Asset allocation under shortfall constraints

Finding a balance between seeking gains and defending against adverse performance.

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Over the long term, equity investors have been richly rewarded for the risks that they have endured. For example, during the 1926–1987 period, the S&P provided an annual return advantage of 6.8%, compared with long-term corporate bonds. Over shorter periods, by contrast, stocks actually underperformed cash on a surprisingly frequent basis. In particular, stocks have underperformed Treasury bills over the past fifteen years in almost 35% of six- to eighteen-month time periods (see Salomon et al. [1990]).

Few professional investors are able to observe calmly and passively while high volatility buffets their portfolio's value over the short run, and most fund sponsors control overall risk by adjusting the extent of their equity position. By adding cash or bonds and thereby lowering equity exposure, fund sponsors reduce portfolio volatility. At the same time, they give up a portion of the risk premium that equity offers; decreased exposure to equity leads to a reduction in expected returns.

In this article we focus on the balance between risky and risk-free assets. Although we use equity as the proxy for *all* the risky assets in a portfolio, our methodology applies equally to any basket of risky assets. We offer a simple model of how to quantify risk tolerance and then use it to determine the maximal equity investment.

We measure downside risk by the "shortfall probability" relative to a minimum return threshold. By specifying both this threshold and a shortfall probability, we can establish a "shortfall constraint" to determine the maximum allocation to risky assets. (See Leibowitz et al. [1990] for full details.)

We also consider the sensitivity of the risky asset allocation to changes in volatility, equity risk premium, return threshold, and shortfall probability. Finally, we show how this methodology can be applied to multi-year investment horizons.

THE EFFICIENT FRONTIER FOR AN EQUITY/CASH PORTFOLIO

A portfolio manager with a well-established horizon always has a continuum of choices between risky and riskless assets. For example, over a one-year investment horizon, the one-year Treasury STRIP provides a riskless return equal to its yield; that is, this "cash" asset has no return volatility. However, modern theory suggests that a holder of risky assets should be compensated for the associated volatility (risk) by means of a positive increment in expected return — the so-called risk premium. Current estimates of the equity risk premium for U.S. equities range from a 4% expected return advantage to a 6% expected return advantage.

Because cash does not have any return volatility, the volatility in an equity/cash portfolio reflects entirely the proportion of equity in that portfolio, and the portfolio manager can control volatility risk by adjusting the equity/cash balance. As the percentage of equity increases, so does both portfolio risk and expected return. Figure 1 illustrates the linear rela-

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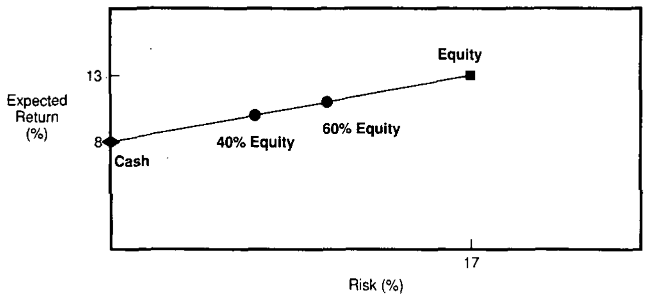
FIGURE 1

THE EFFICIENT FRONTIER FOR AN EQUITY/CASH PORTFOLIO (ONE-YEAR HORIZON)



tionship between the expected return and risk for the full spectrum of equity/cash portfolios over a one-year holding period.

Assumptions for Figure 1 are that the riskless asset yields 8%, the equity risk premium is 5%, and the expected return is equal to the nominal yield. The risk measure is the standard deviation of returns, which we assume is 17% for equity. If "equity" is taken to represent the market portfolio of risky assets, the straight line in Figure 1 can be interpreted as the "efficient frontier" that represents portfolios that provide the maximal return for any given level of risk. The leftmost point of the efficient frontier represents a portfolio with 0% equity, while the far right represents 100% equity. We indicate the location of those portfolios that consist of 40% equity and 60% equity.

THE SHORTFALL LINE

The equity portfolio manager faces a critical strategic decision on the appropriate extent of the equity position. Determination of the "right" equity/cash balance depends ultimately on the fund's risk tolerance. Here we quantify risk tolerance in a simple and intuitive manner by considering first the minimum return that can be tolerated over a given investment horizon. For purposes of exposition, we assume that the plan sponsor believes that it is worth risking a one-year return as low as 3% for the potential gain that can be achieved from equity investment.

While investment in a one-year 8% Treasury STRIP ensures an 8% return, there can be no such minimum return guarantee with an equity investment. Yet by adjusting the equity/cash balance, it is possible to lower the probability of failing to meet the 3% minimum return objective. In particular, we seek to fulfill the following "shortfall constraint" under the assumption that returns are normally distributed:

There must be a probability of 10% or less that returns fall below a 3% threshold over the one-year horizon.[1]

This shortfall constraint will lead to a "shortfall line" that divides the return/risk diagram into two regions. All portfolios that have return/risk characteristics that place them in the upper region will meet or exceed the shortfall constraint. Those portfolios that fall in the lower region will fail to satisfy the shortfall constraint.

To understand how the shortfall line is constructed, we first consider all portfolios that have an expected return of 3%. Such portfolios are represented in Figure 2 by the horizontal line at the 3% return level. Each point on this line represents a different degree of volatility, with higher volatilities leading to more spread out distributions. Thus, as illustrated, the distribution that corresponds to a standard deviation of 5% has a higher concentration of returns near 3% than the distribution that corresponds to a standard deviation of 8%.

In all cases, however, 50% of the returns fall below the expected value of 3%; that is, there is a 50% shortfall probability. The lower tail of the distribution, which is shaded in Figure 2, is called the "shortfall region." The size of the shortfall region corresponds to the shortfall probability.

Now we focus our attention on the portfolio with a standard deviation of 5%. To reduce the size of the shortfall region to 10%, we must push up the distribution (that is, raise the expected return to 9.4%), so that only 10% of the returns fall below 3%. In a similar manner, by sufficiently raising the expected return at all risk levels, we create the 10% shortfall line in Figure 3.

FIGURE 2

PORTFOLIOS WITH A 50% PROBABILITY OF EXCEEDING A 3% RETURN

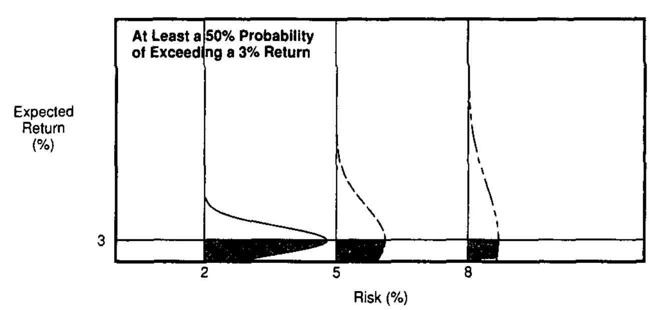
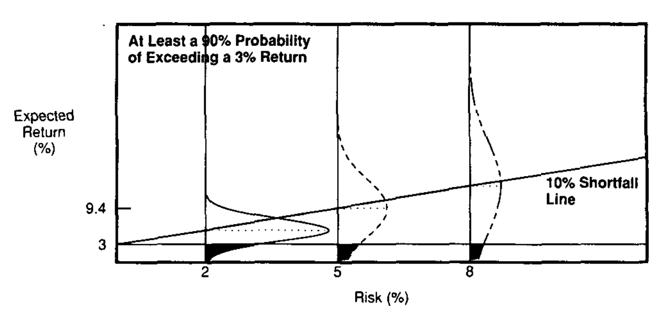


FIGURE 3

PORTFOLIOS WITH A 90% PROBABILITY OF EXCEEDING A 3% RETURN



It can be shown that, under a wide range of conditions, the shortfall constraint always leads to a straight line in the expected return/risk diagram. Comparing Figure 2 with Figure 3, you can see that both shortfall lines emanate from the threshold point of 3% on the vertical axis.

Of course, the 50% shortfall line of Figure 3 is horizontal (that is, it has a slope of 0), while the 10% shortfall line in Figure 4 has a positive slope. Generally, more stringent shortfall probabilities require more steeply sloped shortfall lines.

FIGURE 4

THE SHORTFALL LINE

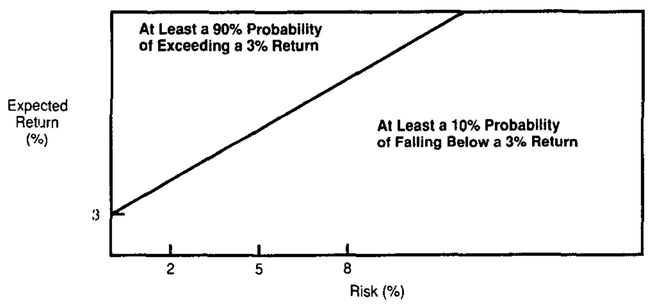


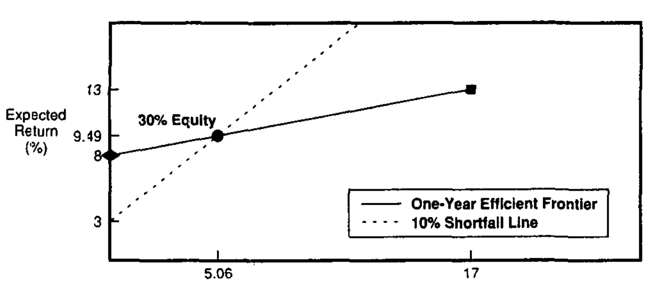
Figure 4 presents the shortfall line on a different scale. Note that all portfolios above the line have sufficiently large expected returns so that they offer at least a 90% probability of a 3% or greater return. Similarly, all portfolios below the line have less than a 90% probability of producing returns above 3%.

THE SHORTFALL CONSTRAINT AND THE EFFICIENT FRONTIER

We achieve our goal of locating portfolios that meet or exceed the shortfall constraint by superimposing the shortfall line of Figure 4 on the efficient frontier in Figure 1. In Figure 5, note that *all* points on the efficient frontier that lie above the shortfall line will meet or exceed the requirement of at most a 10% probability of returns below the 3% threshold.

FIGURE 5

THE SHORTFALL CONSTRAINT AND THE EFFICIENT FRONTIER (ONE-YEAR HORIZON)



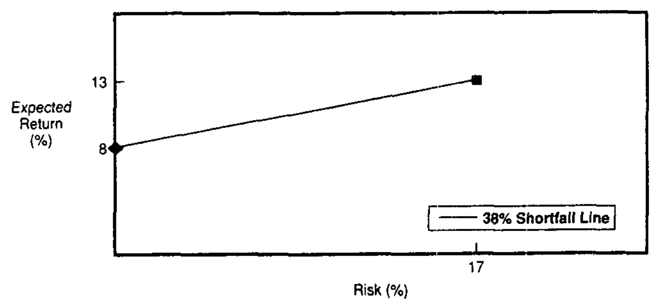
The maximum equity holding that is consistent with this shortfall constraint is found at the intersection of the shortfall line and the efficient frontier. As the graph indicates, this intersection point corresponds to a 30%/70% equity/cash portfolio. The expected return of this portfolio is 9.49%, and its standard deviation is 5.06%.

The low percentage of equity in the portfolio at first may seem counter-intuitive. Actually, it reflects the powerful impact of the high volatility of equity over a one-year horizon. Much larger equity percentages become feasible as we move to longer investment horizons.

Further insight into equity allocation may be gained by observing that the efficient frontier in Figure 5 is itself a shortfall line that corresponds to an 8% minimum return threshold, because it emanates from the 8% point on the return axis. In fact, the slope of the efficient frontier corresponds to a 38% probability of shortfall (see Figure 6).

FIGURE 6

A SHORTFALL INTERPRETATION OF THE EFFICIENT FRONTIER



Such a shortfall line implies that *all* portfolios with greater than 0% equity have a 38% probability of a one-year return below the risk-free rate of 8%. In this context, it is not surprising that a portfolio manager would want to hold only a limited amount of equity, given a *strict* one-year horizon (and with no market view other than that implied by the expected return estimates).

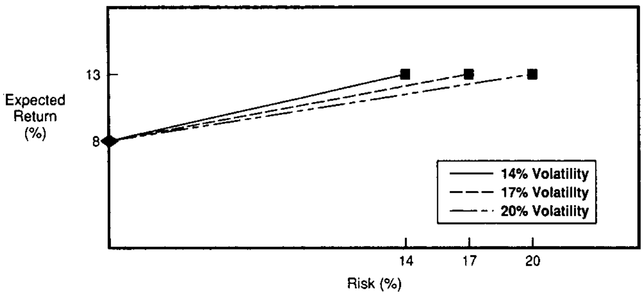
SENSITIVITY TO ALTERNATIVE VOLATILITY AND RISK PREMIUM ESTIMATES

Our example assumes that equity volatility is 17% over a one-year period. Because volatility is, in fact, not constant but varies with changing market conditions, we must test the sensitivity of the equity allocation to variations in volatility.

The impact of changes in volatility is illustrated in Figure 7, where the end point of the efficient frontier shifts horizontally as volatility varies. Observe that lower volatilities increase the slope of the efficient frontier. Consequently, with lower volatility, as we

FIGURE 7

THE EFFICIENT FRONTIER WITH ALTERNATIVE VOLATILITY ESTIMATES



should expect, the maximum admissible equity allocation increases. This increase in equity allocation is evident in Figure 8, which superimposes the shortfall line of Figure 4 on the efficient frontiers from Figure 7.

Note that the effect of volatility on the allowable equity holding is asymmetric. A 3% increase in the volatility estimate lowers the equity percentage by 6%, while a 3% decrease in volatility raises the equity percentage by 9%.

Next we consider the impact of changes in estimates of the risk premium on the equity allocation. Figure 9 shows both the shortfall line and the efficient frontiers for risk premiums of 3%, 5%, and 7%. Changing the risk premium moves the end point of the efficient frontier vertically, yet its slope undergoes only a modest change. Consequently, for the one-year horizon, the equity allocation is fairly insensitive to

FIGURE 8

THE IMPACT OF EQUITY VOLATILITY ON THE MAXIMUM EQUITY HOLDING

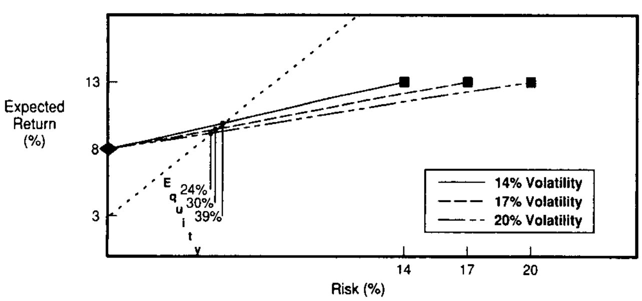
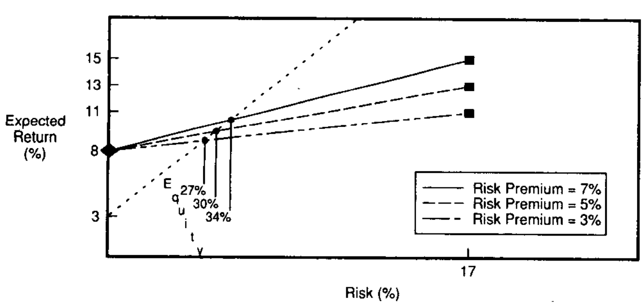


FIGURE 9

THE IMPACT OF ALTERNATIVE RISK PREMIUM ESTIMATES



the risk premium. In fact, it varies only from a low of 27% at a 3% risk premium to a high of 34% at a 7% risk premium.

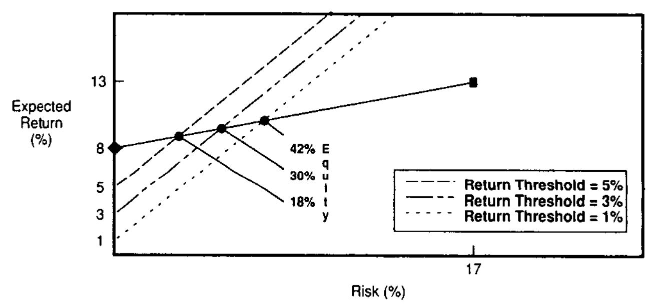
In summary, over one-year horizons, the equity allocation is moderately sensitive to the volatility estimate and fairly insensitive to the risk premium estimate. This is fortuitous, because market estimates of volatility tend to be more stable than estimates of the risk premium. Thus, for the one-year horizon, the shortfall constraint itself, rather than the market estimates, most strongly influences equity allocation.

SENSITIVITY TO VARIATIONS IN THE SHORTFALL CONSTRAINT

The shortfall constraint consists of both a minimum return threshold and a shortfall probability. In Figure 10, we illustrate the impact of changes in the minimum return threshold on the equity allocation.

FIGURE 10

THE IMPACT OF THE MINIMUM RETURN THRESHOLD ON THE MAXIMUM EQUITY HOLDING

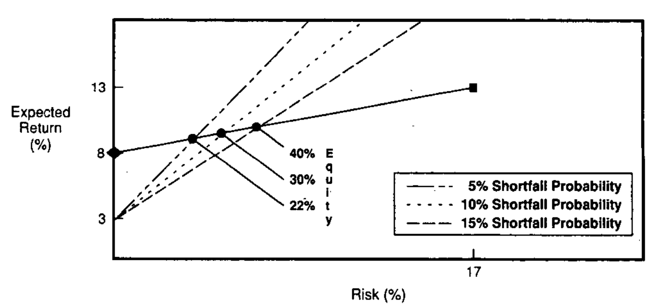


Because the shortfall line always emanates from the threshold value on the vertical axis, the changing minimum return threshold simply results in a parallel shift of the shortfall line. Observe that a 2% change in the minimum return threshold results in a 12% change in equity allocation. For example, a 1% minimum return threshold allows for an increase in equity allocation from 30% to 42%.

Figure 11 illustrates the impact of changes in the shortfall probability. As we noted earlier, the more stringent probabilities lead to steeper slopes for the

FIGURE 11

THE IMPACT OF SHORTFALL PROBABILITY



shortfall lines (vice versa for more liberal probabilities). Thus, if we require only a 15% shortfall probability relative to a 3% return threshold, the lower slope allows an increase in the maximum equity holding to 40%. If we demand a more stringent 5% shortfall probability, the slope is steeper, and the maximum equity holding falls to 22%.

THE MULTI-YEAR INVESTMENT HORIZON

What is the impact of extending the investment horizon on the equity/cash mix? Analysis uses the expected annualized compound return as the return measure and the standard deviation of annualized returns as the risk measure. These choices of annualized return/risk measures enable us to use the same shortfall line as we did for a one-year horizon.

Figure 12 shows the efficient frontier for a one-, three-, and five-year horizon. Here we assume that, for any horizon, there is a riskless asset with an 8% expected return (one-year, three-year and five-year STRIPs). Note that the efficient frontier steepens significantly as the horizon increases, because the *annualized* volatility of returns decreases dramatically from 17% to 7.7% as we lengthen the horizon from one year to five years.[2]

Now we superimpose the shortfall constraint on the efficient frontiers for the three different time periods (see Figure 13). The maximum equity allocation increases dramatically as the horizon increases.

FIGURE 12

THE EFFICIENT FRONTIER FOR MULTI-YEAR HORIZONS

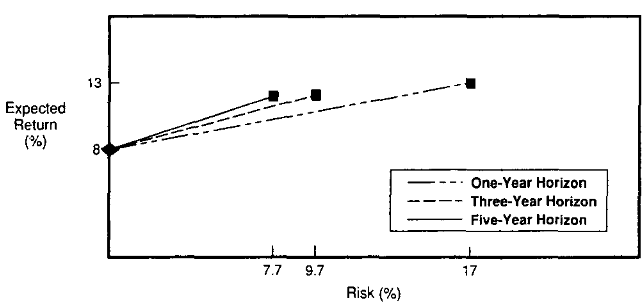
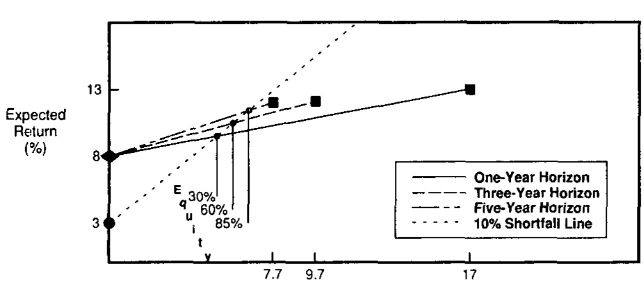


FIGURE 13

THE MULTI-YEAR SHORTFALL CONSTRAINT WITH A 3% RETURN THRESHOLD

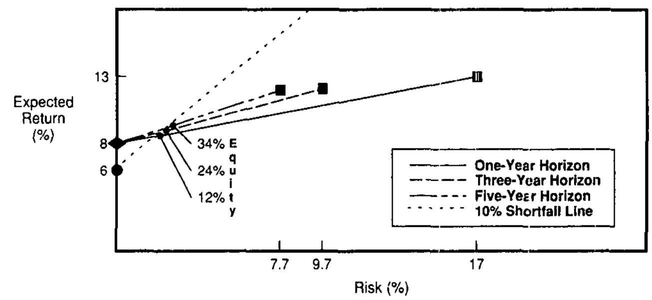


Specifically, it extends from 30% over a one-year horizon to 60% for a three-year horizon and to 85% for a five-year horizon. For any horizon that is longer than about six years, our shortfall constraint allows a 100% equity allocation.

Of course, over a five-year horizon, the 3% threshold is probably too generous. A more realistic threshold at the 6% level dramatically reduces the maximal equity allocation from 85% to only 34%, as shown in Figure 14.

FIGURE 14

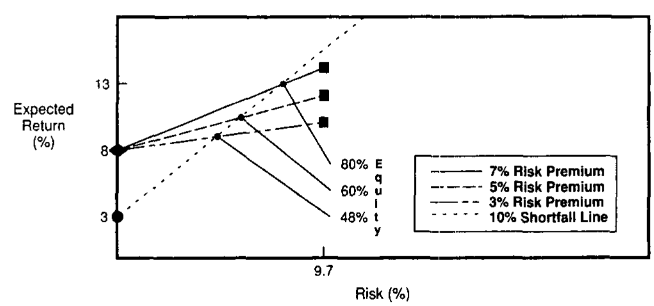
THE MULTI-YEAR SHORTFALL CONSTRAINT WITH A 6% RETURN THRESHOLD



Longer horizons offer a greater opportunity to capture more fully the benefits of high risk premiums. Thus, we should expect the maximum equity allocation to become sensitive to the risk premium estimate. For a fixed three-year horizon, this sensitivity is illustrated in Figure 15.

FIGURE 15

THE THREE-YEAR SHORTFALL CONSTRAINT WITH ALTERNATIVE RISK PREMIUM ESTIMATES



Here we observe that an increase in the risk premium from 5% to 7% leads to a rise in the maximum equity allocation from 60% to 80%. We also observe that the sensitivity to the risk premium is asymmetric. Note that a 2% decrease in the risk premium drops the maximum equity allocation from 60% to only 48%.

SUMMARY AND CONCLUSION

We have described a simple shortfall methodology to gain insight into the maximal allocation of risky assets. The analysis has three critical ingredients: 1) the investment horizon; 2) the minimum return threshold; and 3) the allowable probability that returns will fall below this threshold.

Surprisingly, we find that with a 10% shortfall probability, a 3% return threshold, and a one-year horizon, only 30% of the portfolio should be in risky assets. Over the short term, the volatility of risky assets creates a high probability of poor returns. In effect, there is insufficient time to allow reliable capture of the risk premium that these assets offer. As long as we focus on a one-year horizon, this result holds across a wide range of risk premiums.

As the horizon increases, by contrast, there is a marked decrease in annualized return volatility, and the allowable equity allocation increases dramatically. In our example, over a five-year horizon, the risky asset allocation could be increased to 85% for a minimum return threshold of 3%. Moreover, the multiyear allocations are more sensitive to the risk premium, with higher risk premiums leading to substantially greater equity allocations.

The strength of our shortfall model lies in its ability to capture the allocation impact of a simply stated measure of risk tolerance across one or more investment horizons. This shortfall approach therefore should help fund sponsors address the delicate problem of finding a balance between seeking long-term gains and defending against the risk of adverse performance.

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[1] The shortfall probability is incomplete, because it cannot indicate how bad the shortfall will be in the event that one should occur. For a more fully developed theory of shortfall analysis, see Harlow and Rao [1989] and Bawa and Lindenberg [1977].

[2] The decrease in annualized return volatility reflects the standard random walk model, where the volatility of cumulative return increases with the square root of elapsed time. As a result, the volatility of the *annualized* returns over the investment horizon actually declines as the horizon period lengthens, but the expected return on equity decreases for a five-year horizon ("volatility drag"). For a detailed discussion of this concept, see Martin L. Leibowitz et al. [1989].