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The Pricing of Hedging Longevity Risk with the Help of Annuity Securitizations: An Application to the German Market

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Abstract

Prolongation of life expectancy implies a severe risk for annuity providers. Insurance companies can use securitization transactions to address this longevity risk in their portfolios. Securitization can serve as a substitute for classic reinsurance since it also transfers risk to third parties. We develop a model to hedge annuity portfolios against increases in life expectancy. By doing so, we forecast future mortality rates with the Lee-Carter-model and use the Wang-transformation to incorporate insurance risk. Based on the percentile tranching method, where individual tranches are aligned to Standard & Poor's ratings, we price an inverse survivor bond. This bond offers fix coupon payments to investors while the principal payments are at risk and depend on the survival rate within the underlying portfolio. Finally, we apply this securitization structure to calculate the securitization prices for a sample portfolio from a large life insurance company.

Key words Life Securitization · Longevity Risk · Insurance Pricing and Hedging · Inverse Survivor Bond · Percentile Tranching
JEL Classification G12 · G17 · G22 · G23

1 Introduction

Across the globe, and in the industrial nations in particular, people have seen an unprecedented increase in their life expectancy over the last decades (see, for example, MacDonald et al., 1998). The benefits of this apply to the individual, but the dangers apply to annuity providers. Insurance companies often possess no effective tools to address the longevity risk inherent in their annuity portfolio. However, they can use the financial markets to mitigate the negative implications from an increase in life expectancy to their risk position by securitizing parts of their portfolio. The synthetic securitization acts as a hedge for the insurer's portfolio: the securitization deal transfers the risks of the portfolio to third parties and serves as a substitute for reinsurance.

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In contrast to non-life segments, where securitization transactions such as CAT bonds are an established instrument now and account for high volumes, the life securitization market is still in its infancy (see, e.g., Deutsche Bank, 2010). The first mortality-linked securitization transaction was carried out by Swiss Re in 2003. The issue of the Vita Capital bond for a three-year time period reduced Swiss Re's exposure resulting from shifts in mortality. This bond had principal payments which were dependent on the development of a predefined mortality index (Blake et al., 2006). Another attempt at a longevity bond occurred in 2004. The European Investment Bank tried to issue, with the help of BNP Paribas, a bond with coupon payment depending on the survival of English and Welsh males aged 65 in the year 2002. The duration of the contract was intended to be 25 years and the total volume to be £ 540 mn. However, the transaction did not attract enough investors and was abandoned in 2005 (Chen and Cummins, 2010). After these initial attempts, the life securitization market gained momentum with increasing volumes and transactions until 2007. However, with the beginning of the financial crisis, the issuing of life securitizations dropped significantly and is still in a recovery phase today (for a detailed review, see Section 2).

The literature offers a variety of models for hedging longevity risk. Among them, Wang et al. (2011) apply the concept of reverse mortgages to hedge longevity risk for life insurance companies. Wang et al. (2010) use a model to determine the optimal annuity product mix to receive a natural hedge against longevity risk, while Luciano et al. (2012) make use of Delta-Gamma hedging. In the paper by Gatzert and Wesker (2012), the authors consider a "natural" hedge between life insurance and annuities in order to reduce net portfolio exposure to systemic mortality risk. Another approach is the annuity securitization: Kim and Choi (2011) present a method for securitizing longevity risk with the help of percentile tranching using a coupon-at-risk calculation (see also, for instance, Wills and Sherris, 2010, or Bae et al., 2009). The authors apply the concept of an inverse survivor bond to a fictional portfolio of Australian annuity contracts. Their focus lies on the yield that investors can achieve with an investment in such a bond. Our approach is closest to that of Kim and Choi (2011). We make use of a percentile tranching method directly linking chosen percentiles to Standard & Poor's (S&P) rating classes for insurance-linked securities. By doing so, investors receive a clear picture of the quality of the securitization tranche they are investing in. In addition, the issuer has the possibility to approach specific investor types in terms of risk-attitude directly with its securitization offer. Typically, the coupon payments are put at risk within an inverse survivor bond. The insurer receives on the issuing date cash flow from investors which can be considered a loan. This loan is then repaid to the creditors with an annual principal amount over the entire maturity of the bond. The principal payments are of equal size for each period; however, their full repayment remains subject to the actual survival rates within the underlying portfolio. In addition, the investors receive at each time period the respective coupon rate, which is tranche specific. Within this securitization structure, the insurer receives the loan from the investors and can use this money to generate capital returns. At the end of the transaction, the full loan has been paid back to the investors and the coupon payments depend on the portfolio's survival rates. In the current low interest capital market environment, it is doubtful if this structure is still advantageous for the insurer since the ability to generate sufficient capital returns with the borrowed money to cover potential additional payments from longevity increases might be limited. Thus, and in order to ensure that enough capital is available, our model is based on the payment of fixed coupon rates and variable repayments of the principal (principal-at-risk approach).

In this paper, we extend on methods insurer's can use to hedge their annuity portfolio against longevity risk with the help of annuity securitization. To do so, we take the perspective of the issuing insurance company and calculate the costs of hedging in a four step process. First, we calculate future mortality rates using the classic Lee-Carter-model (Lee and Carter, 1992). In a second step, we adapt the forecasted mortality rates with the help of the transformation by Wang (2000) in order to incorporate the price of risk in the insurance contract. We then apply the transformed death rates to the annuitants of our sample portfolio. Third, we slice the forecasted annuity portfolio into different tranches with the help of the percentile tranching method (analogous to Kim and Choi, 2011). Thereby, we use attachment and detachment points for the individual tranches corresponding to S&P ratings for insurance-linked securities. In a last step, we price the different tranches of our annuity securitization with the help of classical bond pricing. The principal payments that investors receive are a random variable and depend on the survival distribution of the underlying portfolio. If the number of actual survivors is higher than expected, the amount of principal payment is reduced according to the determined tranche levels. To illustrate the implication of this bond structure, we finally conduct several sensitivity tests before we apply our pricing model to the retail sample annuity portfolio from a leading German life insurer. The contribution to the academic literature is threefold. On the theoretical side, building on the work of Kim and Choi (2011), we adapt their pricing model to the current market situation. Putting the principal at risk instead of the coupon payments, the insurer is supplied with sufficient capital in order to cover additional costs due to longevity. On the empirical side, we specify the method for the German market. Inserting specific country data into the model, we analyze price sensitivities of the presented securitization model. Finally, in a case study, we apply the procedure to the annuity portfolio of a large German life insurer and calculate the price of hedging longevity risk.

The remainder of this paper is organized as follows. In Section 2, the theoretical background of securitization in general, and life insurance securitization in particular, is discussed as well as the related literature review. The concept of inverse survivor bonds is explained in detail. Section 3 presents the mathematical calculus of an annuity securitization, from the definition of future mortality rates with the help of the Lee-Carter-model up to the pricing of single securitization tranches. Section 4 presents numerical outcomes and shows the dynamics and sensitivities of our model. Afterwards, Section 5 applies the model to a real-world annuity portfolio from a large life insurance company. Section 6 summarizes the presented results and concludes.

2 Literature review and securitization market overview

With the recent financial crisis in mind – and its being caused by unsound structuring of mortgage loans – securitization is still of major interest for financial service providers in general and insurance companies in particular. In contrast to the banking sector where securitization, via asset-backed securities or collateralized debt obligations for instance, hit huge levels before the financial crisis, life insurance companies are still very cautious about these transactions despite potential benefits (see, for example, De Mey, 2007, or Beltratti and Corvino, 2008, for a review). In general, securitization can be defined as the "repackaging and trading of cash flows that traditionally would have been held on-balance-sheet" (Cummins and Weiss, 2009, p. 515). The purpose of this trading lies in the diversification of risk.

There are manifold explanations why securitization transactions are beneficial for life insurers. According to Cowley and Cummins (2005), a reduction of costs of capital as well as an increase in the return on equity are the main advantages. Through securitization, unlocking embedded profits in the balance sheet becomes possible for the issuer. In addition, this financial instrument can serve as an alternative possibility to financing. Furthermore, the transparency of many on-balance-sheet assets and liabilities – traditionally characterized by a certain degree of illiquidity, complexity, and informational opacity – is improved by securitization (Cowley and Cummins, 2005, p. 194). Other players in the financial market can also benefit from these transactions. Life securitization introduces a new asset class with almost no correlation to other investments (see also Lin and Cox, 2008). The true driver of life portfolios lies in the life expectancy of the policyholders – and this is not affected by the development of yield curves or stock market returns. Consequently, institutional investors should be able to reduce their overall portfolio risk by adding life securitization to their investments (see, for example, Litzenberger et al., 1996, and Deng et al., 2012). From a regulatory perspective, it is expected that securitization will be treated under Solvency II analogous to reinsurance when it comes to the determination of standard capital requirements (Cummins and Trainar, 2009, p. 489). At the same time, it enables third parties to invest in insurance products without possessing an insurance license by acting as investors to the securitization.

Historic life securitization transactions

Figure 1 shows an aggregation of the issued life securitizations from 1999 to 2011. The deals are clustered into three different categories: regulation XXX/aXXX securitization deals,[1] mortality securitizations and embedded value securitizations. Historically, the maximum total amount of life securitization was achieved in 2007 with a volume of \$ 8.1 bn. The amount of transaction volumes then drops significantly with the start of the financial crisis in 2008 to \$ 0.6 bn. In comparison with the assets on the balance sheets of life insurers, the volume of these transactions is totally negligible even in years with extensive usage of this instrument. On average, the securitized deal had a size of \$ 417 mn while the median of the transaction volumes is \$ 300 mn. The top three issuing companies in terms of total volume are Aegon (\$ 3.7 bn), Scottish Re (\$ 3.6 bn), and Genworth (\$ 3.5 bn). In total, insurance companies undertook 71 life securitization transactions from 1999 to 2011, of which 31 were embedded value, 31 regulation XXX/aXXX transactions and 9 mortality transactions. In that period, primary insurers securitized life products in 48 deals with a value of \$ 21.6 bn and reinsurance companies transferred a volume of \$ 8.4 bn with the help of 23 transactions. Further information on observed life securitization deals can be found in, e.g., Lorson (2012).

An explanation for the relatively low transaction volumes in comparison to the balance sheet assets of life insurers can be found in D'Arcy and France (1992). The concerns for the usage of insurance futures based on catastrophy loss indices stated by the author in 1992 still hold true today for the securitization

[1] Issuing of debt securities on insurer's capital reserve requirements for term life products. More details on the different categories can be found in, e.g., Kampa (2010) or Cowley and Cummins (2005).

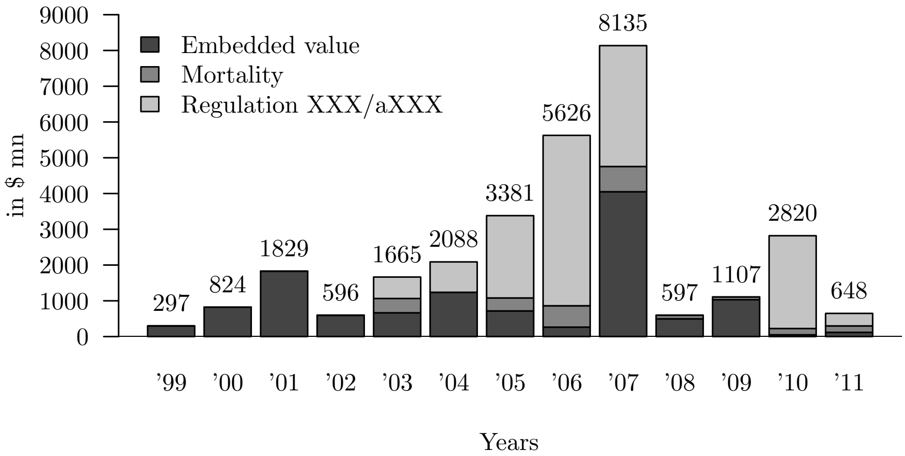


Figure 1: Historic life securitization transactions from 1999 to 2011. Figures are in million US dollars and categorized by regulation XXX/aXXX securitization, mortality securitizations, and embedded value securitizations. The data on the issues is compiled from Swiss Re (2006), Artemis (`http://www.artemis.bm`), Trading Risk (`http://www.trading-risk.com`), and Factiva press research (`www.dowjones.com/factiva`). See also Lorson (2012).

of life insurance portfolios: concerns about the counterparty risk, uncertainties about the treatment of contracts by the regulator, and lack of insurer know-how or expertise. Before we can argue about the three concerns, an important difference between “normal” securitizations and the securitization of life insurance portfolios has to be pointed out. “Normal” securitizations often represent asset securitization such as mortgages or credit card receivables (see also Cowley and Cummins, 2005). The borrower receives an initial amount of money or funding and then begins to make payments (both interest and principal) over time to repay the loan. Consequently, the originator faces the credit risk of the borrower but no risk exposure exists vice versa. By any securitization deal, the economic situation and the risk of the borrower is not affected. However, when it comes to life insurance securitization, the situation is different: life securitization is a securitization of liabilities and implies a different logic. In this case, one counterparty of the original contract (the policyholder) makes payments over a long period of time and receives a payment from the other counterparty (the insurer) in the future at the end of the contract. The policyholder is consequently facing the credit risk of the counterparty and its economic position is severely affected by the counterparty, especially by the originator’s default risk (see Cowley and Cummins, 2005). Insurers who are willing to pursue a life portfolio securitization have to select the buyer of their portfolio very carefully in order to avoid an increase of the counterparty risk for the policyholder. Even if the selection is done carefully and the economic situation of the policyholder will not be deteriorated, it is doubtful whether the regulator will allow such a transaction and how the regulatory environment looks like (see for example Cummins and Weiss, 2009).

Classification of securitization deals

When it comes to classifying securitization deals, Cummins and Weiss (2009) divide the securitization primarily into two different categories: the asset-backed securities such as securities backed by corporate bonds or mortgages, and the non-asset-backed products with examples like futures and options. The first are usually backed by the underlying asset that is securitized, the latter are normally guaranteed by the counterparty of the transaction and/or by an exchange. Both types of securitizations have in common that they can be traded in an organized exchange environment as well as over-the-counter. Through securitization, investors that do not possess an insurance license have access to insurance cash flows (Cox and Schwebach, 1992). This further facilitates the diversification of risk in the entire financial market.

An insurer deciding to undergo a life insurance securitization has, at least theoretically, two possibilities to proceed. The first one is a true sale of the assets which should be securitized, and the second one is a synthetic securitization. In the case of a "true" securitization the insurer would securitize its portfolio (or parts of it) by selling the contracts to investors in different tranches. In contrast to the banking sector where true sale transactions occur daily, this type of securitization does not take place in the insurance industry. The main reason for this is legal issues. As mentioned before, in life insurance the economic position of the policyholder is severely affected by the counterparty since the fulfillment of the service promise lies in the future. It is very doubtful if a regulatory authority would allow such transactions in which the counterparty obligation fades from a supervised insurance company to third party investors. In addition, such a deal would result in a giant time horizon of commitment for the investors. Contract durations of more than 60 years are not uncommon (for instance in the case of classic pension products). Only few investors might be interested in engaging in such long transactions. Furthermore, a potential investor would face several operational problems in a true sale life securitization: he must provide the necessary systems for the premium collection, for instance, or take care of the asset management. Consequently, to the best of our knowledge, there has been no true sale life securitization so far. The second option, "synthetic" securitization (or "synthetic" sale) is the common method for annuity securitization. Within this approach, accounting for the contracts and carrying of risks remains with the insurance company. In most cases, a special purpose vehicle is established to serve as an intermediary for cash flows between the issuer and the investors. The transaction is simply a securitization of cash flows.

Models for life securitization and pricing

Several models for life securitization approaches can be found in the literature. Blake and Burrows (2001) proposes the issuing of survivor bonds whereby the coupon payment which investors receive depend on the mortality rates of a specified population and are thus a random variable (see also Dowd, 2003, and Blake, 2003). This construct of survivor bonds is also applied by Denuit et al. (2007). Other possibilities to hedge longevity risk are survivor options or swaps and futures, which are discussed, for example, by Cox and Lin (2007). Survivor swaps comprise the exchange of cash flows between two parties with a fixed cash flow in the present and an uncertain future cash flow depending on the mortality of a specified group (see, for example, Dowd et al., 2006). Cox et al. (2006) consider the first mortality securitization by Swiss Re in 2003 and use multivariate exponential tilting to price this transaction. Lin and Cox (2005) develop a model for analyzing mortality-based securities, in particular mortality bonds and swaps, and price these contracts (by applying the one factor Wang transformation). Cox et al. (2010) integrate permanent longevity jump processes as well as a temporary jump process in mortality to the Lee-Carter-model and use the derived forecasted mortality rates to price a longevity option with indifference pricing. Lin and Cox (2008) use a two-factor Wang transformation to develop an asset pricing model for securities based on mortality. Bauer et al. (2010) review existing pricing methods for longevity-linked securities and present a longevity derivative that provides an option-type payoff. Also Dowd et al. (2006) use the idea of survivor swaps to exchange future cash flows between the insurer and third parties based on a reference survivor index. Wills and Sherris (2010) securitize a longevity bond with the help of Australian mortality data. They calculate different tranches by percentage cumulative loss and price these tranches. Biffis and Blake (2010) address in their study the effects of asymmetric information on longevity securitizations and how they should be tranched under partial information. For further references on longevity securitization research, we refer to the review by Blake et al. (2011).

Inverse survivor bonds

An inverse survivor bond is a special case of survivor bonds that insurance companies can use to hedge their annuity portfolio against mortality changes. The structure of these bonds is illustrated in Figure 2.

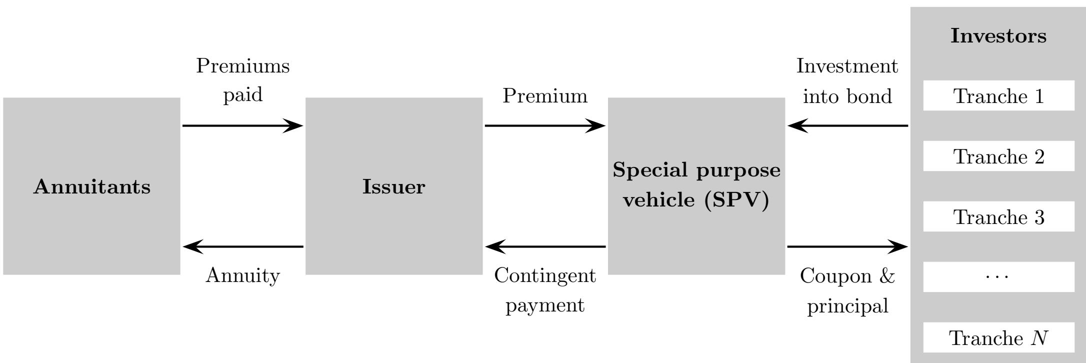


Figure 2: Structure of an Inverse Survivor Bond (adapted from Wills and Sherris, 2010, and Kim and Choi, 2011).

At the center of the bond structure stands a special purpose vehicle (SPV). This legal entity brings together the issuer (the insurance company) and the investors who want to engage in the annuity portfolio. The insurer – while paying out the annuities to its annuitants – transfers the premiums received from the annuity sales to the SPV. Following the investors participation in the bond, the SPV provides contingent payments to the insurance company (securing thus the annuity payments). On the investor side, the SPV issues a survivor bond which pays regular coupon payments. Often, the principal is paid as a lump sum at the maturity of the bond, while the coupon payment is paid annually and is subject to the survival rate of the underlying portfolio, and thus at risk. The investor faces the risk that partial future coupon rates or even all future coupon rates are lost if less annuitants than ex ante expected die within the portfolio. In our consideration, as mentioned before, the principal payment will be at risk whereas the coupon payments are secured.

In order to attract more investors and to offer a tailored risk profile, the survivor bond is sliced into different tranches. Lane and Beckwith (2007) argue that tranching becomes more popular when related to insurance-linked securities. Such bonds usually possess less than ten tranches, which can differ significantly in their risk profile. The investors in the first tranche are provided with the least coverage. They are the first to lose their coupon or principal payments if the number of actual survivors in the portfolio is higher than expected. If the first tranche is exceeded, the loss moves to the second tranche, and so on. Due to the different risk characteristics, each tranche has a different price within the survivor bond. In order to signal to investors the quality of the securitized portfolio, the first loss position (the first tranche) is often kept by the issuer (see, for example, Gale and Hellwig, 1985, or Riddiough, 1997). When maintaining the first loss position, the issuer has an incentive to closely monitor the correct and timely premium collection and to price the portfolio conscientiously.

3 Calculation of the securitization of an annuity portfolio

This section describes the calculus used to synthetically securitize a life annuity portfolio. We adapt the model from Kim and Choi (2011) for an inverse survivor bond with the core difference that we set the principal payment at risk and not coupon payment. Figure 3 illustrates our approach.

Based on historic life tables, Section 3.1 describes the application of the Lee-Carter-model in order to forecast future mortality rates. Afterwards in Section 3.2, we adjust the derived survival probabilities for the uncertainty in the mortality tables as well as the uncertainty of the annuitant's lifetime. Therefore, we use the Wang-transformation (Wang, 2000) to derive the adjusted survival probability distribution for all ages x of annuitants and future years $t = 1, \dots, T$, where T denotes the duration of the securitization contract. In Section 3.3 the different attachment and detachment points for each observed tranche are introduced. We define N tranches with the help of percentiles based on cumulative survival probabilities. Based on these tranche limits, we derive the value of each individual tranche j, $j = 1, \dots, N$, for each time t (until T) and the face value FV of the entire inverse survivor bond. Using this face value we price the individual tranches in Section 3.4 by calculating prices for the coupon payments as well as for the principal payments for all contract years t. We assume that coupons and principals are paid once a year at the end of the period. Finally, we derive the total price of securitization for a given annuitant aged x by summing all tranches and contract years.

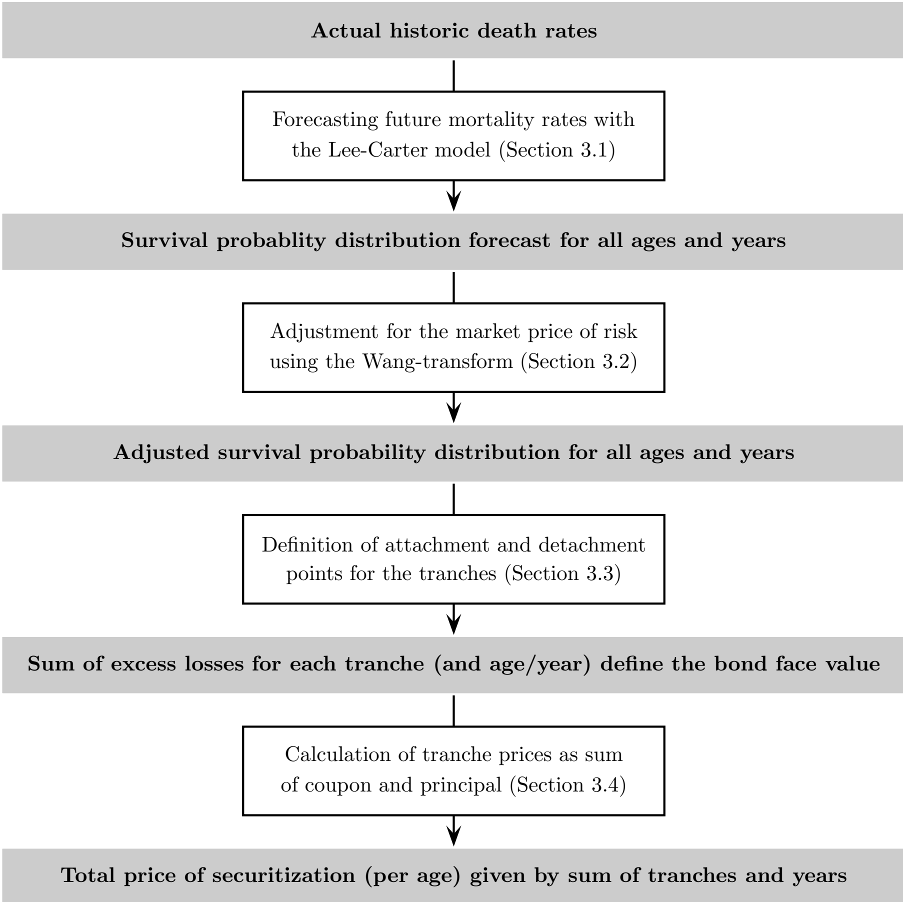


Figure 3: Approach for calculating the price of an annuity securitization.

3.1 Forecasting future mortality rates with the Lee-Carter model

A range of stochastic models for forecasting mortality rates have been developed in the last decade. The earliest model was developed by Lee and Carter (Lee and Carter, 1992) and several researchers built on and extended their work.[2] The Lee-Carter-model is is still the most popular model and widely used in the literature for valuating life portfolios and as a basis for insurance securitization (see, for example, Denuit et al., 2007, or Kim and Choi, 2011).

The Lee-Carter-model is a discrete time series model that uses historic mortality rates to project the future trend of mortality. It is based on the assumption that future mortality will continue to change at the same rates as it did in the past. In the model, the fitted death rates $m(x,t)$ for ages x at times t (see

[2] The most prominent models are those of Renshaw and Haberman (2006), De Jong and Tickle (2006), Delwarde et al. (2007), Czado et al. (2005), or Cairns et al. (2006).

Lee and Carter, 1992, p. 660) follow the log-linear expression:

$$\ln m(x,t) = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t}. \tag{1}$$

The death rates $m(x,t)$ are described by two age-specific constants, α_x and β_x, and a time-varying factor κ_t that represents an index of the level of mortality. The error-term $\epsilon_{x,t}$ captures the age-specific effects that are not contained in the model. The fitted parameter α_x describes the average age over time (independently of the time index). The coefficient β_x is the age-specific component describing how death rates vary to changes in the index κ_t. We consider in our model ages from $x = 0$ (less than one year) up to $x = 100$ years for the fitting. Our analyses include a separate assessment of male and female death rates, as well as of the whole population without gender-differentiation. By doing so, we choose the widely adopted method[3] of Lee and Miller (2001) who adjust the original Lee-Carter-model in three ways. First, concerning the fitting horizon, the latter half of the last century is chosen with the goal of reducing structural shifts. In addition, the mortality index k_t is adjusted to the life expectancy e_0 instead of total deaths. Finally, the "jump-off" error is eliminated through forecasting using observed instead of fitted rates.

Applying the Lee-Carter-model requires a two step process. First, the model parameters in Equation (1) are estimated based on the observed mortality rates, and, second, the projections for the future are performed. Following Lee and Carter (1992), a singular value decomposition is performed on the matrix $\ln m(x,t) - \alpha_x$ with resulting estimates of β_x and κ_t. Subsequently, an iteration process re-estimates the values for κ_t until the modeled number of deaths equals the actual amount of deaths. In order to forecast future mortality rates, Lee and Carter assume that α_x and β_x remain constant over time. To forecast the values of the mortality index κ_t, a standard univariate time series model with a random walk with drifts is used. Under these assumptions and parameterization, Equation (1) evaluates future death rates per age group, which forms the basis for all following calculations. In the following we will refer to the cumulated survival rates for a time-span of t years for individuals aged x at the time of securitization. Thus we transform the obtained mortality rates $m(x,t)$ into survival rates $1 - m(x,t)$, and derive the (simulated) t-year survival probabilities for x years aged individuals (at the time of securitization), which we denote by $\widetilde{p}_{x,t}$. We use the notation $\widetilde{\cdot}$ to denote random variables.

3.2 Adjustment for the market price of risk

Having calculated the future survival probabilities (Section 3.1), we now address the insurance risk inherent in a contract. Following, e.g., Kim and Choi (2011), Denuit et al. (2007), Dowd et al. (2006), and Lin and Cox (2005), we apply the Wang-transformation to our forecasted mortality rates.[4] Wang (2000) has developed a method to price the insurance risk inherent in an insurance contract which combines the classical financial as well as insurance pricing theory. Thus, we will use observed annuity prices to estimate the market price of risk for annuity mortality and then use this distribution to price

[3]See, for example, Hanewald et al., 2011, or Booth and Tickle, 2008.
[4]Chen et al. (2010) prove that for long maturities, the Wang transformation is the preferable method for longevity risk pricing.

bonds (see Lin and Cox, 2005). By doing so, the previously obtained survival rates $\widetilde{p}_{x,t}$ are adapted for the uncertainty in the mortality tables as well as the uncertainty in the lifetime of the annuitant.

If Φ denotes the standard normal cumulative distribution function and Φ^{-1} its inverse, Wang (2000, p. 20) introduces the distortion operator described by

$$g_\lambda(u) = \Phi\left[\Phi^{-1}(u) + \lambda\right], \tag{2}$$

for all u with $0 < u < 1$. The real-valued parameter λ can be interpreted as the market price of risk. Thus a function with given cumulative distribution function, F with values of $-\infty < F < +\infty$ can be transformed into a distorted distribution function F^* with the help of the market price of risk λ by the following equation,

$$F^* = 1 - g_\lambda(1 - F) = \Phi\left[\Phi^{-1}(1 - F) + \lambda\right]. \tag{3}$$

In our framework, we apply the above formula and let F stand for the cumulative distribution function of the future survival probabilities $\widetilde{p}_{x,t}$ for ages x and years t. In the sequel we denote with $\widetilde{p}^{\lambda}_{x,t}$ the so-transformed survival probabilities.

3.3 Tranche definition and excess loss per tranche

The next step involves the definition of individual tranches to be securitized and the calculation of the excess loss for each tranche. We consider N different tranches in our inverse survivor bond. The attachment points $p^{\lambda(j-1)}_{x,t}$ and detachment points $p^{\lambda(j)}_{x,t}$ for tranche j, $j = 1, \ldots, N$, are defined as a percentile of the cumulative survival distribution based on the Wang-transformed values. Here our approach differs from Kim and Choi (2011, p. 15, Fig. 12). These authors define the attachment point of the first tranche $p^{\lambda(0)}_{x,t}$ at the median survival probability or 50%-tile. In order to derive results from financial markets practice, we link the attachment points to the tranches according to the S&P default table for insurance-linked securities.[5] S&P provides cumulative default probabilities for different rating classes and different maturities. In Table 9 (see Appendix 7.2) we report the S&P cumulative default probabilities for insurance-linked securitizations corresponding to the rating classes AAA to B– and maturities ranging from $T = 1$ to 30 years. In our application, the detachment point of the last tranche is defined as the 100%-tile of the survival distribution, i.e. $p^{\lambda(N)}_{x,t} = 1$ (100%-tile). The attachment and detachment points for the remaining tranches can be flexible and defined according to the intended tranche composition of the securitization. The percentile below the attachment point of the first tranche $p^{\lambda(0)}_{x}$ is the part retained by the issuer. It can be considered as a first loss position. In our further applications, we will, e.g., define the attachment $p^{\lambda(0)}_{x,t}$ equal to the percentile of the S&P B+ tranche, which corresponds to an approximately 30% cumulative default probability within 10 years. For further tranches we consider BBB–, A, and AAA in our reference case (see Table 4). The face value FV of the bond can also be readily calculated. It corresponds to the difference between the 100%-tile of the survival distribution and the attachment point of the first tranche ($p^{\lambda(0)}_{x,t}$).

[5] See S&P default tables from 2008, `http://www.standardandpoors.com`.

To calculate the excess loss for each tranche, we assume that the insurer pays 1 (one) currency unit to each annuitant who survives each year. In the following, we suppose the attachment points $p_{x,t}^{\lambda(j-1)}$ and the detachment points $p_{x,t}^{\lambda(j)}$ for each tranche j, $j = 1, \ldots, N$ given. Let l_x represent the initial population of annuitants of age x at the beginning of the securitization ($t = 0$). Thus the actual number of survivors in each group (tranche) is a random variable driven by the numerically simulated distribution of survival rates $\widetilde{p}_{x,t}$, and $\tilde{l}_{x+t}^{\lambda} = l_x \cdot \widetilde{p}_{x,t}^{\lambda}$ describes the actual number of survivors at age $x + t$. Furthermore, and with the yearly annuity payment set to unity, it follows that $l_{x+t}^{(j)} = l_x \cdot p_{x,t}^{\lambda(j)}$ denotes the actual amount of losses linked to the attachment/detachment points (see also Kim and Choi, 2011, pp. 11 and 16). With the above notations, we are able to derive the calculation of the excess of loss. We illustrate our proceeding in Figure 4. Each dot in Figure 4 represents one realization of the simulation run for the life expectancy of an x years aged individual at a given time t after securitization. The dashed lines indicate the attachment and detachment points of the observed tranche j (at time t). If a survival realization lies within the borders of the loss that the considered tranche j has to bear – in line with the definition of the respective attachment and detachment points – the tranche is triggered by the grey shaded loss. Thus the excess loss for the jth tranche for individuals aged x at the time of securitization and t years after securitization, denoted by $\tilde{L}_{x+t}^{(j)}$, is a random variable and can be described by

$$\widetilde{L}_{x+t}^{(j)} = \left[\tilde{l}_{x+t}^{\lambda} - l_{x+t}^{(j-1)}\right]^{+} - \left[\tilde{l}_{x+t}^{\lambda} - l_{x+t}^{(j)}\right]^{+}, \tag{4}$$

where $[\cdot]^{+}$ stands for $\max(0, \cdot)$.

Having determined the excess loss for the tranche, the cash flows to the investors have to be calculated accordingly. A reduction by the excess loss can be applied to the coupon rate or the principal payment of the inverse survivor bond. While existing literature often discusses the case of a reduction of the coupon rate, both models can be found in practice. In our model, if a tranche is triggered by an amount of survivors that is higher than expected, the principal payment of this tranche is reduced proportionally. Therefore, a proportion factor of default is introduced for each tranche j and time t that is defined by

$$\widetilde{\Lambda}_{x+t}^{(j)} = \frac{\widetilde{L}_{x+t}^{(j)}}{l_{x+t}^{(j)} - l_{x+t}^{(j-1)}}, \tag{5}$$

where we have $0 \leq \widetilde{\Lambda}_{x+t}^{(j)} \leq 1$. This factor represents the loss percentage that is inherent in the jth tranche, given the amount of survivors for the observed realization. It corresponds to the ratio of the realized excess loss $\widetilde{L}_{x+t}^{(j)}$ in the tranche and the "width" of the tranche given by $l_{x+t}^{(j)} - l_{x+t}^{(j-1)}$.

3.4 Pricing of the inverse survivor bond

The mechanism used for pricing the single tranches and the annuity securitization is as follows. As with any other bond, the price of the inverse survivor bond consists of two parts, the coupon payment and the principal payment.

The bond is structured in a way that the back payment of the *nominal* face value FV is distributed over the duration of the contract T with T equally sized payments defined by FV/T. Since the principal

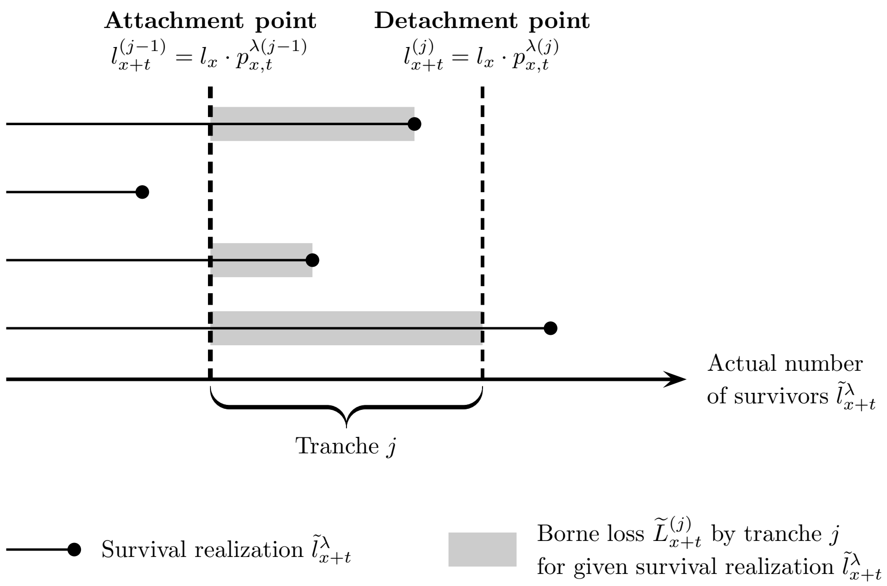


Figure 4: Illustration of the excess loss $\widetilde{L}_{x+t}^{(j)}$ calculation (see Equation 4) for tranche j at time t for annuitants aged $x+t$ (age x at the time of securitization).

payment is put at risk, it becomes a random variable which is expressed by

$$\widetilde{\mathcal{P}}_{x+t}^{(j)} = \left(1 - \widetilde{\Lambda}_{x+t}^{(j)}\right) \cdot \frac{FV}{T}, \tag{6}$$

for the tranches $j = 1, \ldots, N$ and the annuitants' ages $x+t$ ($t = 1, \ldots, T$). The principal is inversely proportionate to the amount of losses or, framed differently, to the amount of survivors in our portfolio. The principal's value can vary between 0, in the case where the actual survival rate is greater than the detachment points of the jth tranche, and the full payment of FV/T, in the case where the actual survival rate is lower than the corresponding attachment point of the jth tranche, i.e., we have $0 \leq \widetilde{\mathcal{P}}_{x+t}^{(j)} \leq \frac{FV}{T}$.

The coupon payment is based on the outstanding principal and is paid annually. The applied annual interest rate $c^{(j)}$ for each tranche j is comprised of a reference yield y such as, for instance, LIBOR or EURIBOR, and a tranche-specific spread $s^{(j)}$, i.e., $c^{(j)} = y + s^{(j)}$. The basis for the calculation of the interest amount is defined by the outstanding amount of debt towards the investor given by $D_t = FV - (t-1) \cdot FV/T = (T-t+1) \cdot FV/T$. Thus the coupon payment is given by

$$\mathcal{C}_t^{(j)} = D_t \cdot c^{(j)} = (T-t+1) \cdot \frac{FV}{T} \cdot \left(y + s^{(j)}\right), \tag{7}$$

for all tranches $j = 1, \ldots, N$ in times $t = 1, \ldots, T$.

On the basis of the principal and coupon payments for each year and tranche along to the realized excess loss, the price of the inverse survivor bond can be derived with the help of the general bond equation.

The bond price $P_x^{(j)}$ in time $t = 0$ for an x years aged individual (at the time of securitization) and for tranches $j = 1, \ldots, N$, corresponds to the sum of the present value of (or expected value of discounted) principal payments $P_x^{(j),\mathcal{P}}$ and the sum of all discounted coupon $P_x^{(j),\mathcal{C}}$ payments until contract maturity T. The price $P_x^{(j)}$ is given by

$$\begin{aligned} P_x^{(j)} &= P_x^{(j),\mathcal{P}} + P_x^{(j),\mathcal{C}} \\ &= \sum_{t=1}^{T} E\left(\widetilde{\mathcal{P}}_{x+t}^{(j)} \cdot (1 + r_{\mathrm{f}})^{-t}\right) + \sum_{t=1}^{T} \mathcal{C}_t^{(j)} \cdot (1 + r_{\mathrm{f}})^{-t} \\ &= \sum_{t=1}^{T} \frac{1}{(1 + r_{\mathrm{f}})^t} \cdot \left[E\left(\widetilde{\mathcal{P}}_{x+t}^{(j)}\right) + \mathcal{C}_t^{(j)}\right] \\ &= \frac{FV}{T} \cdot \sum_{t=1}^{T} \frac{1}{(1 + r_{\mathrm{f}})^t} \cdot \left[E\left(1 - \widetilde{\Lambda}_{x+t}^{(j)}\right) + (T - t + 1) \cdot \left(y + s^{(j)}\right)\right], \end{aligned} \tag{8}$$

with r_{f} the risk-free interest and $E(\cdot)$ denoting the expected value operator.

4 Numerical implementation and results

The aim of this section is to apply the previously introduced model step-by-step to the German market and to link the definition of the tranches to S&P ratings. Thus we will be able to compare the prices for different portfolio structures (different definition of the tranches) and perform selected sensitivity analysis on the example of the results for a 65 year-old annuitant at the moment of securitization and unity (€ 1) annual pension claims.

4.1 Forecasting future mortality rates for Germany

In all industrial societies, the life expectancy of the population has risen continuously over the last decades. This also holds true for Germany. As the basis for our calculations, we use German historic mortality rates from the years 1960 to 2009. Thereby we combine the reported values of the Federal Republic of Germany and the German Democratic Republic.[6] For our study we proceed as described in Section 3.1. For the analysis of the data on deaths and exposures as well as for the Lee-Carter forecasting of mortality rates we use the `R` package 'demography'.[7] Figure 5 shows the historic age-specific death rates in log-values. One can clearly observe that death rates across all ages continuously declined over the last decades. Assuming that this trend continues in the future, we use the Lee-Carter model to calculate future mortality rates for the population. First the model is fitted to the historic death rates. Figures 6 and 7 report the values of α_x, β_x, and the forecasted κ_t for the entire German population, i.e. without regard to gender (gender-indifferent case). Separate results for the male and female population are provided in Figures 14 to 17 in the Appendix 7.1.

[6] The data is derived from the Human Mortality Database and available for download at `http://www.mortality.org`.
[7] See `http://cran.r-project.org/web/packages/demography/demography.pdf` for the documentation.

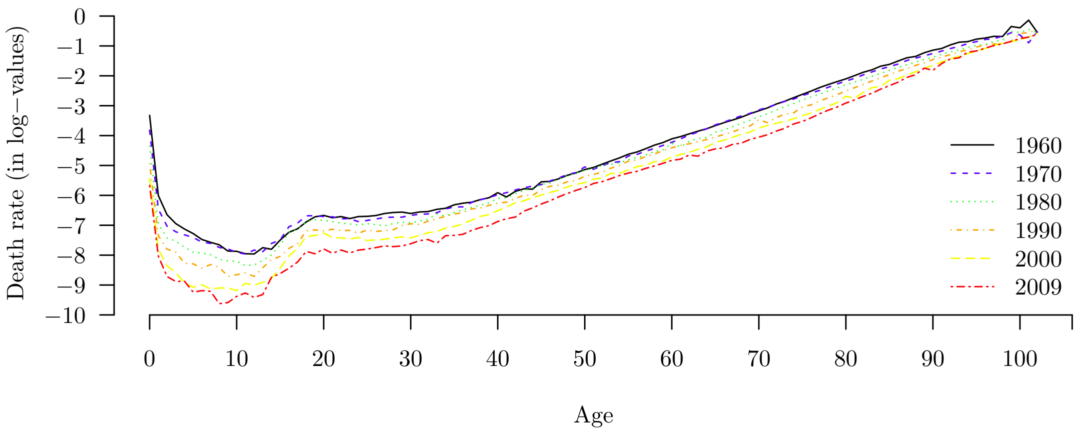


Figure 5: Illustration of German historic death rates (in log-values) for ages 0 to 102 from 1960 to 2009 using data from the Human Mortality Database available at `http://www.mortality.org`.

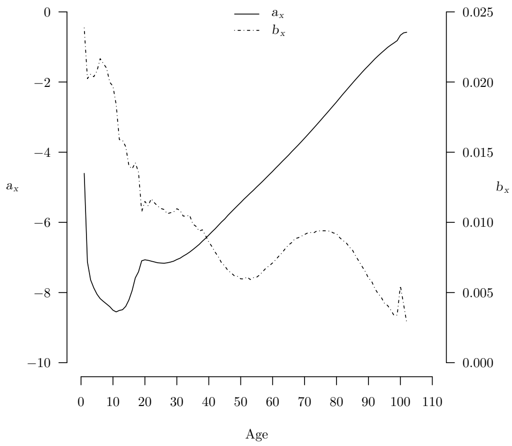


Figure 6: Fitted values for a_x and b_x for ages 0 to 101 for the German population.

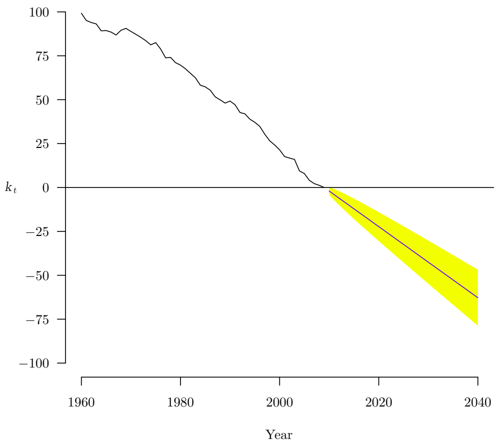


Figure 7: Forecasted values of k_t for the years 2010 to 2040 for the German population.

4.2 Adjustment for the market price of risk

After having determined the predictions on future mortality rates, the next step in securitizing annuity contracts incorporates the adjustment of these rates for insurance risk inherent in the contract with the help of the Wang-transformation as described in Section 3.2. Thus we first estimate the market price of risk λ for the German life insurance market. In order to do so, we use four different quotes of leading German life insurance companies (Allianz, AXA, Generali, and R&V) for an annuity product with an initial payment of € 100 000 and a duration of 10 years. The underlying annuitant is assumed to be a male and a female with the age at 65. For both male and female quotes, we derive the market value of risk λ by numerically solving the following equation:[8]

$$\text{net single premium} = \text{annuity quote} \cdot \psi^{\lambda}_{65,10}. \tag{9}$$

The net single premium is the market price net of annuity expenses, i.e., the net single premium times $(1 - \text{cost loading})$. The cost loading is calculated individually for each company based on the disclosed acquisition and administration costs in the offer. The total cost loading is close to 6% in the analyzed offers. $\psi^{\lambda}_{65,10}$ represents the actuarial present value of a 10-year immediate annuity at age 65. The value is calculated using the risk-adjusted survival probability $p^{\lambda}_{65,10} = \left(1 - \Phi\left[\Phi^{-1}\left(q_{65,10}\right) - \lambda\right]\right)$, where $q_{65,10}$ stands for the 10-year mortality rate of a 65 year-old man or woman provided by the German Federal Statistical Office. Furthermore the discounting of the individual annuity quotes is done with an interest rate of 2.5% as an average of historic two-year government bond returns. The resulting values for the market price of risk λ differ slightly from insurer to insurer. The detailed results for the market price of risk for the different insurers are shown in Table 1.

Market price of risk λ	Allianz	AXA	Generali	R&V	Average
Male	0.3039	0.1957	0.0668	0.5243	0.2727
Female	0.1736	0.0207	0.0571	0.4906	0.1855
Average	–	–	–	–	0.2291

Table 1: Market price of risk λ for different life insurers in Germany.

Our findings for the German market are close to the values reported for other markets in the literature (see, for example, Kim and Choi, 2011) where often a value of 0.2 is used for λ. Since the average result of analysis is near to the values observed by other authors and in order to ensure comparability with existing literature, we will use the parameter value $\lambda = 0.2$ for the Wang-transformation.

4.3 Tranche definition in a reference case

In general, an annuity securitization – as any other securitization transaction – can be performed with different tranche ratings and thus a different inherent quality of the single tranches. In our model, we use the rating definitions of S&P for insurance-linked securities. S&P provides cumulative default probabilities for different time horizons for each rating category. Exemplary values for insurance-linked securities with a maturity of ten years are plotted in Figure 8. A detailed list of these probabilities is provided in Table 9 in Appendix 7.2. We use these values to evaluate the attachment and detachment points for the different tranches of the securitization. According to S&P, there are 16 different ratings with

[8] This proceeding follows, for example, Kim and Choi (2011), or Lin and Cox (2005).

investment grade, ranging from the lowest rating of B− to the top rating of AAA; we only consider these ratings and no ratings for speculative assets. In addition to the different cumulative default probabilities, each rating class possess also an individual yearly yield or return. We define in our model the yield of a rating tranche as a spread in basis points (bp), i.e. hundredth of %, over a reference rate y such as LIBOR or EURIBOR. Figure 9 presents the spreads that we apply for the respective ratings (for details, see Table 9). These spreads have been adapted to observed spreads both in the securitization market as well as in the literature (see, for example, Mählmann, 2012, or Cowley and Cummins, 2005, p. 221).

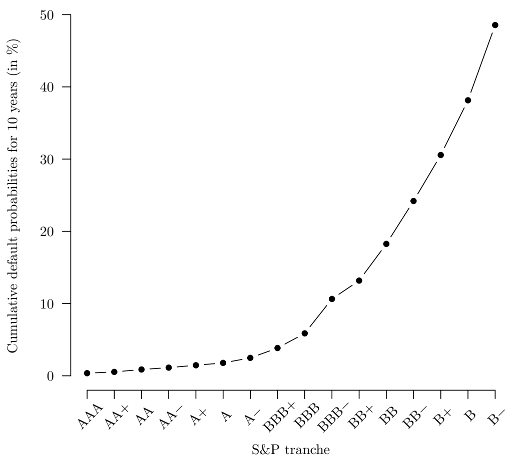


Figure 8: S&P cumulative default probabilities for insurance-linked securities for a 10-year maturity.

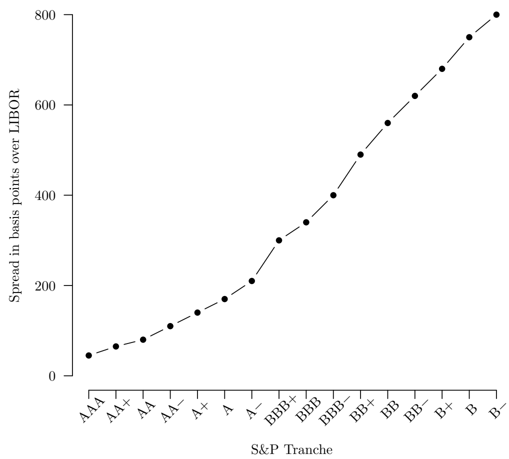


Figure 9: Applied spreads in basis points over reference yield LIBOR for different S&P tranches.

In order to specify the attachment points $p_{x,t}^{\lambda(j-1)}$ and detachment points $p_{x,t}^{\lambda(j)}$ of the tranches, we need to fix further parameters of the contract. Let us introduce a reference case through a portfolio with one annuitant aged 65 at the issuing of the securitization transaction. We set the market price of risk $\lambda = 0.2$ as derived in Section 4.2. Furthermore, for the reference case, we apply a securitization with $N = 4$ rated tranches, namely B+, BBB−, A, and AAA. This can be considered as a structure of tranches with "medium" quality. Following Table 9, the cumulative default probability of the B+ tranche, i.e. the first tranche that is accessible for investors, over a 10-year maturity is 30.565% with the corresponding survival probability of 69.435%.

It is reasonable to assume that the issuing insurer has built up reserves for its annuity portfolio over time which cover the 50%-tile of a potential survival distribution. This means that usually the reserves of annuitants dying earlier can be used to cover additional expenses by the insurers for annuitants who live longer than expected. This is close to the usual logic behind the risk pooling or pricing in insurance contracts, often refering to the expect value of claims or out-payments as measure. Given the proposed securitization structure, the insurer will still have to cover the annuity payments between the reserved 50%-tile and the percentile where the cumulative default probability of the first securitized tranche kicks in. Only from there on the risk is transferred to investors. In our example of a 10-year securitization with a B+ tranche as the most junior, this layer goes from the 50%-tile to the 69.435%-tile. This approach means that the first loss position – under the assumption of a reserve policy of the 50%-tile – is borne by the issuer. This approach is also in line with the literature postulating that the first loss piece is kept by the issuer in order to signal towards investors about the quality of the portfolio (see, for example, Riddiough, 1997, or Gale and Hellwig, 1985).

Table 2 shows the attachment points corresponding to the reference securitization. The numerical findings are based on the Wang-transformed survival rates derived from the Lee-Carter model and are compiled from 100 000 simulation runs.

T	$p_{65}^{\lambda(B+)}$	$p_{65}^{\lambda(BBB-)}$	$p_{65}^{\lambda(A)}$	$p_{65}^{\lambda(AAA)}$
1	0.9935	0.9937	0.9940	0.9942
2	0.9862	0.9866	0.9870	0.9873
3	0.9778	0.9784	0.9791	0.9796
4	0.9691	0.9699	0.9709	0.9715
5	0.9593	0.9604	0.9617	0.9624
6	0.9490	0.9503	0.9519	0.9529
7	0.9378	0.9396	0.9415	0.9428
8	0.9255	0.9276	0.9300	0.9316
9	0.9123	0.9148	0.9177	0.9196
10	0.8973	0.9003	0.9038	0.9061

Table 2: Illustration of the attachment points $p_{65}^{\lambda(j)}$ for four selected tranches B+, BBB−, A, and AAA, and maturities $T = 1, \ldots, 10$ (in years) in the reference case. The market price of risk is set to $\lambda = 0.2$.

4.4 Pricing of inverse survivor bond in a reference case

In order to derive prices for the bond, we need to define further parameters for the reference setting. For the following analysis, we assume the maturity of the inverse survivor bond to be $T = 10$ years. The considered annuitant in the reference case possesses an annual pension claim of € 1 per year and is aged $x = 65$ at the time of securitization. For our securitization, we define the face value FV of the bond equal to $T \cdot$ € 1 = € 10 which is a *nominal* value. The discounted face value of the bond then equals a present value of € 8.53. Furthermore, the coupon payments of the bond are interpreted as spreads over the reference yield y. We set $y = 1\%$. The discounting is done with a risk-free interest rate $r_{\mathrm{f}} = 3\%$. The forceasting of future mortality with the help of the Lee-Carter-model is done on the basis of the whole population. Table 3 summarizes the parameter assumptions for the reference case.

With these input parameters, we calculate the tranche prices $P_{65}^{(j)}$ for the securitization of the reference case. The results obtained are presented in Table 4. According to our model, the hedging of the annuity contract with an annual pension payment of € 1 for a 65 year old yields a bond price for the issuing insurer of € 10.057 (present value). This represents costs of 17.9% (1.527/8.53) based on the present

Parameter	Variable	Value
Annuitants' age at issuing	x	65 years
Population/Gender	–	male & female
Maturity	T	10 years
Market price of risk	λ	0.2
Reference yield	y	1%
Risk-free interest	r_{f}	3%
Annual pension	–	€ 1

Table 3: Model parameterization in the reference case.

value € 8.53 (present face value of the bond). The results show further that the principal amount of the securitization, which is initially distributed among the four tranches in an equal manner (see Section 3.3), differs significantly by the quality of the tranches. The most riskiest tranche – in our simulation the B+ rated tranche – suffers the highest losses and receives a principal payment of $P_{65}^{(B+),\mathcal{P}} =$ € 1.715 instead of the present value of the tranche's face value 2.133 (8.53/4). The amount of coupon payments for each tranche shows exactly the opposite development than the principal. Since the spreads follow an exponential increase according to the riskiness of the tranches (see Figure 9), the B+ tranche receives the highest coupon payments, i.e. y + 680 bp, and can compensate the principal loss with the guaranteed yields. Thus the B+ tranche yields overall annually 2.3% over the risk free rate when comparing the total price $P_{65}^{(B+)} = 2.671$ to the invested € 2.133. The more senior tranche AAA suffers in the principal losses of 0.002, which is neglegible and is still renumerated with additional coupon payments yielding a total payment of 2.309 corresponding to an annual 0.8% return over the risk free rate.

Tranches j		B+	BBB−	A	AAA	Sum
Attachment point	$p_{65}^{\lambda(j)}$	69.435%	89.363%	98.218%	99.638%	
Principal payments	$P_{65}^{(j),\mathcal{P}}$	1.715	2.022	2.113	2.131	7.981
Coupon payments	$P_{65}^{(j),\mathcal{C}}$	0.955	0.612	0.331	0.178	2.076
Bond price (total)	$P_{65}^{(j)}$	2.671	2.634	2.444	2.309	10.057

Table 4: Tranche prices (in €) for the parameterization of the reference case (see Table 3).

4.5 Sensitivity analyses of the bond prices

Before we step to the valuation of the retail insurance portfolio in Section 5, we analyze the effects and sensitivities inherent to the synthetic securitization model. For this we will use fictional portfolios or contracts. In the following paragraphs we study the variations in the results, i.e., the principal and coupon payments as well as the total bond price, when changing selected contract parameters. Our first sensitivity analyses include variations in the bond maturity T and in the structure of the portfolio, i.e.

the rating of the tranches and the number N of tranches. Further analysis include changes in the values of the reference yield y and of the risk-free interest r_{f} (or discount factor). Finally we also vary the underlying population considered and its historical mortality rates, i.e., we consider the total (male and female) population as well as males and females separately.

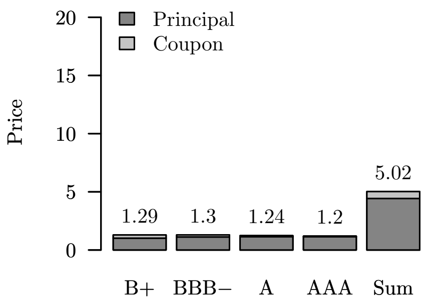
Bond maturity

First we address the differences in the bond prices for several time horizons T of the securitization transaction. For this we use the parameterization of the reference case specified in Table 3 and let T take the values 5, 10, 15, and 20 years. Table 5 presents the results of the sensitivity analysis. The results for the impact of different maturities are also illustrated through Figures 10(a)–(d).

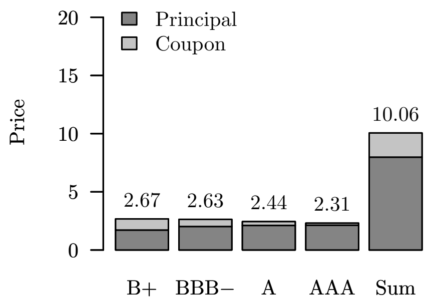
T	Tranches j		B+	BBB−	A	AAA	Sum
5	Attachment point	$p_{65}^{\lambda(j)}$	79.913%	95.641%	99.380%	99.900%	
	Principal payments	$P_{65}^{(j),\mathcal{P}}$	1.021	1.122	1.142	1.145	4.429
	Coupon payments	$P_{65}^{(j),\mathcal{C}}$	0.273	0.175	0.095	0.051	0.594
	Bond price (total)	$P_{65}^{(j)}$	1.294	1.297	1.236	1.195	5.023
10 (reference case)	Attachment point	$p_{65}^{\lambda(j)}$	69.435%	89.363%	98.218%	99.638%	
	Principal payments	$P_{65}^{(j),\mathcal{P}}$	1.715	2.022	2.113	2.131	7.981
	Coupon payments	$P_{65}^{(j),\mathcal{C}}$	0.955	0.612	0.331	0.178	2.076
	Bond price (total)	$P_{65}^{(j)}$	2.671	2.634	2.444	2.309	10.057
15	Attachment point	$p_{65}^{\lambda(j)}$	61.904%	84.582%	96.136%	98.963%	
	Principal payments	$P_{65}^{(j),\mathcal{P}}$	2.211	2.725	2.919	2.979	10.833
	Coupon payments	$P_{65}^{(j),\mathcal{C}}$	1.990	1.276	0.689	0.370	4.325
	Bond price (total)	$P_{65}^{(j)}$	4.201	4.001	3.608	3.349	15.159
20	Attachment point	$p_{65}^{\lambda(j)}$	56.802%	80.409%	93.507%	97.825%	
	Principal payments	$P_{65}^{(j),\mathcal{P}}$	2.576	3.264	3.570	3.704	13.113
	Coupon payments	$P_{65}^{(j),\mathcal{C}}$	3.330	2.134	1.153	0.619	7.236
	Bond price (total)	$P_{65}^{(j)}$	5.905	5.398	4.723	4.323	20.349

Table 5: Results of the sensitivity analysis of the securitization prices with regard to different maturities $T = 5, 10, 15, 20$ of the transaction. Values shown are in € for the parameterization (except for T) from the reference case given in Table 3.

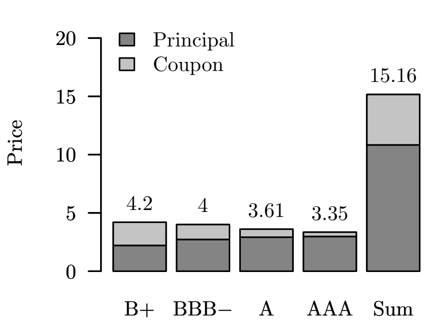
When it comes to the maturity of the inverse survivor bond, several effects can be noticed. The longer the duration is, the more important become the coupon payments $P_{65}^{(j),\mathcal{C}}$ for the total bond price $P_{65}^{(j)}$. For example, within a $T = 20$ year transaction, the coupon payments of the most junior tranche, the



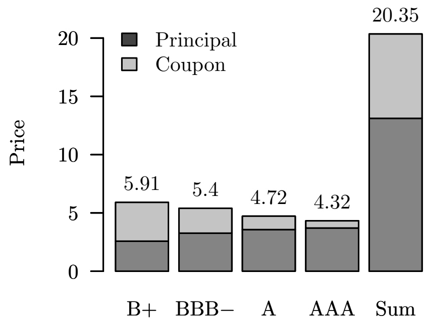
(a) Maturity $T = 5$



(b) Maturity $T = 10$



(c) Maturity $T = 15$



(d) Maturity $T = 20$

Figure 10: Results of the sensitivity analysis of the securitization prices with regard to different maturities $T = 5, 10, 15, 20$ of the transaction. Values shown are in € for the parameterization (except for T) from the reference case given in Table 3.

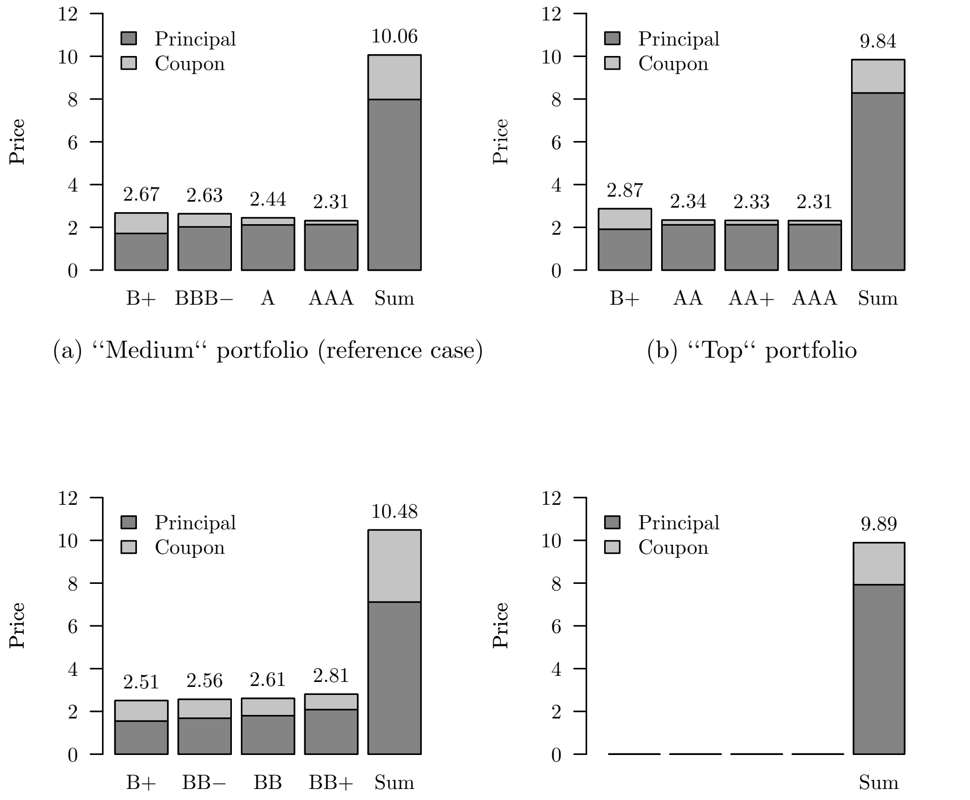
B+ tranche, are in total higher than the principal payment that is made (€ 3.33 compared to € 2.58). Over all tranches and in the case $T = 20$, the sum of all coupon payments $\sum_j P_{65}^{(j),\mathcal{C}}$ represent 35.6% (= 7.236/20.349) from the total bond price $\sum_j P_{65}^{(j)}$. This ratio is only 20.6% (= 2.076/10.057) when the time horizon is $T = 10$ years. This effect can be expected since the coupon payments are stable over the entire time period while the principal payments are at risk each year anew. Furthermore, a doubling of the duration results in a total bond price that is slightly higher than twice the initial one. Prolonging the duration, for instance, from $T = 10$ years to $T = 20$ years, the total price of the transaction $\sum_j P_{65}^{(j)}$ rises from € 10.06 to € 20.35. However, this effect can almost be neglected. Comparing the annual costs – € 1.01 over 10 years with € 1.02 over 20 years – reveals an increase of only 1.2% is observed.

Portfolio structure

Next, we address the impact of different portfolio structures on the prices of the securitization. For our study, we typically consider four different tranches in our securitization transaction. In order to test the impact of different tranche layouts on the price of the portfolio hedging, we apply in the following four different portfolio securitizations. First, we reconsider the "medium"-rated securitization with AAA, A, BBB−, and B+ tranches introduced in the reference case (see Section 4.3). The second approach is a synthetic securitization with "top"-rated tranches, i.e. the portfolio is sliced in a AAA, AA+, AA, and B+ tranche. The third securitization structure represents a "poor"-rated deal with BB+, BB, BB−, and B+ tranches. The last calculation is based on a securitization that entails each S&P rating class between B+ and AAA, i.e. 14 different tranches. If the insurer targets more risk-averse investors with its securitization, it is more likely that the company will pursue a strategy that generates more highly rated tranches ("top" structure). However, if more risk-affine investors are addressed, a synthetic securitization with a higher proportion of riskier tranches is structured ("poor" structure). In Table 6 we report the results from the sensitivity analysis for the structures "medium", "top", and "poor". Figures 11(a)–(d) illustrate the tranche prices for all four different securitization structures.

"Medium" (reference case)	Tranches j		B+	BBB−	A	AAA	Sum
	Attachment point	$p_{65}^{\lambda(j)}$	69.435%	89.363%	98.218%	99.638%	
	Principal payments	$P_{65}^{(j),\mathcal{P}}$	1.715	2.022	2.113	2.131	7.981
	Coupon payments	$P_{65}^{(j),\mathcal{C}}$	0.955	0.612	0.331	0.178	2.076
	Bond price (total)	$P_{65}^{(j)}$	2.671	2.634	2.444	2.309	10.057
"Top"	Tranches j		B+	AA	AA+	AAA	Sum
	Attachment point	$p_{65}^{\lambda(j)}$	69.44%	99.13%	99.46%	99.64%	
	Principal payments	$P_{65}^{(j),\mathcal{P}}$	1.913	2.118	2.123	2.131	8.285
	Coupon payments	$P_{65}^{(j),\mathcal{C}}$	0.955	0.220	0.202	0.178	1.556
	Bond price (total)	$P_{65}^{(j)}$	2.868	2.338	2.325	2.309	9.840
"Poor"	Tranches j		B+	BB−	BB	BB+	Sum
	Attachment point	$p_{65}^{\lambda(j)}$	69.435%	75.803%	81.742%	86.821%	
	Principal payments	$P_{65}^{(j),\mathcal{P}}$	1.550	1.682	1.799	2.085	7.116
	Coupon payments	$P_{65}^{(j),\mathcal{C}}$	0.955	0.882	0.808	0.723	3.368
	Bond price (total)	$P_{65}^{(j)}$	2.505	2.564	2.608	2.807	10.484

Table 6: Results of the sensitivity analysis of the securitization prices with regard to different tranche structures of the transaction. The portfolio structure "medium" corresponds to the reference tranches introduced in Section 4.3. Values shown are in € for the parameterization from the reference case given in Table 3.



(a) "Medium" portfolio (reference case)

(b) "Top" portfolio

(c) "Poor" portfolio

(d) Portfolio with tranches B+ to AAA

Figure 11: Results of the sensitivity analysis of the securitization prices with regard to different tranche structures of the transaction. The portfolio structure "medium" corresponds to the reference tranches introduced in Section 4.3. Case (d) reports the total bond price when $N = 14$ tranches are created including all ratings from B+ to AAA. Values shown are in € for the parameterization from the reference case given in Table 3.

Given the parameterization of the reference case given in Table 3, the calculations reveal that a more poorly rated portfolio with tranches B+, BB−, BB, and BB+ – which represent the first four tranches starting from B+ – yields the highest securitization price with $\sum_j P_{65}^{(j)} =$ € 10.48, whereas a portfolio consisting of highly rated tranches (B+, AA, AA+, and AAA) accounts for the lowest price with € 9.84. This is a difference of 6.5%. The reference case with a securitization structure of medium quality accounts for a price of € 10.06. A securitization including all 14 S&P tranches from B+ to AAA is priced € 9.89. The ratio of all principal payments to the total bond price, $\sum_j P_{65}^{(j),\mathcal{P}} / \sum_j P_{65}^{(j)}$ varies from 67.9% ("poor" structure) to 84.2% ("top" structure) and is strongly related to the rating and the default probabilities of the tranches.

Reference yield, risk-free interest and underlying population

The objective of this paragraph is to summarize the findings from studying the impact of changes in the reference yield y, in the risk-free interest rate r_{f} (used for discounting the payments, see Section 3.4), and in the considered subset of the population underlying our mortality forecasts.

Table 7 reports the results from our sensitivity tests. The input parameters of the reference yield y, the discount factor r_{f}, and the effect of gender consideration in mortality forecasting are analyzed. Testing for the impact of the reference yield y on which the tranche spreads are applied, as expected we observe that the values of the principal payments $P_{65}^{(j),\mathcal{P}}$ remain unaffected. The more senior tranches, such as A or AAA, in particular benefit from an increase in the reference yield. While the coupon payments $P_{65}^{(\mathrm{B+}),\mathcal{C}}$ for the B+ tranche show a total increase of 12.9% from € 0.96 ($y = 1\%$) to € 1.08 ($y = 2\%$), the additional coupon value for the AAA tranche yields € 0.12 (difference in $P_{65}^{(\mathrm{AAA}),\mathcal{C}}$ when y changes from 1% to 2%), which corresponds to an increase of 68.5%. Rising the reference yield y from 1% to 4%, the B+ tranches receives an increase of 38.5% while the coupon payments are more than tripled for the most senior AAA tranche (from € 0.178 to € 0.545). Changes in the risk-free interest r_{f} also show expected effects with all prices decreasing for higher values of the discount rate r_{f}. Finally, the data shows that the consideration of gender in the forecasting of future mortality rates, and thus also the valuation of the securitization, is of minor importance. The tranche prices only differ at the second digit.

5 Case Study: Application to a real insurance portfolio

After having calculated the individual tranche prices for one underlying age of the annuitant in the previous section, we apply now the numerical results to a real portfolio comprising annuitants of different ages. Furthermore annual annuity payments are no more € 1. In order to simulate our tranching of life annuity products, we consider a retail data sample of randomly selected (active) life contracts from a leading German life insurer. The portfolio consists of 20 544 contracts with a total volume of annual guarantied pension payments of € 89.8 mn (nominal annuities). Figures 12 and 13 give a detailed overview on portfolio composition. Since the portfolio contains only annuity contracts in the pay-out phase, the majority of the annuitants is over 60 years old. This effect is illustrated in the distribution of annual payouts per age (see Figure 13). The highest payments are made for the group of retired policyholders over 60 years.

Our objective is to price the inverse survivor bond linked to a transaction of the described retail portfolio, and thus calculate the price of hedging for the insurer. Thereby, we calculate tranche prices for every age within in the portfolio and sum the individual prices up to the entire portfolio value. The different age groups yield similar tranche prices since attachment and detachment points are evaluated on the basis of the survival probabilities of the respective age. The results of the portfolio valuation are presented in Table 8. As for the reference case in Section 4, we calculate the inverse survivor bond for a maturity of $T = 10$ years using the value $\lambda = 0.2$ for the market price of risk.

The insurer effectively receives a payment of € 766.0 mn (€ 89.8 mn for T years, discounted to $t = 0$). The bond price equals $\sum_j P_{\mathrm{all}}^{(j)} =$ € 903.1 mn which yields costs of € 137.1 corresponding to 17.9% of the

y	r_f	Pop.	Tranches j		B+	BBB−	A	AAA	Sum
1%	3%	m.&f.	Principal	$P_{65}^{(j),\mathcal{P}}$	1.715	2.022	2.113	2.131	7.981
			Coupon	$P_{65}^{(j),\mathcal{C}}$	0.955	0.612	0.331	0.178	2.076
			Bond price	$P_{65}^{(j)}$	2.671	2.634	2.444	2.309	10.057
2%	3%	m.&f.	Principal	$P_{65}^{(j),\mathcal{P}}$	1.719	2.027	2.112	2.131	7.989
			Coupon	$P_{65}^{(j),\mathcal{C}}$	1.078	0.735	0.453	0.300	2.566
			Bond price	$P_{65}^{(j)}$	2.797	2.762	2.565	2.431	10.555
3%	3%	m.&f.	Principal	$P_{65}^{(j),\mathcal{P}}$	1.719	2.026	2.112	2.131	7.987
			Coupon	$P_{65}^{(j),\mathcal{C}}$	1.200	0.857	0.576	0.423	3.056
			Bond price	$P_{65}^{(j)}$	2.919	2.883	2.688	2.553	11.043
4%	3%	m.&f.	Principal	$P_{65}^{(j),\mathcal{P}}$	1.717	2.026	2.112	2.131	7.986
			Coupon	$P_{65}^{(j),\mathcal{C}}$	1.323	0.980	0.698	0.545	3.546
			Bond price	$P_{65}^{(j)}$	3.040	3.006	2.810	2.676	11.532
1%	**1%**	m.&f.	Principal	$P_{65}^{(j),\mathcal{P}}$	1.907	2.250	2.344	2.366	8.867
			Coupon	$P_{65}^{(j),\mathcal{C}}$	1.031	0.661	0.357	0.192	2.240
			Bond price	$P_{65}^{(j)}$	2.938	2.910	2.701	2.557	11.107
1%	**2%**	m.&f.	Principal	$P_{65}^{(j),\mathcal{P}}$	1.809	2.134	2.224	2.244	8.410
			Coupon	$P_{65}^{(j),\mathcal{C}}$	0.992	0.636	0.343	0.184	2.156
			Bond price	$P_{65}^{(j)}$	2.801	2.770	2.567	2.428	10.566
1%	**4%**	m.&f.	Principal	$P_{65}^{(j),\mathcal{P}}$	1.632	1.926	2.008	2.026	7.592
			Coupon	$P_{65}^{(j),\mathcal{C}}$	0.921	0.590	0.319	0.171	2.001
			Bond price	$P_{65}^{(j)}$	2.553	2.516	2.327	2.197	9.593
1%	3%	**m.**	Principal	$P_{65}^{(j),\mathcal{P}}$	1.715	2.021	2.113	2.131	7.979
			Coupon	$P_{65}^{(j),\mathcal{C}}$	0.955	0.612	0.331	0.178	2.076
			Bond price	$P_{65}^{(j)}$	2.670	2.633	2.443	2.309	10.055
1%	3%	**f.**	Principal	$P_{65}^{(j),\mathcal{P}}$	1.714	2.020	2.113	2.131	7.978
			Coupon	$P_{65}^{(j),\mathcal{C}}$	0.955	0.612	0.331	0.178	2.076
			Bond price	$P_{65}^{(j)}$	2.669	2.633	2.444	2.309	10.054

Table 7: Results from the sensitivity analysis of the securitization prices with regard to different values of the reference yield $y = 1\%, 2\%, 3\%, 4\%$, of the risk-free interest $r_\mathrm{f} = 1\%, 2\%, 3\%, 4\%$, and with regard to different underlyings for the (male and/or female) population in mortality forecasting. Values shown are in € for the parameterization from the reference case given in Table 3. The first setting with $y = 1\%$, $r_\mathrm{f} = 3\%$ and including the male and female population corresponds to the reference case. Parameters differing from the reference case are printed in bold. "Pop." stands for population, "m." for the male, and "f." for the female population.

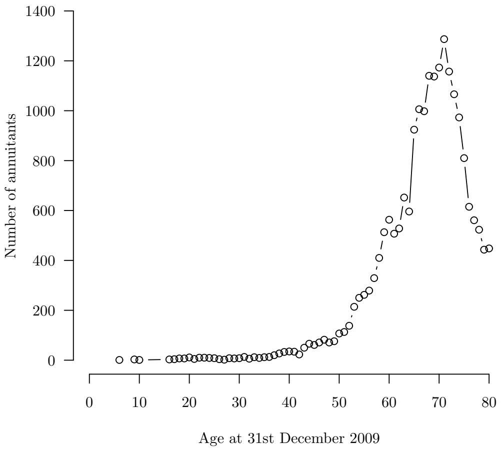


Figure 12: Number of annuitants per age as of 31st December 2009.

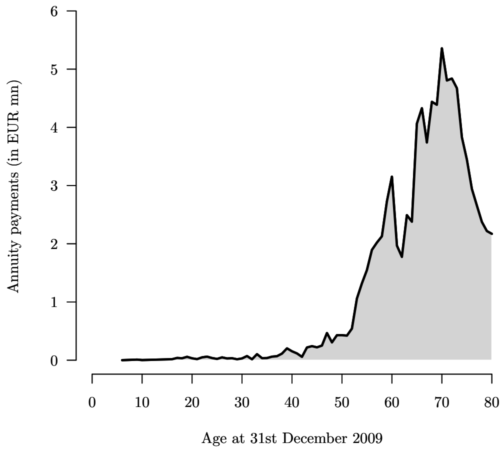


Figure 13: Annual nominal annuity payments in € mn per age as of 31st December 2009. The grey shaded area cumulates to € 89.8 mn in annuity payments.

payment in $t = 0$ (see also Section 4.4). At first glance, this figure seems quite high. However, there a several effects that cushion the issuer's cost. First, the insurer receives the entire investment, i.e. the face value of the bond in time $t = 0$ and has the possibility to generate investment returns with the money that is not paid back to the investors, respectively retained for paying annuities in the event of higher survival rates. In order to compensate the costs, the insurer needs to invest the available capital at an annural return of 4.2%, to be put in relation to the risk-free rate parameterized as $r_{\mathrm{f}} = 3\%$. Second, the proposed hedge structure starts at the cumulative default probability of the B+ tranche (about 70%-tile) and reaches to the 100%-tile. Even if the insurer keeps the first loss position, a large proportion of the risk is transferred to third parties. However, the issuer can still leverage the reserves from annuitants dying earlier than expected. We do not quantify this effects in this paper. From the investor perspective, the B+ tranche is renumerated with an annual return of 2.3% over the risk free rate. The most senior tranche AAA is still able to generate revenues at 0.8% over the risk free rate.

6 Conclusion

This paper introduces a model for an annuity securitization with the help of an inverse survivor bond. We have taken the perspective of the issuing insurer and calculated the price of hedging for the company. We applied a tranching approach for the securitization based on the percentile tranching method and designed the bond in a way that the principal payments are risk, i.e. depending on the survival rates of the underlying portfolio of annuitants. In order to do so, we first used the Lee-Carter-model for Germany and calculated estimates on future mortality rates. We forecast the survival distribution using 100 000

Tranches j		B+	BBB−	A	AAA	Sum
Total prices of gender-indifferent securitization						
Principal payments	$P_{\text{all}}^{(j),\mathcal{P}}$	154.06	181.52	189.73	191.37	716.69
Coupon payments	$P_{\text{all}}^{(j),\mathcal{C}}$	85.79	54.99	29.70	15.95	186.43
Bond price (total)	$P_{\text{all}}^{(j)}$	**239.85**	**236.52**	**219.43**	**207.32**	**903.11**
Securitization of male annuitants in portfolio						
Principal payments	$P_{\text{male}}^{(j),\mathcal{P}}$	88.04	103.65	108.37	109.31	409.37
Coupon payments	$P_{\text{male}}^{(j),\mathcal{C}}$	49.00	31.41	16.96	9.11	106.49
Bond price (total)	$P_{\text{male}}^{(j)}$	137.04	135.06	125.34	118.42	515.86
Securitization of female annuitants in portfolio						
Principal payments	$P_{\text{female}}^{(j),\mathcal{P}}$	66.03	77.81	81.36	82.06	307.26
Coupon payments	$P_{\text{female}}^{(j),\mathcal{C}}$	36.79	23.58	12.73	6.84	79.94
Bond price (total)	$P_{\text{female}}^{(j)}$	102.82	101.39	94.09	88.90	387.20
Total prices of gender-specific securitization						
Principal payments	$P_{\text{all}}^{(j),\mathcal{P}}$	154.07	181.46	189.73	191.37	716.63
Coupon payments	$P_{\text{all}}^{(j),\mathcal{C}}$	85.79	54.99	29.70	15.95	186.43
Bond price (total)	$P_{\text{all}}^{(j)}$	**239.86**	**236.45**	**219.43**	**207.31**	**903.05**

Table 8: Pricing of hedging longevity risk in the retail portfolio with total annual annuity payments of € 89.9mn over 10 years. Reported values are in € mn and represent results for gender-indifferent and gender-specific underlying populations and forecasted future mortality rates.

simulation runs. In a second step, we applied the Wang-transformation to the derived rates in order to incorporate the risk inherent in the annuitant's contract. This distribution serves as the basis for our tranching by slicing the portfolio into different rated tranches according to S&P ratings for insurance-linked securities. Thereby, the first loss position is kept by the issuer and four different tranches are transferred to investors on the capital market. In a next step, we used a classical bond equation to determine the individual tranche prices and the overall costs for the insurer for the securitization of an annuity portfolio. We applied this pricing model to a reference case and perform sensitivity analyses. Finally, in an application to a sample retail portfolio we determined the price for hedging the contracts against longevity risk.

The results show that changes in the reference yield of the coupon payments have a severe impact on the bond price; this effect can be epically observed for highly-rated tranches. Changing the reference yield (such as LIBOR or EURIBOR) from 1% (our reference case) to 2%, causes the coupon payment of a AAA tranche to more than triple on a 10-year maturity. The effect on the bond price due to structural changes in the securitization outlay are also of importance. On a 10-year time period, the total bond price between a "top" rated and "poor" rated portfolio differs by 6.5%.

Considering the issuer as well as the investor perspective, the reference case yields total costs of

17.9% for the insurer from the securitization. At first, this cost loading seems high; however, due to the presented structure of the bond, an annual return of 4.2% on the received capital, which the insurer should invest, is enough to compensate the costs. The transaction is also advantegous for the investors. Investments into the minor B+ tranche are renumerated with an annual yield of 2.3% over the risk-free rate, while the most senior AAA tranche – which bears the lowest default risk – still receives annually 0.8% over the risk-free rate.

Our findings contribute to the current discussion about how insurers can face longevity risk within their annuity portfolios. The fact that the rating structure has such a severe impact on the overall hedging costs for the insurer implies that companies which are willing to undergo an annuity securitization should consider their deal structure very carefully. In addition, we have pointed out that in imperfect markets, the retention of the equity tranche by the originator might be advantageous. Never the less, one has to bear in mind that by this behavior, the insurer is able to reduce the overall default risk in his balance sheet by securitizing a life insurance portfolio; however, the fraction of first loss pieces from defaults increases more than proportionally. The insurer has to take care to not be left with large, unwanted remaining risk positions in his books.

7 Appendix

7.1 Forecasting future mortality rate for Germany: additional results

Figures 14 and 15 show the values of α_x, β_x, and the forecasted κ_t for the male German population. Figures 16 and 17 report the values for the female German population.

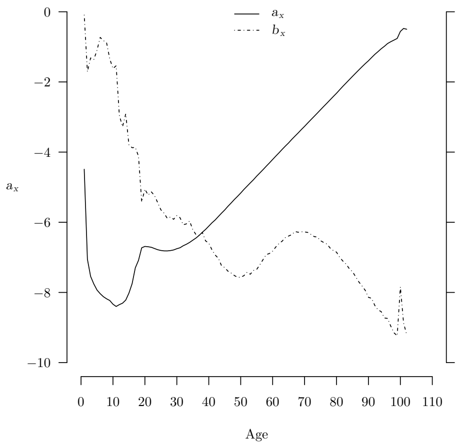


Figure 14: Fitted values for a_x and b_x for ages 0 to 101 for the German male population.

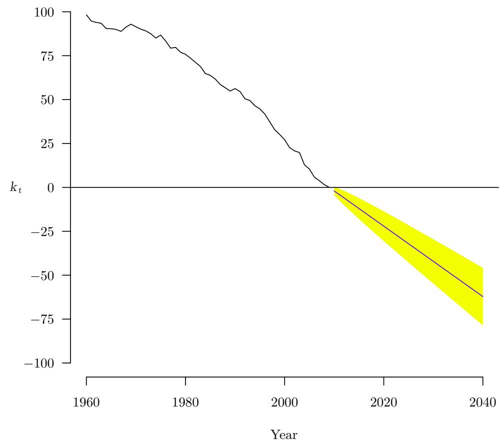


Figure 15: Forecasted values of k_t for the years 2010 to 2040 for the German male population.

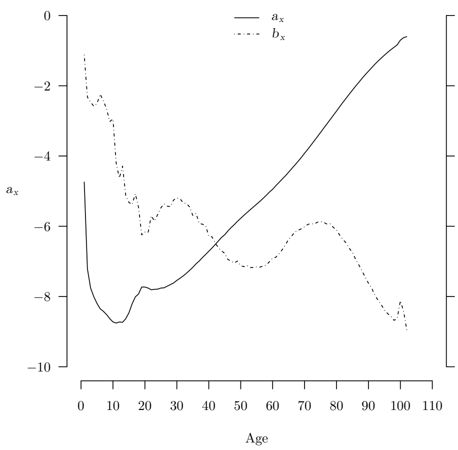


Figure 16: Fitted values for a_x and b_x for ages 0 to 101 for the German female population.

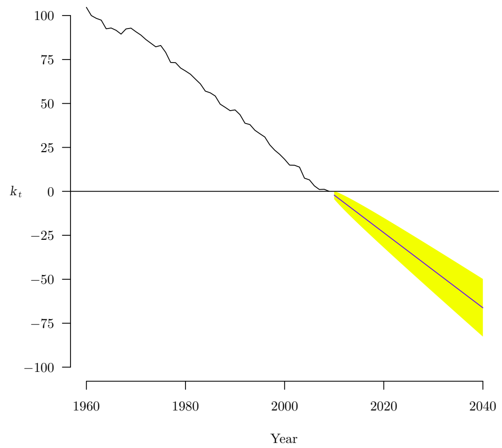


Figure 17: Forecasted values of k_t for the years 2010 to 2040 for the German female population.

7.2 S&P rating classes for insurance-linked securitizations

Table 9 shows the cumulative default probabilities of the S&P rating for insurance-linked securities according to their maturity.

Maturity	AAA	AA+	AA	AA−	A+	A	A−	BBB+
1	0.003	0.010	0.015	0.025	0.040	0.060	0.085	0.234
2	0.027	0.048	0.074	0.106	0.150	0.200	0.264	0.514
3	0.052	0.085	0.133	0.188	0.260	0.340	0.443	0.850
4	0.076	0.123	0.191	0.269	0.370	0.480	0.621	1.246
5	0.100	0.160	0.250	0.350	0.480	0.620	0.800	1.704
6	0.122	0.192	0.310	0.397	0.531	0.655	0.966	1.805
7	0.144	0.224	0.420	0.543	0.719	0.887	1.287	2.261
8	0.204	0.311	0.549	0.713	0.937	1.152	1.648	2.756
9	0.276	0.414	0.700	0.909	1.184	1.451	2.047	3.284
10	0.362	0.536	0.872	1.130	1.458	1.782	2.479	3.842
15	1.037	1.447	2.078	2.617	3.237	3.864	5.051	6.936
20	2.175	2.893	3.858	4.690	5.586	6.493	8.068	10.279
25	3.804	4.853	6.133	7.223	8.337	9.454	11.284	13.667
30	5.885	7.241	8.781	10.066	11.329	12.580	14.553	17.003
$s^{(j)}$	45	65	80	110	140	170	210	300

Maturity	BBB	BBB−	BB+	BB	BB−	B+	B	B−
1	0.353	0.547	1.632	2.525	3.518	4.510	5.824	8.138
2	0.825	1.279	3.211	4.946	6.915	8.885	11.751	16.674
3	1.405	2.177	4.758	7.230	10.095	12.960	17.152	24.004
4	2.073	3.213	6.276	9.380	13.037	16.694	21.921	30.025
5	2.812	4.359	7.763	11.403	15.745	20.087	26.089	34.945
6	2.980	6.316	8.327	12.175	16.832	21.462	27.947	38.234
7	3.672	7.434	9.598	13.826	18.895	24.083	30.999	41.476
8	4.390	8.529	10.831	15.387	20.800	26.457	33.680	44.209
9	5.127	9.598	12.025	16.862	22.563	28.610	36.046	46.543
10	5.876	10.637	13.179	18.258	24.197	30.565	38.145	48.559
15	9.684	15.418	18.383	24.234	30.849	38.096	45.822	55.592
20	13.414	19.591	22.777	28.944	35.737	43.198	50.706	59.851
25	16.980	23.300	26.570	32.808	39.556	46.958	54.169	62.789
30	20.367	26.665	29.933	36.108	42.709	49.936	56.845	65.022
$s^{(j)}$	340	400	490	560	620	680	750	800

Table 9: S&P default table for insurance-linked securitizations reporting cumulative default probabilities (in %) for different ratings and maturities (in years). For each rating the corresponding (yearly) spread $s^{(j)}$ is indicated in basis points (1 bp = 0.01%). See also S&P at `http://www.standardandpoors.com`.

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