Prospective mortality table with a small size dataset: estimation risk measure

2011-05-30

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Context of the study

We consider here a (relatively) small group observed during a few years (ten) and we want to build a prospective mortality table.

Because of the small sample size, sampling fluctuations appear (*cf*. Olivieri [2001]) and we have to measure the estimation risk.

We must also use a reference mortality table to built our mortality table and there is a risk to use an non proper reference (*cf*. Planchet & Lelieur [2007]). We need to quantify this “expert opinion risk” also.

We choose not ta deal with the model risk, but it’s an very rich and complex topic which could leads to interrestinf further research (*cf*. Booth & Tickle [2008]).

Context of the study

We focus here on the estimation risk associated with sampling fluctuations and we will also address the expert opinion risk in choosing the reference table.

Both risk are systematic ones and are dangerous. Their impact on reserves and solvency evaluations for the insurance undertaker may be important.

A reflexion is essential then to the measure of these risks within the framework of the construction of prospective mortality tables.

The issue of non independence of the risk sources between periods or ages (*cf*. Booth & Tickle [2008], Loisel & Serant [2007]) is not dealt with in this work.

AGENDA

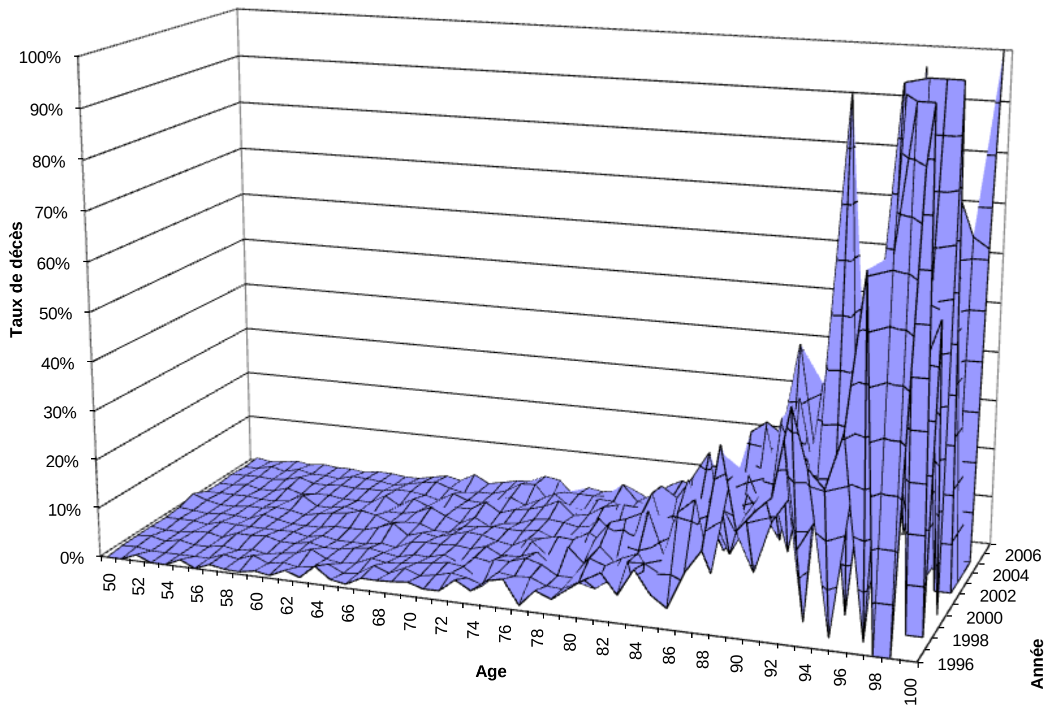
Estimation risk measure

1. **Presentation of the mortality table**
2. Estimation risk measure
3. Consequences for an annuity plan
4. Impact of the choice of a reference

1. Presentation of the mortality table

The data

We use a dataset from a pension plan from 1996 to 2007. The risk exposure is concentrated between ages 50 and 100 years. The whole yearly exposure equals 20.000 years. The sex-ratio is 40%.



1. Presentation of the mortality table

Method of construction

The pension plan mortality tables (M/F) are built using the Brass model. The regulatory French TGH/F 05 have been chosen as a reference, because they describe life annuitants survival in an insurance framework (*cf*. Planchet [2006]). Regression is made with instantaneous data form 1996 to 2007:

$$\ln\left(\frac{\hat{q}_{xt}}{1-\hat{q}_{xt}}\right) = a \times \ln\left(\frac{q_{xt}^{réf}}{1-q_{xt}^{réf}}\right) + b + \varepsilon_{xt}$$

The optimization criterion which is minimized is:

$$D = \sum_{x,t} R_{xt}\left(\hat{q}_{xt} - q_{xt}\right)^2$$

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2. Estimation risk measure

Re-sampling of adjusted mortality rates

We want to evaluate the impact of the sampling fluctuations on the estimation on the model parameters. We first re-sampled the crude death rates using the “direct rates simulation” method proposed in Kamega et Planchet [2010].

It consists in simulating death numbers in the binomial distribution $B\left(R_{xt};\hat{q}_{xt}\right)$ and then to infer the simulated death rates:

$$\hat{q}_{xt}^{k}=d_{xt}^{k}/R_{xt}$$

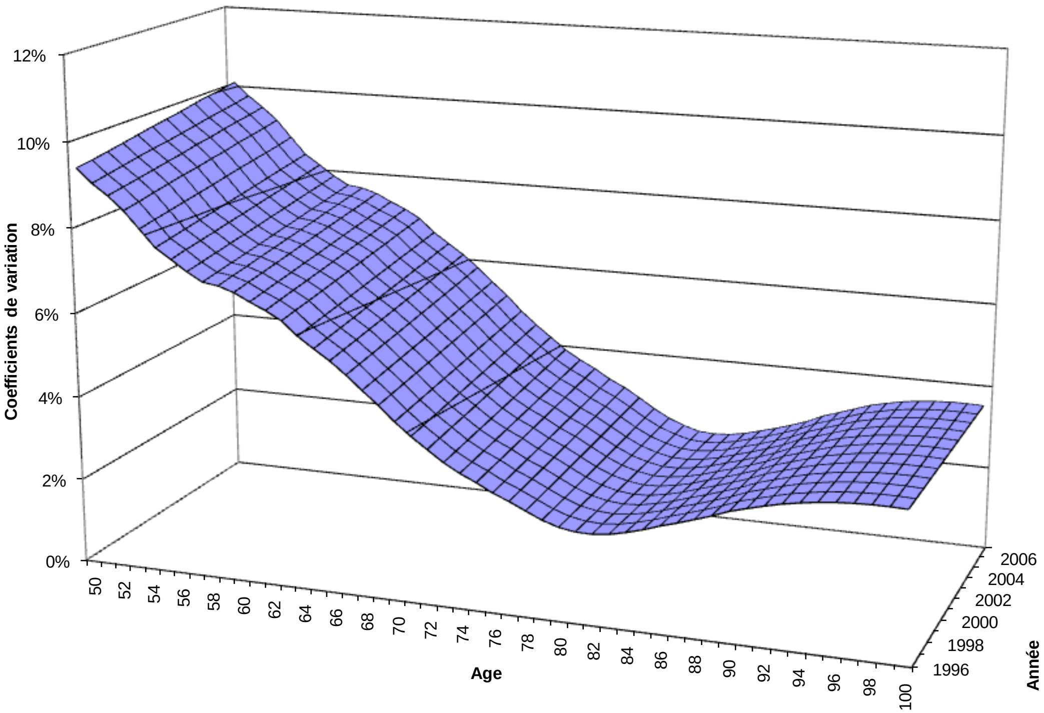
Then we can compute a new estimation of the parameter (*a*,*b*) with the Brass-model.

Re-sampling of adjusted mortality rates

We compute simulated adjusted mortality rates, with 5.000 realizations; the variation coefficients of the simulated adjusted rates are:

$$y_{xt}^{k} = \hat{a}^{k} \ln\left(\frac{q_{xt}^{réf}}{1-q_{xt}^{réf}}\right) + \hat{b}^{k}$$

$$q_{xt}^{k} = \frac{\exp\left(y_{xt}^{k}\right)}{1+\exp\left(y_{xt}^{k}\right)}$$



Computation of the confidence interval

We want to use the re-sampled rates to build a lower and an upper bound for the adjusted mortality table.

To achieve this goal we consider a functional associated to each mortality table and then we compute a confidence interval for this functional.

The use of the (partial) life expectancy appears to be a “natural” one:

$$EV_{xt|n} = \sum_{h=1}^{n} \prod_{u=0}^{h-1} \left(1 - q_{x+u,t+u}\right)$$

Computation of the confidence interval

We compute a lower and an upper value for the partial life expectancy using the equation:

$$P\left(EV_{xt}^{i} < EV_{xt}^{k} \leq EV_{xt}^{s}, x = x_0\right) = 1 - \alpha$$

We use empirical estimators of those values:

$$EV_{xt}^{i} = \mathbf{inf}\left\{EV_{xt}^{g} \in \left[EV_{xt}^{1}, \dots, EV_{xt}^{K}\right] \mid P\left(EV_{xt}^{k} \leq EV_{xt}^{g}\right) \geq \alpha/2, x = x_0\right\}$$

$$EV_{xt}^{s} = \mathbf{inf}\left\{EV_{xt}^{g} \in \left[EV_{xt}^{1}, \dots, EV_{xt}^{K}\right] \mid P\left(EV_{xt}^{k} \leq EV_{xt}^{g}\right) \geq 1 - \alpha/2, x = x_0\right\}$$

We chose age 67 as reference and compute the life expectancy up to 95 years.

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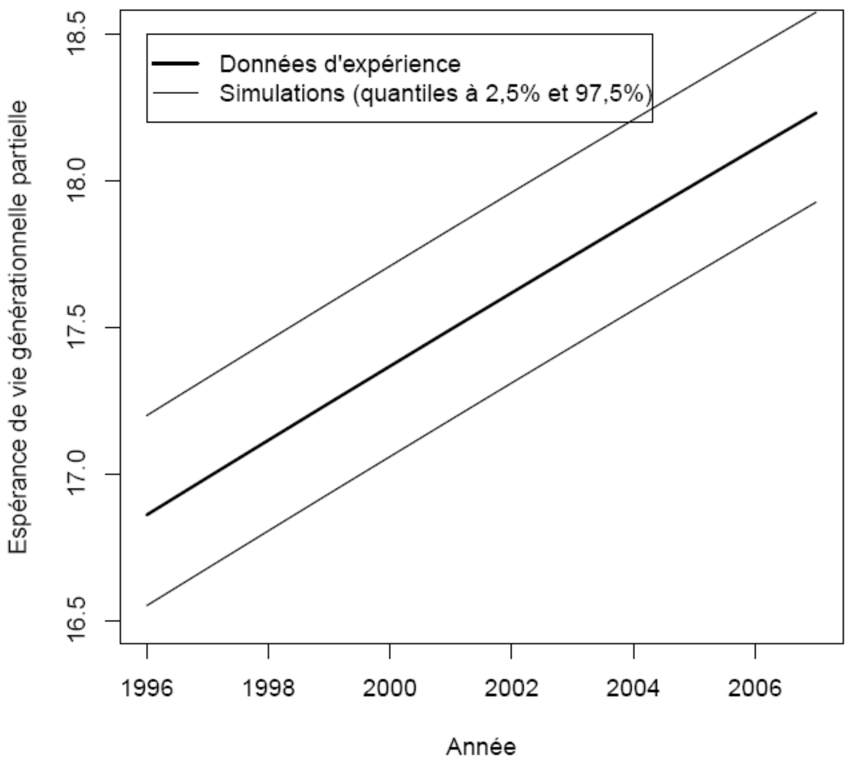
3. Consequences for an annuity plan

Confidence intervals for partial life expectancies and reserves

The typical results are

Esp, de vie gén, partielle (pour t=1996)	Données d'expérience (1)	Simulations à partir des données d'expérience			
		Quantile à 2,5% (2)	Ecart relatif (2)/(1)-1	Quantile à 97,5% (3)	Ecart relatif (3)/(1)-1
Age: 50 / Age fin: 90	32,7	32,2	-1,6%	33,3	1,7%
Age: 60 / Age fin: 90	22,7	22,3	-1,7%	23,1	1,9%
Age: 67 / Age fin: 90	16,1	15,8	-1,7%	16,4	1,9%
Age: 70 / Age fin: 90	13,5	13,2	-1,7%	13,7	1,8%
Age: 50 / Age fin: 95	33,9	33,3	-1,7%	34,5	1,9%
Age: 60 / Age fin: 95	23,6	23,2	-1,8%	24,0	2,0%
Age: 67 / Age fin: 95	**16,9**	**16,5**	**-1,8%**	**17,2**	**2,0%**
Age: 70 / Age fin: 95	14,2	13,9	-1,9%	14,5	2,0%



We denote for life expectancies an almost 2% variation and this variation is stable with the reference age used.

The variation for the best estimate provision of the pension plan liabilities is close, about 1.5% (the actualization effect leads to a smallest variation).

Solvency requirement

The solvency capital requirement is computed in the Solvency II framework: the undertaker must avoid ruin for a one year time period with a probability level greater than 99.5%.

We analyze the risk sub-module “longevity risk” in the “life” module: *the risk of loss, or of adverse change in the value of insurance liabilities, resulting from changes in the level, trend, or volatility of mortality rates, where a decrease in the mortality rate leads to an increase in the value of insurance liabilities.*

We compare the value of the solvency capital requirement between the standard approach (case one) and an internal model approach (case two). In the second case we will compute the capital associated with the estimation risk.

3. Consequences for an annuity plan

Solvency requirement

The calculus is the standard model (QIS5 framework) is straightforward: it consists in a reduction of 20% of the conditional death rates.

For the internal model approach we refer to the framework described in Guibert & al. [2010]:

$$\text{Capital\_MIP}_0 \approx \frac{\dfrac{VaR_{99,5\%}(\chi)}{L_0}-1}{1+\alpha\left(D_0-\dfrac{VaR_{99,5\%}(\chi)}{L_0}(D_0-1)\right)}L_0$$

with $\chi = \dfrac{P_1+L_1}{1+R_1}$

3. Consequences for an annuity plan

Solvency requirement

The results in the standard model framework are:

Provision - D. d'expérience (1)	Charge capital (FS - Longévité) (2)	Poids de la charge de capital (2)/(1)
4 752	316	6,7%

With the internal model approach:

Provision - D. d'expérience (1)	Charge capital (MIP - Risque estimation) (2)	Poids de la charge de capital (2)/(1)
4 752	92	1,9%

The capital charge associated with the estimation risk (volatility risk of the adjusted rates) is about 2% of the best estimate value of the provisions. As a consequence, this capital charge is about 30% of the longevity risk sub module capital charge in the QIS5.

AGENDA

Estimation risk measure

4. Impact of the choice of a reference

Because the mortality table used for the valuation of the provisions is built by positioning the crude rates compared to a reference (the Brass model) it is important to evaluate the robustness of the result compared to this assumption.

We used the French TGH/F 05. They have been built by positioning experimental data to the general French population observed over the period 1962 / 2000.

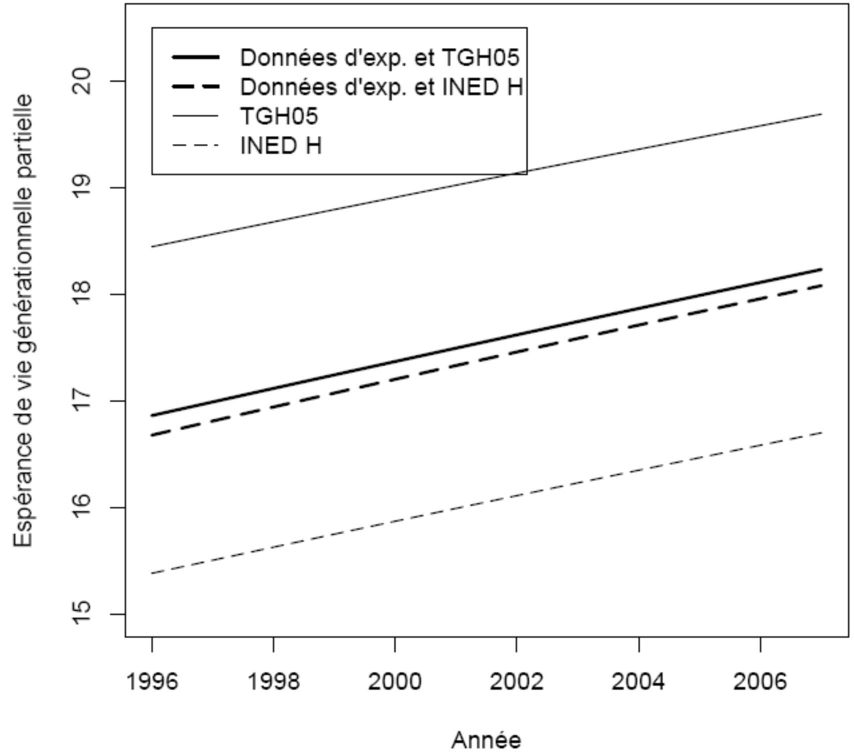
In this section we compare the model adjustment with two assumptions for the reference:

- TGH/F 05 (insured people) ;
- INED prospective tables (French population).

4. Impact of the choice of a reference

The impact is about 1% of the partial life expectancy, which is half of the estimation risk associated with the sampling fluctuations.

Esp. de vie gén. partielle (pour t=1996) Age: 67 / Age fin: 95		Ecart relatif avec (1)
Données d'expérience / Positionnement table population assurée (TGH05) (1)	16,9	0,0%
Quantile à 2,5% (simulations et positionnement TGH05)	16,5	-1,8%
Quantile à 97,5% (simulations et positionnement TGH05)	17,2	2,0%
Données d'expérience / Positionnement table population générale (INED H)	16,7	-1,1%
Table population assurée (TGH05)	18,4	9,4%
Table population générale (INED H)	15,4	-8,8%



Conclusion

One highlighted a variation from approximately 1.5% between the provision calculated starting from the table resulting from the data and that calculated starting from the tables corresponding at the boundaries of the interval of probability at 95% of the tables resulting from simulations.

This variation is close to that observed for the life expectancies, but is slightly weaker taking into account in particular the actualization effect.

Concerning the load of capital, it appears that the need for capital under the only risk of estimate related to the fluctuations of sampling accounts for approximately 30% of the need for capital under the risk of longevity of the Solvency II project (QIS5).

On this point, one recalls that the risk of longevity of the Solvency Ii project reflects the risk related on the level, the tendency and the volatility of the rates of deaths adjusted starting from the data.

Conclusion

With that is added a risk associated with the choice with the reference, of which the order of magnitude is half of the precedent.

Thus, half of the requirement in capital intended to cover the risk of longevity in the QIS 5 is “consumed” by uncertainties associated with construction with the best estimate reserving tables.

The explicit quantification of these risks thus appears essential within the framework of the building of a partial internal model.

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