal(1.42, 2.38) as the severity loss distribution and Poisson(2) as the frequency distribution and by applying Monte Carlo we arrive to calculate the VaR corresponding to the Operational risk for a specific risk type (let's say internal fraud).

To explain a bit the example given, we took into consideration the Poisson and Lognormal as the weekly loss frequency and severity distributions respectively. For the aggregate loss distribution we generate n number of loss each time regarding the Poisson distribution and n losses according the Lognormal distribution and so by summing the losses X_i , $i = 1, \dots, n$ and repeating the same steps 52 times we obtain S which would be the one annual total loss.

At the end, we repeat the same steps over and over again 100,000 times, we obtain the aggregate loss distribution on which we calculate the Value at Risk.

The programming was done using Matlab software and it resulted the output and calculations below:

VaR(99.9%)	0.1%	118,162
Mean VaR		207,885

Table 2.1: The VaR and Mean VaR calculation

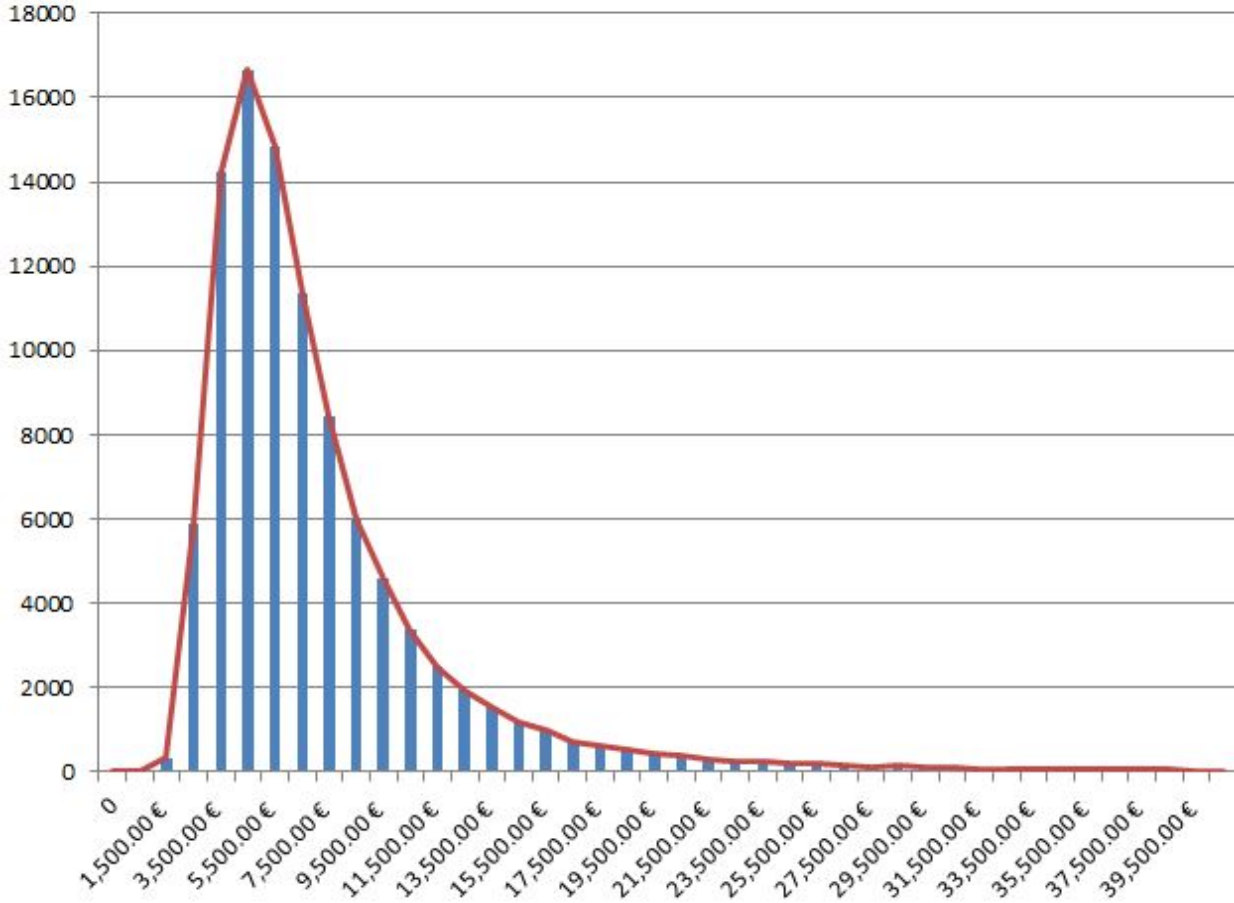


Figure 2.5: Annual Aggregate Loss Distribution

2.3 Treatment of Truncated Data

Generally, not all operational losses are declared. Databases are recorded starting from a threshold of a specific amount (for example, 5,000 €). This phenomenon, if not properly addressed, may create unwanted biases of the aggregate loss since the parameter estimation regarding the fitted distributions would be far from reality.

In this section, we will discuss the various approaches used in dealing with truncated data.

Data are said to be truncated when observations that fall within a given set are excluded. Left-truncated data is when the numbers of a set are less than a specific value, which means that neither the frequency nor the severity of such observations have been recorded (cf. Chernobai *et al.* [2005]).

In general, there are four different kinds of approaches that operational risk managers apply to estimate the parameters of the frequency and severity distributions in the absence of data due to truncation.

Approach 1

For this first approach, the missing observations are ignored and the observed data are treated as a complete data set in fitting the frequency and severity distributions. This approach leads to the highest biases in parameter estimation. Unfortunately, this is also the approach used by most practitioners.

Approach 2

The second approach is divided into two steps:

- Similar to the first approach, unconditional distributions are fitted to the severity and frequency distribution
- The frequency parameter is adjusted according to the estimated fraction of the data over the threshold u

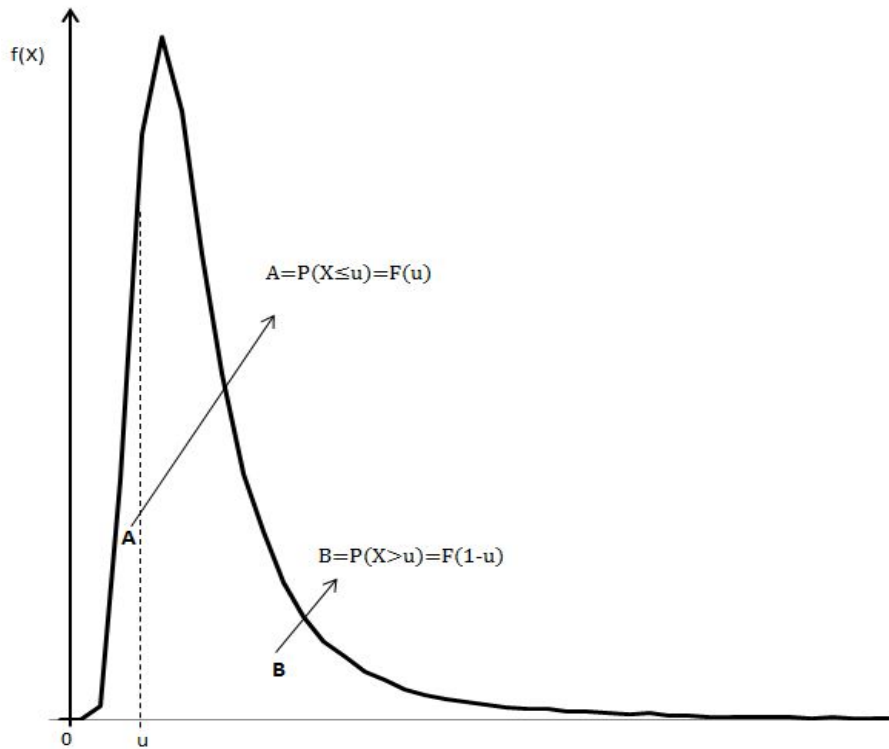


Figure 2.6: Fraction of missing data A and observed data B (cf. Chernobai *et al.* [2006])

In the end, the adjusted frequency distribution parameter is expressed by:

$$\hat{\lambda}_{adj} = \frac{\hat{\lambda}_{obs}}{1 - \hat{F}_{cond}(u)}$$

where $\hat{\lambda}_{adj}$, represents the adjusted (complete data) parameter estimate, $\hat{\lambda}_{obs}$ is the observed frequency parameter estimate, and $\hat{F}_{cond}(u)$ depicts the estimated conditional severity computed at threshold u .

Approach 3

This approach is different from previous approaches since the truncated data is explicitly taken into account in the estimation of the severity distribution to fit conditional severity and unconditional frequency.

The density of the truncated severity distribution would result in:

$$f_{cond}(x) = \begin{cases} \frac{f(x)}{(1 - F(u))} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

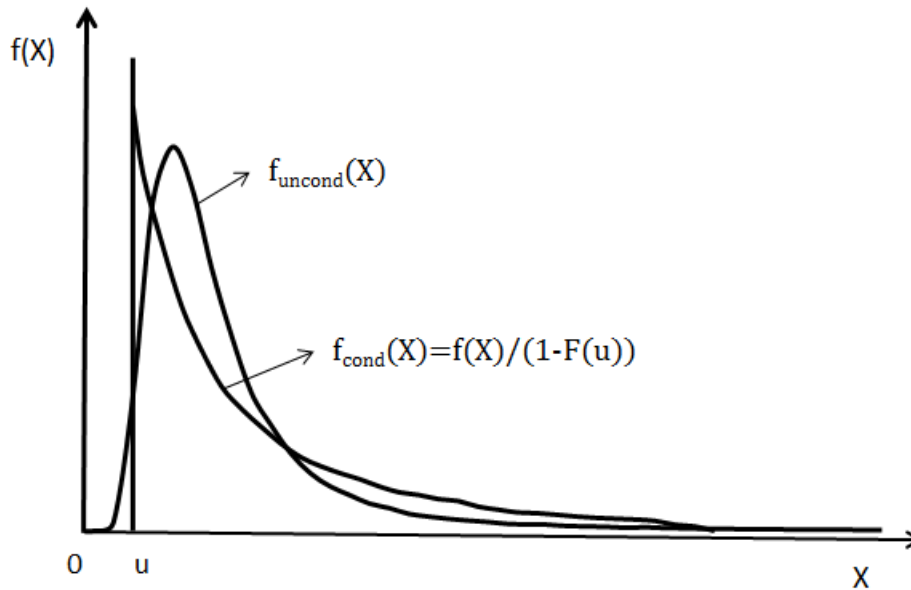


Figure 2.7: Unconditional and conditional severity densities (cf. Chernobai *et al.* [2006])

Approach 4

The fourth approach is deemed the best in application as it combines the second and third procedures by taking into account the estimated severity distribution and, as in Approach 2, the frequency parameter adjustment formula $\hat{\lambda}_{adj}$.

In modelling operational risk, this is the only relevant approach out of the four proposed as it addresses both the severity and the frequency of a given distribution.

2.3.1 Estimating Parameters using MLE

The MLE method can then be applied to estimate our parameters. To demonstrate, let's define (x_1, \dots, x_n) as losses exceeding the threshold u so the conditional Maximum Likelihood can be written as follows:

$$\prod_{i=1}^n \frac{f(x_i)}{\mathbb{P}(X_i \geq u)} = \prod_{i=1}^n \frac{f(x_i)}{1 - F_{X_i}(u)}$$

and the log-Likelihood would be:

$$\sum_{i=1}^n \ln \left(\frac{f(x_i)}{1 - F_{X_i}(u)} \right) = \sum_{i=1}^n \ln(f(x_i)) - n \ln(1 - F_{X_i}(u))$$

When losses are truncated, the frequency distribution observed has to be adjusted to consider the particular non-declared losses. For each period i , let's define n_i as the number of losses which have to be added to m_i , which is the number of estimated losses below the threshold, so that the adjusted number of losses is $n_i^a = n_i + m_i$.

To reiterate, the ratio between the number of losses below the threshold, m_i , and the observed loss number, n_i , is equal to the ratio between the left and right severity functions:

$$\frac{m_i}{n_i} = \frac{\hat{F}(u)}{1 - \hat{F}(u)}$$

where \hat{F} is the truncated cumulative distribution function with parameters estimated using MLE.

Finally, we have:

$$n_i^a = n_i + m_i = n_i + \frac{n_i \times \hat{F}(u)}{1 - \hat{F}(u)} = \frac{n_i}{1 - \hat{F}(u)}$$

2.3.2 Kolmogorov-Smirnov test adapted for left truncated Data

The Kolmogorov-Smirnov (KS) test, measures the absolute value of the maximum distance between empirical and fitted distribution function and puts equal weight on each observation. so regarding the truncation criteria KS test has to be adapted (cf. Chernobai *et al.* [2005]).

For that, let us assume the random variables (X_1, \dots, X_n) iid following the unknown probability distribution P_X .

The null hypothesis related would be:

$$H_0 : P_X \text{ has a cumulative distribution } F_0^*, \text{ where } F_0^* = \frac{F_0(x) - F_0(u)}{1 - F_0(u)}$$

Let's note: $y_j = F_0(x_j)$ and $y_u = F_0(u)$ so that KS_{obs} is:

$$KS_{obs}^* = \max\{KS^{+*}, KS^{-*}\}$$

where,

$$KS^{+*} = \frac{\sqrt{n}}{1 - y_u} \sup_j \left(y_u + \frac{j}{n}(1 - y_u) - y_j \right)$$

$$KS^{-*} = \frac{\sqrt{n}}{1 - y_u} \sup_j \left(y_j - \left(y_u + \frac{j-1}{n}(1 - y_u) \right) \right)$$

The p-value associated is then calculated using Monte-Carlo simulation.

2.4 Introduction to Robust Statistics

Operational risk Data is one of the most challenging problems both as to quantity and as to quality. Presence of outliers, left truncation, parameter instability and the limited number of events.

In 2001, the Basel Committee made the following recommendation (cf. BCBS [2001b], Annex 6, pp. 26):

”data will need to be collected and robust estimation techniques (for event impact, frequency, and aggregate operational loss) will need to be developed”.

While Classical estimators as MLE may produce biased estimates of parameters leading to unreasonably high estimates of mean, variance and the operational risk capital measure, robust statistics display less bias, greater efficiency and far more robustness than does MLE.

This section determine influence functions for Maximum Likelihood estimation for a number of severity distributions, both truncated and not. This work is based on the study of Opdyke & Cavallo [2012].

2.4.1 Maximum Likelihood Estimation (MLE)

According to Basel II, MLE is the basis for deriving estimators for parameters in operational risk.

To maintain its statistical properties known to be asymptotically normally distributed, asymptotically efficient (minimum variance) and asymptotically unbiased, MLE requires the following assumptions:

- Loss severities are identically distributed within unit of measure(perfect homogeneity)
- Individual loss severities are independent from one another
- The probability model of the severity distribution is well specified

Now given an i.i.d. sample of losses (x_1, x_2, \dots, x_n) and knowledge of the true probability density function $f(x|\theta)$, the MLE parameter estimates are the values $\hat{\theta}$ that maximize the likelihood function:

$$L(\theta|x) = \prod_{i=1}^n f(x_i|\theta)$$

The same numeric result is obtained by maximizing the log-likelihood function:

$$l(\theta|x_1, x_2, \dots, x_n) = \ln[L(\theta|x)] = \sum_{i=1}^n \ln[f(x_i|\theta)]$$

M-class estimators include a wide range of statistical models for which the optimal values of the parameters are determined by computing sums of sample quantities and MLE is

amongst the class of M-estimators.

M-estimators are solutions of θ which minimize, $\sum_{i=1}^n \rho(x_i, \theta)$

Particularly, MLE estimator is an M-class estimator with $\rho(x, \theta) = -\ln[f(x, \theta)]$

$$\text{so, } \hat{\theta} = \arg \min_{\theta \in \Theta} \sum_{i=1}^n \rho(x_i, \theta) = \arg \min_{\theta \in \Theta} \sum_{i=1}^n -\ln f(x_i, \theta)$$

Putten as an example in Appendix E the MLE estimators for the parameters of the Lognormal distribution.

2.4.2 Influence Function

An essential tool in robust statistics is the use of Influence function that allows to analytically determine the sensitivity of parameter estimates to specific deviations from assumed statistical model. It describes how parameter estimates of the assumed severity distribution are affected if some portion of the data follows another unspecified distribution at a particular severity value.

let δ_x be a distribution where the value x occurs with probability 1.

If Y has a distribution δ_x , then $\mathbb{P}(Y \leq y) = 0$ if $y < x$, and the mean of Y is $\mathbb{E}(Y) = x$.

$$\delta_x(y) = \begin{cases} 1 & \text{if } y \geq x \\ 0 & \text{otherwise} \end{cases}$$

Next, we consider a mixture of two distributions where an observation is randomly sampled from distribution F with probability $1 - \epsilon$, otherwise sampling is from the distribution δ_x . That is, with probability ϵ , the observed value is x . The resulting distribution is $F_{x,\epsilon} = (1 - \epsilon)F + \epsilon\delta_x$.

δ_x is known as the cumulative distribution function of the dirac delta function Δ_x defined as,

$$\Delta_x(y) = \begin{cases} 1 & \text{if } y = x \\ 0 & \text{otherwise} \end{cases}$$

A statistical functional, is the function $T(F(y, \theta)) = T(F)$ that estimates specific statistics as moments or parameters of a model given an assumed distribution $F(y, \theta)$ and a specific estimator (such as MLE or method of moments, ...).

In the statistics literature, the derivative of a functional, $T(F)$, is called the influence function of T at F , which was introduced by Hampel (1968, 1974). Roughly, the influence function measures the relative extent a small perturbation in F has on $T(F)$. Put another way, it reflects the limiting influence of adding one more observation, x , to a large sample.

As so, the influence function for T at F is:

$$IF(x|T, F) = \lim_{\epsilon \rightarrow 0} \left[\frac{T\{(1 - \epsilon)F + \epsilon\delta_x\} - T(F)}{\epsilon} \right] = \lim_{\epsilon \rightarrow 0} \left[\frac{T(F_\epsilon) - T(F)}{\epsilon} \right]$$

Now if the Influence Function is bounded over the domain of F , then the estimator is said to be B-Robust for the distribution and in case where the Influence Function is unbounded well then the estimator is not B-Robust.

As an example of the above, let's formulate the Influence Function for the mean:

$$\begin{aligned} IF(x|T, F) &= \lim_{\epsilon \rightarrow 0} \left[\frac{T\{(1 - \epsilon)F + \epsilon\delta_x\} - T(F)}{\epsilon} \right] \\ &= \lim_{\epsilon \rightarrow 0} \left[\frac{\int y d\{(1 - \epsilon)F + \epsilon\delta_x\}(y) - \int y dF(y)}{\epsilon} \right] \\ &= \lim_{\epsilon \rightarrow 0} \left[\frac{(1 - \epsilon) \int y dF(y) + \epsilon \int y d\delta_x(y) - \int y dF(y)}{\epsilon} \right] \\ &= \lim_{\epsilon \rightarrow 0} \left[\frac{\epsilon x - \epsilon \mu}{\epsilon} \right] \\ &= x - \mu \end{aligned}$$

As seen, the Influence Function for the mean is unbounded therefore not B-Robust. In contrast, the Influence Function for the median is bounded (cf. Hampel *et al.* [1986]).

2.4.2.1 The Influence Function for MLE

M-class estimators are defined as any estimator whose optimized function satisfies (cf. Opdyke & Cavallo [2012]): $\sum_{i=1}^n \varphi(X_i, \theta) = 0$, and similarly, $\hat{\theta} = \arg \min_{\theta} \left(\sum_{i=1}^n \rho(x_i, \theta) \right)$,

where $\varphi(x, \theta) = \frac{\partial \rho(x, \theta)}{\partial \theta}$ and most specifically,

$$\rho(x, \theta) = -\ln[f(x, \theta)]. \text{ So, } \varphi(x, \theta) = \frac{\partial \rho(x, \theta)}{\partial \theta} = -\frac{\frac{\partial f(x, \theta)}{\partial \theta}}{f(x, \theta)}$$

$$\text{and } \varphi'_\theta = \frac{\partial \varphi_\theta(x, \theta)}{\partial \theta} = \frac{\partial^2 \rho(x, \theta)}{\partial \theta^2} = \frac{-\frac{\partial^2 f(x, \theta)}{\partial \theta^2} \cdot f(x, \theta) + \left[\frac{\partial f(x, \theta)}{\partial \theta} \right]^2}{[f(x, \theta)]^2}.$$

Now for the Influence Function for the M-class estimators is,

$$IF_\theta(x, \theta) = \frac{\varphi_\theta(x, \theta)}{-\int_a^b \varphi'_\theta(y, \theta) dF(y)}$$

and so,

$$IF_{\theta}(x, \theta) = \frac{\frac{\partial f(x, \theta)}{\partial \theta}}{f(x, \theta)} - \frac{\int_a^b \left[\frac{\partial f(y, \theta)}{\partial \theta} \right]^2 - \frac{\partial^2 f(y, \theta)}{\partial \theta^2} \cdot f(y, \theta)}{f(y, \theta)} dy \quad (2.1)$$

where a and b define the endpoints of the distribution F .

The correlation between the parameters must be taken into account in case of distributions with more than one parameter and the equation (2.1) would result in:

$$IF_{\theta}(x, \theta) = A(\theta)^{-1} \varphi_{\theta} = \begin{bmatrix} -\int_a^b \frac{\partial \varphi_{\theta_1}}{\partial \theta_1} dF(y) & -\int_a^b \frac{\partial \varphi_{\theta_1}}{\partial \theta_2} dF(y) \\ -\int_a^b \frac{\partial \varphi_{\theta_2}}{\partial \theta_1} dF(y) & -\int_a^b \frac{\partial \varphi_{\theta_2}}{\partial \theta_2} dF(y) \end{bmatrix}^{-1} \begin{bmatrix} \varphi_{\theta_1} \\ \varphi_{\theta_2} \end{bmatrix} \quad (2.2)$$

2.4.2.2 The Influence function for MLE case of the truncated Severity Distributions

As seen in previous sections, since not all operational losses are declared most banks record their losses above a certain threshold H ($H = 5,000\text{€}$, $10,000\text{€}$ or $20,000\text{€}$), so the data with smaller losses is usually not available.

As so, the severity distribution becomes left truncated, with pdf and cdf the following:

$$g(x, \theta) = \frac{f(x, \theta)}{1 - F(H, \theta)} \text{ and } G(x, \theta) = 1 - \frac{1 - F(x, \theta)}{1 - F(H, \theta)}$$

Now the terms of the Influence Function for the MLE estimator become:

$$\rho(x, \theta) = -\ln(g(x, \theta)) = -\ln\left(\frac{f(x, \theta)}{1 - F(H, \theta)}\right) = \ln(1 - F(H, \theta)) - \ln(f(x, \theta))$$

and,

$$\varphi_{\theta}(x, H, \theta) = \frac{\partial \rho(x, \theta)}{\partial \theta} = -\frac{\frac{\partial f(x, \theta)}{\partial \theta}}{f(x, \theta)} - \frac{\frac{\partial F(H, \theta)}{\partial \theta}}{1 - F(H, \theta)}$$

$$\varphi'_{\theta}(x, H, \theta) = \frac{\partial \varphi_{\theta}(x, H, \theta)}{\partial \theta} = \frac{\partial^2 \rho(x, \theta)}{\partial \theta^2} = \frac{-\frac{\partial^2 f(x, \theta)}{\partial \theta^2} \cdot f(x, \theta) + \left[\frac{\partial f(x, \theta)}{\partial \theta}\right]^2}{[f(x, \theta)]^2} - \frac{\frac{\partial^2 F(H, \theta)}{\partial \theta^2} \cdot [1 - F(H, \theta)] + \left[\frac{\partial F(H, \theta)}{\partial \theta}\right]^2}{[1 - F(H, \theta)]^2}$$

Finally,

$$IF_{\theta}(x, \theta) = \frac{-\frac{\frac{\partial f(x, \theta)}{\partial \theta}}{f(x, \theta)} - \frac{\frac{\partial F(H, \theta)}{\partial \theta}}{1 - F(H, \theta)}}{-\frac{1}{1 - F(H, \theta)} \int_a^b \frac{-\frac{\partial^2 f(y, \theta)}{\partial \theta^2} \cdot f(y, \theta) + \left[\frac{\partial f(y, \theta)}{\partial \theta}\right]^2}{f(y, \theta)} dy + \frac{\frac{\partial^2 F(H, \theta)}{\partial \theta^2} \cdot [1 - F(H, \theta)] + \left[\frac{\partial F(H, \theta)}{\partial \theta}\right]^2}{[1 - F(H, \theta)]^2}}$$

The structure of the multi-parameter version correspond to the previous non truncated severity with the difference of the cumulative left truncated distribution cdf $G()$.

$$IF_{\theta}(x, \theta) = A(\theta)^{-1} \varphi_{\theta} = \begin{bmatrix} -\int_a^b \frac{\partial \varphi_{\theta_1}}{\partial \theta_1} dG(y) & -\int_a^b \frac{\partial \varphi_{\theta_1}}{\partial \theta_2} dG(y) \\ -\int_a^b \frac{\partial \varphi_{\theta_2}}{\partial \theta_1} dG(y) & -\int_a^b \frac{\partial \varphi_{\theta_2}}{\partial \theta_2} dG(y) \end{bmatrix}^{-1} \begin{bmatrix} \varphi_{\theta_1} \\ \varphi_{\theta_2} \end{bmatrix} \quad (2.3)$$

2.4.2.3 General Example for the LogNormal Distribution

The LogNormal distribution is characterized by its pdf, $f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma x} \exp\left\{-\frac{1}{2}\left(\frac{\ln(x) - \mu}{\sigma}\right)^2\right\}$, for $0 < x < \infty$ and $0 < \sigma < \infty$

As seen above for the calculation of the Influence Function, we are in need of the first and second order derivatives:

$$\begin{aligned} \frac{\partial}{\partial \mu} f(x; \mu, \sigma) &= \left[\frac{\ln(x) - \mu}{\sigma^2} \right] f(x; \mu, \sigma) \\ \frac{\partial}{\partial \sigma} f(x; \mu, \sigma) &= \left[\frac{(\ln(x) - \mu)^2}{\sigma^3} - \frac{1}{\sigma} \right] f(x; \mu, \sigma) \\ \frac{\partial^2}{\partial \mu^2} f(x; \mu, \sigma) &= \left[\frac{(\ln(x) - \mu)^2}{\sigma^4} - \frac{1}{\sigma^2} \right] f(x; \mu, \sigma) \\ \frac{\partial^2}{\partial \sigma^2} f(x; \mu, \sigma) &= \left(\left[\frac{1}{\sigma^2} - \frac{3(\ln(x) - \mu)^2}{\sigma^4} \right] + \left[\frac{(\ln(x) - \mu)^2}{\sigma^3} - \frac{1}{\sigma} \right]^2 \right) f(x; \mu, \sigma) \\ \frac{\partial}{\partial \mu \partial \sigma} f(x; \mu, \sigma) &= \left[\frac{\ln(x) - \mu}{\sigma^2} \right] \left[\frac{(\ln(x) - \mu)^2}{\sigma^3} - \frac{3}{\sigma} \right] f(x; \mu, \sigma) \end{aligned}$$

Now by integrating the above into (2.2):

$$\varphi_{\theta} = \begin{bmatrix} \varphi_{\mu} \\ \varphi_{\sigma} \end{bmatrix} = \begin{bmatrix} \frac{\partial \rho(x, \theta)}{\partial \mu} \\ \frac{\partial \rho(x, \theta)}{\partial \sigma} \end{bmatrix} = \begin{bmatrix} \frac{\frac{\partial f(x, \theta)}{\partial \mu}}{f(x, \theta)} \\ \frac{\frac{\partial f(x, \theta)}{\partial \sigma}}{f(x, \theta)} \end{bmatrix} = \begin{bmatrix} \frac{\mu - \ln(x)}{\sigma^2} \\ \frac{1}{\sigma} - \frac{(\ln(x) - \mu)^2}{\sigma^3} \end{bmatrix}$$

Now for the Fisher information matrix we have:

$$\bullet -\int_0^{\infty} \frac{\partial \varphi_{\mu}}{\partial \mu} dF(y) = -\int_0^{\infty} \left[\frac{(\ln(y) - \mu)^2}{\sigma^2} \right] - \left[\frac{(\ln(y) - \mu)^2}{\sigma^4} - \frac{1}{\sigma^2} \right] f(y) dy = -\int_0^{\infty} \frac{1}{\sigma^2} f(y) dy = -\frac{1}{\sigma^2}$$

$$\begin{aligned}
 & \bullet - \int_0^\infty \frac{\partial \varphi_\sigma}{\partial \sigma} dF(y) = - \int_0^\infty \left(\frac{3(\ln(y) - \mu)^2}{\sigma^4} - \frac{1}{\sigma^2} \right) f(y) dy = \frac{1}{\sigma^2} - \frac{3}{\sigma^4} \int_0^\infty (\ln(y) - \mu)^2 f(y) dy = \frac{-3\sigma^2}{\sigma^4} + \frac{1}{\sigma^2} = -\frac{2}{\sigma^2} \\
 & \bullet - \int_0^\infty \frac{\partial \varphi_\mu}{\partial \sigma} dF(y) = - \int_0^\infty \frac{\partial \varphi_\sigma}{\partial \mu} dF(y) = \\
 & \int_0^\infty \left(\left[\frac{\ln(y) - \mu}{\sigma^2} \right] \left[\frac{(\ln(y) - \mu)^2}{\sigma^3} - \frac{1}{\sigma} \right] - \left[\frac{\ln(y) - \mu}{\sigma^2} \right] \left[\frac{(\ln(y) - \mu)^2}{\sigma^3} - \frac{1}{\sigma} \right] \right) f(y) dy = 0
 \end{aligned}$$

The cross-Partial derivatives indicate the uncorrelation between the 2 parameters.

The Influence Function would result into:

$$IF_\theta(x; \theta) = A(\theta)^{-1} \varphi_\theta = \begin{bmatrix} \frac{-1}{\sigma^2} & 0 \\ 0 & \frac{2}{\sigma^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\mu - \ln(x)}{\sigma^2} \\ \frac{1}{\sigma} - \frac{(\ln(x) - \mu)^2}{\sigma^3} \end{bmatrix} = \begin{bmatrix} -\sigma^2 & 0 \\ 0 & -\frac{\sigma^2}{2} \end{bmatrix} \begin{bmatrix} \frac{\mu - \ln(x)}{\sigma^2} \\ \frac{1}{\sigma} - \frac{(\ln(x) - \mu)^2}{\sigma^3} \end{bmatrix} = \begin{bmatrix} \frac{\ln(x) - \mu}{2\sigma} \\ \frac{(\ln(x) - \mu)^2 - \sigma^2}{2\sigma} \end{bmatrix}$$

We can see that for neither μ nor σ are B-robust since the IFs are unbounded as $x \rightarrow +\infty$, $IF \rightarrow +\infty$.

What's interesting is that the MLE parameter values become large or small under contamination with very small values as $x \rightarrow 0^+$, $IF_\mu \rightarrow -\infty$ and $IF_\sigma \rightarrow +\infty$

2.5 Working with Extremes for Catastrophic Risks

"If things go wrong, how wrong can they go?" is a particular question which one would like to answer (cf. Gilli & Kellezi [2003]).

Extreme Value Theory (EVT) is a branch of statistics that characterizes the lower tail behavior of the distribution without tying the analysis down to a single parametric family fitted to the whole distribution.

This theory was pioneered by Leonard Henry Caleb Tippett³ and was codified by Emil Julius Gumbel⁴ in 1958. We use it to model the rare phenomena that lie outside the range of available observations.

The theory's importance has been heightened by a number of publicised catastrophic incidents related to operational risk:

- In February 1995, the Singapore subsidiary of Barings, a long-established British bank, lost about \$1.3 billion because of the illegal activity of a single trader, Nick Leeson. As a result, the bank collapsed and was subsequently sold for one pound.
- At Daiwa Bank, a single trader, Toshihide Igushi, lost \$1.1 billion in trading over a period of 11 years. These losses only became known when Iguchi confessed his activities to his managers in July 1995.

In all areas of risk management, we should put into account the extreme event risk which is specified by low frequency and high severity.

³Leonard Tippett was an English physicist and statistician.

⁴Emil Julius Gumbel was a German mathematician.

In financial risk, we calculate the daily value-at-risk for market risk and we determine the required risk capital for credit and operational risks. As with insurance risks, we build reserves for products which offer protection against catastrophic losses.

Extreme Value Theory can also be used in hydrology and structural engineering, where failure to take proper account of extreme values can have devastating consequences.

Now, back to our study, operational risk data appear to be characterized by two attributes: the first one, driven by high-frequency low impact events, constitutes the body of the distribution and refers to expected losses; and the second one, driven by low-frequency high-impact events, constitutes the tail of the distribution and refers to unexpected losses. In practice, the body and the tail of data do not necessarily belong to the same underlying distribution or even to distributions belonging to the same family.

Extreme Value Theory appears to be a useful approach to investigate losses, mainly because of its double property of focusing its analysis only on the tail area (hence reducing the disturbance on small- and medium-sized data) as well as treating the large losses by a scientific approach such as the one driven by the Central Limit Theorem for the analysis of the high-frequency low-impact losses.

We start by briefly exploring the theory:

EVT is applied to real data in two related ways. The first approach deals with the maximum (or minimum) values that the variable takes in successive periods, for example months or years. These observations constitute of the extreme events, also called block (or per-period) maxima. At the heart of this approach is the "three-types theorem" (Fisher and Tippett, 1928), which states that only three types of distributions can arise as limiting distributions of extreme values in random samples: the Weibull, the Gumbel and the Frechet distribution. This result is important as the asymptotic distribution of the maxima always belongs to one of these three distributions, regardless of the original distribution.

Therefore the majority of the distributions used in finance and actuarial sciences can be divided into these three categories as follows, according to the weight of their tails (cf. Smith [2002]):

- Light-tail distributions with finite moments and tails, converging to the Weibull curve (Beta, Weibull);
- Medium-tail distributions for which all moments are finite and whose cumulative distribution functions decline exponentially in the tails, like the Gumbel curve (Normal, Gamma, Log-Normal);
- Heavy-tail distributions, whose cumulative distribution functions decline with a power in the tails, like the Frechet curve (T-Student, Pareto, Log-Gamma, Cauchy).

The second approach to EVT is the Peaks Over Threshold (POT) method, tailored for the analysis of data bigger than the preset high thresholds. The severity component of the POT method is based on the Generalized Pareto Distribution (GPD). We discuss the details of these two approaches in the following segments.

2.5.1 Generalized Extreme Value Distribution: Basic Concepts

Suppose X_1, X_2, \dots, X_n are independent random variables, identically distributed with common distribution $F(x) = \mathbb{P}(X \leq x)$ and let $S_n = X_1 + X_2 + \dots + X_n$ and $M_n = \text{Max}(X_1, X_2, \dots, X_n)$.

We have the following two theorems (cf. Smith [2002]):

Theorem 1

$$\lim_{n \rightarrow +\infty} \mathbb{P}\left(\frac{S_n - a_n}{b_n} \leq x\right) = \Phi(x)$$

Where $\Phi(x)$ is the distribution function of the normal distribution, $a_n = nE(X_1)$ and $b_n = \sqrt{\text{Var}(X_1)}$.

Theorem 2

If there exists suitable normalizing constants $c_n > 0$, $d_n \in \mathbb{R}$ and some non-degenerate distribution function H such that:

$$\mathbb{P}\left(\frac{M_n - d_n}{c_n} \leq x\right) = F_{M_n}(a_n x + b_n) \xrightarrow{d} H(x)$$

Then H belongs to one of the three standard extreme value distributions (cf. Gilli & Kellezi [2003]):

- Gumbel:

$$\Lambda(x) = e^{-e^{-x}} \quad \text{if } x \in \mathbb{R}$$

- Fréchet:

$$\Phi_\alpha(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ e^{-x^{-\alpha}} & \text{if } x > 0 \text{ and } \alpha > 0 \end{cases}$$

- Weibull:

$$\Psi_\alpha(x) = \begin{cases} e^{-(-x)^{-\alpha}} & \text{if } x \leq 0 \text{ and } \alpha > 0 \\ 1 & \text{if } x > 0 \end{cases}$$

Jenkinson and Von Mises generalize the three functions by the following distribution function:

$$H_\xi(x) = \begin{cases} e^{-(1+\xi x)^{-\frac{1}{\xi}}} & \text{if } \xi \neq 0 \\ e^{-e^{-x}} & \text{if } \xi = 0 \end{cases}$$

where $1 + \xi x > 0$, a three parameter family is obtained by defining $H_{\xi, \mu, \sigma}(x) = H_\xi\left(\frac{x - \mu}{\sigma}\right)$ for a location parameter $\mu \in \mathbb{R}$ and a scale parameter $\sigma > 0$.

The case $\xi > 0$ corresponds to Fréchet with $\alpha = \frac{1}{\xi}$, $\xi < 0$ to Weibull with $\alpha = -\frac{1}{\xi}$, and the limit $\xi \rightarrow 0$ to Gumbel.

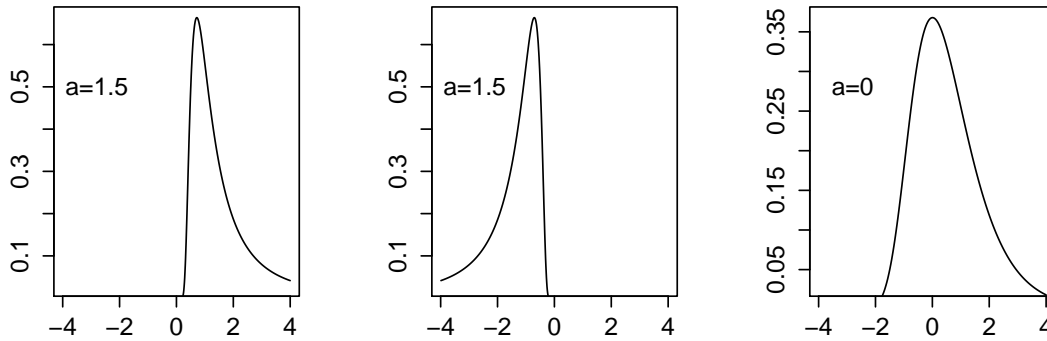


Figure 2.8: Densities for the Fréchet, Weibull and Gumbel functions

2.5.2 Block Maxima Method

Observations in the block maxima method are grouped into successive blocks and the maxima within each block are selected. The theory states that the limit law of the block maxima belongs to one of the three standard extreme value distributions mentioned before.

To use the block-maxima method, a succession of steps need to be followed. First, the sample must be divided into blocks of equal length. Next, the maximum value in each block (maxima or minima) should be collected. Then, we fit the generalized extreme value distribution.

2.5.3 Generalized Pareto Distribution

The Generalized Pareto (GP) Distribution has a distribution function with two parameters:

$$G_{\xi,\sigma}(x) = \begin{cases} 1 - \left(1 + \frac{\xi x}{\sigma}\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - e^{-\frac{x}{\sigma}} & \text{if } \xi = 0 \end{cases}$$

where $\sigma > 0$, and where $x \leq 0$ when $\xi \leq 0$ and $0 \leq x \leq -\frac{\sigma}{\xi}$ when $\xi < 0$.

The value of ξ determines the type of distribution: for $\xi < 0$, the model gives the type II Pareto distribution; for $\xi = 0$, we get the exponential distribution; and for $\xi > 0$, we get a reparameterised Pareto distribution.

For $X > 0$, we have the following formula:

$$\mathbb{E}(X^p) = p \int_0^{+\infty} y^{p-1} P(X > y) dy$$

We use this formula to calculate the mean (cf. Appendix D):

For $\sigma > 0, 0 < \xi < 1$ and $x \leq 0$:

$$\mathbb{E}(X) = \frac{\sigma}{1 - \xi}$$

and we calculate the variance for $\xi < \frac{1}{2}$:

$$V(X) = \frac{\sigma^2}{(\xi - 1)^2(1 - 2\xi)}$$

2.5.4 Excess Loss Distribution

Excess losses are defined as those losses that exceed a threshold. So, given a threshold value for large losses, the excess loss technique can be applied to determine the amount of provisions needed to provide a reserve for large losses. We consider a distribution function F of a random variable X which describes the behavior of the operational risk data in a certain Business Line (BL). We are interested in estimating the distribution function F_u of a value x above a certain threshold u (cf. Medova & Kariacou [2002]).

The distribution F_u is called the conditional excess distribution function and is formally defined as:

$$F_u(y) = \mathbb{P}(X - u \leq y \mid X > u) \text{ for } y = x - u > 0.$$

We verify that F_u can be written in terms of F as:

$$\begin{aligned} F_u(y) &= \mathbb{P}(X - u \leq y \mid X > u) \\ &= \frac{\mathbb{P}(X - u \leq y; X > u)}{\mathbb{P}(X > u)} \\ &= \frac{\mathbb{P}(u \leq X \leq y + u)}{1 - \mathbb{P}(X \leq u)} \\ &= \frac{F_X(y + u) - F_X(u)}{1 - F_X(u)} \\ &= \frac{F_X(x) - F_X(u)}{1 - F_X(u)} \end{aligned}$$

For a large class of underlying distribution function F the conditional excess distribution function $F_u(y)$ for a large u is approximated by:

$$F_u(y) \approx G_{\xi, \sigma}(y) \quad u \rightarrow +\infty$$

where

$$G_{\xi, \sigma}(y) = \begin{cases} 1 - \left(1 + \frac{\xi}{\sigma}y\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - e^{-\frac{y}{\sigma}} & \text{if } \xi = 0 \end{cases}$$

is the Generalized Pareto Distribution.

We will now derive an analytical expression for VaR_q and ES_q . First, we define $F(x)$ as:

$$F(x) = (1 - F(u))G_{\xi,\sigma}(x) + F(u) \quad \text{for } x > u$$

Then, we estimate $F(u)$ by $\frac{n - N_u}{n}$ where n is the total number of observations and N_u the number of observations above the threshold u . So, we have:

$$F(x) = \frac{N_u}{n} \left(1 - \left(1 + \frac{\xi}{\sigma}(x - u) \right)^{-\frac{1}{\xi}} \right) + \left(1 - \frac{N_u}{n} \right)$$

which simplifies to:

$$F(x) = 1 - \frac{N_u}{n} \left(1 + \frac{\xi}{\sigma}(x - u) \right)^{-\frac{1}{\xi}}$$

Inverting the last equation, we have:

$$\begin{aligned} 1 - q &= 1 - \frac{N_u}{n} \left(1 + \frac{\xi}{\sigma}(VaR_q - u) \right)^{-\frac{1}{\xi}} \\ \left(\frac{nq}{N_u} \right)^{-\xi} &= 1 + \frac{\xi}{\sigma}(VaR_q - u) \\ VaR_q &= u + \frac{\sigma}{\xi} \left(\left(\frac{n}{N_u}q \right)^{-\xi} - 1 \right) \end{aligned}$$

For the calculation of the expected shortfall, we notice that

$$P(X - VaR_q | X > VaR_q) = F_{VaR_q}(y) = G_{\xi,\sigma+\xi(VaR_q-u)}(y)$$

Since we have $F_u(y) \approx G_{\xi,\sigma}(y)$ and as ξ is the shape parameter, we can immediately conclude that:

$$E(X - VaR_q | X > VaR_q) = \frac{\sigma + \xi(VaR_q - u)}{1 - \xi}$$

And now, we estimate the expected shortfall:

$$\begin{aligned} ES_q &= VaR_q + E(X - VaR_q | X > VaR_q) \\ &= VaR_q + \frac{\sigma + \xi(VaR_q - u)}{1 - \xi} \\ &= \frac{VaR_q}{1 - \xi} + \frac{\sigma - \xi u}{1 - \xi} \end{aligned}$$

2.5.5 The Peak Over Threshold

The POT method considers observations exceeding a given high threshold. As an approach, it has increased in popularity as it uses data more efficiently than the block maxima method. However, the choice of a threshold can pose a problem.

To use the peak over threshold methods, we first select the threshold. Then, we fit the Generalised Pareto Distribution function to any exceedences above u . Next, we compute the point and interval estimates for the Value-at-Risk and the expected shortfall (cf. Medova & Kariacou [2002]).

Selection of the threshold:

While the threshold should be high, we need to keep in mind that with a higher threshold, fewer observations are left for the estimation of the parameters of the tail distribution function.

So, it's better to select the threshold manually, using a graphical tool to help us with the selection. We define the *sample mean excess plot* by the points:

$$(u, e_n(u)), \quad x_1^n < u < x_n^n$$

where $e_n(u)$ is the sample mean excess function defined as:

$$e_n(u) = \frac{\sum_{i=k}^n (x_i^n - u)}{n} \sum_{i=1}^n \mathbb{I}_{\{x_i^n > u\}}$$

and where $x_1^n, x_2^n, \dots, x_n^n$ represent the increasing order of the n observations.

Fitting the GPD function to the exceedances over u :

As defined in the previous sections, the distribution of the observations above the threshold in the right tail and below the threshold in the left tail should be a generalized Pared distribution. The best method to estimate the distribution's parameters is the Maximum Likelihood estimation method, explained below.

For a sample $y = \{y_1, \dots, y_n\}$ the log-likelihood function $L(\xi, \sigma | y)$ for the GPD is the logarithm of the joint density of the n observations.

$$L(\xi, \sigma | y) = \begin{cases} -n \ln \sigma - \left(\frac{1}{\xi} + 1\right) \sum_{i=1}^n \ln \left(1 + y_i \frac{\xi}{\sigma}\right) & \text{if } \xi \neq 0 \\ -n \ln \sigma - \frac{1}{\sigma} \sum_{i=1}^n y_i & \text{if } \xi = 0 \end{cases}$$

2.6 Application to a Legal Events Database

To check and understand the concepts, let's apply them to an exercise using the four distributions: Exponential, Lognormal, Weibull and Pareto.

The table 2.2 below shows a legal event database provided by Julie Gamonet, (cf. Gamonet [2009]) depicting four years' of losses. The units are in €.

14/04/2004	323.15	12/04/2006	42.59	15/06/2007	71.02
04/06/2004	305.8	20/04/2006	4,746.8	22/06/2007	3,030
06/09/2004	66,000	04/05/2006	2,437.98	06/07/2007	50,000
10/11/2004	5,476.53	04/05/2006	92.84	10/08/2007	673.12
25/01/2005	798.82	05/07/2006	55,500	28/08/2007	132.56
17/02/2005	4.34	18/07/2006	1,000,000	17/10/2007	2.4
22/02/2005	91.38	10/08/2006	103.66	17/10/2007	31.11
07/04/2005	1,924.78	21/09/2006	193.16	29/10/2007	21,001.82
10/11/2005	2.23	13/12/2006	5,795.84	30/11/2007	4.54
10/11/2005	3.3	31/12/2006	1,035.62	06/12/2007	31.74
29/11/2005	93.66	27/02/2007	1,001	19/12/2007	32.39
30/12/2005	176.64	13/03/2007	1,428.45	28/12/2007	2.12
07/01/2006	3.5	11/05/2007	1,738	28/12/2007	15,000
28/02/2006	412.82	22/05/2007	3,455	31/12/2007	1,283.04

Table 2.2: Database of Legal loss events (cf. Gamonet [2009])

Average	29,630.57
Standard Deviation	154,118.645
Skewness	6.16
Kurtosis	39.27

Table 2.3: First four moments of the sample

An initial analysis calculates the average, standard deviation, skewness and kurtosis of the database and shows that the database is leptokurtic as the skewness is greater than 3. So, given the heavy tail, it would be a good idea to start testing the database with exponential distributions.

2.6.1 Some Probability Distributions

We will be applying the four distributions: Exponential, Lognormal, Weibull and Pareto to the database in an attempt to fit and estimate the parameter of the distributions. But, before doing that, let's take a quick look at the four types of distributions.

2.6.1.1 Exponential Distribution

We say that X has an exponential distribution with parameter λ if it has a PDF of the form:

$$f(x) = \lambda e^{-\lambda x} \text{ for } x \geq 0$$

The expected value and variance of an exponentially distributed random variable X with rate parameter λ is given by:

$$\begin{aligned} \mathbb{E}[X] &= \frac{1}{\lambda} \\ \text{Var}[X] &= \frac{1}{\lambda^2} \end{aligned}$$

The cumulative distribution function is:

$$F(x) = 1 - e^{-\frac{x}{\lambda}}$$

And the moment estimation for the one-parameter case is simply calculated by:

$$\hat{\lambda} = \frac{1}{\sum_{i=1}^n \frac{X_i}{n}}$$

2.6.1.2 Lognormal Distribution

If X is a random variable with a normal distribution, then $Y = \exp(X)$ has a log-normal distribution. Likewise, if Y is lognormally distributed, then $X = \log(Y)$ is normally distributed.

The probability density function (PDF) of a log-normal distribution is:

$$f_X(x, \mu, \sigma) = \frac{1}{x\sigma} \sqrt{2\pi} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}$$

Where μ and σ are called the location and scale parameter, respectively. So if X is a lognormally distributed variable, then $\mathbb{E}[X] = e^{-\frac{1}{2}\sigma^2}$ and $\text{Var}[X] = (e^{\sigma^2} - 1)e^{2\mu + \sigma^2}$

2.6.1.3 Weibull Distribution

The Weibull distribution is a continuous probability distribution. It is named after Waloddi Weibull who described it in detail in 1951, although it was first identified by Fréchet in 1927 and first applied by Rosin & Rammler in 1933 to describe the size distribution of particles. This is the distribution that has received the most attention from researchers in the past quarter century.

The probability density function (PDF) of a Weibull random variable x is:

$$f(x) = \frac{b}{a^b} x^{b-1} e^{(-\frac{x}{a})^b} \text{ for } x \geq 0$$

The cumulative distribution function (CDF) is given by:

$$F(x) = 1 - e^{-\left(\frac{x}{a}\right)^b}$$

The mean and variance of a Weibull random variable can be expressed as:

$$\mathbb{E}[X] = a\Gamma\left(1 + \frac{1}{b}\right) \text{ and } Var[X] = a^2 \left[\Gamma\left(1 + 2\frac{1}{b}\right) - \Gamma\left(1 + \frac{1}{b}\right)^2 \right]$$

2.6.1.4 Pareto Distribution

The Pareto distribution was named after the economist Vilfredo Pareto, who formulated an economic law (Pareto's Law) dealing with the distribution of income over a population. The Pareto distribution is defined by the following functions:

$$\text{CDF: } F(x) = 1 - \left(\frac{k}{x}\right)^\alpha; k \leq x < \infty; \alpha, k > 0$$

$$\text{PDF: } f(x) = \frac{\alpha k^\alpha}{x^{\alpha+1}}; k \leq x < \infty; \alpha, k > 0$$

A few well known properties are:

$$\mathbb{E}[X] = \frac{\alpha k}{(\alpha - 1)}, \alpha > 1$$

$$Var[X] = \frac{\alpha k^2}{(\alpha - 1)^2(\alpha - 2)}, \alpha > 2$$

2.6.1.5 Output Analysis

The four distributions have been fitted to the database and the parameters were estimated according to the Maximum Likelihood Estimation. Also, a QQ-plot has been graphed to see how well the distributions fit the data. The Kolmogorov-Smirnov test was also carried out to see how well the distributions compare to the actual data.

As we will see in the outputs generated, the best model is the Lognormal as it does not differ much from the data set. However, we also observe that none of these models deal very well with the largest of events, which confirms that we need to apply extreme value theory.

Distribution	Parameter(s)
Exponential	$\lambda = 0.00003375$
Lognormal	$\mu = 5.9461, \sigma = 3.1642$
Weibull	$a = 1860.8, b = 0.3167$
Pareto	$k = 3.10, \alpha = 88.01$

Table 2.4: Estimation of the Parameters for the Exponential, Lognormal and Weibull distributions

As we have seen before, a Q-Q plot is a plot of the quantiles of two distributions against each other. The pattern of points in the plot is used to compare the two distributions. Now, while graphing the Q-Q plots to see how the distributions fit the data, the results shows that the Lognormal, Weibull and Pareto distributions are the the best models since the points of those three distributions in the plot approximately lie on the straight line of $y = x$, as seen in figure 2.9. Nevertheless, Kolmogorov-Smirnov test clearly depicts that the Lognormal distribution is better in accepting the null hypothesis that the data comes from the same continuous distribution.

Kolmogorov-Smirnov	
Exponential	$2.6654e - 007$
Lognormal	0.9095
Weibull	0.5642
Pareto	0.7520

Table 2.5: KS formal test result

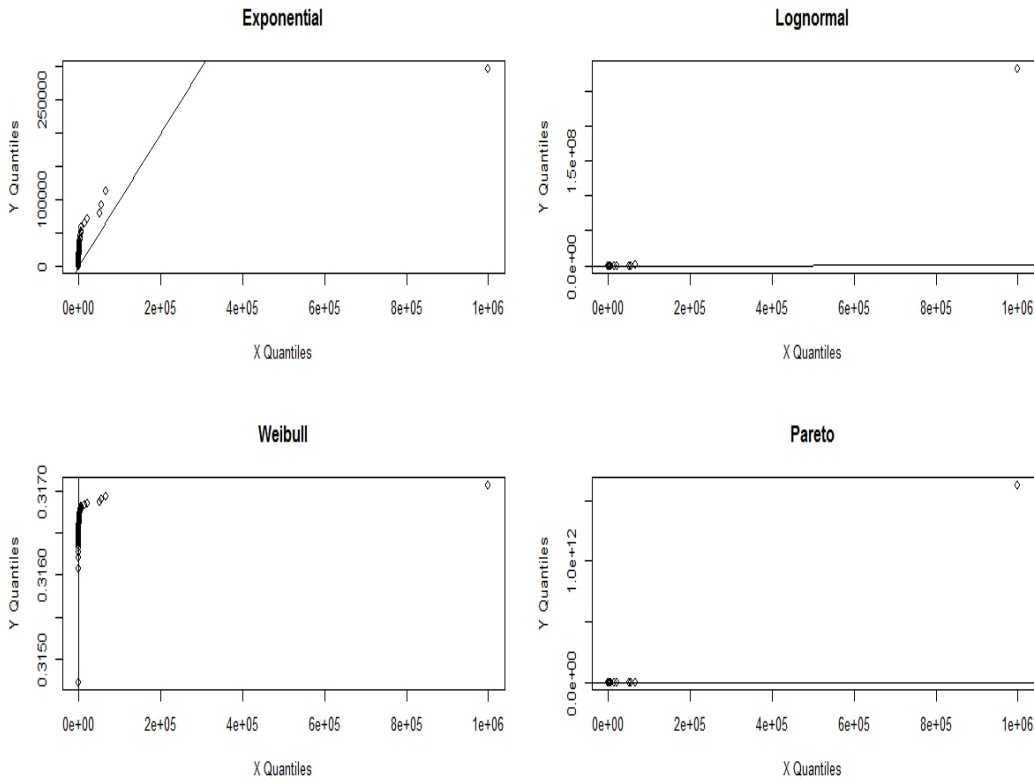


Figure 2.9: QQ-plots for fitted distributions

2.6.2 LDA and Application of Extreme Value Theory

As seen in previous sections, the Loss Distribution Approach has many appealing features since it is expected to be much more risk-sensitive. It is necessary to remember that VaR is calculated for a specific level of confidence and a given period of time, assuming normal conditions, which means that VaR does not include all aspects of risks. So one cannot estimate or predict losses due to extreme movements, such as losses encountered in major companies throughout the years (see table 2.6). For that, Extreme value theory is applied to characterize the tail behavior and model the rare phenomena that lie outside the range of available observations.

Loss Report	Event Description
<p><i>Enron Corporation</i> Start: 1 January 1998 End: 31 December 2001 Settlement: 15 July 2005 Loss Amount: 1,595 million USD Event Type: Clients, Products and Business Practices Business Line: Trading and Sales</p>	<p>On 15 July 2005, it was announced that Enron reached a \$1,595 million settlements with authorities in California, Washington and Oregon to settle allegations that the company was involved in market manipulation and price gouging during the energy crisis on the West Coast in 2000 2001. On 17 October 2005, an energy trader working for Enron pled guilty to one count of fraud, when he admitted to manipulating California's energy market through a fraudulent scheme during the period 1998 2001.</p>
<p><i>Barings Bank</i> Start: 1 February 1995 End: 27 February 1995 Settlement: 27 February 1995 Loss Amount: 1.3 billion Event Type: Internal Fraud Business Line: Trading and Sales</p>	<p>Barings Bank, a 233 years old British bank, suffered a 1.3 billion loss as a result of the unauthorised trading activity of Nick Leeson who was based in Singapore. The loss was greater than the bank's entire capital base and reserves, which created an extreme liquidity shortage. As a result, Barings declared bankruptcy and subsequently got acquired by the Dutch bank ING.</p>
<p><i>Société Générale</i> Start: 1 January 2005 End: 18 January 2008 Settlement: 24 January 2008 Loss Amount: \$7.2 billion Event Type: Internal Fraud Business Line: Trading and Sales</p>	<p>On 24 January 2008, the French bank, Société Générale, announced a EUR4.9 billion loss as a result of the unauthorised activities of a rogue trader on positions in stock index futures worth EUR50 billion. The rogue trader, Jerome Kerviel, managed to breach five levels of controls, having gained knowledge of how to circumvent the banks control systems from his position in the back office. The fraud was discovered after he made a mistake in his attempt to cover up fictitious trades.</p>

Table 2.6: Highly publicized loss events

In this section, we are going to take an internal loss database for a particular business line and apply to it the Loss Distribution Approach and calculate the VaR by using the Extreme Value Theory.

2.6.2.1 Application to an internal Loss Data

Our internal Database was provided by a Lebanese bank, the bank defines a reportable incident as any unusual event, operational in nature, which caused or had the potential to cause damage to the bank, whether tangibly or not, in readily measurable form (with financial impact, even in the bank’s favor) or as an estimate (in economic or opportunity cost terms). In simple terms, operational risk events are anything that went wrong or that could go wrong.

Hence, given our data we were able to compute the Severity and Frequency distributions related to a particular Business Line \times Event Type, we have to mention that some of the data was simulated so our application is not based on the real data:

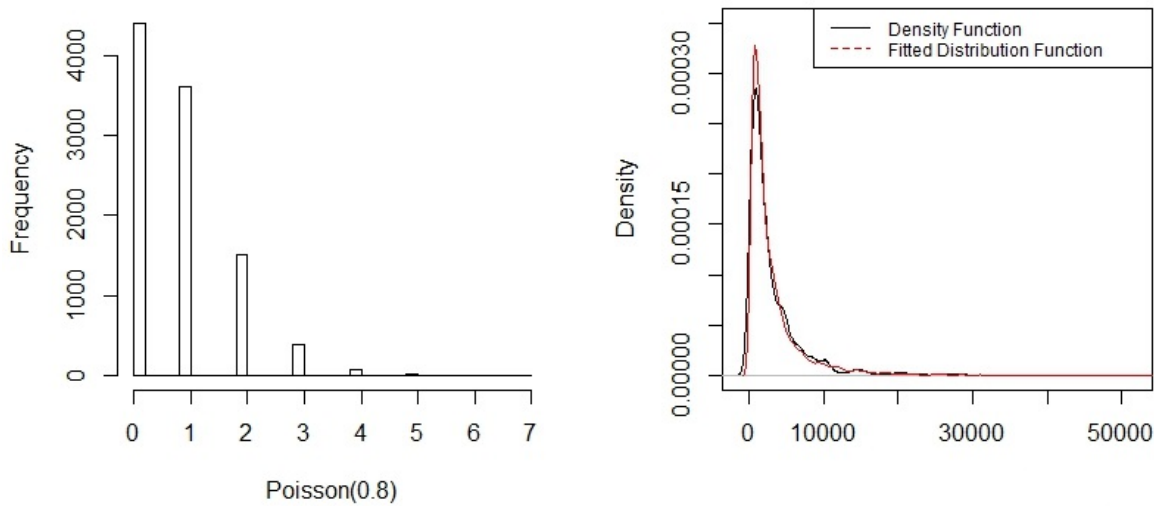


Figure 2.10: Frequency and severity distributions of Poisson and Lognormal \mathcal{LN}

As so, and by using Monte Carlo method treated in section 2.2.3.2, for Poisson $\mathcal{P}(0.8)$ and $\mathcal{LN}(7.5, 1.12)$ as our Lognormal Severity Distribution, we obtained our aggregated annual loss with the density function shown in Figure 2.11:

Our Value at Risk would result into:

VaR(99.9%)	0.1%	150,100\$
Mean VaR		189,084.3\$

Table 2.7: The VaR and Mean VaR calculation

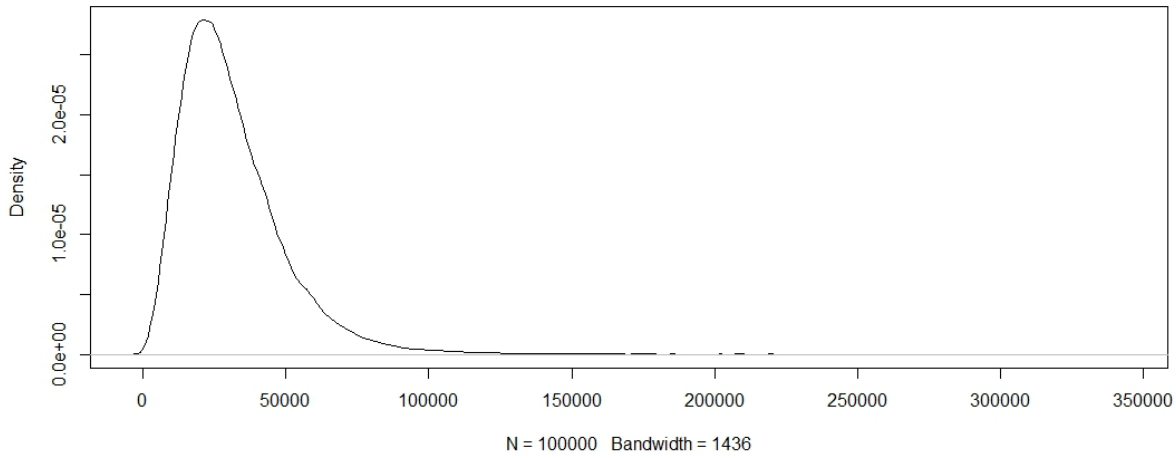


Figure 2.11: Aggregate Loss Distribution for a particular Business Line \times Event Type.

2.6.2.2 Application of the Extreme Value theory

Now, by applying the Extreme Value Theory explained in section 2.4 for the excess loss distribution and by setting our upper threshold as the 99% quantile, we could obtain a more robust Value at risk calculation that could mitigate our risk in a more precise manner.

Fitting the Generalized Pareto Distribution:

$$G_{\xi,\sigma}(y) = \begin{cases} 1 - \left(1 + \frac{\xi}{\sigma}y\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - e^{-\frac{y}{\sigma}} & \text{if } \xi = 0 \end{cases}$$

and calculating the VaR and Expected Shortfall related :

$$\begin{aligned} VaR_q &= u + \frac{\sigma}{\xi} \left(\left(\frac{\binom{n}{N_u} q}{\binom{n}{N_u}} \right)^{-\xi} - 1 \right) \\ ES_q &= \frac{VaR_q}{1 - \xi} + \frac{\sigma - \xi u}{1 - \xi} \end{aligned}$$

We obtain:

VaR	152,040 \$
Expected Shortfall	191,661.2 \$

Table 2.8: The VaR and Expected Shortfall with the use of Extreme Value Theory

Yet, if the calibration of severity parameters ignores external data, then the severity distribution will likely be biased towards low-severity losses, since internal losses are typically lower than those recorded in industry-wide databases. As so, LDA would be more accurate if both internal and external data are merged together in the calibration process, this point is illustrated in FRACHOT & RONCALLI [2002] for Mixing internal and external data for managing operational risk and DAHEN & DIONNE [2008] for Scaling for the Severity and frequency of External Operational Loss Data.

2.7 The Bayesian Approach: Internal Data, External Data and Expert Opinion

The Basel Committee has mentioned explicitly that (cf. BCBS [2005], paragraph 675): *“A bank must use scenario analysis of expert opinion in conjunction with external data to evaluate its exposure to high-severity events. This approach draws on the knowledge of experienced business managers and risk management experts to derive reasoned assessments of plausible severe losses. For instance, these expert assessment could be expressed as parameters of an assumed statistical loss distribution.”*

As mentioned earlier, the Basel Committee has authenticated an operational risk matrix of 8×7 risk cells (see Appendix A for reference). Each of these 56 risk cells leads to the modelling of loss frequency and loss severity distribution by financial institutions. Let’s focus on a one risk cell at a time.

After choosing a corresponding frequency and severity distribution, the managers estimate the necessary parameters. Let γ refer to the company’s risk profile which could accord to the location, scale, or shape of the severity distribution. While γ needs to be estimated from available internal information, the problem is that a small amount of internal data does not lead to a robust estimation of γ . Therefore the estimate needs to include other considerations in addition to external data and expert opinions.

For that, the risk profile γ is treated as the adjustment of a random vector Γ which is calibrated by the use of external data from market information. Γ is therefore a random vector with a known distribution, and the best prediction of our company specific risk profile γ would be based on a transformation of the external knowledge represented by the random Γ vector. The distribution of Γ is called a prior distribution. To explore this aspect further, before assessing any expert opinion and any internal data study, all companies have the same prior distribution Γ generated from market information only. Company specific operational risk events $X = (X_1, \dots, X_N)$ and expert opinions $\zeta = (\zeta^{(1)}, \dots, \zeta^{(M)})$ are gathered over time. As a result, these observations influence our judgment of the prior distribution Γ and therefore an adjustment has to be made to our company specific parameter vector γ . Clearly, the more data we have on X and ζ , the better the prediction of our vector γ and the less credibility we give to the market. So in a way, the observations X and the expert opinion ζ transform the market prior risk profile Γ into a conditional distribution of Γ given X and ζ denoted by $\Gamma|X, \zeta$ (cf. Lambrigger *et al.* [2007]).

Γ	γ
Parameter representing the whole industry	Company specific parameter
Considers external market data only	Considers internal data X and expert opinion ζ
Random variable	Realization of Γ , hence deterministic
With known distribution	Unknown, estimated by $\mathbb{E}[\Gamma X, \zeta]$

Table 2.9: Internal data and expert opinion (X, ζ) transform the prior risk profile of the whole industry Γ into an individual company specific γ (cf. Lambrigger *et al.* [2008]).

We Denote:

$\pi_{\Gamma}(\gamma)$, the unconditional parameter density.

$\hat{\pi}_{\Gamma|X, \zeta}(\gamma)$, the conditional parameter density also called posterior density.

And let's assume that observations and expert opinions are conditionally independent and identically distributed (i.i.d.) given γ , so that:

$$\begin{aligned} h_1(X|\gamma) &= \prod_{i=1}^N f_1(X_i|\gamma) \\ h_2(X|\gamma) &= \prod_{m=1}^M f_2(\zeta^{(m)}|\gamma) \end{aligned}$$

where f_1 and f_2 are the marginal densities of a single observation and a single expert opinion, respectively.

Bayes theorem gives for the posterior density of $\Gamma|X, \zeta$:

$$\hat{\pi}_{\Gamma|X, \zeta}(\gamma) = c\pi_{\Gamma}(\gamma)h_1(X|\gamma)h_2(X|\gamma)$$

$$\hat{\pi}_{\Gamma|X, \zeta}(\gamma) \propto \pi_{\Gamma}(\gamma)h_1(X|\gamma)h_2(X|\gamma)$$

where c is the normalizing constant not depending on γ . At the end, the company specific parameter γ can be estimated by the posterior mean $\mathbb{E}[\Gamma|X, \zeta] = \int \gamma \hat{\pi}_{\Gamma|X, \zeta}(\gamma) d\gamma$.

2.7.1 A simple Model

Let loss severities be distributed according to a lognormal-normal-normal model for an example. Given this model, we hold the following assumptions to be true (cf. Lambrigger *et al.* [2008]):

- Market Profile: Let Δ be normally distributed with parameters of mean μ_{ext} and standard deviation σ_{ext} , estimated from external sources, i.e. market data.
- Internal Data: Consider the losses of a given institution $i = 1, \dots, N$, conditional on (Δ) , to be i.i.d. lognormal distributed: $X_1, \dots, X_N | \Delta \hookrightarrow \mathcal{LN}(\Delta, \sigma_{int})$ where σ_{int} is assumed as known. That is, $f_1(\cdot | \Delta)$ corresponds to the density of a $\mathcal{LN}(\Delta, \sigma_{int})$ distribution.

- **Expert Opinion:** Suppose we have M experts with opinion ζ^m around the parameter Δ , where $1 \leq m \leq M$. We let $\zeta^{(1)}, \dots, \zeta^{(M)} | \Delta \text{ i.i.d. } \hookrightarrow \mathcal{N}(\Delta, \sigma_{exp})$ where σ_{exp} is the standard deviation denoting expert uncertainty. That is, $f_2(\cdot | \Delta)$ corresponds to the density of a $\mathcal{N}(\Delta, \sigma_{exp})$ distribution.

Moreover, we assume expert opinion ζ and internal data X to be conditionally independent given a risk profile Δ .

We adjust the market profile Δ to the individual company's profile by taking into consideration internal data and expert opinion to transform the distribution to be company specific. The mean and standard deviation of the market are determined from external data (for example, using maximum likelihood or the method of moments) as well as by expert opinion.

μ_{ext} and σ_{ext} for the market profile distribution are estimated from external data (Maximum likelihood or the method of moments).

Under the model assumption, we have the credibility weighted average theorem (cf. Appendix F).

With $\overline{\log X} = \frac{1}{N} \sum_{i=1}^N \log X_i$, the posterior distribution $\Delta | X, \zeta$ is a normal distribution $\mathcal{N}(\hat{\mu}, \hat{\sigma})$ with parameters

$$\hat{\sigma}^2 = \left(\frac{1}{\sigma_{ext}^2} + \frac{N}{\sigma_{int}^2} + \frac{M}{\sigma_{exp}^2} \right)^{-1}$$

and

$$\hat{\mu} = \mathbb{E}[\Delta | X, \zeta] = \omega_1 \mu_{ext} + \omega_2 \overline{\log X} + \omega_3 \bar{\zeta}$$

Where the credibility weights are given by $\omega_1 = \frac{\hat{\sigma}^2}{\sigma_{ext}^2}$, $\omega_2 = \frac{\hat{\sigma}^2 N}{\sigma_{int}^2}$ and $\omega_3 = \frac{\hat{\sigma}^2 M}{\sigma_{exp}^2}$

The theorem provides a consistent and unified method to combine the three mentioned sources of information by weighting the internal observations, the relevant external data and the expert opinion according to their credibility. If a source of information is not believed to be very plausible, it is given a smaller corresponding weight, and vice versa. As expected, the weights ω_1 , ω_2 , ω_3 add up to 1.

This theorem not only gives us the company's expected risk profile, represented by $\hat{\mu}$, but also the distribution of the risk, which is $\Delta | X, \zeta \hookrightarrow \mathcal{N}(\hat{\mu}, \hat{\sigma})$ allowing us to quantify the risk and its corresponding uncertainty.

2.7.2 Illustration of the Bayesian Approach

Assuming that a Bank models its risk according to the lognormal-normal-normal model and the three assumptions mentioned above, with scale parameter $\sigma_{int} = 4$, external parameters $\mu_{ext} = 2$, $\sigma_{ext} = 1$ and the expert opinion of the company given by $\bar{\zeta} = 6$ with $\sigma_{exp} = 3/2$. The observations of the internal operational risk losses sampled from a

$\mathcal{LN}(\mu_{int} = 4, \sigma_{int} = 4)$ distribution are given below:

Loss i	1	2	3	4	5	6	7	8	9	10	...
Severity X_i	20.45	360.52	1.00	7,649.77	1.92	11.60	1,109.01	24.85	0.05	209.48	...

Table 2.10: Sampled Losses from a $\mathcal{LN}(4, 4)$

So to reiterate, we have the following parameters:

M	1
μ_{ext}	2
μ_{exp}	6
μ_{int}	4
σ_{int}	4
σ_{ext}	1
σ_{exp}	1.50

Table 2.11: Parameters assumption

Now we can calculate the estimation and the credibility weights using the formulas given previously (see table 2.12):

$\overline{\log X}$	4.20
$\hat{\sigma}^2$	0.17
$\hat{\mu}$	3.96
ω_1	0.1718
ω_2	0.7518
ω_3	0.07637

Table 2.12: Parameter calculation output

In the end, we compare the classical maximum likelihood estimator to the estimator without expert opinion corresponding to $M = 0$ and the Bayes estimator, as shown in figure 2.12 below:

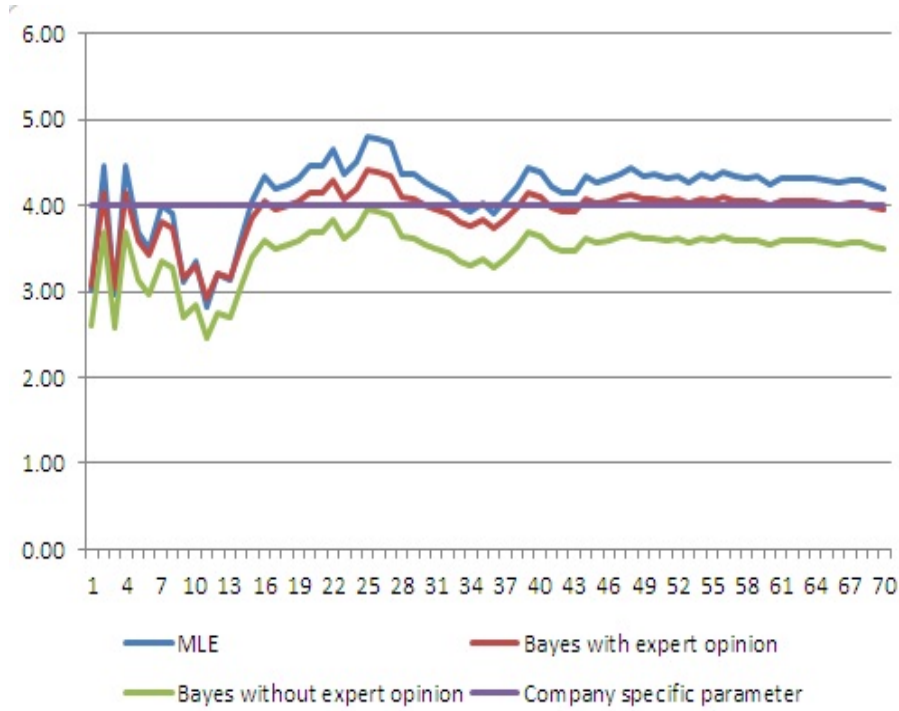


Figure 2.12: Comparison of the MLE estimator $\hat{\mu}_k^{MLE}$, to the Bayesian estimator $\hat{\mu}_k$ and the no expert opinion Bayes estimator given by Shevchenko and Wüthrich $\hat{\mu}_k^{SW}$

The figure 2.12 shows that the Bayesian approach has a more stable behavior around the true value of $\mu_{int} = 4$ even when just a few data points are available, which is not the case with the MLE and the SW estimators.

In this example we see that in combining external data with the expert opinions, we stabilize and smooth our estimators, in a way that works better than the MLE and the no expert opinion estimators. This shows the importance of the Bayesian approach for estimating the parameters and calculating the capital requirement under Basel II or Solvency II for Operational Risk.

In the next chapter, we will apply the Bayesian updating technique to a Lebanese bank by trying to combine our experts opinion to an internal loss database.

Chapter 3

Combining Internal Data with Scenario Analysis

3.1 Introduction

Under the regulations of Basel Accords and Solvency Directive, to be able to estimate their aggregate operational risk capital charge, many financial institutions have adopted a Loss Distribution Approach (LDA), consisting of a frequency and a severity distribution, based on its own internal losses. Yet, basing our models on historical losses only might not be the perfect approach since no future attention is being taken into consideration which can generate a biased capital charge, defined as the 0.01 % quantile of the loss distribution, facing reality. On the other hand, adding scenario analysis given by the experts provide to some extent a future vision.

The main idea in this chapter is the following: A Bayesian inference approach offers a methodical concept that combines internal data with scenario analysis. We are searching first to integrate the information generated by the experts with our internal database; by working with conjugate family distributions, we determine a prior estimate. This estimate is then modified by integrating internal observations and experts' opinion leading to a posterior estimate; risk measures are then calculated from this posterior knowledge. See Shevchenko [2011] for more on the subject.

On the second half, we use Jeffreys non-informative prior and apply Monte Carlo Markov Chain with Metropolis Hastings algorithm, thus removing the conjugate family restrictions and developing, as the chapter shows, a generalized application to set up a capital evaluation. For a good introduction to non-informative prior distributions and MCMC see Robert [2007].

Combining these different information sources for model estimation is certainly one of the main challenges in operational risk and as we are going to see throughout this chapter, that a good expert judgment is of great importance, if not well treated estimation risk consequence is severe. More on Bayesian Inference techniques could be found in Berger [1985].

3.2 Bayesian techniques in combining two data sources: Conjugate prior

In our study, our data related to retail banking business line and external fraud event type is of size 279, collected in \$ over 4 years from January 2008 till December 2011. The data fits the $Poisson(5.8)$ as a frequency distribution, and $\mathcal{LN}(\mu = 6.7, \sigma = 1.67)$ as the severity distribution.

Applying Monte Carlo simulation (cf. Frachot *et al.* [2001]), with $\lambda_{ID} = 5.8$, $\mu_{ID} = 6.7$, and $\sigma_{ID} = 1.67$, we obtained a Value-at-Risk of $VaR_{ID} = 1,162,215.00$ at 99.9%, using internal losses only. The 99.9% confidence has been chosen throughout this chapter based on Basel specification regarding banks.

On the other hand, working with the scenario analysis, our experts gave us their assumptions for the frequency parameter λ . As for the severity, we used the histogram approach (cf. SHEVCHENKO [2011] pp. 119-121) and (BERGER [1985]) where our experts represent a probability reflecting that a loss is in an interval of losses (see table 3.1 below).

Losses Interval in \$	Expert Opinion
[0, 5000[65%
[5000, 20000[19%
[20000, 50000[10%
[50000, 100000[3.5%
[100000, 250000[1.5%
[250000, 400000[0.7%
≥ 400000	0.3%

Table 3.1: Scenario analysis

If we consider our severity distribution being Lognormal with parameters μ and σ^2 , the objective is to find the parameters $(\mu_{exp}, \sigma_{exp})$ that adjust our histogram in a way to approach as much as possible the theoretical lognormal distribution. For this we can use chi-squared statistic that allows us to find (μ, σ) that minimize the chi-squared distance:

$$\tilde{T} = \sum_{i=1}^n \frac{(E_i - O_i)^2}{E_i},$$

where E_i and O_i are respectively the empirical and theoretical probability.

Our experts provided $\lambda = 2$, and by applying chi-squared, we obtained our lognormal parameters: $(\mu = 7.8, \sigma = 1.99)$ with the $VaR(99.9\%) = 6,592,086.00$.

We can see the high spread between the two values which can cause a problem in allocating a non-biased capital requirement. In the next sections, we will apply the Bayesian inference techniques, thus joining our internal observations with the experts opinion.

3.2.1 Modelling Frequency distribution: The Poisson Model

We are going to work with the Poisson and Lognormal distributions since they are the most used distributions in Operational Risk (cf. Shevchenko [2011]).

Consider the annual number of events N for a risk in a bank modelled as a random variable from the Poisson distribution $\mathcal{P}(\lambda)$, where Λ is considered as a random variable with the prior distribution $Gamma(a, b)$. So we have: $\mathbb{P}(N = n) = e^{-\lambda} \frac{\lambda^n}{n!}$, and λ has a prior density:

$$\Pi(\Lambda = \lambda) = \frac{\left(\frac{\lambda}{b}\right)^{a-1}}{\Gamma(a)b} e^{-\frac{\lambda}{b}}, \quad \lambda > 0, a > 0, b > 0$$

As for the likelihood function, given the assumption that n_1, n_2, \dots, n_T are independent, for $N = n$:

$$h(n|\lambda) = \prod_{i=1}^T e^{-\lambda} \frac{\lambda^{n_i}}{n_i!},$$

where n is the number of historical losses and n_i is the number of losses in month i .

Thus, the posterior density would be: $\Pi(\lambda|N = n) = \frac{h(n|\lambda)\Pi(\lambda)}{h(n)}$, but since $h(n)$ plays the role of a normalizing constant, $\Pi(\lambda|N = n)$ could be rewritten as:

$$\Pi(\lambda|N = n) \propto h(n|\lambda)\Pi(\lambda) \propto \frac{\left(\frac{\lambda}{b}\right)^{a-1}}{\Gamma(a)b} e^{-\frac{\lambda}{b}} \prod_{i=1}^T e^{-\lambda} \frac{\lambda^{n_i}}{n_i!} \propto \frac{\lambda^{\sum_{i=1}^T n_i + a - 1}}{b} e^{-\lambda(T + \frac{1}{b})} \propto \lambda^{a_T - 1} e^{-\frac{\lambda}{b_T}}.$$

Which is $Gamma(a_T, b_T)$, i.e. the same as the prior distribution with $a_T = \sum_{i=1}^T n_i + a$ and

$$b_T = \frac{b}{(1 + Tb)}$$

So we have:

$$\mathbb{E}(\lambda|N = n) = a_T b_T = \omega \bar{N} + (1 - \omega)(ab) = \omega \bar{N} + (1 - \omega)\mathbb{E}(\Lambda), \quad \text{with } \omega = \frac{n}{n + \frac{1}{b}}$$

To apply this, and since the only unknown parameter is λ that is estimated by our experts with, $\mathbb{E}(\lambda) = 2$.

The experts may estimate the expected number of events, but cannot be certain of the estimate. Our experts specify $\mathbb{E}[\lambda]$ and an uncertainty that the "true" λ for next month is within the interval $[a_0, b_0] = [0.5, 8]$ with a probability $p = 0.7$ that $\mathbb{P}(a_0 \leq \lambda \leq b_0) = p$, then we obtain the below equations:

$$\begin{aligned} \mathbb{E}[\lambda] &= a \times b = 2 \\ \mathbb{P}(a_0 \leq \lambda \leq b_0) &= \int_{a_0}^{b_0} \pi(\lambda|a, b) d\lambda = F_{a,b}^{(G)}(b_0) - F_{a,b}^{(G)}(a_0) = 0.7 \end{aligned}$$

Where $F_{a,b}^{(G)}(\cdot)$ is the $Gamma(a, b)$ cumulative distribution function.

Solving the above equations would give us the prior distribution parameters $\lambda \hookrightarrow \text{Gamma}(a = 0.79, b = 2.52)$, and by using the formulas stated, we obtain: $a_T = 279.8$ and $b_T = 0.02$ as our posterior parameters distribution. At the end, we calculate a $\text{VaR}(99.9\%) = 1,117,821.00$ using Monte Carlo simulation:

- 1- Using the estimated Posterior $\text{Gamma}(a_T, b_T)$ distribution, generate a value for λ ;
- 2- Generate n number of monthly loss regarding the frequency of loss distribution $\text{Poisson}(\lambda)$
- 3- Generate n losses X_i , ($i = 1, \dots, n$) regarding the loss severity distribution $\mathcal{LN}(\mu, \sigma^2)$;
- 4- Repeat steps 2 and 3 for $N = 12$. Summing all the generated X_i to obtain S which is the annual loss;
- 5- Repeat steps 1 to 4 many times (in our case 10^5) to obtain the annual aggregate loss distribution.
- 6- The VaR is calculated taking the 99.9th percentile of the aggregate loss distribution.

We notice that our Value-at-Risk is close to the VaR generated by the internal losses alone, since the only thing took as unknown was λ , both parameters μ and σ are equal to (μ_{ID}, σ_{ID}) .

3.2.2 Modelling severity distribution: Lognormal $\mathcal{LN}(\mu, \sigma)$ distribution with unknown μ

Assume that the loss severity for a risk is modelled as a random variable from a lognormal distribution $\mathcal{LN}(\mu, \sigma)$ and we consider $\mu \hookrightarrow \mathcal{N}(\mu_0, \sigma_0^2)$ as a prior distribution.

So we have,
$$\Pi(\mu) = \frac{1}{\sigma_0 \sqrt{2\pi}} \exp \left\{ -\frac{(\mu - \mu_0)^2}{2\sigma_0^2} \right\}.$$

Taking $Y = \ln X$, we calculate the posterior distribution as previously by:

$$\Pi(\mu|\mu_0, \sigma_0^2) \propto \Pi(\mu)h(Y|\mu, \sigma) \propto \frac{e^{-\frac{(\mu-\mu_0)^2}{2\sigma_0^2}}}{\sigma_0\sqrt{(2\pi)}} \prod_{i=1}^n \frac{e^{-\frac{(y_i-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

since we are using a conjugate prior distribution, we know that the posterior distribution will follow a Normal distribution with parameters (μ_1, σ_1^2) , where:

$$\Pi(\mu|\mu_0, \sigma_0^2) \propto e^{-\frac{(\mu-\mu_1)}{2\sigma_1^2}}$$

By identification we obtain:
$$\begin{cases} \frac{1}{2\sigma_1^2} = \frac{1}{2\sigma_0^2} + \frac{n}{2\sigma^2} \\ \frac{\mu_1}{\sigma_1^2} = \frac{\mu_0}{\sigma_0^2} + \frac{\sum_{i=1}^n y_i}{\sigma^2} \end{cases}$$

$$\text{So, } \mu_1 = \frac{\mu_0 + \omega_0 \sum_{i=1}^n y_i}{1 + n\omega_0} = \omega \bar{Y} + (1 - \omega)\mu_0, \quad \sigma_1^2 = \frac{\sigma_0^2}{1 + n\omega_0}, \quad \text{with } \omega_0 = \frac{\sigma_0^2}{\sigma^2}, \quad \text{and } \omega = \frac{n\omega_0}{1 + n\omega_0}$$

Assuming that the loss severity for a risk is modelled as a random variable from a lognormal distribution $X \hookrightarrow \mathcal{LN}(\mu, \sigma)$, $\Omega = \mathbb{E}[X|\mu, \sigma] = e^{\mu + \frac{1}{2}\sigma^2} \hookrightarrow \mathcal{LN}(\mu_0 + \frac{1}{2}\sigma^2, \sigma_0^2)$ and we consider $\mu \hookrightarrow \mathcal{N}(\mu_0, \sigma_0^2)$ as a prior distribution.

Since the only thing unknown is μ , we already have $\sigma = 1.67$ and $\lambda = 5.8$, and the experts gave us:

$$\begin{aligned} \mathbb{E}[\Omega] &= e^{\mu_0 + \frac{1}{2}\sigma^2 + \frac{1}{2}\sigma_0^2} = 15,825 \text{ \$} \\ \mathbb{P}(1 \leq \Omega \leq 250,000) &= \Phi\left(\frac{\ln 250,000 - \frac{1}{2}\sigma^2 - \mu_0}{\sigma_0}\right) - \Phi\left(\frac{\ln 1 - \frac{1}{2}\sigma^2 - \mu_0}{\sigma_0}\right) = 99\% \end{aligned}$$

Where Φ is the cumulative distribution function of the standard normal distribution.

Solving these two equations, we find that the prior distribution of μ is: $\mu \hookrightarrow \mathcal{N}(\mu_0 = 8.15, \sigma_0^2 = 0.25)$.

Hence using the formulas stated above where, $\mu_1 = \frac{\mu_0 + \omega_0 \sum_{i=1}^n y_i}{1 + n\omega_0} = 6.72$, $\sigma_1^2 = \frac{\sigma_0^2}{1 + n\omega_0} = 0.0096$, and $\omega_0 = \frac{\sigma_0^2}{\sigma^2} = 0.0898$, with $n = 279$ is the total number of historical losses.

We find out that the posterior distribution: $\mu \hookrightarrow \mathcal{N}(\mu_1 = 6.72, \sigma_1 = 0.1)$.

At the end, using the posterior μ distribution and Monte Carlo method, we calculate the 99.9% Value-at-Risk: $VaR(99.9\%) = 1,188,079.00$.

The same analysis goes here as well, since the only unknown parameter is μ , $(\lambda, \sigma) = (\lambda_{ID}, \sigma_{ID})$, the VaR calculated will be closer to our Internal Data Value-at-Risk.

3.2.3 Modelling frequency and severity distributions: Unknown Poisson(λ) parameter and Lognormal $\mathcal{LN}(\mu, \sigma)$ distribution with unknown μ

In the two previous subsections, we illustrated the case of modelling frequency and severity distributions with unknown λ that follows a $Gamma(a, b)$ distribution and μ that follows a $\mathcal{N}(\mu_0, \sigma_0^2)$ respectively.

Joining these two distributions is relatively simple since we have the hypothesis of independence between frequency and severity, which allows us to estimate independently the two posterior distributions and estimate the parameters.

As so, we have already demonstrated the fact that our posterior density $\Pi(\lambda|N = n)$

follows the $Gamma(a_T, b_T)$ distribution, with $a_T = \sum_{i=1}^T n_i + a$ and $b_T = \frac{b}{(1 + nb)}$

and, $\Pi(\mu|\mu_0, \sigma_0^2) \hookrightarrow \mathcal{N}(\mu_1, \sigma_1^2)$, with $\mu_1 = \frac{\mu_0 + \omega_0 \sum_{i=1}^n y_i}{1 + n\omega_0}$, $\sigma_1^2 = \frac{\sigma_0^2}{1 + n\omega_0}$, with $\omega_0 = \frac{\sigma_0^2}{\sigma^2}$. Since we have the hypothesis of independence between frequency and severity, which allows us to estimate independently the two posterior distributions, which have been already calculated for the parameter λ we took the gamma distribution and for the μ parameter, the posterior distribution was normal with:

$$\begin{aligned}\lambda &\hookrightarrow \text{Gamma}(279.8, 0.02) \\ \mu &\hookrightarrow \mathcal{N}(6.72, 0.1)\end{aligned}$$

By simulating those two laws using Monte Carlo simulation (cf. section 3.2.1), we obtain a Value-at Risk of 1, 199, 000.00 using the estimated posterior Gamma and Normal distributions.

This result is highly interesting, since with two unknown parameters λ and μ , the VaR is still closer to VaR_{ID} . This states that the parameter σ is the key parameter in this application, as we are going to see throughout this chapter.

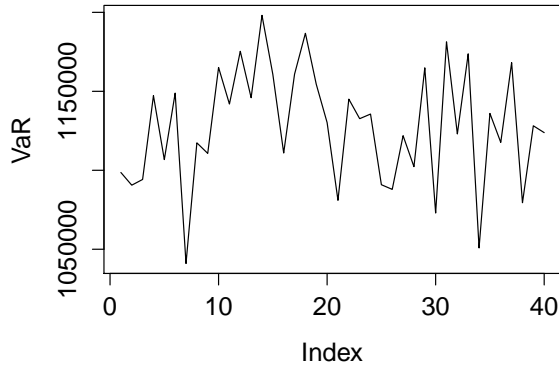
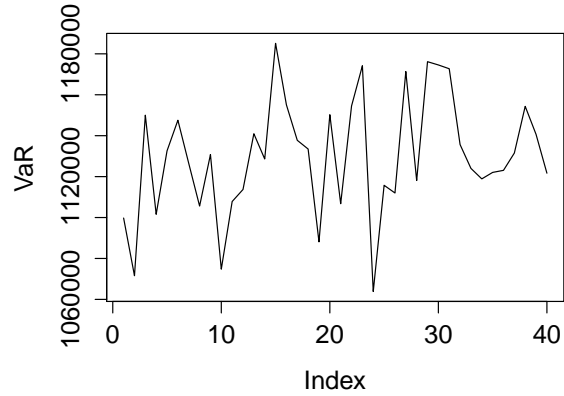
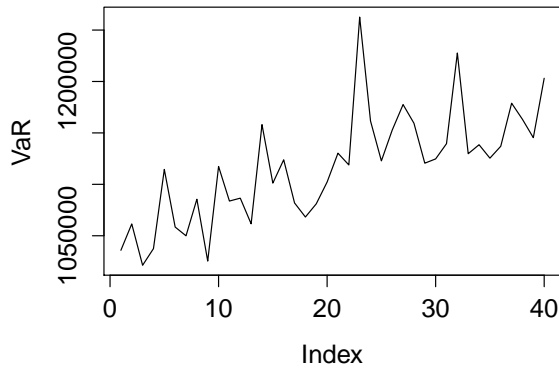
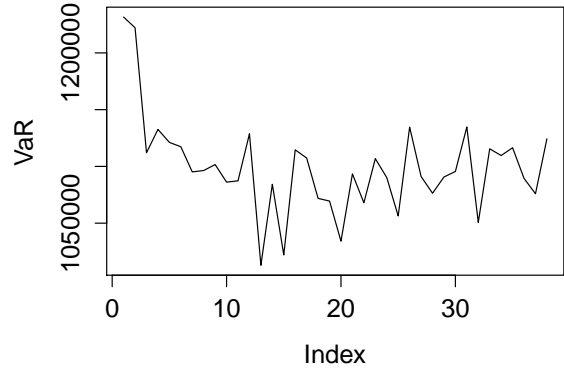
The general case where all parameters are unknown will not be treated in this section since it is more complex to tackle it with the use of conjugate prior distributions (cf. Shevchenko [2011] pp. 129-131) for details.

3.2.4 Sensitivity Analysis

Working with this method, is generally simple since conjugate prior is involved, yet one of the main questions is how to ensure that experts opinion are consistent, relevant, and capture well the situation, which might in a way, cause a model error. In this work we did not take into consideration this aspect and the experts opinion were treated as correct. To improve our results, we can do a sensitivity test regarding our prior parameters. On the other hand, we only have the mean of the number of losses, given by our expert. So it appears difficult to obtain the distribution of λ with this only information. So in a way, we are immensely relying on our prior parameters which in reality don't give us a banking sense and are not easily comprehensive.

We are going to test the prior parameters given by the experts and highlight the direct consequence on our Capital Required. To start with the first case, where we are working with unknown λ ; our experts gave us a value of $a_0 = 0.5$ and $b_0 = 8$ as seen in section 3.2.1, so taking into consideration a step of 0.035 for an interval $[0.1, 1.5]$ and 0.1 for $[6, 10]$, respectively, we obtain the following VaR results (check Figures 3.1 and 3.2). As for the cases of unknown μ and unknown σ , we took an interval for $\mu_0 \in [6, 10]$ and $\sigma_0 \in [0.3, 4.1]$ with a step of 0.1 (see Figures 3.3 and 3.4).

The following figures show the stability of our Value-at-Risk calculations regardless of all changes in our prior parameters $(a_0, b_0, \mu_0, \sigma_0)$. Relying on the choice of our intervals, we notice that the boundaries of our VaR are in an acceptable range.


 Figure 3.1: Sensitivity for a_0

 Figure 3.2: Sensitivity for b_0

 Figure 3.3: Sensitivity for μ_0

 Figure 3.4: Sensitivity for σ_0

3.3 Bayesian techniques in combining two data sources: MCMC-Metropolis Hastings algorithm

In this section, we will use a noninformative prior and more particularly the Jeffreys prior, (cf. Jeffreys [1946]), that attempts to represent a near-total absence of prior knowledge that is proportional to the square root of the determinant of the Fisher information:

$$\pi(\omega) \propto \sqrt{|I(\omega)|},$$

where $I(\omega) = -\mathbb{E} \left(\frac{\partial^2 \ln \mathcal{L}(X|\omega)}{\partial \omega^2} \right)$.

Then we are going to apply an MCMC model to obtain a distribution for the parameters and generate our capital required at 99.9%. This will allow us to compare both methods' results and develop a generalized application to set up our capital allocation, since no restrictions is made regarding the distributions. As for the parameter σ , it will no longer

be fixed as in the previous sections. For more details on the Jeffreys prior and MCMC-Metropolis Hastings algorithm check ROBERT [2007].

3.3.1 MCMC with the Poisson(λ) distribution

Assuming that the parameter λ is the only thing unknown, the Jeffreys prior distribution is: $\pi(\lambda) \propto \frac{\sqrt{\lambda}}{\lambda}$ (see Appendix G), thus finding the posterior distribution $f(\lambda|n_{SA}, n_{ID})$ with the use of experts Scenario Analysis and Internal Data would be:

$$f(\lambda|n_{SA}, n_{ID}) \propto \overbrace{\pi(\lambda)}^{\text{Jeffreys prior}} \underbrace{\mathcal{L}(n_{SA}, \lambda)\mathcal{L}(n_{ID}, \lambda)}_{\text{Likelihood functions}}.$$

So by applying Metropolis Hastings algorithm, (check appendix H.1 for full support on detailed algorithm), with the objective density:

$$\begin{aligned} f(\lambda|n_{SA}, n_{ID}) &\propto \frac{1}{\sqrt{\lambda}} \prod_{k=1}^{n_{SA}} \frac{e^{-\lambda}\lambda^k}{k!} \prod_{k=1}^{n_{ID}} \frac{e^{-\lambda}\lambda^k}{k!} \\ &\propto \frac{1}{\sqrt{\lambda}} \prod_{k=1}^{n_{SA}} e^{-\lambda}\lambda^k \prod_{k=1}^{n_{ID}} e^{-\lambda}\lambda^k \\ &\propto \frac{1}{\sqrt{\lambda}} e^{n_{SA}\lambda} \lambda^{\sum_k^{n_{SA}} k} e^{n_{ID}\lambda} \lambda^{\sum_k^{n_{ID}} k} \end{aligned}$$

and with a uniform proposal density: $U(\lambda_{SA}, \lambda_{ID})$, we obtain the parameter λ distribution see Figure 3.5.

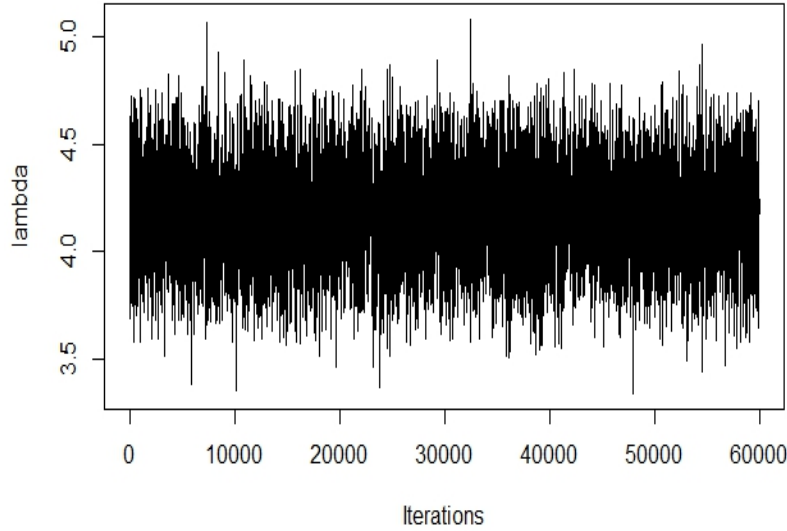
We have removed the first 3000 iterations so that the chain is stationary (burn-in iterations effect), (cf. Gilks *et al.* [1996] pp. 5-6). We obtain a 99.9 % Value-at-Risk of 1,000,527.00.

The result is close to the VaR considered with the use of conjugate family.

3.3.2 MCMC with Unknown Poisson(λ) parameter and Lognormal $\mathcal{LN}(\mu, \sigma)$ distribution with unknown μ

Assuming that the parameters λ and μ are the only things unknown, we will treat them independently and since the Poisson(λ) case has already been treated, the Jeffreys prior distribution for μ is: $\pi(\mu) \propto \frac{1}{\sigma} \propto 1$ (see Appendix G), thus finding the posterior distribution $f(\mu|x, y)$ with the use of experts Scenario Analysis and Internal Data would be:

$$f(\mu|x, y) \propto \overbrace{\pi(\mu)}^{\text{Jeffreys prior}} \underbrace{\mathcal{L}(x_1, x_2, \dots, x_{n_{SA}}|\mu, \sigma_{SA})\mathcal{L}(y_1, y_2, \dots, y_{n_{ID}}|\mu, \sigma_{ID})}_{\text{Likelihood functions}}.$$


 Figure 3.5: MCMC for the parameter λ

So by applying Metropolis Hastings algorithm, (check Appendix *H.2* for full support on detailed algorithm), with the objective density:

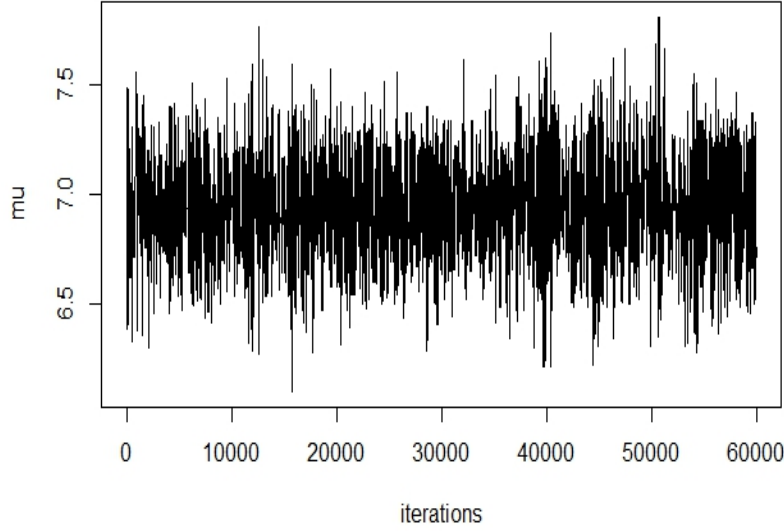
$$\begin{aligned} f(\mu|x, y) &\propto \prod_{i=1}^{n_{SA}} \frac{1}{x_i \sqrt{2\pi\sigma_{SA}^2}} \exp\left\{-\frac{(\ln x_i - \mu)^2}{2\sigma_{SA}^2}\right\} \prod_{i=1}^{n_{ID}} \frac{1}{y_i \sqrt{2\pi\sigma_{ID}^2}} \exp\left\{-\frac{(\ln y_i - \mu)^2}{2\sigma_{ID}^2}\right\} \\ &\propto \exp\left\{-\sum_i \frac{(\ln x_i - \mu)^2}{2\sigma_{SA}^2}\right\} \exp\left\{-\sum_i \frac{(\ln y_i - \mu)^2}{2\sigma_{ID}^2}\right\} \end{aligned}$$

and with a uniform proposal density: $U(0, 12)$, we obtain the parameter μ distribution see Figure 3.6.

We obtain a Value-at-Risk of 1,167,060.00.

Comparing this to the same case generated with conjugate prior, we can check the closeness of both values.

In the next subsection, we will tackle the general case, where all parameters are unknown, this case was not treated with conjugate prior distributions since it would be more complicated.


 Figure 3.6: MCMC for the parameter μ

3.3.3 General case: MCMC with Unknown Poisson(λ) parameter and Lognormal $\mathcal{LN}(\mu, \sigma)$ distribution with unknown μ and σ

We are going to assume the general case, where all the parameters are unknown λ , μ and σ , we will treat them independently and since the Poisson(λ) case has already been employed, the Jeffreys prior distribution for $\omega = (\mu, \sigma)$ is: $\pi(\omega) \propto \frac{1}{\sigma^3}$ (cf. Appendix G), thus finding the posterior distribution $f(\omega|x, y)$ with the use of experts Scenario Analysis and Internal Data would be:

$$f(\omega|x, y) \propto \underbrace{\pi(\omega)}_{\text{Jeffreys prior}} \underbrace{\mathcal{L}(x_1, x_2, \dots, x_{n_{SA}}|\mu, \sigma)\mathcal{L}(y_1, y_2, \dots, y_{n_{ID}}|\mu, \sigma)}_{\text{Likelihood functions}}.$$

So by applying Metropolis Hastings algorithm, (check appendix H.3 for full support on detailed algorithm), with the objective density:

$$\begin{aligned} f(\omega|x, y) &\propto \frac{1}{\sigma^3} \prod_{i=1}^{n_{SA}} \frac{1}{x_i \sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(\ln x_i - \mu)^2}{2\sigma^2}\right\} \prod_{i=1}^{n_{ID}} \frac{1}{y_i \sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(\ln y_i - \mu)^2}{2\sigma^2}\right\} \\ &\propto \frac{1}{\sigma^3} \frac{1}{\sigma^{n_{SA}}} \exp\left\{-\sum_i \frac{(\ln x_i - \mu)^2}{2\sigma^2}\right\} \frac{1}{\sigma^{n_{ID}}} \exp\left\{-\sum_i \frac{(\ln y_i - \mu)^2}{2\sigma^2}\right\} \end{aligned}$$

and with a uniform proposal density: $U(0, 12)$ and $U(0, 7)$ for μ and σ respectively,

we obtain the parameters μ and σ distributions, illustrated in Figure 3.7.

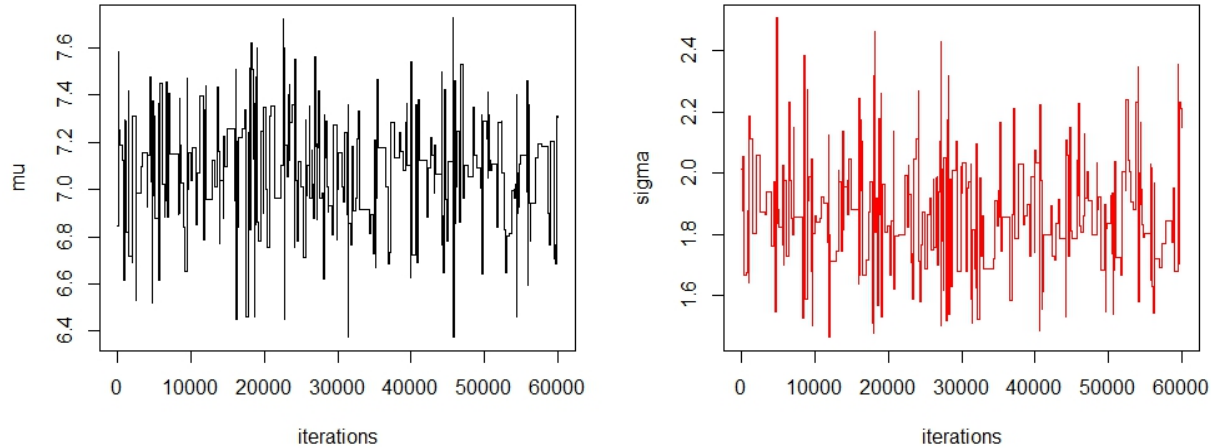


Figure 3.7: MCMC for the parameters μ and σ

We have removed as well, the first 3000 iterations so that the chain is stationary (burn-in iteration effect). We obtain a Value-at-Risk of 3,061,151.00. The general case generates a good combination between internal data and experts' opinion with a capital requirement of 3,061,151 \$.

3.3.4 Confidence Interval calculation

To recapitulate on all the calculations, table 3.2 summarizes all Value-at-Risk generated. As for the calculation of the confidence interval, since we are working with order statistics, the interval (x_l, x_u) would cover our quantile x_p with a 95% probability that depends on the lower bound l , upper bound u , number of steps n and confidence level p .

In our calculations, we took $n = 10^5$, $p = 99.9\%$ and our integers (l, u) , were constructed using the normal approximation $\mathcal{N}(np, np(1-p))$ to the binomial distribution $\mathcal{B}(n, p)$, (since n is large). Then a simple linear interpolation has been made to obtain the values of (x_l, x_u) , (cf. David & Nagaraga [2003] pp. 183-186), for more details and demonstrations.

Table 3.2 shows the helpful use of the Bayesian inference techniques. The results of both methods are close and comparable; though conjugate prior is simple but the distributions are restricted to the conjugate family, yet with the Jeffreys non-informative prior and MCMC-Metropolis Hastings algorithm, we will have a wider options and generate a good combination between internal data and experts' opinions.

<i>Case</i>	<i>Confidence Interval</i>		<i>VaR (99.9%)</i>	<i>Length</i>
<i>Aggregate</i>	\$ 1,040,697.00	\$ 1,230,492.00	\$ 1,162,215.00	15.42%
<i>Scenario Analysis</i>	\$ 6,094,853.00	\$ 7,171,522.00	\$ 6,592,086.00	15.01%
<i>Bayesian unknown λ</i>	\$ 1,053,861.00	\$ 1,184,129.00	\$ 1,117,821.00	11.00%
<i>Bayesian unknown μ</i>	\$ 1,097,195.00	\$ 1,268,136.00	\$ 1,188,079.00	13.48%
<i>Bayesian unknown λ and μ</i>	\$ 1,141,767.00	\$ 1,318,781.00	\$ 1,199,000.00	13.42%
<i>MCMC λ</i>	\$ 944,793.10	\$1,101,274.00	\$1,000,527.00	14.21%
<i>MCMC λ, μ</i>	\$1,098,930.00	\$1,244,564.00	\$1,167,060.00	11.70%
<i>MCMC λ, μ, σ</i>	\$2,839,706.00	\$3,310,579.00	\$3,061,151.00	14.22%

Table 3.2: Value at Risk and Confidence intervals for all cases treated

In addition, we are going to use the Monte Carlo confidence intervals approach (cf. Buckland [1984]) in order to compare it with the previous approach and ensure the similarities. Consider a parameter X with its consistent estimator Y , with cumulative distribution function $F(y|x)$ generated by some process which can be simulated. Number of simulations n and y values are independently generated and then ordered from largest to smallest. An approximate $100(1 - 2\alpha)$ confidence level for X is given by (y_j, y_k) where j and k represent respectfully the lower and upper bound of the interval and they are set as: $j = (n + 1)\alpha$ and $k = (n + 1)(1 - \alpha)$. Usually j and k will not be integer; therefore we can simply round it to the nearest integer values or even use linear interpolation. We seek to calculate (y_j, y_k) , this may be found, using a conventional $100(1 - 2\alpha)\%$ confidence level, by solving: $F(Y|X = y_j) = 1 - \alpha$ and $F(Y|X = y_k) = \alpha$.

The actual confidence level has a beta distribution with parameters $k - j$ and $n - k + j + 1$, this is concluded when percentiles of the distribution $F(Y|X = y)$ are estimated by simulation.

Respecting that B has a beta distribution, $E(B) = \frac{k - j}{n + 1}$ and $Var(B) = \frac{(k - j)(n - k + j + 1)}{(n + 1)^2(n + 2)}$

In our case, by using the confidence level of 99.9% and by applying the previous calculations we have obtained an approximation 95% interval for actual confidence level with $n = 10^5$, $p = 99.9\%$, $j = 50$, $k = 99951$, $\alpha = 0.05\%$, $\zeta = 2.5\%$, $\sigma = \sqrt{Var(B)}$ and by moving 1.96 standard errors in either direction from the estimate we obtain our confidence interval: $[p - \Phi^{-1}(\zeta)\sigma, p + \Phi^{-1}(\zeta)\sigma] = [99.88\%, 99.92\%]$, which is very close to the previous interval calculation in table 3.2.

Furthermore, figures 3.8, 3.9 and 3.10 illustrate the Value-at-Risk calculation for different confidence level. It shows the presence of the general case between both Internal and Scenario Analysis curves. On the other hand, the conjugate prior figure 3.10, regarding all 3 unknown variables, point out the closeness of the curves which add to our previous analysis that σ is our key parameter.

Moreover, the concept of Scenario Analysis with the expert opinion deserves more clarification. Roughly speaking, when we refer to experts judgments, we express the idea that banks' experts and experienced managers have some reliable intuitions on the riskiness of their business and that these intuitions are not entirely reflected in the bank's historical, internal data. In our case, experts' intuitions were directly plugged into severity and frequency estimations through building a histogram approach.

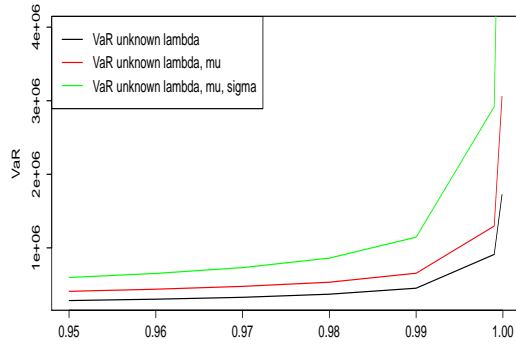


Figure 3.8: Different VaR Calculation for all MCMC Cases with unknown parameters

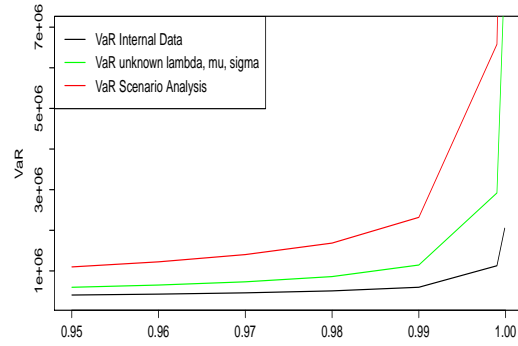


Figure 3.9: Different VaR Calculation for Internal Data, MCMC General Case and Scenario Analysis

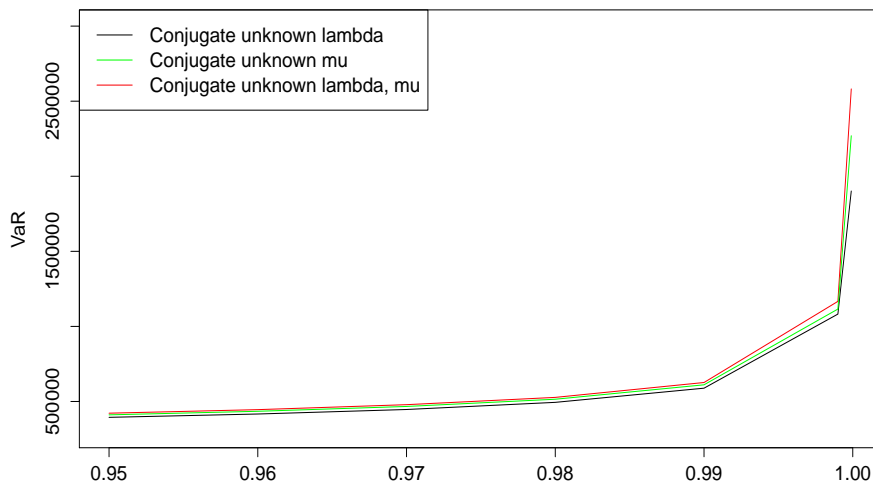


Figure 3.10: Different VaR Calculation for all conjugate prior cases

3.4 Bayesian approach reviewed

In this part, we are going to replace the experts opinions, by assuming that the experts' parameters are set using the Basel II standardized approach calculation. Hence, the experts opinion is questionable in the meaning of when it's used, we shift into the Markovian process which can cause problems.

3.4.1 Operational Risk Standardized Approach

In the Standardized Approach (SA), banks' activities are divided into 8 business lines (cf. BCBS [2006]): corporate finance, trading & sales, retail banking, commercial banking, payment & settlements, agency services, asset management, and retail brokerage. Within each business line, there is a specified general indicator that reflects the size of the banks' activities in that area. The capital charge for each business line is calculated by multiplying gross income by a factor β assigned to a particular business line.

Business line (j)	Beta factors(β_j)
$j = 1$, corporate finance	18%
$j = 2$, trading & sales	18%
$j = 3$, retail banking	12%
$j = 4$, commercial banking	15%
$j = 5$, payment & settlement	18%
$j = 6$, agency services	15%
$j = 7$, asset management	12%
$j = 8$, retail brokerage	12%

Table 3.3: Business lines and the Beta factors

The total capital charge is calculated as a three year average over all positive gross income (GI) as follows:

$$K_{SA} = \frac{\sum_{i=1}^3 \max(\sum_{j=1}^8 \beta_j GI^i, 0)}{3}$$

Hence, the application of the Standardized Approach generates a capital requirement of $K_{SA} = 2,760,780$.

3.4.2 Numerical Results for Expert opinion treated as SA

Setting the parameters to give us the same Standardized approach capital requirement and treating them as the expert parameters gave us: $\lambda = 6.8$, $\mu = 7.3$ and $\sigma = 1.72$.

Case	Confidence Interval		VaR (99.9%)	Length
Aggregate	\$ 1,040,697.00	\$ 1,230,492.00	\$ 1,162,215.00	15%
Scenario Analysis	\$ 2,570,297.00	\$ 2,876,469.00	\$ 2,759,640.00	12%
Bayesian unknown λ	\$ 1,084,427.00	\$ 1,257,414.00	\$ 1,172,801.00	16%
Bayesian unknown μ	\$ 1,045,412.00	\$ 1,183,887.00	\$ 1,118,045.00	13%
Bayesian unknown λ and μ	\$ 1,114,267.00	\$ 1,249,999.00	\$ 1,175,326.00	12%
MCMC λ	\$ 1,025,132.00	\$ 1,188,600.00	\$ 1,083,511.00	16%
MCMC λ, μ	\$ 1,169,519.00	\$ 1,347,836.00	\$ 1,253,938.00	15%
MCMC λ, μ, σ	\$ 1,678,124.00	\$ 1,912,897.00	\$ 1,769,198.00	14%

Table 3.4: Value at Risk and Confidence intervals for all cases treated

We note that the Standardized approach from Basel II is the one to rely on when it comes to calculate the VaR with its confidence interval. It is interesting to compare both results in table 3.2 and 3.4 where we notice that the VaR results in the cases of Bayesian unknown λ , unknown μ and unknown λ, μ are very close to the result in the Aggregate case.

As for the MCMC approach where experts' opinions are respected, in the case of unknown λ, μ and σ the results are close to the Scenario Analysis VaR. We conclude that, the expert opinion used parameters can be uncertain and cause an issue because it can lead to a disruption in the Markovian process. Combining these different data sources, highlights the importance of experts' opinion coherence which generates an estimation risk that affects our capital required calculation.

3.5 Conclusion

Using the information given by the experts, we were able to determine all the parameters of our prior distribution, leading to the posterior distributions with the use of internal data, which allowed us to compute our own capital requirement. This approach offers a major simplicity in its application through the employment of the conjugate distributions. Therefore, allowing us to obtain explicit formulas to calculate our posterior parameters. Yet, the application of this approach could not be perfected since it's restricted to the conjugate family.

On the other hand, Jeffreys prior with MCMC-Metropolis Hastings algorithm provided us with wider options and generated a satisfactory result regarding all three unknown variables λ, μ and σ , with the only difference of using complex methods. Taking σ unknown as well, was very essential in reflecting the credibility of estimating our capital requirement.

Yet, treating the experts' outputs the same as Basel's operational risk Standardized approach, illustrated the necessity of calling attention to the judgments given. In our application, judgments were needed to make sensible choices but these choices will influence the results. Understanding this influence, should be an important aspect of capital calculation, since it created an estimation risk that has highly influenced our capital requirement.

Chapter 4

Operational Risk Management in a Bank

We have seen in previous chapters the measurement of Operational risk with the use of Advanced Measurement Approach (AMA) and more particularly the Loss Distribution Approach LDA. Using LDA for modelling Operational risk cannot be perfected unless a vision of the past losses with the internal losses, present showing the current performance with the use of RCSA and KRIs and future giving us a future-looking view with the use of External Losses and Scenario analysis, has been conducted and incorporated in the Capital requirement calculation. For that, in this chapter we will outline the management of operational risk in this same particular qualitative aspect for banks and more particularly for Bank Audi aiming to apply more advanced models and management techniques in its Operational Risk department. The sections in this part are divided as the following: Risk and Control Self Assessment (RCSA), Incident reporting, Key Risk Indicators (KRIs), Incorporation of External Data, Scenario analysis, Insurance covering operational risks and at the end, we will scale our severity for external loss data and normalize the external losses to our Lebanese Bank.

4.1 Introduction

Operational risk can be the most devastating and at the same time, the most difficult to anticipate. Its appearance can result in sudden and dramatic reductions in the value of a firm. It cannot be managed successfully with a few spreadsheets or databases developed by an internal risk management department. In fact, one of the biggest mistakes an institution can make is to rely on simplistic and traditional solutions, which can lead to less than ideal choices about managing operational risk.

For the purpose of managing operational risk, it is mostly the potential for large, unexpected losses, either on a per event basis or within a set time period (e.g. a year). The operational risk management of a Lebanese bank formalizes the approach of government. It is meant at a minimum to comply with the qualifying qualitative criteria for the use of more advanced measurement approaches as Standardized and Advanced Measurement Approach as well as for local and cross-border regulatory requirements for defining,

measuring and managing operational risk. Figure 4.1 depicts the key components of the bank’s Operational Risk Management Framework.

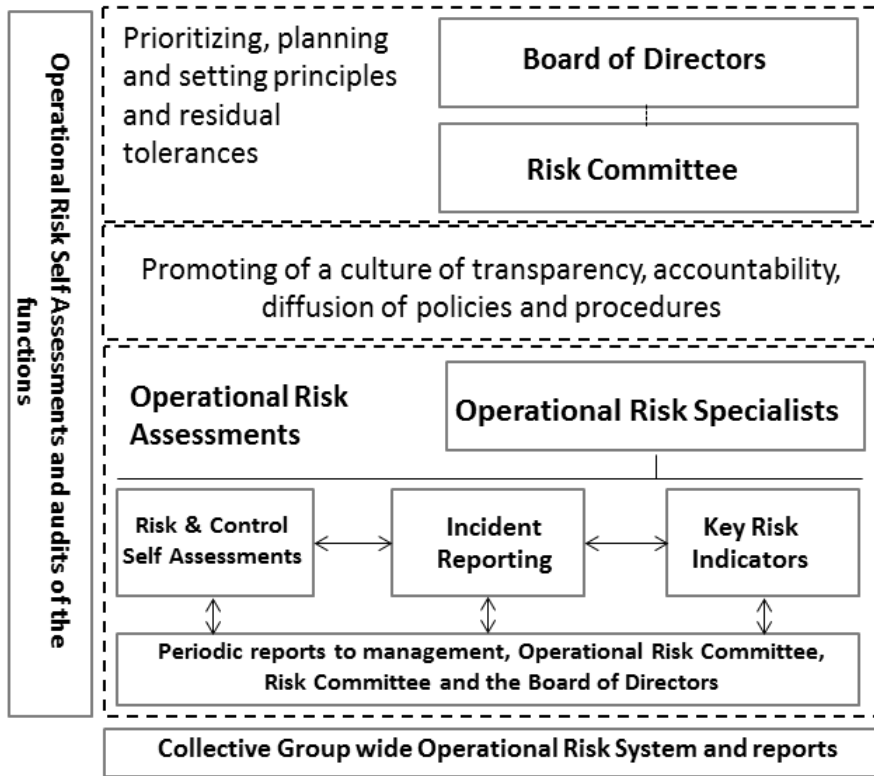


Figure 4.1: Operational Risk Framework

4.2 Risk and Control Self Risk Assessment

The RCSA described in this section, will be used to assist in the identification and assessment of operational risks. The main purpose of the RCSA table is to provide a template for business owners where they can map their Business Units risk scenarios and related controls, assess them and determine the ways to deal with them.

The implementation of the assessment model requires:

- Expressing Risk Tolerance
- Setting the rating scales
- Defining the fields of the RCSA table

4.2.1 Expressing Risk Tolerance

Board members and managers need to express their views with respect to severity and frequency in terms of occurrences per year, at the Business Unit levels respectively. In other words, when we say "High Severity", it is pertinent to express what is meant by "high", ideally in terms of a dollars interval representing an economic loss or opportunity cost; not the potential of an accounting loss. Similarly, when we say "Low frequency", it is useful to express this in terms of potential occurrences per period (per year for example). Also, board members and managers should express what amount of gross operational loss, i.e., before any recovery, from insurance or otherwise, either on an event basis or as a total per year, would call for a major review of the way a department operates and the validity of the controls in place. This is a measure of risk tolerance which will help operational risk management evaluate the effectiveness of the controls as events happen.

4.2.2 Setting the rating scales

Each department should select the severity level that best describes the reality. This is in order to normalize the meaning of qualitative risk ratings.

Loss severity in \$		
Level 1	Insignificant	$\leq 0.5K$
Level 1	Low	$> 0.5K \text{ and } \leq 1K$
Level 1	Medium-Low	$> 1K \text{ and } \leq 5K$
Level 1	Medium	$> 5K \text{ and } \leq 25K$
Level 1	Medium-High	$> 25K \text{ and } \leq 50K$
Level 1	High	$> 50K \text{ and } \leq 250K$
Level 1	Catastrophic	$> 250K \text{ and } \leq 10000K$
Level 2	Insignificant	$\leq 2K$
Level 2	Low	$> 2K \text{ and } \leq 5K$
Level 2	Medium-Low	$> 5K \text{ and } \leq 10K$
Level 2	Medium	$> 10K \text{ and } \leq 50K$
Level 2	Medium-High	$> 50K \text{ and } \leq 100K$
Level 2	High	$> 100K \text{ and } \leq 500K$
Level 2	Catastrophic	$> 500K \text{ and } \leq 15000K$
Level 3	Insignificant	$\leq 5K$
Level 3	Low	$> 5K \text{ and } \leq 25K$
Level 3	Medium-Low	$> 25K \text{ and } \leq 50K$
Level 3	Medium	$> 50K \text{ and } \leq 100K$
Level 3	Medium-High	$> 100K \text{ and } \leq 250K$
Level 3	High	$> 250K \text{ and } \leq 1000K$
Level 3	Catastrophic	$> 1000K \text{ and } \leq 20000K$
Level 4	Insignificant	$\leq 10K$
Level 4	Low	$> 10K \text{ and } \leq 50K$
Level 4	Medium-Low	$> 50K \text{ and } \leq 125K$
Level 4	Medium	$> 125K \text{ and } \leq 250K$
Level 4	Medium-High	$> 250K \text{ and } \leq 500K$
Level 4	High	$> 500K \text{ and } \leq 2500K$
Level 4	Catastrophic	$> 2500K \text{ and } \leq 25000K$
Level 5	Insignificant	$\leq 25K$
Level 5	Low	$> 25K \text{ and } \leq 75K$
Level 5	Medium-Low	$> 75K \text{ and } \leq 250K$
Level 5	Medium	$> 250K \text{ and } \leq 500K$
Level 5	Medium-High	$> 500K \text{ and } \leq 1000K$
Level 5	High	$> 1000K \text{ and } \leq 5000K$
Level 5	Catastrophic	$> 5000K \text{ and } \leq 100000K$

Table 4.1: Potential Operational loss severity

Each department should select the frequency level that best describes the possible frequency rates of potential operational losses in its business.

Loss frequency

Level 1	High	Once a quarter or more
Level 1	Medium-High	Twice a year
Level 1	Medium	Once a year
Level 1	Medium-Low	Once every two years
Level 1	Low	Once every five years
Level 1	very Low	Once every twenty years
Level 2	High	Once a month or more
Level 2	Medium-High	Once a quarter
Level 2	Medium	Twice a year
Level 2	Medium-Low	Once a year
Level 2	Low	Once every two years
Level 2	very Low	Once every twenty years
Level 3	High	Once a week or more
Level 3	Medium-High	Once a month
Level 3	Medium	Once a quarter
Level 3	Medium-Low	Twice a year
Level 3	Low	Once a year
Level 3	very Low	Once every twenty years

Table 4.2: Potential Operational loss frequency

As a result of the two detailed tables above, there are $([7 \text{ severity dollar values} \times 5 \text{ severity levels}] \times [6 \text{ frequency value} \times 3 \text{ frequency levels}]) = 630$ possible combinations of severity expressed in dollars and frequency expressed in events per period.

In order to clarify the reasoning, an extreme example could help: While a low frequency, high severity event in capital markets could be an unauthorized transaction happening once every five years and costing \$1 Million, a low frequency, high severity event in credit cards could be a credit card fraud of \$25,000 happening twice a year.

4.2.3 Defining the fields of the RCSA table

Board members should then respond to the four questions below:

- What is your maximum tolerable operational loss per event?
- What is your maximum tolerable operational loss per year?
- What level of operational loss per event would you call insignificant?
- What level of operational loss per year would you call insignificant?

Once an answer is obtained on these 4 questions, Operational Risk Management will compare Business Owner ratings collected during RCSAs to the risk tolerance levels approved by the Board members.

4.3 Incident Reporting

4.3.1 Definition

A reportable incident is any unusual event, operational in nature, which caused or had the potential to cause damage to the bank, whether tangibly or not, in readily measurable form (with financial impact, even in the bank's favor) or as an estimate (in economic or opportunity cost terms). In simple terms, operational risk events are anything that went wrong or that could go wrong.

Several benefits can result from having an effective incident reporting mechanism. Some of these benefits can be summarized as follows:

- Protect the bank
- Contain incidents and address their root cause.
- Avoid recurrence
- Promote an open and honest environment.

4.3.2 Example of incidents

Incidents pertaining to any activity:

- Medical costs and other compensation related to any accident that occurred on or off the Bank premises for which the bank accepted responsibility.
- A mistake in a funds transfer.
- Loan losses partially or fully attributable to an operational issue such as false information given by the customer, excess over limit allowed without authorization, over-valuation of collateral or other operational issues linked to the management of collaterals.
- Overtime costs in the course of dealing with an operational incident (for example: a system failure).
- Cost of repair in case of accident or breakdown.
- The disappearance of any asset, for any reason, known or suspected.
- Any legal cost incurred in the course of dealing with an operational event.

Incidents generally specific to branches:

- Penalty paid to a regulatory or government body.
- Discovery of an error or a theft having affected a customer's account and our compensation to the customer (for example: debiting a wrong customer account, theft from a client's account).
- Loss due to a system error (for example: crediting the wrong accounts and that we were unable to reverse).
- Loss in foreign exchange or capital markets transactions due to a trading mistake (selling instead of buying or quantity mixed up with price).
- A hold up on or off premises.

Incidents generally specific to central departments:

- An interest penalty paid to a correspondent bank.
- A penalty paid to regulatory authorities.
- A loss in foreign exchange or capital markets due to a wrong entry on the system or to a trader trading above his limit.
- Losses in credit cards resulting from card skimming, ATM fraud, and liability shift.
- Operational losses from HR-related issues, e.g. compensation to an employee following an accident or a mistake.

4.4 Key Risk Indicators

4.4.1 Definition

KRIs are metrics that measure risk built into a specific business process/function or where the implementation of effective controls could reduce the occurrence of potential risk events.

They are tools used to monitor either exposure to Key Risks (on an inherent or residual basis) or controls for Key Risks.

KRIs are carefully selected parameters, tailored to selected business processes or areas, which are agreed to have a potential signaling function regarding changes in the operational risk profile.

As such, they are metrics that alert the organization to impending problems in a timely fashion as well as monitor the banks control culture. In conjunction with RCSAs and the analysis of incidents or loss events, they indicate the level of risk in a particular area of

business or function and may be used to trigger corrective action before the occurrence of events.

To reiterate, KRIs are the measures summarizing the frequency, severity and impact of Operational Risk events or corporate actions occurred in the bank during a reporting period. Table 4.3 shows the different types of KRIs.

Risk dimension	Indicators type
Frequency	Number of risk events
Severity	Volume of risk events Average risk losses Maximum duration of disruptions
Impact	Total amount of risk losses Cost of mitigations

Table 4.3: Key Risk Indicators type

4.4.2 KRIs Process

Following are the KRI standards which cover the entire process of the KRIs implementation in the process of management of operational risk. The KRI process is divided into the following phases: The selection phase followed by the periodical reporting phase and the review phase.

4.4.2.1 Identification of Key Risk Indicators

The identification of KRIs is preceded by the identification of key risks in a specific business unit. The Business Owner, in coordination with Operational Risk Management should develop his own KRIs which cover the operational risk profile of his activities. The developed KRIs can be related to risk drivers, actual incidents, near misses, audit exceptions, etc.

4.4.2.2 Assessment of Key Risks Indicators

Once the KRIs are selected, they must be assessed for their importance with respect to the risks they intend to monitor. Only time and experience are likely to tell whether an indicator presumed as "key" is indeed "key" or whether its predictive value has been overestimated. Assessing KRIs is also to assess the availability of data required to measure KRIs. KRIs not easy to measure due to the unavailability of data may be rejected.

After selection of the KRI, its unit of measurement and its frequency of tracking shall be decided by Business Owner. Key risks are then reviewed periodically for their relevance to the bank by Operational Risk Management in joint consultation with respective business unit.

Examples of Key Risk Indicators throughout departments is shown in table 4.4:

<p>Branch Network: Number of complaints and claims to the Bank Number of lost clients Amount of compensation paid to the client Volume of balances lost / opportunity cost</p> <p>Human resources: Turnover of experienced staff Number of temporary/short term staff Number of employees, attended training courses Number of employees, failed to pass mandatory evaluation</p> <p>Legal department: Number of legal actions against the Bank / third parties Volume of legal actions against the Bank / third parties Number of regulatory enquires / legislation breaches</p>	<p>Consumer Lending Department: Average days of getting loan approval Number of identified fraud cases Client dissatisfaction evidenced by client surveys Number of critical errors detected in credit files</p> <p>IT: Number of failures related to IT system and other equipment Number of calls to help desk on IT system and other equipment Average down-time of IT system and other equipment Increase in transaction load on systems</p> <p>Finance department: Volume of penalties, imposed by regulators Total amount of suspicious transactions Number of late completion or non-completed transactions</p>
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Table 4.4: Key Risk Indicators sample

4.4.2.3 Setting the KRI Scale

The KRI scale is defined as follows:

Zone/Scale	Scale Definition
Green	No action is required. No threshold breach
Amber	A threshold breach. Management attention is required to determine whether action needs to be taken in the near future
Red	May require immediate action. A persistent threshold breach

Table 4.5: KRIs Risk Level

4.4.2.4 Determination of Thresholds

For each selected KRI, a set of threshold values or limits are to be determined. To establish the initial thresholds for each KRI, an assessment of KRI data and trends for at least 6 months is needed. Meanwhile however, the KRI is live as far as trends are concerned.

Once the threshold is set, the performance of the KRI is tested in case an incident occurs. If the KRI value crossed the threshold predicting the occurrence of an incident, then the threshold established might be meaningful to that KRI, otherwise another more relevant threshold should be set.

4.4.3 Periodical KRI Reporting

Once all the characteristics of a KRI are defined and set, generating timely KRI data is required. A periodical KRI report, generated based on the defined frequency of data measurement, will be monitored.

In case an indicator breaches a pre-assigned threshold, remedial measures are necessary regarding sensitive KRI zones. The Operational risk department shall review the KRI report, refer to the Business Owner for information, add recommendations when needed and follow up on remedial measures initiated by the business owner.

4.4.4 Review of KRIs and their Thresholds

KRIs shall be reviewed periodically in terms of their relevance especially when there is change in product, process, technology, activities and other internal and external material changes. In certain cases, a particular KRI may no longer be applicable due to changing internal or external business circumstances. In essence, each KRI identified should be true reflective of the risk and for that an ongoing review shall be done.

Besides regular review of KRIs, the thresholds for the KRIs need to be periodically evaluated to account for the change in the level and quality of controls. If for instance, process controls have been substantially modified, the thresholds for the KRIs have to be changed to factor in the changes in the controls.

Another review of the thresholds is needed based on incident occurrence. For a relevant KRI, in case the KRI value is in the Green zone while an incident occurred, this indicates that the threshold is too high and a lower one should be determined. In case the KRI value is in the red zone for a considerable period of time while no incident occurs, the threshold might be too low and a higher threshold should be determined.

4.5 Incorporation of External Data

In the words of Charles Babbage¹: "Errors using inadequate data are much less than those using no data at all." It seems to be generally accepted in the finance industry that internal loss data alone is not sufficient for obtaining a comprehensive understanding of the risk profile of a bank. This is the reason why additional data sources have to be used, in particular external losses. There are many ways to incorporate external data into the calculation of operational risk capital. External data can be used to supplement an internal loss data set, to modify parameters derived from the internal loss data, and to improve the quality and credibility of scenarios. External data can also be used to validate the results obtained from internal data or for benchmarking.

Bank Audi's internal management, has recently bought SAS OpRisk software giving them as well, in addition to a lot of features, the opportunity to add SAS OpRisk Global Data package. SAS OpRisk Global Data is one of the largest, most comprehensive and most accurate repository of information on publicly reported operational losses in excess of 100,000 \$. The solution documents more than 25,000 events across all industries world-

¹Charles Babbage, (1791-1871) was an English polymath. He was a mathematician, philosopher, inventor and mechanical engineer, who is best remembered now for originating the concept of a programmable computer

wide and provides up to 50 descriptive, categorization and financial elements. Empirical evidence suggests that loss magnitude is to some extent a function of firm size. That is, if the same loss event occurred simultaneously at two firms of different sizes, the larger firm would experience the larger loss, all else being equal. As a result, external data must be scaled to be proportional to the size of the firm being analyzed. It provides five types of data on which scaling can be performed – revenues, assets, net income, number of employees and shareholder equity. So by using a powerful scaling algorithm or methodology, the bank can easily tailor external data to the organization’s exact analysis requirements.

External data is used as additional data source for modelling tails of severity distributions. The obvious reason is that extreme loss events at each bank are so rare that no reliable tail distribution can be constructed from internal data only. Yet external losses do not reflect Bank Audi’s risk profile as accurately as internal events but they surely can significantly improve the quality of the model.

4.6 Scenario Analysis

According to Basel, scenario analysis is a process of obtaining expert opinion of business line and risk managers to identify potential operational risk events and assess their potential outcome. Scenario analysis is an effective tool to consider potential sources of significant operational risk and the need for additional risk management controls or mitigation solutions. Given the subjectivity of the scenario process, a robust governance framework is essential to ensure the integrity and consistency of the process (cf. BCBS [2011]).

4.6.1 Scenario Analysis Algorithm and Procedures

The following algorithm has been extracted from the book *Operational Risk Management* (cf. Moosa [2007]):

- 1- Defining and structuring the task, specifying the area of interest and identifying the major relevant features of this area.
- 2- Describing important external factors and their influence on the area of interest. These factors form the influence fields.
- 3- Identifying major descriptors for each field and making assumptions about their future trends.
- 4- Checking the consistency of possible combinations of alternative assumptions regarding the critical descriptors and identifying assumption bundles.
- 5- Combining assumptions with the trend assumptions regarding the uncritical descriptors, resulting in a scenario for each field.
- 6- Making assumptions with respect to possible interfering events and their probabilities as well as their impacts on the field.

- 7- Assessing the impact of the field scenarios on the area of interest and its depiceters. Respective scenarios are constructed.
- 8- Identifying strategies that could promote or impede the developments described in the scenarios.

So as a scenario analysis procedure for the bank, we would have the following chart, moving from the Scenario risk drivers and Assumptions formulation to the Scenario selection, capital planing and follow-up (see figure 4.2):

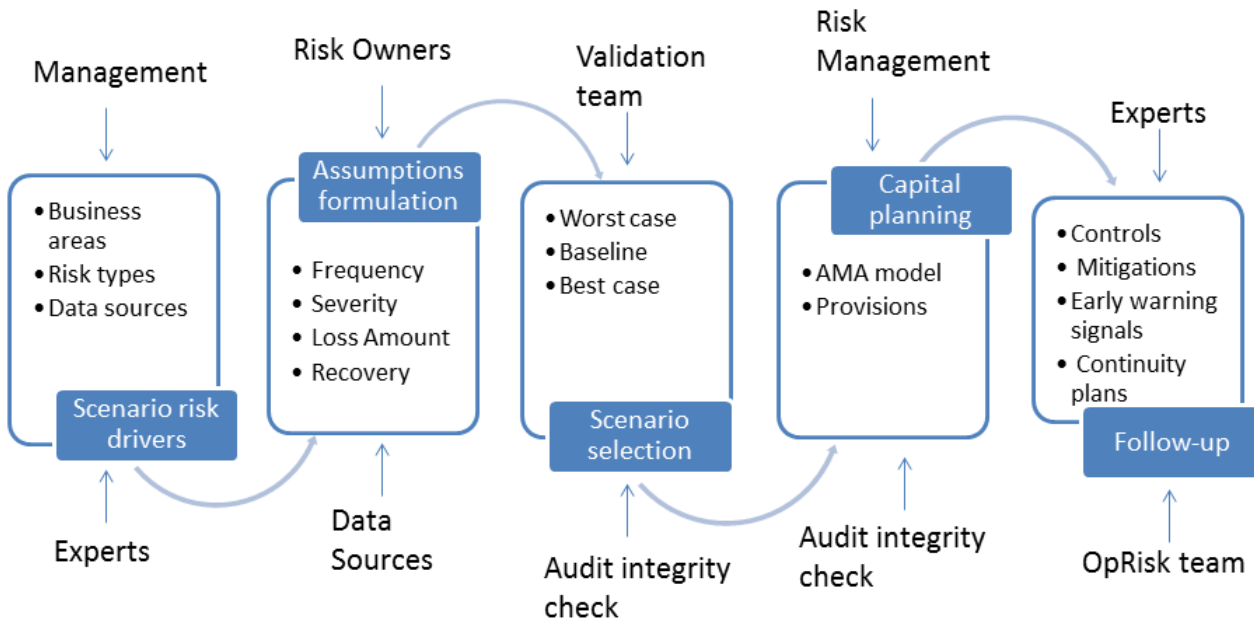


Figure 4.2: Scenario Analysis Procedure

As for the data collection, we have many data sources:

- External loss data
- Internal loss data
- RCSA
- KRI
- Expert opinions (imaginative thinking)

4.6.2 Scenario risk drivers

RCSA and KRIs may help as well to identify the business lines and event types of high impact, as we are going to see in table 4.6 below. Given this RCSA, we can see for example, put in bold, some of the high impact type of scenarios that might influence the bank.

	Internal Fraud	External Fraud	Employment Practices & Workplace Safety	Clients Products & Business practices	Damage to physical Assets	Business disruption and system failure	Execution delivery & Process Management
Corporate Finance	Failure to follow procedures/limits	Misrepresentation of information	Occupational accident	Regulatory breach	Business continuity failure	IT system failure	Transaction error
Trading & Sales	Unauthorized trading	Forgery	Occupational accident	compromised client information	Damage to premises	IT system failure	Data entry error
Retail Banking	Embezzlement	Credit card robbery	Environmental issue	Negative media publications	Terrorist attack	Utility outage	Inaccurate/Incomplete contract
commercial Banking	Theft of customer funds	Fraudulent transfer of funds	Discrimination	Client suitability	Natural disaster	IT system failure	Lost loan documentation
Payment and settlement	theft of client funds or assets	Payment fraud	Discrimination	Noncompliance with AML rules	Business continuity failure	Failure of payment channels	Failure to follow procedures
Agency services	Abuse of duties	Forgery	Wrongful termination	Mis-selling	Business continuity failure	IT system failure	Processing error
Asset Management	Unauthorized trading activities	Cybercrime	Occupational accident	Fiduciary breach	Business continuity failure	IT system failure	Mismanagement of account assets
Retail Brokerage	Insider trading	Forgery	Occupational accident	compromised client information	Business continuity failure	IT system failure	Tax noncompliance

Table 4.6: RCSA according the business lines and event types

Another set of high severity scenarios examples is the following (cf. Chernobai *et al.* [2007]):

- Large loan or card fraud (internal/external)
- High-scale unauthorized trading
- Legislation non-compliance or incomplete disclosure (banking, tax, AML regulation)
- Massive technology failure or new system migration
- Servers disruptions / network shutdown that lead to outages and loss of information
- Mergers and acquisitions with other banks
- Doubling the bank’s maximum historical loss amount
- Increase/decrease of loss frequency by 20%
- Increase/decrease if loss severity by 50%/100%

A wide variety of risks exist, thus necessitating their regrouping in order to categorize and evaluate their threats for the functioning of any given business. The concept of a risk matrix, coined by Richard Prouty (1960), allowed us in the previous chapters to highlight which risks can be modeled as was seen in the figure 2.1. For the same purpose, we have divided the different types of scenarios and events in a severity frequency matrix exposing the internal loss data, external loss data, RCSA, KRIs, Audit findings and Scenario Analysis, see figure 4.3.

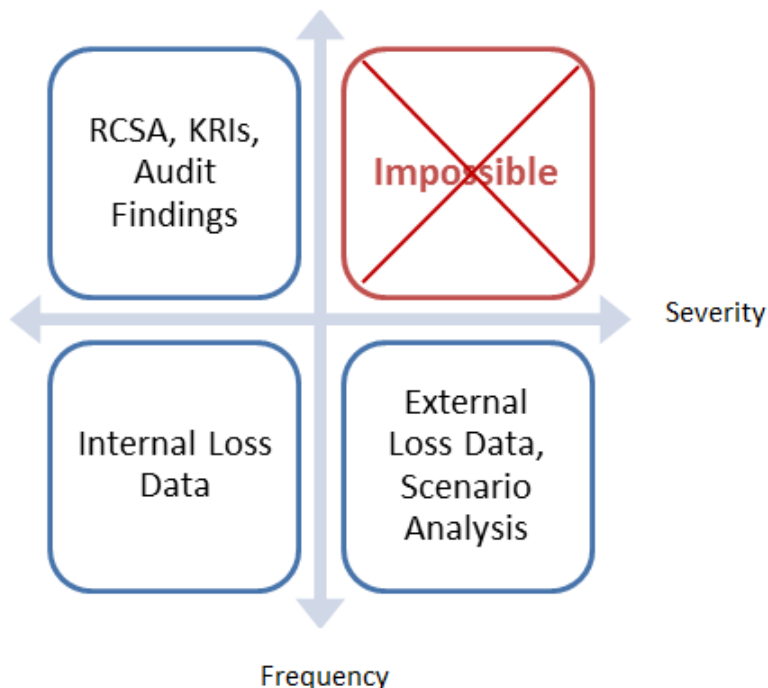


Figure 4.3: Scenario Matrix

4.7 Insurance Covering Operational Risks

The role that insurance plays in diminishing the financial impact of operational losses of a bank is highly important. The transfer of a risk to an insurer can contribute to a better performance preventing critical situation and covering a variety of losses. The Basel Committee approved that insurance can be used as a tool to reduce the financial impact of operational risks for banks, meaning that a specific type of insurance against operational risks can lead to a lower level of minimal capital allocated to a particular risk category.

While the purchase of insurance covering operational risks is still in its early stages of development, it would allow banks to replace operational risk by counterparty risk.

4.7.1 Insurance Policies and Coverage

Many types and categories of insurance can be purchased, each with a specific clause and price regarding the demand of the insured customer. Following, we explain the different types of insurance policies with their respective coverages and exclusions. This information is obtained from a local Lebanese bank.

4.7.1.1 Bankers Blanket Bond Insurance

Intended for banks and other institutions that are engaged in providing financial services, the policy indemnifies the assured for direct monetary losses due to loss, damage, and

misplacement during the policy period (which is usually one year).

Scope of Cover

- Clause 1 (Infidelity of Employees) Covers loss of property due to dishonest or fraudulent acts of one or more employees of the insured resulting in unethical financial gains.
- Clause 2 (On Premises) Covers loss of the insured or the customers' property on the insured's premises due to theft, burglary, damage, destruction or misplacement.
- Clause 3 (In Transit) Covers loss or damage to property from any cause while in transit either in the custody of the assured's employees or the custody of any Security Company or its vehicles but excluding property in mail and property subject to amounts recoverable from a Security Company under the latter's own insurance.
- Clause 4 (Forged Cheques et al) Covers loss due to forgery or fraudulent alteration of any financial instrument or payment on the above basis.
- Clause 5 (Counterfeit Currency) Covers the insured's loss due to acceptance in good faith of any counterfeit or fraudulently altered currency or coins.
- Clause 6 (Damage to Offices and Contents) Covers loss or damage suffered to all contents owned by the assured in their offices (excluding electronic equipment) due to theft, robbery, hold-up vandalism, etc.

Limit of Indemnity

As per sums agreed by both parties according to nature, size and volume of business handled by the insured in all their offices and branches. Usually specifies amounts for every loss under the insuring clauses and sometimes on an aggregate or overall annual basis.

- XXX US\$ any one loss in respect of Infidelity of Employees
- XXX US\$ any one loss in respect of On Premises
- XXX US\$ any one loss in respect of In Transit
- XXX US\$ any one loss in respect of Forgery or Alteration
- XXX US\$ any one loss in respect of Counterfeited Currency
- XXX US\$ any one loss in respect of Offices and Contents
- XXX US\$ any one loss in respect of Securities or Written Instruments
- XXX US\$ any one loss in respect of Books of Accounts and Records
- XXX US\$ any one loss in respect of Legal Fees

All in excess of:

XXX US\$ each loss however reducing to:

- *XXX* US\$ every loss in respect of insuring In transit, Offices & Contents, and Legal Fees
- *XXX* US\$ on aggregate in respect of insuring Counterfeited Currency

Premium Rating

A sum rated on the basis of amounts and limits of indemnity agreed, deductibles, claims history and insured, etc.

Exclusions

Loss or damage due to war risks, etc.
Loss not discovered during the policy period
Acts of directors' defaults
Shortage, cashier's error or omissions

4.7.1.2 Directors and Officers Liability Insurance

The following insurance covers are applied solely for claims first made against an insured during the period and reported to the insurer as required by the policy.

Management Liability

- **Individuals:** The insurer shall pay the loss of each insured person due to any wrongful act.
- **Outside Entity Directors:** The insurer shall pay the loss of each outside entity director due to any wrongful act.
- **Company Reimbursement:** If a company pays the loss of an insured person due to any wrongful act of the insured person, the insurer will reimburse the company for such loss.

Special excess protection for non-executive directors

The insurer will pay the non-indemnifiable loss of each and every non-executive director due to any wrongful act when the limit of liability, all other applicable insurance and all other indemnification for loss have all been exhausted.

Exclusions

The insurer shall not be liable to make any payment under any extension or in connection with any claim of:

- A wrongful act intended to secure profit gains or advantages to which the insured was not legally entitled.
- The intentional administration of fraud.
- Bodily injury, sickness, disease, death or emotional distress, or damage to destruction, loss of use of any property provided.

Limit of liability

XXX US\$ - Aggregate

- Per non-executive director special excess limit: separate excess aggregate limit for each non-executive director of the policyholder *XXX* US\$ each
- Investigation: 100% of the limit of liability under the insurance covers of Company Reimbursement, Management Liability, and 10% of the Per non-executive director special excess limit

4.7.1.3 Political Violence Insurance

This kind of policy indemnifies the insured with the net loss of any one occurrence up to but not exceeding the policy limit against:

- Physical loss or damage to the insured's buildings and contents directly caused by one or more of the following perils occurring during the policy period:
 - Act of Terrorism;
 - Sabotage;
 - Riots, Strikes and/or Civil Commotion;
 - Malicious Damage;
 - Insurrection, Revolution or Rebellion;
 - War and/or Civil War;
- Expenses incurred by the insured in the removal of debris directly caused by any one or more of the Covered Causes of Loss.

Exclusions

- Loss or damage arising directly or indirectly from nuclear detonation, nuclear reaction, radiation or radioactive contamination.
- Loss or damage directly or indirectly caused by seizure, confiscation, nationalization, requisition, detention, legal or illegal occupation of any property insured.
- Any loss arising from war between any two or more of the following: China, France, Russia, United States of America and the United Kingdom.
- Loss or damage arising directly or indirectly through electronic means including computer hacking or viruses.
- Loss or damage arising directly or indirectly from theft, robbery, house-breaking, mysterious or unexplained disappearance of property insured.

Limitations

- In respect of loss or damage suffered under this extension, the underwriters' maximum liability shall never be more than the Business Interruption Policy Limit (if applicable), or the Policy Limit (if applicable) where this Policy Limit is a combined amount for losses arising from both physical loss or physical damage and Business Interruption, for any one occurrence
- To clarify, when a business interruption policy limit applies to losses suffered under this extension, it shall apply to the aggregate of all claims by all insureds and in respect of all insured locations hereunder, and underwriters shall have no liability in excess of the business interruption policy limit whether insured losses are sustained by all of the insureds or any one or more of them, or whether insured losses are sustained at any one or more of the insured locations.
- With respect to loss under this extension resulting from damage to or destruction of film, tape, disc, drum, cell and other magnetic recording or storage media for electronic data processing, the length of time for which underwriters shall be liable hereunder shall not exceed:
 - Thirty (30) consecutive calendar days or the time required with exercised due diligence and dispatch to reproduce the data thereon from duplicates or from originals of the previous generation, whichever is less; or the length of time that would be required to rebuild, repair or reinstate such property but not exceeding twelve (12) calendar months, whichever is greater.

4.7.1.4 Electronic and Computer Crime Policy

This kind of policy covers electronic and computer crimes related to the following:

Computer Systems

Loss due to the fraudulent preparation, modification or input of electronic data into computer systems, a service bureau's computer system, an electronic fund transfer system or a customer communication system.

Electronic Data, Electronic Media, Electronic Instruction

- Losses due to the fraudulent modification of electronic data or software programs within computer systems;
- Losses due to robbery, burglary, larceny or theft of electronic data or software programs;
- Losses due to the acts of a hacker causing damage or destruction to electronic data or software programs;
- Losses due to damage or destruction of electronic data or software programs using computer virus.

Electronic Communications

Loss due to the transfer of funds as a result of unauthorized and fraudulent electronic communications from customers, a clearing house, custodians or financial institutions.

Insured's Service Bureau Operations

Loss due to a customer transferring funds as a result of fraudulent entries of data whilst the insured is acting as a service bureau for customers.

Electronic Transmissions

Loss due to the transfer of funds on the faith of any unauthorized and fraudulent customer voice initiated funds transfer instructions.

Customer Voice Initiated Transfers

Loss due to the transfer of funds on the faith of any unauthorized and fraudulent customer voice initiated funds transfer instructions.

Extortion

Loss by a third party who has gained unauthorized access into the insured's computer systems threatening to cause the transfer of funds, disclosure of confidential security codes to third parties, or damage to electronic data or software programs.

Limit of Indemnity

XXX US\$ any one loss and in the aggregate for all clauses

The amount of the deductible under this policy for each and every loss is in excess of XXX US\$

4.7.1.5 Plastic Card Insurance Policy

These kinds of policies will indemnify the insured against losses sustained through alteration, modification or forgery in any Visa Electron Card, Bankernet, Visa and MasterCard issued by the insured or issued on his behalf and resulting from cards that have been lost, stolen, or misused by an unauthorized person.

Exclusions

The policy does not cover:

- Loss for which the assured obtained reimbursement from its cardholder, any financial institution, plastic card association or clearing house representing the assured.
- Loss not discovered during the policy period.
- Loss which arises directly or indirectly by reason of or in connection with war, invasion, act of foreign enemy, hostilities, or civil war.
- Loss resulting from the issue of any plastic card to guarantee the cashing of any cheque.
- Loss resulting wholly or partially, directly or indirectly from any fraudulent or dishonest act performed alone or with others, by an officer, director or employee of the assured or by any organization that authorizes, clears, manages, or interchanges transactions for the assured.

Limit of Indemnity

XXX US\$ - per card per year

XXX US\$ - in the annual aggregate for all cards

Deductible

XXX US\$ in the annual aggregate

4.7.2 Advantages of Insuring Operational Risks

In general, the role of insurance is to transfer the financial impact of a risk from one entity to another. However, transferring risk is not the same as controlling it as we do not avoid, prevent or reduce the actual risk itself. Nevertheless, insurance as a risk reduction tool helps the bank avoid or optimize the loss by buying a policy related to operational risk for which the bank pays an insurance premium in exchange for a guarantee of compensation in the event of the materialization of a certain risk. This means that insuring against operational risks enables a bank to eliminate or reduce large fluctuations of cash flow caused by high and unpredictable operational losses. By doing so, the bank benefits by improving income and increasing its market value, allowing it to avoid severe situations that would lead to insolvency.

- A variety of factors influence banks to purchase insurance to cover operational risks: The size of a bank matters as smaller banks have lower equity and free cash flows, thus making them more vulnerable to losses from operational risks. Consequently, large banks have the resources to manage their operational risks, though they also purchase insurance policies to protect themselves from any type of major loss, especially when it affects investors' confidence or would result in extreme negative effects.
- The time horizon also has its effect: the extent to which a bank can cover the immediate expense of an insurance premium in exchange for a benefit that may materialize only in the long run depends on the time horizon over which the bank is willing to pay premiums to cover a risk that may or may not happen in the long term.
- The better the rating, the higher the cost of refinancing: banks with very good rating can opt to finance losses by contracting credits rather than insurance. However, the bank might suffer high losses when it incurs considerable deficits that were not subject to insurance causing restrictions in its access to financing.

4.7.3 Basel II views

The Basel II Committee (cf. BCBS [2003]) stated that any effort to improve risk management should be viewed independently from the request of capital and hence insurance should not affect the required minimum capital. However, many bankers and insurers believe that insurance should be treated as an instrument of reducing the required minimum capital for operational risk. The problem here arises in determining how much of

the insured amount needs to be deducted from the level of required capital.

Moreover, the Basel Committee is against the use of insurance to optimize the capital required for operational risk for banks that use either the Basic Indicator Approach or the Standardized Approach, but a bank using the AMA is allowed to consider the risk mitigating impact of insurance in the measuring of operational risk used for regulatory minimum capital requirements. The recognition of insurance mitigation is limited to 20% of the total operational risk capital charge.

In addition to this, the insurance policy must have an initial term of at least one year. For policies with a residual term of less than one year, the bank must make appropriate haircuts reflecting the declining term of the policy, up to a full 100% haircut for policies with a residual term of 90 days or less. Additionally, the insurance policy should not have exclusions or limitations based upon regulatory action or for the receiver or liquidator of a failed bank.

The insurance coverage must be explicitly mapped to the actual operational risk exposure of the bank and have a minimum claims paying ability rating of A as shown in the table below (cf. BCBS [2010]):

Agency	CPA rating	Descriptive	Ratings category	Definition
S&P	A	Strong	Insurer financial strength rating	Denotes strong financial security characteristics
Moody's	A	Good	Long term insurance financial strength rating	Denotes ability to meet senior policyholder obligations.
Fitch	A	High credit quality	Investment grade	Denotes low expectation of credit risk
AM Best	A	Excellent	Secure best ratings	Denotes strong ability to meet ongoing policyholder obligations

Table 4.7: Agencies equivalent ratings

4.7.4 Capital assessment under insurance on operational losses

In this section, we discuss insurance coverage and its effects on operational losses. Individual operational losses are insured with an external insurer under an excess of loss (XL) contract. So, to include insurance contracts in the operational risk model, we take into consideration many other factors such as deductibles d and policy limit m (cf. Bazzarello *et al.* [2006]).

Let's consider x_{ij} as the i^{th} loss drawn from the severity distribution in the year j , and n_j as the number of losses in year j drawn from the frequency distribution. Then the insurance recovery for the individual loss x_{ij} would be:

$$R_{d,m}(x_{ij}) = \min\left(\max(x_{ij} - d, 0), m\right) \quad \forall i = 1, \dots, n_j, \quad j = 1, \dots, J$$

where J is the number of annual losses.

On an annual basis, if we set the aggregated deductibles as D , the aggregated policy limit as M , and we let $X_j = \sum_i x_{ij}$ be the j^{th} annual loss, then the annual recovery loss can be rewritten as:

$$R_{d,m,D,M}(X_j) = \min\left(\max\left(\sum_{i=1}^{n_j} R_{d,m}(x_{ij}) - D, 0\right), M\right), \quad \forall j = 1, \dots, J$$

Hence, the net annual loss would result in:

$$Y_j = X_j - R_{d,m,D,M}(X_j), \quad \forall j = 1, \dots, J$$

Adhering to the Basel II standards for AMA, we take into consideration the following (cf. BCBS [2010]):

- Appropriate haircuts
- Payment uncertainty
- Counterparty risk

4.7.4.1 Appropriate haircuts

For policies with a residual term of less than one year, the bank must make appropriate haircuts reflecting the declining residual term of the policy, up to a full 100% haircut for policies with a residual term of 90 days or less. Accounting for the haircuts, the recovered annual loss can be written as:

$$R_{d,m,D,M}(X_j) = \alpha \min\left(\max\left(\sum_{i=1}^{n_j} R_{d,m}(x_{ij}) - D, 0\right), M\right), \quad \forall j = 1, \dots, J$$

where,

$$\alpha = \begin{cases} \min\left(\frac{\text{Number of lasting days}}{365}, 1\right) & \text{if Number of lasting days} > 90 \\ 0 & \text{if Number of lasting days} \leq 90 \end{cases}$$

4.7.4.2 Payment uncertainty

Payment uncertainty occurs when the insuring party cannot commit to its contractual obligations on a timely basis. To account for such deviations from full recovery, we use β ($0 \leq \beta \leq 1$) as the average recovery rate to discount the insurance payments. Beta can be estimated from internal data as:

$$\beta = \frac{\text{Actual recovered loss amount in one year}}{\text{Potential recovered loss amount}}$$

We then integrate this factor into our calculation for the recovery on annual loss:

$$R_{d,m,D,M}(X_j) = \beta \alpha \min \left(\max \left(\sum_{i=1}^{n_j} R_{d,m}(x_{ij} - D, 0) \right), M \right), \forall j = 1, \dots, J$$

4.7.4.3 Counterparty risk

Counterparty risk occurs when the insurance company fails to fulfill its payment obligations.

To model this particular risk, let's consider pd as the probability of default and γ as the recovered loss given default. So, if J is the number of years containing annual losses, then the full insurance recoveries can be obtained for only $(1 - pd)J$ years as we expect a coverage only when the insurer is in good financial health. Now, for the remaining $pd J$ years, insurance recoveries must be discounted using the factor γ according to the formula:

$$R_{d,m,D,M}(X_j) = \begin{cases} \gamma \beta \alpha \min \left(\max \left(\sum_{i=1}^{n_j} R_{d,m}(x_{ij} - D, 0) \right), M \right) & \text{if } j \in YD, \forall j = 1, \dots, J \\ \beta \alpha \min \left(\max \left(\sum_{i=1}^{n_j} R_{d,m}(x_{ij} - D, 0) \right), M \right) & \text{if } j \in YND, \forall j = 1, \dots, J \end{cases}$$

where, YD is the set of simulated years where the insurer has defaulted and YND is the set of simulated years where the insurer has not defaulted.

4.8 Scaling Severity for External Loss Data

We have seen in the previous sections the management of Operational Risk in the banking sector and more particularly at a Lebanese Bank. Yet given the importance of scenario analysis and according to Basel II accords, the use of external data is absolutely indispensable to the implementation of an advanced method for calculating operational capital. This section investigates how the severity of external losses are scaled for integration with internal data (cf. DAHEN & DIONNE [2008]). The model based on OLS, generate a normalization function to be able to join external loss data with specific internal loss data for Bank Audi Lebanon, so a similar approach of Dahen and Dionne was treated taking into account firm size, location, business lines and risk types to calculate internal loss equivalent to an external loss, which might occur in a given bank. The estimation results show that the variables took into consideration have significant power in explaining the loss amount. They are used to develop a normalization formula.

4.8.1 Loss data and scenarios

As we have mentioned in previous chapters Loss data is the foundation of an Advanced Measurement Approach based on loss distributions. This is one of the main reasons for undertaking Operational Risk loss data collection. It is not just to meet regulatory requirements, but also to develop one of the most important sources of operational risk management information. Yet we recognize that internal loss data has some weaknesses as a foundation for risk exposure measurement, including:

- Loss data is a backward-looking measure, which means it will not immediately capture changes to the risk and control environment.
- Loss data is not available in sufficient quantities in any financial institution to permit a reasonable assessment of exposure, particularly in terms of assessing the risk of extreme losses.

These weaknesses can be addressed in a variety of ways, including the use of statistical modelling techniques, as well as the integration of the other AMA elements, i.e. external data, scenario analysis all of which have been previously discussed in the above sections. For the application of the LDA approach (cf. chapter 2) at Bank Audi for example aiming to use advanced models, the following data can be used:

- Internal loss data: Bank Audi started the collection of loss data in 2008. Hence, a loss history of more than five years is now available for all business lines in the bank.
- External data: The data we will work on is generated from 2008 till 2012 by SAS OpRisk Global Data.
- Generated scenarios: Specified by experts in departments, control and support functions and regions in RCSA and KRIs.

The main idea in this section is to provide a process for normalizing the external loss data to be able finally to feed it into the LDA model.

4.8.2 Description of the External Data

SAS OpRisk Global Data is one of the largest, repository of information on publicly reported operational losses in excess of 100,000 \$. The solution documents more than 25,000 events across all industries worldwide and provides up to 50 descriptive, categorization and financial elements.

The database contains the following elements:

- The name of the parent company and of the subsidiary
- A full description of the event
- Event risk Category according to Basel II with its sub risk category
- Business unit according to Basel business line - Level 1 and 2
- Country of Incident and legal entity
- The loss amount in local currency, American dollars, and its real value (counting inflation)
- currency conversion rate and currency code
- Date of incident and settlement date
- Industry: Either financial services or public administration
- An industry sector code and name
- Information on the institution where the loss occurred: total revenues, total assets, net income, total deposits, shareholder equity and number of employees

In our study, we took external losses from 2008 till 2012 accounting for 1062 external loss.

We are going as well to suppose the following hypothesis:

- We suppose that the loss amounts recorded in the base as reported in the media are exact and factorial. The evaluation of losses is thus based neither on rumors nor predictions.
- We suppose that all types of losses are as likely to be recorded in the base; there is thus no media effect related to certain types of risk.
- We suppose that the external base provides all the losses of more than a 100,000 dollars for the financial institutions contained in it.
- We suppose that there is no correlation between the amount of the loss and the probability of its being reported. The severity and frequency distributions are thus supposed to be independent.

4.8.3 Model specification

4.8.3.1 Theoretical scaling model

We will not go through the full theoretical background for the model, for detailed support of the theory behind the model see DAHEN & DIONNE [2008].

The scaling mechanism depends on three fundamental hypotheses. The first is that the monetary loss can be broken down into two components: common and idiosyncratic or specific. The second stipulates a non-linear relation between the idiosyncratic component and the different factors composing it. The third and last hypothesis states that, aside from the factors controlled for the purpose of scaling, all the other non-observable factors (quality of control environment, etc.) are supposed to remain the same for all banks. Thus our scaling formula would be:

$$\ln(Loss_i) = \ln(Comp_{common}) + a \ln(assets_i) + \sum_j b_j factor_{s_{ij}},$$

and in order to explain the variability of the losses and to construct the scaling model, the different elements of the idiosyncratic component must be identified, since they play a role as factors explaining the severity of losses.

4.8.3.2 Description of the variables

The endogenous variable would be the logarithm of the loss amount taken greater than 100,000 USD. The statistics shows that the average by loss event is evaluated at 37 million USD, with a standard deviation of 320 million. The maximum of the losses is 6 billion. The loss amounts thus vary widely from quite substantial to catastrophic.

As for the exogenous variables, according to the literature, the size, location, business lines and event types must be taken into consideration.

Many information on size are available, such as: total equity, total revenues, total assets and number of employees. We have chosen total assets as the estimator for size, since all these variables are correlated,. In our database, losses reported differ greatly in size, varying from the smallest bank (with total assets of 0.4 million USD) to the largest institution (with assets of 3,783,173.10 million USD).

As for the location variable it has been classified in three categories: United States (USA), Europe and Other countries (Others) since losses do not all occur in the same country, a variable capturing the effect of location must be incorporated. Seeing differences in environment, legislation, etc., we expect this variable to be significantly linked to loss amounts. It is worth noting that 32.30% of the losses occurred in USA, while 28.25% in Europe and 39.45% in all the other countries. This variation can be explained by the fact that the number of banks in the United States is big.

In table 4.8, we present the number of events, the average, and the standard deviation for losses according to location in addition to the total assets divided in those three locations.

	USA	Europe	Others
Number of losses	343	300	419
Average of losses (Millions of dollars)	48.98	51.6	16.7
Standard deviation (Millions of dollars)	422.1	382.3	96.8
Average of Assets (Millions of dollars)	759,603.96	876,565.89	339,262.04
Standard deviation (Millions of dollars)	919,482.20	930,666.66	651,653.37

Table 4.8: Statistics on operational losses according to location of event. Locations have been classified in three categories: United States (USA), Europe and Other countries (Others)

In addition, business lines should have impact on the losses, that's why we have added all 8 business lines, see Appendix B for details. Table 4.9 shows all business line category: RB: Retail Banking, CB: Commercial Banking, PS: Payment and Settlement, CF: Corporate Finance, AM: Asset Management, AS: Agency Services, TS: Trading & Sales, Rb: Retail brokerage, with their number of losses, average of losses and standard deviation.

	RB	CB	PS	CF	AM	AS	TS	Rb
Number of losses	640	198	13	27	29	5	96	54
Average of losses (Millions of dollars)	39.41	26.18	10.13	65.72	12.72	55.026	62.37	6.79
Standard deviation (Millions of dollars)	394.91	107.07	19.44	103.96	22.72	44.71	260.83	25.18

Table 4.9: Statistics of operational losses according to business lines in which the losses occurred. We have selected the classification proposed by Basel II, including 8 lines of business

At the end, we have added all 7 event types, see Appendix B for details, since certain risk types are infrequent but extremely severe, whereas others are very frequent but of relatively weak severity. Table 4.10, display all 7 event types: CPBP: Clients, Products & Business Practices, IF: Internal Fraud, DPA: Damage to Physical Assets, EF: External Fraud, EPWS: Employment Practices and Workplace Safety, BDSF: Business Disruption and System Failures, EDPM: Execution, Delivery & Process Management.

	CPBP	IF	DPA	EF	EPWS	BDSF	EDPM
Number of losses	199	277	32	453	36	9	56
Average of losses (Millions of dollars)	94.87	31	73.025	18.12	7.66	23.03	14.55
Standard deviation (Millions of dollars)	553.47	176.87	224.31	284.85	15.32	52.30	58.38

Table 4.10: Statistics of operational losses according to event types in which the losses occurred. We have selected the classification proposed by Basel II, including 7 event types

4.8.3.3 Linear regression

To be able to evaluate the common and specific components for each loss amount. The previous equation:

$$\ln(Loss_i) = \ln(Comp_{common}) + a \ln(assets_i) + \sum_j b_j factors_{ij},$$

would regress to the following:

$$\ln(Loss_i) = \underbrace{a_0}_{\ln(Comp. Comm.)} + a_1 Size_i + a_2 USA_i + a_3 Europe_i + \underbrace{\sum_{j=4}^{10} a_j BL_{ij} + \sum_{j=11}^{16} a_j RT_{ij}}_{\ln g(Comp. Idiosyncratic)} + \epsilon_i$$

Where,

- a_0 : Common Component;
- $Size_i = \ln(Assets_i)$;
- USA_i and $Europe_i$ are both Binary variables taking the values of 1 if the loss occurred in those countries and 0 if not. The variable omitted is Others for its not applicability in the regression's output;
- BL_{ij} : Binary variable assuming the value 1 if the loss occurred in the business unit j, otherwise 0. The category omitted is Asset Management for its not applicability in the regression's output;
- RT_{ij} : Binary variable assuming the value 1 if the loss is of the risk type j, otherwise 0. The category omitted is Execution, Delivery & Process Management type of risk for its not applicability in the regression's output.
- ϵ_i : Error term variable representing the non-observable specific component which is supposed to follow a normal distribution with parameters $(0, \sigma^2)$.

4.8.3.4 Simple Regression Results

Table 4.11 shows the Simple Regression results regarding all the variables took into consideration. The Ordinary Least Squares (OLS) method is used to estimate the parameters. We notice that we obtain an adjusted R-squared of 21.8% which will be accepted since it is difficult to capture certain non-observable factors, which are not present in the external base and mentioning that it is better than the 5% found in the literature to date (Shih et al., 2000) and the 10.63% found in (DAHEN & DIONNE [2008]).

Variable	Coefficient	P-value
Constant	12.357	0.000
Ln(Assets)	0.05	0.00811
Retail Banking	-0.87942	0.0165
Trading and Sales	0.34315	0.37843
Agency services	2.561	0.00366
Commercial Banking	0.67	0.07526
Retail brokerage	-0.973	0.02247
Payment and Settlement	0.3684	0.54845
Corporate Finance	1.07	0.02903
United States	0.476	0.0005
Europe	0.16233	0.25507
External Fraud	0.088	0.7527
Internal Fraud	0.792	0.00518
Damage to Physical Assets	2.002	4.80e-06
Business Disruption and System Failures	1.6991	0.00974
Clients, Products & Business Practices	1.1802	4.24E-05
Employment Practices and Workplace Safety	0.8791	0.02473
R-Squared	0.2297	
Adjusted R-Squared	0.2179	

Table 4.11: Results obtained from estimating the linear regression coefficients with the ordinary least squares method

It is worth noting that with a 90% confidence level, the variables we took into consideration and are valuable to us were: Constant variable, Ln(Assets), Retail Banking, Agency services, Commercial Banking, Retail brokerage, Corporate Finance, United States, Internal Fraud, Damage to Physical Assets, Business Disruption and System Failures, Clients Products & Business Practices, Employment Practices and Workplace Safety. We mention that the three variables: Others, Asset Management and Executive, Delivery & Process Management were omitted since the variables are a linear combination of other variables.

As a robustness check, we started by integrating the variables one by one and compare their impact on our regression table 4.12, shows the different cases treated. Case 3 contains only the significant variables in the scaling model. Whereas Cases 1 and 2 are used to test the stability of each category of variables in the basic model. The figures in parenthesis are the P-value statistics. We notice that at first in case 1 where only the size ($\ln(Assets)$) is placed our Adjusted R-Squared was 0.5%, yet with the addition of the business lines and the location variables we have seen a major improve to 16%. On case 3, we put all significant variables to check the improvement which increased to 21.93%.

Variables	Case 1	Case 2	Case 3
Constant	12.694 (0.000)	13.236 (0.000)	12.61452 (0.000)
Ln(Assets)	0.0521 (0.0095)	0.062 (0.00098)	0.05668 (0.00197)
Retail Banking		-1.476 (0.000)	-1.1518 (0.000)
Trading and Sales		2.361 (0.00567)	2.31181 (0.005)
Agency services			
Commercial Banking			0.40666 (0.0505)
Retail brokerage		-1.366 (0.000)	-1.28293 (0.000)
Payment and Settlement			
Corporate Finance		0.975 (0.0134)	0.8076 (0.0347)
United States		0.366 (0.0035)	0.40684 (0.0009)
Europe			
External Fraud			
Internal Fraud			0.70285 (0.000)
Damage to Physical Assets			1.8994 (0.000)
Business Disruption and System Failures			1.6021 (0.0088)
Clients, Products & Business Practices			1.10116 (0.000)
Employment Practices and Workplace Safety			0.80723 (0.012)
R-Squared	0.006329	0.163	0.2281
Adjusted R-Squared	0.005391	0.1622	0.2193

Table 4.12: Robustness test for different cases of the variables treated

4.8.3.5 Normalization formula and resulted output

We have captured in section 4.8.3.3 the regression formula with $\ln(Loss_i)$ as the dependent variable. Now as the common component is constant for all loss amounts, it is possible to re-write the regression equation as follows:

$$Comp_{Comm.} = \frac{Loss_A}{g(Comp_{idio})_A} = \frac{Loss_B}{g(Comp_{idio})_B} = \dots = \frac{Loss_N}{g(Comp_{idio})_N}.$$

At the end, if we suppose that we have a loss which occurred in an external bank B and that we want to know its equivalent value if it occurred in a local bank A. Based on the analysis above, we multiply the coefficients already estimated by the corresponding value of the different variables to find the idiosyncratic or specific component:

$$Loss_A = \frac{g(Comp_{idio})_A}{g(Comp_{idio})_B} \times Loss_B.$$

To apply all of the preceding to Bank Audi, we present three examples of losses extracted from the external database SAS OpRisk Global Data and we show in detail how the scaling is done and how the losses are normalized to Bank Audi. For that, table 4.13 illustrates the amount of losses with the Total Assets (per M\$) of the banks where the different losses have been produced.

		External Loss					Total Assets	Coefficient
Citibank	2011	0.750	USA	Retail Banking	EF	1,913,902	2.75	
ING Group	2008	0.59	Europe	Asset Management	IF	1,918,955	9.10	
Bank of Canada	2009	1.960	Europe	Retail Banking	EF	595,353	1.61	

		Loss Audi					Total Assets	Coefficient
Bank Audi	2011	0.379	Lebanon	Retail Banking	EF	28,737	1.39	
Bank Audi	2008	0.470	Lebanon	Asset Management	IF	20,385	7.25	
Bank Audi	2009	1.677	Lebanon	Retail Banking	EF	26,486	1.38	

Table 4.13: Examples of Normalized external losses extracted from SAS Oprisk Global Data

We take into example Citibank in 2011 suffered from an External Fraud (EF) in its Retail Banking for 0.750 $M\$$, now by applying the previous theory with the required variables needed, we can calculate its coefficient: $g(Com.Idio.)$, which would be in this case:

$$\begin{aligned}
 g(Com.Idio.)_{ext} &= \exp(0.050 \times \ln(Assets_{ext}) - 0.879 \times Retail\ Banking + 0.476 \times USA) \\
 &= \exp(0.050 \times \ln(1,913,902 \times 10^6) - 0.879 \times 1 + 0.476 \times 1) \\
 &= 2.75
 \end{aligned}$$

We then apply the same calculation technique for Bank Audi, we obtain a coefficient of $g(Com.Idio.)_{Audi} = 1.39$.

Since,

$$\begin{aligned}
 g(Com.Idio.)_{Audi} &= \exp(0.050 \times \ln(Assets_{Audi}) - 0.879 \times Retail\ Banking + 0.476 \times 0) \\
 &= \exp(0.050 \times \ln(28,737 \times 10^6) - 0.879 \times 1 + 0.476 \times 0) \\
 &= 1.39
 \end{aligned}$$

At the end, the normalization formula would be:

$$\begin{aligned}
 Loss_{Audi} &= \frac{g(Com.Idio.)_{Audi}}{g(Com.Idio.)_{ext}} \times External\ Loss \\
 &= \frac{1.39}{2.75} \times 0.75 \\
 &= 0.379\ M\$
 \end{aligned}$$

4.8.4 Data requirements for specifying frequency distributions

We have seen previously all the requirements to set and specify the severity distribution yet no work has been done regarding the specification of the frequency distribution.

In LDA model, the specification of frequency distributions could as well be entirely based on internal loss data, in contrast to FRACHOT & RONCALLI [2002] who suggest to use internal and external frequency data and DAHEN & DIONNE [2008] who completed their work by scaling model for frequency of external losses. The main reasons for using only internal data are:

- Internal loss data reflects loss profile most accurately;
- It is difficult to ensure completeness of loss data from other financial institutions. However, data completeness is essential for frequency calibration;
- Data requirements are lower for calibrating frequency distributions than for calibrating severity distributions (in particular, if Poisson distributions are used).

In this chapter we will stop the study to this limit, and for further readings regarding best practices, (cf. AUE & KALKBRENER [2007]), as for the scaling models for the Severity and Frequency of External Operational Loss Data, see DAHEN & DIONNE [2008] and FRACHOT & RONCALLI [2002].

Part I - Conclusion

This part's objective has featured the different perspectives of operational risk, that have risen to the point of holding a significant position in risk assessment.

Operational risk quantification is a challenging task both in terms of its calculation as well as in its organization. Regulatory requirements (Basel Accords for Banks and Solvency Directive for insurance companies) are put in place to ensure that financial institutions optimize this risk.

The different qualitative and quantitative approaches for operational risk in addition to the importance use of insurance policies, have been highlighted throughout the four chapters. At the end, since the use of external data is absolutely important to the implementation of an advanced method for calculating operational capital like LDA, we scaled the severity of external losses for integration with internal data, a similar approach as DAHEN & DIONNE [2008] was used, and finished by presenting three examples of losses extracted from our external loss database that showed in detail how the scaling is done and how are the losses normalized to bank Audi.

We managed as well to illustrate the various mathematical and actuarial techniques for mitigating this risk, and emphasized the qualitative management of operational risk which has set grounds and gave continuity for the second part, focusing on the estimation risk behind operational risk and its influence on our capital requirement.

Part II

Addressing Estimation Risk

Chapter 5

Estimation Errors and SCR Calculation

5.1 Introduction

Measuring the Value-at-Risk of the own funds is a central topic in insurance with the new Solvency II framework and finance regarding Basle II and III accords.

Many banks and financial institutions, develop models to compute the value-at-risk and resulting capital requirement, but we know that any model is by definition an imperfect simplification and in some cases a model will produce results that are bias due to parameter estimation errors. For instance, this point is illustrated in BOYLE AND WINDCLIFF [2004] for pension plans investment and in PLANCHET & THEROND [2012] for the determination of Solvency capital in the Solvency II Framework. As a direct consequence of parameter estimation risk, the capital requirement may be underestimated.

Article 101 of the European directive states that the Solvency Capital Requirement (SCR), shall correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period.

We are interested in this chapter, in assessing the potential loss of accuracy from estimation error when calculating the SCR, we expand this analytical framework where an insurer must calculate a VaR to a confidence level of 99.5% on a distribution which we must estimate the parameters, now this estimation might lead to important differences in the numerical results. To be able to illustrate this situation we took the very particular simple case of the only market risk for an asset consisting of a zero coupon bond, we are thus led to study the distribution of $P(1, T - 1)$ which is the value of the asset at $t = 1$ and we highlight the possible undervaluation of the Solvency Capital Requirement if a special attention is not given to the risk of parameter estimation. At the end, we are going to check the effect of adding another zero coupon bond on our Capital estimation. The following chapter adds to the value of the section 2.4 in chapter 2 about the influence function for MLE as an introduction to robust statistics and it has been subject to publication at the Bulletin Français d'Actuariat see KARAM & PLANCHET [2013].

5.2 Model Presentation

For sake of simplicity, we use in this chapter the classical one-factor interest rate Vasicek model (cf. Vasicek [1977]). The key point here is that we need closed form solutions for the zero-coupon bond price. Moreover, the distribution of the ZC price at a future time is known (and is log normal one). This will help us to compute very easily the quantile of the price distribution.

5.2.1 Simulating Interest rates: The Vasicek model

We consider an asset consisting of a zero-coupon bond now Vasicek model, assumes that $r(t)$ is an adapted process on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{0 < t < T})$ that satisfies the following stochastic differential equation:

$$dr_t = k(\theta - r_t)dt + \sigma dW_t$$

where k , θ and σ are non-negative constants and r_t is the current level of interest rate. The parameter θ is the long run normal interest rate. The coefficient $k > 0$ determines the speed of pushing the interest rate towards its long run normal level, and W is a standard Brownian motion.

The solution of the $r(t)$ stochastic differential equation would generate (cf. Bayazit [2004]):

$$r_t = r_0 e^{-kt} + \theta(1 - e^{-kt}) + \sigma e^{-kt} \int_0^t e^{ku} dW_u \quad (5.1)$$

and for $0 \leq s \leq t \leq T$, $r_t = r_s e^{-k(t-s)} + \theta(1 - e^{-k(t-s)}) + \sigma \int_s^t e^{-k(t-u)} dW_u$

If one wants to take the market price of risk λ into account, (for the purpose of this chapter and to minimize our uncertainty, we took a constant market price of risk $\lambda(t, r_t) = \lambda$, but in general a market price of risk has a more complex structure, more on this topic could be found in CAJA AND PLANCHET [2011]), which allows us to switch between the real universe \mathbb{P} to the risk-neutral world \mathbb{Q} , then with the respect of the new probability measure \mathbb{Q} , we can rewrite the stochastic equation as follows (cf. Van Elen [2010]):

$$dr_t = k(\theta^* - r_t)dt + \sigma dW_t^*$$

The structure of this equation in the risk-neutral world is comparable to that in the real universe where, $\theta^* = \theta - \frac{\sigma\lambda}{k}$.

In particular, we can use the explicit formula seen previously with,

$$r_t = r_0 e^{-kt} + \theta^*(1 - e^{-kt}) + \sigma e^{-kt} \int_0^t e^{ku} dW_u^*.$$

A zero-coupon bond of maturity T is a financial security paying one unit of cash at a prespecified date T in the future without intermediate payments. The price at time $t \leq T$

is denoted by $P(t, T)$.

The general case of a zero coupon bond at time t with maturity T is (cf. Bayazit [2004]):

$$P(t, T) = A(t, T)e^{-B(t, T)r(t)},$$

$$\text{where, } \begin{cases} A(t, T) = \exp\left[\left(\theta^* - \frac{\sigma^2}{2k^2}\right)(B(t, T) - (T - t)) - \frac{\sigma^2}{4k}B(t, T)^2\right] \\ B(t, T) = \frac{1 - e^{-k(T-t)}}{k} \end{cases}$$

5.2.2 Parameter Estimation

For the Vasicek model, we have $dr_t = k(\theta - r_t)dt + \sigma dW_t$ with the solution already calculated that generate $r_t = r_s e^{-k(t-s)} + \theta(1 - e^{-k(t-s)}) + \sigma e^{-kt} \int_s^t e^{ku} dW_u$,

now r_t is normally distributed with mean and variance:

$$\begin{cases} \mathbb{E}[r_t | \mathcal{F}_s] = r_s e^{-k(t-s)} + \theta(1 - e^{-k(t-s)}) \\ \text{Var}[r_t | \mathcal{F}_s] = \frac{\sigma^2}{2k}(1 - e^{-2k(t-s)}) \end{cases}$$

The properties of the integral of a deterministic function relative to a Brownian motion lead to the exact discretization (cf. Planchet & Thérond [2005]):

$$r_{t+\delta} = r_t e^{-k\delta} + \theta(1 - e^{-k\delta}) + \sigma \sqrt{\frac{1 - e^{-2k\delta}}{2k}} \epsilon,$$

where ϵ is a random variable that follows the standard normal distribution and δ is the discretization step.

To calibrate this short rate model, let's rewrite it in more familiar regression format:

$$y_t = \alpha + \beta x_t + \sigma_1 \epsilon_t$$

$$\text{Where, } y_t = r_{t+\delta}, \alpha = \theta(1 - e^{-k\delta}), \beta = e^{-k\delta}, x_t = r_t \text{ and } \sigma_1 = \sigma \sqrt{\frac{1 - e^{-2k\delta}}{2k}}$$

The OLS regression provides the maximum likelihood estimator for the parameters: α , β and σ_1 (cf. Brigo *et al.* [2007]).

One can for instance compare the formulas stated above with the estimators derived directly via maximum likelihood given our Log-Likelihood function: $\ln(\mathcal{L}) = \ln\left(\frac{1}{\sigma_1 \sqrt{2\pi}}\right)^n -$

$\frac{1}{2\sigma_1^2} \sum_{i=1}^n (r_i - r_{i-1}\beta - \theta(1 - \beta))^2$ which are of the form (cf. Brigo & Mercurio [2006]):

$$\hat{\beta} = \frac{n \sum_{i=1}^n r_i r_{i-1} - \sum_{i=1}^n r_i \sum_{i=1}^n r_{i-1}}{n \sum_{i=1}^n r_{i-1}^2 - \left(\sum_{i=1}^n r_{i-1}\right)^2}$$

$$\hat{\theta} = \frac{\sum_{i=1}^n [r_i - \hat{\beta}r_{i-1}]}{n(1 - \hat{\beta})}$$

$$\hat{\sigma}_1^2 = \frac{1}{n} \sum_{i=1}^n [r_i - \hat{\beta}r_{i-1} - \hat{\theta}(1 - \hat{\beta})]^2$$

Parameter estimation is an important stage in the simulation of trajectories of a continuous process because it can cause a bias as we will see in the next section.

Given the parameters estimated all what is left is to find the market price of risk λ which give us the right price of the zero coupon. Since $P(0, T)$ is usually known (given by the market at time $t = 0$), and $P(0, T)$ is a function of k , θ , σ and λ as seen previously, λ would be calculated as:

$$\lambda = \left(\left(\theta - \frac{\sigma^2}{2k^2} \right) - \frac{\left(\frac{\sigma^2}{4k} B(0, T) + r_0 \right) B(0, T) + \ln(P(0, T))}{B(0, T) - T} \right) \frac{k}{\sigma}$$

5.2.3 Calculation of the SCR

Article 101 of the European directive (cf. CEIOPS [2009]) states that the Solvency Capital Requirement (SCR), shall correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period.

In our case the SCR would be calculated as follows:

$r_t = r_0 e^{-kt} + \theta^*(1 - e^{-kt}) + \sigma e^{-kt} \int_0^t e^{ku} dW_u^*$ is normally distributed with mean and variance:

$$\begin{cases} \mathbb{E}[r_t | \mathcal{F}_0] = r_0 e^{-kt} + \theta^*(1 - e^{-kt}) \\ \text{Var}[r_t | \mathcal{F}_0] = \frac{\sigma^2}{2k} (1 - e^{-2kt}) \end{cases}$$

In addition, $A(t, T)$ and $B(t, T)$ are deterministic in the the zero coupon price fundamental equation:

$$P(t, T) = A(t, T) e^{-B(t, T)r(t)},$$

So, $P(t, T)$ would follow the LogNormal distribution since if X is normally distributed with mean μ_x and variance σ_x^2 , ae^{-bX} would follow the Lognormal distribution with:

mean: $e^{\ln(a) - b\mu_x + \frac{1}{2}b^2\sigma_x^2}$ and variance: $(e^{b^2\sigma_x^2} - 1)e^{-2b\mu_x + b^2\sigma_x^2}$.

So in our case,

$$\mathbb{E}[P(t, T)] = \exp \left\{ \ln(A(t, T)) - B(t, T) \left(r_0 e^{-kt} + \theta^*(1 - e^{-kt}) \right) + \frac{1}{2} B(t, T)^2 \left(\frac{\sigma^2}{2k} (1 - e^{-2kt}) \right) \right\}$$

$$\text{Var}[P(t, T)] = \left(e^{B(t, T)^2 \frac{\sigma^2}{2k} (1 - e^{-2kt})} - 1 \right) \exp \left\{ -2B(t, T) (r_0 e^{-kt} + \theta^*(1 - e^{-kt})) + B(t, T)^2 \frac{\sigma^2}{2k} (1 - e^{-2kt}) \right\}$$

To reiterate, since we have $P(t, T) \hookrightarrow \mathcal{LN}\left(\mu_L = \ln(A(t, T)) - B(t, T)\mathbb{E}[r_t|\mathcal{F}_0], \sigma_L^2 = B(t, T)^2 \text{Var}[r_t|\mathcal{F}_0]\right)$, if we denote x_p^{LN} our quantile, p the critical value and Φ the cdf of a standardized gaussian random variable we would have:

$$x_p^{LN} = VaR_p(P(t, T)) = F_0^{-1}(p) = \exp(\sigma_L \underbrace{\Phi^{-1}(p)}_{1-\alpha} + \mu_L)$$

At the end, the Solvency Capital Requirement (SCR) of our zero coupon bond would be:

$$SCR = P(0, T) - x_p(e^{-R_1} P(1, T - 1))$$

In practice, $p = 0.5\%$ and R_1 is the spot rate for $t = 1$.

5.3 Estimation Risk

To show the estimation risk we are going to take the very particular simple case of the only market risk for an asset consisting of a zero coupon bond, we are thus led to study the distribution of $P(1, T - 1)$ which is the value of the asset at $t = 1$ and then we will add to it one more zero coupon bond and check how estimation risk affects the Solvency Capital Requirement if a special attention is not given to the risk of parameter estimation.

5.3.1 Case of one zero coupon bond

For the case of one zero coupon bond, we are going to follow the below steps to calculate our Solvency Capital requirement and show the estimation error behind:

1. We fix the Vasicek parameters set $\vartheta = (k_0, \theta_0, \sigma_0, \lambda_0)$ by applying the Maximum Likelihood estimation technique stated previously in section 2.2.
2. Let $\omega = (k, \theta, \sigma)$, given the asymptotic normality of MLE, we have: $\hat{\omega} \hookrightarrow \mathcal{N}(\underbrace{(k_0, \theta_0, \sigma_0)}_{\omega_0}, \Sigma)$,

where Σ is calculated using the inverse of fisher information matrix: $\Sigma = \left(-\mathbb{E} \left[\frac{\partial^2 \ln \mathcal{L}}{\partial \omega \partial \omega'} \right] \right)_{\omega=\omega_0}^{-1}$.

3. We estimate the market price of risk λ since $P(0, T)$ is known at $t = 0$, which allows us to switch between the real measure \mathbb{P} to the risk-neutral measure \mathbb{Q} see section 2.2;
4. Calculate the Solvency capital Requirement (SCR) of the Zero-Coupon bond at 0.5%, and its Relative difference $\epsilon = \frac{\hat{SCR} - SCR}{SCR}$;
5. Repeat steps 2 to 5 on a number N of times (in our example, we took $N = 10^5$ iterations) to be able to analyze the empirical cumulative distribution function of $\epsilon = (\epsilon_1, \dots, \epsilon_N)$, where $\epsilon_i = \frac{\hat{SCR}_i - SCR}{SCR}$.

As for the calculation of Fisher's inverse information Matrix:

$$\Sigma = \left(-\mathbb{E} \left[\frac{\partial^2 \ln \mathcal{L}}{\partial \omega \partial \omega'} \right]_{\omega=\omega_0} \right)^{-1},$$

we used the log-likelihood function: $\ln(\mathcal{L}) = \ln \left(\frac{1}{\sigma_1 \sqrt{2\pi}} \right)^n - \frac{1}{2\sigma_1^2} \sum_{i=1}^n (r_i - r_{i-1}\beta - \theta(1-\beta))^2$
see appendix A for the full support.

5.3.2 Estimation Risk of a Portfolio consisting of two Zero-Coupon bonds

In this section, we will be interested in capturing the effect of parameter estimation when we add another zero-coupon bond to our portfolio; we took the two zero coupon bonds maturing respectively at T_1 and T_2 months.

Let P_0 be the Price at $t = 0$ of our Portfolio, $P_0 = P(0, T_1) + P(0, T_2)$ with the only difference that P will not follow a LogNormal Distribution in that case.

Following the same estimation technique of parameters from above, we explain how we computed the market price of risk λ which give us the right price of the Portfolio.

Now with $P(0, T) = e^{\left(\theta - \frac{\lambda\sigma}{k} - 1/2 \frac{\sigma^2}{k^2}\right) \left(\frac{1-e^{-Tk/12}}{k} - T/12\right) - 1/4 \frac{\sigma^2(1-e^{-Tk/12})^2}{k^3} - \frac{(1-e^{-Tk/12})r\theta}{k}}$, already illustrated above we would have:

$$\begin{aligned} P_0 &= P(0, T_1) + P(0, T_2) \\ P_0 &= e^{\left(\theta - \frac{\lambda\sigma}{k} - 1/2 \frac{\sigma^2}{k^2}\right) \left(\frac{1-e^{-T_1 k/12}}{k} - T_1/12\right) - 1/4 \frac{\sigma^2(1-e^{-T_1 k/12})^2}{k^3} - \frac{(1-e^{-T_1 k/12})r\theta}{k}} + \\ &\quad e^{\left(\theta - \frac{\lambda\sigma}{k} - 1/2 \frac{\sigma^2}{k^2}\right) \left(\frac{1-e^{-T_2 k/12}}{k} - T_2/12\right) - 1/4 \frac{\sigma^2(1-e^{-T_2 k/12})^2}{k^3} - \frac{(1-e^{-T_2 k/12})r\theta}{k}} \end{aligned}$$

We can solve this equation numerically and find λ by using the dichotomy method.

The sum of lognormal variables

We often encounter the sum of lognormal variables in financial modeling, a lot of methods are used to approximate the sum into a lognormal distribution but in our study, we are going to apply Fenton-Wilkinson approach (cf. Fenton [1960]) the most used for its simplicity. We place ourselves in the case where we have l lognormal distributions $X_i = e^{Y_i}$ where $Y_i \hookrightarrow \mathcal{N}(\mu_i, \sigma_i)$, the approach states that the sum of l lognormal distributions

$S = \sum_{i=1}^l e^{Y_i}$ can be approximated by a lognormal distribution e^Z , $Z \hookrightarrow \mathcal{N}(\mu_Z, \sigma_Z^2)$ where,

$$\begin{aligned}\mu_Z &= 2\ln(m_1) - 1/2\ln(m_2) \\ \sigma_Z^2 &= \ln(m_2) - 2\ln(m_1)\end{aligned}$$

and,

$$\begin{aligned}m_1 = \mathbb{E}(S) &= e^{\mu_Z + \sigma_Z^2/2} = \sum_{i=1}^l e^{\mu_{Y_i} + \sigma_{Y_i}^2}, \\ m_2 = \mathbb{E}(S^2) &= e^{2\mu_Z + 2\sigma_Z^2} = \sum_{i=1}^l e^{2\mu_{Y_i} + 2\sigma_{Y_i}^2} + 2 \sum_{i=1}^{l-1} \sum_{j=i+1}^l e^{\mu_{Y_i} + \mu_{Y_j}} e^{1/2(\sigma_{Y_i}^2 + \sigma_{Y_j}^2 + 2\rho_{ij}\sigma_{Y_i}\sigma_{Y_j})},\end{aligned}$$

are the first and second moment of S (cf. El Faouzi & Maurin [2006])¹.

As a result, and by applying the same previous steps as in section 6.3.1, where

$$SCR = P - x_p \left(\underbrace{P(12, T - 12) + P(12, T' - 12)}_{P_1} \right),$$

with x_p our quantile estimated, P_1 is the price of our Portfolio at $t = 12$ months and typically in the Solvency II context $p = 0.5\%$, we were able to estimate our Capital Requirement.

5.3.3 Numerical Results

In this section, we apply the preceding theoretical discussion of our estimation technique to the problem at hand. From the Federal Reserve (FR), we took our interest rates, dated from January 1982 till August 2008 ($n = 320$ months), and compared our results with smaller data dated from July 2001 till August 2008, which gives us 86 months overall. We estimated the model parameters of Vasicek then we applied the asymptotic normality of Maximum Likelihood to generate various outcomes. This simulation example is intended to indicate how parameter estimation can affect directly the Solvency Capital Requirement for one zero coupon bond of maturity 120 months and two zero coupon bonds of respective maturity of 60 and 120 months. Now since the estimation of the SCR is executed on simulated values, the simulations and the estimation of the SCR has to be effected on a large size sample (we took $N = 10^5$ simulations).

As so, and with our fixed values parameters shown in table 1 below, we have been able to estimate our Solvency Capital Requirement and its relative difference $\epsilon = (\epsilon_1, \dots, \epsilon_N)$,

$$\text{where } \epsilon_i = \frac{S\hat{C}R_i - SCR}{SCR}.$$

¹Corrections of the formulas given in this paper have been made regarding the second moment of S : $\mathbb{E}(S^2)$

Fixed Values		
n	320	86
$r_0 =$	1.75%	1.75%
$k =$	0.219	0.153
$\theta =$	3.388%	0.971%
$\sigma =$	1.104%	0.8189%
$\lambda =$	-0.4189	-0.9954
$P(0, 120) =$	0.678	0.678
$P(0, 60) =$	0.854	0.854
$P =$	1.532	1.532

Table 5.1: Vasicek parameter model

With the given Inverse Fisher information matrix for $n = 320$:

$$\begin{bmatrix} 0.00004655 & -0.00004988 & -2.7924 \times 10^{-13} \\ -0.00004988 & 0.0001488 & 8.7048 \times 10^{-13} \\ -2.7924 \times 10^{-13} & 8.7048 \times 10^{-13} & 1.5591 \times 10^{-8} \end{bmatrix}$$

and for $n = 86$:

$$\begin{bmatrix} 0.0002959 & -0.0003975 & -1.1661 \times 10^{-12} \\ -0.0003975 & 0.0009364 & 2.69943 \times 10^{-12} \\ -1.1661 \times 10^{-12} & 2.69943 \times 10^{-12} & 3.2427 \times 10^{-8} \end{bmatrix}$$

Now we are interested in the particular case of $\epsilon < 0$ and more particularly where $\epsilon < \epsilon_{max}$ where the SCR is underestimated by more than 3%. For that, table 2 compares the results regarding, $\mathbb{P}(\epsilon \leq -0.03)$, for our one and two zero coupon bonds of maturity 60 and 120 months.

$n = 320$		$n = 86$	
$P(0,120)$	$P(0,60)+P(0,120)$	$P(0,120)$	$P(0,60)+P(0,120)$
45.3%	42.9%	48.4%	47.8%

Table 5.2: Underestimation of the SCR

We can easily check that whichever the case treated, on average, 45% of the simulations underestimated our solvency capital requirement by more than 3%, ($\mathbb{P}(\epsilon \leq -3\%) = 45\%$). Figure 6.2, shows us the empirical cumulative distribution function of ϵ of our Portfolio composed of one zero coupon bond maturing in 120 months, for $n = 320$. We notice that, 45.3% of our cases underestimated the Solvency Capital Requirement by more than 3% given a probability $\mathbb{P}(\epsilon < -3\%) = 45.3\%$.

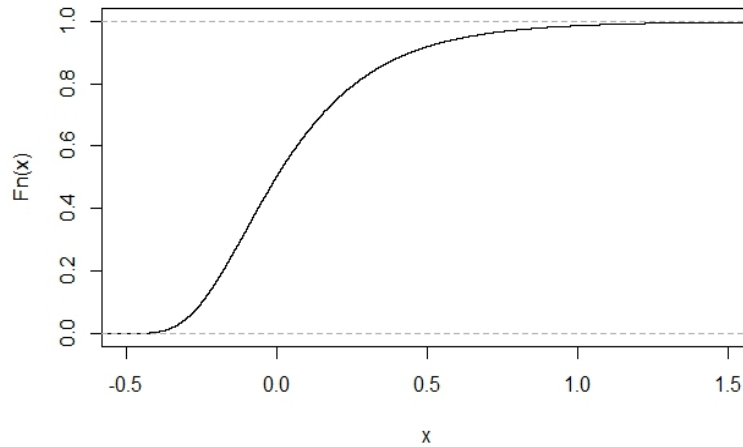


Figure 5.1: Empirical Cumulative Distribution Function of ϵ

5.4 Conclusion

In this chapter, we have focused on the possible underestimation of the Solvency Capital Requirement, by taking into consideration the very simple case of an asset consisting of one and two zero coupon bonds. Applying the Vasicek model, enabled us to illustrate the direct consequence of parameter estimation risk on the capital requirement. For example BOYLE AND WINDCLIFF [2004] shows a similar consequence by working on the pension plans investment.

In practice we do not know the true value of our parameters and estimated values are usually treated as correct, yet if we take into consideration the impact of parameter estimation risk, then our capital requirement have a 50% chance to be underestimated by more then 3% as the study shows us.

So it would be more appropriate in reality to privilege simple and prudent models and avoid complexing them, in a way, to prevent more estimation errors that might have severe influences in some cases on our Solvency Capital Requirement.

Moreover, such simplified models should be used to comply to the Own Risk Solvency Assessment (ORSA) required by Solvency II.

Chapter 6

An Alternative Yield Curve Estimation

6.1 Introduction

Article 101 of the European directive, (cf. Official Journal of the European Union [2009]), states that the Solvency Capital Requirement (SCR), shall correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period. Our focus in this chapter is on interest rate risk. The Nelson-Siegel (1987) curve fitting model is widely used by central banks as a method for the term structure of interest rates, they employ government bonds in the estimations since they carry no default risk. The model as well, can also be valuable for forecasting the term structure (see Diebold & Li [2006]). We have many ways and optimization techniques for calibrating the model: from the Ordinary Least Squares (OLS) or the maximization of the Adjusted \mathcal{R}^2 .

We are going to propose a new method of calibration by using Maximum Likelihood Estimation technique (MLE) then showing that the estimation risk is low. To be able to illustrate this situation, we consider the monthly market rates of French government bonds dated between August 2011 and July 2012, and calculate the capital constituting a life Annuity Immediate at age of 65 in arrears, with the use of the French mortality table TH00-02 and we highlight the robustness of our estimation technique and the decrease of the estimation risk behind.

6.2 Model Presentation

The relationship between the yields of default-free zero coupon bonds and their length to maturity is defined as the term structure of interest rates and is shown visually in the yield curve. We use in this work, the famous Nelson-Siegel model to be able to estimate the term structure of interest rates. This will help us to compute the price of a life Annuity Immediate easily.

6.2.1 Curve Fitting: The Nelson-Siegel model

The Nelson-Siegel (cf. Nelson & Siegel [1987]), is a parametric model, widely used in reality for the term structure of interest rates (cf. Le Maistre & Planchet [2013]). The instantaneous forward rate at maturity m is given by the solution to a second-order differential equation with real and equal roots.

The function form is:

$$f(m, \theta) = \beta_0 + \beta_1 \exp\left(\frac{-m}{\lambda}\right) + \beta_2 \frac{m}{\lambda} \exp\left(\frac{-m}{\lambda}\right),$$

In this equation, $\theta = (\beta_0, \beta_1, \beta_2)$ denote respectively the long-term value of the interest rate, the slope and curvature parameter. β_2 and λ are responsible for the hump, where λ determines the position of the hump representing the scale parameter that measures the rate at which the short-term and medium term components decay to zero.

The spot interest rate for maturity m can be derived by integrating the previous equation. The resulting function is expressed as follows (cf. BIS Papers [2005]):

$$R(m, \theta) = \beta_0 + \beta_1 \left(\frac{1 - e^{-m\tau}}{m\tau} \right) + \beta_2 \left(\frac{1 - e^{-m\tau}}{m\tau} - e^{-m\tau} \right)$$

where, β_0 describing the long run is positive, $\beta_0 + \beta_1$ determines the starting value of the curve at maturity zero must be positive as well and $\tau = \frac{1}{\lambda} > 0$

We have many ways and optimization techniques for calibrating the model. R project uses two types of estimation treated in the two packages: 'YieldCurve¹' and 'fBonds²'. Where the first tries to maximize the adjusted \mathcal{R}^2 and the second function finds a global solution and start values for the β 's are solved exactly as a function of λ using OLS.

On the other hand, our estimation technique focuses on Maximizing the Likelihood function by respecting the parameters' constraints. The idea is as follows: $R_{t,j} = \hat{R}_{t,j} + \epsilon_{t,j}$, and the estimated Nelson-Siegel function is of the form:

$$\hat{R}_{t,j}(m_j, \hat{\theta}) = \hat{\beta}_0 + \hat{\beta}_1 \left(\frac{1 - e^{-m_j \hat{\tau}}}{m_j \hat{\tau}} \right) + \hat{\beta}_2 \left(\frac{1 - e^{-m_j \hat{\tau}}}{m_j \hat{\tau}} - e^{-m_j \hat{\tau}} \right)$$

where, $\hat{\theta} = (\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\tau})$, t is the date and m_j is the j -th maturity. with the constraints of:

$$\begin{cases} \hat{\beta}_0 > 0 \\ \hat{\beta}_1 + \hat{\beta}_2 > 0 \\ \hat{\tau} > 0 \end{cases}$$

and the hypothesis that: $\epsilon_{t,j} \hookrightarrow \mathcal{N}(0, \sigma^2)$, where $\hat{\sigma} = \sqrt{\frac{1}{n} \sum_{j=1}^n (R_{t,j} - \hat{R}_{t,j})^2}$, is an estimation for σ .

¹URL: <http://cran.r-project.org/web/packages/YieldCurve/YieldCurve.pdf>

²URL: <http://cran.r-project.org/web/packages/fBonds/fBonds.pdf>

Hence, the Log-Likelihood function would be: $\max_{\hat{\theta}} \left[\frac{-n}{2} \ln(2\pi\sigma^2) - \frac{1}{2} \sum_{j=1}^n \left(\frac{R_{t,j} - \hat{R}_{t,j}}{\sigma} \right)^2 \right]$,

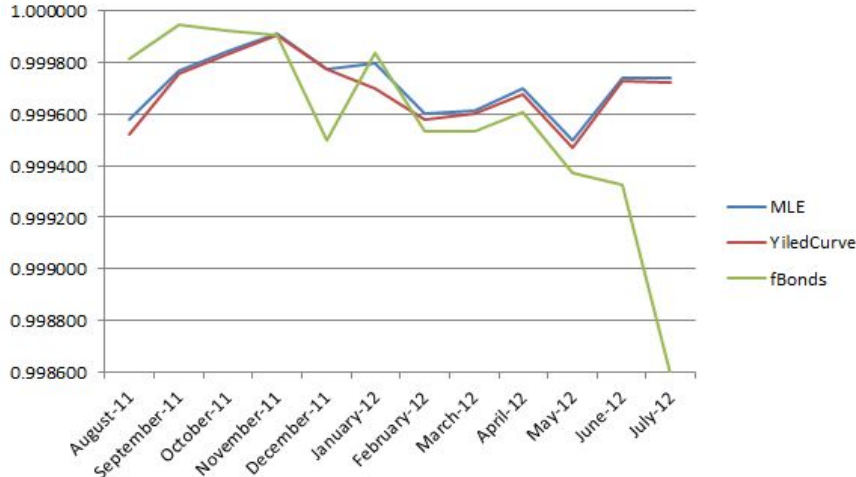
and maximizing this function would supply us with the estimators we need for our curve fit.

Table 6.1 below compares the three used estimation methods for particular Yield Curve dates, we can see that the MLE method proposed by us, with respect to the constraints stated previously, fits very well compared to the others. The R 'fBonds' methods gives the global solution without taking into consideration the constraints and on the other hand 'YieldCurve' method has a lower adjusted \mathcal{R}^2 .

August-11			
	MLE	R "YieldCurve" package	R "fBonds" package
β_0	0.04724	0.04711	-2.09087
β_1	-0.03976	-0.03952	2.09640
β_2	-0.04207	-0.04279	2.60792
τ	0.46968	0.47819	0.01266
Adjusted \mathcal{R}^2	99.958%	99.952%	99.982%
July-12			
	MLE	R "YieldCurve" package	R "fBonds" package
β_0	0.04159	0.04110	-11.96394
β_1	-0.03920	-0.03824	11.96146
β_2	-0.05982	-0.06202	13.12383
τ	0.52439	0.55179	0.00541
Adjusted \mathcal{R}^2	99.974%	99.972%	99.859%

Table 6.1: Comparing the three estimation methods

As for Figure 6.1 below it shows the adjusted \mathcal{R}^2 for all the months calculation, where it clearly demonstrate the effectiveness of MLE estimation regarding the others. Noting that the "fBonds" used estimation technique does not respect the constraints given by the model with an \mathcal{R}^2 higher than MLE. Furthermore, applying MLE allows us to emphasize the robustness of our estimated parameters by the use of the asymptotic normality. A property that is not applied by the other methods.


 Figure 6.1: Adjusted \mathcal{R}^2

6.2.2 Estimation Risk

To show the estimation risk, we are going to take the monthly market rates of French government bonds dated between August 2011 and July 2012, fit them using the MLE method explained previously and calculate the capital constituting of a life Annuity Immediate at age of 65 in arrears, with the use of the French mortality table TH00-02 and we highlight the robustness of our estimation technique.

To do this, we are going to follow the below steps to calculate our Value-at-Risk and show the estimation error behind:

1. We fix, for a given date, the Nelson-Siegel parameters set $\omega_0 = (\beta_0, \beta_1, \beta_2, \tau)$ by applying the Maximum Likelihood estimation technique stated previously in section 6.2.1.

2. We calculate the life annuity immediate $a_x^0 = \sum_{h \geq 1} P(0, h) \frac{L_{x+h}}{L_x}$, where $P(0, h) = \exp(-R(h, \omega_0)h)$, and $\frac{L_{x+h}}{L_x}$ given by TH00-02;

3. Given the asymptotic normality of MLE, we have: $\hat{\omega} \hookrightarrow \mathcal{N}(\omega_0, \Sigma)$, where Σ is calculated using the inverse of fisher information matrix: $\Sigma = \left(-\mathbb{E} \left[\frac{\partial^2 \ln \mathcal{L}_\omega}{\partial \omega \partial \omega'} \right] \right)_{\omega=\omega_0}^{-1}$;

4. Calculate the life annuity immediate for the given $\hat{\omega}$;

5. Repeat steps 3 and 4 on a number n of times (in our case, we took $n = 10^5$ iterations) to be able to analyze the empirical cumulative distribution function of $a_x = \{a_x^i, i = 1, \dots, n\}$;

6. Repeat steps 1 to 5 for each date.

As for the calculation of Fisher's inverse information Matrix:

$$\Sigma = \left(-\mathbb{E} \left[\frac{\partial^2 \ln \mathcal{L}_\omega}{\partial \omega \partial \omega'} \right] \right)_{\omega=\omega_0}^{-1},$$

we used the log-likelihood function: $\ln(\mathcal{L}) = \frac{-n}{2} \ln(2\pi\sigma^2) - \frac{1}{2} \sum_{j=1}^n \left(\frac{R_{t,j} - \hat{R}_{t,j}}{\sigma} \right)^2$ see appendix J for the full support.

6.2.3 Numerical Results

In this section, we apply the preceding theoretical discussion of our estimation technique to the problem at hand. From Bloomberg, we took monthly market rates of French government bonds dated between August 2011 and July 2012 ($n = 12$ months). For each date, we estimated the model parameters of Nelson-Siegel then we simulated various outcomes from Nelson-Siegel using the known parameter set and proceed to estimate the model parameters. This simulation example is intended to indicate how parameter estimation can affect directly our calculated capital constituting of a life Annuity Immediate for a person at the age of 65 in arrears, with the use of the French mortality table TH00-02.

Now since the estimation of the VaR is executed on simulated values, the simulations and the estimations has to be effected on a large size sample (we took 10^5 simulations).

As for the calculation of the confidence interval, since we are working with order statistics, the interval (x_l, x_u) would cover our quantile x_p with a 99.5% probability that depends on the lower bound l , upper bound u , number of steps n and confidence level p .

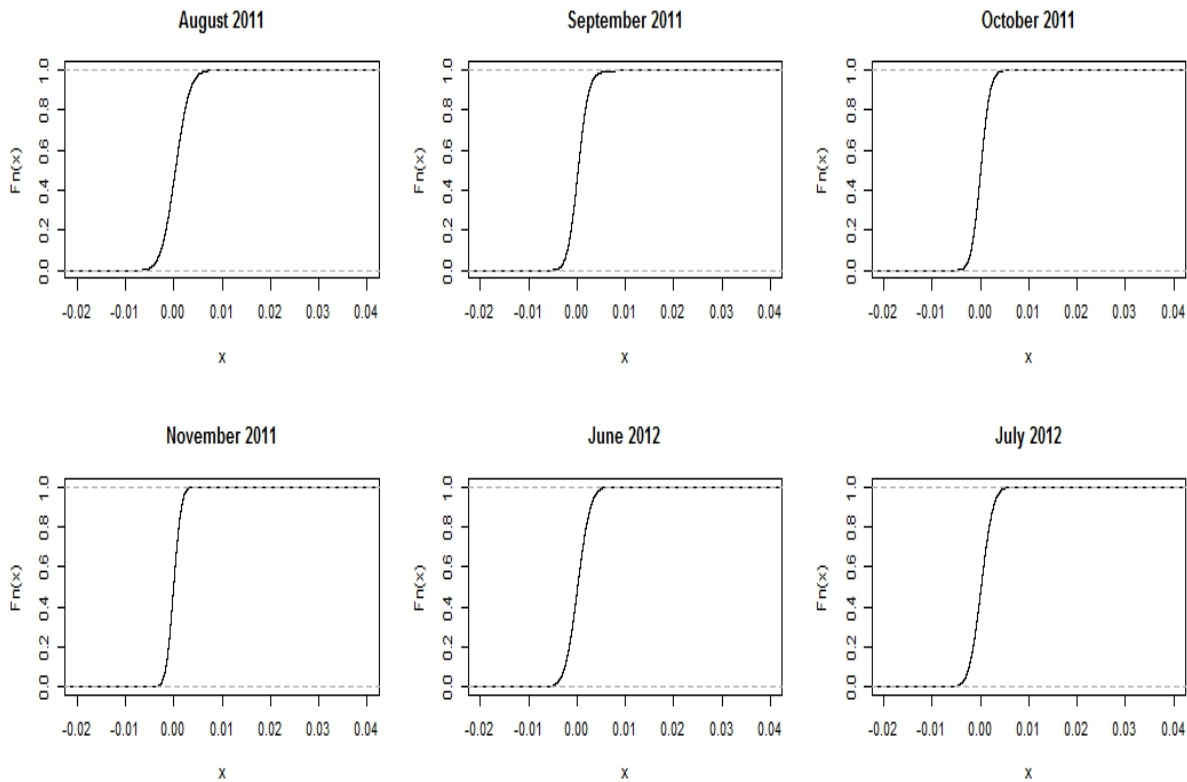
In our calculations, we took $n = 10^5$, $p = 99.5\%$ and our integers (l, u) , were constructed using the normal approximation $\mathcal{N}(np, np(1-p))$ to the binomial distribution $\mathcal{B}(n, p)$, (since n is large). Then a simple linear interpolation has been made to obtain the values of (x_l, x_u) , (cf. David & Nagaraga [2003] pp. 183-186), for more details and demonstrations.

As so, and with our fixed values parameters, for each month, we have been able to estimate the VaR(99.5%) of our annuity shown in table 6.2.

Table 6.2 displays the non-existence of consequences in estimated parameters with a relative change on the VaR/Annuity not more then 0.8% throughout the months. Furthermore, calculating the relative difference for a given date: $\epsilon = (\epsilon_1, \dots, \epsilon_n)$, where $\epsilon_i = \frac{\hat{a}_x^i - a_x^0}{a_x^0}$. Figure 6.2, shows the Empirical Cumulative Distribution Function of ϵ which clearly displays the robustness of our estimation.

	Date	Annuity	VaR(99.5%)	$\Delta(\text{VaR}/\text{Annuity})$	CI(99.5%)	$\Delta(\text{UB}/\text{Mean})$
1	August-11	11.907	11.986	0.67%	11.985 11.988	0.64%
2	September-11	12.123	12.204	0.67%	12.201 12.208	0.68%
3	October-11	11.586	11.634	0.42%	11.634 11.635	0.43%
4	November-11	11.379	11.415	0.32%	11.415 11.415	0.32%
5	December-11	11.771	11.871	0.84%	11.869 11.873	0.82%
6	January-12	11.676	11.730	0.46%	11.729 11.731	0.48%
7	February-12	11.877	11.973	0.80%	11.971 11.974	0.78%
8	March-12	11.921	12.005	0.70%	12.004 12.007	0.69%
9	April-12	11.872	11.948	0.64%	11.947 11.949	0.62%
10	May-12	12.511	12.607	0.77%	12.606 12.609	0.75%
11	June-12	12.071	12.139	0.57%	12.139 12.140	0.56%
12	July-12	12.697	12.765	0.54%	12.764 12.767	0.53%

Table 6.2: Nelson Siegel's Annuity Output


 Figure 6.2: Empirical Cumulative Distribution Functions for ϵ , for several dates

6.3 Conclusion

We have developed in this work, a new estimation approach for fitting a yield curve that leads to a robust assessment of risk related to a Maximum Likelihood Estimation of Nelson-Siegel parameters. Compared to other used estimation methods it performs

efficiently, respecting as well, all of the constraints given by the model.

To expose the robustness, we took monthly market rates of French government bonds dated between August 2011 and July 2012 ($n = 12$ months). For each date, we estimated the model parameters of Nelson-Siegel then we simulated various outcomes from Nelson-Siegel using the known parameter set and proceed to estimate the model parameters, then we calculated a life Annuity Immediate for a person at the age of 65 in arrears, with the use of the French mortality table TH00-02.

We have featured the nonexistence of consequences in estimating the parameters with a relative change on the VaR/Annuity not more than 0.8% throughout the months. Furthermore, calculating the relative difference for a given date, clearly indicates the robustness of our estimation.

The result of this chapter will help us illustrate a different type of estimation risk related to the fluctuation of our yield curves that will be treated in the next chapter. This study gave us a sort of assertiveness that fitting our yield curve using MLE method will not generate additional instability and uncertainty regarding the parameters estimated. Yet as we are going to see further on, which is so different then what have we illustrated in the previous chapter where our uncertainty was directly linked to the parameters estimated, the instability we had in our model would be the result of the conceptual framework that our calculations were based on.

Chapter 7

Market Consistency and Economic Evaluation

7.1 Introduction

Solvency II has chosen a framework for the valuation of technical provisions consistent with market values, with the obligation of taking provision for risk consistent with market values, where a yield curve is used for discounting purposes. Accordingly, assets are to be valued at their market value, whereas liabilities are discounted at the risk-free yield curve rate. This principle of market consistency, introduced by the regulators, requires the use of financial markets as reference to mathematical models at the date of assets and liabilities valuation. This implementation in practice leads to important volatility (cf. EIOPA [2013]). Adjustments being tested as part of the exercise Long-Term Guarantees Assessment (LTGA), (cf. KPMG [2013]), particular objective to remedy this situation. The consequence of high volatility would impact our balance sheet which will be subject to the variation of the interest rates and not to forget the errors generated from the estimated parameters with which the estimation method have been calibrated.

In our study, we are fitting yield curves using Nelson-Siegel model (cf. NELSON & SIEGEL [1987]), but to reduce the parameter estimation error, we applied an MLE method explained in details in the previous chapter. Thus, the only problem we are left with, is the variation of the financial markets that is creating high volatilities on our balance sheet.

Moreover, the assessment of balance sheet is taking into account risks that insurers are not really exposed to, as we are going to see in this chapter. Hence, fluctuation of interest rate curve add to this phenomenon and leads to volatility assessments of technical provisions, (spread risk is increasing given that the risks we are not held responsible of are being taken into consideration). Then, this volatility could compromise the Solvency Capital Requirement (SCR) by underestimating it, (cf. IFRGAN [2013]) for more on this. Our assets role are essential to cover the liability cash flows, but since our assets are being valued at their market value, this will create the following problem: an increase of spread risk on the asset side will lead to a mismatching between assets and liabilities. Therefore, we will illustrate a simple proposal of correction, using the monthly annuity immediate calculation, where we apply the moving average technique to stabilize the volatility and

smooth our curve. In addition, we try to reconstruct our yield curve in a way to attempt and stabilize the balance sheet exposure, by trying to synchronize the volatility of assets with the one of liabilities.

At the end, we will test the behavior of our Asset Liability Management by applying a stress test in order to show the interaction between the Asset side with the Liability and the consequence on the Equity.

7.2 Market Consistency

The current low yield environment and the regulators market consistent approach has exposed the balance sheet of many insurers to high volatility. We investigate in this section, a market-consistent valuation of insurance liabilities that neutralizes changes and stabilize the balance sheet exposure.

7.2.1 Economic Evaluation

Since Solvency II has chosen a framework for the evaluation of technical provisions consistent with market values that implementation in practice leads to important volatility, we are going to provide a simple proposal of correction, where we apply the moving average technique to stabilize the volatility and smoothing our curve.

To illustrate this, we used the same monthly market rates of French government bonds dated between August 2011 and July 2012, and calculate the capital constituting a life Annuity Immediate at age of 65 in arrears, with the use of the French mortality table TH00-02, as done in the previous section. The rates are as well fitted using Nelson-Siegel (cf. Nelson & Siegel [1987]), with the Maximum Likelihood Estimation method (cf. chapter 6) that gives us more robustness in parameter estimation. We complete this, by calculating the average curve of the 12 and 24 months prior to the evaluation see Table 7.1.

	Date	Annuity	Smoothing 12 months	Smoothing 24 months
1	August-11	11.907	11.574	11.565
2	September-11	12.122	11.565	11.599
3	October-11	11.585	11.530	11.615
4	November-11	11.378	11.496	11.616
5	December-11	11.772	11.509	11.639
6	January-12	11.675	11.533	11.655
7	February-12	11.878	11.569	11.672
8	March-12	11.922	11.631	11.691
9	April-12	11.872	11.678	11.700
10	May-12	12.512	11.767	11.723
11	June-12	12.071	11.833	11.733
12	July-12	12.697	11.920	11.766

Table 7.1: 12 months and 24 months smoothing calculations

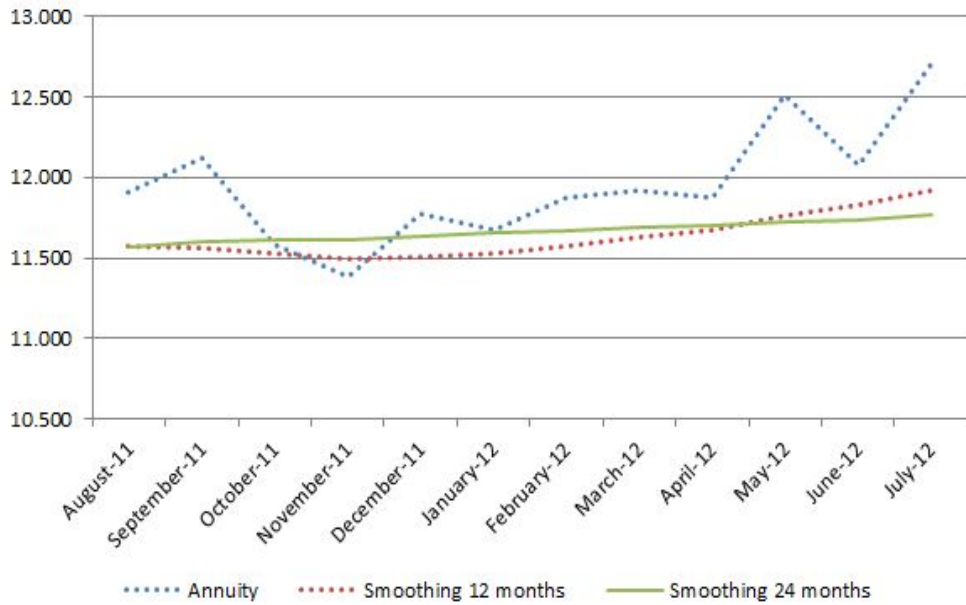


Figure 7.1: Economic Valuation

Figure 7.1 shows the different smoothing curves. The blue curve shows that the erratic nature of the assessment of commitment varies from one month to the next within a range of $\pm 5\%$. Showing the high volatility as months progress.

To reiterate, in addition to EIOPA solutions (cf. Planchet & Leroy [2013]), applying a simple smoothing technique as seen in the Figure 7.1, lead us to a much more usable solution in terms of managing risks. When looking at the curves, we logically note a stabilization in the assessments, that evolves now by a trend, which reflects in this case a decline in interest rates. We can as well note that, the smoothed 12 and 24 curves are below the annuity curve for the majority of the corresponding months, this is related to the severe decrease in interest rates throughout the months (see Figure 7.2) due to the Eurozone crisis and most specifically, Greek government-debt crisis that has widely affected the European Union.

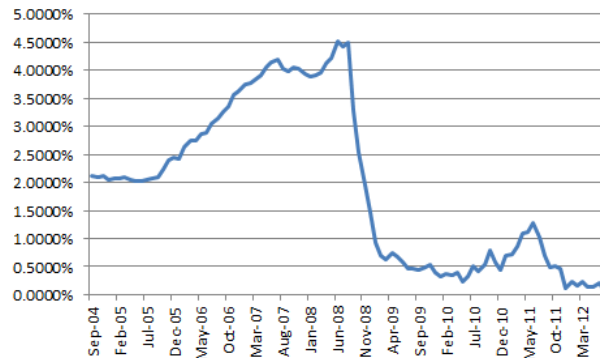


Figure 7.2: Monthly Spot Rates

7.2.2 Balance Sheet Exposure

Under the context of Solvency II Framework and the new international accounting standard¹, IAS19 states that the pension liabilities, must be defined by market valuation and that the converse of pension liabilities to be taken up in the financial statement of the company. Of course, this means that any volatility in the assets will be felt in the corporation's financial.

The interest-rate exposure arises because rates fluctuate from day to day and continuously over time. Where an asset is marked at a fixed rate, a rise in rates will reduce its NPV and so reduce its value to the insurance company. This is intuitively easy to understand, because the asset is now paying a below-market rate of interest. Or we can think of it as a loss due to opportunity cost foregone, since the assets are earning below what they could earn if they were employed elsewhere in the market. The opposite applies if there is a fall in rates: this causes the NPV of the asset to rise. Even for the simplest insurance operation, we can check that this will produce a net mismatch between assets and liabilities as we are going to see further in this work.

The objective is to eliminate the volatility of liabilities caused by the assets backing up the insurance commitments. Indeed, in the case of long-duration liabilities, invested assets are typically composed of bonds which market values are subject to credit risk, liquidity risk and default risk. But these assets are purchased in a primary objective to neutralize interest rate risk and not to be sold. Fluctuations in market value due to movements in interest rates does not constitute an efficient economic information to manage our Balance Sheet exposure since only the default of the counterparty is the real risk we are exposed to.

Our interest in this section, is to attempt and stabilize the balance sheet exposure, by trying to synchronize the volatility of assets with the one of liabilities. For the actual balance sheet of the company, we are going to identify the Net-Asset-Value which is defined as the difference between the market-consistent value at time t , $V_t(A_t)$ of the assets A_t and the discounted value (risk-free yield curve) $V_t(L_t)$ of the liabilities L_t , $NAV_t = V_t(A_t) - V_t(L_t)$.

In this work, our asset portfolio is being invested on $n = \frac{L_{x+h}}{L_x}$ number of zero-coupon

bonds, where the $\sum_{h \geq 1} P(0, h) \frac{L_{x+h}}{L_x}$ represent the price of our portfolio discounted by

monthly market rates of French government bonds dated between August 2011 and July 2012 (12 months). As for the liability part, we calculate the capital constituting of a life Annuity Immediate at age of 65 in arrears, with the use of the French mortality table TH00-02 and discounted using Government bond, nominal, all triple A issuer companies provided by European Central Bank² for the same dates. The use of two different yield curves generates an Asset/Liability mismatch scenario. Not to forget that the payment stream from the issuer to the annuitant has an unknown duration based principally upon the date of death of the annuitant, where the contract will be terminated.

¹URL: www.ifrs.org

²URL: www.ecb.europa.eu

The rates are as well fitted using Nelson-Siegel estimation technique . We complete this, by calculating the average curve of the 24 months prior to the evaluation and calculate the NAV at time t related to:

$$\begin{cases} NAV_1 : Asset - Liability \\ NAV_2 : Asset\ smoothed - Liability\ smoothed \\ NAV_3 : Asset - Liability\ smoothed \end{cases}$$

The high instability generated in the Balance Sheet, tends to complicate the situation of the insurance company and its risk control. In a way, we are investing our Asset in zero-coupon bond that present some spread compared to the market, which is usually explained by credit risk, default risk and liquidity risk of the bond issuer. On the other hand, seeing that we are tending to hold our bond until maturity, the only risk assigned to it would be the default of the counterparty, (Downgrading the bond value would not influence us in any risky way and selling the bond before maturity is of no importance as well). Moreover, all these risks are reflected in the market price of the assets. Therefore, if assets are valued at market value then they will be underrated because the insurer is only exposed to the default risk of the issuer. However, an increase in the spread leads to a decrease in our Asset side, but our Liability, (discounted by the use of the risk-free yield curve), stays the same, which creates a deficiency that we are not responsible of. At the end, and since in our case, only counterparty default is the real risk, the coverage of commitments is submitted to variations which the cause is only partially linked to our risk effectively supported (cf. Planchet & Leroy [2013]).

Date	Asset		Liability		NAV		
	Instantaneous	Smoothed	Instantaneous	Smoothed	NAV_1	NAV_2	NAV_3
Aug-11	11.907	11.565	12.376	11.778	-0.469	-0.213	0.129
Sep-11	12.122	11.599	12.695	11.824	-0.572	-0.225	0.298
Oct-11	11.585	11.615	12.382	11.863	-0.797	-0.248	-0.277
Nov-11	11.378	11.616	12.145	11.891	-0.767	-0.274	-0.513
Dec-11	11.772	11.639	12.627	11.945	-0.855	-0.306	-0.173
Jan-12	11.675	11.655	12.622	11.991	-0.947	-0.336	-0.316
Feb-12	11.878	11.672	12.661	12.033	-0.783	-0.361	-0.155
Mar-12	11.922	11.691	12.699	12.076	-0.777	-0.386	-0.155
Apr-12	11.872	11.700	12.791	12.119	-0.919	-0.419	-0.246
May-12	12.512	11.723	13.484	12.171	-0.972	-0.449	0.341
Jun-12	12.071	11.733	12.932	12.200	-0.861	-0.467	-0.128
Jul-12	12.697	11.766	13.466	12.296	-0.768	-0.530	0.401

Table 7.2: Balance Sheet asset, Liability and NAV calculation

Figure 7.3 illustrates the different NAV related to smoothed and non-smoothed Asset Liability calculation through all 12 Months.

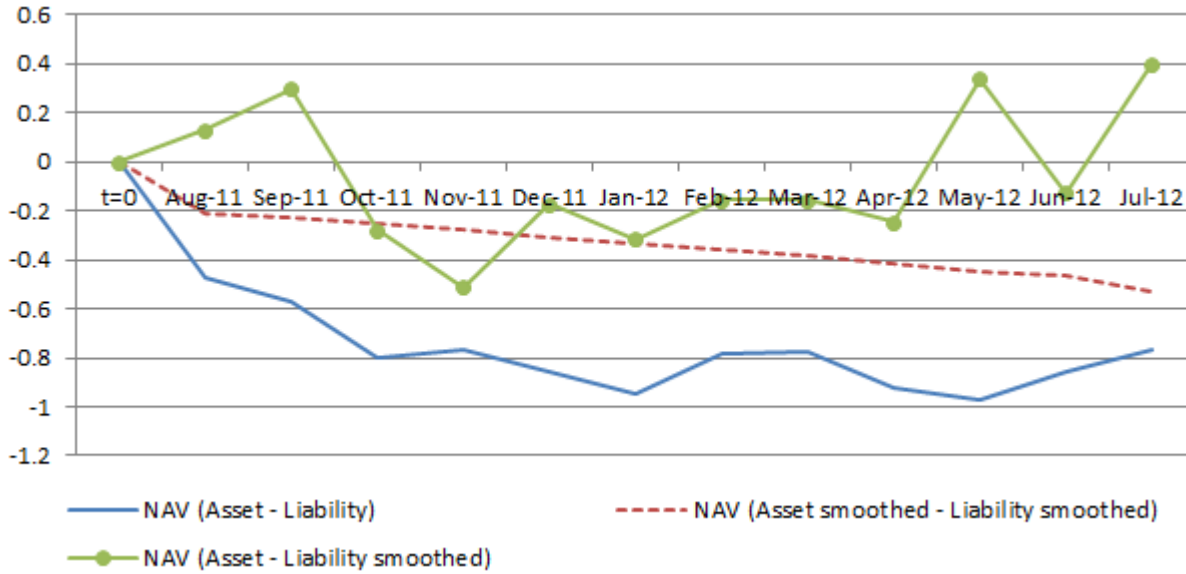


Figure 7.3: NAV Calculation

7.2.2.1 Restructuring the Yield Curve

The volatility of interest rates have resulted a debate amongst insurers on how to evaluate their Balance Sheet. In the previous sections, we have already emphasized the role of risks encountered in reality and their effect on our Balance Sheet. In this part, we are going to change the weights given to our yield curves and expose another way of tackling the subject.

We have an insurance Liability labeled by the ECB spot rates and we are investing on a number of zero-coupon bonds evaluated by the use of the French Government rates.

We used the monthly market rates dated between September 2004 and July 2012 and calculated the capital constituting a life Annuity Immediate at age of 65 in arrears, with the use of the French mortality table TH00-02, this capital a_x is then invested in zero-coupon bonds, so that at $t = 0$ our Assets are equal to the Liability. To be able to reconstruct our yield curve, we are going to assume that our Liability is always stable with a life annuity immediate for a 65 years in arrears.

In another way, for a month to the other, our balance is being compensated which means that every exit is being replaced by an entry invested in the yield curve month. Hence, our entry flow α for a given month t would be:

$$\alpha_t = f\left(\frac{1}{12}q_x, Curve_t\right) = \frac{1}{12}q_x \cdot a_{x_t}$$

Where,

- $\frac{1}{12}q_x = 1 - (1 - q_x)^{1/12}$, is the probability that a person at the age of $x = 65$, would die within a month;

- $a_{x_t} = \sum_{h \geq 1} P(0, h) \frac{L_{x+h}}{L_x}$, for the month t in consideration.

At the end, we would have the following recurrence relation:

$$\begin{cases} \overline{Curve}_0 = \text{Yield Curve of the first month September 2004} \\ \overline{Curve}_t = (1 - \alpha_t)\overline{Curve}_{t-1} + \alpha_t Curve_t, \quad \text{for } t \geq 1 \end{cases}$$

This application, generates a new reconstructed yield curve, allowing our liability to be discounted with the use of a new yield curve that is much coherent with our Portfolio selected and can neutralize the variations and exposure of our balance sheet as Table 7.3 and Figure 7.4 show us, by only representing the last 12 months of our calculations. We have to mention that the rates generated are as well fitted using Nelson-Siegel ML estimation technique. The NAV calculated are related to:

Date	Asset		Liability		NAV		
	Instantaneous	Smoothed	Instantaneous	Smoothed	NAV_1	NAV_2	NAV_3
Aug-11	11.907	11.179	12.269	11.337	-0.362	-0.159	0.570
Sep-11	12.122	11.192	12.569	11.361	-0.447	-0.169	0.761
Oct-11	11.585	11.201	12.271	11.379	-0.685	-0.178	0.206
Nov-11	11.378	11.209	12.093	11.389	-0.715	-0.181	-0.012
Dec-11	11.772	11.216	12.583	11.420	-0.811	-0.204	0.352
Jan-12	11.675	11.225	12.618	11.425	-0.943	-0.200	0.250
Feb-12	11.878	11.231	12.603	11.446	-0.725	-0.215	0.432
Mar-12	11.922	11.248	12.703	11.475	-0.782	-0.228	0.446
Apr-12	11.872	11.257	12.749	11.499	-0.877	-0.242	0.373
May-12	12.512	11.277	13.443	11.532	-0.931	-0.255	0.980
Jun-12	12.071	11.290	12.848	11.549	-0.777	-0.259	0.522
Jul-12	12.697	11.316	13.396	11.590	-0.699	-0.274	1.107

Table 7.3: NAV Calculation by restructuring the Yield Curve

$$\begin{cases} NAV_1 : \text{Asset} - \text{Liability} \\ NAV_2 : \text{Asset smoothed} - \text{Liability smoothed} \\ NAV_3 : \text{Asset} - \text{Liability smoothed} \end{cases}$$

Figure 7.5, displays all resulted curves in a way to compare the different NAV calculated. Reconstructing the Yield curve in the way explained in this section, granted the balance sheet throughout the months, less exposure by trying and synchronizing the asset and liability sides.

On the other hand, Figure 7.6 shows the instantaneous and restructured smoothed capital calculation which have been discounted with the use of French Government bonds OAT from September 2004 till July 2012.

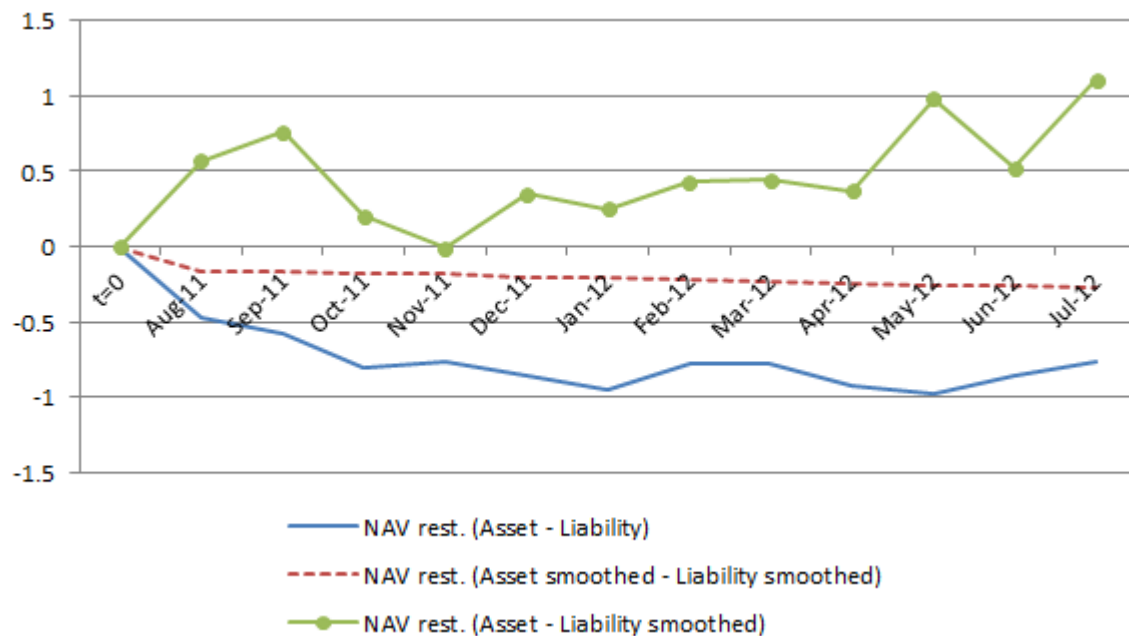


Figure 7.4: NAV Calculated by restructuring our Yield Curve

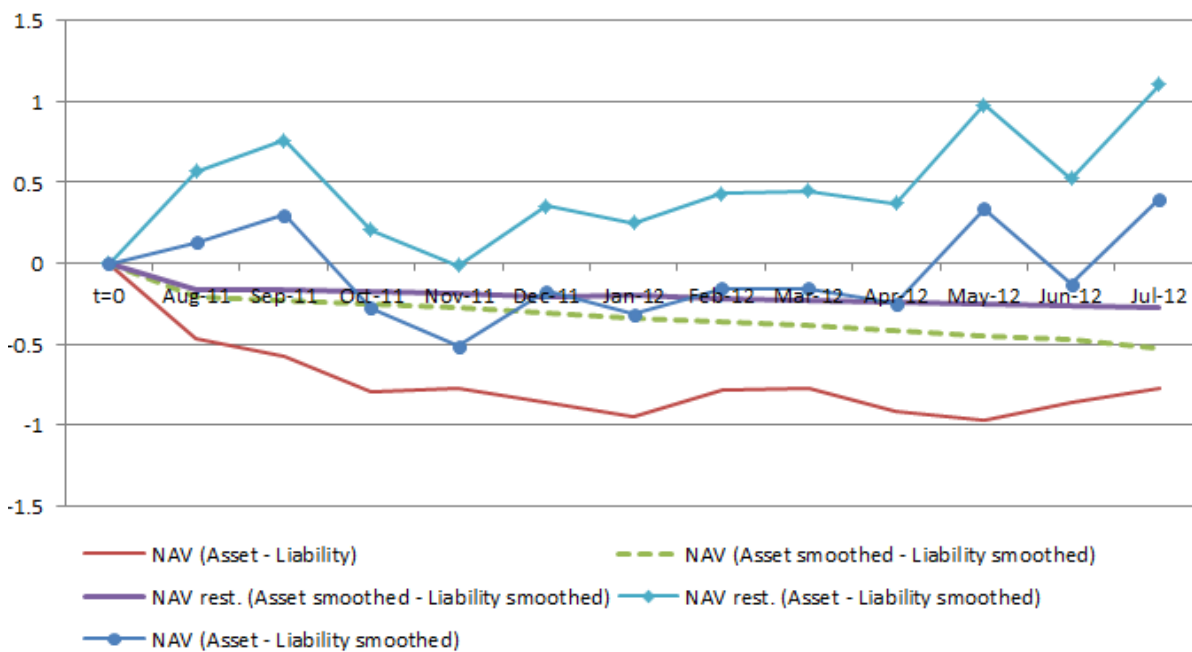


Figure 7.5: Compared NAV Curves

7.2.2.2 Sensitivity to the age of the portfolio

To check the influence of the weights calculated to our yield curves, we will take 3 different categories of ages with $x = 65$, $x = 75$ and $x = 85$.

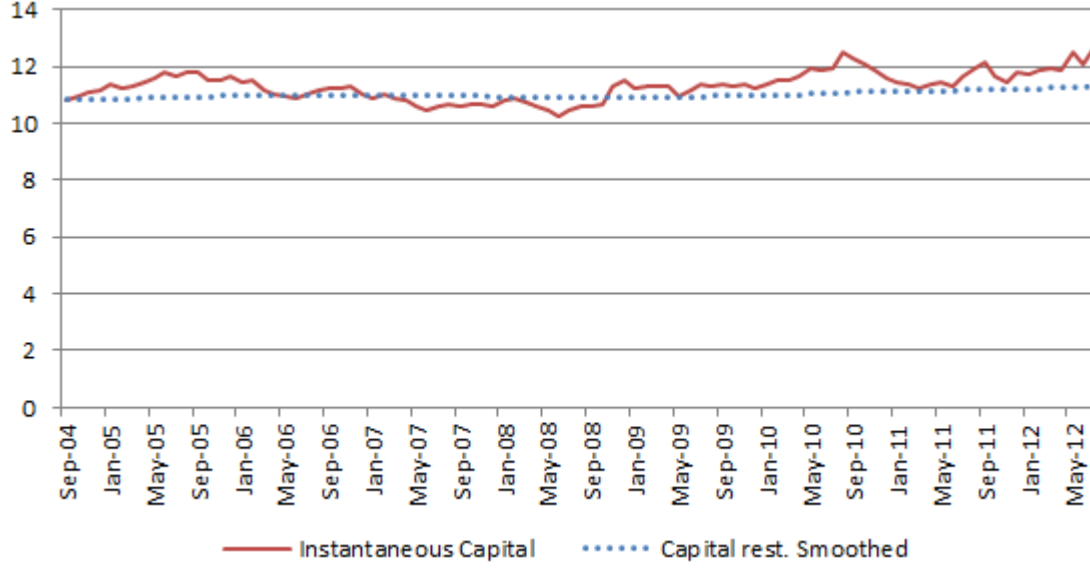


Figure 7.6: NAV Calculated by restructuring our Yield Curve

We have:

$$\begin{aligned}
 \overline{Curve}_t &= (1 - \alpha_t)\overline{Curve}_{t-1} + \underbrace{\alpha_t}_{\text{weight}} Curve_t \\
 &= (1 - \alpha_t)(1 - \alpha_{t-1})\overline{Curve}_{t-2} + \underbrace{(1 - \alpha_t)\alpha_{t-1}}_{\text{weight}} Curve_{t-1} + \alpha_t Curve_t \\
 &= (1 - \alpha_t)(1 - \alpha_{t-1})(1 - \alpha_{t-2})\overline{Curve}_{t-3} + \underbrace{(1 - \alpha_t)(1 - \alpha_{t-1})\alpha_{t-2}}_{\text{weight}} Curve_{t-2} + \\
 &\quad (1 - \alpha_t)\alpha_{t-1} Curve_{t-1} + \alpha_t Curve_t \\
 &= \vdots \\
 &= \prod_{i=1}^t (1 - \alpha_i)\overline{Curve}_0 + \underbrace{\sum_{i=1}^{t-1} \alpha_{t-i} \prod_{k=t-i+1}^t (1 - \alpha_k)}_{\text{weight}} Curve_{t-i} + \alpha_t Curve_t, \quad \text{for } t \geq 1
 \end{aligned}$$

So we would have the weights for each Curve:

$$\begin{aligned}
 t &\longrightarrow \alpha_t \\
 t-1 &\longrightarrow (1 - \alpha_t)\alpha_{t-1} \\
 t-2 &\longrightarrow (1 - \alpha_t)(1 - \alpha_{t-1})\alpha_{t-2} \\
 &\vdots \\
 t-i &\longrightarrow \alpha_{t-i} \prod_{k=t-i+1}^t (1 - \alpha_k)
 \end{aligned}$$

At the end, Figure 7.7 shows the sensitivity to different ages of the Portfolio, highlighting as well the fact that by age growth, the weights given to past curves decrease and

those given to recent curves increase. As for the weighted durations $D_x = \sum_{i=0}^n weight_i \times i$, we obtain: $D_{65} = 26.9$, $D_{75} = 25.3$ and $D_{85} = 19.6$.

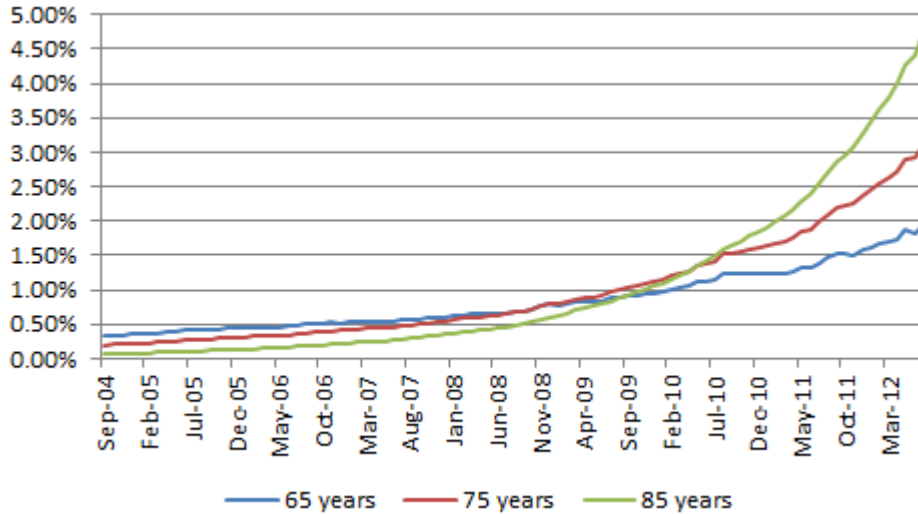


Figure 7.7: Age Sensitivity

7.2.2.3 Stress Test

To test the behavior of our Asset Liability Management, we will stress test our Liability Portfolio consisting of a Life annuity immediate stable for the age of 65. The idea is that, given a particular month, 30% of our Portfolio will be extracted. Hence, showing the direct effect on our Balance Sheet. Table 7.4, shows the interaction between the Asset(A) side with the Liability(L) and the consequence on the Equity(E).

Asset (A)	Liability (L)
3- Required liquidation $\alpha\%$ of Assets to cover step 1	1- Shock by 30% on a given month
4- $(1 - \alpha)\%A$ must cover the remaining payments of L	2- $70\%L$ is the remaining value of our Liability
Equity (E)	
i) If $\alpha = 30\% \rightarrow E$ is stable	
ii) If $\alpha < 30\% \rightarrow E$ is increased	
iii) If $\alpha > 30\% \rightarrow E$ is decreased	

Table 7.4: Balance Sheet exposure with a shock

We will assume that on September 2006, an unexpected exit of 30% of our Liability Portfolio has been made, (having the reconstructed yield curve see section 2.2.1). The insurer has in this case to sell a part $\alpha\%$ of his assets in a way to pay for the unplanned departures. Regardless of our mortality table, Figure 7.8 displays the amount of yearly

payments to be made given that a shock happened, we see that on September 2006 where the stress scenario appeared, the insurer had to pay about 4.33 €, which is decomposed of the yearly one euro amount that has to be regularly made and the market rate value of the 30% exit that generates an amount of 3.33 €.

On the other hand, we must as well sell a part of our Zero-Coupon, that generates the same amount. Hence, given the market value at the date: $\alpha = 29.6\%$, which means that 29.6% of our Assets have to be sold, enabling us to absorb the shock.

At the end we must say that, since the market was in our favor with an $\alpha < 30\%$, we have gained a bit from this situation, but this is not the case all the time. So in a way, we are exposed to a pure interest rate risk.

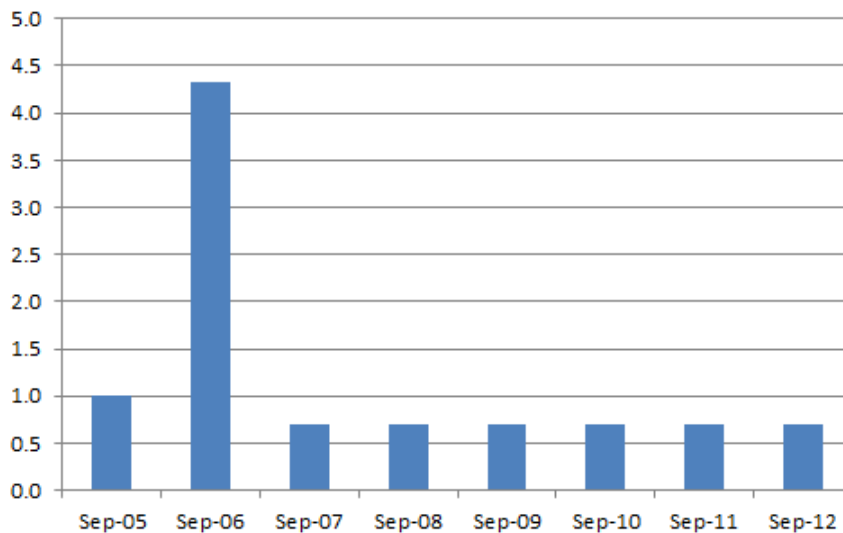


Figure 7.8: Yearly payments given a shock of 30%

Now if we consider that our Portfolio is not stable by taking into consideration the mortality table required at the age of 65, this situation will complicate a bit more our stress test. Additionally, having ECB yield curve for our Liability and French Government for the Asset, will change our calculations. On one hand, given our market values at the stress date (September 2006), we would be obliged to sell 30.2% of our Assets allowing us to cover the unplanned payments. On the other hand, we find ourselves unable to cover the yearly payments afterwards which influence our Equity, as already explained in Table 7.4. Now since we took mortality into consideration, the new scheme would differ, and instead of paying a fixed amount throughout the years, our payments would vary regarding the number of living people $\frac{L_{x+h}}{L_x}$ at year h , as we will see in Figure 7.9, the yearly payments would correspond to:

$$\left\{ \begin{array}{l}
 \text{September 2005 : } \frac{L_{x+1}}{L_x} \\
 \text{September 2006 : } \frac{L_{x+2}}{L_x}(1 + 0.3a_{x+2}) \\
 \text{September 2007 : } 0.7 \frac{L_{x+3}}{L_x} \\
 \text{September 2008 : } 0.7 \frac{L_{x+4}}{L_x} \\
 \dots \qquad \qquad \dots
 \end{array} \right.$$

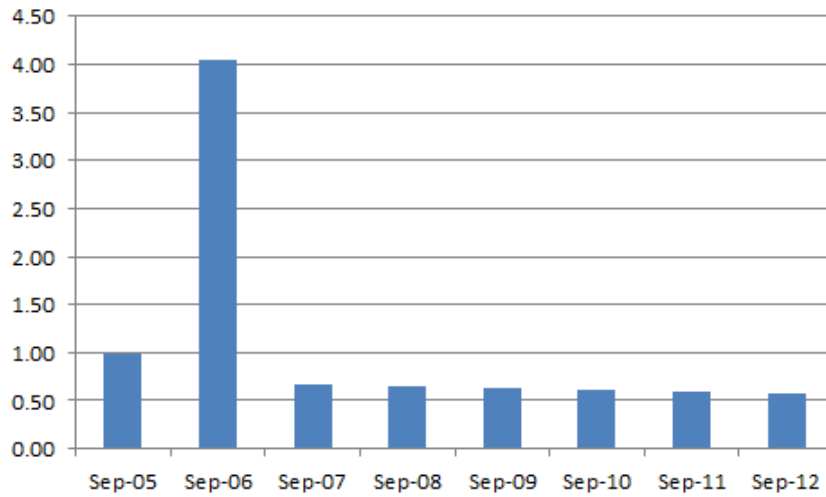


Figure 7.9: Yearly variable payments given a shock of 30%

7.3 Conclusion

In this chapter, we have emphasized the high impact of volatility, seen in market values, on our calculations. Targeting to solve the volatility issue, the EIOPA proposed many solutions. Accordingly, we have decided in this work to focus more on the conceptual framework of our calculation and try to limit the volatility of our estimation by providing a simple proposal of correction, where we apply the moving average technique to stabilize the volatility and smooth our jumping curves.

On the second part, we proposed another method to synchronize our balance sheet, where we reconstructed the yield curve in a way to investigate a market-consistent valuation of insurance liabilities that neutralize changes and stabilize the balance sheet exposure.

At the end, we showed the behavior of our Asset Liability Management by applying a stress test in order to spot the interaction between the Asset side with the Liability and the consequence on the Equity.

Hence, calling attention to the reflections on assumptions of the calculation instead of focusing on the so called hypothesis "consistent with market values", would be more appropriate and effective than to complicate models and generate additional instability.

General Conclusion

Operational risk quantification is a challenging task both in terms of its calculation as well as in its organization. Regulatory requirements, (Basel III for banks and solvency II for insurance companies), are put in place to ensure that financial institutions mitigate this risk. Many calculation criteria have been developed, ranging from Basic to Standardized to Advanced Measurement Approaches, in addition to a wide range of qualitative regulatory framework which enhance the whole set of managing, mitigating and quantifying operational risk.

In this thesis, we have provided a full overview of operational risk by presenting both qualitative aspects and quantitative theories and approaches for some financial institutions wishing to model this type of risk. Additionally, we displayed the highly importance in putting attention on the consequences of estimation risk in model development, which is a particular case of operational risk.

The first part was dedicated to point out the different perspectives of operational risk, that have risen to the point of holding a significant position in risk assessment, as seen by the fact that many banking failures was the result of operational losses. This has incited the regulators both Basel Accords and Solvency Directive, to propose some quantification methods ensuring that banks and insurance companies are mitigating this risk. Additionally, a full overview of the quantification techniques and an outline management of operational risk in a qualitative aspect is given throughout the chapters of this part. Not to forget the role that insurance plays in diminishing the financial impact of operational losses of a bank. The transfer of a risk to an insurer can contribute to a better performance preventing critical situation and covering a variety of losses which has been as well noted in the last chapters of this part. Moreover, we show the necessity of a good judgment to make sensible choices but these choices will influence results. Understanding this influence, should be important. We concluded this part, by highlighting the use of external data which is absolutely important to the implementation of an advanced method for calculating operational capital charge like LDA, we try to scale the severity of external losses for integration with internal data and finished by presenting three examples of losses extracted from our external loss database that showed in detail how the scaling is done and how should the losses be normalized.

The risk of not accurately estimating the amount of future losses is an essential issue in risk measurements. Sources of estimation risk include errors in estimation of parameters which can affect directly the VaR precision. Estimation risk in itself is related to operational risk in a way that the losses are arising from estimation errors. Based on this, we were interested in exposing the estimation risk in its different aspects from measuring the error induced on the SCR by the estimation error of the parameters to check the market consistency and economic valuation. Also, and since Solvency Directive has chosen a framework for the evaluation of technical provisions consistent with market values: the interest rate curve is to be used for discounting purposes. The implementation of this framework in practice leads to important volatility assessments that might have

high consequences on balances. Furthermore, we presented a new approach for fitting a yield curve, that led to a robust assessment of risk. Thus, eliminating the potential loss of accuracy from estimation error when calculating a Value-at-Risk. We illustrated, a simple proposal of correction, where we apply the moving average technique and try to reconstruct a new Yield Curve in a way, to stabilize the volatility and smooth our curve.

To reiterate, quantifying operational risk is not easy at all, an attention has to be made regarding all measurement techniques and methods used, so that a robust capital allocation is generated. Meeting the requirements of advanced models is challenging, but given the importance of Operational risk it is a must. We have drawn the attention in our study to the importance of operational risk and more particularly to the estimation risk that we are facing. Many banks and financial institutions, developed models to compute the value-at-risk and resulting capital requirement, but we know that any model is by definition an imperfect simplification and in some cases a model will produce results that are bias due to parameter estimation errors. For instance, this point is illustrated in BOYLE AND WINDCLIFF [2004] for pension plans investment and in PLANCHET AND THEROND [2012] for the determination of Solvency capital in the Solvency II Framework. One of the consequences of estimation risk, is that the capital requirement may be underestimated. So it would be more appropriate in reality to privilege simple and prudent models and avoid complexing them, in a way to prevent more estimation errors that might have severe influences on our models.

At the end, the work described in this thesis opens the way to other perspectives. We have indeed shown the importance of estimation risk in our work, yet we did not treat any mitigation of that risk. In addition, since a model can at the best be an approximate representation of the real world, risk always exist and must be carefully treated. We need to be aware of the impact of model uncertainty on our models. Model risk should be treated with extreme caution. We can mention the paper, Difficult Risk and Capital Models, FRANKLAND *et al.* [2013], which focuses on four specific types of error in model risk.

Another perspective for this work might be to check the judgment exercise of the experts which create more biases in problem solving then solutions, since decisions are usually based on beliefs concerning the likelihood of uncertain events. In our work, judgments were needed to make sensible choices but these choices have influenced our results. Understanding this influence, should be an important aspect of capital calculations, since it created an estimation risk that has highly influenced our capital requirement. More on this topic could be found with TVERSKY & KAHNEMAN [1974], [2000] and KAHNEMAN [2003] which focuses on experts judgment under uncertainty.

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Appendix A

Qualifying Criteria for Standardized Approach for Operational Risks

- The bank must have an operational risk management system with clear responsibilities assigned to an operational risk management function. The operational risk management function is responsible for developing strategies to identify, assess, monitor and control/mitigate operational risk; for codifying firm-level policies and procedures concerning operational risk management and controls; for the design and implementation of the firms operational risk assessment methodology; for the design and implementation of a risk-reporting system for operational risk.
- As part of the banks internal operational risk assessment system, the bank must systematically track relevant operational risk data including material losses by business line. Its operational risk assessment system must be closely integrated into the risk management processes of the bank. Its output must be an integral part of the process of monitoring and controlling the banks operational risk profile. For instance, this information must play a prominent role in risk reporting, management reporting, and risk analysis. The bank must have techniques for creating incentives to improve the management of operational risk throughout the firm.
- There must be regular reporting of operational risk exposures, including material operational losses, to business unit management, senior management, and to the board of directors. The bank must have procedures for taking appropriate action according to the information within the management reports.
- The banks operational risk management system must be well documented. The bank must have a routine in place for ensuring compliance with a documented set of internal policies, controls and procedures concerning the operational risk management system, which must include policies for the treatment of non-compliance issues.
- The banks operational risk management processes and assessment system must be subject to validation and regular independent review. These reviews must include

both the activities of the business units and of the operational risk management function.

- The banks operational risk assessment system (including the internal validation processes) must be subject to regular review by external auditors and/or supervisors.

For more details on Standardized Approach qualifying criteria and measurements see BCBS[2005].

Appendix B

Business Lines and Event types

<i>Basel II Business Lines (BL)</i>	<i>Basel II Event Types</i>
Corporate finance ($\beta_1 = 0.18$)	Internal fraud
Trading & Sales ($\beta_2 = 0.18$)	External fraud
Retail banking ($\beta_3 = 0.12$)	Employment practises and workplace safety
Commercial banking ($\beta_4 = 0.15$)	Clients, products and business practices
Payment & Settlement ($\beta_5 = 0.18$)	Damage to physical assets
Agency Services ($\beta_6 = 0.15$)	Business disruption and system failures
Asset management ($\beta_7 = 0.12$)	Execution, delivery and process management
Retail brokerage ($\beta_8 = 0.12$)	

Table B.1: Basel II 8 Business Lines \times 7 Event Types

Appendix C

Detailed loss event type classification

Event-Type Category (Level 1)	Definition	Categories (Level 2)	Activity Examples (Level 3)
Internal fraud	Losses due to acts of a type intended to defraud, misappropriate property or circumvent regulations, the law or company policy, excluding diversity/discrimination events, which involves at least one internal party.	Unauthorised Activity	Transactions not reported (intentional) Trans type unauthorised (w/monetary loss) Mismarking of position (intentional)
		Theft and Fraud	Fraud / credit fraud / worthless deposits Theft / extortion / embezzlement / robbery Misappropriation of assets Malicious destruction of assets Forgery Check kiting Smuggling Account take-over / impersonation / etc. Tax non-compliance / evasion (wilful) Bribes / kickbacks Insider trading (not on firm's account)
External fraud	Losses due to acts of a type intended to defraud, misappropriate property or circumvent the law, by a third party	Theft and Fraud	Theft/Robbery Forgery Check kiting
		Systems Security	Hacking damage Theft of information (w/monetary loss)
Employment Practices and Workplace Safety	Losses arising from acts inconsistent with employment, health or safety laws or agreements, from payment of personal injury claims, or from diversity / discrimination events	Employee Relations	Compensation, benefit, termination issues Organised labour activity
		Safe Environment	General liability (slip and fall, etc.) Employee health & safety rules events Workers compensation
		Diversity & Discrimination	All discrimination types
Clients, Products & Business Practices	Losses arising from an unintentional or negligent failure to meet a professional obligation to specific clients (including fiduciary and suitability requirements), or from the nature or design of a product.	Suitability, Disclosure & Fiduciary	Fiduciary breaches / guideline violations Suitability / disclosure issues (KYC, etc.) Retail consumer disclosure violations Breach of privacy Aggressive sales Account churning Misuse of confidential information Lender Liability
		Improper Business or Market Practices	Antitrust Improper trade / market practices Market manipulation Insider trading (on firm's account) Unlicensed activity Money laundering

Table C.1: Detailed loss event classification 1

Event-Type Category (Level 1)	Definition	Categories (Level 2)	Activity Examples (Level 3)
		Product Flaws	Product defects (unauthorised, etc.) Model errors
		Selection, Sponsorship & Exposure	Failure to investigate client per guidelines Exceeding client exposure limits
		Advisory Activities	Disputes over performance of advisory activities
Damage to Physical Assets	Losses arising from loss or damage to physical assets from natural disaster or other events.	Disasters and other events	Natural disaster losses Human losses from external sources (terrorism, vandalism)
Business disruption and system failures	Losses arising from disruption of business or system failures	Systems	Hardware Software Telecommunications Utility outage / disruptions
Execution, Delivery & Process Management	Losses from failed transaction processing or process management, from relations with trade counterparties and vendors	Transaction Capture, Execution & Maintenance	Miscommunication Data entry, maintenance or loading error Missed deadline or responsibility Model / system misoperation Accounting error / entity attribution error Other task misperformance Delivery failure Collateral management failure Reference Data Maintenance
		Monitoring and Reporting	Failed mandatory reporting obligation Inaccurate external report (loss incurred)
		Customer Intake and Documentation	Client permissions / disclaimers missing Legal documents missing / incomplete
		Customer / Client Account Management	Unapproved access given to accounts Incorrect client records (loss incurred) Negligent loss or damage of client assets
		Trade Counterparties	Non-client counterparty misperformance Misc. non-client counterparty disputes
		Vendors & Suppliers	Outsourcing Vendor disputes

Table C.2: Detailed loss event classification 2

Appendix D

Moments calculation for the Generalized Pareto Distribution

The Generalized Pareto Distribution, has two parameters with the distribution function:

$$G_{\xi,\sigma}(x) = \begin{cases} 1 - \left(1 + \frac{\xi x}{\sigma}\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - e^{-\frac{x}{\sigma}} & \text{if } \xi = 0 \end{cases}$$

where $\sigma > 0$, and where $x \leq 0$ when $\xi \leq 0$ and $0 \leq x \leq -\frac{\sigma}{\xi}$ when $\xi < 0$.

The mean and variance of the Generalized Pareto distribution calculation $\mathbb{E}(X)$ and $V(X)$ are:

For $\sigma > 0$, $0 < \xi < 1$ and $x \leq 0$,

$$\begin{aligned} \mathbb{E}(X) &= \int_0^{+\infty} (1 - F(x)) dx \\ &= \frac{\sigma}{\xi} \int_0^{+\infty} \frac{\xi}{\sigma} \left(1 + x \frac{\xi}{\sigma}\right)^{-\frac{1}{\xi}} dx \\ &= \frac{\sigma}{\xi} \left[\frac{\left(1 + x \frac{\xi}{\sigma}\right)^{-\frac{1}{\xi} + 1}}{-\frac{1}{\xi} + 1} \right]_0^{+\infty} \\ &= \frac{\sigma}{\xi} \left(\frac{1}{\frac{1}{\xi} - 1} \right) \\ &= \frac{\sigma}{1 - \xi} \end{aligned}$$

and we calculate the variance for $\xi < \frac{1}{2}$,

$$\begin{aligned}
 \mathbb{E}(X^2) &= 2 \int_0^{+\infty} y P(X > y) dy \\
 &= 2 \int_0^{+\infty} y (1 - F(y)) dy \\
 &= 2 \int_0^{+\infty} y \left(1 + y \frac{\xi}{\sigma}\right)^{-\frac{1}{\xi}} dy \\
 &= 2 \left(\left[\frac{y\sigma}{\xi - 1} \left(1 + y \frac{\xi}{\sigma}\right)^{1 - \frac{1}{\xi}} \right]_0^{+\infty} - \frac{\sigma}{\xi - 1} \int_0^{+\infty} \left(1 + y \frac{\xi}{\sigma}\right)^{1 - \frac{1}{\xi}} dy \right) \\
 &= \frac{-2\sigma}{\xi - 1} \int_0^{+\infty} \left(1 + y \frac{\xi}{\sigma}\right)^{1 - \frac{1}{\xi}} dy \\
 &= \frac{-2\sigma^2}{\xi(\xi - 1)} \left[\frac{\left(1 + y \frac{\xi}{\sigma}\right)^{\frac{2\xi - 1}{\xi}}}{\frac{2\xi - 1}{\xi}} \right]_0^{+\infty} \\
 &= \frac{2\sigma^2}{(\xi - 1)(2\xi - 1)}
 \end{aligned}$$

which give us,

$$\begin{aligned}
 V(X) &= \mathbb{E}(X^2) - [\mathbb{E}(X)]^2 \\
 &= \frac{2\sigma^2}{(\xi - 1)(2\xi - 1)} - \frac{\sigma^2}{(\xi - 1)^2} \\
 &= \frac{(\xi - 1)(2\sigma^2) - (2\xi - 1)\sigma^2}{(\xi - 1)^2(2\xi - 1)} \\
 &= \frac{\sigma^2}{(\xi - 1)^2(1 - 2\xi)}
 \end{aligned}$$

Appendix E

Derivation of MLE Estimators for LogNormal Parameters

Assuming i.i.d. sample of n loss observations x_1, x_2, \dots, x_n from the LogNormal distribution.

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma x}} \exp\left\{-\frac{1}{2}\left(\frac{\ln(x) - \mu}{\sigma}\right)^2\right\}, \text{ for } x, \sigma \in \mathbb{R}^{+*}$$

We maximize the function $l(\theta|x) = \sum_{i=1}^n \ln[f(x_i|\mu, \sigma)]$ by finding $\hat{\mu}$ and $\hat{\sigma}$ such that $\frac{\partial l(\theta|x)}{\partial \mu} = 0$ and $\frac{\partial l(\theta|x)}{\partial \sigma} = 0$.

We have,

$$\begin{aligned} l(\theta|x) &= \sum_{i=1}^n \ln[f(x_i|\mu, \sigma)] \\ &= \sum_{i=1}^n \ln(1) - \ln(\sqrt{2\pi}\sigma x_i) - \frac{1}{2}\left(\frac{\ln(x_i) - \mu}{\sigma}\right)^2 \\ &= \sum_{i=1}^n -\ln(\sqrt{2\pi}) - \ln(\sigma) - \ln(x_i) - \frac{[\ln(x_i) - \mu]^2}{2\sigma^2} \end{aligned}$$

$$0 = \frac{\partial l(\theta|x)}{\partial \mu} = \sum_{i=1}^n \frac{\partial}{\partial \mu} \left(-\frac{[\ln(x_i) - \mu]^2}{2\sigma^2} \right) = \sum_{i=1}^n \frac{2[\ln(x_i) - \mu]}{2\sigma^2} = \sum_{i=1}^n \frac{[\ln(x_i) - \mu]}{\sigma^2} =$$
$$\frac{1}{\sigma^2} \sum_{i=1}^n \ln(x_i) - \frac{n\mu}{\sigma^2}$$

$$n\mu = \sum_{i=1}^n \ln(x_i) \text{ so, } \hat{\mu} = \frac{\sum_{i=1}^n \ln(x_i)}{n}$$

and,

$$\begin{aligned}
0 &= \frac{\partial l(\theta|x)}{\partial \sigma} = \sum_{i=1}^n \frac{\partial}{\partial \sigma} \left(-\ln(\sigma) - \frac{[\ln(x_i) - \mu]^2}{2\sigma^2} \right) = \sum_{i=1}^n -\frac{1}{\sigma} - \frac{(-2)[\ln(x_i) - \mu]^2}{2\sigma^3} = \\
&\sum_{i=1}^n \frac{[\ln(x_i) - \mu]^2}{\sigma^3} - \frac{n}{\sigma} \\
\frac{n}{\sigma} &= \frac{1}{\sigma^3} \sum_{i=1}^n [\ln(x_i) - \mu]^2, \text{ so } n\sigma^2 = \sum_{i=1}^n [\ln(x_i) - \mu]^2 \\
\text{at the end, } \hat{\sigma}^2 &= \frac{\sum_{i=1}^n [\ln(x_i) - \hat{\mu}]^2}{n}
\end{aligned}$$

Appendix F

Simple Credibility Model

Given $\Theta = \theta$, random variables X_1, X_2, \dots, X_n are independent and identically distributed with

$$\mu(\theta) = \mathbb{E}[X_j | \Theta = \theta]$$

and,

$$\sigma^2(\theta) = \text{Var}[X_j | \Theta = \theta].$$

Θ is a random variable with $\mu_0 = \mathbb{E}[\mu(\Theta)]$, and $\tau^2 = \text{Var}[\mu(\theta)]$.

The aim of credibility estimators is to find an estimator of $\mu(\Theta)$ which is linear in X_1, X_2, \dots, X_n :

$$\widehat{\mu(\Theta)} = \hat{a}_0 + \hat{a}_1 X_1 + \dots + \hat{a}_n X_n,$$

and minimize quadratic loss function:

$$\{\hat{a}_0, \dots, \hat{a}_n\} = \min_{a_0, \dots, a_n} \mathbb{E} [(\mu(\theta) - a_0 - a_1 X_1 - \dots - a_n X_n)^2]$$

The invariance of the distribution of X_1, \dots, X_n under permutations of X_j , gives $\hat{a}_1 = \hat{a}_2 = \dots = \hat{a}_n := b$

Then, by solving the minimization problem for two parameters a_0 and b by setting corresponding partial derivatives with respect to a_0 and b to zero, we obtain:

$$\widehat{\mu(\Theta)} = \omega \bar{X} + (1 - \omega) \mu_0$$

where,

$$\omega = \frac{n}{n + \frac{\sigma^2}{\tau^2}}$$

and,

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i.$$

Appendix G

Jeffreys prior distribution

Jeffreys prior attempts to represent a near-total absence of prior knowledge that is proportional to the square root of the determinant of the Fisher information:

$$\pi(\omega) \propto \sqrt{|I(\omega)|},$$

where $I(\omega) = -\mathbb{E}\left(\frac{\partial^2 \ln \mathcal{L}(X|\omega)}{\partial \omega^2}\right)$.

Jeffreys prior for Poisson(λ) and Lognormal(μ, σ) distributions

Let $N \hookrightarrow \mathcal{P}(\lambda)$, the poisson density function is: $f(k|\lambda) = \mathbb{P}(N = k) = \frac{e^{-\lambda} \lambda^k}{k!}$ with,

$$\frac{\partial^2 \ln f(k|\lambda)}{\partial \lambda^2} = -\frac{k}{\lambda^2}$$

and consequently, $\pi(\lambda) \propto \frac{\sqrt{\lambda}}{\lambda}$.

Let $X \hookrightarrow \mathcal{LN}(\mu, \sigma^2)$, with $f_X(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(\ln x - \mu)^2}{2\sigma^2}\right\}$.

Hence, by letting $\omega = (\mu, \sigma)$ and calculating the corresponding partial derivatives to $\ln f_X(x)$ we obtain:

$$I(\omega) = \begin{bmatrix} \sigma^{-2} & 0 \\ 0 & 1/2 \sigma^4 \end{bmatrix}$$

As a consequence, $\pi(\mu) \propto \frac{1}{\sigma^2} \propto 1$ and $\pi(\omega) = \frac{1}{\sqrt{2\sigma^6}} \propto \frac{1}{\sigma^3}$

Appendix H

MCMC Metropolis-Hastings algorithm

H.1 Applying MCMC with Metropolis-Hastings algorithm for λ

- 1- Initialize $\lambda_0 = \frac{\lambda_{ID} + \lambda_{SA}}{2}$
- 2- Update from λ_i to λ_{i+1} ($i = 1, \dots, n$) by
 - Generating $\lambda \leftrightarrow U(\lambda_{SA}, \lambda_{ID})$
 - Define $\zeta = \min\left(\frac{f(\lambda|n_{SA}, n_{ID})}{f(\lambda_i|n_{SA}, n_{ID})}, 1\right)$
 - Generate $Rnd \leftrightarrow U(0, 1)$
 - If $Rnd \leq \zeta$, $\lambda_{i+1} = \lambda$, else $\lambda_{i+1} = \lambda_i$
- 3- Remove the first 3000 iterations, so that the chain is stationary (burn-in effect).

H.2 Applying MCMC with Metropolis-Hastings algorithm for μ

- 1- Initialize $\mu_0 = \mu_{ID}$
- 2- Update from μ_i to μ_{i+1} ($i = 1, \dots, n$) by
 - Generating $\mu \leftrightarrow U(0, 12)$

- Define $\zeta = \min \left(\frac{f(\mu|x, y)}{f(\mu_i|x, y)}, 1 \right)$
 - Generate $Rnd \hookrightarrow U(0, 1)$
 - If $Rnd \leq \zeta$, $\mu_{i+1} = \mu$, else $\mu_{i+1} = \mu_i$
- 3- Remove the first 3000 iterations, so that the chain is stationary (burn-in effect).

H.3 Applying MCMC with Metropolis-Hastings algorithm for $\omega = (\mu, \sigma)$

- 1- Initialize $\mu_0 = \mu_{ID}$ and $\sigma_0 = \sigma_{ID}$
- 2- Update from μ_i to μ_{i+1} and σ_i to σ_{i+1} , ($i = 1, \dots, n$) by
- Generating $\mu \hookrightarrow U(0, 12)$ and $\sigma \hookrightarrow U(0, 7)$
 - Define $\zeta = \min \left(\frac{f(\mu, \sigma|x, y)}{f(\mu_i, \sigma_i|x, y)}, 1 \right)$
 - Generate $Rnd \hookrightarrow U(0, 1)$
 - If $Rnd \leq \zeta$, $\mu_{i+1} = \mu$ and $\sigma_{i+1} = \sigma$ else $\mu_{i+1} = \mu_i$ and $\sigma_{i+1} = \sigma_i$
- 3- Remove the first 3000 iterations from both distributions, so that the chains is stationary (burn-in effect).

For more on Markov Chain Monte Carlo and Metropolis-Hastings algorithm see Robert[2007].

Appendix I

The Inverse of Fisher Information Matrix

The calculation of the Inverse Fisher Information Matrix: $\Sigma = \left(-\mathbb{E} \left[\frac{\partial^2 \ln \mathcal{L}_\omega}{\partial \omega \partial \omega'} \right] \right)_{\omega=\omega_0}^{-1}$, was done by the use of the Log-Likelihood function:

$$\ln(\mathcal{L}) = \ln \left(\frac{1}{\sigma_1 \sqrt{2\pi}} \right)^n - \frac{1}{2\sigma_1^2} \sum_{i=1}^n (r_i - r_{i-1}\beta - \theta(1 - \beta))^2$$

Now by deriving the Log-Likelihood function we obtain the following:

$$\begin{aligned} \frac{\partial^2}{\partial \beta^2} \ln(\mathcal{L}) &= -1/2 \frac{2n\theta^2 + \sum_{i=1}^n 2r_{i-1}^2 - 4r_{i-1}\theta}{\sigma_1^2} \\ \frac{\partial^2}{\partial \theta^2} \ln(\mathcal{L}) &= -1/2 \frac{2n - 4n\beta + 2n\beta^2}{\sigma_1^2} \\ \frac{\partial^2}{\partial \sigma_1^2} \ln(\mathcal{L}) &= \frac{n}{\sigma_1^2} - 3 \frac{n\theta^2 - 2n\theta^2\beta + n\theta^2\beta^2 + \sum_{i=1}^n r_i^2 - 2r_i r_{i-1}\beta - 2r_i\theta + 2r_i\beta\theta + r_{i-1}^2\beta^2 + 2r_{i-1}\beta\theta - 2r_{i-1}\beta^2\theta}{\sigma_1^4} \\ \frac{\partial^2}{\partial \theta \partial \beta} \ln(\mathcal{L}) &= -1/2 \frac{-4n\theta + 4n\theta\beta + \sum_{i=1}^n 2r_i + 2r_{i-1} - 4r_{i-1}\beta}{\sigma_1^2} \\ \frac{\partial^2}{\partial \sigma_1 \partial \beta} \ln(\mathcal{L}) &= \frac{-2n\theta^2 + 2n\theta^2\beta + \sum_{i=1}^n -2r_i r_{i-1} + 2r_i\theta + 2r_{i-1}^2\beta + 2r_{i-1}\theta - 4r_{i-1}\beta\theta}{\sigma_1^3} \\ \frac{\partial^2}{\partial \sigma_1 \partial \theta} \ln(\mathcal{L}) &= \frac{2n\theta - 4n\theta\beta + 2n\theta\beta^2 + \sum_{i=1}^n -2r_i + 2r_i\beta + 2r_{i-1}\beta - 2r_{i-1}\beta^2}{\sigma_1^3} \\ \frac{\partial^2}{\partial \beta \partial \theta} \ln(\mathcal{L}) &= -1/2 \frac{-4n\theta + 4n\theta\beta + \sum_{i=1}^n 2r_i + 2r_{i-1} - 4r_{i-1}\beta}{\sigma_1^2} \end{aligned}$$

$$\frac{\partial^2}{\partial\beta\partial\sigma_1}\ln(\mathcal{L}) = \frac{-2n\theta^2 + 2n\theta^2\beta + \sum_{i=1}^n -2r_i r_{i-1} + 2r_i\theta + 2r_{i-1}^2\beta + 2r_{i-1}\theta - 4r_{i-1}\beta\theta}{\sigma_1^3}$$

$$\frac{\partial^2}{\partial\theta\partial\sigma_1}\ln(\mathcal{L}) = \frac{2n\theta - 4n\theta\beta + 2n\theta\beta^2 + \sum_{i=1}^n -2r_i + 2r_i\beta + 2r_{i-1}\beta - 2r_{i-1}\beta^2}{\sigma_1^3}$$

We can easily check that the matrix is symmetric, since:

$$\frac{\partial^2}{\partial\theta\partial\beta}\ln(\mathcal{L}) = \frac{\partial^2}{\partial\beta\partial\theta}\ln(\mathcal{L}), \quad \frac{\partial^2}{\partial\sigma_1\partial\beta}\ln(\mathcal{L}) = \frac{\partial^2}{\partial\beta\partial\sigma_1}\ln(\mathcal{L}) \quad \text{and} \quad \frac{\partial^2}{\partial\sigma_1\partial\theta}\ln(\mathcal{L}) = \frac{\partial^2}{\partial\theta\partial\sigma_1}\ln(\mathcal{L}).$$

Appendix J

The Inverse of Fisher Information Matrix for Nelson-Siegel Maximum Likelihood Estimation Technique

The calculation of the Inverse Fisher Information Matrix: $\Sigma = \left(-\mathbb{E} \left[\frac{\partial^2 \ln \mathcal{L}_\omega}{\partial \omega \partial \omega'} \right] \right)_{\omega=\omega_0}^{-1}$, was done by the use of the Log-Likelihood function:

$$\ln(\mathcal{L}) = \frac{-n}{2} \ln(2\pi\sigma^2) - \frac{1}{2} \sum_{i=1}^n \left(\frac{R_{t,i} - \hat{R}_{t,i}}{\sigma} \right)^2$$

Now by deriving the Log-Likelihood function we obtain the following:

$$\frac{\partial^2}{\partial \beta_0^2} \ln(\mathcal{L}) = -\frac{n}{\sigma^2}$$

$$\frac{\partial^2}{\partial \beta_1^2} \ln(\mathcal{L}) = -1/2 \sum_{i=1}^n 2 \frac{1}{\sigma^2 m_i^2 \tau^2} - 4 \frac{e^{-m_i \tau}}{\sigma^2 m_i^2 \tau^2} + 2 \frac{(e^{-m_i \tau})^2}{\sigma^2 m_i^2 \tau^2}$$

$$\frac{\partial^2}{\partial \beta_2^2} \ln(\mathcal{L}) = -1/2 \sum_{i=1}^n 2 \frac{(-1 + e^{-m_i \tau} m_i \tau + e^{-m_i \tau})^2}{\sigma^2 m_i^2 \tau^2}$$

$$\frac{\partial^2}{\partial \tau^2} \ln(\mathcal{L}) = -1/2 \sum_{i=1}^n 2 r_i \beta_2 m_i^3 e^{-m_i \tau} \tau^3 - \beta_0 \beta_2 m_i^3 e^{-m_i \tau} \tau^3 - 2 \beta_0 \beta_1 e^{-m_i \tau} m_i^2 \tau^2 + 2 r_i \beta_1 e^{-m_i \tau} m_i^2 \tau^2 + \dots$$

$$\frac{\partial^2}{\partial \beta_0 \partial \beta_1} \ln(\mathcal{L}) = -1/2 \sum_{i=1}^n -2 \frac{e^{-m_i \tau}}{\sigma^2 m_i \tau} + 2 \frac{1}{\sigma^2 m_i \tau}$$

$$\frac{\partial^2}{\partial \beta_0 \partial \beta_2} \ln(\mathcal{L}) = -1/2 \sum_{i=1}^n -2 \frac{e^{-m_i \tau}}{\sigma^2} - 2 \frac{e^{-m_i \tau}}{\sigma^2 m_i \tau} + 2 \frac{1}{\sigma^2 m_i \tau}$$

$$\frac{\partial^2}{\partial\beta_0\partial\tau}\ln(\mathcal{L}) = -1/2 \sum_{i=1}^n 2 \frac{m_i\beta_2 e^{-m_i\tau}}{\sigma^2} + 2 \frac{\beta_1 e^{-m_i\tau}}{\sigma^2 m_i \tau^2} + 2 \frac{\beta_1 e^{-m_i\tau}}{\sigma^2 \tau} + 2 \frac{\beta_2 e^{-m_i\tau}}{\sigma^2 m_i \tau^2} + 2 \frac{\beta_2 e^{-m_i\tau}}{\sigma^2 \tau} - 2 \frac{\beta_1}{\sigma^2 m_i \tau^2} - 2 \frac{\beta_2}{\sigma^2 m_i \tau^2}$$

$$\begin{aligned} \frac{\partial^2}{\partial\beta_1\partial\tau}\ln(\mathcal{L}) = & -1/2 \sum_{i=1}^n -4 \frac{\beta_1}{\sigma^2 m_i^2 \tau^3} + 2 \frac{\beta_0 e^{-m_i\tau}}{\sigma^2 m_i \tau^2} + 2 \frac{\beta_0 e^{-m_i\tau}}{\sigma^2 \tau} - 2 \frac{r_i e^{-m_i\tau}}{\sigma^2 m_i \tau^2} - 2 \frac{r_i e^{-m_i\tau}}{\sigma^2 \tau} + 8 \frac{\beta_2 e^{-m_i\tau}}{\sigma^2 m_i^2 \tau^3} + \\ & 6 \frac{\beta_2 e^{-m_i\tau}}{\sigma^2 m_i \tau^2} + 2 \frac{\beta_2 e^{-m_i\tau}}{\sigma^2 \tau} - 4 \frac{(e^{-m_i\tau})^2 \beta_2}{\sigma^2 m_i^2 \tau^3} - 6 \frac{(e^{-m_i\tau})^2 \beta_2}{\sigma^2 m_i \tau^2} - 4 \frac{(e^{-m_i\tau})^2 \beta_2}{\sigma^2 \tau} + 2 \frac{r_i}{\sigma^2 m_i \tau^2} - 2 \frac{\beta_0}{\sigma^2 m_i \tau^2} - 4 \frac{\beta_2}{\sigma^2 m_i^2 \tau^3} + \\ & 8 \frac{\beta_1 e^{-m_i\tau}}{\sigma^2 m_i^2 \tau^3} + 4 \frac{\beta_1 e^{-m_i\tau}}{\sigma^2 m_i \tau^2} - 4 \frac{\beta_1 (e^{-m_i\tau})^2}{\sigma^2 m_i^2 \tau^3} - 4 \frac{\beta_1 (e^{-m_i\tau})^2}{\sigma^2 m_i \tau^2} \end{aligned}$$

$$\begin{aligned} \frac{\partial^2}{\partial\beta_2\partial\tau}\ln(\mathcal{L}) = & -1/2 \sum_{i=1}^n 8 \frac{\beta_2 e^{-m_i\tau}}{\sigma^2 m_i \tau^2} - 8 \frac{\beta_2 (e^{-m_i\tau})^2}{\sigma^2 m_i \tau^2} + 6 \frac{\beta_1 e^{-m_i\tau}}{\sigma^2 m_i \tau^2} - 6 \frac{\beta_1 (e^{-m_i\tau})^2}{\sigma^2 m_i \tau^2} - 2 \frac{r_i e^{-m_i\tau}}{\sigma^2 \tau} - \\ & 4 \frac{\beta_1 (e^{-m_i\tau})^2}{\sigma^2 m_i^2 \tau^3} + 8 \frac{\beta_2 e^{-m_i\tau}}{\sigma^2 m_i^2 \tau^3} - 4 \frac{\beta_2 (e^{-m_i\tau})^2}{\sigma^2 m_i^2 \tau^3} - 2 \frac{r_i e^{-m_i\tau}}{\sigma^2 m_i \tau^2} + 2 \frac{\beta_0 e^{-m_i\tau}}{\sigma^2 m_i \tau^2} + 8 \frac{\beta_1 e^{-m_i\tau}}{\sigma^2 m_i^2 \tau^3} + 2 \frac{\beta_0 m_i e^{-m_i\tau}}{\sigma^2} - \\ & 4 \frac{\beta_2 (e^{-m_i\tau})^2 m_i}{\sigma^2} + 2 \frac{r_i}{\sigma^2 m_i \tau^2} - 2 \frac{\beta_0}{\sigma^2 m_i \tau^2} - 4 \frac{\beta_1}{\sigma^2 \tau^3 m_i^2} + 4 \frac{\beta_2 e^{-m_i\tau}}{\sigma^2 \tau} - 8 \frac{\beta_2 (e^{-m_i\tau})^2}{\sigma^2 \tau} + 2 \frac{\beta_0 e^{-m_i\tau}}{\sigma^2 \tau} + 2 \frac{\beta_1 e^{-m_i\tau}}{\sigma^2 \tau} - \\ & 4 \frac{\beta_1 (e^{-m_i\tau})^2}{\sigma^2 \tau} - 4 \frac{\beta_2}{\sigma^2 \tau^3 m_i^2} - 2 \frac{r_i m_i e^{-m_i\tau}}{\sigma^2} \end{aligned}$$

$$\frac{\partial^2}{\partial\beta_1\partial\beta_2}\ln(\mathcal{L}) = -1/2 \sum_{i=1}^n -4 \frac{e^{-m_i\tau}}{\sigma^2 m_i^2 \tau^2} - 2 \frac{e^{-m_i\tau}}{\sigma^2 m_i \tau} + 2 \frac{(e^{-m_i\tau})^2}{\sigma^2 m_i^2 \tau^2} + 2 \frac{(e^{-m_i\tau})^2}{\sigma^2 m_i \tau} + 2 \frac{1}{\sigma^2 m_i^2 \tau^2}$$

We can easily check the symmetry between the derivatives by calculating:

$$\frac{\partial^2}{\partial\beta_1\partial\beta_0}\ln(\mathcal{L}), \frac{\partial^2}{\partial\beta_2\partial\beta_0}\ln(\mathcal{L}), \frac{\partial^2}{\partial\beta_2\partial\beta_1}\ln(\mathcal{L}), \frac{\partial^2}{\partial\tau\partial\beta_0}\ln(\mathcal{L}), \frac{\partial^2}{\partial\tau\partial\beta_1}\ln(\mathcal{L}), \frac{\partial^2}{\partial\tau\partial\beta_2}\ln(\mathcal{L}).$$

With the calculated Inverse Fisher information matrix for January, May and June 2012 respectively:

$$\begin{bmatrix} 0.0002073524868 & -0.0002092749170 & -0.0003758479225 & 0.0001206084079 \\ -0.0002092749170 & 0.0002113310112 & 0.0003791704355 & -0.0001220492405 \\ -0.0003758479225 & 0.0003791704355 & 0.0006828312441 & -0.0002192919973 \\ 0.0001206084079 & -0.0001220492405 & -0.0002192919973 & 0.00007284896082 \end{bmatrix}$$

$$\begin{bmatrix} 0.00000077287583 & -0.00000097142986 & 0.00000060673452 & -0.00002646070 \\ -0.00000097142986 & 0.0000021353495 & -0.0000042267498 & 0.000054060676 \\ 0.00000060673452 & -0.0000042267498 & 0.000019597771 & -0.00015296904 \\ -0.00002646070 & 0.000054060676 & -0.00015296904 & 0.0019702643 \end{bmatrix}$$

$$\begin{bmatrix} 0.00000045258754 & -0.000000563330 & 0.00000019165506 & -0.000011855585 \\ -0.000000563330 & 0.0000011466567 & -0.0000018390743 & 0.000022846680 \\ 0.00000019165506 & -0.0000018390743 & 0.0000084197638 & -0.000053622260 \\ -0.000011855585 & 0.000022846680 & -0.000053622260 & 0.00064884444 \end{bmatrix}$$

List of Tables

1.1	Business lines and the Beta factors	10
1.2	Simple example related to the BIA and SA calculation criteria	11
1.3	Ratio of Operational Risk Economic Capital to Overall Economic Capital and to Minimum Regulatory Capital	13
1.4	Analysis of QIS data: BI Approach (Based on 12% of Minimum Regulatory Capital)	14
1.5	Analysis of QIS data: the Standardized Approach (Based on 12% of Mini- mum Regulatory Capital)	15
1.6	Size Ranking Across Three Measures of "Typical" Beta by Business Lines .	16
1.7	Correlation Matrix for the different risks	24
1.8	QIS5 Factors	25
2.1	The VaR and Mean VaR calculation	36
2.2	Database of Legal loss events (cf. Gamonet [2009])	52
2.3	First four moments of the sample	52
2.4	Estimation of the Parameters for the Exponential, Lognormal and Weibull distributions	54
2.5	KS formal test result	55
2.6	Highly publicized loss events	56
2.7	The VaR and Mean VaR calculation	57
2.8	The VaR and Expected Shortfall with the use of Extreme Value Theory . .	58
2.9	Internal data and expert opinion (X, ζ) transform the prior risk profile of the whole industry Γ into an individual company specific γ (cf. Lambrigger <i>et al.</i> [2008]).	60
2.10	Sampled Losses from a $\mathcal{LN}(4, 4)$	62
2.11	Parameters assumption	62
2.12	Parameter calculation output	62
3.1	Scenario analysis	66
3.2	Value at Risk and Confidence intervals for all cases treated	76
3.3	Business lines and the Beta factors	78
3.4	Value at Risk and Confidence intervals for all cases treated	78
4.1	Potential Operational loss severity	83
4.2	Potential Operational loss frequency	84
4.3	Key Risk Indicators type	87

4.4	Key Risk Indicators sample	88
4.5	KRIs Risk Level	88
4.6	RCSA according the business lines and event types	92
4.7	Agencies equivalent ratings	100
4.8	Statistics on operational losses according to location of event. Locations have been classified in three categories: United States (USA), Europe and Other countries (Others)	106
4.9	Statistics of operational losses according to business lines in which the losses occurred. We have selected the classification proposed by Basel II, including 8 lines of business	106
4.10	Statistics of operational losses according to event types in which the losses occurred. We have selected the classification proposed by Basel II, including 7 event types	106
4.11	Results obtained from estimating the linear regression coefficients with the ordinary least squares method	108
4.12	Robustness test for different cases of the variables treated	109
4.13	Examples of Normalized external losses extracted from SAS Oprisk Global Data	110
5.1	Vasicek parameter model	124
5.2	Underestimation of the SCR	124
6.1	Comparing the three estimation methods	129
6.2	Nelson Siegel's Annuity Output	132
7.1	12 months and 24 months smoothing calculations	136
7.2	Balance Sheet asset, Liability and NAV calculation	139
7.3	NAV Calculation by restructuring the Yield Curve	141
7.4	Balance Sheet exposure with a shock	144
B.1	Basel II 8 Business Lines \times 7 Event Types	159
C.1	Detailed loss event classification 1	161
C.2	Detailed loss event classification 2	162

List of Figures

1.1	Illustration of the Loss Distribution Approach method (LDA) (cf. Maurer [2007])	18
1.2	Operational Risk Capital-at-Risk (CaR) (cf. Maurer [2007])	20
1.3	Overview of the sbAMA	23
1.4	Solvency Capital Requirement (SCR)	24
2.1	Risk Matrix	27
2.2	Loss severity of a Beta distribution	29
2.3	Loss severity of a Lognormal distribution	30
2.4	Loss Frequency Distribution	33
2.5	Annual Aggregate Loss Distribution	36
2.6	Fraction of missing data A and observed data B (cf. Chernobai <i>et al.</i> [2006])	37
2.7	Unconditional and conditional severity densities (cf. Chernobai <i>et al.</i> [2006])	38
2.8	Densities for the Fréchet, Weibull and Gumbel functions	48
2.9	QQ-plots for fitted distributions	55
2.10	Frequency and severity distributions of Poisson and Lognormal \mathcal{LN}	57
2.11	Aggregate Loss Distribution for a particular Business Line \times Event Type. .	58
2.12	Comparison of the MLE estimator $\hat{\mu}_k^{MLE}$, to the Bayesian estimator $\hat{\mu}_k$ and the no expert opinion Bayes estimator given by Shevchenko and Wüthrich $\hat{\mu}_k^{SW}$	63
3.1	Sensitivity for a_0	71
3.2	Sensitivity for b_0	71
3.3	Sensitivity for μ_0	71
3.4	Sensitivity for σ_0	71
3.5	MCMC for the parameter λ	73
3.6	MCMC for the parameter μ	74
3.7	MCMC for the parameters μ and σ	75
3.8	Different VaR Calculation for all MCMC Cases with unknown parameters .	77
3.9	Different VaR Calculation for Internal Data, MCMC General Case and Scenario Analysis	77
3.10	Different VaR Calculation for all conjugate prior cases	77
4.1	Operational Risk Framework	82
4.2	Scenario Analysis Procedure	91
4.3	Scenario Matrix	93

5.1 Empirical Cumulative Distribution Function of ϵ 125

6.1 Adjusted \mathcal{R}^2 130

6.2 Empirical Cumulative Distribution Functions for ϵ , for several dates 132

7.1 Economic Valuation 137

7.2 Monthly Spot Rates 137

7.3 NAV Calculation 140

7.4 NAV Calculated by restructuring our Yield Curve 142

7.5 Compared NAV Curves 142

7.6 NAV Calculated by restructuring our Yield Curve 143

7.7 Age Sensitivity 144

7.8 Yearly payments given a shock of 30% 145

7.9 Yearly variable payments given a shock of 30% 146