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# Product pricing and solvency capital requirements for long-term care insurance

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This paper presents a comprehensive assessment of premiums, reserves and solvency capital requirements (SCRs) for long-term care (LTC) insurance policies using Activities of Daily Living and US data. We compare stand-alone policies, whole life insurance policies with LTC benefit riders (LTC insurance combined with whole life insurance), life care annuities (LTC insurance combined with annuities) and shared LTC insurance in terms of net premium cost and SCRs. Net premiums and best-estimate reserves for base LTC insurance policies are determined using Thiele's differential equation. Product features such as the elimination period and the maximum benefit period are compared using a simulation-based model. We show how a maximum benefit period can reduce costs and risks for LTC insurance products. SCRs for longevity risk and disability risk are based on the Solvency II standard formula. We quantify the extent to which whole life insurance policies with LTC benefit riders and life care annuities provide lower SCRs than stand-alone LTC insurance policies.

*Keywords*: activities of daily livings; Solvency II; longevity risk; disability risk

## 1. Introduction

Long-term care (LTC) costs have shown a significant increase over recent decades and the increasing trend is projected to continue in future (e.g. Congressional Budget Office 2004, Productivity Commission of Australia 2013, Shi & Zhang 2013). LTC expenses are significantly higher if the insured moves into LTC facilities (see Fong *et al*. 2012, for a discussion on nursing home admittance). The primary funding for LTC costs in Australia is the lifetime stop-loss mechanism funded through the pay-as-you-go scheme (Productivity Commission of Australia 2011, 2013). In the USA, the base funding programme for LTC costs is Medicaid (Meiners 2008). In particular, in the USA, community-based LTC costs are primarily funded through two public programmes: Medicaid and Medicare; institutionalised LTC expenses are primarily funded through Medicaid and personal co-payments (Kaye *et al*. 2010).

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Organisation for Economic Co-operation and Development (2005) and Colombo *et al*. (2011) provide a comprehensive review on the LTC funding systems in OECD countries including Australia and the USA.

Recent discussions in Australia and many other countries have focused on developing the private LTC insurance market as an important supplement for public funding sources (see e.g. Glendinning *et al*. 2004, Colombo *et al*. 2011, Productivity Commission of Australia 2011, 2013). Though the private insurance is an important source, the share of the private market is small. In the USA, only 4% of LTC costs are reimbursed from private insurance (Brown & Finkelstein 2008). Motivated by the small share of private LTC insurance, Brown & Finkelstein (2008) investigated the interaction of the public Medicaid programme and private LTC insurance. They find that Medicaid has a very large crowd-out effect due to the implicit tax imposed on the benefits of private LTC insurance. Against this background, a comprehensive analysis of LTC insurance in terms of premium costs, policy designs and solvency capital will allow a more informed consideration of the role and effectiveness of private LTC insurance.

An LTC insurance policy entitles the insured to receive benefits when the insured becomes functionally disabled according to the definition pre-specified in the policy (Haberman & Pitacco 1999). LTC insurance policies, however, do not have a uniform definition for the benefit eligibility in the market. The most frequently used criteria for defining functional disability in LTC insurance are the number of activities of daily livings (ADLs) that individuals cannot perform independently and cognitive impairment (Haberman & Renshaw 1996, Murtaugh *et al*. 2001, Pritchard 2006). The Australian Bureau of Statistics defines individuals' functional disability based on the Core Activity Restrictions that can be linked to scales of ADLs (Leung 2004, 2006). We focus on ADLs as the basis for a private LTC insurance contract.

LTC insurance policies can be categorised into four different types (Haberman & Pitacco 1999, Leung 2006): fixed benefit policies sold to healthy individuals, fixed benefit policies sold to the elderly entering or already staying in LTC facilities, indemnity-based benefit policies and policies that allow the insured to choose between fixed benefit and LTC service. The fixed benefit policy is the most typical and widely used type in the private LTC insurance market. Fixed benefit LTC insurance can be stand-alone policies, included as a rider benefit in the whole life insurance or life care annuities (Haberman & Pitacco 1999).

A stand-alone policy pays out the predetermined benefit when the insured becomes functionally disabled. In practice, LTC policies can be combined with other forms of insurance. LTC cover included as a rider benefit in a whole life insurance policy is a financial product that allows the insured to draw the death benefit for LTC costs before death (Haberman & Pitacco 1999). With this policy, the insured is eligible for LTC benefits when becoming functionally disabled and also becomes eligible upon death for the death benefit net of drawn LTC benefits. Another version is to directly include LTC benefits in a whole life insurance where the death benefit is a fixed amount (Leung 2006). This paper focuses on the type in Leung (2006).

LTC insurance can also be combined with annuities, which is usually referred to as the life care annuity (Murtaugh *et al*. 2001, Warshawsky 2007, Brown & Warshawsky 2013). The life care annuity reduces the adverse selection problem by pooling annuitants who are vulnerable to longevity risk and LTC insurance policyholders who are vulnerable to disability risk (Murtaugh *et al*. 2001). This risk pooling of the life care annuity provides a natural hedge and therefore reduces insurance premiums (Murtaugh *et al*. 2001, Warshawsky 2007, Brazell *et al*. 2008). Such annuities offer a valuable product structure for both the insurer and the insured, especially, given the increasing need for individuals to fund their own retirement income and the potential role of a private annuity market.

The base LTC insurance in this paper is an LTC insurance policy with no elimination period or maximum benefit period. In order to minimise adverse selection and to make LTC insurance more affordable, insurers usually include an elimination period and a maximum benefit period in the product. The elimination period is the required minimum number of consecutive payment periods before the insured becomes eligible for benefits. The elimination period can span from three months to two years. Most LTC insurers provide a lifetime elimination period, which means that the insured does not have to go through the elimination period each time before he or she is eligible for receiving benefits. The maximum benefit period is also a useful tool in managing risks and making the product more affordable. Analogous to the upper limit in property and casualty insurance, the maximum benefit period is the maximum periods of payment that the insured can possibly receive. The commonly used maximum benefit periods are three, four and five years. These alternative product designs have implications for both costs and capital requirements.

This paper considers product design for LTC and analyses lump sum and periodic premiums for a broad range of fixed benefit LTC insurance policies taking into account product features such as the elimination period and the maximum benefit period. Different combinations of the elimination period and the maximum benefit period are analysed to highlight how more affordable products can be offered. We show that the maximum benefit period is effective in reducing costs and idiosyncratic risk. Solvency capital requirements (SCRs) under Solvency II for different types of LTC insurance policies are assessed to determine the extent of capital reductions in stand-alone policies for disabled lives compared to healthy lives. The results show that the SCR as proportion of best-estimate reserve is lower for LTC insurance sold to disabled individuals than healthy individuals. Capital reductions for combined LTC insurance with life insurance and annuities are quantified. For example, under the Solvency II standard formula framework, it is shown that 80% less solvency capital per unit premium at policy issue is required for life care annuities compared to stand-alone insurance for policies sold to 65-year-old healthy males.

The paper is arranged as follows. Section 2 describes the Markov model framework for health dynamics. Section 3 presents the methodology on pricing, reserving and deriving capital requirements for LTC insurance policies. The first part of Section 3 outlines Thiele's differential equation approach used for the pricing and reserving of base LTC insurance policies. The second part focuses on policies with flexible product features which require a simulation-based approach to derive premiums and reserves. The third part discusses the SCR in the Solvency II Directive (European Insurance and Occupational Pension Authority 2011). Section 4 briefly describes the data used to derive health dynamics and presents the transition rates assumed for the analysis. The demographic characteristics of the experience assumed in the analysis are also provided in Section 4. Section 5 presents results for premiums based on the methods described in Section 3 and compares premiums for different types of LTC insurance policies. Section 6 gives results for the best-estimate liabilities and SCRs for the different types of policies. Section 7 concludes.

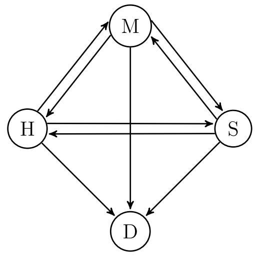


Figure 1. Four-state Markov transition diagram.

## 2. Markov model framework for health dynamics

An LTC insurance policy pays benefits to the insured when the insured becomes functionally disabled, i.e. the benefits are dependent on the current health state. We employ a four-state continuous time Markov model to describe the health dynamics of retirees. As pointed out in Wolf & Gill (2009), using Markov models for disability results in underestimations of short duration disability. As our study mainly focuses on LTC disability that is typically chronic and comes with long durations, this limitation is not expected to cause large bias in the results (Brown & Warshawsky 2013).

The health states are categorised based on the number of difficulties in independently performing ADLs. The transition diagram is shown in Figure 1, where 'H' denotes the healthy state, 'M' denotes the mildly disabled state (defined as having 1–2 ADL difficulties),[1] 'S' denotes the severely disabled state (defined as having 3–6 ADL difficulties) and 'D' denotes the dead state. As shown in the diagram, the paper allows for recovery from disability, which is in line with prior studies (e.g. Robinson 1996, Leung 2006, Pritchard 2006, Ameriks *et al.* 2011, Brown & Warshawsky 2013). Some prior studies (such as Ferri & Olivieri 2000, Olivieri & Pitacco 2001) do not take into account recovery based on the argument that LTC disability has the chronic characteristic that makes it very hard to recover from the disabled state. Based on Fong *et al.* (2015), the recovery rate is comparatively high and should be taken into account in the analysis.

The transition rates and probabilities are age and sex dependent. Let $\Omega_\chi = \{H, M, S, D\}$ denote the state space, $x$ the individual's age last birthday and $\chi(x) \in \Omega_\chi$ the health state at age $x$. For $t \geq 0$ and $i, j \in \Omega_\chi$, the transition probability from state $i$ at age $x$ into state $j$ at age $x + t$ is defined as follows:

$$p_{ij}(x, x+t) = \Pr\{\chi(x+t) = j \mid \chi(x) = i\}. \tag{1}$$

Instantaneous transition intensities are assumed to be integrable on compact intervals. For $i \neq j$, the transition intensity is defined as:

$$\mu_{ij}(x) = \lim_{\Delta x \to 0^+} \frac{p_{ij}(x, x+\Delta x)}{\Delta x}. \tag{2}$$

[1]Typical private LTC insurance, particularly in North America, pays a benefit when the insured has difficulties with either 2+ or 3+ ADLs. This paper uses 3+ ADLs as a base and shows the effect of 2+ ADLs in a sensitivity analysis in Table 9.

We use a generalised linear model (GLM) with a log link function and a Poisson distribution to graduate the estimated transition rates. In this model, the number of transitions is assumed to follow a Poisson distribution with mean that depends on age in the following function:

$$m_x = e_x \sum_{s=0}^{k} \beta_s x^s, \tag{3}$$

where $e_x$ denotes the exposure for $x$-year-old individuals, $\beta_s$ denotes the coefficients for the $s$th-order polynomial function of age and $k$ is the maximum polynomial order. We select the optimal $k$ based on AICc, BIC and model deviance (see Fong *et al*. 2015, for the detailed methodology). The graduated transition rates are then used as inputs to the pricing, reserving and capital requirements of LTC insurance.



## 3. Methodology

The focus of this paper is to consider how prices, reserves and SCRs can be impacted by different product features, so loadings for expenses and profit are not included in the premiums and best-estimate reserves in this paper. We also use the same biometric and financial bases for calculating premiums and best-estimate reserves.

### *3.1. Thiele's differential equation*

The base LTC insurance policy has no elimination period or maximum benefit period. Since we assume a Markov process for our transitions and benefits, we can compute premiums with the generalised Thiele's differential equation (Hoem 1969, Leung 2006, Christiansen *et al.* 2014). Thiele's differential equation approach, first published in Gram (1910), provides a set of simultaneous differential equations that are used to calculate premiums and reserves for life insurance policies, where only alive and dead states are involved. The generalised Thiele's differential equation has been applied to life contingencies that involve multiple health states in prior studies (see, e.g. Norberg 1992, 1995, Linnemann 1993).

We use $V_i(t, T)$ to denote the time $t$ expected present value of benefits paid to an individual in state $i$ within the period $(t, T)$, where $i \in \Omega_\chi$ is the health state at time $t$ and $T$ is the terminal period. As discussed in Fong *et al.* (2015), it is difficult to extrapolate the transition rates past age 100 due to limited exposure at very old ages. Therefore, the maximum attainable age is assumed to be 100. Based on the four-state health transition diagram defined in Figure 1, the expected present value is given by:

$$V_i(t, T) = \int_t^T e^{-\int_t^s \left(\delta + \sum_{j \neq i} \mu_{ij}(x+u)\right) du} \left[ b_i(s) + \sum_{j \neq i} \mu_{ij}(x+s) \Big( B_{ij}(s) + V_j(s, T) \Big) \right] ds, \tag{4}$$

where $i, j \in \Omega_\chi = \{\text{H, M, S, D}\}$ denote health states as defined in Section 2, $\delta$ is the continuously compounded interest rate, $b_i(s)$ is the annuity payment to the insured while in state $i$ at time $s$,

$\mu_{ij}(x+s)$ is the transition intensity from state $i$ to state $j$ for individuals aged $x+s$ and $B_{ij}(s)$ is the benefit payment upon transitions from state $i$ to state $j$ at time $s$. When the insured dies, the reserve becomes zero after the payment of death benefit, if any, i.e. $V_i(t, T) \equiv 0$ when $i = D$. $b_i(s)$ is usually referred to in the literature as the sojourn benefit and $B_{ij}(s)$ as the transition benefit (Christiansen *et al.* 2014). Since this paper focuses on fixed benefit products, the sojourn and transition benefits are fixed for base stand-alone policies, life insurance policies with LTC benefit riders of the type in Leung (2006) and life care annuities.

Differentiating both sides of Equation (4) with respect to $t$, the generalised Thiele's differential equation is derived and expressed as follows:

$$\frac{\mathrm{d}V_i(t, T)}{\mathrm{d}t} = \delta V_i(t, T) - b_i(t) - \sum_{j \neq i} \mu_{ij}(x+t)\Big(B_{ij}(t) + V_j(t, T) - V_i(t, T)\Big), \quad (5)$$

where notations are consistent with those in Equation (4). Equation (5) explicitly shows that the change in the reserve during an infinitesimal time can be decomposed into four parts: the accrued interest, the paid-out annuity to individuals staying in certain states if any, the benefit payment upon transitions if any and the jump in the reserve if the individual transitions into a different state.

When the transition intensities are simple functions with respect to age, the above simultaneous differential equations can be easily solved to derive a closed formula for the reserve function. Based on the assumed values for the interest rate $\delta$ and the graduated transition rate $\mu_{ij}(x+t)$, the reserve function evaluated at time 0 is the lump sum premium of a base LTC insurance policy.

Since graduated transition rates are parametrised as exponential polynomial functions with respect to age, the reserve functions cannot be directly solved for in the simultaneous equations as in Equation (5). We use Euler's rule to derive a numerical solution of the reserve function $V_i(t, T)$. Euler's rule is a commonly used approach in discretising differential equations. The d$t$ term in Equation (5) is replaced by a very short period of time $h$, such as a day. Rearranging the discretised version of Equation (5), the process of calculating the reserve for a previous period is as follows:

$$\begin{aligned} V_i(T, T) &= 0, \quad \forall\, i \in \Omega_\chi, \qquad (6) \\ V_i\big(T-(u+1)h, T\big) &= V_i(T-uh, T)\left(1 - h\delta - h\sum_{j\neq i}\mu_{ij}(x+T-uh)\right) + hb_i(T-uh) \\ &\quad + h\sum_{j\neq i}\mu_{ij}(x+T-uh)\Big(B_{ij}(T-uh) + V_j(T-uh, T)\Big), \end{aligned}$$

where $i, j \in \Omega_\chi$, $u \in \{0, 1, 2, \ldots, \frac{T}{h} - 1\}$ is a non-negative integer and other notations are consistent with those in Equation (5). An implicit assumption is that the transition intensity is constant within the short period of time $h$. Numerical values are then solved for based on backward iterations of Equation (6) starting from the terminal period.

The lump sum premium of a base LTC insurance sold to an individual in state $i$ is equal to the expected present value of future benefits, based on the principle of equivalence. The expected present value of a unit benefit while the insured is in the severely disabled state can be calculated as the reserve function evaluated at the present time, which can be expressed as follows:

$$v_{\mathrm{S}} = V_i(0, T), \tag{7}$$

where $i \in \Omega_\chi$, $\forall t \in (0, T)$, $b_{\mathrm{S}}(t) = 1$, $b_k(t) = 0$ for $k \neq \mathrm{S}$ and $B_{ij}(t) \equiv 0$. The lump sum premium of insurance policies sold to individuals in state $i$, denoted by $P_i^L$, is then calculated in the following equation:

$$P_i^L = y v_{\mathrm{S}}, \tag{8}$$

where $y$ is the predetermined annual amount of LTC insurance benefits paid while the insured is in the severely disabled state.

Continuously paid premiums are also of interest in the actuarial field. In particular, premiums paid on a very frequent basis, such as weekly premiums, are usually approximated as continuous premiums. The expected present value of a unit payment while the individual stays in the healthy or mildly disabled state is first calculated based on the discretised simultaneous differential equations, as shown in Equation (6), with a very small step size such as 0.001. In mathematical expressions, the first step is to calculate the following expected present value:

$$v_{\mathrm{H,M}} = V_i(0, T), \tag{9}$$

where $i \in \Omega_\chi$, $\forall t \in (0, T)$, $b_{\mathrm{H}}(t) = b_{\mathrm{M}}(t) = 1$, $b_{\mathrm{S}}(t) = b_{\mathrm{D}}(t) = 0$ and $B_{ij}(t) \equiv 0$. The expected present value of a unit payment while the individual stays in the severely disabled state is then calculated using the same specifications as in the case of the lump sum premium, as shown in Equation (7). Based on the principle of equivalence, the continuously paid premium per annum of insurance policies sold to individuals in state $i$, denoted by $\bar{P}_i$, is solved for in the following equation:

$$\bar{P}_i v_{\mathrm{H,M}} = y v_{\mathrm{S}}. \tag{10}$$

Instead of paying lump sum premiums at the outset or continuously paying insurance premiums, policyholders usually choose to pay LTC insurance premiums on an annual, quarterly or monthly basis while the insured is not eligible for receiving LTC insurance benefits.[2] Thiele's differential equation is a very useful tool in dealing with these periodic premiums.

For periodic premiums of a base LTC insurance policy, the unit payment while the insured stays in the healthy or the mildly disabled state is calculated using the discretised version of Thiele's differential equation, as shown in Equation (6), with a step size of the corresponding frequency, for example $h = \frac{1}{12}$ for premiums paid on a monthly basis. Let $v'_{\mathrm{H,M}}$ denotes the expected present value of a unit payment while the insured is in the healthy or the mildly disabled state at the beginning of each assessment interval (for example, at the beginning of each month for premiums on a monthly basis). $v_{\mathrm{S}}$ is calculated using the same specifications on

[2]For some types of LTC insurance policies, only lump sum premiums are taken into account since periodic or continuous premiums are not feasible, e.g. policies sold to individuals who are already severely disabled and life care annuities.

the benefit payments as in Equation (7). Based on the principle of equivalence, the premium on a regular basis of insurance policies sold to individuals in state $i$, denoted by $P_i^{(f)}$, can be solved for in the following equation:

$$f P_i^{(f)} v'_{\mathrm{H,M}} = y v_{\mathrm{S}}, \tag{11}$$

where $f$ denotes the number of payments in a year. For example, $f = 12$ and 1 for monthly and annual premiums, respectively.

### *3.2. Simulation-based approach*

The generalised Thiele's differential equation is widely used for pricing and reserving of insurance in a Markov model framework. When flexible product features such as the elimination period and the maximum benefit period are allowed, the LTC benefit depends on durations in one or multiple states and the benefit process is therefore no longer Markovian. Consequently, the generalised Thiele's differential equation approach cannot be directly used for LTC insurance policies with more flexible features (Hoem 1969, Christiansen *et al*. 2014). Based on age- and duration-specific transition rates, non-simulation methods can be used to take into account these product features in the calculation of premiums and best-estimate reserves (see, e.g. Waters 1989, CMI 1991, Cordeiro 2002). However, due to the lack of available long-period data for LTC disability in the Health and Retirement Study (HRS) data, duration-specific rates are not accurate. In addition, the two-year window between consecutive waves does not allow for an accurate estimation of the durations in disabled states.

To take into account these flexible product features, we use a simulation-based approach, which is to simulate health trajectories of a large number of homogeneous and independent individuals and to calculate the expected present values of benefit payments based on the simulated health trajectories.

Given graduated annual transition rates from Fong *et al.* (2015), transition probabilities for a short period of time can be calculated as the matrix exponential of the annual transition rate matrix multiplied by the short period, assuming that transition rates are constant within integer ages. Based on the calculated transition probabilities for a short period, an individual's health state in the next period, given his or her current health state, follows a multinomial distribution. The health transition trajectories of a large number of $N$ homogeneous individuals aged $x$ are simulated using the multinomial distribution. Based on the simulated health trajectories, the present value of future benefit payments to the insured currently in state $i$ at time $t$ is:

$$PV_i(l,t) = \sum_{s=t}^{\omega-x} \Delta_i(l,t) B(l,0,1,\ldots,s) e^{-\delta(s-t)}, \tag{12}$$

where $i \in \Omega_\chi$ is the health state, $l$ denotes the $l$th individual, $\omega$ is the maximum attainable age, $\Delta_i(l,t)$ is an indicator variable that equals 1 if the $l$th insured is in state $i$ at time $t$ and equals 0 otherwise, $B(l,0,1,\ldots,s)$ is the benefit payment to the $l$th insured based on the simulated historical and current health status up to time $s$ and $\delta$ is the continuously compounded interest rate. The benefit payment function $B(l,0,1,\ldots,s)$ takes into account the elimination period and the maximum benefit period. In particular, the value of $B(l,0,1,\ldots,s)$ is set to zero for those periods spent in functional disability before the duration surpasses the elimination period; the value of $B(l, 0, 1, \dots, s)$ is set to zero if the total number of payments exceeds the maximum benefit period.

The lump sum premium of insurance policies sold to individuals in state $i$, denoted by $\tilde{P}_i^L$, is calculated as the sample mean of the present values of benefits across all the simulated homogeneous individuals:

$$\tilde{P}_i^L = \frac{1}{N} \sum_{l=1}^{N} PV_i(l, 0), \tag{13}$$

where $N$ is the number of simulations. In addition, the estimation standard error is calculated as the standard deviation divided by the square root of the number of simulations. The premium paid on a regular basis of insurance policies sold to individuals in state $i$, denoted by $\tilde{P}_i^{(f)}$, can be calculated in the following equation:

$$\tilde{P}_i^{(f)} \tilde{a}_{\text{HM}}^{(f)}(0) = \frac{1}{N} \sum_{l=1}^{N} PV_i(l, 0), \tag{14}$$

where $\tilde{a}_{\text{HM}}^{(f)}(0)$ is the time-0 present value of unit payments at the beginning of each $\frac{1}{f}$ year while the insured is healthy or mildly disabled.

The premium of LTC insurance calculated using the simulation approach is only an approximation to the price calculated using Thiele's differential equation approach. If the number of simulations is very large and the step size $h$ of Euler's approximation approach is small enough in deriving numerical solutions to the differential equations, the premiums of base LTC insurance policies calculated using the two approaches should give very close results.

### *3.3. Best-estimate reserves and SCRs*

LTC insurance providers take risks that span a long period of time. Accurate estimations of premiums and reserves are therefore critical in the risk management for product providers. The best-estimate reserve is the expected present value of future liabilities. The solvency capital held on top of best-estimate reserve and risk margins ensures that the insurer survives losses resulting from extreme events that occur with low probabilities.

#### *3.3.1. Best-estimate reserves based on Thiele's differential equation*

Best-estimate reserves for individuals in each alive health state are determined from Equation (6) using Thiele's differential equation approach. Let $\chi(t) \in \Omega_\chi$ denote the health state that the individual stays at time $t$. The time $t$ best-estimate reserve for an LTC insurance policy issued to an individual in state $k$, denoted by $V(t, T|\chi(0) = k)$, is the expected value of best-estimate reserves for individuals in different health states. The time $t$ best-estimate reserve is determined as follows:

$$V\big(t, T|\chi(0) = k\big) = \sum_i \Pr\big(\chi(t) = i|\chi(0) = k\big) V_i(t, T), \tag{15}$$

where $i, k \in \Omega_\chi$, $\Pr(\chi(t) = i|\chi(0) = k)$ is the probability of staying in heath state $i$ at time $t$, given the insured is in state $k$ at the outset, and $V_i(t, T)$ is the best-estimate reserve for an insured in state $i$ at time $t$ which is calculated from Equation (6).

### 3.3.2. *Best-estimate reserves based on the simulation approach*

Let $\tilde{V}_i(t, T)$ denote best-estimate reserves for an insured in state $i$ at time $t$ calculated using the simulation approach. $\tilde{V}_i(t, T)$ can be estimated as follows:

$$\tilde{V}_i(t, T) = \frac{\sum_{l=1}^{N} PV_i(l, t)}{\sum_{l=1}^{N} \Delta_i(l, t)}, \tag{16}$$

where $i \in \Omega_\chi$, $N$ is the number of individuals at the outset, $PV_i(l, t)$ is calculated using Equation (12) and $\Delta_i(l, t)$ is the indicator variable that equals 1 if the $l$th insured is in state $i$ at time $t$ and equals 0 otherwise. Based on the simulation approach, the best-estimate reserve for an LTC insurance policy issued to an individual in state $k$ at the outset, denoted by $\tilde{V}\big(t, T|\chi(0) = k\big)$, can be calculated as follows:

$$\tilde{V}\big(t, T|\chi(0) = k\big) = \frac{\sum_{i \in \Omega_\chi} \sum_{l=1}^{N} PV_i(l, t)}{N} = \sum_{i \in \Omega_\chi} \frac{\Delta_i(l, t)}{N} \tilde{V}_i(t, T), \tag{17}$$

where the simulation is on a cohort of $N$ individuals who are in state $k$ at the outset.

Distributional risk measures, such as the Value-at-Risk (VaR), are widely used in actuarial practice, in particular in the context of estimating SCRs. Based on the simulation approach, these distributional risk measures of the reserve can be easily calculated. The $\mathrm{VaR}_\alpha$ of liabilities for the insured in state $i$ at time $t$ is calculated as the $100\alpha\%$ quantile of $PV_i(l, t)$ across all simulated individuals at time $t$, which is expressed as follows:

$$\begin{aligned} &VaR_\alpha\Big(t, T|\chi(0) = k\Big) \\ &\quad = \arg\min_x \left\{ \forall\, l \in \{1, 2, \ldots, N\},\ \Pr\left( \sum_{i \in \Omega_\chi} PV_i(l, t) > x \right) \leq 1 - \alpha \right\}. \end{aligned} \tag{18}$$

### 3.3.3. *Solvency capital requirements*

The SCR is defined under Solvency II as the amount of capital required to cover losses that occur with a probability of 99.5% over one year (Olivieri & Pitacco 2009, Kochanski 2010, Meyricke & Sherris 2014). SCRs are required to take into account a broad range of risks that insurers are faced with, including longevity risk, the risk of higher disability rates, the risk of lower recovery rates, interest rate risk, etc. Planchet & Tomas (2014b) consider the impact of mortality risk of disabled lives on SCRs for LTC insurance. Pitacco (2015) uses a sensitivity analysis to assess the impact of mortality risk and disability risk on premiums of LTC insurance, which provides insights into the level of solvency capital for product providers.

Let $\mathrm{NAV}_t$ denote the net asset value at time $t$, which is calculated as the difference between the value of assets and the best estimate of liability at time $t$. The SCR at time $t$ is then defined as the smallest amount of capital held at time $t$, so that the probability of a positive NAV next year is no less than 0.995. This can be expressed as follows:

$$\mathrm{SCR}_t = \arg\min_x \left\{ \Pr\left(\mathrm{NAV}_{t+1} > 0 | \mathrm{NAV}_t = x\right) \geq 99.5\% \right\}. \tag{19}$$

An equivalent expression frequently adopted in practice for the SCR is shown as follows (Börger 2010, Meyricke & Sherris 2014):

$$\mathrm{SCR}_t = \arg\min_x \left\{ \Pr\left(\mathrm{NAV}_t - \mathrm{NAV}_{t+1} e^{-\delta_{t+1}} > x\right) \leq 0.5\% \right\}, \tag{20}$$

where $\delta_{t+1}$ is the continuously compounded interest rate per annum from time $t$ to $t+1$.

An alternative approach to the above framework is to use the standard formula in Solvency II. The SCR for a specific shock in the standard formula is calculated as the negative change in $\mathrm{NAV}_t$ in the presence of a shock that represents a one-in-two-centuries crisis. The SCR calculated using the standard formula, denoted by $\mathrm{SCR}_t^{\mathrm{Shock}}$, can be expressed as follows:

$$\mathrm{SCR}_t^{\mathrm{Shock}} = \mathrm{NAV}_t - \mathrm{NAV}_t^{\mathrm{Shock}}, \tag{21}$$

where $\mathrm{NAV}_t^{\mathrm{Shock}}$ is the net asset value at time $t$ if the one-off permanent shock occurs.

For periodic premium LTC insurance, we also take into account management actions in terms of increasing premiums when shocks occur. The delay of management actions in adjusting premiums has an impact on the value of $\mathrm{NAV}_t^{\mathrm{Shock}}$.

The paper focuses on the analysis of two major risks that LTC insurance providers are faced with: longevity risk and disability risk. Under Solvency II, longevity risk is assessed as a permanent 20% decrease in mortality rates at all ages; disability risk is assessed as an increase of 35% in disability rates at all ages for the next year, a permanent increase of 25% at all ages for the following years and a permanent decrease of 20% in recovery rates at all ages (European Insurance and Occupational Pension Authority 2011). The SCR for the aggregate risk is to calculate the SCR for each risk and to aggregate these SCRs via a correlation matrix (Kochanski 2010). Based on the assumption of zero correlation between longevity risk and disability risk, the SCR for longevity risk and disability risk is calculated as follows:

$$\mathrm{SCR}_t^S = \sqrt{\left(\mathrm{SCR}_t^{\mathrm{Longevity}}\right)^2 + \left(\mathrm{SCR}_t^{\mathrm{Disability}}\right)^2}, \tag{22}$$

where $\mathrm{SCR}_t^{\mathrm{Longevity}}$ and $\mathrm{SCR}_t^{\mathrm{Disability}}$ are the SCRs for longevity risk and disability risk, respectively, which are calculated using Equation (21).

In the standard formula, insurers are also required to hold a risk margin in addition to the best-estimate reserve in order to cover residual risks associated with those captured in SCRs (Olivieri & Pitacco 2009, European Insurance and Occupational Pension Authority 2011). The risk margin also represents the fair value amount that another insurer would require to take over the liabilities (Meyricke & Sherris 2014). The risk margin, denoted by $\mathrm{RM}_t$, is linked to current and future SCRs and is determined as follows:

$$\mathrm{RM}_t = \sum_{k=0}^{m} c \frac{\mathrm{SCR}_{t+k}^{S}}{(1+r_f)^k}, \tag{23}$$

where $m$ is the time to exhaustion of the portfolio of LTC insurance policies, $c$ is the cost of capital and $r_f$ is the risk-free interest rate.

## 4. Health dynamics

### 4.1. Data description

In order to provide realistic estimates, we use health transitions data from the University of Michigan HRS, which is a US nationally representative ongoing survey of people aged 50 and above. Starting from 1992, the survey has been conducted biennially to collect information on physical and mental health functioning, health insurance, health expenditures, retirement plans and assets. Since there was an inconsistent structure of questions asked before wave 1998, we use data from wave 1998 onward to the latest available wave 2010, when this research paper was written.

The data have detailed information on self-reported difficulties in six ADLs and an assessment of mental functioning. The six ADLs are dressing, walking, bathing, eating, transferring and toileting. There is also information on whether the respondent moves into a nursing home, but information on the respondent thereafter is no longer tracked. Based on the number of ADLs that the individuals cannot perform independently, we categorise alive health states into healthy, mildly disabled, severely disabled and dead (see Section 2, for the detailed definition of each state). Exposure years and the number of transitions are summarised in Table 1.

### 4.2. Graduated transition rates

Table 2 compares goodness of fit for the nested specifications of the relationship between transition rates and age, i.e. Equation (3), based on three selection criteria: AICc, BIC and the difference between model residual deviances. We find that a quadratic specification is optimal for two and seven health transition rates for males and females, respectively. Likelihood ratio tests of the difference in residual deviance also indicate that it is beneficial to include the quadratic age term for some transition rates. These selection criteria support the use of a linear function of age for the remaining health transition rates.

Transition rates are then graduated based on the optimal specification for each transition. Table 3 shows the fitted parameters for these graduated rates.

### 4.3. Demographic characteristics of the simulated individuals

In order to provide a large enough sample, we simulate health trajectories of 40,000 males and 40,000 females starting at various ages and from the healthy, mildly disabled and severely disabled states based on the monthly transition probabilities. Monthly transition probabilities

Table 1. Summary of exposure years and the number of transitions.

| Age band | Exposure years | | | Number of transitions | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | H | M | S | H-M | H-S | H-D | M-H | M-S | M-D | S-H | S-M | S-D |
| *Males* | | | | | | | | | | | | |
| 50–54 | 1946.6 | 165.6 | 85.7 | 32 | 4 | 8 | 15 | 7 | 3 | 4 | 7 | 3 |
| 55–59 | 6602.6 | 589.3 | 209.8 | 143 | 27 | 60 | 111 | 22 | 10 | 13 | 22 | 18 |
| 60–64 | 12, 139.0 | 1147.7 | 408.2 | 267 | 42 | 159 | 234 | 69 | 54 | 30 | 59 | 31 |
| 65–69 | 13, 963.7 | 1361.0 | 530.1 | 347 | 62 | 270 | 301 | 75 | 80 | 27 | 60 | 69 |
| 70–74 | 11, 766.0 | 1444.9 | 628.1 | 410 | 80 | 349 | 250 | 87 | 105 | 28 | 44 | 87 |
| 75–79 | 8881.3 | 1431.0 | 717.0 | 397 | 87 | 394 | 224 | 97 | 103 | 17 | 37 | 117 |
| 80–84 | 5557.1 | 1195.4 | 693.6 | 337 | 89 | 356 | 182 | 96 | 150 | 11 | 35 | 166 |
| 85–89 | 2564.4 | 787.2 | 624.4 | 213 | 92 | 257 | 103 | 84 | 132 | 11 | 30 | 156 |
| 90–94 | 786.2 | 328.2 | 310.8 | 85 | 56 | 131 | 36 | 39 | 96 | 8 | 8 | 125 |
| 95–100 | 129.6 | 92.5 | 89.5 | 21 | 21 | 39 | 9 | 7 | 30 | 0 | 3 | 47 |
| Total | 64, 336.3 | 8542.7 | 4297.5 | 2252 | 560 | 2023 | 1465 | 583 | 763 | 149 | 305 | 819 |
| *Females* | | | | | | | | | | | | |
| 50–54 | 4539.7 | 381.6 | 171.1 | 67 | 21 | 8 | 52 | 13 | 2 | 10 | 13 | 4 |
| 55–59 | 10, 855.1 | 1202.5 | 494.3 | 280 | 40 | 55 | 212 | 69 | 27 | 37 | 63 | 16 |
| 60–64 | 15, 767.3 | 1932.4 | 887.5 | 458 | 74 | 114 | 436 | 129 | 37 | 42 | 112 | 36 |
| 65–69 | 16, 652.7 | 2293.9 | 971.2 | 553 | 112 | 193 | 474 | 147 | 86 | 41 | 145 | 79 |
| 70–74 | 14, 029.9 | 2170.3 | 1096.9 | 575 | 107 | 226 | 441 | 178 | 97 | 53 | 95 | 86 |
| 75–79 | 10, 853.4 | 2267.5 | 1216.2 | 579 | 144 | 257 | 349 | 157 | 116 | 41 | 95 | 171 |
| 80–84 | 7546.3 | 2227.3 | 1377.6 | 570 | 162 | 315 | 338 | 190 | 166 | 37 | 94 | 242 |
| 85–89 | 3905.6 | 1907.7 | 1558.5 | 445 | 172 | 302 | 235 | 211 | 212 | 36 | 82 | 312 |
| 90–94 | 1250.9 | 1016.1 | 1060.3 | 218 | 92 | 160 | 86 | 156 | 172 | 18 | 50 | 296 |
| 95–100 | 240.7 | 273.8 | 429.4 | 52 | 24 | 51 | 18 | 76 | 75 | 3 | 13 | 174 |
| Total | 85, 641.6 | 15, 673.2 | 9262.8 | 3797 | 948 | 1681 | 2641 | 1326 | 990 | 318 | 762 | 1416 |

Notes: For reporting purposes, we report these summaries for five-year age groups. Data for lives aged 101 and above are truncated.

are calculated as the matrix exponential of the graduated annual transition rates divided by 12. For illustrative purposes, simulations for the 65-year-old male and female cohorts starting from the healthy state are as follows.

Figure 2 shows the number of mildly disabled and severely disabled individuals among the simulated independent and homogeneous 65-year-old healthy males and females, respectively. It can be seen from Figure 2 that there are fewer mildly disabled and severely disabled individuals for males than females at all ages in the simulated cohorts. In addition, the ages at which the numbers of mildly disabled and severely disabled individuals, respectively, reach the peak are smaller for males than females. For both males and females, the number of mildly disabled individuals is larger than that of the severely disabled at all ages, except after around age 93 from which severely disabled individuals start to dominate. This is also confirmed in Table 4 that shows the proportion of survivors in each health state on a five-year basis for the simulated 65-year-old healthy individuals.

Based on the simulated health trajectories for the 80,000 individuals (40,000 males and 40,000 females) who are healthy at age 65, relevant demographic characteristics can be computed (Brown & Warshawsky 2013). The results are shown in Table 5. These demographic characteristics include the life expectancy, time spent in each level of disability, share of people ever becoming disabled and the average age of first disability conditional on becoming disabled. Life expectancy is the expected survival time of the cohort plus 65; expected time in each health state

Table 2. Poisson GLM: goodness of fit of nested models.

| | Males | | | | Females | | | |
|---|---|---|---|---|---|---|---|---|
| $k$ | AICc | BIC | $D_c$ | $\Delta D_c$ | AICc | BIC | $D_c$ | $\Delta D_c$ |
| **Disability rates** | | | | | | | | |
| $\mu_{\mathrm{HM}}$: *healthy to mildly disabled* | | | | | | | | |
| 1 | 256.77 | **260.38** | **33.91** | | 334.42 | 338.03 | 87.15 | |
| 2 | **256.37** | 261.66 | 31.26 | 2.65 | 305.15 | **310.44** | **55.63** | **31.52***** |
| 3 | 258.66 | 265.52 | 31.19 | 0.07 | **304.20** | 311.05 | 52.31 | 3.32* |
| $\mu_{\mathrm{HS}}$: *healthy to severely disabled* | | | | | | | | |
| 1 | 237.77 | 241.38 | 62.62 | | 260.51 | 264.13 | 64.70 | |
| 2 | **217.44** | **222.73** | **40.04** | **22.59***** | 248.16 | **253.45** | **50.09** | **14.61***** |
| 3 | 219.55 | 226.40 | 39.78 | 0.25 | **246.87** | 253.73 | 46.44 | 3.65* |
| $\mu_{\mathrm{MS}}$: *mildly disabled to severely disabled* | | | | | | | | |
| 1 | **214.46** | **218.07** | **43.54** | 314.73 | 318.34 | 99.05 | | |
| 2 | 215.97 | 221.25 | 42.79 | 0.75 | 278.90 | **284.19** | **60.96** | **38.09***** |
| 3 | 215.29 | 222.15 | 39.75 | 3.04* | **278.50** | 285.36 | 58.21 | 2.76 |
| **Recovery rates** | | | | | | | | |
| $\mu_{\mathrm{MH}}$: *mildly disabled to healthy* | | | | | | | | |
| 1 | **244.31** | **247.92** | **44.62** | | 300.61 | 304.23 | 72.82 | |
| 2 | 245.44 | 250.72 | 43.49 | 1.13 | **290.77** | **296.06** | **60.72** | **12.10***** |
| 3 | 247.71 | 254.57 | 43.41 | 0.08 | 293.05 | 299.91 | 60.64 | 0.08 |
| $\mu_{\mathrm{SH}}$: *severely disabled to healthy* | | | | | | | | |
| 1 | **147.94** | **151.55** | **42.01** | | 182.83 | 186.45 | 41.31 | |
| 2 | 149.37 | 154.65 | 41.18 | 0.83 | **179.44** | **184.73** | **35.66** | **5.65**** |
| 3 | 151.59 | 158.45 | 41.04 | 0.14 | 181.80 | 188.66 | 35.66 | 0.00 |
| $\mu_{\mathrm{SM}}$: *severely disabled to mildly disabled* | | | | | | | | |
| 1 | **184.84** | **188.45** | **45.95** | | **239.36** | **242.97** | **58.64** | |
| 2 | 185.53 | 190.82 | 44.38 | 1.56 | 240.57 | 245.85 | 57.60 | 1.05 |
| 3 | 187.88 | 194.74 | 44.37 | 0.01 | 242.76 | 249.62 | 57.43 | 0.17 |
| **Mortality rates** | | | | | | | | |
| $\mu_{\mathrm{HD}}$: *healthy to dead* | | | | | | | | |
| 1 | 248.70 | **252.31** | **21.86** | | 271.48 | 275.09 | 50.02 | |
| 2 | **247.81** | 253.09 | 18.70 | 3.15* | **264.39** | **269.67** | **40.67** | **9.35***** |
| 3 | 248.28 | 255.14 | 16.82 | 1.89 | 266.24 | 273.10 | 40.17 | 0.51 |
| $\mu_{\mathrm{MD}}$: *mildly disabled to dead* | | | | | | | | |
| 1 | 228.91 | 232.53 | 36.62 | | 245.42 | 249.04 | 43.72 | |
| 2 | **223.39** | **228.68** | **28.84** | **7.78**** | **242.61** | **247.90** | **38.65** | **5.07**** |
| 3 | 225.11 | 231.96 | 28.19 | 0.65 | 242.68 | 249.54 | 36.36 | 2.29 |
| $\mu_{\mathrm{SD}}$: *severely disabled to dead* | | | | | | | | |
| 1 | **239.71** | **243.32** | **47.29** | | **245.43** | **249.04** | **30.06** | |
| 2 | 241.70 | 246.98 | 47.02 | 0.27 | 247.67 | 252.95 | 30.04 | 0.02 |
| 3 | 243.49 | 250.35 | 46.46 | 0.56 | 247.28 | 254.13 | 27.29 | 2.75 |

Notes: AICc is the Akaike information criterion corrected for sample size, BIC is the Bayesian information criterion and $D_c$ is the residual deviance statistic. $k = 1$ implies age term only; $k = 2$ implies age and age-squared terms; and $k = 3$ implies age, age-squared and age-cubed terms. The optimal criteria value is bolded for each set of nested models. * is for statistic that is significant at the 10% level, ** at the 5% level, *** at the 1% level.

is calculated as the mean of total time spent in each state across all simulated individuals. Share with disability is the proportion of people who are ever disabled. Share with severe disability is the proportion of people who are ever severely disabled.

It can be seen from Table 5 that 65-year-old healthy females are expected to live three years longer than their male counterparts. The expected time of the remaining life spent in the severely disabled state for a healthy female aged 65 is nearly twice that for a 65-year-old male. More than half of males and nearly three-quarters of females in the simulated 65-year-old healthy

Table 3. Parameter estimates of the Poisson GLM with log link.

| Intensity | Males | | | Females | | |
|---|---|---|---|---|---|---|
| | $\beta_0$ | $\beta_1$ | $\beta_2(\times 10^{-2})$ | $\beta_0$ | $\beta_1$ | $\beta_2(\times 10^{-2})$ |
| *Disability rates* | | | | | | |
| $\mu_{HM}$ | −7.12*** | 0.05*** | – | −2.73*** | −0.06*** | 0.08*** |
| $\mu_{HS}$ | 1.32 | −0.25*** | 0.23*** | −1.87 | −0.16*** | 0.16*** |
| $\mu_{MS}$ | −4.59*** | 0.03*** | – | 1.22 | −0.13*** | 0.11*** |
| *Recovery rates* | | | | | | |
| $\mu_{MH}$ | −0.75*** | −0.01*** | – | −7.64*** | 0.18*** | −0.14*** |
| $\mu_{SH}$ | 0.06 | −0.05*** | – | −4.82** | 0.08 | −0.08* |
| $\mu_{SM}$ | 0.18 | −0.04*** | – | −0.05 | −0.03*** | – |
| *Mortality rates* | | | | | | |
| $\mu_{HD}$ | −9.71*** | 0.09*** | – | −7.67*** | 0.01 | 0.06*** |
| $\mu_{MD}$ | −4.36** | −0.01 | 0.05 | −4.20*** | −0.03 | 0.06** |
| $\mu_{SD}$ | −5.47*** | 0.05*** | – | −6.62*** | 0.06*** | – |

$^*p < 0.10$; $^{**}p < 0.05$; $^{***}p < 0.01$.

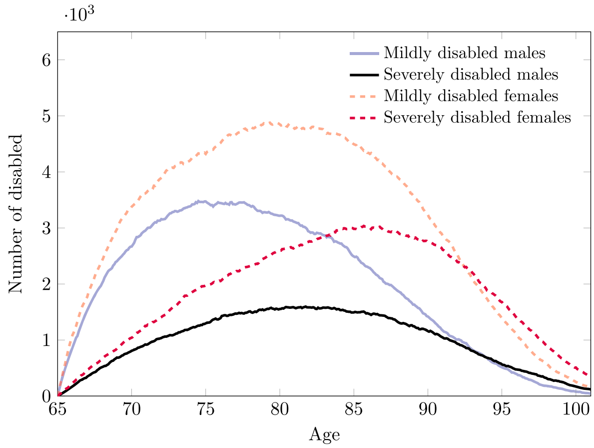


Figure 2. Number of disabled among the simulated cohorts of 65-year-old healthy males and females.

cohorts are expected to experience mild or severe disability. Among those males who are ever disabled, nearly half of them are expected to experience severe disability. For females, nearly 60% of those who are ever disabled are expected to ever become severely disabled. Given that a 65-year-old healthy individual ever becomes disabled in the remaining life, disability after age 65 is expected to first strike at age 76 for both males and females. Conditional on ever becoming severely disabled, the expected age of first severe disability after age 65 for females is about one year older than that for males.

## 5. LTC insurance premiums

This section presents the estimated premiums based on the approaches covered in Section 3. The results for the base case analysis for base LTC insurance policies, i.e. products without the

Table 4. Proportion of survivors in each health state for the simulated 65-year-old healthy male and female cohorts.

| Age | Survivors | Healthy (%) | Mildly disabled (%) | Severely disabled (%) |
|---|---|---|---|---|
| *Males* | | | | |
| 65 | 40, 000 | 100.00 | 0.00 | 0.00 |
| 70 | 35, 834 | 90.30 | 7.45 | 2.25 |
| 75 | 29, 735 | 83.96 | 11.68 | 4.36 |
| 80 | 22, 129 | 78.44 | 14.54 | 7.02 |
| 85 | 13, 912 | 71.37 | 17.91 | 10.72 |
| 90 | 6612 | 61.04 | 21.46 | 17.50 |
| 95 | 2064 | 47.77 | 24.90 | 27.33 |
| 100 | 340 | 27.35 | 22.94 | 49.71 |
| *Females* | | | | |
| 65 | 40, 000 | 100.00 | 0.00 | 0.00 |
| 70 | 37, 597 | 88.25 | 8.98 | 2.77 |
| 75 | 33, 587 | 81.26 | 12.86 | 5.88 |
| 80 | 27, 735 | 73.17 | 17.48 | 9.35 |
| 85 | 20, 001 | 62.62 | 22.47 | 14.91 |
| 90 | 11, 375 | 47.41 | 28.44 | 24.15 |
| 95 | 4273 | 28.32 | 32.44 | 39.25 |
| 100 | 855 | 11.46 | 29.12 | 59.42 |

Table 5. Demographic characteristics of the simulated 65-year-old healthy individuals (40,000 males and 40,000 females).

| Demographic characteristics | Males | Females |
|---|---|---|
| Mean years of life after age 65 | 16.33 | 19.43 |
| Mean years with mild disability | 1.78 | 2.80 |
| Mean years with severe disability | 0.89 | 1.68 |
| Share with disability | 56.43% | 72.70% |
| Share with mild disability | 47.89% | 63.37% |
| Share with severe disability | 26.82% | 42.39% |
| Average age of first disability, conditional on becoming disabled | 76.23 | 76.52 |
| Average age of first mild disability, conditional on becoming mildly disabled | 75.83 | 76.38 |
| Average age of first severe disability, conditional on becoming severely disabled | 80.51 | 81.70 |

elimination period or the maximum benefit period, are presented. These products include stand-alone policies sold to the healthy, the mildly disabled and the severely disabled, life insurance policies with LTC benefit riders and life care annuities. Different combinations of the elimination period and the maximum benefit period are taken into account. Thiele's differential equation and the simulation approach are also compared.

### 5.1. Base policies

The premiums of base stand-alone long-term insurance policies sold to the healthy, the mildly disabled and severely disabled are calculated using Thiele's differential equation approach as described in Section 3. The base stand-alone policy pays \$100 per day when the insured is severely disabled and no elimination period is included in the base policy. The benefit is unlimited as long as the insured stays in the severely disabled state since there is no maximum benefit

Table 6. Premiums ($) of base stand-alone LTC insurance policies sold to individuals in different health states and at different ages.

| | Males | | | | Females | | | |
|---|---|---|---|---|---|---|---|---|
| Age | Lump sum | Continuous | Annual | Monthly | Lump sum | Continuous | Annual | Monthly |
| *Stand-alone policies sold to the healthy* | | | | | | | | |
| 55 | 15,923 | 1138 | 1126 | 95 | 27,526 | 1825 | 1806 | 152 |
| 60 | 16,766 | 1350 | 1333 | 112 | 28,913 | 2127 | 2101 | 177 |
| 65 | 17,448 | 1619 | 1596 | 135 | 30,313 | 2535 | 2501 | 211 |
| 70 | 17,915 | 1964 | 1933 | 163 | 31,469 | 3084 | 3036 | 257 |
| 75 | 18,193 | 2428 | 2383 | 202 | 32,099 | 3824 | 3753 | 318 |
| 80 | 18,403 | 3094 | 3025 | 257 | 31,924 | 4828 | 4719 | 402 |
| *Stand-alone policies sold to the mildly disabled* | | | | | | | | |
| 55 | 28,694 | 2326 | 2295 | 194 | 48,865 | 3647 | 3607 | 304 |
| 60 | 31,230 | 2935 | 2892 | 244 | 47,727 | 3977 | 3926 | 331 |
| 65 | 32,622 | 3639 | 3581 | 303 | 47,391 | 4550 | 4482 | 379 |
| 70 | 32,590 | 4417 | 4340 | 368 | 47,163 | 5412 | 5318 | 450 |
| 75 | 31,096 | 5242 | 5139 | 436 | 46,333 | 6615 | 6483 | 550 |
| 80 | 28,328 | 6075 | 5942 | 505 | 44,260 | 8188 | 8001 | 681 |
| *Stand-alone policies sold to the severely disabled* | | | | | | | | |
| 55 | 130,655 | – | – | – | 157,337 | – | – | – |
| 60 | 136,521 | – | – | – | 159,954 | – | – | – |
| 65 | 136,771 | – | – | – | 159,412 | – | – | – |
| 70 | 131,552 | – | – | – | 154,487 | – | – | – |
| 75 | 121,918 | – | – | – | 144,742 | – | – | – |
| 80 | 109,382 | – | – | – | 130,743 | – | – | – |

Notes: The base stand-alone LTC insurance pays $100 per day while the insured is severely disabled.

period in the base policy. The base policies are assumed sold to healthy individuals aged 55, 60, 65, 70, 75 and 80, respectively, in exchange for lump sum premiums or premiums paid on a regular basis. The continuously compounded interest rate is assumed to be a constant 4% per annum.

Lump sum, continuous, annual and monthly premiums of the above base stand-alone policy sold to individuals in different initial health states are shown in Table 6. It can be seen from the table that LTC insurance premiums for females are considerably higher than those for their male counterparts. This is due to the dual effects of females' higher disability rates and lower mortality rates compared to those of males. Higher disability rates increase the probability of claiming LTC insurance benefits, whereas lower mortality rates decrease the probability of ceasing benefit payments due to deaths.

The premium generally increases as the age at policy issue goes up, except for very old females who pay lump sum premiums. The impact of policy purchase age on the premium amount results from the combined effects of disability, recovery and mortality. On the one hand, older individuals have higher probabilities of getting disabled and lower probabilities of recovery that increase the amount of premiums. On the other hand, older individuals also have higher death probabilities that result in higher probabilities of ceasing benefit payments and therefore lower the amount of premiums. For females ageing from 75 to 80, the effects of the increasing mortality rate slightly outweigh the joint effects of the higher disability rate and the lower recovery rate. Subsequently the lump sum premiums charged to 80-year-old females are slightly higher than 75-year-old females, as shown in Table 6. Since the probability of staying in or coming back to the healthy state decreases as the age goes up, the present value of unit payments while the insured is in the healthy state, such as $\upsilon_{\text{H}}$ in Equation (10) and $\upsilon'_{\text{H}}$ in Equation (11), is lower for older ages. Consequently, the continuous, annual and monthly premiums charged to 80-year-old females are higher than the corresponding premiums charged to 75-year-old females.

Premiums of base stand-alone policies sold to mildly disabled and severely disabled individuals are shown in the second and third panels of Table 6. The base stand-alone LTC insurance sold to disabled individuals also pays $100 per day while the insured is severely disabled. Benefit payment amount, eligibility for receiving benefits, interest rate and other parameters are the same as in the analysis of stand-alone policies sold to the healthy. The only difference is the starting health state in which the insured stays. It can be seen from the table that policies sold to the mildly disabled and the severely disabled are considerably more expensive than those sold to the healthy since individuals already in the disabled state have higher probabilities of staying longer in the severely disabled state than healthy individuals.

Premiums of life insurance policies with LTC benefit riders and life care annuities calculated using Thiele's differential equation approach are shown in Table 7. The whole life insurance policy with an LTC benefit rider has a fixed death benefit of $500,000 and pays the LTC benefit of $100 per day while the insured is severely disabled. Since the death benefit is a large component and males have higher mortality rates than their female counterparts, the calculated premium of the whole life insurance policy with an LTC benefit rider charged to the male insured is larger than the premium charged to their female counterparts.[3] Due to the dominating component of the death benefit, life insurance policies with LTC benefit riders become very expensive. In particular, when the insured is disabled, expected values of LTC and death benefits both increase by a large amount. Therefore, life insurance policies with LTC benefit riders sold to disabled individuals are not feasible in the market.

The life care annuity pays $50 per day while the insured is healthy or mildly disabled and the benefit is upgraded to $100 per day if the insured is in the severely disabled state. It can be seen from Table 7 that life care annuities are more affordable as the insured becomes older. It is also interesting to note that premiums for life care annuities sold to individuals who are mildly disabled are lower than those charged to the healthy and the severely disabled. The results provide evidence for the insurability of LTC costs for old and impaired individuals and also provide insights into the design of more affordable LTC insurance policies.

Inflation protection is a typical feature included in most LTC insurance policies. LTC insurance policies with inflation protection provide the insured with benefits that increase with inflation. We assume a continuously compounded inflation rate of 3% per annum, instead of including an indexing model to link benefits to actual inflation. Lump sum premiums for base LTC insurance policies with inflation protection are shown in Table 8. It can be seen that including inflation protection leads to a large increase in the premium of all types of base insurance policies. This increase in the premium reduces as the purchasing age is older or the insured is in a worse health state. In addition, including inflation protection makes the insurance policies more expensive for females than males of the same age and health condition.

[3]The discrepancy between premiums of life insurance policies with LTC benefit riders that are charged to males and females becomes larger when the death benefit is increased due to higher mortality rates of males.

Table 7. Premiums ($) of base life insurance policies with LTC benefit riders and life care annuities.

| Age | Males | | | | Females | | | |
|---|---|---|---|---|---|---|---|---|
| | Lump sum | Continuous | Annual | Monthly | Lump sum | Continuous | Annual | Monthly |
| *Rider benefit policies sold to the healthy* | | | | | | | | |
| 55 | 226,927 | 16,219 | 16,042 | 1350 | 209,708 | 13,906 | 13,759 | 1158 |
| 60 | 258,649 | 20,826 | 20,570 | 1734 | 239,785 | 17,637 | 17,426 | 1468 |
| 65 | 291,614 | 27,053 | 26,675 | 2252 | 272,847 | 22,820 | 22,509 | 1900 |
| 70 | 324,797 | 35,615 | 35,044 | 2964 | 307,940 | 30,183 | 29,708 | 2512 |
| 75 | 357,067 | 47,658 | 46,767 | 3965 | 343,570 | 40,930 | 40,171 | 3406 |
| 80 | 387,212 | 65,096 | 63,649 | 5415 | 377,597 | 57,100 | 55,821 | 4750 |
| *Life care annuities sold to the healthy* | | | | | | | | |
| 55 | 267,773 | – | – | – | 298,983 | – | – | – |
| 60 | 240,319 | – | – | – | 273,634 | – | – | – |
| 65 | 211,479 | – | – | – | 245,530 | – | – | – |
| 70 | 182,067 | – | – | – | 215,110 | – | – | – |
| 75 | 153,053 | – | – | – | 183,191 | – | – | – |
| 80 | 125,472 | – | – | – | 150,957 | – | – | – |
| *Life care annuities sold to the mildly disabled* | | | | | | | | |
| 55 | 250,787 | – | – | – | 290,061 | – | – | – |
| 60 | 222,786 | – | – | – | 263,741 | – | – | – |
| 65 | 194,002 | – | – | – | 234,859 | – | – | – |
| 70 | 165,388 | – | – | – | 204,025 | – | – | – |
| 75 | 137,878 | – | – | – | 172,404 | – | – | – |
| 80 | 112,263 | – | – | – | 141,551 | – | – | – |
| *Life care annuities sold to the severely disabled* | | | | | | | | |
| 55 | 270,261 | – | – | – | 323,363 | – | – | – |
| 60 | 239,606 | – | – | – | 292,932 | – | – | – |
| 65 | 208,682 | – | – | – | 259,514 | – | – | – |
| 70 | 178,927 | – | – | – | 224,654 | – | – | – |
| 75 | 151,417 | – | – | – | 190,228 | – | – | – |
| 80 | 126,759 | – | – | – | 157,930 | – | – | – |

Notes: The base life insurance policy with an LTC benefit rider pays $100 per day while the insured is severely disabled and pays a death benefit of $500,000 when the insured dies. The base life care annuity pays $50 per day while the insured is alive and additional $50 per day while the insured is severely disabled.

We also show lump sum premiums for base LTC insurance policies where the insured become eligible for LTC benefits when they have difficulties with two or more ADLs. The results are also compared with prior results where the definition for receiving LTC benefits is to have difficulties with three or more ADLs. The results are shown in Table 9. It can be seen that relaxing the LTC disability to 2+ ADLs makes base stand-alone LTC insurance much more expensive, but the impact on life insurance policies with LTC benefit riders and life care annuities is minimal. The increase in premium of stand-alone policies reduces for older ages and where the insured is in a worse health state. This is slightly larger for males than females in the healthy state and the mildly disabled state and is larger for females than their male counterparts in the severely disabled state.

### 5.2. *Policies with typical product features*

Based on the simulated health trajectories of the different individuals, premiums and reserves for policies with typical product features are calculated using the simulation approach described in Section 3.2. Different combinations of the elimination period and the maximum benefit

Table 8. Lump sum premiums for base LTC insurance policies with inflation protection.

| | Males | | | Females | | |
|---|---|---|---|---|---|---|
| Age | Without inflation | With inflation | Increase (%) | Without inflation | With inflation | Increase (%) |
| *Stand-alone policies sold to the healthy* | | | | | | |
| 55 | 15,923 | 29,708 | 86.57 | 27,526 | 54,264 | 97.14 |
| 60 | 16,766 | 28,945 | 72.64 | 28,913 | 52,697 | 82.26 |
| 65 | 17,448 | 27,930 | 60.08 | 30,313 | 50,946 | 68.07 |
| 70 | 17,915 | 26,681 | 48.94 | 31,469 | 48,790 | 55.04 |
| 75 | 18,193 | 25,308 | 39.11 | 32,099 | 46,042 | 43.44 |
| 80 | 18,403 | 24,008 | 30.46 | 31,924 | 42,560 | 33.32 |
| *Stand-alone policies sold to the mildly disabled* | | | | | | |
| 55 | 28,694 | 44,193 | 54.02 | 48,865 | 78,748 | 61.16 |
| 60 | 31,230 | 45,339 | 45.18 | 47,727 | 73,980 | 55.01 |
| 65 | 32,622 | 44,992 | 37.92 | 47,391 | 69,990 | 47.69 |
| 70 | 32,590 | 42,958 | 31.82 | 47,163 | 66,041 | 40.03 |
| 75 | 31,096 | 39,360 | 26.57 | 46,333 | 61,471 | 32.67 |
| 80 | 28,328 | 34,556 | 21.98 | 44,260 | 55,767 | 26.00 |
| *Stand-alone policies sold to the severely disabled* | | | | | | |
| 55 | 130,655 | 154,702 | 18.40 | 157,337 | 197,103 | 25.27 |
| 60 | 136,521 | 159,375 | 16.74 | 159,954 | 196,134 | 22.62 |
| 65 | 136,771 | 157,321 | 15.03 | 159,412 | 191,237 | 19.96 |
| 70 | 131,552 | 149,008 | 13.27 | 154,487 | 181,256 | 17.33 |
| 75 | 121,918 | 135,968 | 11.52 | 144,742 | 166,109 | 14.76 |
| 80 | 109,382 | 120,155 | 9.85 | 130,743 | 146,862 | 12.33 |
| *Rider benefit policies sold to the healthy* | | | | | | |
| 55 | 226,927 | 240,711 | 6.07 | 209,708 | 236,446 | 12.75 |
| 60 | 258,649 | 270,828 | 4.71 | 239,785 | 263,569 | 9.92 |
| 65 | 291,614 | 302,097 | 3.59 | 272,847 | 293,480 | 7.56 |
| 70 | 324,797 | 333,564 | 2.70 | 307,940 | 325,262 | 5.63 |
| 75 | 357,067 | 364,183 | 1.99 | 343,570 | 357,513 | 4.06 |
| 80 | 387,212 | 392,818 | 1.45 | 377,597 | 388,233 | 2.82 |
| *Life care annuities sold to the healthy* | | | | | | |
| 55 | 267,773 | 386,968 | 44.51 | 298,983 | 450,833 | 50.79 |
| 60 | 240,319 | 332,297 | 38.27 | 273,634 | 394,043 | 44.00 |
| 65 | 211,479 | 280,065 | 32.43 | 245,530 | 337,567 | 37.48 |
| 70 | 182,067 | 231,296 | 27.04 | 215,110 | 282,469 | 31.31 |
| 75 | 153,053 | 186,933 | 22.14 | 183,191 | 230,037 | 25.57 |
| 80 | 125,472 | 147,737 | 17.74 | 150,957 | 181,654 | 20.33 |
| *Life care annuities sold to the mildly disabled* | | | | | | |
| 55 | 250,787 | 357,529 | 42.56 | 290,061 | 430,790 | 48.52 |
| 60 | 222,786 | 303,319 | 36.15 | 263,741 | 374,509 | 42.00 |
| 65 | 194,002 | 252,675 | 30.24 | 234,859 | 318,576 | 35.65 |
| 70 | 165,388 | 206,604 | 24.92 | 204,025 | 264,409 | 29.60 |
| 75 | 137,878 | 165,758 | 20.22 | 172,404 | 213,778 | 24.00 |
| 80 | 112,263 | 130,384 | 16.14 | 141,551 | 168,436 | 18.99 |
| *Life care annuities sold to the severely disabled* | | | | | | |
| 55 | 270,261 | 363,643 | 34.55 | 323,363 | 455,549 | 40.88 |
| 60 | 239,606 | 307,057 | 28.15 | 292,932 | 393,576 | 34.36 |
| 65 | 208,682 | 255,889 | 22.62 | 259,514 | 332,848 | 28.26 |
| 70 | 178,927 | 211,169 | 18.02 | 224,654 | 275,812 | 22.77 |
| 75 | 151,417 | 173,076 | 14.30 | 190,228 | 224,554 | 18.04 |
| 80 | 126,759 | 141,166 | 11.37 | 157,930 | 180,252 | 14.13 |

period are allowed for in calculating premiums based on the simulation approach. For illustrative purposes, lump sum premiums of stand-alone policies, life insurance policies with LTC benefit riders and life care annuities issued to 65-year-old healthy individuals are shown in

Table 9. Lump sum premiums for base LTC insurance policies with different LTC disability definitions: 2+ ADLs vs. 3+ ADLs.

| | Males | | | Females | | |
|---|---|---|---|---|---|---|
| Age | 3+ ADLs | 2+ ADLs | Difference from 3+ ADLs | 3+ ADLs | 2+ ADLs | Difference from 3+ ADLs |
| *Stand-alone policies sold to the healthy* | | | | | | |
| 55 | 15,923 | 26,657 | 67.41 | 27,526 | 45,000 | 63.48 |
| 60 | 16,766 | 27,487 | 63.95 | 28,913 | 46,157 | 59.64 |
| 65 | 17,448 | 28,059 | 60.82 | 30,313 | 47,473 | 56.61 |
| 70 | 17,915 | 28,296 | 57.95 | 31,469 | 48,459 | 53.99 |
| 75 | 18,193 | 28,202 | 55.02 | 32,099 | 48,615 | 51.45 |
| 80 | 18,403 | 27,852 | 51.35 | 31,924 | 47,493 | 48.77 |
| *Stand-alone policies sold to the mildly disabled* | | | | | | |
| 55 | 28,694 | 45,261 | 57.74 | 48,865 | 72,121 | 47.59 |
| 60 | 31,230 | 47,794 | 53.04 | 47,727 | 69,362 | 45.33 |
| 65 | 32,622 | 48,756 | 49.46 | 47,391 | 68,184 | 43.88 |
| 70 | 32,590 | 47,836 | 46.78 | 47,163 | 67,330 | 42.76 |
| 75 | 31,096 | 45,006 | 44.73 | 46,333 | 65,596 | 41.57 |
| 80 | 28,328 | 40,503 | 42.98 | 44,260 | 61,956 | 39.98 |
| *Stand-alone policies sold to the severely disabled* | | | | | | |
| 55 | 130,655 | 146,985 | 12.50 | 157,337 | 187,343 | 19.07 |
| 60 | 136,521 | 150,330 | 10.11 | 159,954 | 181,076 | 13.21 |
| 65 | 136,771 | 148,445 | 8.54 | 159,412 | 175,380 | 10.02 |
| 70 | 131,552 | 141,413 | 7.50 | 154,487 | 168,242 | 8.90 |
| 75 | 121,918 | 130,106 | 6.72 | 144,742 | 157,759 | 8.99 |
| 80 | 109,382 | 115,896 | 5.96 | 130,743 | 142,987 | 9.36 |
| *Rider benefit policies sold to the healthy* | | | | | | |
| 55 | 226,927 | 237,845 | 4.81 | 209,708 | 227,297 | 8.39 |
| 60 | 258,649 | 269,470 | 4.18 | 239,785 | 257,161 | 7.25 |
| 65 | 291,614 | 302,290 | 3.66 | 272,847 | 290,170 | 6.35 |
| 70 | 324,797 | 335,271 | 3.22 | 307,940 | 325,128 | 5.58 |
| 75 | 357,067 | 367,248 | 2.85 | 343,570 | 360,311 | 4.87 |
| 80 | 387,212 | 396,926 | 2.51 | 377,597 | 393,395 | 4.18 |
| *Life care annuities sold to the healthy* | | | | | | |
| 55 | 267,773 | 272,975 | 1.94 | 298,983 | 307,601 | 2.88 |
| 60 | 240,319 | 245,590 | 2.19 | 273,634 | 282,118 | 3.10 |
| 65 | 211,479 | 216,727 | 2.48 | 245,530 | 253,939 | 3.42 |
| 70 | 182,067 | 187,176 | 2.81 | 215,110 | 223,396 | 3.85 |
| 75 | 153,053 | 157,906 | 3.17 | 183,191 | 191,206 | 4.37 |
| 80 | 125,472 | 129,966 | 3.58 | 150,957 | 158,477 | 4.98 |
| *Life care annuities sold to the mildly disabled* | | | | | | |
| 55 | 250,787 | 261,300 | 4.19 | 290,061 | 303,750 | 4.72 |
| 60 | 222,786 | 233,782 | 4.94 | 263,741 | 277,020 | 5.03 |
| 65 | 194,002 | 205,040 | 5.69 | 234,859 | 247,994 | 5.59 |
| 70 | 165,388 | 176,002 | 6.42 | 204,025 | 216,992 | 6.36 |
| 75 | 137,878 | 147,647 | 7.09 | 172,404 | 184,884 | 7.24 |
| 80 | 112,263 | 120,854 | 7.65 | 141,551 | 152,969 | 8.07 |
| *Life care annuities sold to the severely disabled* | | | | | | |
| 55 | 270,261 | 291,262 | 7.77 | 323,363 | 346,964 | 7.30 |
| 60 | 239,606 | 260,166 | 8.58 | 292,932 | 314,357 | 7.31 |
| 65 | 208,682 | 227,793 | 9.16 | 259,514 | 279,606 | 7.74 |
| 70 | 178,927 | 195,657 | 9.35 | 224,654 | 243,408 | 8.35 |
| 75 | 151,417 | 165,134 | 9.06 | 190,228 | 207,043 | 8.84 |
| 80 | 126,759 | 137,230 | 8.26 | 157,930 | 172,111 | 8.98 |

Tables 10 and 11. When the elimination period is zero and the maximum benefit period is unlimited, the products are base policies, i.e. the first two rows in the first panels of Tables 10 and 11.

Table 10. Premiums ($) of stand-alone policies with different combinations of the elimination period and the maximum benefit period issued to 65-year-old healthy individuals.

| Elimination | Males | | | Females | | |
|---|---|---|---|---|---|---|
| period | Lump sum | Annual | Monthly | Lump sum | Annual | Monthly |
| *Unlimited benefit period* | | | | | | |
| 0-day | 17,018 | 1510 | 131 | 29,843 | 2392 | 207 |
| | *(219)* | *(31)* | *(5)* | *(287)* | *(35)* | *(6)* |
| 30-day | 16,561 | 1470 | 128 | 29,155 | 2337 | 202 |
| | *(216)* | *(30)* | *(5)* | *(284)* | *(35)* | *(6)* |
| 60-day | 16,116 | 1430 | 124 | 28,479 | 2283 | 198 |
| | *(213)* | *(30)* | *(5)* | *(281)* | *(34)* | *(6)* |
| 90-day | 15,680 | 1391 | 121 | 27,817 | 2230 | 193 |
| | *(210)* | *(29)* | *(5)* | *(278)* | *(34)* | *(6)* |
| *5-year maximum benefit period* | | | | | | |
| 0-day | 13,837 | 1228 | 107 | 22,907 | 1836 | 159 |
| | *(154)* | *(22)* | *(4)* | *(184)* | *(24)* | *(4)* |
| 30-day | 13,473 | 1196 | 104 | 22,391 | 1795 | 155 |
| | *(153)* | *(22)* | *(4)* | *(183)* | *(24)* | *(4)* |
| 60-day | 13,117 | 1164 | 101 | 21,884 | 1754 | 152 |
| | *(151)* | *(22)* | *(4)* | *(181)* | *(23)* | *(4)* |
| 90-day | 12,770 | 1133 | 99 | 21,386 | 1714 | 148 |
| | *(149)* | *(21)* | *(4)* | *(179)* | *(23)* | *(4)* |
| *4-year maximum benefit period* | | | | | | |
| 0-day | 12,512 | 1110 | 97 | 20,470 | 1641 | 142 |
| | *(135)* | *(20)* | *(4)* | *(159)* | *(21)* | *(4)* |
| 30-day | 12,183 | 1081 | 94 | 20,013 | 1604 | 139 |
| | *(133)* | *(19)* | *(4)* | *(157)* | *(21)* | *(4)* |
| 60-day | 11,861 | 1053 | 92 | 19,564 | 1568 | 136 |
| | *(132)* | *(19)* | *(4)* | *(156)* | *(20)* | *(4)* |
| 90-day | 11,548 | 1025 | 89 | 19,122 | 1533 | 133 |
| | *(130)* | *(19)* | *(3)* | *(155)* | *(20)* | *(4)* |
| *3-year maximum benefit period* | | | | | | |
| 0-day | 10,700 | 950 | 83 | 17,237 | 1382 | 120 |
| | *(111)* | *(16)* | *(3)* | *(128)* | *(17)* | *(3)* |
| 30-day | 10,418 | 924 | 80 | 16,854 | 1351 | 117 |
| | *(109)* | *(16)* | *(3)* | *(127)* | *(17)* | *(3)* |
| 60-day | 10,142 | 900 | 78 | 16,476 | 1321 | 114 |
| | *(108)* | *(16)* | *(3)* | *(126)* | *(17)* | *(3)* |
| 90-day | 9873 | 876 | 76 | 16,106 | 1,291 | 112 |
| | *(107)* | *(16)* | *(3)* | *(125)* | *(16)* | *(3)* |

Notes: The stand-alone LTC insurance pays $100 per day while the insured is severely disabled. Standard errors are shown in brackets under the corresponding premium estimates.



It can be seen from Table 10 that the elimination period and the maximum benefit period are effective tools in making the premium more affordable. For example, premiums of base stand-alone policies sold to 65-year-old females would be 46% cheaper if a 90-day elimination period and a maximum benefit period of 3 years are included. The elimination period and the maximum benefit period are not effective in making life insurance policies with LTC benefit riders and life care annuities more affordable as shown in Table 11 because LTC benefits account for only a small proportion in life insurance policies with LTC benefit riders and life care annuities.

We also investigate the premiums for shared LTC insurance issued to couples. The $n$-year shared LTC insurance pools a couple's LTC insurance into one policy and increases the maximum benefit period for the pooled policy to $2n$. For example, a three-year shared LTC insurance policy

Table 11. Lump sum premiums ($) of life insurance policies with LTC benefit riders and life care annuities with different combinations of the elimination period and the maximum benefit period issued to 65-year-old healthy individuals.

| Elimination | Rider benefit policies | | Life care annuities | |
|---|---|---|---|---|
| period | Males | Females | Males | Females |
| *Unlimited benefit period* | | | | |
| 0-day | 291,875 | 273,536 | 211,475 | 245,982 |
| | *(469)* | *(465)* | *(441)* | *(440)* |
| 30-day | 291,419 | 272,848 | 211,247 | 245,638 |
| | *(468)* | *(463)* | *(441)* | *(439)* |
| 60-day | 290,973 | 272,173 | 211,024 | 245,301 |
| | *(468)* | *(462)* | *(440)* | *(438)* |
| 90-day | 290,538 | 271,510 | 210,806 | 244,969 |
| | *(467)* | *(461)* | *(439)* | *(438)* |
| *5-year maximum benefit period* | | | | |
| 0-day | 288,695 | 266,600 | 209,885 | 242,514 |
| | *(454)* | *(431)* | *(429)* | *(416)* |
| 30-day | 288,331 | 266,084 | 209,703 | 242,256 |
| | *(453)* | *(431)* | *(429)* | *(416)* |
| 60-day | 287,975 | 265,577 | 209,525 | 242,003 |
| | *(453)* | *(430)* | *(429)* | *(416)* |
| 90-day | 287,627 | 265,079 | 209,351 | 241,754 |
| | *(453)* | *(430)* | *(428)* | *(415)* |
| *4-year maximum benefit period* | | | | |
| 0-day | 287,370 | 264,163 | 209,222 | 241,296 |
| | *(451)* | *(426)* | *(426)* | *(411)* |
| 30-day | 287,041 | 263,706 | 209,058 | 241,067 |
| | *(451)* | *(426)* | *(426)* | *(411)* |
| 60-day | 286,719 | 263,257 | 208,897 | 240,843 |
| | *(451)* | *(426)* | *(426)* | *(411)* |
| 90-day | 286,406 | 262,815 | 208,740 | 240,622 |
| | *(450)* | *(426)* | *(425)* | *(410)* |
| *3-year maximum benefit period* | | | | |
| 0-day | 285,558 | 260,930 | 208,316 | 239,679 |
| | *(448)* | *(423)* | *(422)* | *(405)* |
| 30-day | 285,276 | 260,547 | 208,175 | 239,488 |
| | *(448)* | *(423)* | *(422)* | *(405)* |
| 60-day | 284,999 | 260,169 | 208,037 | 239,299 |
| | *(448)* | *(423)* | *(422)* | *(405)* |
| 90-day | 284,730 | 259,799 | 207,902 | 239,114 |
| | *(448)* | *(422)* | *(422)* | *(405)* |

Notes: The whole life insurance policy with an LTC benefit rider pays $100 per day while the insured is severely disabled and pays a death benefit of $500,000 when the insured dies. The life care annuity pays $50 per day while the insured is alive and additional $50 per day while the insured is severely disabled. Standard errors are shown in brackets under the corresponding premium estimates.

Table 12. Lump sum premiums for shared and separately purchased LTC insurance, a couple of a male and a female both aged 65.

| Product | 3-year, separate | 3-year, shared | 6-year, separate |
|---|---|---|---|
| Premium | 27,937 | 37,450 | 39,507 |
| s.e. | *(239)* | *(240)* | *(375)* |
| Product | 2-year, separate | 2-year, shared | 4-year, separate |
| Premium | 21,171 | 30,569 | 32,982 |
| s.e. | *(173)* | *(182)* | *(294)* |

gives each spouse the potential to use six year's LTC benefits as long as their total benefits do not exceed six years.

Let $\gamma_1$ and $\gamma_2$ denote the random number of periods of eligible LTC benefits for the husband and wife, respectively. The number of periods for receiving eligible LTC benefits under an $n$-year shared LTC insurance policy is therefore $\min\{\gamma_1 + \gamma_2, 2n\}$. We have

$$\min\{\gamma_1, n\} + \min\{\gamma_2, n\} \leq \min\{\gamma_1 + \gamma_2, 2n\} \leq \min\{\gamma_1, 2n\} + \min\{\gamma_2, 2n\}. \qquad (24)$$

As a result, an $n$-year shared LTC insurance policy allows each spouse to access his or her partner's unused LTC funds, but is cheaper than buying two separate stand-alone policies each with $2n$-year maximum benefit period. For a couple of a male and a female who are both 65 years old, the lump sum premiums for shared LTC insurance policies are compared with premiums if they purchase LTC insurance separately. The results are shown in Table 12. We can see that the premium for a two-year (three-year) shared LTC insurance policy is about 7% (5%) cheaper than the total premium of two four-year (six-year) stand-alone LTC insurance policies separately purchased by a couple.

## 6. Reserves and capital requirements

### *6.1. Best-estimate reserves*

Best-estimate reserves for individuals in each alive state, i.e. $V_i(t, T)$ and $\tilde{V}_i(t, T)$ for any $i \in \{\mathrm{H}, \mathrm{M}, \mathrm{S}\}$ calculated in Equations (6) and (16) respectively, in base stand-alone LTC insurance policies purchased by 65-year-old individuals with lump sum premiums are shown in Figure 3. We focus on policies paid with lump sum premiums.

As shown in Figure 3, the best-estimate reserve for the healthy initially increases from accrued interest and then decreases as expected large benefit payments are made. When the individual becomes disabled, the reserve has a sharp increase. Reflecting the impact of disability, reserves for females are larger than those for their male counterparts, regardless of the health state they are in.

To calculate the best-estimate reserves for policies that are sold to disabled individuals, we also simulate 80,000 65-year-old individuals (including 40,000 males and 40,000 females) who are in the mildly disabled state at the outset and in the severely disabled state at the outset, respectively. Figure 4 shows best-estimate reserves for base stand-alone policies paid with lump sum premiums. These base stand-alone policies are sold to 65-year-old individuals in different health states at the outset. The best-estimate reserves, i.e. $V\big(t, T|\chi(0) = k\big)$ and $\tilde{V}\big(t, T|\chi(0) = k\big)$, are calculated using Equations (6) and (15), respectively. Figure 4 shows that although reserves for policies sold to more disabled individuals are larger at the outset, they decline much faster.

The VaR of liabilities can be used to assess the idiosyncratic risk, in particular for small insurance providers. The VaR of liabilities for lump sum premium base stand-alone policies sold to the 65-year-old healthy individuals is calculated and shown in Figure 5. The 99.5% VaR is

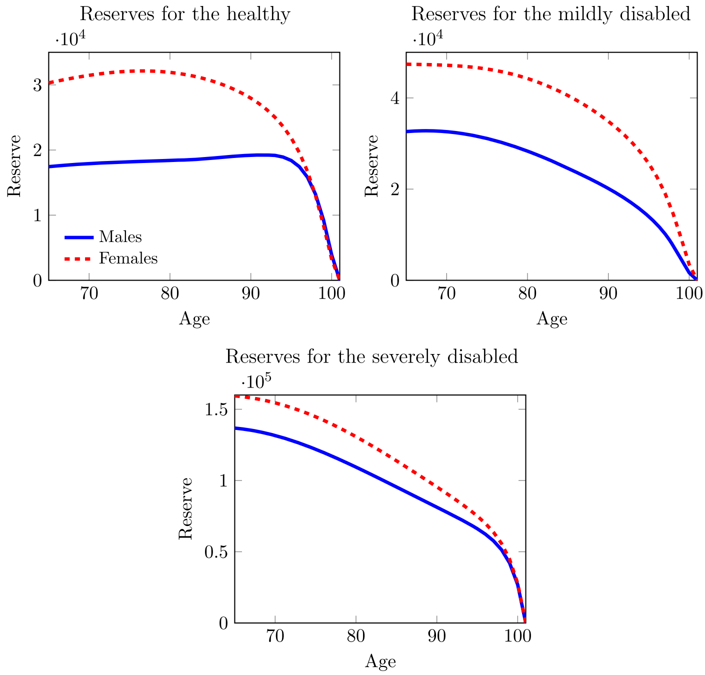


Figure 3. Best-estimate reserves for individuals in each alive state in a base stand-alone policy sold to the 65-year-old with lump sum premiums.

much higher than the best-estimate reserve, and this reflects large idiosyncratic risk as the lives reduce in number at the older ages.

The difference between the 99.5% VaR of future liabilities and the best-estimate reserve as a proportion of the best-estimate reserve, which can be expressed as $\mathrm{VaR}_{99.5\%}\big(t, T\,|\,\chi(0) = k\big)/\tilde{V}\big(t, T\,|\,\chi(0) = k\big) - 1$, is calculated for stand-alone policies with different elimination periods and maximum benefit periods. The results are shown in Figure 6. The top two panels show results for policies with different elimination periods. The bottom two panels show results for policies with different maximum benefit periods.

Figure 6 shows that the maximum benefit period is effective in reducing extremely large losses, but the elimination period is not effective in reducing idiosyncratic risk.

### *6.2. SCRs under Solvency II*

This section assesses the impact of longevity risk and disability risk on SCRs under the Solvency II standard formula framework. In the Solvency II standard formula, the SCR for longevity risk and disability risk is calculated using Equation (22). In addition to SCRs, insurers are required to hold a risk margin that is calculated according to Equation (23). In the following results shown in this section, it is assumed that the cost of capital (denoted by $c$) is 6% per annum and the risk-free interest rate (denoted by $r_f$) is the annual effective rate equivalent to a continuous compounding rate of 4% per annum.

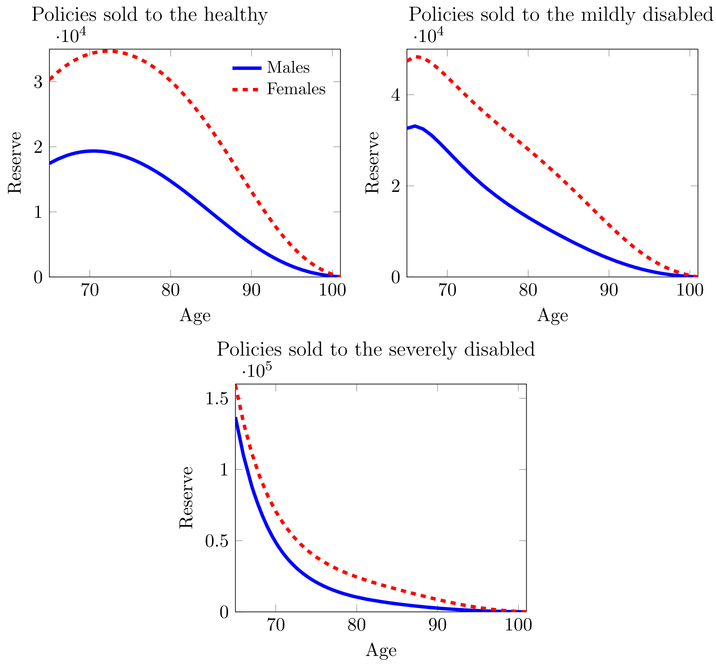


Figure 4. Best-estimate reserves for base stand-alone policies issued to a 65-year-old in different health states with lump sum premiums.

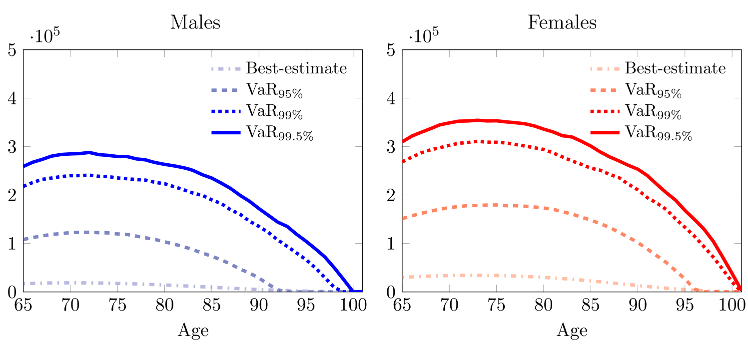


Figure 5. VaR of liabilities of base lump sum premium stand-alone policies issued to 65-year-old healthy individuals.

The ratio of solvency capital requirement to the best-estimate reserve is used to assess the relative level of capital required for a unit premium under the Solvency II standard formula framework. The ratio, denoted by $\eta_k(t)$, is calculated in Equation (25):

$$\eta_k(t) = \frac{\mathrm{SCR}_t^S}{V\big(t, T|\chi(0) = k\big)}, \tag{25}$$

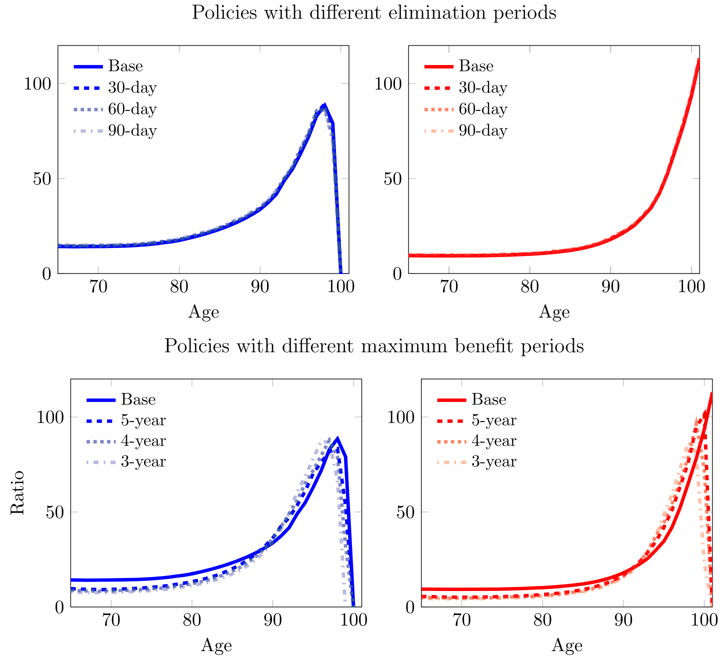


Figure 6. VaR minus best-estimate reserves as a proportion of best-estimate reserves for lump sum premium stand-alone policies with different elimination periods and maximum benefit periods. The policies are issued to 65-year-old healthy males (left) and females (right).

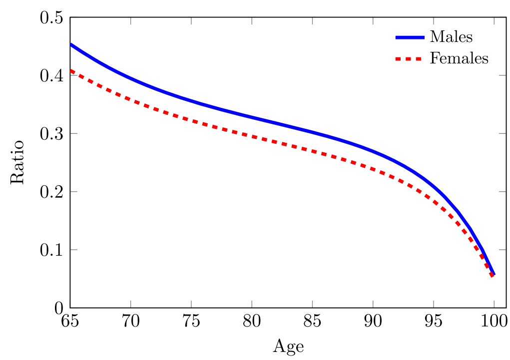


Figure 7. The ratio of solvency capital requirement to the best-estimate reserve for base lump sum premium stand-alone policies sold to 65-year-old healthy individuals.

where $k \in \Omega_\chi$ is the insured's health state at policy issue, $V\left(t, T \,|\, \chi(0) = k\right)$ is the best-estimate reserve calculated in Equation (15) and $\mathrm{SCR}_t^S$ is the SCR calculated in Equation (22). The ratio, $\eta_{\mathrm{H}}(t)$, for a base lump sum premium stand-alone LTC insurance policy sold to the healthy is shown in Figure 7.

Under Solvency II, solvency capital requirement as a proportion of the best-estimate reserve, $\eta_{\mathrm{H}}(t)$, for a lump sum premium base stand-alone policy sold to 65-year-old healthy individuals

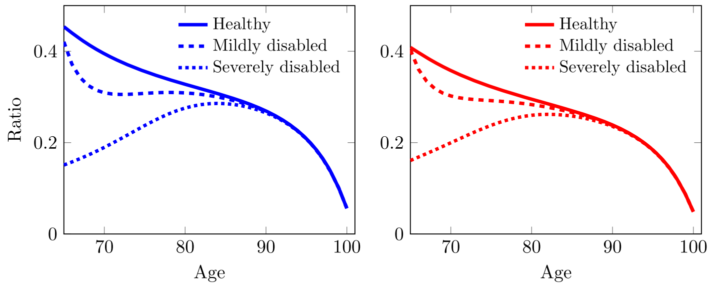


Figure 8. The ratio of solvency capital requirement to the best-estimate reserve for base lump sum premium stand-alone policies sold to 65-year-old healthy, mildly disabled and severely disabled males (left) and females(right).

decreases significantly as the insured becomes older. At age 80, solvency capital requirement is 33% of the best-estimate reserve for males and 30% for females. Figure 7 shows that base stand-alone policies sold to healthy males require slightly more capital than those sold to healthy females for a unit premium. The difference between capital requirements for base stand-alone policies sold to the two genders diminishes as the insured reach very old ages.

The ratio of solvency capital requirement to the best-estimate reserve is compared across base stand-alone policies issued to individuals in different health states. The results are shown in Figure 8. Base stand-alone policies sold to healthy individuals require high levels of capital per unit premium. Base stand-alone policies sold to severely disabled individuals are very expensive and require high levels of reserves (as shown in Table 6), but they require lower amounts of capital per unit premium compared to those sold to the healthy and mildly disabled. Differences in $\eta_k(t)$ for policies issued to individuals in different health states diminishes after around 25 years since policy inception.

The impact of longevity risk and disability risk on capital requirements based on the ratios of risk-specific SCRs to the best-estimate reserve is shown in Figure 9. For policies issued to healthy and mildly disabled individuals, disability risk capital requirements are higher than for longevity risk for males (females) in the first 8 (12) years. After that longevity risk dominates capital requirements. In general, disability risk has more impact on capital requirements for policies issued to disabled females than to disabled males, but the effects of disability risk are very similar for policies sold to healthy males and healthy females. Longevity risk has more impact on the capital requirements for policies sold to males than to females.

The $\eta_{\mathrm{H}}(t)$ ratios for life insurance policies with LTC benefit riders and for life care annuities are shown in the left and right panels of Figure 10, respectively. The existing natural hedge in life insurance policies with LTC benefit riders and life care annuities results in solvency capital requirements per unit premium for these two types of policies being lower than those for stand-alone policies. There are considerable capital reductions in policies that combine LTC insurance with life insurance or annuities as noted in Zhou-Richter & Gründl (2011). We show the extent to which life care annuities reduce the required capital level for LTC insurance providers.

The ratios of risk-specific SCRs to the best-estimate reserve for life insurance policies with LTC benefit riders and for life care annuities are shown in Figure 11. Longevity shocks impact the components of life insurance policies with LTC benefit riders in opposite directions. Mortality improvements reduce the reserve for the whole life insurance benefit, while longevity risk

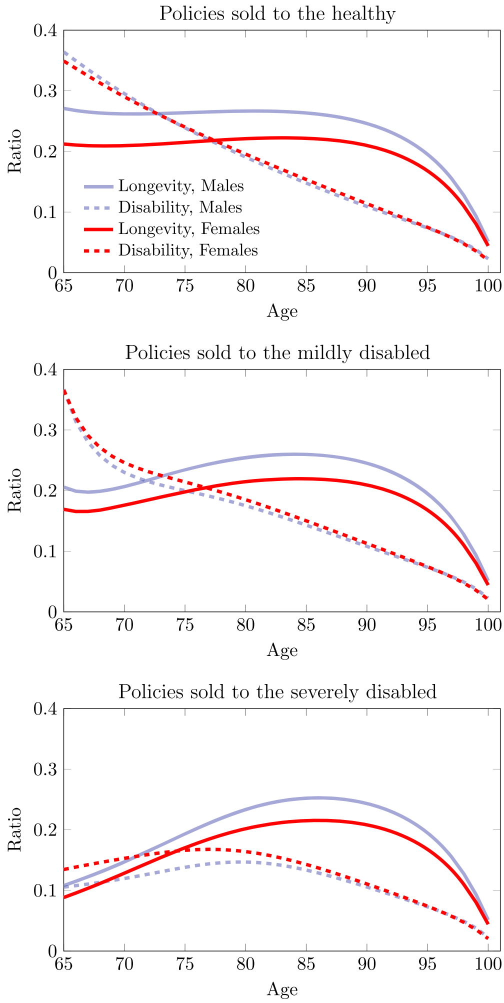


Figure 9. The ratio of SCR for longevity risk and for disability risk, respectively, to the best-estimate reserve, for base lump sum premium stand-alone policies sold to the healthy, the mildly disabled and the severely disabled.

increases the reserve for the LTC insurance as shown in Figure 9. Overall longevity risk, as in the Solvency II standard formula framework, results in a significant reduction in the liability for life insurance policies with LTC benefit riders.[4]

Disability risk impacts the components of life care annuities in the opposite directions as well. Higher disability rates and lower recovery rates increase the expected value of liabilities for LTC benefits but also increase the average mortality rates, which results in a lower expected value of liabilities for annuity payments. The effect of disability shock on the liabilities of life care annuities is dominated by that of longevity shock.

[4]In calculating the aggregate SCRs for life insurance policies with LTC benefit riders and for life care annuities, negative values of the risk-specific SCRs are set to zero.

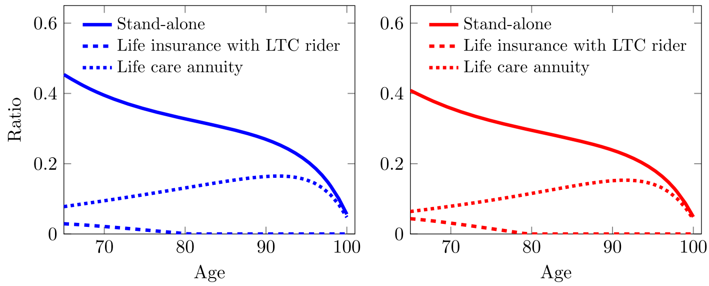


Figure 10. The ratio of solvency capital requirement to the best-estimate reserve for base lump sum premium LTC insurance policies sold to 65-year-old healthy males (left) and females (right).

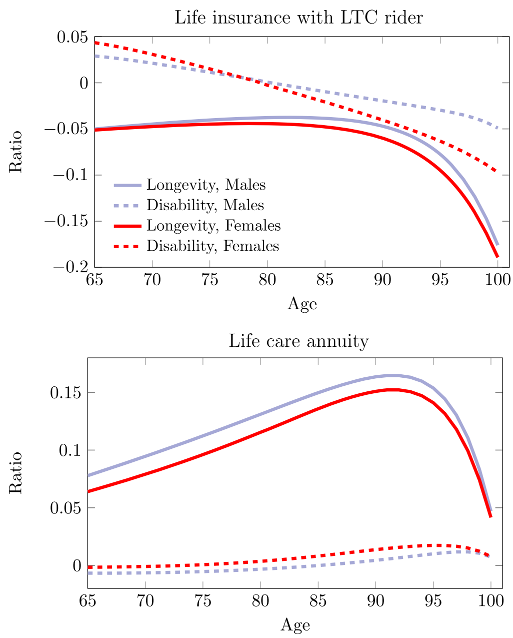


Figure 11. The ratio of the SCR for longevity risk and disability risk, respectively, to the best-estimate reserve, for base life insurance policies with LTC benefit riders and life care annuities sold to 65-year-old healthy individuals.

When calculating SCRs for periodic premium LTC insurance policies, an important practical issue is that the management can take actions, such as raising premiums, in the presence of adverse situations. For illustrative purposes, we compare SCRs per dollar provision for base annual premium stand-alone LTC insurance taking into account different periods of delay in management actions in terms of adjusting annual premiums when shocks occur. The results are shown in Figure 12. It shows how SCRs can be substantially reduced if immediate management actions are allowed when shocks occur. These results are in line with Planchet & Tomas (2014a).

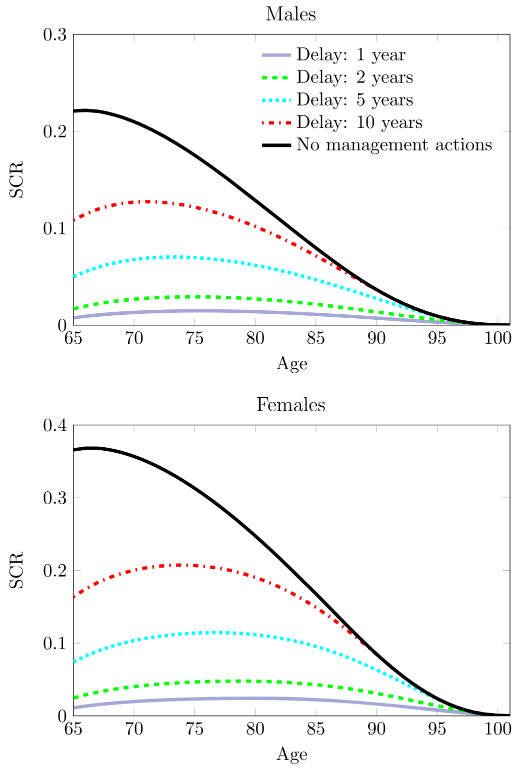


Figure 12. SCRs per dollar provision for base annual premium LTCI policies sold to 65-year-old healthy males (top panel) and females (bottom panel), with different periods of delay in management actions.



## 7. Conclusions

This paper has assessed premiums, best-estimate reserves and SCRs for a broad range of LTC insurance policies sold to individuals in different health states. Thiele's differential equation approach and a simulation-based method are applied to a range of policy designs. LTC insurance policies considered are stand-alone policies, life insurance policies with LTC benefit riders (LTC insurance combined with whole life insurance), life care annuities (LTC insurance combined with annuities) and shared LTC insurance.

Policies providing reasonable levels of fixed benefits are relatively affordable for healthy lives. Whereas premiums of stand-alone policies are very high for disabled and older individuals, in particular for those who are severely disabled, life care annuities that combine LTC insurance and annuities are more affordable for disabled and older individuals as well as for healthy lives. Policy design can be used to enhance the insurability of LTC expenses for individuals with impaired health.

The simulation-based approach is required to assess premiums and reserves for policies with different combinations of elimination periods and maximum benefit periods. This also allows the distributional measures of future liabilities, such as the VaR, to be estimated. The elimination period and maximum benefit period are shown to be effective in making an LTC product more affordable. The maximum benefit period is effective in reducing idiosyncratic risk arising from reduced numbers of policies at the older ages.

The Solvency II standard formula framework shows that SCRs are high for LTC insurance taking into account both longevity risk and disability risk. Stand-alone policies issued to the more disabled require less capital per unit premium compared to healthy lives. Interestingly, life insurance policies with LTC benefit riders and life care annuities show substantial reductions in the required capital per premium compared to stand-alone LTC insurance, reinforcing the potential benefits of these combined products. This is in line with the results in Pitacco (2015).

We have provided a thorough analysis of premiums, reserves and capital of LTC polices. We have used the US individual data to provide mortality and disability assumptions to allow a realistic assessment. The analysis presented provides valuable insights into the product design of more affordable products and an analysis of the solvency risks faced by LTC insurance providers.



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## Disclosure statement

No potential conflict of interest was reported by the authors.

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