Asset allocation of a pension scheme during the decumulation phase

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Context (1)

The determination of the asset allocation is a central topic in life insurance: especially for the « with-profits » contracts and additional pension plans.

In the case of a retirement plan, the duration of the plan allows the Asset Liability Managers to develop long-term strategies independent of short-term speculations (which are relevant of the Tactical Asset Allocation).

Context (2)

The first models of asset allocation to integrate risk related to financial placements were inspired by financial techniques, in particular of the Markowitz criterion [1952].

Boyle [2004] shows the operational limits of these approaches when parameters of the asset models have to be estimated which breeds estimation errors.

Since 2001 some authors like Battochio, Menoncin, Scaillet, Milevsky or Boulier developed models integrating insurance constraints (in particular the mortality risk), but their practical implementation is delicate (choice of a utility function for the policy-holders).

Context (3)

In the spirit of « Solvency 2 » thoughts, ruin probability models have been proposed. The idea is to determine the asset allocation which controls the ruin probability at the desired level or, in other words, the ability of the insurance company to face its engagements.

The objective of this work is to propose a criterion of asset allocation which integrates specific insurance constraints, and which does not require to fix a priori the probability of ruin (which is to be controlled ex post).

We implement this criterion in the case of a defined benefit plan.

Presentation of the pension scheme (1)

→ Defined Benefit Pension plan of a French company.

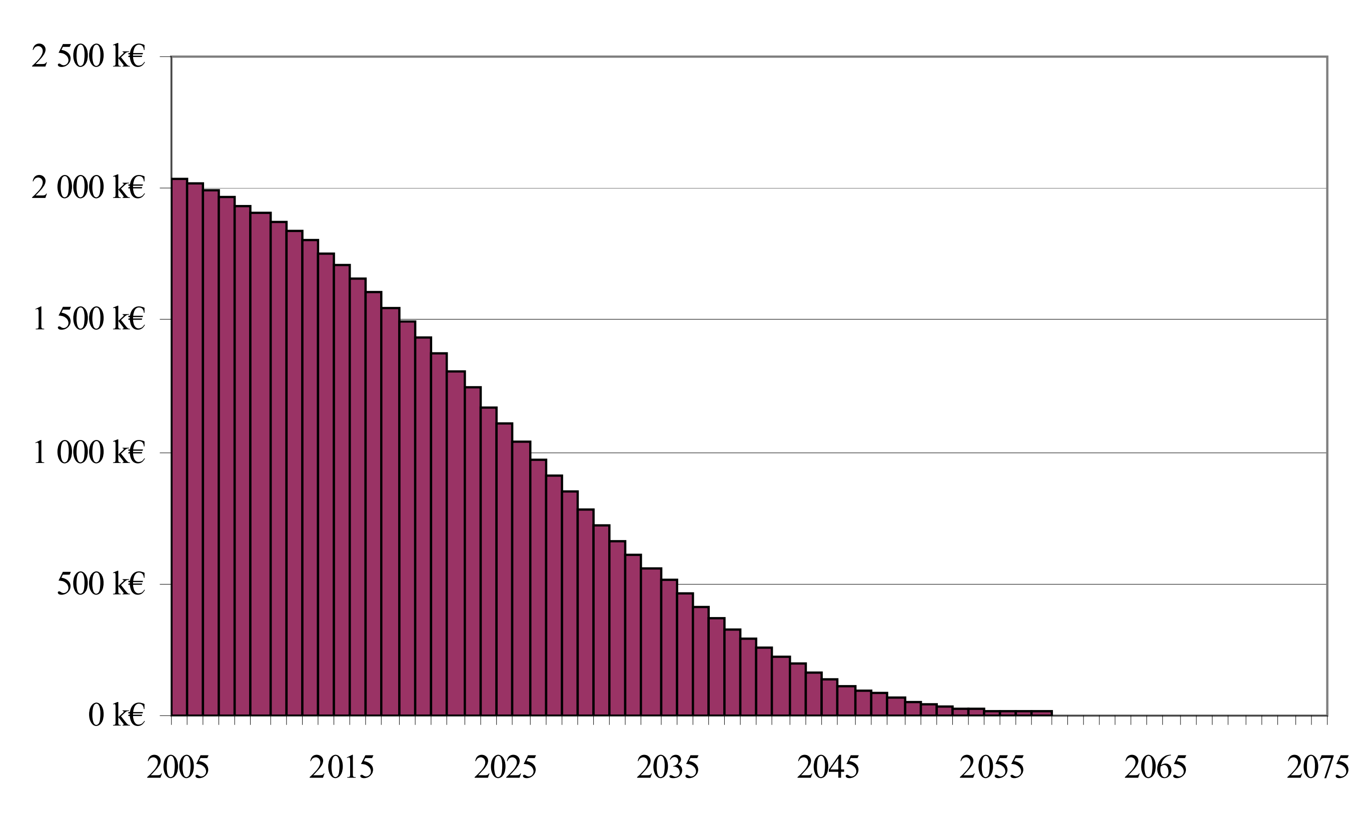
<u>Main characteristics:</u>

- Number of pensioners = 374
- Average age = 64 years
- Average annual pension = 5,5 k€

Using the French mortality table TV 2000 and a discount rate equal to 2,5% :

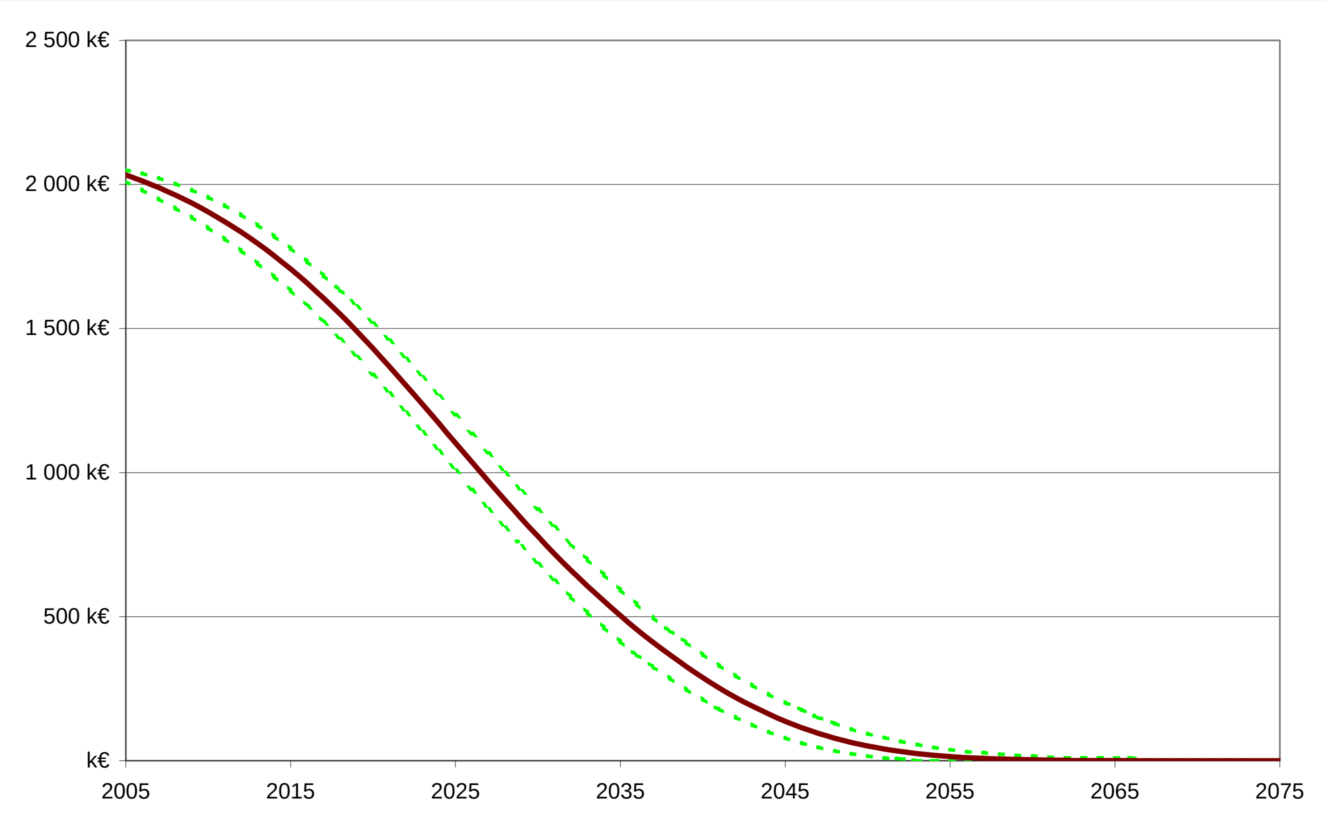
- Initial mathematical reserve = 32,8 M€
- Duration = 12,3 years

Presentation of the pension scheme (2)



Expected future benefits

Presentation of the pension scheme (3)



Expected future benefits – 95% confidence interval

Problem

During the decumulation phase, the insurer does not receive any more premium. So he has to manage his financial portfolio in order to be able to pay the future benefits.

We deal with this problem when the manager has to compose his asset portfolio with a riskless asset and a risky asset.

LIABILITY	
French GAAP	**Economic**
Equities = Solvency Margin	Economic Equities
Mathematical Reserve (discount rate fixed by the law)	Economic Mathematical Reserve (discount rate = expected rate of return of the assets)

Notation

- E_t : amount of Equity of the company at time t,
- L_t : amount of mathematical reserve of the company at time t,
- A_t : value of the asset of the company at time t,
- $\tilde{F}_t$: benefit cash-flow to be paid at time t,
- i : discount rate of the mathematical reserve,
- r : constant, riskless interest rate,
- **P** : historical probability measure and Φ the associated filtration,
- **J** : set of pensioners,
- $x(j)$: age at 0 of the j^{th} pensioner and r_j : amount of his pension.

Evolution of the liabilities

At time 0, we estimate the expected future cash-flows : $(F_t)_{t\geq 1}$

$$F_t = \mathbf{E}\left[\tilde{F}_t \mid \Phi_0\right]$$

With :

- $\tilde{F}_t = \sum_{j\in\mathbf{J}} r_j * \mathbf{1}_{]t;\infty[}\left(T_{x(j)}\right)$
- $T_{x(j)}$ is the (random) date of death of a pensioner aged $x(j)$ years.

Using the traditional notation of life insurance, we have : $F_t = \sum_{j\in\mathbf{J}} r_j * \frac{l_{x(j)+t}}{l_{x(j)}}$

Then we can determine the initial amount of mathematical reserve L_0 by :

$$L_0 = \sum_{t=1}^{\infty} F_t (1+i)^{-t}$$

Evolution of the Balance Sheet

Let us suppose that the pensions are paid at the beginning of each year, the evolution of the balance of the insurer is given by :

$$\begin{cases} L_t = \sum_{k=t+1}^{\infty} \frac{\mathbf{E}\left[\tilde{F}_k \,\middle|\, \Phi_t\right]}{(1+i)^{k-t}} \\ A_t = \left(1+\tilde{R}_t\right) A_{t-1} - \tilde{F}_t \\ E_t = A_t - L_t \end{cases}$$

where $\tilde{R}_t$ is the return of the financial portfolio between *t*-1 and *t*.

Evolution of the Asset (1)

Let us suppose that the insurer has to compose his portfolio with two assets:

- a riskless asset which price is Y_t at time t : $Y_t = Y_0\, e^{rt}$
- a risky asset with price X follows a geometric brownian motion :

$$\frac{dX_t}{X_t} = \mu\, dt + \sigma\, dB_t$$

where B is a standard brownian motion.

These models are very simple but our objective is to illustrate the use of the « maximizing economic equities » asset allocation criterion. This criterion could be used with more complex models: a diffusion process (CIR, Vasicek) for the interest rate of the « riskless » asset and a jump diffusion process for the risky asset for example.

Evolution of the Asset (2)

Without loss of generality, let us set:

$$X_0 = Y_0 = 1$$

We will also make the two following natural assumptions:

- the expected return of the risky asset is higher than the return of the riskless asset,
- the discount rate of the mathematical reserve (fixed by the law) is lower than the riskless interest rate.

$$\mu \geq r \geq i \geq 0$$

Investment Withdrawals (1)

When the insurer has to pay benefits, we will suppose that he sells the two assets proportionally to their respective market value in the portfolio. It is the same as considering that the insurer initially composes a fund with the two assets, and that he does not recompose it. Each time the insurer has to pay benefits, he sells some shares of this fund.

$$A_{t+1} = \frac{\theta\, X_{t+1} + (1-\theta) Y_{t+1}}{\theta\, X_t + (1-\theta) Y_t} A_t - F_{t+1}$$

By induction, we get :

$$A_t = \left(\theta\, X_t + (1-\theta) Y_t\right) \left[A_0 - \sum_{s=1}^{t} \frac{F_s}{\theta\, X_s + (1-\theta) Y_s} \right]$$

where θ is the initial proportion of risky asset.

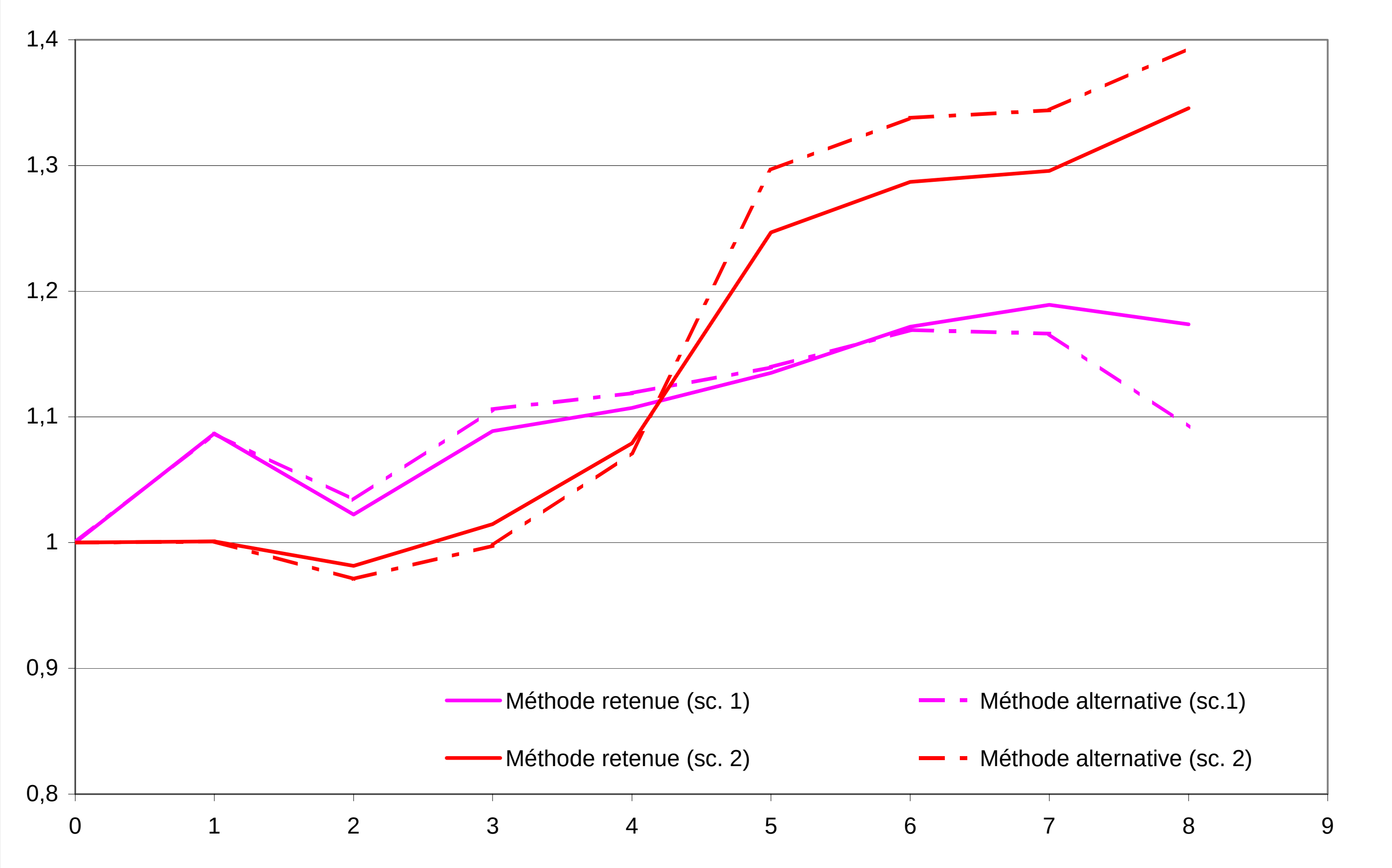
Investment Withdrawals (2)

Let us note that the adopted approach: $$A_{t+1} = \frac{\theta\, X_{t+1} + (1-\theta) Y_{t+1}}{\theta\, X_t + (1-\theta) Y_t} A_t - F_{t+1}$$

is not equivalent to: $$A_{t+1} = \left(\theta \frac{X_{t+1}}{X_t} + (1-\theta) \frac{Y_{t+1}}{Y_t} \right) A_t - F_{t+1}$$

In this alternative approach, at the beginning of each year, the insurer recomposes its financial portfolio in order to maintain the initial allocation.

Investment Withdrawals (3)



Evolution of the Asset of the insurance company (2 trajectories)

Determination of an asset allocation criterion (1)

The French regulation imposes to the insurer:

- To discount the mathematical reserve a smaller rate than

$$\mathbf{Min}\left\{ 60\% * \mathrm{TME} ; 3,5\% \right\}.$$

- To have a minimal level of equities greater than the solvency margin (4% of the the mathematical reserve in our example).

In addition, the profit of the insurer comes from:

- The financial return of the equities,
- The surplus of financial products (when the financial return is greater than the discount rate) generated by the mathematical reserve.

N.B. We supposed that the contract does not contain any profit option.

Determination of an asset allocation criterion (2)

Let us denote: $\Lambda_\theta = \sum_{t=1}^{\infty} \frac{\tilde{F}_t}{\theta X_t + (1-\theta) Y_t}$

This random variable represents the amount necessary at 0 to pay all the pensions.

$\mathbf{E}\left[\Lambda_\theta\right]$ can be interpreted like the "economic mathematical reserve" (discount rate = return of the assets).

The insurer is likely to try to minimize this quantity.

Indeed, to minimize $\mathbf{E}\left[\Lambda_\theta\right]$ is equivalent to :

- choose the asset allocation which will damp the future flows of pension as well as possible,
- maximize the initial economic equities: $\left(E_0 + L_0 - \mathbf{E}\left[\Lambda_\theta\right]\right)$

Risk analysis (1)

Let us notice that it is possible to analyze the total risk supported by the company by the means of Λ_θ.

The variance of Λ_θ can indeed be proposed as an indicator of the total risk of the pension scheme. Its decomposition enables us to appreciate the respective shares of the financial risks and mortality:

$$\mathbf{V}\left[\Lambda_\theta\right] = \mathbf{E}\left[\mathbf{V}\left(\Lambda_\theta \middle| X\right)\right] + \mathbf{V}\left[\mathbf{E}\left(\Lambda_\theta \middle| X\right)\right]$$

where X is the price of the risky asset.

Then :

$\mathbf{E}\left[\mathbf{V}\left(\Lambda_\theta \middle| X\right)\right]$ represents the financial risk,

$\mathbf{V}\left[\mathbf{E}\left(\Lambda_\theta \middle| X\right)\right]$ represents the mortality risk.

Risk analysis (2)

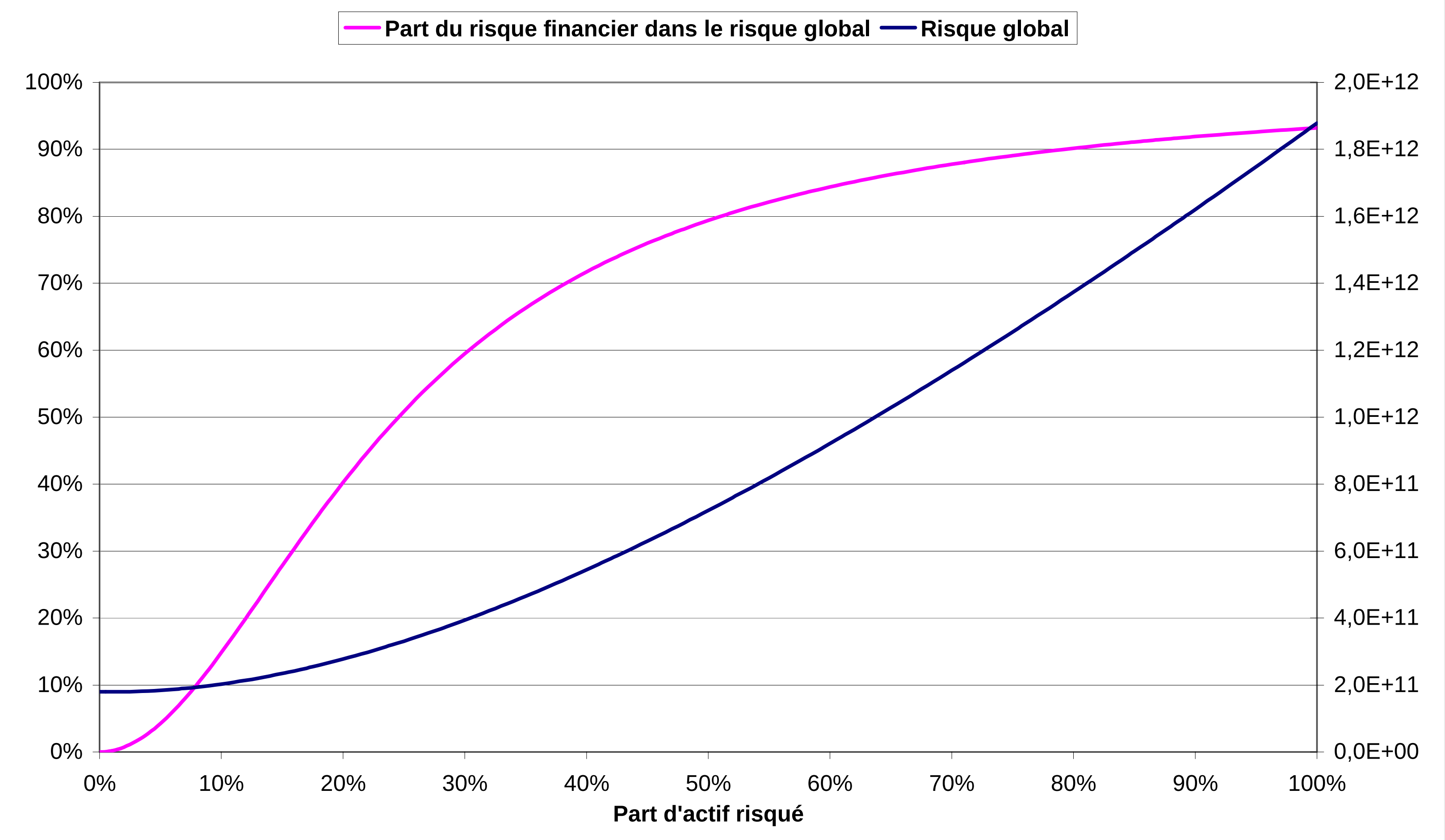
In order to estimate theses variables, we have simulated paths of the risky asset and of the mortality. Then we have used their empirical estimators.

For the following numerical illustrations, we used the parameters:

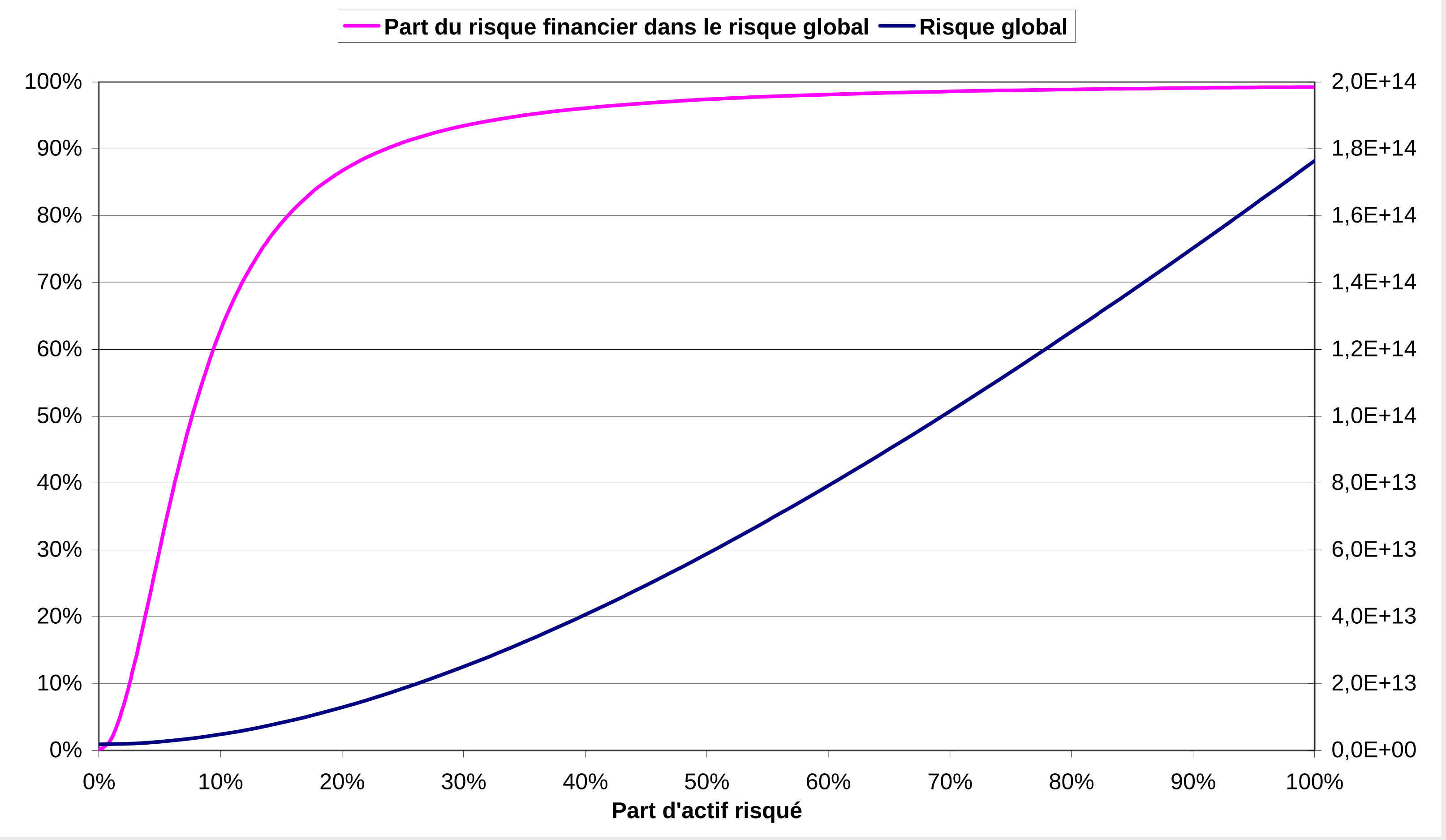
$$E_0 = 4\% * L_0 \qquad r = \ln\left\{1 + 4{,}62\%\right\} \approx 4{,}52\%$$

$$\sigma = 25\% \qquad \mu = \ln\left(1 + 6\%\right)$$

Risk analysis (3)



Risk analysis – 10 times as many pensioners



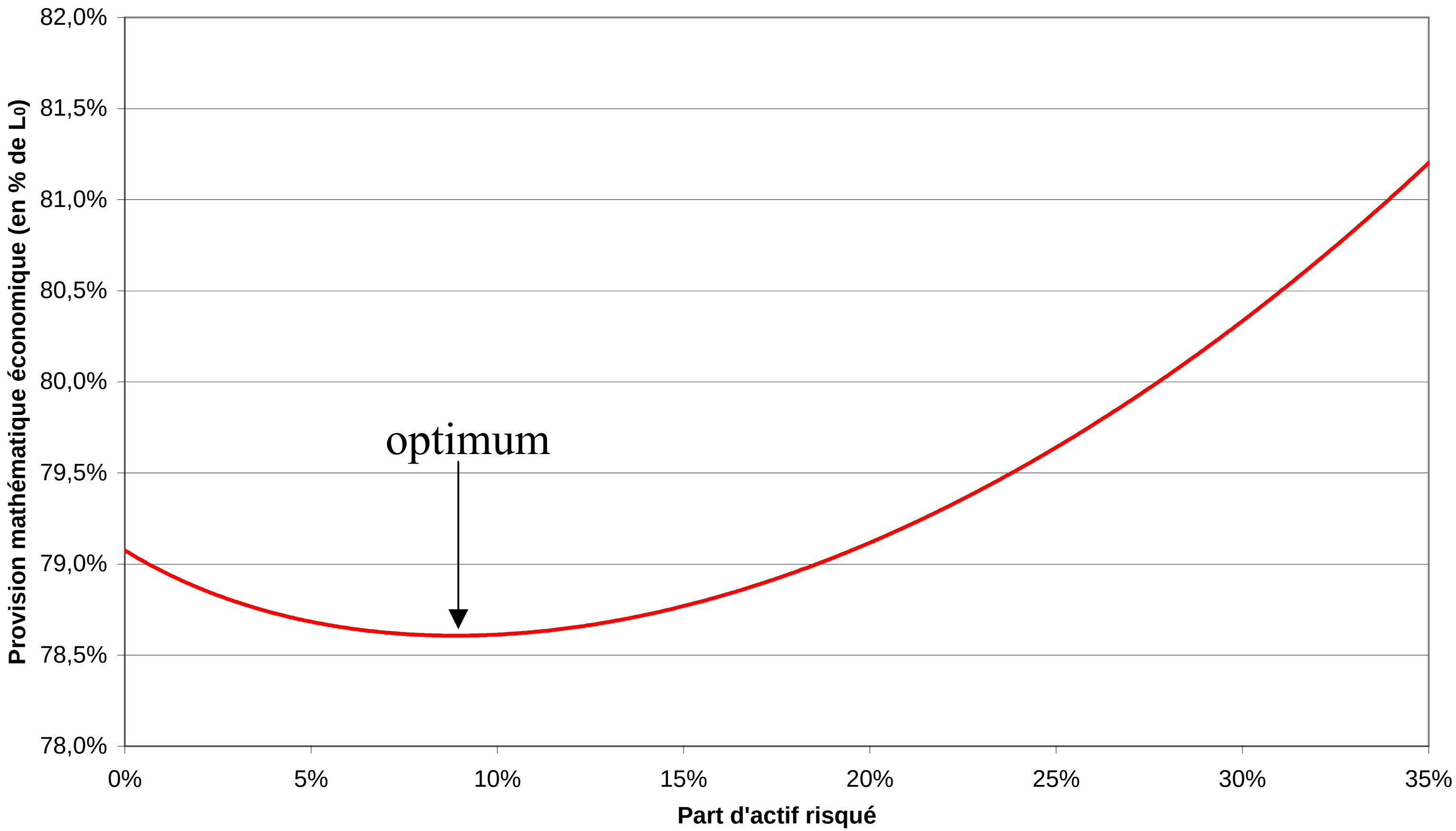
Optimization of the economic equities (1)

Let us consider the following optimization program:

$$\begin{cases} \textbf{Inf } \mathbf{E}\left[\Lambda_\theta\right] \\ \theta \in \left[0;1\right] \end{cases}$$

The simulation techniques enable us to solve this problem. After having generated "many" trajectories of the Asset and of the Liability, $\mathbf{E}\left[\Lambda_\theta\right]$ will be estimated by the average of realizations of Λ_θ.

Optimization of the economic equities (2)



Optimization of the economic equities (3)

In our illustration, the optimum is given for θ = 8,85%.

Assuming that the demographic risk and the financial risk are independent, the implementation of this criterion gives you the same allocation taking the mortality risk into account or not. Indeed:

$$\mathbf{E}\left[\Lambda_\theta\right] = \mathbf{E}\left[\sum_{t=1}^{\infty} \frac{\tilde{F}_t}{\theta\, X_t + (1-\theta) Y_t}\right] = \sum_{t=1}^{\infty} \mathbf{E}\left[\frac{1}{\theta\, X_t + (1-\theta) Y_t}\right] * \mathbf{E}\left[\tilde{F}_t\right] = \sum_{t=1}^{\infty} F_t * \mathbf{E}\left[\frac{1}{\theta\, X_t + (1-\theta) Y_t}\right]$$

So when $\left(F_t\right)_{t\geq 1}$ has been calculated, it remains to simulate the asset trajectories

→ It is an important saving of computing time.

Alternative approach: ruin probability

A traditional approach in insurance consists in choosing the asset allocation which controls the probability of ruin of the company at a desired level.

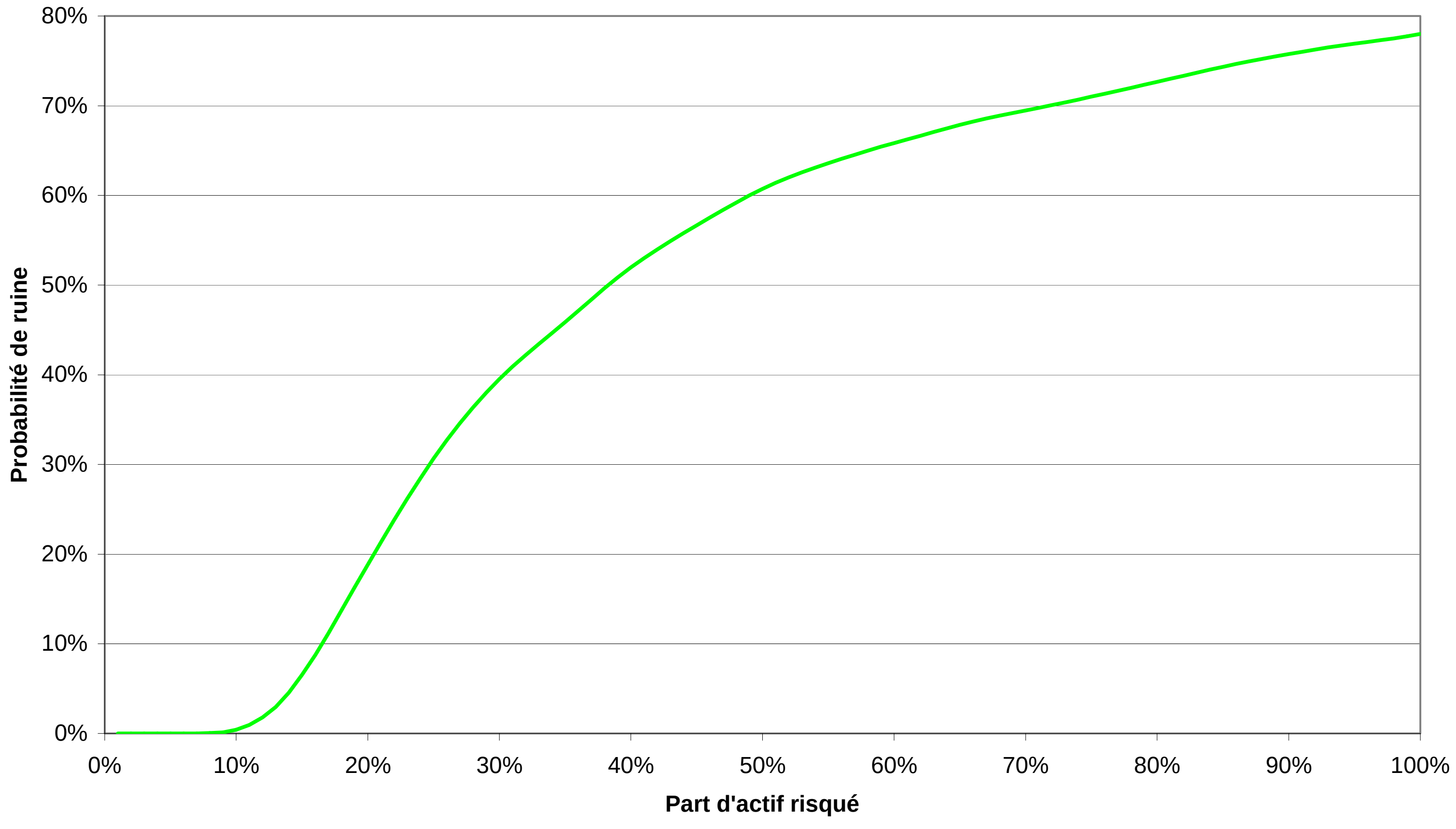
Let us denote τ the time to ruin of the company : $\tau = \mathbf{Inf}\left\{ t \in \mathbf{N} \,\middle|\, E_t < 0 \right\}$

For the insurance company, the expected profit coming from the pension scheme is increasing in the proportion invested in the risky asset. So we have to solve:

$$\mathbf{Sup}\left\{ \theta \in [0;1] \,\middle|\, \mathbf{P}_\theta(\tau < \infty) \leq \pi_{max} \right\}$$

where π_{max} is the maximum ruin probability that the company can accept.

Ruin probability



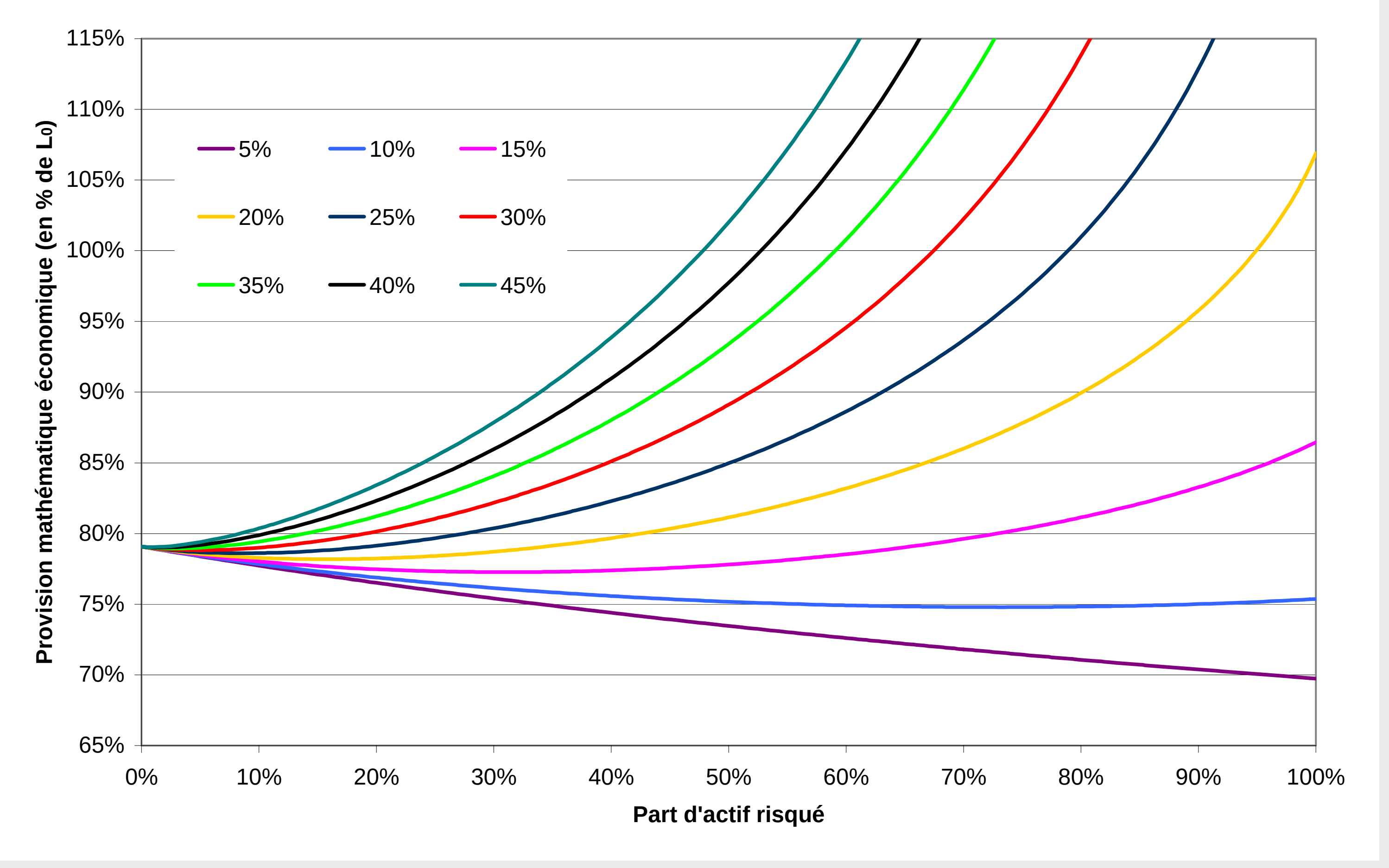
Comparison of the results

	θ	Ruin Probability	Proportion of the financial risk in the global risk	Proportion of the financial risk in the global risk (10 times as many pensioners)
Maximization of the economic equities	8,85%	0,30%	12%	53%
Ruin probability < 1%	10,47%	1%	16%	65%

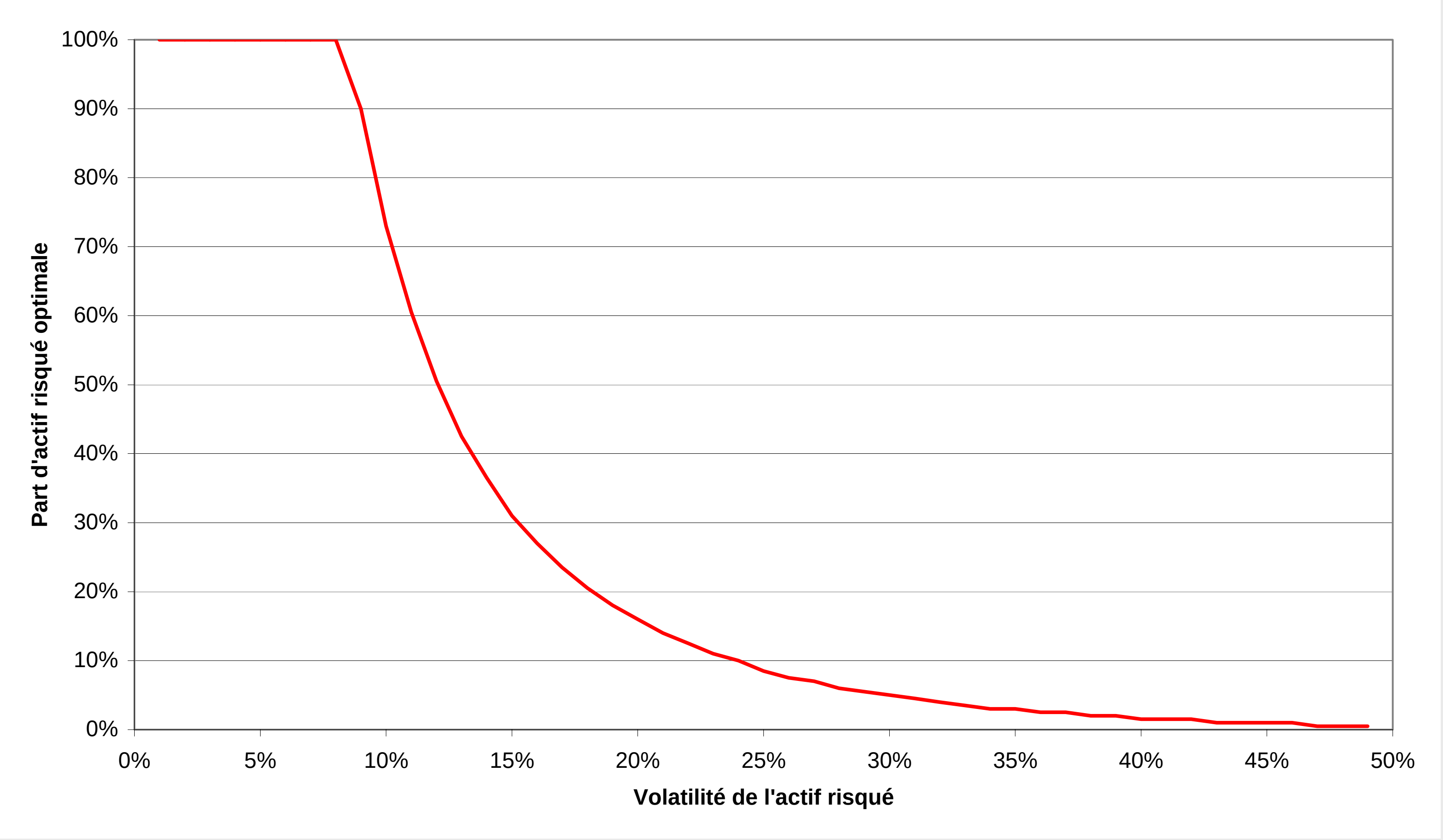
In our example, the asset allocation determined by the criterion of maximization of the economic equities stocks leads to a careful allocation.

We may note that on a small insurance portfolio, the mortality risk is non-negligible.

Sensitivity to the volatility of the risky asset (1)



Sensitivity to the volatility of the risky asset (2)



Pension Adjustment (1)

Now we will suppose that the pensions are indexed on inflation. Let us denote I_t the price index at time t.

If the instantaneous inflation rate follows an Ornstein-Ulhenbeck Process :

$$I_{t+h} = I_t * \mathbf{exp} \int_t^{t+h} \left(j + x_s\right) ds$$

where :

$$dx_s = -a\, x_s\, ds + \sigma_I\, dB_s$$

we have:

$$\mathbf{E}\left[\left.\frac{I_{t+\delta}}{I_t}\right| \Phi_t\right] = \mathbf{exp}\left\{ j\delta + x_t \frac{1-e^{-a\delta}}{a} + \frac{\sigma_I^2}{2a^2}\left(\delta - \frac{1-e^{-a\delta}}{a} - \frac{\left(1-e^{-a\delta}\right)^2}{2a}\right)\right\}$$

For the numerical illustrations, we will make the assumption that the evolution of the prices is independent of the evolution of the risky asset X. This assumption can be easily relaxed.

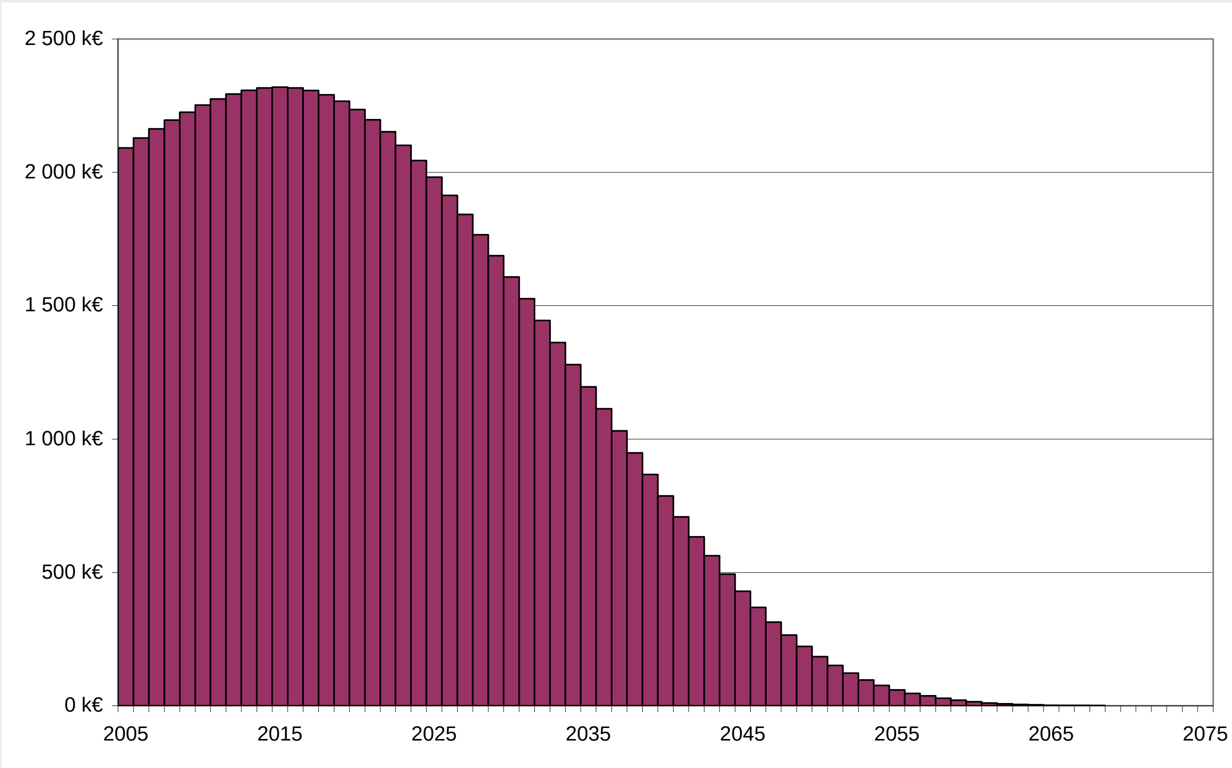
Pension Adjustment (2)

The pension adjustment is an additional engagement for the insurer. We thus will integrate it in the mathematical reserve.

We simulated the evolution of inflation using the parameters estimated on the French data by Fargeon & Nissan [2003] :

$$j = 0{,}0279 \qquad a = 0{,}2631 \qquad \sigma_I = 0{,}0056 \qquad x_0 = 0$$

Pension Adjustment (3)



Expected Future Benefits

Pension Adjustment (4)

The volatility of these flows is important because there is no mutualization of the inflation risk.

Adjusted Pensions – 95% confidence interval

Pension Adjustment (5)

Taking into account the pension adjustment results in :

- Increase of the initial mathematical reserve,

  → increase of the equities (4% of the mathematical reserve)

- Increase of the duration of the Liability.

This approach thus appears careful.

The increase of the duration of the liability has a direct impact on the asset allocation.

Pension Adjustment (6)

The increase in the global risk due to the random nature of the revalorization can be observed thanks to the decomposition of the risk previously suggested:

$$\Lambda_\theta^I = \sum_{t=1}^{\infty} \frac{\tilde{F}_t * I_t}{\theta\, X_t + (1-\theta) Y_t}$$

If the global risk is measured by the variance of Λ_θ^I we have :

$$\mathbf{V}\left[\Lambda_\theta^I\right] = \mathbf{E}\left[\mathbf{V}\left(\Lambda_\theta^I \middle| X, I\right)\right] + \mathbf{V}\left[\mathbf{E}\left(\Lambda_\theta^I \middle| X, I\right)\right]$$

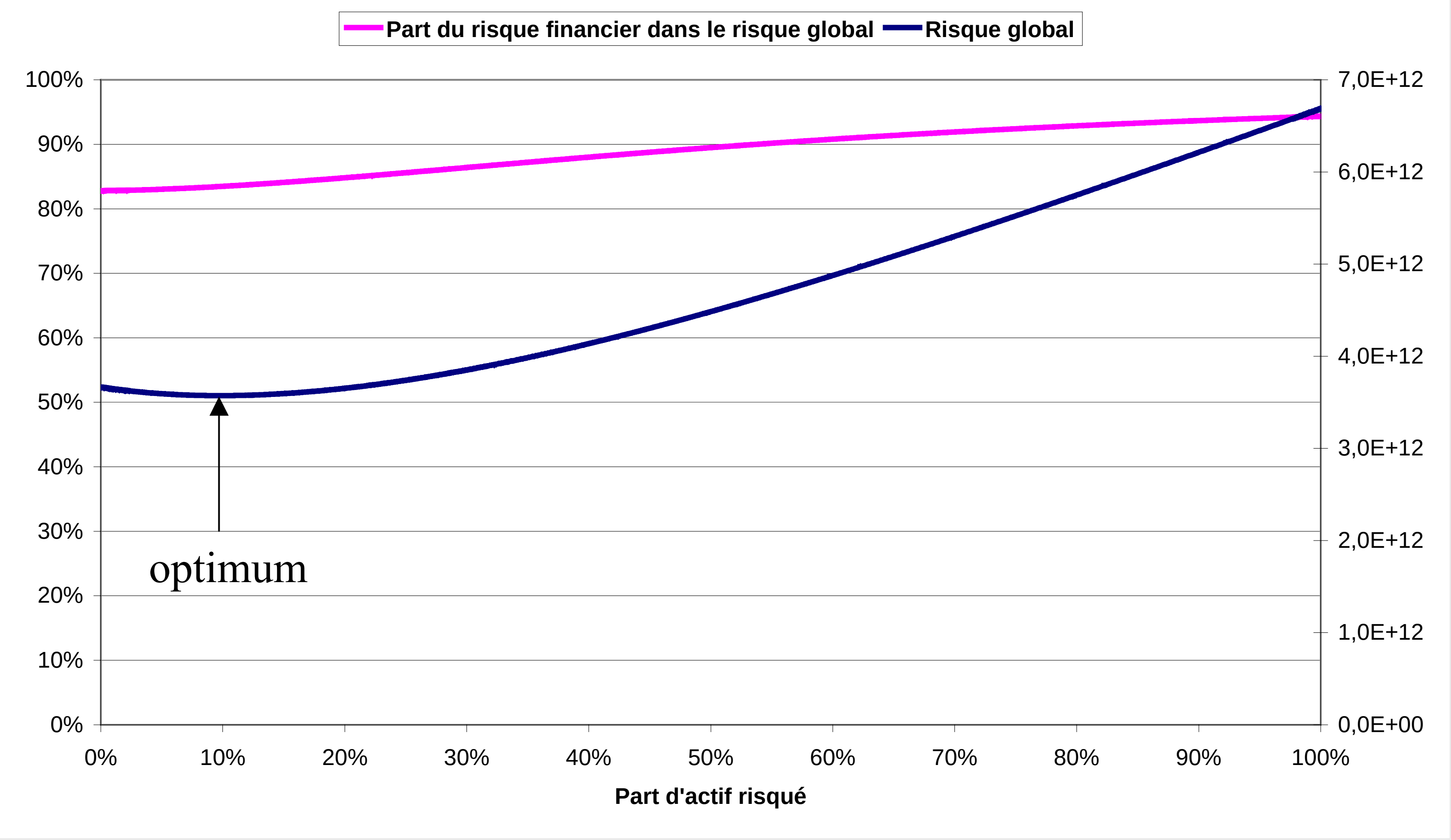
where X is the price of the risky asset and I the price index.

Then :

$\mathbf{E}\left[\mathbf{V}\left(\Lambda_\theta^I \middle| X, I\right)\right]$ represents the financial risk (risky asset + inflation),

$\mathbf{V}\left[\mathbf{E}\left(\Lambda_\theta^I \middle| X, I\right)\right]$ represents the mortality risk.

Pension Adjustment (7)



Pension Adjustment (8)

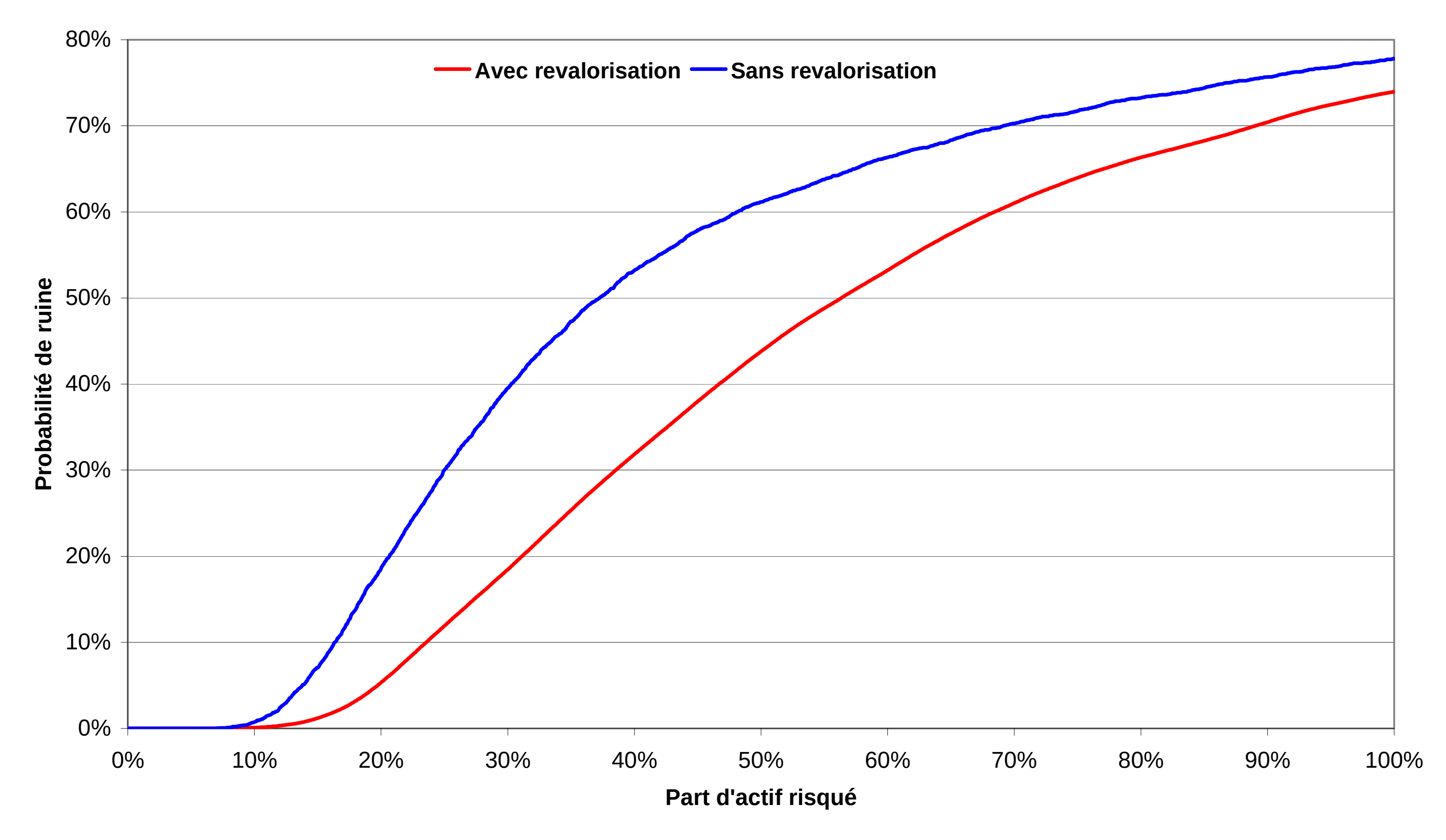
Whatever the selected allocation is, the share of the financial risk in the global risk is higher than 80%.

There is a non-trivial allocation for which the global risk reaches a minimum. Indeed the insurer has to invest in the risky asset if he wants to have a financial portfolio with an expected return higher than the discount rate increased by the average inflation rate:

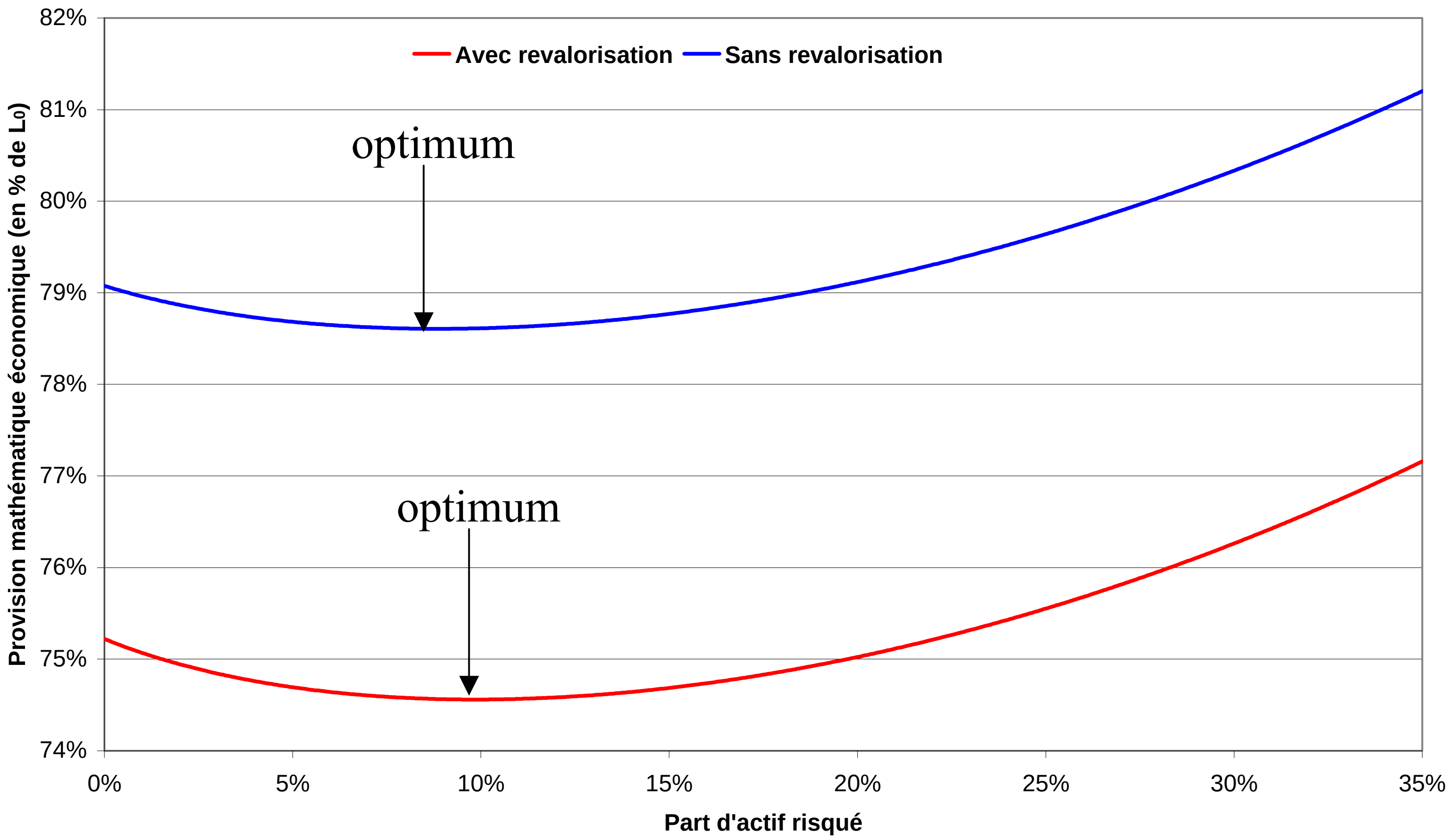
$$e^{j-r} = 0{,}983 > 0{,}976 = (1+i)^{-1}$$

Thus the insurer will have to invest 9,75% of his initial Asset in the risky asset to minimize the risk measured by the variance of the economic reserve.

Pension adjustment – Ruin probability



Pension adjustment – Maximizing economic equities



Conclusion

The « maximizing economic equities » criterion:

- is easy to implement,
- is adaptable to complex financial models thanks to the contribution of Monte-Carlo simulations,
- does not require an external parameter whose determination is not obvious (ex: the level of the probability of ruin: 0,1%, 1%, 5).

Very sensitive to the variability of the financial assets, the allocation determined by the criterion of maximization of the economic equities requires a good knowledge of the volatility of the financial assets.

Further development of the model

The following developments are in progress:

- To integrate the dependence between the inflation and the stock price (directly in the existing model or via a global model like the Wilkie model).
- To test the relevance of the « maximizing economic equities » criterion on the reinvestments.
- To adapt this criterion to a pension plan during the accumulation phase.
- To integrate an analysis of the surplus for the shareholders of the insurance company in order to measure the risk premium of the pension plan.

References

BATTOCHIO P., MENONCIN F., SCAILLET O. [2004] « Optimal asset allocation for pension funds under mortality risk during the accumulation and decumulation phases », forthcoming in *Annals of Operations Research*.

BOYLE P., WINDCLIFF H. [2004] « The 1/n pension investment puzzle », submitted to *North American Actuarial Journal*.

FARGEON L., NISSAN K. [2003] « Recherche d'un modèle actuariel d'analyse dynamique de la solvabilité d'un portefeuille de rentes viagères », Mémoire d'actuariat ENSAE.

JACQUEMIN J., PLANCHET F. [2003] « Méthodes de simulation en assurance», *Bulletin Français d'Actuariat*, Vol. 6, n°11.

LAMBERTON D., LAPEYRE B. [1997] *Introduction au calcul stochastique appliqué à la finance*, 2nde édition. Paris : Ellipses.

MARKOWITZ H. [1952] « Portfolio Selection », *Journal of Finance*, Vol.7, n°1.

PLANCHET F., THEROND P. [2004a] « Simulation de trajectoires de processus continus », Cahiers de recherche de l'ISFA, WP 2024.

PLANCHET F., THEROND P. [2004b] « Critères d'allocation d'actifs pour un régime de rentiers », Conférence scientifique de l'Institut des Actuaires, Paris, 1er juillet 2004.

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