Portfolio selection in defined contribution pension plans

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Outline

- Importance of defined contribution(DC) plans
- Nature of DC plan
- Investment risk transferred to the employee
- Overview of the portfolio selection problem
- Finance models: Markowitz model: continuous time models
- Model predictions not observed in actual investment behavior
- Two pension puzzles observed in DC plan

  1. The $\frac{1}{n}$ rule

  2. The own company stock puzzle

- Basic insight: importance of estimation risk

This talk

- Some background
- Use simple intuitive approach
- One period Markowitz model
- Optimal solution hyper sensitive to mean vector:Estimation risk
- Equally weighted portfolio is a simple heuristic
- Preference for own company stock related to home bias to familiar assets
- Cheer for the home team(Roma? Inter? Juventus? Perugia?)
- Boyle Uppal and Wang develop a model to rationalize observed over investment in own company stock.
- Some final thoughts

Trend from DB to DC

- Worldwide trend from defined benefit to defined contribution plans
- Corporations contend that DB plan expenses are too volatile (and too high). Widespread trend to DC Plans
- Public sector plans also switching.
- Several European countries have modified their government-sponsored pension plans from a pure DB to DC-like plans; eg Sweden, Germany
- 700,000 corporate DC plans in U.S. covering 56 m workers
  56,000 DB plans covering 23 m employees
- Total assets of defined contribution plans in the USA are about 2 trillion dollars.

Nature of DC plans

- Under a DC plan employee (and often employer) pay a fixed percentage of salary
- These contributions invested in a portfolio of assets
- Benefit at retirement is market value of this portfolio
- DC plan often has a range of asset classes.
- Employee normally makes the investment choices
- Under a DC plan the employee bears the investment risk
- Switch from DB to DC transfers risk from employer to employee
- The employee now has to make the asset allocation decision

Features of *Full Fledged* Asset Allocation Model

- Which Asset Classes
- Construct *good* economic model of returns
- Model the dynamics of these assets
- Decide upon the relevant state variables
- Estimate parameters of model
- Model investor's preferences
- Formulate as an optimization problem
- Solve the optimization problem
- Seems like a Herculean task for the average employee
- Average employee is not Hercules

Background: practice

- Asset allocation is key investment decision for institutions
- Stock bond relative weights more important than which stocks
- Most of the performance of pension funds is determined by asset allocation decision
- Rule of thumb 60% in stocks 40% in bonds
- Very important decision for members of of defined contribution pension plans.
- Recall Enron collapse. Employee pensions heavily invested in Enron stock
- Can finance theory help? Is there any scientific guidance ?

Models

- Markowitz(1952) showed how to select optimal portfolios based on mean and variance. One period model
- Arrow model(1952) of contingent claims
- Merton(1969,1971) used continuous time framework: analyzed consumption investment problem. Solved portfolio problem using methods of optimal stochastic control
- Explicit solutions in a few cases: assumptions about the investor's utility function and the asset price dynamics.
- Hard to obtain solutions under the dynamic programming approach and difficult to incorporate constraints.

Markowitz Approach to Portfolio Selection

Assume N risky assets. Define a portfolio to be a vector:

$$\mathbf{x} = (x_1, x_2, \ldots, x_N)^T \qquad \text{subject to:} \quad \mathbf{e}^T\mathbf{x} = 1, \tag{1}$$

where x_i represents the fraction of wealth held in asset i and $\mathbf{e} = (1, 1, \ldots, 1)^T$ is the N by one vector consisting of N ones.

Given a risk tolerance level, $\tau \in [0, \infty)$, the optimal portfolio can be found by solving the parametric quadratic program :

$$\max_{\mathbf{x}} \left(2\tau\boldsymbol{\mu}^T\mathbf{x} - \mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right) \tag{2}$$

$$\text{subject to: } \mathbf{e}^T\mathbf{x} = 1, \tag{3}$$

where:

$$\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_N)^T \ , \tag{4}$$

and μ_i is the expected return on asset i, and:

$$\boldsymbol{\Sigma} = \left[\rho_{ij}\sigma_i\sigma_j\right], \qquad i, j = 1, \ldots, N \ , \tag{5}$$

where ρ_{ij} is the correlation between assets i and j, and σ_i is the volatility of asset i. We assume that the matrix $\boldsymbol{\Sigma}$ is positive definite.

This problem has a simple closed form solution. The optimal portfolio:

$$\mathbf{x}^*(\tau) = \mathbf{x}_{min} + \tau \Delta \mathbf{x}_{risk} \ , \tag{6}$$

where:

$$\mathbf{x}_{min} = \frac{1}{\mathbf{e}^T \boldsymbol{\Sigma}^{-1} \mathbf{e}} \boldsymbol{\Sigma}^{-1} \mathbf{e} \ , \tag{7}$$

$$\Delta \mathbf{x}_{risk} = \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} - \frac{\mathbf{e}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}}{\mathbf{e}^T \boldsymbol{\Sigma}^{-1} \mathbf{e}} \ \boldsymbol{\Sigma}^{-1} \mathbf{e} \ . \tag{8}$$

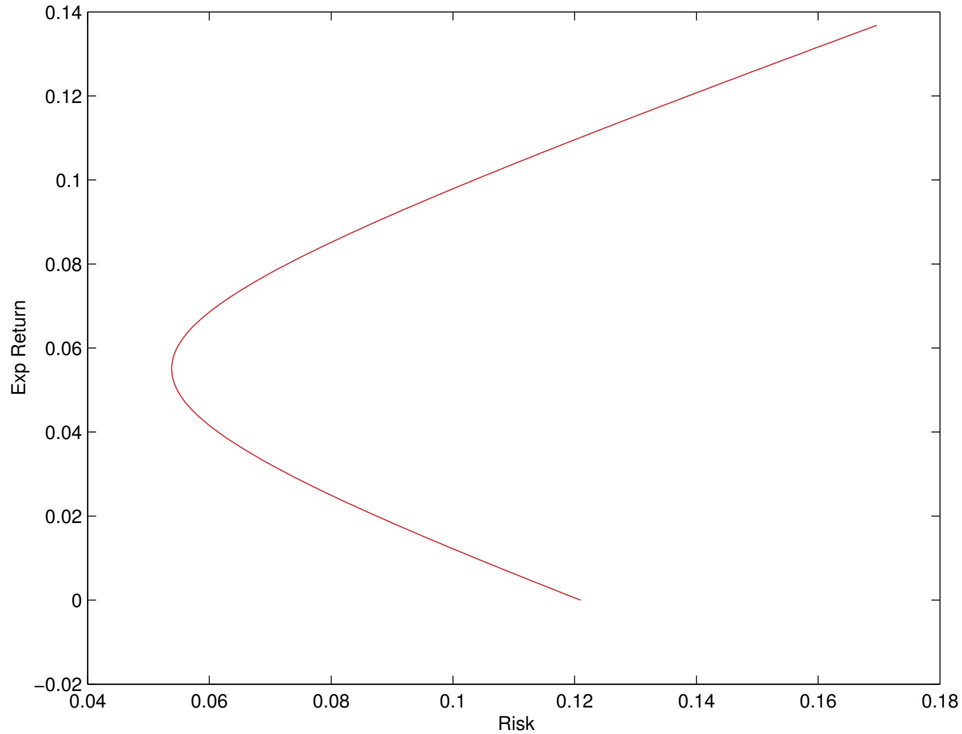

Figure 1: The efficient frontier based on the true population parameters. Frontier in red. Vertical axis expected return. Horizontal axis standard deviation. Short sales are allowed. We assume investor knows true values of μ, Σ

Specimen investments

We use a five asset universe to illustrate some of these ideas. Assume that an investor wishes to select an efficient portfolio based on five different asset classes

- The S&P 500 Large Cap (Large Cap),
- The S&P Mid Cap 400 (Mid Cap),
- The Russell 2000 Small Cap (Small Cap),
- The Morgan Stanley World Equity excluding US (World Index),
- The Lehman Brothers long-term government bond index (Long Bond Index).

Representative of those offered to many pension plan participants within in the US. Estimate parameters from 15 years history.

Asset Name	μ	σ			ρ		
Large Cap	.1165	.1449	1.0000	0.9180	0.8341	0.4822	0.3873
Mid Cap	.1288	.1549	0.9180	1.0000	0.9386	0.4562	0.3319
Small Cap	.0968	.1792	0.8341	0.9386	1.0000	0.4206	0.2002
World Index	.0921	.1715	0.4822	0.4562	0.4206	1.0000	0.2278
Long Bond Index	.0547	.0558	0.3873	0.3319	0.2002	0.2278	1.0000

Table 1: The expected returns, volatilities and correlations for the distribution of returns of the assets available in the pension plan described in this section.

Assume these are the true Parameters

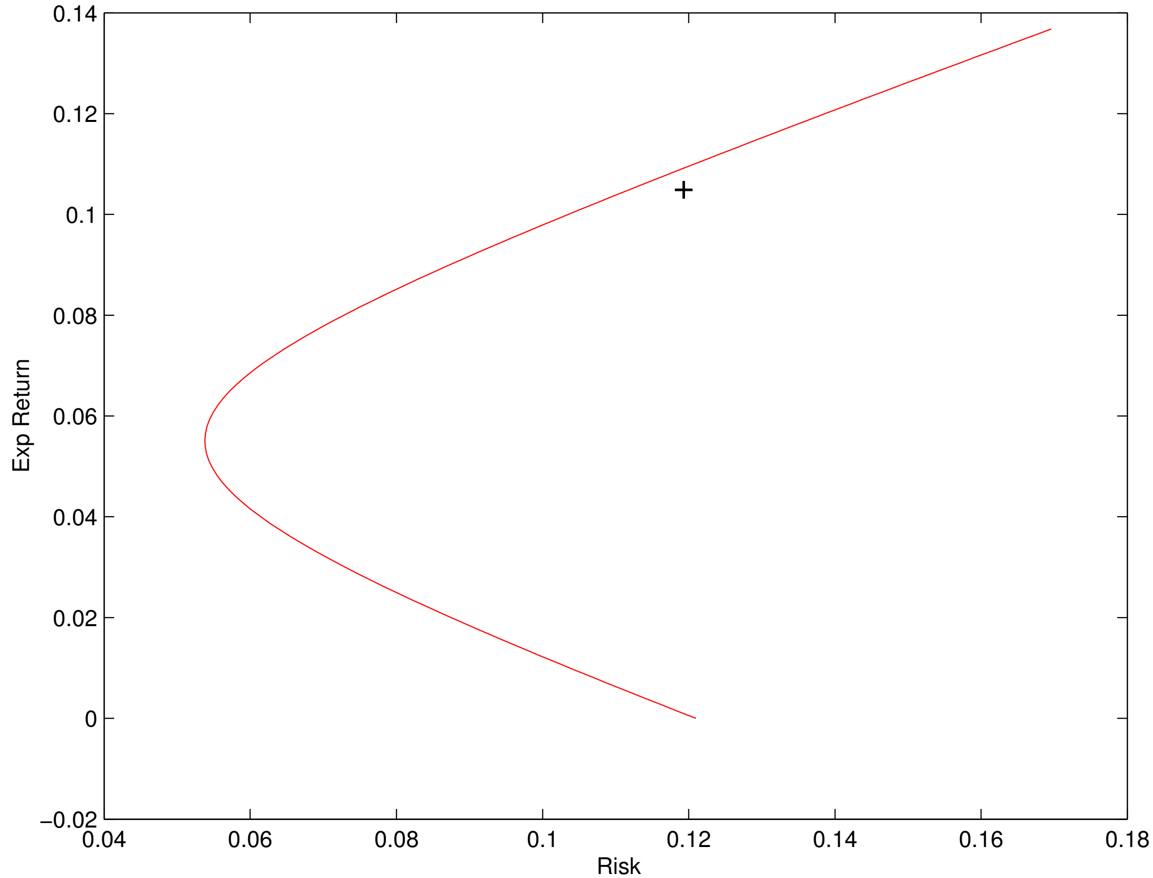

Figure 2: The efficient frontier based on the true population parameters. Equally weighted portfolio marked with a + sign. Note that it lies inside the efficient frontier. Short sales are allowed

Estimation Risk

- DC plan member will not know the true values of the expected return vector $\boldsymbol{\mu}$ and the variance-covariance matrix $\boldsymbol{\Sigma}$.
- need to estimate $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$. Various methods. One way is to estimate them from historical data.
- Historical estimates for $\boldsymbol{\Sigma}$ are robust, but that the estimates for $\boldsymbol{\mu}$ are very noisy, even in a stationary market.
- This estimation risk has important consequences for the optimal portfolio selection problem.
- Black and Litterman (1992) suggested using reverse optimization and implied expected returns in lieu of the historically estimated returns.
- Other methods involve using CAPM models to infer reasonable values expected return vector.

Parameter Estimation Risk

The efficient frontier calculation assumes that the investor knows the both the expected return vector, $\boldsymbol{\mu}$ and the variance-covariance matrix, $\boldsymbol{\Sigma}$.

We denote the mean-variance optimal portfolio based on the true population parameters by $\mathbf{x_{true}}$.

Suppose that the estimates of the expected return and variance-covariance matrix based on sample of M historical observations are denoted by $\widehat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{\Sigma}}$. Denote the mean-variance optimal portfolio based on these estimated parameters by $\mathbf{x_{est}}$. We can distinguish three frontiers:

Three frontiers

- *The true frontier*. In this case the expected return and variance are computed as follows:

$$\mathbf{x_{true}}^T \boldsymbol{\mu}, \qquad \mathbf{x_{true}}^T \boldsymbol{\Sigma}\, \mathbf{x_{true}} \ . \tag{9}$$

  True frontier is unattainable since we do not know the true parameters, and hence cannot compute $\mathbf{x_{true}}$.

- *The estimated frontier*. In this case the expected return and variance are computed as follows:

$$\mathbf{x_{est}}^T \widehat{\boldsymbol{\mu}}, \qquad \mathbf{x_{est}}^T \widehat{\boldsymbol{\Sigma}}\, \mathbf{x_{est}} \ . \tag{10}$$

  The estimated frontier is based on the portfolio $\mathbf{x_{est}}$, which is derived using the estimated weights, and hence can be calculated. It does not provide a sensible comparison since in reality the future returns will be drawn from the true distribution and not the distribution given by the sample estimates.

- *The actual frontier*. In this case the expected return and variance are computed as follows:

$$\mathbf{x_{est}}^T \boldsymbol{\mu}, \qquad \mathbf{x_{est}}^T \boldsymbol{\Sigma}\, \mathbf{x_{est}} \; . \tag{11}$$

  The actual frontier depicts the performance of the portfolios constructed using the sample estimates, $\mathbf{x_{est}}$, under the true distribution of returns, $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$.

The true frontier must by construction dominate the actual frontier. We note that the true frontier is the unattainable ideal, the estimated frontier is illusory, and the actual frontier is the most realistic one for many comparison purposes.

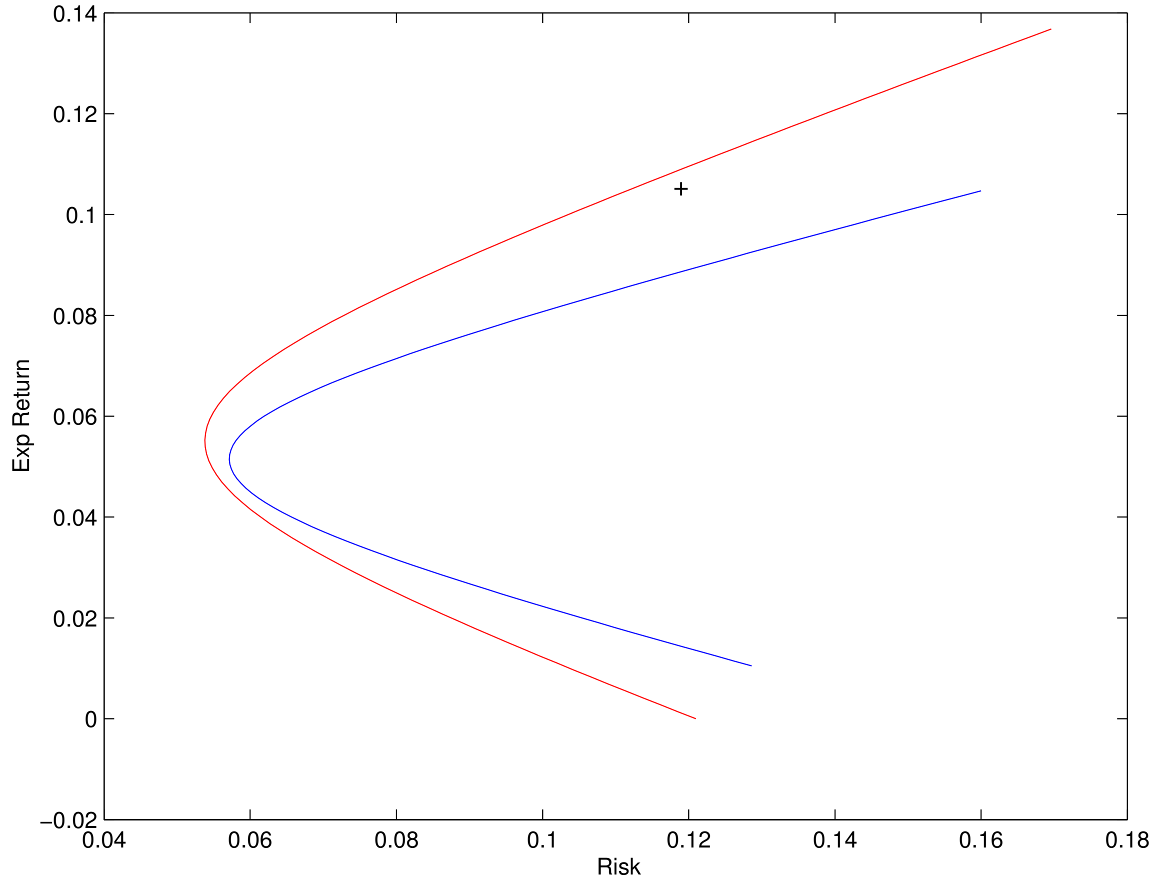

Figure 3: The efficient frontier based on the true population parameters. Actual frontier is in blue. Equally weighted portfolio marked with a + sign. Short sales are allowed

DC Plan case

Since we analyze investment decisions of employees in defined contribution pension plans where short sales are not permitted we impose the condition:

$$\mathbf{x} \geq \mathbf{0}\ . \tag{12}$$

Now we do not obtain an analytical solution for the optimal portfolio weights. However we can solve the optimization problem using numerical quadratic programming methods. Details are given by Best and Grauer(1990). Of course negative weights are ruled out and this affects the solutions. In the next graph we take the average of one hundred actual portfolios.
The true frontier is in red. We estimate values for $\widehat{\boldsymbol{\mu}}$ and $\widehat{\boldsymbol{\Sigma}}$ using one hundred simulated samples generated using the the true population parameters. Each sample consists of sixty monthly observations.

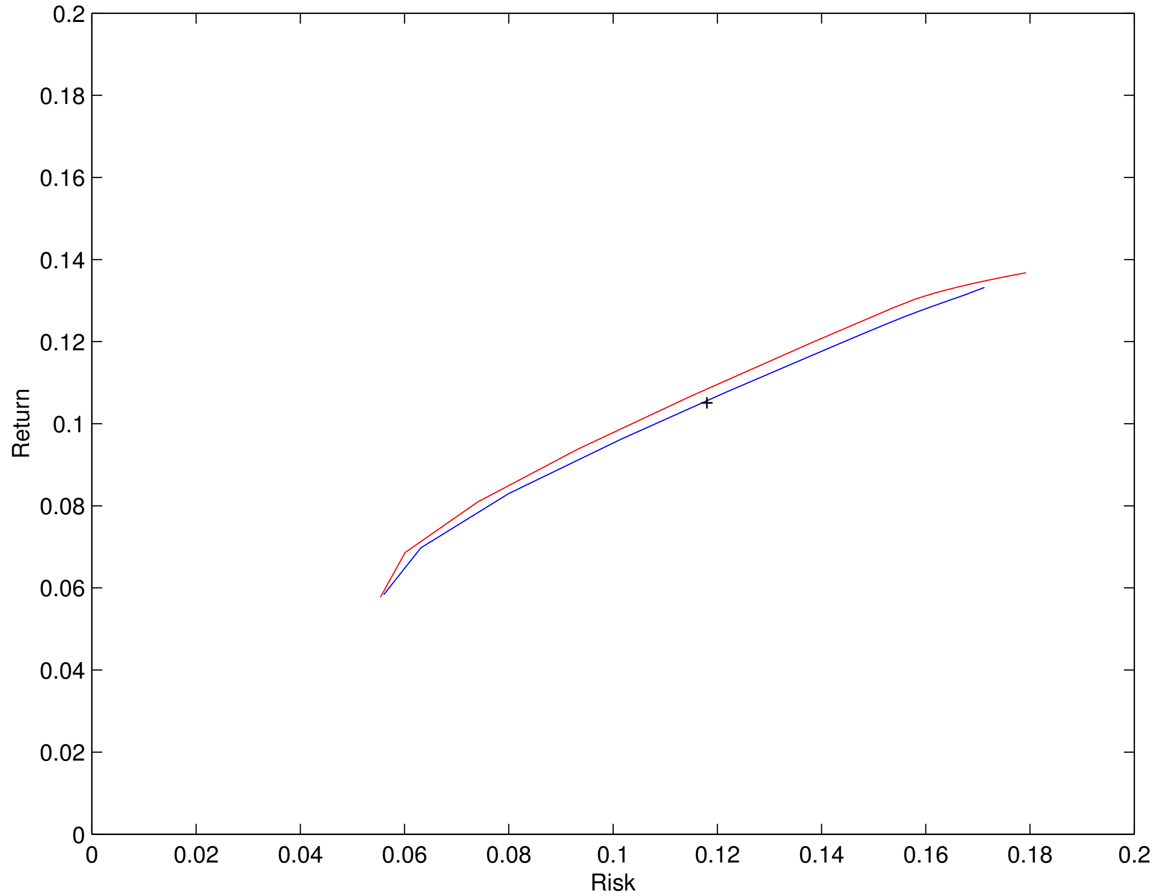

Figure 4: The top curve denotes the true frontier. The lower curve represents the average of one hundred actual frontiers based on one hundred simulated samples. The equally weighted portfolio is denoted by the + sign. No short sales.

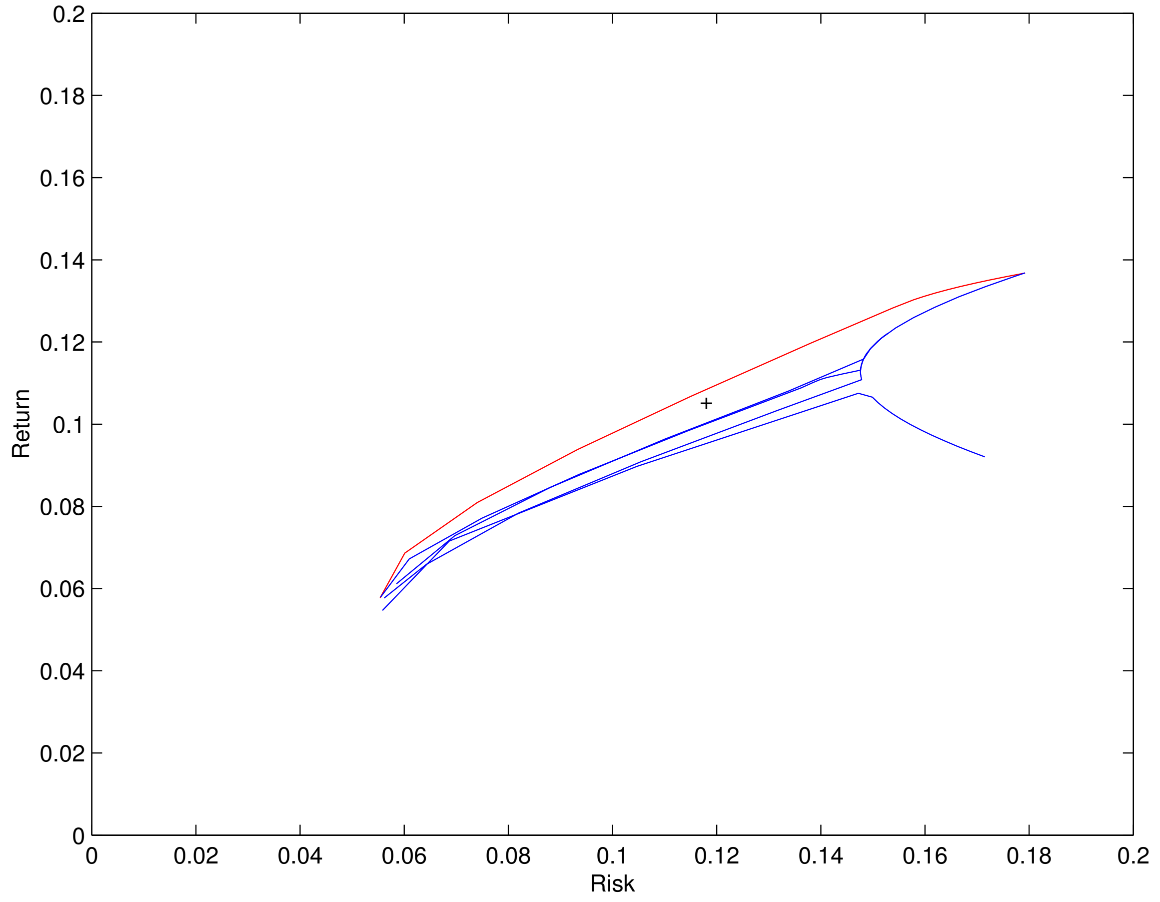

Figure 5: The true frontier is the topmost curve and the other four curves represent the four worst actual frontiers based on one hundred simulated samples. The equally weighted portfolio is denoted by the + sign. No short sales.

Conclusions: Sub sole nihil novi est

- Some DC plan members use $\frac{1}{n}$ investment strategy
- This is a very simple heuristic. Easy to implement
- Markowitz optimal portfolio very sensitive to estimation of the mean
- The $\frac{1}{n}$ rule protects investor against bad outcomes
- Jobson and Korkie (1980) state that *"naive formation rules such as the equal weight rule can outperform the Markowitz rule."*
- Michaud(1998) notes because of that estimation risk *an equally weighted portfolio may often be substantially closer to the true MV optimality than an optimized portfolio.*

Puzzle: Own Company Stock

The own company stock puzzle

- 29% of assets of defined contribution plans are in company stock
  - Mitchell and Utkus (2002); Meulbroek (2002)

- 33% of assets in retirement plans for a sample of S&P 500 firms are invested in company stock
  - Benartzi (2001)

- 25% of **discretionary contributions** are invested in company stock
  - Benartzi (2001)

Inefficient to put a large amount in any one stock especially in own company stock

US Plans offering own-company stock in plan

Mitchell and Utkus (2002)

- 42% of all defined contribution plan participants
- 59% of total plan assets
- 23 million employees have this investment option available
- Total assets of these plans is approximately $1.2 trillion
- Higher percentage in own company stock for plans of large firms

Standard Investment Models

- Markowitz(1952) one period mean variance model
- Merton continuous time framework: used optimal stochastic control
  1. Lognormal asset prices
  2. Power utility. Relative risk aversion γ
  3. Constant interest rate $\bar{r}$
- For N risky assets, the portfolio weights π are

$$\pi = \frac{1}{\gamma}\Sigma^{-1}(\mu - \bar{r}e)$$

where $\pi, \; e, \; \mu$ are $N \times 1$ vectors and Σ is the variance covariance matrix

A Simple Example to show the intuition

- Consider a simplified example.
- This will help interpret our findings: Provides a metric
- Suppose we have N stocks
- All stocks have the same mean μ.
- They all have the same variance σ^2.
- Pairwise correlation between all stocks is ρ -a constant.
- The variance of the equally weighted portfolio is

$$\frac{\sigma^2}{N} + (1 - \frac{1}{N})\rho\sigma^2 \approx \rho\sigma^2$$

Example

$$\Sigma = \begin{bmatrix} \sigma^2 & \rho\sigma^2 & \cdots & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 & \cdots & \rho\sigma^2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho\sigma^2 & \rho\sigma^2 & \cdots & \sigma^2 \end{bmatrix}. \tag{13}$$

Assume

$$\mu = \begin{bmatrix} .12 \\ .12 \\ \vdots \\ .12 \end{bmatrix}$$

$r = .05$ and $\rho = .5$.

Suppose $N = 100$ then we can compute that the optimal portfolio holding is $\frac{1}{N} = .01$ in each stock.

Increase return on first stock

- Make one change
- Investor expects that the return on the first stock is 13% instead of 12%
- Recompute the optimal portfolio weights using the Markowitz(Merton) model
- In this case the investor puts 15.3% of her total investment(in risky assets) in the first stock and spreads the balance of 84.7% evenly over the other (99) stocks.

Impact of different returns on first stock

Here we see what happens to optimal weights as we vary the return on the first risky asset.

Excess Return on first stock %pa	Percentage invested in first stock	Percentage in rest
0	1	99
1	15.3	84.7
2	29.5	70.5
3	43.7	56.3

Model

Boyle Uppal and Wang(2003), Ambiguity Aversion and own Company Stock in DC Pension Plans

Boyle's law

Dedicated to Hans Gerber. When working with co authors you should ensure

1. That they are smarter than you
2. That they are younger than you(not necessary)
3. That they are further down the alphabet

For example one of my favorite coauthors is Robert Zvan ie Zvan Robert

Overview of Model

Boyle Uppal and Wang: Basic Idea

- Investor is uncertain about the true model
- Worries that model she uses is mis-specified
- Wants decision to be robust against model mis-specification
- Evaluates worst case alternative model against a reference model
- Parameter measures strength of preference for robustness
- Maximize expected utility over worst case scenarios
- Gilboa and Schmeidler(1989), Andersen Hansen and Sargeant(1999), Maenhout(1999), Uppal and Wang(2003)

Quick Outline

- Continuous time model
- Utility defined recursively
- Agent can have multiple priors
- Use relative entropy as a penalty function to capture concern about model misspecifications
- Agents use reference model to differentiate among priors
- This formulation leads to enough differentiability to use the HJB approach
- Solve for an equilibrium model where it is optimal for agents to hold own company stock

Notation

- Investor can invest in both own company stock and market
- Variance of market is σ^2_{mar} and variance of own company stock is σ^2_{own}
- Assume all stocks have the same variance, σ^2, and all pairwise correlations are equal to ρ

$$\sigma^2_{mar} = \rho\sigma^2$$

- Subjective parameter reflecting investor's aversion to ambiguity about own company stock is $\phi_{own} \geq 0$
- Subjective parameter reflecting investor's aversion to ambiguity about the market is $\phi_{mar} \geq 0$

Closed Form expression for Portfolio weights

The expression for the optimal portfolio is

$$\pi = \frac{1}{\gamma} (\Omega A)^{-1} (\mu - r\mathbf{1}) \tag{14}$$

where A is a two by two matrix that includes impact of ambiguity.

$$\Omega A = \begin{bmatrix} \sigma_{own}^2(1 + \phi_{own}) & \sigma_{mar}^2 \\ \sigma_{mar}^2 & \sigma_{mar}^2(1 + \phi_{mar}) \end{bmatrix} \tag{15}$$

Note if $\phi_{own} = \phi_{mar} = 0$ we recover the Merton result: no investment in own company stock.

Parameter values

- σ_{mar} is assumed to be 20% p.a.
- Based on empirical work of Chan, Karceski and Lakonishok (1999) we assume that
  - Average volatility of large firms is 28.3%,
  - Average volatility of medium firms is 36.1%,
  - Average volatility of small firms is 44.7%,
- We characterize company profiles by selecting $\sigma = \{ \underbrace{28.3\%}_{\text{Table 1}}, , , , \underbrace{36.1\%}_{\text{Table 2}}, \quad \underbrace{40\%}_{\text{Table 3}} \}$
- We can calibrate the model and relate the $\phi' s$ to observed proportions of own company stock. Can also express as equivalent drift and variance adjustments.

Table 2: Asset allocations for large firms: $\sigma = \sigma_{own} = 0.283$

ϕ_{mar}	Risk adjustment	Drift adjustment	Portfolio weights $\frac{\pi_{own}}{\pi_{own}+\pi_m}$	$\frac{\pi_m}{\pi_{own}+\pi_m}$
Panel with $\phi_{own} = 0$				
0.00	1.00	0.000	**0.00**	1.00 ◀
0.25	**1.25**	**0.012**	**0.20**	0.80 ◀
0.50	**1.50**	**0.018**	**0.33**	0.67 ◀

Table 3: Asset allocations for medium firms: $\sigma = \sigma_{own} = 0.361$

ϕ_{mar}	Risk adjustment	Drift adjustment	Portfolio weights $\frac{\pi_{own}}{\pi_{own}+\pi_m}$	$\frac{\pi_m}{\pi_{own}+\pi_m}$

- Panel with $\phi_{own} = 0$

0.00	1.00	0.000	**0.00**	1.00 ◀
0.25	**1.25**	**0.013**	**0.10**	0.90 ◀
0.50	**1.50**	**0.020**	**0.18**	0.82 ◀

Summary of Model

- Simple model for portfolio choice when investors are uncertain about true distribution of asset returns.
  - Model allows investors to incorporate their ambiguity about one class of assets relative to others.

- If agents are ambiguous about market return then they will over invest in own-company stock.
  - True even if ambiguous about own-company stock returns.

- Calibration from $\phi's$ to simple adjustments

- Model generates realistic numbers consistent with empirical observations.

- Model predicts that own company stock holdings will be highest for large firms

Suum cuique pulchrum est(Cicero)

- To each his own is beautiful
- Chacun trouve ses propres oeuvres belles

Some thoughts

- Pensions are increasingly important:Increasing longevity
- DC plans transfer investment risk to employees
- Unsophisticated agents must make the asset allocation decision
- Need a basic paradigm
- Very tough problem: these features
  1. Multi period
  2. Which assets to select
  3. Determine return dynamics
  4. Model non stationary
  5. Estimation of model and parameters
  6. Importance of financial econometrics
  7. Optimization
  8. Implementation
- Little wonder agents use simple heuristics

The reality

- Employee face a jungle of professional jargon
- Good advice is hard to find and how does employee know it is good.
- Analogy.
  1. Eye surgery requires considerable training, deep technical knowledge and experience.
  2. So does asset allocation
  3. Imagine giving someone a brochure and expecting her to be a successful eye surgeon
  4. Why should we expect the brochure approach to work for asset allocation?
- Actuaries insurance experts, academics and finance folks have an opportunity(responsibility?) to produce simple sensible approaches that can address this complex task