Realistic Tables with Competing Risks

Application to Inception Rates Estimation for Long Term Care Insurance

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Agenda

Long Term Care Insurance Framework

- In France, LTC insurance provides benefits for elderly people suffering from a loss of mobility and autonomy in their activity of daily living.
- In addition to the social benefits.
- LTC insurance may be individual or collective.
- Payment of benefits depends to the level of dependency.

Long Term Care Insurance Framework

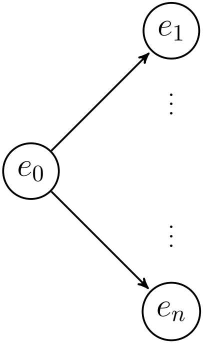
- Pricing, reserving and managing LTC risks strongly depend to the tables selected.
- LTC risks are tricky to estimate:
  - the first french products dates from the early of 80's,
  - only a few insurers have reliable data with sufficient amount (but for higher levels of dependency and not at older ages),
  - definitions and grids are not unique,
  - covariate effects (gender, place, pathology, etc.) are rarely taking into account.
- Forecasting LTC risks (longevity, disability and mortality) are also a very difficult exercice.
- Pratitioners often use empirical methods and expert opinions.

Motivation

- Constructing realistic tables for inception rates in dependency is an important challenge in Solvency II perspective.
- Distinguing entry by pathology is very usefull as:
  - waiting periods in contracts depend on the type of desease,
  - pathologies have a major role in the survival of LTC claimants.
- In presence of competing risks, practioners often use techniques based on latent failure times and arbitrary choices for modelling dependence between them.
- Aims of our approach:
  - use a more relevant multistate approach to estimate inception rates with right censored data,
  - measure the biais that the common approach used by prationers comprises.
- Limits of our approach:
  - a non-parametric approach requires a signicant amount of reliable data and does not permit forecasting to older ages,
  - longitudinal data are required.

Competing Risks Model

- A competing risks process models adequately both the entry in dependency according to different pathologies and other exit causes (e.g. death, cancellation).



Example of exit causes with 4 types of pathology.

	Exit causes
e_1	Neurologic pathologies
e_2	Various pathologies
e_3	Terminal cancers
e_4	Dementia
e_5	Death
e_6	Cancel

- Benefits and premiums are paid according to the papern of states of the policyholder.

Quantities of Interest

We introduce a Markov process $(X_t)_{t\geq 0}$ with finite state space $\mathcal{S} = \{e_0, e_1, \ldots, e_n\}$ where the state e_j represents the j-th exit cause of the initial state. T represents the survival time in initial state and C the right-censoring time.

- Transition probability

$$\begin{aligned} p_{0j}(s,t) &= \mathbb{P}\left(X_t = e_j \mid X_s = e_0\right) \\ &= \mathbb{P}\left(T \leq t, X_T = e_j \mid T > s\right). \end{aligned}$$

- Cause-specific hazard

$$\mu_{0j}(t) = \lim_{\Delta t \to 0} \frac{p_{0j}(t, t+\Delta t)}{\Delta t}.$$

- Latent failure time

$$T_{0j} = \inf_{t\geq 0}(X_t = e_j).$$

Technical provisions

- Technical provisions correspond to the expectation of future discounted cash-flows relating to the contract.
- With a policyholder aged x years at t_0, reserves at time $t \geq t_0$ are

$$\sum_{j \neq e_0} \int_{t-t_0}^{\infty} B\left(t, t_0+\tau\right) p_{00}\left(x, x+\tau\right) \mu_{0j}\left(x+\tau\right) c_j\left(x+\tau\right) d\tau$$

$$-\int_{t-t_0}^{\infty} B\left(t, t_0+\tau\right) p_{00}\left(x, x+\tau\right) b\left(x+\tau\right) d\tau.$$

- Inception rate $q_j\left(t\right)$ appears when we approximate the above formula

$$\approx \sum_{j \neq e_0} \sum_{k=t-t_0}^{\infty} B\left(t, t_0+k+1\right) p_{00}\left(x, x+k\right) q_j\left(x+k\right) c_j\left(x+k\right)$$

$$-\sum_{k=t-t_0}^{\infty} B\left(t, t_0+k\right) p_{00}\left(x, x+k\right) b\left(x+k\right).$$

Classical Methods

- A extensive literature is dedicated to infer competing risks model:
  - methods based on latent failure times (e.g. Prentice *et al.* (1978)),
  - proportional hazards models (e.g. Fine and Gray (1999)),
  - multistate approaches (e.g. Andersen *et al.* (2002)).
- Practioners often use latent failure times approachs (e.g. Deléglise *et al.* (2009)) but:
  - latent failure time variable T_{0j} is artificial,
  - dependence structure between latent failure times is unknow.
- These last estimators may overestimate the inception rates (see Gooley *et al.* (1999)).

Multistate Approach

We aim to estimate inception rates with a non-parametric multistate approach (see Andersen *et al.* (1993)).

- We observe continuously M independent policyholders $\left(\bar{T}^m, V^m\right)_{m=1,\ldots,M}$ where $\bar{T}^m = \min\left(T^m, C^m\right)$ and $V = X_T$ the failure cause.
- The cumulative cause-specific hazards are estimated with Nelson-Aalen estimator.
- The inception rates are consequently obtained with the Aalen-Johansen estimator

$$\hat{q}_j\left(t\right) = \sum_{\left\{m,t<\bar{T}^{(m)}\leq t+1\right\}} \hat{S}\left(\bar{T}^{(m)}-\right) \frac{\mathbb{1}_{\left\{V^{(m)}=j\right\}}}{Y_0\left(\bar{T}^{(m)}\right)}.$$

- These estimators are asymptotically normally distributed.

Latent Failure Times Approach

- Praticitioners usually assume latent failure times are independent and then we have

$$\mathbb{P}\left(T_{0j} > t\right) = \exp\left(-\int_0^t \mu_{0j}\left(\tau\right) d\tau\right).$$

- Consequently, the inception rates per event are estimated with Kaplan-Meier estimator and noted $q_j^*\left(t\right)$.
- Then, arbitrary dependence rule are applied on $q_j^*\left(t\right)$ to verify the egality

$$1 - \sum_{j=1}^{n} q_j\left(t\right) = p_{00}\left(t, t+1\right).$$

Latent Failure Times Approach

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  - $\ldots$
  - $\breve{q}_{j_n}(t) = \hat{q}^*_{j_n}(t)\prod_{k=1}^{n-1}\left(1 - \hat{q}^*_{j_k}(t)\right).$

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- The new estimator $\breve{q}_j(t)$ has upper and lower bounds

$$b_j^-(t) = \hat{q}^*_j(t)\prod_{k\neq j}(1 - \hat{q}^*_k(t)) \leq \breve{q}_j(t) \leq \hat{q}^*_j(t) = b_j^+(t).$$

Estimation with Multistate Approach

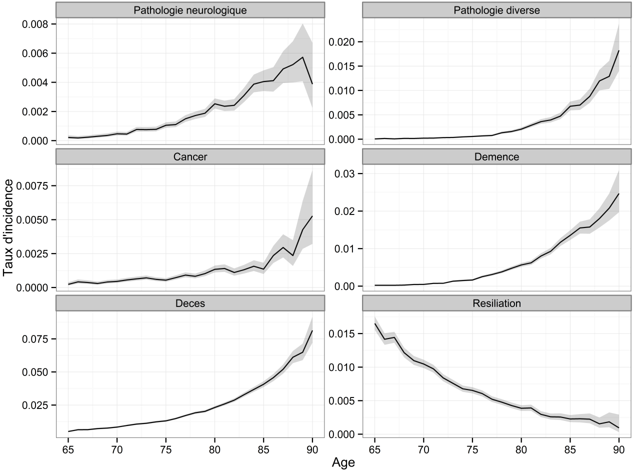

Figure: Inception rates estimate with approximate pointwise 95% confidence intervals

Smoothed Rates

- We smooth the crude inception rates with non-parametric Whittaker-Henderson model.
- Smoothing parameters are selected on the basis of residuals analysis and by regarding the following fitting criteria:
    - cross validation and generalized cross validation,
    - AIC and AICC.

Smoothed Rates

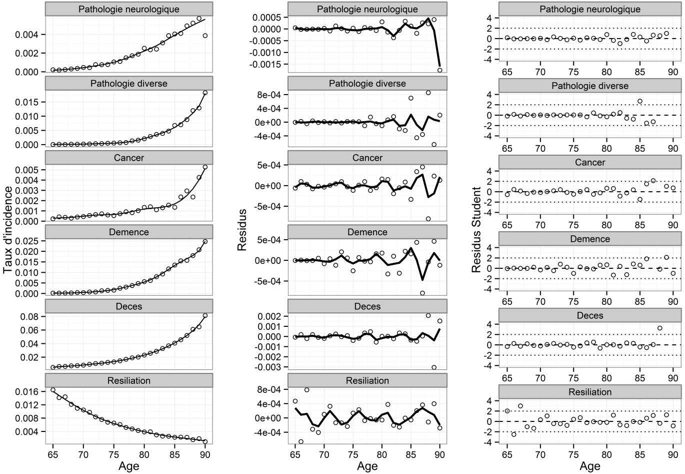

Figure: Smoothed rates, residuals and Student residuals

Measuring Estimation Risk

- We consider systematic risks induced by the construction of such tables and resulting from:
  - estimation of crude rates (due to sampling variation),
  - parameters estimation.
- These risks are taken into account with non-parametric bootstrap.

Measuring Estimation Risk

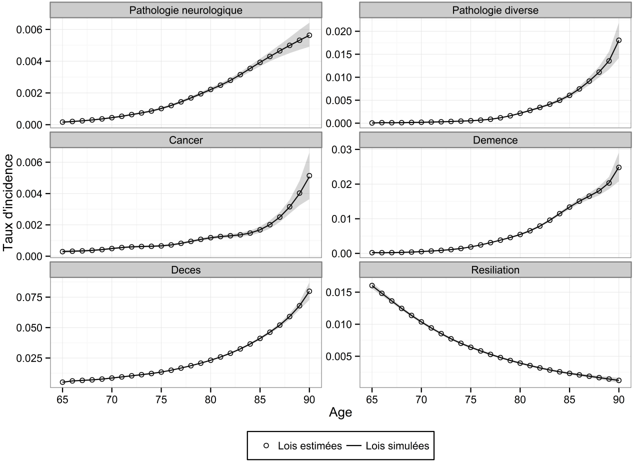

Figure: Simulated rates ($K = 1000$) with 95% simulated confidence intervals

Measuring Estimation Risk

- We compute dispersion coefficients

$$c\left(\psi_{jx}\right) = \frac{\sqrt{\sum_{k=1}^{K}\left(\tilde{q}_j^k\left(x\right) - \tilde{q}_j\left(x\right)\right)^2}}{\tilde{q}_j\left(x\right)}.$$

- The risk is relatively important and should be considered carefully for technical provisions valuation.

Exit causes	**Average** $c\left(\psi_{jx}\right)$
Neurologic pathologies	6.01%
Various pathologies	12.12%
Terminal cancers	7.94%
Dementia	6.53%
Death	2.05%
Cancel	3.76%

Comparing with the latent failure times approach

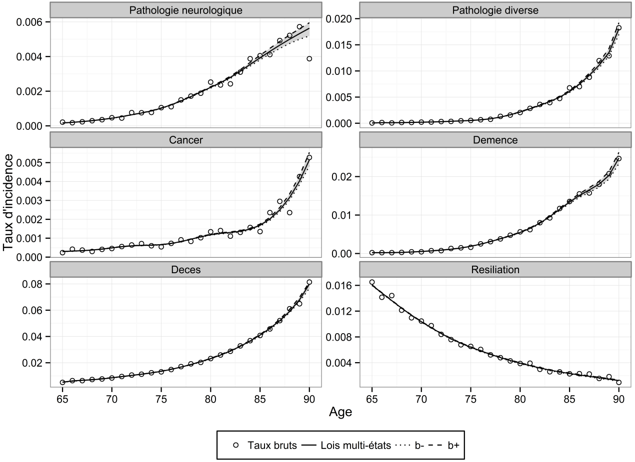

Figure: Comparing multistate and latent failure times approachs

Comparing with the latent failure times approach

- We compare 4 differents priority orders.
- At 65 years old, the larger gap measured on technical provisions is around 2.5%.
- Priority orders should be selected soundly!

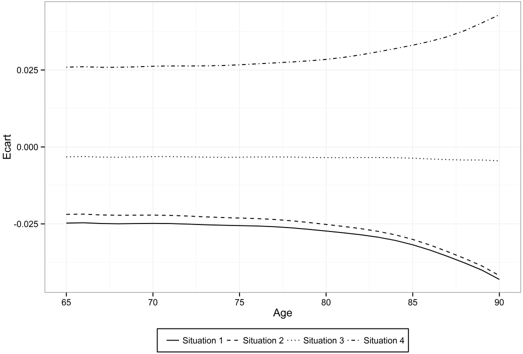

Figure: Gap on technical provisions with the both approachs

Summary

- Multistate approaches are rarely used by pratictioners which prefer Kaplan-Meier estimators and marginal approachs for crude rates estimation.
- Multistate approachs are a little more complex but theorically more appropriate for competing risks as the joint distribution of latent failure times are unobserved.
- A non-parametric approach may difficult to implement due to the lack of available data but:
  - it provides an realistic fit that one can expected in Solvency II perspective.
  - it can be used to perform goodness-of-fit tests.
- Biais observed with latent failure times approach depends on the treatment of priorities applied on each cause. In our application, a sound choice may reduce significatively this biais and justify the use of latent failure times approach.

- Outlook
  - Extrapoling inception rates to older ages.
  - Taking into account other sources of heterogeneity.

Thank you for your kind attention.

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