Short-Term Interest Rates as Predictors of Inflation

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Irving Fisher pointed out that with perfect foresight and a well-functioning capital market, the one-period nominal rate of interest is the equilibrium real return plus the fully anticipated rate of inflation. In a world of uncertainty where foresight is imperfect, the nominal rate of interest can be thought of as the equilibrium expected real return plus the market's assessment of the expected rate of inflation.

The relationships between interest rates and inflation have been tested extensively.[1] In line with Fisher's initial work, the almost universal finding is that there are no relationships between interest rates observed at a point in time and rates of inflation subsequently observed. Although the market does not do well in predicting inflation, the general finding is that there are relationships between current interest rates and past rates of inflation. This is interpreted as evidence in favor of the Fisherian view. Thus Fisher concludes:

> We have found evidence, general and specific, . . . that price changes do, generally and perceptibly, affect the interest rate in the direction indicated by *a priori* theory. But since forethought is imperfect, the effects are smaller than the theory requires and lag behind price movements, in some periods, very greatly. [p. 451]

[1] For a summary, see Richard Roll.

Fisher's empirical evidence, and that of most other researchers, is in fact inconsistent with a well-functioning or "efficient" market.[2] An efficient market correctly uses all relevant information in setting prices. If the inflation rate is to some extent predictable, and if the one-period equilibrium expected real return does not change in such a way as to exactly offset changes in the expected rate of inflation, then in an efficient market there will be a relationship between the one-period nominal interest rate observed at a point in time and the one-period rate of inflation subsequently observed. If the inflation rate is to some extent predictable and no such relationship exists, the market is inefficient: in setting the nominal interest rate, it overlooks relevant information about future inflation.

This paper is concerned with efficiency in the market for one- to six-month U.S. Treasury Bills. Unlike Fisher and most of the rest of the literature, the results presented here indicate that, at least during the 1953–71 period, there are definite relationships between nominal interest rates and rates of inflation subsequently observed. Moreover, during this period the bill market seems to be efficient in the sense that nominal interest rates summarize all the information about future inflation rates that is in time-series of past inflation rates. Finally, another interesting result is that the substantial variation in nominal bill rates during the 1953–71

[2] For a discussion of the theory of efficient capital markets and related empirical work, see the author.

period seems to be due entirely to variation in expected inflation rates; in other words, expected real returns on bills seem to be constant during the period.

The theory and tests of bill market efficiency are first presented for one-month bills. The results are then extended to bills with longer maturities.

I. Inflation and Efficiency in the Bill Market: Theory

A. *Returns and the Inflation Rate*

The nominal return from the end of month $t-1$ to the end of month t on a Treasury Bill with one month to maturity at $t-1$ is

$$(1) \quad R_t = \frac{v_t - v_{t-1}}{v_{t-1}} = \frac{\$1,000 - v_{t-1}}{v_{t-1}}$$

where $v_t = \$1,000$ is the price of the bill at t, and v_{t-1} is its price at $t-1$. Since the bill has one month to maturity at $t-1$, once v_{t-1} is set, R_t is known and can be interpreted as the one-month nominal rate of interest set in the market at $t-1$ and realized at t.

Let p_t be the price level at t, that is, p_t is the price of consumption goods in terms of money, so that the purchasing power of a unit of money, the price of money in terms of goods, is $\pi_t = 1/p_t$. The real return from $t-1$ to t on a one-month bill is then

$$(2) \quad \tilde{r}_t = (v_t \tilde{\pi}_t - v_{t-1}\pi_{t-1})/v_{t-1}\pi_{t-1}$$

$$(3) \quad = R_t + \tilde{\Delta}_t + R_t\tilde{\Delta}_t$$

where tildes (~) are used to denote random variables, and

$$(4) \quad \tilde{\Delta}_t = (\tilde{\pi}_t - \pi_{t-1})/\pi_{t-1}$$

is the rate of change in purchasing power from $t-1$ to t. In monthly data, R_t and $\tilde{\Delta}_t$ are close to zero, so that although the equality only holds as an approximation, no harm is done if (3) is reduced to

$$(5) \quad \tilde{r}_t = R_t + \tilde{\Delta}_t$$

Thus the real return from the end of month $t-1$ to the end of month t on a Treasury Bill with one month to maturity at $t-1$ is the nominal return plus the rate of change in purchasing power from $t-1$ to t.

The fact that $\tilde{r}_t$ is a random variable at $t-1$ only because $\tilde{\Delta}_t$ is a random variable explains why bills are attractive for studying how well the market uses information about future inflation in setting security prices. It seems reasonable to assume that investors are concerned with real returns on securities. Since all uncertainty in the real return on a one-month bill is uncertainty about the change in the purchasing power of money during the month, one-month bills are the clear choice for studying how well the market absorbs information about inflation one month ahead. For the same reason, n-month bills are best for studying n-month predictions of inflation.

B. *The General Description of an Efficient Market*

Market efficiency requires that in setting the price of a one-month bill at $t-1$, the market correctly uses all available information to assess the distribution of $\tilde{\Delta}_t$. Formally, in an efficient market,

$$(6) \quad f_m(\Delta_t \mid \phi_{t-1}^m) = f(\Delta_t \mid \phi_{t-1})$$

where ϕ_{t-1} is the set of information available at $t-1$, ϕ_{t-1}^m is the set of information used by the market, $f_m(\Delta_t \mid \phi_{t-1}^m)$ is the market assessed density function for $\tilde{\Delta}_t$, and $f(\Delta_t \mid \phi_{t-t})$ is the true density function implied by ϕ_{t-1}.

When the market sets the equilibrium price of a one-month bill at $t-1$, R_t is also set. Given the relationship among $\tilde{r}_t$, R_t, and $\tilde{\Delta}_t$ of (5), the market's assessed distribution for $\tilde{r}_t$ is implied by R_t and its assessed distribution for $\tilde{\Delta}_t$. If (6) holds, then the market's assessed distribution for $\tilde{r}_t$ is the true distribution

(7) $f_m(r_t \mid \phi_{t-1}^m, R_t) = f(r_t \mid \phi_{t-1}, R_t)$

In short, if the market is efficient, then in setting the nominal price of a one-month bill at $t-1$, it correctly uses all available information to assess the distribution of $\tilde{\Delta}_t$. In this sense v_{t-1} fully reflects all available information about $\tilde{\Delta}_t$. Since an equilibrium value of v_{t-1} implies an equilibrium value of R_t, the one-month nominal rate of interest set in the market at $t-1$ likewise fully reflects all available information about $\tilde{\Delta}_t$. Finally, when an efficient market sets R_t, the distribution of the real return $\tilde{r}_t$ that it perceives is the true distribution.

C. A Simple Model of Market Equilibrium

The preceding specification of market efficiency is so general that it is not testable. Since we cannot observe $f_m(\Delta_t \mid \phi_{t-1}^m)$, we cannot determine whether (6) holds, and so we cannot determine whether the the bill market is efficient. What the model lacks is a more detailed specification of the link between $f_m(\Delta_t \mid \phi_{t-1}^m)$ and v_{t-1}; that is, we must specify in more detail how the equilibrium price of a bill at $t-1$ is related to the market-assessed distribution of $\tilde{\Delta}_t$. This is a common feature of tests of market efficiency. A test of efficiency must be based on a model of equilibrium, and any test is simultaneously a test of efficiency and of the assumed model of equilibrium.

The first assumption of the model of bill market equilibrium is that in their decisions with respect to one-month bills, the primary concern of investors is the distribution of the real return on a bill. A market equilibrium depends visibly on a market-clearing value of the nominal price v_{t-1}, but it is assumed that what causes investors to demand the outstanding supply of bills is the implied "equilibrium distribution" of the real return. Testable propositions about market effi-

ciency then require propositions about the characteristics of the market assessed distribution $f_m(r_t \mid \phi_{t-1}^m, R_t)$ that results from an equilibrium price v_{t-1} at $t-1$. As is common in tests of market efficiency, we concentrate on the mean of the distribution, and the proposition about $E_m(\tilde{r}_t \mid \phi_{t-1}^m, R_t)$ is that for all t and ϕ_{t-1}^m,

(8) $E_m(\tilde{r}_t \mid \phi_{t-1}^m, R_t) = E(\tilde{r})$

Thus the model of bill market equilibrium is the statement that each month the market sets the price of a one-month bill so that it perceives the expected real return on the bill to be $E(\tilde{r})$. In short, the equilibrium expected real return on a one-month bill is assumed to be constant through time.

II. Testable Implications of Market Efficiency When the Equilibrium Expected Real Return Is Constant Through Time

A. The Real Return

In an efficient market (7) holds, and (7) implies

(9) $E_m(\tilde{r}_t \mid \phi_{t-1}^m, R_t) = E(\tilde{r}_t \mid \phi_{t-1}, R_t)$

If market equilibrium is characterized by (8), then with (9) we have

(10) $E(\tilde{r}_t \mid \phi_{t-1}, R_t) = E(\tilde{r})$

Thus at any time $t-1$ the market sets the price of a one-month bill so that its assessment of the expected real return is the constant $E(\tilde{r})$. Since an efficient market correctly uses all available information, $E(\tilde{r})$ is also the true expected real return on the bill.

The general testable implication of this combination of market efficiency with a model of market equilibrium is that there is no way to use ϕ_{t-1}, the set of information available at $t-1$, or any subset of ϕ_{t-1}, as the basis of a correct assessment of the expected real return on a one-month bill

which is other than $E(\tilde{r})$. One subset of ϕ_{t-1} is the time-series of past real returns. If (10) holds,

$$(11) \quad E(\tilde{r}_t \mid r_{t-1}, r_{t-2}, \ldots) = E(\tilde{r})$$

That is, there is no way to use the time-series of past real returns as the basis of a correct assessment of the expected real return which is other than $E(\tilde{r})$. If (11) holds, the autocorrelations of $\tilde{r}_t$ for all lags are equal to zero, so that sample autocorrelations provide tests of (11).

Sample autocorrelations of $\tilde{r}_t$ are presented later, but it is well to make one point now. The autocorrelations are joint tests of market efficiency and of the model for the equilibrium expected real return. Thus nonzero autocorrelations of $\tilde{r}_t$ are consistent with a world where the equilibrium expected real return is constant and the market is inefficient, but nonzero autocorrelations are also consistent with a world where the market is efficient and equilibrium expected real returns change through time as a function of the sequence of past real returns. Market efficiency in no way rules out such behavior of the equilibrium expected return.

B. The Nominal Interest Rate as a Predictor of Inflation

There are tests that distinguish better between the hypothesis that the market is efficient and the hypothesis that the expected real return is constant through time. From (5), the relationship between the market's expectation of the rate of change in purchasing power, the nominal rate of interest, and the market's expectation of the real return is

$$(12) \quad E_m(\tilde{\Delta}_t \mid \phi_{t-1}^m) = E_m(\tilde{r}_t \mid \phi_{t-1}^m, R_t) - R_t$$

If the expected real return is the constant $E(\tilde{r})$, then (12) becomes

$$(13) \quad E_m(\tilde{\Delta}_t \mid \phi_{t-1}^m) = E(\tilde{r}) - R_t$$

If the market is also efficient,

$$(14) \quad E(\tilde{\Delta}_t \mid \phi_{t-1}) = E(\tilde{r}) - R_t$$

Thus a constant expected real return implies that all variation through time in the nominal rate R_t is a direct reflection of variation in the market's assessment of the expected value of $\tilde{\Delta}_t$. If the market is also efficient, then all variation in R_t mirrors variation in the best possible assessment of the expected value of $\tilde{\Delta}_t$. Moreover, once R_t is set at time $t-1$, the details of ϕ_{t-1}, the information that an efficient market uses to assess the expected value of $\tilde{\Delta}_t$, become irrelevant. The information in ϕ_{t-1} is summarized completely in the value of R_t. In this sense, the nominal rate R_t observed at $t-1$ is the best possible predictor of the rate of inflation from $t-1$ to t.

To test these propositions, it is convenient to introduce a new class of models of market equilibrium that includes (8) as a special case. Suppose that at any time $t-1$ the market always sets the price of a one-month bill so that it perceives the expected real return to be

$$(15) \quad E_m(\tilde{r}_t \mid \phi_{t-1}^m, R_t) = \alpha_0 + \gamma R_t$$

If the market is also efficient, we have

$$(16) \quad E(\tilde{r}_t \mid \phi_{t-1}, R_t) = \alpha_0 + \gamma R_t$$

With (5), (15) and (16) imply that

$$(17) \quad E_m(\tilde{\Delta}_t \mid \phi_{t-1}^m) = \alpha_0 + \alpha_1 R_t,$$
$$\alpha_1 = \gamma - 1$$

$$(18) \quad E(\tilde{\Delta}_t \mid \phi_{t-1}) = \alpha_0 + \alpha_1 R_t,$$
$$\alpha_1 = \gamma - 1$$

In the new model, γ is the proportion of the change in the nominal rate from one month to the next that reflects a change in the equilibrium expected real return, and $-\alpha_1 = 1 - \gamma$ is the proportion of the change in R_t that reflects a change in the expected value of $\tilde{\Delta}_t$. In the special case where the expected real return is constant through

time, $\gamma = 0$, $\alpha_1 = -1$, and all variation in R_t mirrors variation in $E(\tilde{\Delta}_t | \phi_{t-1})$.

Estimates of α_0 and α_1 in (18) can be obtained by applying least squares to

$$(19) \qquad \tilde{\Delta}_t = \alpha_0 + \alpha_1 R_t + \tilde{\epsilon}_t$$

If the coefficient estimates are inconsistent with the hypothesis that

$$(20) \qquad \alpha_0 = E(\tilde{r}) \quad \text{and} \quad \alpha_1 = -1$$

the model of a constant equilibrium expected real return is rejected. The more general interpretation of (15), that is, with unrestricted values of the coefficients, can then be taken as the model for the equilibrium expected real return, and other results from the estimates of (19) can be used to test market efficiency. Thus, like (14), (18) says that in an efficient market R_t summarizes all the information about the expected value of $\tilde{\Delta}_t$ which is in ϕ_{t-1}. For example, given R_t, the sequence of past values of the disturbance $\tilde{\epsilon}_t$ in (19) should be of no additional help in assessing the expected value of $\tilde{\Delta}_t$ which implies that the autocorrelations of the disturbance should be zero for all lags.

The approach is easily generalized to obtain other tests of (14). For example, one item of information available at $t-1$ is Δ_{t-1}. If periods of inflation or deflation tend to persist, then Δ_{t-1} is relevant information for assessing the expected value of $\tilde{\Delta}_t$. If the information in Δ_{t-1} is not correctly used by the market in setting R_t, then the coefficient α_2 in

$$(21) \quad \tilde{\Delta}_t = \alpha_0 + \alpha_1 R_t + \alpha_2 \Delta_{t-1} + \tilde{\epsilon}_t$$

is nonzero. On the other hand, if (14) holds, the market is efficient and the value of R_t set at $t-1$ summarizes all the information available about the expected value of $\tilde{\Delta}_t$, which includes any information in Δ_{t-1} and any information in the past values of $\tilde{\epsilon}_t$. Thus, in this case, $\alpha_2 = 0$ and the autocorrelations of the disturbance $\tilde{\epsilon}_t$ in (21)

are zero for all lags. Moreover, if (14) holds, the expected real return is constant through time, so that the values of α_0 and α_1 in (21) are as in (20). All of these propositions are tested below with least squares estimates of (21).

C. *Reinterpretation of the Proposed Tests*

It is well to recognize that all of the tests of market efficiency are different ways to examine whether in assessing the expected value of $\tilde{\Delta}_t$, the market correctly uses any information in the past values Δ_{t-1}, Δ_{t-2}, ... The point is obvious with respect to tests based on the coefficient α_2 in (21). The argument is also direct for the autocorrelations of the disturbances $\tilde{\epsilon}_t$ in (19) and (21). The disturbance $\tilde{\epsilon}_t$ in (19) is the deviation of $\tilde{\Delta}_t$ from the market's assessment of its conditional expected value, when $E_m(\tilde{\Delta}_t | \phi_{t-1}^m)$ is given by (17). The autocorrelations of $\tilde{\epsilon}_t$ tell us whether the past values of these deviations are used correctly by the market when it assesses the expected value of $\tilde{\Delta}_t$. Nonzero autocorrelations imply that the market is inefficient; one can improve on the market's assessment of the expected value of $\tilde{\Delta}_t$ by making correct use of information in past values of Δ_t. Likewise the disturbance $\tilde{\epsilon}_t$ in (21) is the deviation of $\tilde{\Delta}_t$ from its conditional expected value when the latter is allowed to be a function of Δ_{t-1} as well as of R_t. Finally, if the equilibrium expected real return is constant through time, then the market's assessment of the expected value of $\tilde{\Delta}_t$ is described by (13). From (5) it then follows that

$$(22a) \quad \tilde{r}_t - E(\tilde{r}) = \tilde{\Delta}_t + R_t - E(\tilde{r})$$

$$(22b) \qquad\qquad = \tilde{\Delta}_t - E_m(\tilde{\Delta}_t | \phi_{t-1}^m)$$

Thus, the deviation of $\tilde{r}_t$ from its expected value is the deviation of $\tilde{\Delta}_t$ from the market's assessment of its expected value, when the latter is described by (13). Tests of market efficiency based on the autocorrelations of $\tilde{r}_t$, like all the other proposed

tests, are concerned with whether the market correctly uses any information in the time-series of past values, Δ_{t-1}, Δ_{t-2}, . . . , when it assesses $E_m(\tilde{\Delta}_t|\phi^m_{t-1})$ on which the nominal rate R_t is then based. Any such test must assume some model of market equilibrium, that is, some proposition about the equilibrium expected real return $E_m(\tilde{r}_t|\phi^m_{t-1})$, which in turn implies some proposition about $E_m(\tilde{\Delta}_t|\phi^m_{t-1})$, and this is where the tests differ.

There is, however, no need to apologize for the fact that the tests of market efficiency concentrate on the reaction of the market to information in the time-series of past rates of change in the purchasing power of money. Beginning with the pioneering work of Fisher, researchers in this area have long contended, and the results below substantiate the claim, that past rates of inflation are important information for assessing future rates. Moreover, previous work almost uniformly suggests that the market is inefficient; in assessing expected future rates of inflation, much of the information in past rates is apparently ignored. This conclusion, if true, indicates a serious failing of a free market. The value of a market is in providing accurate signals for resource allocation, which means setting prices that more or less fully reflect available information. If the market ignores the information from so obvious a source as past inflation rates, its effectiveness is seriously questioned. The issue deserves further study.

III. The Data

The one-month nominal rate of interest R_t used in the tests is the return from the end of month $t-1$ to the end of month t on the Treasury Bill that matures closest to the end of month t. The data are from the quote sheets of Salomon Brothers. In computing R_t from (1), the average of the bid and asked prices at the end of month

$t-1$ is used for the nominal price v_{t-1}. The Bureau of Labor Statistics Consumer Price Index (CPI) is used to estimate Δ_t, the rate of change in the purchasing power of money from the end of month $t-1$ to the end of month t. The use of any index to measure the level of prices of consumption goods can be questioned. There is, however, no need to speculate about the effects of shortcomings of the data on the tests. If the results of the tests seem meaningful, the data are probably adequate.

The tests cover the period from January 1953 through July 1971. Tests for periods prior to 1953 would be meaningless. First, during World War II and up to the Treasury-Federal Reserve Accord of 1951, interest rates on Treasury Bills were pegged by the government. In effect, a rich and obstinate investor saw to it that Treasury Bill rates did not adjust to predictable changes in inflation rates. Second, at the beginning of 1953 there was a substantial upgrading of the CPI.[3] The number of items in the Index increased substantially, and monthly sampling of major items became the general rule. For tests of market efficiency based on monthly data, monthly sampling of major items in the CPI is critical. Sampling items less frequently than monthly, the general rule prior to 1953, means that some of the price changes for month t show up in the Index in months subsequent to t. Since nominal prices of goods tend to move together, spreading price changes for month t into following months creates spurious positive autocorrelation in monthly changes in the Index. This gives the appearance that there is more information about future inflation rates in past inflation rates than is really the case. Since the spurious component of the information in measured inflation rates is not easily isolated, test of market efficiency on pre-

[3] See ch. 10 of the *BLS* reference.

1953 data would be difficult to interpret.

The values of the *CPI* from August 1971 to the present (mid-1974) are also suspect. During this period the Nixon Administration made a series of attempts to fix prices. The controls were effective in creating "shortages" of some important goods (who can forget the gas queues of the winter of 1973–74?), so that for this period there are nontrivial differences between the observed values of the *CPI* and the true costs of goods to consumers. For this reason, the tests concentrate on the "clean" precontrols period January 1953 to July 1971.

IV. Results for One-Month Bills

Table 1 shows sample autocorrelations $\hat{\rho}_\tau$ of Δ_t for lags τ of from one to twelve months. The table also shows sample means and standard deviations of Δ_t, and

$$(23) \qquad \sigma(\hat{\rho}_1) = 1/(T-1)^{1/2}$$

where $T-1$ is the number of observations used to compute $\hat{\rho}_1$, and $\sigma(\hat{\rho}_1)$ is the approximate standard error of $\hat{\rho}_1$ under the hypothesis that the true autocorrelation is zero. Table 2 shows sample autocorrelations and other statistics for the real return r_t. Although, for simplicity, the de-

velopment of the theory is in terms of the approximation given by (5), the exact expression (3) is used to compute r_t in the empirical work.

Table 3 shows summary statistics for the estimated version of (19). In addition to the least squares regression coefficient estimates a_0 and a_1, the table shows the sample standard errors of the estimates $s(a_0)$ and $s(a_1)$; the coefficient of determination, adjusted for degrees of freedom; $s(e)$, the standard deviation of the residuals; and the first three residual autocorrelations, $\hat{\rho}_1(e)$, $\hat{\rho}_2(e)$, and $\hat{\rho}_3(e)$. Table 4 shows similar summary statistics for the estimated version of (21).

A. The Information in Past Inflation Rates

The market efficiency hypothesis to be tested is that the one-month nominal interest rate R_t set in the market at the end of month $t-1$ is based on correct utilization of all the information about the expected value of $\tilde{\Delta}_t$, which is in the time-series of past values $\Delta_{t-1}, \Delta_{t-2}, \ldots$. The hypothesis is only meaningful, however, if past rates of change in purchasing power do indeed have information about the expected future rate of change. The predominance

TABLE 1—AUTOCORRELATIONS OF Δ_t:
ONE-MONTH INTERVALS

	1/53–7/71	1/53–2/59	3/59–7/64	8/64–7/71
$\hat{\rho}_1$	.36	.21	−.09	.35
$\hat{\rho}_2$	.37	.28	−.09	.34
$\hat{\rho}_3$	.27	.10	−.25	.26
$\hat{\rho}_4$	.30	.16	−.05	.23
$\hat{\rho}_5$	.29	.01	.03	.33
$\hat{\rho}_6$	.29	−.01	.09	.30
$\hat{\rho}_7$	.25	.05	−.06	.18
$\hat{\rho}_8$	.34	.18	−.20	.37
$\hat{\rho}_9$	.36	.21	.13	.24
$\hat{\rho}_{10}$	.34	.20	.04	.21
$\hat{\rho}_{11}$	.27	.09	−.09	.18
$\hat{\rho}_{12}$	.37	.18	.17	.30
$\sigma(\hat{\rho}_1)$	.07	.12	.13	.11
$\bar{\Delta}$	−.00188	−.00111	−.00108	−.00321
$s(\Delta)$	.00234	.00258	.00169	.00195
$T-1$	222	73	64	83

TABLE 2—AUTOCORRELATIONS OF r_t:
ONE-MONTH BILLS

	1/53–7/71	1/53–2/59	3/59–7/64	8/64–7/71
$\hat{\rho}_1$	.09	.11	−.04	.10
$\hat{\rho}_2$	.13	.17	.01	.08
$\hat{\rho}_3$	−.02	−.02	−.20	−.01
$\hat{\rho}_4$	−.01	.01	−.06	−.10
$\hat{\rho}_5$	−.02	−.14	.00	.08
$\hat{\rho}_6$	−.02	−.18	.07	.07
$\hat{\rho}_7$	−.07	−.09	−.09	−.15
$\hat{\rho}_8$	.04	.05	−.23	.17
$\hat{\rho}_9$	.11	.11	.09	.04
$\hat{\rho}_{10}$	.10	.12	.07	−.02
$\hat{\rho}_{11}$	.03	.03	−.10	−.07
$\hat{\rho}_{12}$	.19	.16	.19	.15
$\sigma(\hat{\rho}_1)$	.07	.12	.13	.11
$\bar{r}$	.00074	.00038	.00111	.00075
$s(r)$	.00197	.00240	.00172	.00168
$T-1$	222	73	64	83

TABLE 3—REGRESSION TESTS ON ONE-MONTH BILLS
$$\Delta_t = a_0 + a_1 R_t + e_t$$

Period	a_0	a_1	$s(a_0)$	$s(a_1)$	Coefficient of Determination	$s(e)$	$\hat{\rho}_1(e)$	$\hat{\rho}_2(e)$	$\hat{\rho}_3(e)$
1/53–7/71	.00070	− .98	.00030	.10	.29	.00196	.09	.13	− .02
1/53–2/59	.00116	−1.49	.00069	.42	.14	.00240	.09	.15	− .05
3/59–7/64	− .00038	− .33	.00095	.42	− .01	.00168	− .09	− .08	− .26
8/64–7/71	.00118	−1.10	.00083	.20	.26	.00167	.09	.06	− .02

of large estimated autocorrelations of Δ_t in Table 1 indicates that this is the case.

In fact, especially for the longer periods 1/53–7/71 and 8/64–7/71, the sample autocorrelations of Δ_t for different lags are similar in size with individual estimates in the neighborhood of .30. This finding is discussed later when the behavior through time of Δ_t is studied in more detail.

B. *Market Efficiency*

Given that the equilibrium expected real return is constant through time, the market efficiency hypothesis says that the autocorrelations of the real return $\tilde{r}_t$ are zero for all lags. The sample autocorrelations of r_t in Table 2 are close to zero. Recall from (5) that the real return r_t is approximately the rate of change in purchasing power Δ_t plus the nominal interest rate R_t. The evidence from the sample autocorrelations of Δ_t and r_t in Tables 1 and 2 is that adding R_t to Δ_t brings the substantial autocorrelations of Δ_t down to values close to zero. This is consistent with the hypothesis that R_t, the nominal rate set at t−1, summarizes completely

the information about the expected value of $\tilde{\Delta}_t$ which is in the time-series of past values, Δ_{t-1}, Δ_{t-2}, ...

Tables 3 and 4 give further support to the market efficiency hypothesis. When applied to (21), the hypothesis says that α_2, the coefficient of Δ_{t-1}, is zero, and the autocorrelations of the disturbance $\tilde{\epsilon}_t$ are likewise zero for all lags. The residual autocorrelations in Table 4 are close to zero. The values of a_2, the sample estimates of α_2 in (21), are also small and always less than two standard errors from zero. When applied to (19), the market efficiency hypothesis is again that the autocorrelations of the disturbance $\tilde{\epsilon}_t$ should be zero. The residual autocorrelations in Table 3 are close to zero. Moreover, comparing the results for the estimated versions of (19) and (21) in Tables 3 and 4 shows that dropping Δ_{t-1} from the model has almost no effect on the coefficients of determination, which is consistent with the implication of market efficiency that the value of R_t set at time t−1 summarizes any information in Δ_{t-1} about the expected value of $\tilde{\Delta}_t$.

Closer inspection of the tables seems to

TABLE 4—REGRESSION TESTS ON ONE-MONTH BILLS
$$\Delta_t = a_0 + a_1 R_t + a_2 \Delta_{t-1} + e_t$$

Period	a_0	a_1	a_2	$s(a_0)$	$s(a_1)$	$s(a_2)$	Coefficient of Determination	$s(e)$	$\hat{\rho}_1(e)$	$\hat{\rho}_2(e)$	$\hat{\rho}_3(e)$
1/53–7/71	.00059	− .87	.11	.00030	.12	.07	.30	.00195	− .05	.13	− .04
1/53–2/59	.00108	−1.40	.11	.00069	.44	.11	.14	.00238	− .09	.17	− .07
3/59–7/64	− .00054	− .30	− .08	.00097	.42	.13	− .02	.00170	− .01	− .11	.25
8/64–7/71	.00073	− .89	.14	.00084	.24	.11	.24	.00164	− .04	.05	− .01

provide slight evidence against market efficiency. Except for the 3/59–7/64 period, the first-order sample autocorrelations of r_t, though small, are nevertheless all positive. The estimated regression coefficients a_2 of Δ_{t-1} in Table 4 are likewise small but generally positive, as are the first-order residual autocorrelations in Table 3. It is well to note, however, that even after the upgrading of the *CPI* in 1953, there are some items whose prices are sampled less frequently than monthly; and items that are sampled monthly are not sampled at the same time during the month. Again, since prices of goods tend to move together, these quirks of the sampling process induce spurious positive autocorrelation in measured rates of change in purchasing power. Since an efficient market does not react to "information" that is recognizably spurious, the small apparent discrepancies from efficiency provide more "reasonable" evidence in favor of the efficiency hypothesis than if the data suggested that the hypothesis does perfectly well.

C. *The Expected Real Return*

The evidence is also consistent with the hypothesis that the expected real return on a one-month bill is constant during the 1953–71 period. First, the sample autocorrelations of the real return r_t are joint tests of the hypotheses that the market is efficient and that the expected real return is constant through time. Since the sample autocorrelations of r_t in Table 2 are close to zero, the evidence is consistent with a world where both hypotheses are valid.

The regression coefficient estimates for (19) and (21) in Tables 3 and 4 are, however, more direct evidence on the hypothesis that the expected value of $\tilde{r}_t$ is constant. The hypothesis implies that in (19) and (21), the intercept α_0 is the constant expected real return $E(\tilde{r})$ and the coefficient α_1 of R_t is -1.0. The coefficient

estimates a_1 of α_1 in (19) and (21) are always well within two standard errors of -1.0. And statistical considerations aside, the estimate $a_1 = -.98$ for (19) for the overall period 1/53–7/71 is impressively close to -1.0. Given estimates a_1 of α_1 in (19) and (21) that are close to -1.0, and given the earlier observation that the estimates a_2 of α_2 in (21) are close to zero, equation (5) and the least squares formulas guarantee that the intercept estimates a_0 for (19) and (21) in Tables 3 and 4 are close to the sample means of the real return in Table 2.

Finally, the sample autocorrelations of r_t in Table 2 and the regression coefficient estimates a_0 and a_1 in Tables 3 and 4 are consistent with the world of equation (13) where the equilibrium expected real return is constant and all variation through time in the nominal interest rate R_t mirrors variation in the market's assessment of the expected value of $\tilde{\Delta}_t$. There is, however, another interesting way to check this conclusion. From the discussion of (22) it follows that the standard deviation of the real return $\tilde{r}_t$ is the standard deviation of the disturbance $\tilde{\epsilon}_t$ in (19) when the coefficients α_0 and α_1 in (19) are constrained to have the values $\alpha_0 = E(r)$ and $\alpha_1 = -1.0$ that are appropriate under the hypothesis that the expected real return is constant through time. If this hypothesis is incorrect, letting the data choose values of α_0 and α_1, as in Table 3, should produce lower estimates of the disturbance variance than when the values of the coefficients are constrained. But the results indicate that, especially for the longer periods, not only are the values of $s(r)$ in Table 2 almost identical to the values of $s(e)$ in Table 3, but the sample autocorrelations of r_t and e_t are almost identical. In short, the hypothesis that the expected real return is constant fits the data so well that the residuals from the estimated version of (19) are more or less identical to the devia-

tions of r_t from its sample mean.

V. The Behavior of $\tilde{\Delta}_t$

The results allow some interesting insights into the behavior through time of $\tilde{\Delta}_t$. The rate of change in purchasing power can always be written as

$$(24) \qquad \tilde{\Delta}_t = E(\tilde{\Delta}_t \mid \phi_{t-1}) + \tilde{\epsilon}_t$$

Since the evidence is consistent with the hypothesis that the expected real return is constant through time, we can substitute (14) into (24) to get

$$(25) \qquad \tilde{\Delta}_t = E(\tilde{r}) - R_t + \tilde{\epsilon}_t$$

The conclusion drawn from the residual autocorrelations in Table 3 and the sample autocorrelations of r_t in Table 2 is that the disturbance $\tilde{\epsilon}_t$ in (25) is uncorrelated through time. The time-series of past values of $\tilde{\epsilon}_t$ is no real help in predicting the next value. Quite the opposite sort of behavior characterizes the expected value of $\tilde{\Delta}_t$ in (24). Since, as stated in (25), variation in R_t through time mirrors variation in the expected value of $\tilde{\Delta}_t$, the time-series properties of R_t are the time-series properties of $E(\tilde{\Delta}_t \mid \phi_{t-1})$. For the 1/53–7/71 period, the first four sample autocorrelations of R_t are all in excess of .93, and only one of the first twenty-four is less than .9. Sample autocorrelations close to 1.0 are consistent with the representation of R_t as a random walk. Thus in contrast with the evidence for the disturbance $\tilde{\epsilon}_t$ in (24), the autocorrelations of R_t indicate that there is much persistence through time in the level of R_t and thus in the level of $E(\tilde{\Delta}_t \mid \phi_{t-1})$. The time-series of past values of R_t has substantial information about future values.

This discussion helps explain the behavior of the sample autocorrelations of $\tilde{\Delta}_t$ in Table 1. As stated in (24), $\tilde{\Delta}_t$ has two components. One component of $\tilde{\Delta}_t$, its expected value, behaves like a random walk. The other component of $\tilde{\Delta}_t$, the dis-

turbance $\tilde{\epsilon}_t$, is essentially random noise. The autocorrelations of its expected value cause the autocorrelations of $\tilde{\Delta}_t$ to likewise have approximately the same magnitude for different lags. The uncorrelated disturbance $\tilde{\epsilon}_t$, however, causes the autocorrelations of $\tilde{\Delta}_t$, unlike those of R_t, to be far below 1.0.

The sample autocorrelations of R_t suggest that the expected value of $\tilde{\Delta}_t$ behaves through time much like a random walk. The sample autocorrelations of the month-to-month changes in R_t, shown in Table 5, suggest, however, that we can improve on this description of the behavior of $E(\tilde{\Delta}_t \mid \phi_{t-1})$. For example, the first-order autocorrelations of $R_t - R_{t-1}$ are consistently negative. From the first-order autocorrelations for the longer periods, the change in R_t might reasonably be represented as

$$(26) \quad \tilde{R}_{t+1} - R_t = - .25(R_t - R_{t-1}) + \tilde{\eta}_t$$

Thus the process that generates the nominal rate is no longer just a random walk. The process is slightly regressive so that on average the change in the expected inflation rate from one month to the next reverses itself by about 25 percent.

TABLE 5—AUTOCORRELATIONS OF $R_t - R_{t-1}$

	1/53–7/71	1/53–2/59	3/59–7/64	8/64–7/71
$\hat{\rho}_1$	$-.25$	$-.14$	$-.41$	$-.18$
$\hat{\rho}_2$	.06	.05	.07	.06
$\hat{\rho}_3$	.01	.07	$-.03$	.00
$\hat{\rho}_4$	.15	.23	.08	.18
$\hat{\rho}_5$	$-.03$	$-.04$	.07	$-.13$
$\hat{\rho}_6$	$-.06$	.01	$-.12$	$-.01$
$\hat{\rho}_7$	$-.13$	$-.35$	$-.11$	$-.05$
$\hat{\rho}_8$	.10	.17	.13	.02
$\hat{\rho}_9$	.06	$-.03$	$-.06$	.18
$\hat{\rho}_{10}$	$-.24$	$-.26$	$-.07$	$-.42$
$\hat{\rho}_{11}$	$-.05$	$-.16$	$-.10$	.08
$\hat{\rho}_{12}$	.09	.13	.06	.04
$\sigma(\hat{\rho}_1)$	.07	.12	.13	.11
$\overline{dR}$	.00001	.00000	.00001	.00001
$s(dR)$	.00032	.00028	.00035	.00033
$T-1$	221	72	63	82

VI. Results for Bills with Longer Maturities

The presentation of theory and tests of bill market efficiency has concentrated so far on one-month bills and one-month rates of change in the purchasing power of money. As far as the theory is concerned, the interval of time over which the variables are measured is arbitrary. In testing the theory, the fact that the *CPI* is only reported monthly limits us to tests based on intervals that cover an integral number of months. Tests are presented now for one- to six-month intervals. Thus, in these tests the interval from t−1 to t is one, or two, . . . , or six months; R_t is the sure one-, or two-, . . . , or six-month nominal rate of interest from t−1 to t on a bill with one, or two, . . . , or six months to maturity at t−1; and the real return $\tilde{r}_t$ and the rate of change in the purchasing power of money $\tilde{\Delta}_t$ are likewise measured for nonoverlapping one- to six-month intervals.

Since the theory and tests are the same for bills of all maturities, the market efficiency hypothesis is that in setting the nominal rate R_t at time t−1, the market correctly uses any information about the expected value of $\tilde{\Delta}_t$ which is in the time-series of past values $\Delta_{t-1}, \Delta_{t-2}, \ldots$. The model of market equilibrium on which the tests are based is the assumption that the expected real returns on bills with one to six months to maturity are constant through time. The tests of these propositions are in Tables 6 to 9, and the tests are the same as those for one-month bills in Tables 1 to 4. Results for the one- to three-month versions of the variables are shown for the 1/53–7/71 and 3/59–7/71 periods. Since the data for four- to six-month bills are only available beginning in March 1959, results for the four- to six-month versions of the variables are only shown for the 3/59–7/71 period.

Implicit in the tests of market efficiency is the assumption that past rates of change in purchasing power have information about expected future rates of change. The autocorrelations of Δ_t in Table 6 support this assumption. The autocorrelations are large for all six intervals used to measure Δ_t. But consistent with the hypotheses that the market is efficient and that the equilibrium expected real returns on bills with different maturities are constant through time, the autocorrelations of the real returns shown in Table 7 are close to zero. Remember from (5) that the *n*-month real return on an *n*-month bill is approximately the *n*-month rate of change in purchasing power plus the *n*-month nominal return on the bill. Thus the evidence from the autocorrelations of Δ_t and r_t in Tables 6 and 7 is that when R_t is added to Δ_t, the substantial autocorrelations of Δ_t drop to values close to zero. This is consistent with a world where R_t, the *n*-month nominal rate set at t−1, summarizes all the information about the expected value of the rate of change in purchasing power over the *n* months from t−1 to t which is in the time-series of past rates of changes in purchasing power.

The model gets further support from the regression tests in Table 8. Consistent with the hypothesis that expected real returns are constant through time, the estimates a_1 of α_1 in (19) in Table 8 are all impressively close to −1.0. Consistent with the hypothesis that the market is efficient, the residual autocorrelations in Table 8 are close to zero for bills of all maturities.

The only hint of evidence against the model is in the estimates of (21) for five- and six-month bills in Table 9. As predicted by the model, the values of a_1 and a_2 for one- to four-month bills are close to −1.0 and 0.0, and the residual autocorrelations are close to 0.0. For the five- and six-month bills, however, the values of a_1 are rather far from −1.0 and the values of a_2

TABLE 6—AUTOCORRELATIONS OF Δ_t: ONE- TO-SIX-MONTH INTERVALS

| | 1/53–7/71 Interval | | | 3/59–7/71 Interval | | | | | |
	1	2	3	1	2	3	4	5	6
$\hat{\rho}_1$	.36	.50	.53	.40	.55	.58	.67	.84	.86
$\hat{\rho}_2$	.37	.39	.57	.39	.50	.74	.72	.78	.83
$\hat{\rho}_3$	.27	.43	.59	.32	.66	.64	.71	.74	.74
$\hat{\rho}_4$	.30	.45	.54	.36	.57	.70	.71	.73	.81
$\hat{\rho}_5$	.29	.52	.48	.43	.58	.66	.63	.76	.90
$\hat{\rho}_6$	.29	.41	.38	.44	.56	.65	.76	.77	1.03[a]
$\hat{\rho}_7$	.25	.40	.39	.34	.53	.65	.61	.89	.98
$\hat{\rho}_8$	.34	.32	.27	.40	.60	.58	.73	.83	.95
$\hat{\rho}_9$	.36	.36	.32	.44	.55	.73	.70	.94	.45
$\hat{\rho}_{10}$	.34	.30	.08	.40	.49	.42	.65	.79	.32
$\hat{\rho}_{11}$	.27	.28	.35	.34	.54	.84	.54	.14	−.07
$\hat{\rho}_{12}$	.37	.28	.29	.47	.56	.55	.82	.11	.23
$\sigma(\hat{\rho}_1)$	.07	.10	.12	.08	.12	.14	.17	.19	.21
$\bar{\Delta}$	−.00188	−.00368	−.00550	−.00228	−.00445	−.00656	−.00881	−.01105	−.01319
$s(\Delta)$	.00234	.00386	.00521	.00211	.00348	.00485	.00628	.00735	.00857
$T−1$	222	110	73	148	73	49	36	29	24

[a] The sample autocorrelations are estimated as linear regression coefficients. Thus the estimates can be greater than 1.0.

are rather far from 0.0. In conducting so many different tests for so many different bills, however, some results are likely to turn out badly even though the model is a valid approximation to the world. This argument gains force from the fact that the autocorrelations of the real returns in Table 7 and the estimates of (19) in Table 8 do not produce evidence for five- and six-month bills that contradicts the model.

VII. Interest Rates as Predictors of Inflation: Comparisons with the Results of Others

In a world where equilibrium expected real returns on bills are constant through

TABLE 7—SAMPLE AUTOCORRELATIONS OF r_t: ONE- TO SIX-MONTH BILLS

| | 1/53–7/71 Bill | | | 3/59–7/71 Bill | | | | | |
	1	2	3	1	2	3	4	5	6
$\hat{\rho}_1$	.09	.15	.00	.05	.03	−.16	−.17	.02	.07
$\hat{\rho}_2$	.13	−.09	.02	.05	−.15	.16	−.06	−.13	.07
$\hat{\rho}_3$	−.02	−.03	.08	−.08	.18	−.14	.20	−.03	−.05
$\hat{\rho}_4$	−.01	.01	.26	−.07	−.06	.25	.14	.03	.26
$\hat{\rho}_5$	−.02	.18	.16	.06	.00	.11	−.08	.15	.11
$\hat{\rho}_6$	−.02	.10	−.09	.10	.10	.04	.30	−.19	.43
$\hat{\rho}_7$	−.07	.15	.06	−.10	.07	.06	−.22	.33	−.04
$\hat{\rho}_8$	.04	−.01	−.01	.00	.14	.02	.17	−.02	.49
$\hat{\rho}_9$	.11	.06	.08	.09	.08	.18	.16	.25	−.60
$\hat{\rho}_{10}$	.10	.00	−.32	.05	−.07	−.33	−.09	.04	.27
$\hat{\rho}_{11}$	.03	.04	.11	−.04	.08	.36	−.02	−.69	.32
$\hat{\rho}_{12}$	.19	.09	.19	.20	.20	.10	.32	.13	.07
$\sigma(\hat{\rho}_1)$	.07	.10	.12	.08	.12	.14	.17	.19	.21
$\bar{r}$	.00074	.00185	.00306	.00090	.00224	.00373	.00514	.00706	.00882
$s(r)$	.00197	.00292	.00371	.00169	.00236	.00307	.00379	.00375	.00444
$T−1$	222	110	73	148	73	49	36	29	24

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TABLE 8—REGRESSION TESTS ON ONE- TO SIX-MONTH BILLS

$\Delta_t = a_0 + a_1 R_t + e_t$

Period	Bill	a_0	a_1	$s(a_0)$	$s(a_1)$	Coefficient of Determination	$s(e)$	$\hat{\rho}_1(e)$	$\hat{\rho}_2(e)$	$\hat{\rho}_3(e)$
1/53–7/71	1	.00070	− .98	.00030	.10	.29	.00196	.09	.13	− .02
	2	.00161	− .96	.00066	.11	.42	.00296	.15	− .08	− .03
	3	.00228	− .92	.00105	.11	.48	.00380	.00	.03	.10
3/59–7/71	1	.00120	−1.09	.00041	.12	.36	.00169	.04	.05	− .08
	2	.00269	−1.08	.00086	.12	.52	.00245	.02	− .16	.14
	3	.00397	−1.03	.00145	.13	.55	.00330	− .16	.12	− .16
	4	.00543	−1.03	.00216	.14	.58	.00413	− .18	− .10	.14
	5	.00635	− .97	.00236	.12	.68	.00416	.01	− .10	− .02
	6	.00879	−1.01	.00344	.14	.65	.00505	.01	− .01	− .11

time, then, aside from the additive constant $E(\tilde{r})$ in (13), the nominal rate R_t set at time $t-1$ is in effect the market's prediction of the rate of change in purchasing power from $t-1$ to t. The coefficients of determination in Table 8 indicate that variation through time in these predictions accounts for 30 percent of the variance of subsequently observed values of Δ_t in the case of one-month bills, and the proportion of the sample variance of Δ_t accounted for by R_t increases to about 65 percent for five- and six-month bills. Thus, nominal interest rates observed at $t-1$ contain nontrivial information about the rate of change in purchasing power from $t-1$ to t. Moreover, the evidence on market efficiency suggests that the mar-

ket's prediction of $\tilde{\Delta}_t$ is the best that can be made on the basis of information available at time $t-1$; or, more precisely, it is the best that can be done on the basis of information in past rates of change in purchasing power.

As noted earlier, the results reported here differ substantially from those of the rest of the literature on interest rates and inflation. In line with the early work of Fisher, the almost universal finding in other studies is that the market does not perform efficiently in predicting inflation. But the earlier studies, including, of course, Fisher's, are based primarily on pre-1953 data, and the negative results on market efficiency may to a large extent just reflect poor commodity price data. By

TABLE 9—REGRESSION TESTS ON ONE- TO SIX-MONTH BILLS

$\Delta_t = a_0 + a_1 R_t + a_2 \Delta_{t-1} + e_t$

Period	Bill	a_0	a_1	a_2	$s(a_0)$	$s(a_1)$	$s(a_2)$	Coefficient of Determination	$s(e)$	$\hat{\rho}_1(e)$	$\hat{\rho}_2(e)$	$\hat{\rho}_3(e)$
1/53–7/71	1	.00059	− .87	.11	.00030	.12	.07	.30	.00195	− .05	.13	− .04
	2	.00115	− .78	.17	.00064	.13	.09	.44	.00280	.03	− .06	.02
	3	.00173	− .79	.11	.00107	.15	.12	.48	.00372	− .06	.07	.05
3/59–7/71	1	.00109	−1.01	.07	.00042	.14	.08	.35	.00169	− .03	.05	− .07
	2	.00252	−1.02	.05	.00094	.18	.12	.51	.00248	− .02	− .16	.15
	3	.00390	−1.06	− .04	.00169	.23	.17	.53	.00334	− .10	.11	− .17
	4	.00520	− .97	.07	.00261	.26	.20	.57	.00423	− .23	− .06	.12
	5	.00359	− .57	.40	.00301	.27	.23	.71	.00404	− .13	− .08	− .02
	6	.00263	− .39	.58	.00406	.28	.23	.72	.00461	− .29	.18	− .32

the same token, the success of the tests reported here is probably to a nonnegligible extent a consequence of the availability of good data beginning in 1953.

Poor commodity price data also probably explain why the empirical literature is replete with evidence in support of the so-called Gibson Paradox—the proposition that there is a positive relationship between the nominal interest rate and the level of commodity prices, rather than the relationship between the interest rate and the rate of change in prices posited by Fisher.[4] With a poor price index, the Fisherian relationship between the nominal interest rate and the true inflation rate can be obscured by noise and by spurious autocorrelation in measured inflation rates. But over long periods of time—and the Gibson Paradox is usually posited as a long-run phenomenon—even a poor index picks up general movements in prices. Thus if inflations and deflations tend to persist (an implication of the evidence presented here that $E(\tilde{\Delta}_t \mid \phi_{t-1})$ is close to a random walk), there may well appear to be a relationship between the level of interest rates and the measured level of prices, which merely reflects the more fundamental Fisherian relationship between the interest rate and the rate of change of prices that is obscured by poor data. In this study, which is based on the relatively clean data of the 1953–71 period, the Fisherian relationship shows up clearly.

[4] For a discussion of the Gibson Paradox and a review of previous evidence, see Roll. A more recent study is Thomas Sargent.

VIII. Conclusions

The two major conclusions of the paper are as follows. First, during the 1953–71 period, the bond market seems to be efficient in the sense that in setting one-to six-month nominal rates of interest, the market correctly uses all the information about future inflation rates that is in time-series of past inflation rates. Second, one cannot reject the hypothesis that equilibrium expected real returns on one-to six-month bills are constant during the period. When combined with the conclusion that the market is efficient, this means that one also cannot reject the hypothesis that all variation through time in one- to six-month nominal rates of interest mirrors variation in correctly assessed one- to six-month expected rates of change in purchasing power.

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