

CALIBRATING LMN MODEL TO COMPUTE BEST ESTIMATES IN LIFE INSURANCE

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ABSTRACT

In this paper, we propose to study a method for calibrating Longstaff, Mithal and Neis model (or LMN model) from a CDS (credit derivative swaps) and bonds associated with the same entity as the CDS. This model is used by many insurers. The calculations will use at each observation date a risk-free rate curve generated through Nelson, Siegel and Svensson method from swap rates vs. six-month Euribor. The process will decompose the spread attributed to the reference entity and will especially evaluate the component associated to default risk and the one associated to liquidity risk. The entity will be Deutsch Bank AG. The study will reveal the existence of a negative liquidity component for some corporate bonds and at some cotation dates, which will show that rate swaps indexed on six-month Euribor include an implied spread.

Calibration data come from cotations provided by Reuters and the studied period spreads from the 5th January 2009 to the 30th December 2011.

The main idea of this paper is that parameters must be determined to ensure the prices from the model represent the best possible prices observed not at a given date, but over a fixed period. The practical implementation of this idea requires the use of a genetic algorithm, which we present here.

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AGENDA

1. Introduction	2
2. Model description.....	4
3. Calibrating LMN model	9
3.1. Choosing and collecting data	10
3.2. Data description.....	13
3.3. Principle of calibration algorithm.....	14
3.3.1. Computing model parameters	15
3.3.2. Measuring the spread components of a corporate bond	18
3.3.3. Alternative method	20
4. Analysing the results	21
4.1. Fitting quality	21
4.2. Decomposing the yield to maturity of a bond	22
4.3. Out of sample test.....	23
5. Conclusion.....	25
6. References	26
7. Appendix	27
7.1. Fitting quality	28
7.2. Decomposing the yield to maturity of a bond	29
7.3. Out of sample test.....	29

1. INTRODUCTION

In the last ten years, we have observed a generalization of the use of “economic valuations” in various frameworks used by insurers: regulation (Solvency II), accounting (IFRS) and financial reporting (MCEV). This led to the use by insurers of methods originally developed for pricing financial instruments to calculate their liabilities. On this occasion, many challenges have emerged, particularly in life insurance, e.g. long duration of life insurance liabilities, no market for insurance liabilities, partially endogenous risk factors and volatility of the value which does not reflect the risks carried.

Practitioners are turning to *ad hoc* approaches by projecting the flow of benefits of the contract with Markov models and obtained numerical results using the following scheme (see BONNIN *et al.* [2014]):

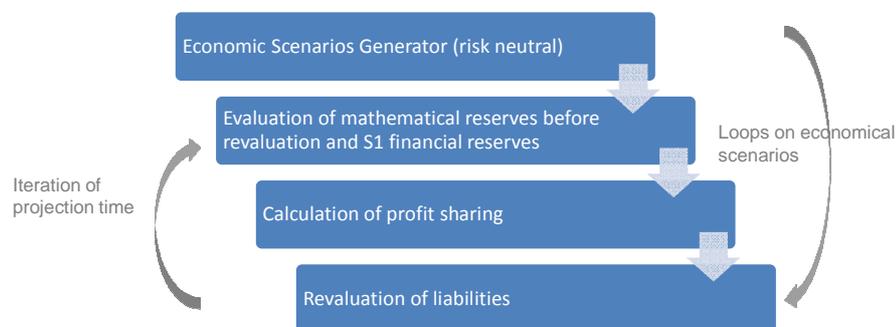


Fig. 1 – Projection structure for best estimate calculation

The risk neutral Economic Scenarios Generator (ESG) is thus an essential component of the calculation (*cf.* PLANCHET *et al.* [2011]). Risks associated with bonds are the most important, given the structure of the assets of an insurer. There is some work on the selection and calibration LMN model calibration

of models for the risk-free interest rate (see e.g. MOUDIKI [2014] and the references therein). But corporate bond yield offers a spread on risk-free rate, composed of default risk rate and a residual spread which can be interpreted as a liquidity indicator. Default risk component of a bond can be extracted from data on CDS premiums. There is however very little work those risks in this very specific context.

To model this spread risk in order to compute savings contract best estimates, LMN model elaborated by LONGSTAFF *et al.* [2005] represents an attractive choice in that it is easy to implement and allows to take efficiently this risk into account. It is therefore often chosen by insurers (see OUAJOU [2010]). It is also the chosen model in ESG package¹.

The reasons of this choice are the following ones:

- ✓ The existence of analytical forms to compute the price of a corporate zero coupon bond, which makes the implementation easier due to the existence of closed forms to compute zero coupon price.
- ✓ Its compatibility with all risk-free rate models. Deduced forms do not depend on the choice of a particular risk-free rate model.

However, if this model turns out to be simple to use, its calibration is difficult. The aim of this paper is to propose a calibration method adapted to insurance context.

More precisely, it is necessary to define a coherent model calibration with both the observed prices and the long-term horizon. Calibration should be relatively stable when updated on nearby dates. The purpose of this paper is to propose a method for a consistent estimation of the parameters of the LMN model that respects this constraint. A method using only the last known prices, as in LONGSTAFF *et al.* [2005] is obviously not relevant here, because it actually induces volatility of the parameters that is not representative of the risks actually incurred. The stochastic processes used for risk factors have constant coefficients and the choice of the parameters must be consistent with this hypothesis over a short period.

We can see that price volatility and therefore the associated calibration is a matter of concern to the regulator. The latest specifications of the standard formula provide a volatility adjustment (see EIOPA [2014]) whose objective is to stabilize the calculation results (that is to say reserves and net asset value). But this adjustment assumes that volatility is the result of the volatility of the single liquidity premium. Our approach does not make this assumption and is therefore complementary to the proposal of the regulator.

The main idea of this paper is that parameters must be determined to ensure the prices from the model represent the best possible prices observed not at a given date, but over a fixed period. The practical implementation of this idea requires the use of a genetic algorithm, which we present here.

¹ <http://cran.r-project.org/web/packages/ESG/index.html>

2. MODEL DESCRIPTION

Like other reduced-form models LMN model does not directly explain default cause. It rather focus on modelling corporate default probability and this default can occur at any time. The figure Fig. 2 describes how this algorithm calculates corporate bond payment flow at a date of coupon detachment. Let:

- ✓ $V(t_0)$, the flow value at date t_0 ,
- ✓ C , the coupon value,
- ✓ c , the coupon value in percentage,
- ✓ M , the nominal amount,
- ✓ t_1, \dots, t_N , the dates of future coupon payments,
- ✓ t_{-M}, \dots, t_{-1} , the dates of past coupon payments,
- ✓ δ , the time interval between two consecutive coupon payments, this time interval being assumed constant,
- ✓ t_0 , the current date.
- ✓ t_N , the bond maturity date.

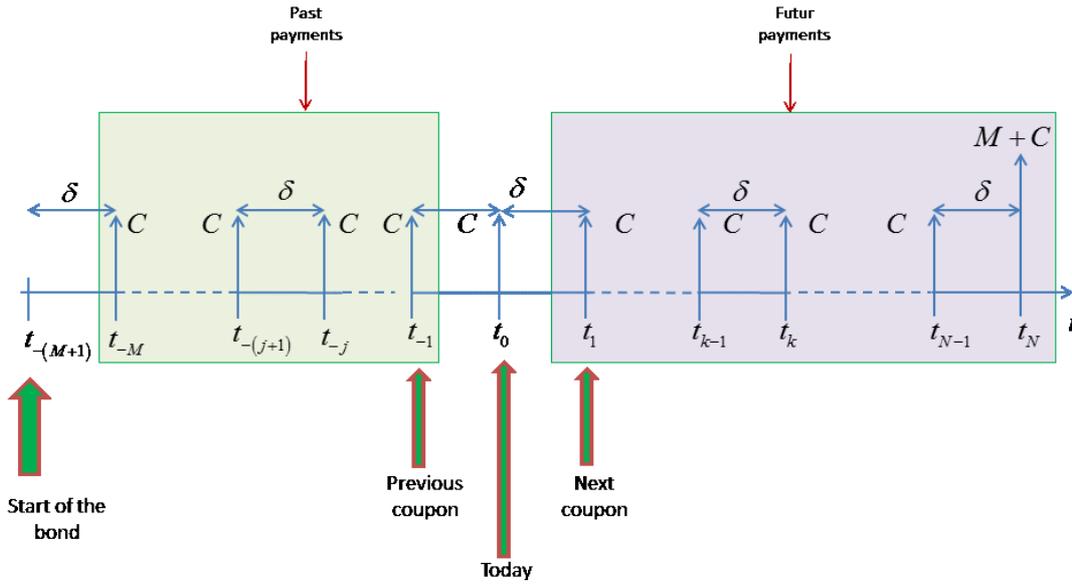
We can write: $C = cM$ et $M + C = M(1 + c)$. It follows:

$$V(t_0) = \begin{cases} P[\tau > t_0] \times F(t_0) & \text{If no default occurs} \\ P[\tau \leq t_0] \times (1 - \omega) \times M & \text{Otherwise} \end{cases}$$

and

$$F(t_0) = \begin{cases} C & \text{If } t_0 < t_N \\ C + M & \text{If } t_0 = t_N \end{cases}$$

Furthermore, we suppose that payment occurs at default moment, which is not necessarily true in practice.



$$V(t_0) = \begin{cases} P[\tau > t_0] \times F(t_0) & \text{If no default occurs} \\ P[\tau \leq t_0] \times (1-\omega) \times M & \text{Otherwise} \end{cases}$$

Fig. 2 – Flows of a corporate bond

We note:

- ✓ r_τ , the risk-free rate,
- ✓ λ_τ , the intensity of the Poisson process governing default,
- ✓ γ_τ , a convenience yield or liquidity process, which will be used to catch the additional return investors may require, in addition to compensating credit risk, from holding a corporate bond rather than a riskless bond with similar characteristics.
- ✓ ω , the recovery rate.

The three process are stochastic and supposed to be decorrelated from each other. The recovery rate is arbitrarily fixed at 50 %. LONGSTAFF *et al.* [2005] assume that those simplifications have little effect on empirical results. The recovery rate of a corporate bond can be formulated as:

$$rc_t = r_t + \lambda_t + \gamma_t \quad (1)$$

There is no need to specify risk-neutral dynamics of risk-free rate to solve for CDS premiums and corporate bond prices. We only need that these dynamics be such that the price of a risk-free zero-coupon bond $P(0, T)$ with maturity T be written as:

$$P(0, T) = E \left[\exp \left(- \int_0^T r_\tau d\tau \right) \right] \quad (2)$$

The risk-neutral dynamics of the intensity process (of CIR type) is given by:

$$d\lambda_\tau = (\alpha - \beta\lambda_\tau) d\tau + \sigma \sqrt{\lambda_\tau} dZ_{\lambda_\tau} \quad (3)$$

Where α , β et σ are positive constants and Z_{λ_τ} a standard Brownian motion. These dynamics allow for both mean reversion and conditional heteroskedasticity in corporate spreads, and ensure that

default intensity keeps positive or zero. The liquidity process follows a risk-neutral dynamics expressed as:

$$d\gamma = \eta dZ_\gamma \quad (4)$$

Where η is a positive constant and Z_γ also a standard Brownian motion. These dynamics allow the liquidity process to take positive and negative values. Following DUFFIE [1998], LANDO [1998] and DUFFIE et al. [1999], it is natural to represent the value of a corporate bond and the values of premium and protection legs of a CDS as simple expectations under the risk-neutral probability. For simplification purpose, we assume that the coupon c is continuously paid as long as no default occurs. The price of a corporate bond with maturity T can be simply expressed as a combination of the different involved process (see LONGSTAFF et al. [2005] whose we will use most closed forms required to calibration realized in this work).

Bond price is given by:

$$\begin{aligned} CB(c, \omega, T) = & c \int_0^T A(\tau) \exp(B(\tau)\lambda) C(\tau) P(0, \tau) e^{-r\tau} d\tau \\ & + A(T) \exp(B(T)\lambda) C(T) P(0, T) e^{-rT} \\ & + (1-\omega) \int_0^T \exp(B(\tau)\lambda) C(\tau) P(0, \tau) (G(\tau) + H(\tau)\lambda) e^{-r\tau} d\tau \end{aligned} \quad (5)$$

and premium CDS is given by:

$$s = \frac{\omega \int_0^T \exp(B(\tau)\lambda) P(0, \tau) (G(\tau) + H(\tau)\lambda) d\tau}{\int_0^T A(\tau) \exp(B(\tau)\lambda) P(0, \tau) d\tau} \quad (6)$$

where:

$$\left\{ \begin{aligned} A(\tau) &= \exp\left(\frac{\alpha(\beta+\phi)}{\sigma^2} \tau\right) \left(\frac{1-\kappa}{1-\kappa e^{\phi\tau}}\right)^{\frac{2\alpha}{\sigma^2}} \\ B(\tau) &= \frac{\beta-\phi}{\sigma^2} + \frac{2\phi}{\sigma^2(1-\kappa e^{\phi\tau})} \\ C(\tau) &= \exp\left(\frac{\eta^2 \tau^3}{6}\right) \\ G(t) &= \frac{\alpha}{\phi} (e^{\phi t} - 1) \exp\left(\frac{\alpha(\beta+\phi)}{\sigma^2} t\right) \left(\frac{1-\kappa}{1-\kappa e^{\phi t}}\right)^{\frac{2\alpha}{\sigma^2}+1} \\ H(\tau) &= \exp\left(\frac{\alpha(\beta+\phi) + \phi\sigma^2}{\sigma^2} \tau\right) \left(\frac{1-\kappa}{1-\kappa e^{\phi\tau}}\right)^{\frac{2\alpha}{\sigma^2}+2} \\ \phi &= \sqrt{2\sigma^2 + \beta^2} \\ \kappa &= \frac{\beta+\phi}{\beta-\phi} \end{aligned} \right. \quad (7)$$

We can note that, if $\lambda = 0$, then $s = 0$, which is consistent with the fact that default cost must be zero, if entity bears no default risk.

Regarding the pricing of a CDS, it is important to remember that swaps are contracts, not securities. Therefore, due to their contractual nature, they are less sensitive to liquidity and convenience yield effects:

- Securities are in fixed supply. By contrast, the notional amount of a CDS may be arbitrarily high, which implies that laws of supply and demand likely to affect corporate bonds are much less likely to affect CDS.
- Generic or fungible nature of payment flows prevent CDS from becoming “special” in a similar way to sovereign bonds or popular stocks on market.
- Since new CDS can always be created, these contracts are much less prone to be “compressed” than the underlying corporate bonds.
- Since CDS look like insurance contracts, a lot of investors purchasing credit protection may intend to do so for a fixed horizon and, consequently, may not intend to close out their position earlier.
- Even though an investor plans to unwind a CDS position, it may be less costly for him to simply enter into a new CDS in the opposite direction than to attempt to unwind his current position. Subsequently, the liquidity of his current position is less relevant due to his capacity to duplicate swap cash flows via other contracts.
- It can sometimes be hard and onerous to sell corporate bonds. However, it is normally easier to sell a protection than buy one on CDS market.
- At last, BLANCO et al. [2004] notice that credit derivative markets are more liquid than corporate bond markets in the meaning that new information is captured more quickly by CDS premiums than by corporate bond prices.

We suppose here that the convenience yield or illiquidity process γ_t can be applied to the cash flows generated by corporate bonds, but not to the cash flows generated by CDS contracts. Alternatively, γ_t can be considered as the differential convenient yield between corporate securities and credit derivative contracts. Thus, if CDS contracts embed a liquidity component, then γ_t may underestimate spread component non relative to corporate default. We denote:

- ✓ s , the premium paid by the protection buyer against default.
- ✓ δ' , the premium payment frequency.
- ✓ $t_{-(M'+1)}$, the CDS starting date.
- ✓ $t_{N'}$, the CDS maturity date.
- ✓ t_0 , the default date of the reference entity.
- ✓ $t_1, \dots, t_{N'}$, the dates of future premium payments.
- ✓ $t_{-M'}, \dots, t_{-1}$, the dates of past premium payments.

The figure Fig. 3 describes the generated cash flows if some default occurs before the maturity of the contract.

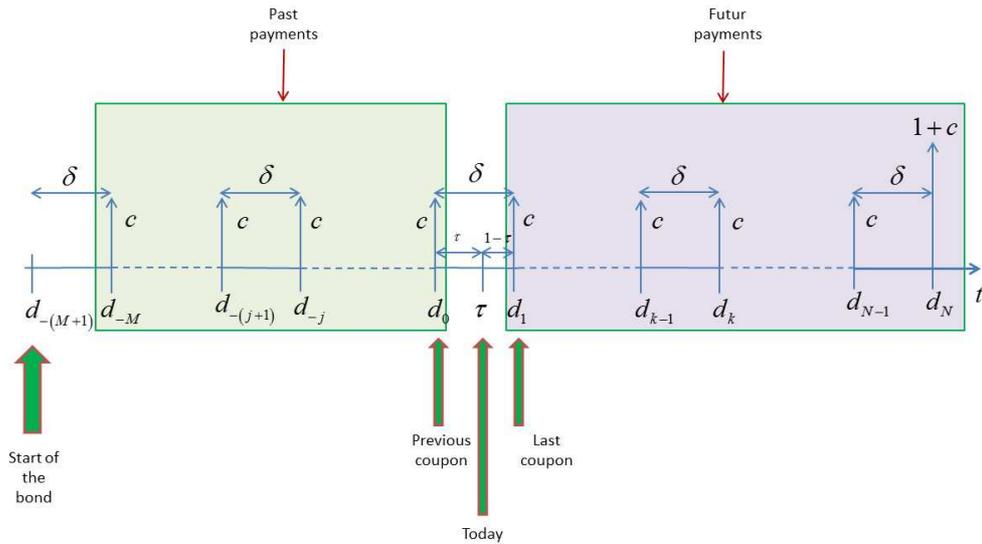


Fig. 3 – Cash flows generated by a CDS when default occurred

Given the dynamics of the intensity and liquidity process, standard results obtained by DUFFIE *et al.* [1999] allow for easily inferring closed forms respectively for corporate bond price and for CDS premium. These forms are also used by LONGSTAFF *et al.* [2005] and we refer the reader to them. Let's examine now some strip bond and some CDS. We note:

- ✓ $y^{(réel)}$, the real bond yield.
- ✓ $r^{(réel)}$, the riskless component of the total yield.
- ✓ $s_{def}^{(réel)}$, the default component of the total yield.
- ✓ $s_{liq}^{(réel)}$, the liquidity component of the total yield.
- ✓ s_{CDS} , the CDS premium.
- ✓ b , the bias or the difference between the sum of the riskless rate and the default component, and the CDS premium.

The bias b comes from using CDS premium to estimate bond spread default component. LONGSTAFF *et al.* [2005] show that CDS premium is a downward biased estimation of the default component of bond yield in stability period, but it becomes an upward biased estimation when reference entity gets close to bankruptcy.

The figure Fig. 4 shows the relationships between the above values.

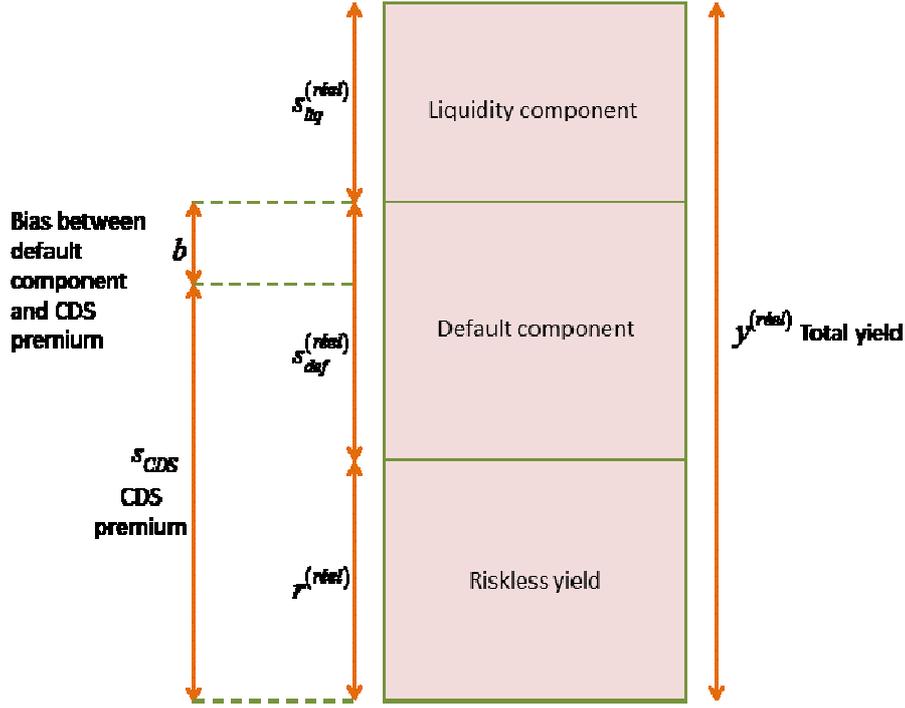


Fig. 4 –decomposition of the yield of a strip coupon bond.

In case where the coupons are paid at a discrete yearly frequency f at the dates $\tau_{i,j} = (i-1)f + j$ ($\tau_{N-1,f} = T$), with $i \in [1, N-1]$ and $j \in [1, f]$, the relation (5) becomes:

$$\begin{aligned}
 CB(c, \omega, T) = & \sum_{k=1}^N A(\tau_i) \exp(B(\tau_{i,j})\lambda) C(\tau_{i,j}) P(\tau_{i,j}) e^{-\gamma\tau_{i,j}} \\
 & + A(T) \exp(B(T)\lambda) C(T) P(0, T) e^{-\gamma T} \\
 & + (1-\omega) \int_0^T \exp(B(\tau)\lambda) C(\tau) P(0, \tau) (G(\tau) + H(\tau)\lambda) e^{-\gamma\tau} d\tau
 \end{aligned} \tag{8}$$

Likewise, if the premiums are paid at a discrete yearly frequency f' at the dates $\tau'_{i,j} = (i-1)f' + j$ ($\tau'_{N'-1,f'} = T$), with $i \in [1, N'-1]$ and $j \in [1, f']$, the relation (6) becomes:

$$s = \frac{\omega \int_0^T \exp(B(\tau)\lambda) P(0, \tau) (G(\tau) + H(\tau)\lambda) d\tau}{\sum_{k=1}^{N'} A(\tau'_{i,j}) \exp(B(\tau'_{i,j})\lambda) P(0, \tau'_{i,j})} \tag{9}$$

3. CALIBRATING LMN MODEL

Once the model is calibrated from a data sample, it can be used to decompose the implied spread in the CDS premium. In the context of this study, we will confine ourselves to the euro, putting so aside the other currencies. The risk-free rate curves used here have been generated by Nelson, Siegel and Svensson interpolation function from swap rates vs. 6-month Euribor. The interpolation method is described in NELSON and SIEGEL [1987] and SVENSSON [1994] and the calibration

process is explained in GILLI et al. [2010]. Besides, the different calibration functions are optimized through a differential evolution algorithm described in STORN and PRICE [1997].

3.1. CHOOSING AND COLLECTING DATA

The first considered approach consists of computing a CDS premium from a sample bearing market's and our portfolio's features. Nevertheless, given the difficulty to find appropriate data for such a method and to take into account the correlations between the different entities, this approach was dropped.

Next, we studied an approach mainly based on bond and CDS indexes. The advantage of an index is to synthesize the evolution of a market or an asset family, to be usable as a standard measure of managers' performance and to serve as an underlying basis for different derivatives.

Bond index families published by Markit Group² (*Markit Credit Indices*) caught our attention:

- **The *Markit iBoxx EUR High Yield index family*.** This family tracks the performance of euro-denominated sub-investment grade corporate debt and aims to provide appropriate reference portfolios. In addition to a global index, index values are broken down according to maturity, rating and sector. The *iBoxx EUR Core High Yield* indices includes the standard bond structures are included. An exhaustive framework is built both at bond and index levels, including yield, duration, convexity, fixed-to-floater bonds, excess returns, spreads, etc. This family includes bonds with the following characteristics:
 - Fixed coupon bonds (“plain vanilla” bonds),
 - Zero coupon bonds,
 - Floating rate notes with EURIBOR as a reference interest rate without cap or floor,
 - Sinking funds with known redemption schedules,
 - Bonds with American and European call options,
 - Bonds with poison put options,
 - Bonds with make-whole call or tax changes call provisions,
 - Event-driven bonds such as rating and registration-sensitive bonds,
 - Payment-in-kind bonds,
 - Callable perpetuals,
 - Callable Fixed-to-floater bonds.

- **The *iBoxx EUR Benchmark index family*.** This index family is published by the International Index Company Limited (IIC) and represents the investment grade fixed-income market for Euro denominated bond. It includes an overall index and four subfamilies of main indexes. The sovereign index subfamily is composed of sovereign debts issued by a government from euro zone and denominated in euro or in a former currency from euro zone. This subfamily include an overall and maturity indexes. An overall index is published for each euro zone country (excepted Luxembourg), namely Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal and Spain. The sovereign index subfamily is complemented by maturity indexes for Germany, France and Italy.

The *iBoxx EUR Non-Sovereigns* index include all bonds which do not meet *the iBoxx EUR Sovereigns* index's criteria. Those bonds are next classified into index sub-groups of sub-sovereign, collateralized and corporate bonds. The *iBoxx EUR Other Sovereigns* index

² <http://www.markit.com/en/>

comprises sovereign bonds issued by countries outside euro zone within sub-sovereign index subgroup. Corporate bond indexes include an overall index and indexes by rating and maturity, with a distinction being drawn between financial and non-financial bonds, and sub-indexes by rating and maturity for each of these sub-categories. In addition, each of them is also broken down into market segments. Senior and subordinated debt indexes are published for financial and non-financial sectors. Financial debt sub-indexes, in accordance with their debt status, are also calculated.

- **The Markit iBoxx EUR Liquid indices.** This index family consists of a subset of the bonds in the *Markit iBoxx EUR* index family of benchmark indices. The liquid indices have been created to provide a convenient basis for OTC, exchange-traded derivatives and Exchange Traded Funds (ETFs).

Performance indices are generally made up of a large number of bonds. Portfolio managers tracking an index from the broader benchmark Markit iBoxx EUR index family will suffer significant costs in replicating or hedging the individual bonds in the portfolio. Moreover, bonds with particular characteristics or reduced amounts outstanding are generally quite illiquid, thus resulting in relatively broad bid-offer spreads.

The *Markit iBoxx EUR Liquid* indices are aimed at overcoming these obstacles by limiting the number of bonds per index and excluding special bond types, diminishing therefore costs in tracking and hedging.

This family is composed of three sub-families: sovereigns, sub-sovereigns and corporates. The *Markit iBoxx EUR Liquid Sovereigns* index is broken down into four maturity buckets: extra short, short, medium. It is a weighted aggregation of the *Markit iBoxx EUR Sovereigns Short*, the *Markit iBoxx EUR Sovereigns Medium* and the *Markit iBoxx EUR Sovereigns Long* index. The *Markit iBoxx EUR Liquid Agencies/Supranationals AAA* index comprises the most liquid government-backed bonds with an AAA rating.

The *Corporates* index sub-category is classified according to market conventions. Along with the overall index, there are separate indexes for financial and non-financial sectors, which are also divided by ratings. Moreover, a panel of economic sector indexes using ICB classification scheme is proposed.

Economic sector indexes for Basic Materials, Oil & Gas, Health Care and Technology are not published, because there are not enough available bonds.

The *Markit iBoxx EUR Liquid* include only fixed-rate bonds with payment at maturity, denominated in euro or in one of domestic currencies converted into euro (conventional bonds). The issuer's domicile is not taken into account. There are two exceptions to this rule so that two types of bonds frequently used by companies, namely rating-indexed bonds and bonds with known flows (such as step-up bonds), are eligible. Nevertheless, amortizing bond or sinking fund are excluded from family, as well as zero-coupon bonds, bonds whose last coupon is calculated on a different period, redeemable bonds, perpetual bonds (including fixed-term and perpetual hybrid loans issued by banks and insurance companies).

This index family seems to be the more relevant for the topic of this study.

- **The Markit iBoxx European ABS index family.** This index family is designed to measure the performance of EUR, GBP and USD denominated asset-backed securities originating from Europe.

Furthermore, the French Bond Association³ (*Comité de Normalisation Obligataire*), on one of its document, draws up an overview of European performance bonds:

- ✓ Europe Bond Investment Grade Corporate Bond Index,
- ✓ Euro Corporate Index (sub-indexes by rating and sector),
- ✓ Barclays Capital Euro Corporate Bond Index,
- ✓ Barclays Capital Euro Corporate Floating Rate Notes Bond Index,
- ✓ Corporate Barclays Index,
- ✓ Barclays Capital Aggregate Bond Index (previously called Lehman Aggregate Bond Index).

As for CDS indices, we can mention *iTraxx* indexes provided by [Markit](#), leading credit derivative index provider in Europe and Asia. Those are synthetic indexes representative of CDS referencing credit quality of firms selected according to Markit criteria. The *iTraxx* Europe “on the run” series offer an exposure to 125 equally weighted credit derivatives whose the underlying issuers are European investment grade firms (“high credit quality” issuers whose rating is higher than BBB- according to Standard & Poor’s or Baa3 according to Moody’s). The list of the underlying instruments of the series is revised twice a year and the new series are issued in March and September by Markit. The index composition is quarterly revised to be always exposed to the latest on-the-run series published by Markit. These indexes are the most liquid and the most representative of the market.

Among those indexes, the index which could be suitable for this study would be the *iTraxx Europe Main 5-year* index, which is representative of the market of 5-year maturity credit derivatives on private European investment-grade issuers. The diversification rules for this index allow for including firms in main areas of economic activity. The *iTraxx* indexes have increased transparency and liquidity on credit derivative markets, on which the *iTraxx Europe Main 5-year* index is a benchmark in Europe.

Moreover, with that perspective, we should ensure that those indexes are available on Reuters. CDS and corporate bond indexes must be consistent in that they must have the same entities basket and include equivalent maturities. The main caveat is to select CDS indexes and corporate bonds compatible in terms of similar features. For instance, if *iTraxx Europe Main 5-year* CDS index is selected (price will depend on market liquidity), it will be necessary to find a bond index with the same issuers and maturities. The drawback is difficulty to find some homogeneity between CDS index and bond index and it is not the case with the above mentioned indexes.

Moreover, it is easier to consider in the first place one or several reference entities representative of a rating or an economic sector and to retrieve:

- The 5-year maturity CDS premium,
- The price of a bond issued by CDS reference entity and also with a 5-year maturity.

³ <http://www.cnofrance.org/>

However, for a given CDS, it is difficult to find a bond with the same maturity and issued by the same entity (and even though such a bond was found, the effects of idiosyncratic noise and pricing errors among data would imply a significant volatility in default probability). That is the reason why we adopt the approach implemented by LONGSTAFF *et al.* [2005], which consist of using bonds issued by the same entity but with maturities bracketing the 5-year horizon of this CDS. The section 3.3 explains this procedure.

We will limit our study to a few issuers in a given rating category. As for the reference entity, we will choose Deutsche Bank AG⁴. Its rating is quite high both short- and long-term, as can be seen in the table Tab. 1.

Rating reference	Long-term rating	Perspective	"Intrinsic" quotation	Short-term rating
Moody's Investors Service	A2	Stable	baa2	P-1
Standard & Poor's	A+	Négative	a-	A-1
Fitch Ratings	A+	Stable	A	F1+

Tab. 1 – Deutsch Bank AG ratings.

We have selected the three fixed-rate bonds below issued by the Deutsche Bank AG:

1. DE000DB5S501
2. DE000DB7URS2
3. DE000DB5S5U8

3.2. DATA DESCRIPTION

Table Tab. 2 describes the bonds used in the calibration process, whereas the table Tab. 3 lists quotation dates from 1 December 2009 to 15 December 2011.

Isin code	Ticket code	Issue currency	Issue date	Maturity date	Reference rate	Coupon (%)
DE000DB5S501	DB	Euro	24-09-2007	24-09-2012	LIBOR	4.875
DE000DB7URS2	DB	Euro	09-06-2009	09-06-2016	EURIBOR	3.75
DE000DB5S5U8	DB	Euro	31/08/2007	31/08/2017	LIBOR	5.125

Tab. 2 – Bonds issued by the Deutsche Bank AG.

⁴ https://www.db.com/index_e.htm

Closing date	CDS premium (pb)	DE000DB5S501		DE000DB7URS2		DE000DB5S5U8	
		Bid (%)	Ask (%)	Bid (%)	Ask (%)	Bid (%)	Ask (%)
01/12/2009	85	106.779	107.061	103.799	104.199	108.467	108.814
04/12/2009	79	106.699	106.98	103.696	104.096	108.207	108.553
20/01/2010	82	106.661	106.929	104.143	104.644	107.703	108.042
25/01/2010	82	106.535	106.802	104.217	104.718	107.768	108.108
24/02/2010	99	107.073	107.203	105.01	105.19	108.662	109.001
25/02/2010	101	107.266	107.397	105.38	105.56	109.152	109.493
10/03/2010	80	107.136	107.265	105.26	105.44	109.296	109.636
19/04/2010	99	107.26	107.384	105.87	106.05	110.253	110.593
15/06/2010	150	106.285	106.515	107.53	107.77	109.343	110.005
17/06/2010	141	106.352	106.582	107.37	107.61	109.556	110.219
24/06/2010	140	106.25	106.477	107.4	107.64	109.958	110.623
13/07/2010	118	105.927	106.149	107.99	108.17	110.334	110.998
25/08/2010	110	106.511	106.83	109.84	110.14	114.045	114.726
26/08/2010	111	106.452	106.77	109.78	110.08	113.972	114.652
01/09/2010	110	106.462	106.777	109.99	110.29	114.253	114.934
03/09/2010	136	106.437	106.752	109.27	109.57	113.483	114.159
09/09/2010	103	106.339	106.651	109.29	109.59	113.535	113.804
17/09/2010	85	106.034	106.341	108	108.3	111.738	112.067
21/09/2010	147	106.034	106.34	108.12	108.42	111.849	112.179
14/10/2010	90	106.01	106.306	108.9	109.2	113.407	113.74
15/10/2010	96	105.931	106.227	108.77	109.07	112.851	113.182
19/10/2010	139	105.798	106.093	108.42	108.72	112.554	112.883
20/10/2010	98	105.698	105.991	108.19	108.49	112.279	112.607
04/11/2010	146	105.633	105.92	107.81	108.11	112.169	112.495
01/12/2010	184	105.616	105.893	106.38	106.68	109.935	110.376
14/12/2010	97	105.417	105.689	105.13	105.43	108.165	108.781
05/01/2011	100	105.472	105.735	106.32	106.62	109.172	109.79
21/01/2011	87	104.554	104.806	104.16	104.56	106.907	107.505
17/02/2011	95	104.272	104.513	103.74	104.44	106.665	106.901
08/03/2011	159	103.863	103.94	103.09	103.39	106.062	106.353
24/03/2011	94	103.844	103.92	103.47	103.77	106.626	106.917
18/05/2011	93	103.313	103.38	103.82	104.02	108.144	108.202
26/05/2011	98	103.371	103.478	104.26	104.46	108.346	108.52
23/06/2011	109	103.183	103.246	104.96	105.16	108.379	108.667
16/08/2011	155	103.234	104.586	107.24	107.54	107.2	107.756
09/09/2011	195	103.264	103.473	108.42	108.72	109.497	109.78
14/10/2011	173	102.47	102.641	106.7	107.7	108.124	109.227
02/11/2011	202	102.501	102.662	107.835	108.235	110.737	111.074
07/12/2011	195	NA	NA	NA	NA	NA	NA
15/12/2011	235	NA	NA	NA	NA	NA	NA

Tab. 3 – Quotations of the bonds used in this study. "NA" means that no data is available at the considered date.

Some few comments:

- ✓ Data about bonds are missing at quotation dates of November 2 and December 15, 2011.
- ✓ For all quotation dates, *i.e.* from December 1, 2009, to November 2, 2011, the three bonds are used in the calibration process.

3.3. PRINCIPLE OF CALIBRATION ALGORITHM

We implement the following approach to compute the corporate spread. For each corporate bond included in the bracketing set, we calculate the yield of a riskless corporate bond with the same maturity and coupon rate. Next, we substrate the resulting rate from the yield on the corporate bond to get the corporate yield spread. To calculate the 5-year yield spread, we regress the yield spreads for the individual bonds on their respective maturities. The resulting value is used as estimation of the spread associated to the studied entity. In addition, we infer the parameters of the forms of corporate bond price (8) and CDS premium (9). In this section, we describe how this

algorithm is implemented. Finally, the authors apply the same procedure to compute corporate spread default components.

3.3.1. Computing model parameters

To begin, we note:

- t_1, \dots, t_N , the quotation dates of Deutsch Bank AG's CDS premiums.
- n_i , the number of available bonds at the date t_i , $\forall i \in \{1, \dots, N\}$.
- $O(t_i, j)$, the bond j included in the set bracketing the date t_i , $\forall i \in \{1, \dots, N\}$.
- $T_{i,j}$, $O(t_i, j)$ bond maturity.
- $bid(t_i, j)$, the bid price of the bond $O(t_i, j)$, $\forall i \in \{1, \dots, N\}$ et $\forall j \in \{1, \dots, n_i\}$.
- $ask(t_i, j)$, the ask price of the bond $O(t_i, j)$, $\forall i \in \{1, \dots, N\}$ et $\forall j \in \{1, \dots, n_i\}$.
- $CB^{(pdc)}(t_i, j)$, the clean price of the bond $O(t_i, j)$, $\forall i \in \{1, \dots, N\}$ et $\forall j \in \{1, \dots, n_i\}$.
- $CB^{(pc)}(t_i, j) = CB(t_i, j)$, the full coupon price (in basis points) of the bond $O(t_i, j)$, $\forall i \in \{1, \dots, N\}$ et $\forall j \in \{1, \dots, n_i\}$.
- $CC(t_i, j)$, the accrued interests of the bond $O(t_i, j)$, $\forall i \in \{1, \dots, N\}$ et $\forall j \in \{1, \dots, n_i\}$.

The clean price is computed as the average price between bid and ask prices observed on the market:

$$CB^{(pdc)}(t_i, j) = \frac{bid(t_i, j) + ask(t_i, j)}{2}, \quad (10)$$

$$\forall i \in \{1, \dots, N\} \text{ and } \forall j \in \{1, \dots, n_i\}$$

The accrued interests are given by:

$$CC(t_i, j) = c\tau = c \frac{J_c}{N_a} \quad (11)$$

$$\forall i \in \{1, \dots, N\} \text{ and } \forall j \in \{1, \dots, n_i\}$$

The full coupon price is the sum of the clean price and the accrued interests:

$$CB^{(pc)}(t_i, j) = CB^{(pdc)}(t_i, j) + CC(t_i, j) \quad (12)$$

$$\forall i \in \{1, \dots, N\} \text{ and } \forall j \in \{1, \dots, n_i\}$$

where:

- J_c is the number of days elapsed since last coupon detachment.
- N_a is the number of days in a year.
- $\tau = \frac{J_c}{N_a}$ is the fraction of year elapsed since last coupon detachment.

We follow the convention that $N_a = 365$ days. We also note, $\forall i \in \{1, \dots, N\}$ et $\forall j \in \{1, \dots, n_i\}$:

- $CB^{(mkt)}(t_i, j)$, the observed full coupon price of the bond $O(t_i, j)$.
- λ_i , the initial value of the intensity process at date t_i .
- γ_i , the initial value of the liquidity process at date t_i .
- $s^{(mkt)}(t_i)$, the observed value of CDS premium at date t_i .
- $s^{(mod)}(t_i, \lambda_i)$, the theoretical value of CDS premium at date t_i .
- $CB^{(mod)}(t_i, j, \lambda_i, \gamma_i)$, the theoretical full coupon price of the bond $O(t_i, j)$.

From CDS premiums $s^{(mkt)}(t_i)$ and bond prices $CB^{(mkt)}(t_i, j)$ observed over the examined period, the algorithm aims at computing, through a differential evolution algorithm, the quadruplet $(\alpha^*, \beta^*, \sigma^*, \eta^*)$ minimizing the equation systems below:

$$s^{(mkt)}(t_i) = s^{(mod)}(t_i, \lambda_i), \forall i \in \{1, \dots, N\} \quad (13)$$

$$\begin{cases} CB^{(mkt)}(t_i, 1) &= CB^{(mod)}(t_i, 1, \lambda_i, \gamma_i) \\ \vdots &= \vdots \\ CB^{(mkt)}(t_i, j) &= CB^{(mod)}(t_i, j, \lambda_i, \gamma_i), \forall i \in \{1, \dots, N\} \\ \vdots &= \vdots \\ CB^{(mkt)}(t_i, n_i) &= CB^{(mod)}(t_i, n_i, \lambda_i, \gamma_i) \end{cases} \quad (14)$$

The calibration algorithm is subsequently executed at two levels:

- **The main level.** It consists of finding the quadruplet $(\alpha^*, \beta^*, \sigma^*, \eta^*)$ minimizing the overall observation error over the examined period.
- **The second level.** It consists of determining the pairs $(\lambda_i^*, \gamma_i^*)$ minimizing observation error at each date t_i ($\forall i \in \{1, \dots, N\}$).

Let:

- $\varepsilon_1(t_i, \lambda_i)$ the quadratic error between the observed and theoretical CDS premiums:

$$\varepsilon_1(t_i, \lambda_i) = \sqrt{\left(\frac{s^{(\text{mod})}(t_i, \lambda_i) - s^{(\text{mkt})}(t_i)}{s^{(\text{mkt})}(t_i)}\right)^2} = \left|\frac{s^{(\text{mod})}(t_i, \lambda_i) - s^{(\text{mkt})}(t_i)}{s^{(\text{mkt})}(t_i)}\right|, \forall i \in \{1, \dots, N\} \quad (15)$$

- $\varepsilon_2(t_i, \lambda_i, \gamma_i)$ the quadratic error between the observed and theoretical bond full coupon prices:

$$\varepsilon_2(t_i, \lambda_i, \gamma_i) = \sqrt{\sum_{j=1}^{n_i} \left(\frac{CB^{(\text{mkt})}(t_i, j) - CB^{(\text{mod})}(t_i, j, \lambda_i, \gamma_i)}{CB^{(\text{mkt})}(t_i, j)}\right)^2}, \forall i \in \{1, \dots, N\} \text{ et } \forall j \in \{1, \dots, n_i\} \quad (16)$$

- $\varepsilon_T(\alpha, \beta, \sigma, \eta)$ the error over the examined period:

$$\varepsilon_T(\alpha, \beta, \sigma, \eta) = \sum_{i=1}^N (\varepsilon_1(t_i, \lambda_i) + \varepsilon_2(t_i, \lambda_i, \gamma_i)), \forall i \in \{1, \dots, N\} \quad (17)$$

The reader will notice that we use relative errors (*i.e.* expressed as percentage) and not absolute errors, since the overall error embeds heterogeneous data, *i.e.* bond prices (expressed as percentage) and CDS premiums (expressed in basis points). That is the reason why, for consistency's and homogeneity's sake, it is required to normalize the processed data.

We take the following approach to determine a given quadruplet $(\alpha, \beta, \sigma, \eta)$. For each date t_i :

- We compute the initial value of the intensity process λ_i minimizing the error $\varepsilon_1(t_i, \lambda_i)$ with the equations (6) and (15):

$$\lambda^*(t_i) = \arg \text{Min}_{\lambda_i \geq 0} \{\varepsilon_1(t_i, \lambda_i)\} \quad (18)$$

- From the value $\lambda^*(t_i)$ calculated at the previous step, we determine the initial value of the intensity process γ_i minimizing the error $\varepsilon_2(t_i, \lambda_i, \gamma_i)$ with the equations (5) and (16):

$$\gamma^*(t_i) = \arg \text{Min}_{\gamma_i \geq 0} \{\varepsilon_2(t_i, \lambda_i^*, \gamma_i)\} \quad (19)$$

Once the previous loop has been completed, we compute the overall error $\varepsilon_T(\alpha, \beta, \sigma, \eta)$ over the observed period with the equation (17).

Thus, the quadruplet $(\alpha^*, \beta^*, \sigma^*, \eta^*)$ to compute is the one minimizing that overall error:

$$(\alpha^*, \beta^*, \sigma^*, \eta^*) = \arg \text{Min}_{\alpha, \beta, \sigma, \eta \in \mathbb{R}} \{\varepsilon_T(\alpha, \beta, \sigma, \eta)\} \quad (20)$$

The above described procedure turns out to be the function to be minimized by the differential evolution algorithm. The figure Fig. 5 summarizes the different calibration steps.

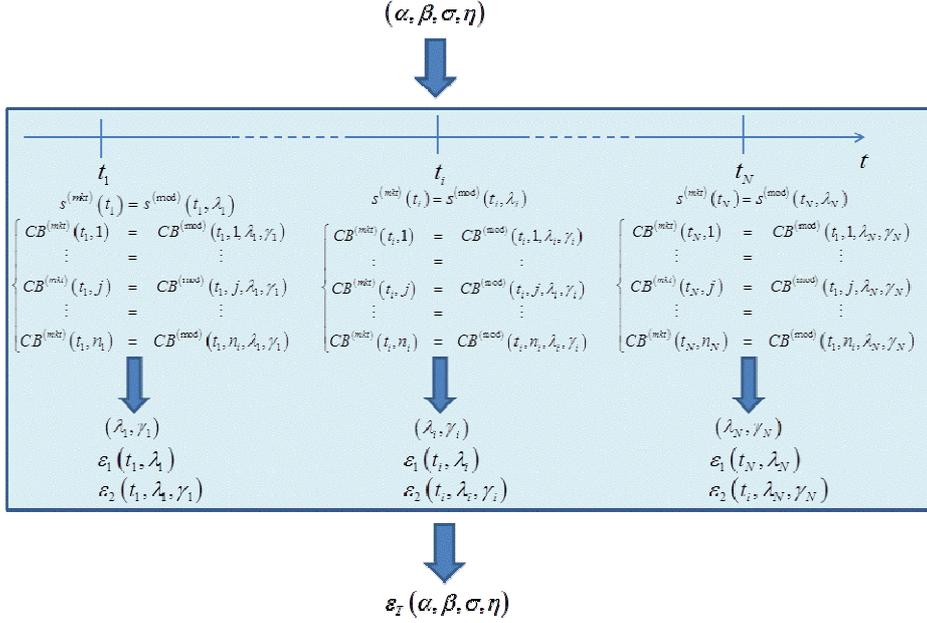


Fig. 5 – How the calibration algorithm operates.

Finally, we impose the following constraints on the parameters:

$$\begin{cases} 0.001 \leq \alpha \leq 0.05 \\ 0.005 \leq \beta \leq 0.91 \\ 0.005 \leq \sigma \leq 0.4 \\ 0.002 \leq \eta \leq 0.4 \end{cases} \quad (21)$$

Note: at a given observation date, residual bond maturity is not necessarily integer.

3.3.2. Measuring the spread components of a corporate bond

We have now to compute 5-year maturity corporate bond spread components at each date t_i of the observation period. For this purpose, we consider the bonds $O(t_i, 1), \dots, O(t_i, n_i)$. We adopt a two-staged approach:

1. We compute for each bond $O(t_i, j)$:
 - a. The equivalent riskless bond price $CB^{(riskless)}(t_i, c, \omega, T_{i,j})$ and its actuarial yield $r^{(mod)}(t_i, T_{i,j})$.
 - b. The equivalent bond price but with no liquidity risk $CB^{(def)}(t_i, c, \omega, T_{i,j})$ and its actuarial yield $y^{(def)}(t_i, T_{i,j})$.
 - c. The equivalent bond price but with no default risk $CB^{(liq)}(t_i, c, \omega, T_{i,j})$ and its actuarial yield $y^{(liq)}(t_i, T_{i,j})$.
2. We deduce from the previous step the features of a 5-year maturity bond through linear regression:
 - a. We linearly regress the prices $CB^{(riskless)}(t_i, c, \omega, T_{i,1}), \dots, CB^{(riskless)}(t_i, c, \omega, T_{i,n_i})$ against the maturities $T_{i,1}, \dots, T_{i,n_i}$ and we then calculate the price of the equivalent 5-year maturity bond $CB^{(riskless)}(t_i, c, \omega, T)$.

- b. Likewise, we linearly regress the yields $r^{(\text{mod})}(t_i, T_{i,1}), \dots, r^{(\text{mod})}(t_i, T_{i,n_i})$ against the maturities $T_{i,1}, \dots, T_{i,n_i}$ and we then calculate the actuarial yield of the equivalent 5-year maturity bond $r^{(\text{mod})}(t_i, T)$.
- c. We similarly operate with the prices $CB^{(\text{def})}(t_i, c, \omega, T_{i,1}), \dots, CB^{(\text{def})}(t_i, c, \omega, T_{i,n_i})$ which we linearly regress against the maturities $T_{i,1}, \dots, T_{i,n_i}$ and we then calculate the price of the equivalent 5-year maturity bond $CB^{(\text{def})}(t_i, c, \omega, T)$.
- d. We linearly regress the yields $y^{(\text{def})}(t_i, T_{i,1}), \dots, y^{(\text{def})}(t_i, T_{i,n_i})$ against the maturities $T_{i,1}, \dots, T_{i,n_i}$ and we then calculate the actuarial yield of the equivalent 5-year maturity bond $y^{(\text{liq})}(t_i, T)$.
- e. We do the same with the prices $CB^{(\text{liq})}(t_i, c, \omega, T_{i,1}), \dots, CB^{(\text{liq})}(t_i, c, \omega, T_{i,n_i})$ which we linearly regress against the maturities $T_{i,1}, \dots, T_{i,n_i}$ and we then calculate the price of the equivalent 5-year maturity bond $CB^{(\text{liq})}(t_i, c, \omega, T)$.
- f. We linearly regress the yields $y^{(\text{liq})}(t_i, T_{i,1}), \dots, y^{(\text{liq})}(t_i, T_{i,n_i})$ against the maturities $T_{i,1}, \dots, T_{i,n_i}$ and we then calculate the actuarial yield of the equivalent 5-year maturity bond $y^{(\text{liq})}(t_i, T)$.

The figures Fig. 6 and Fig. 7 describe how the bond price $P(\tau)$ is computed between two coupon payments.

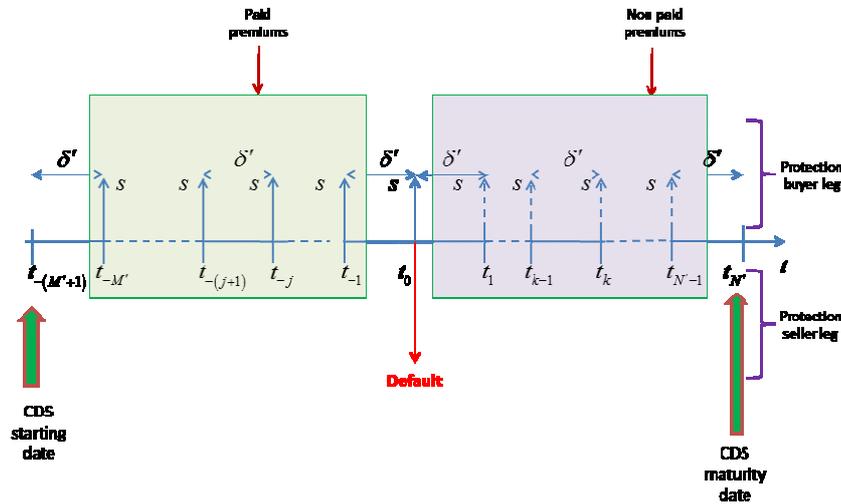


Fig. 6 – Diagram of the flows of a bond with $d_N - d_0 - \tau$ -year residual maturity.

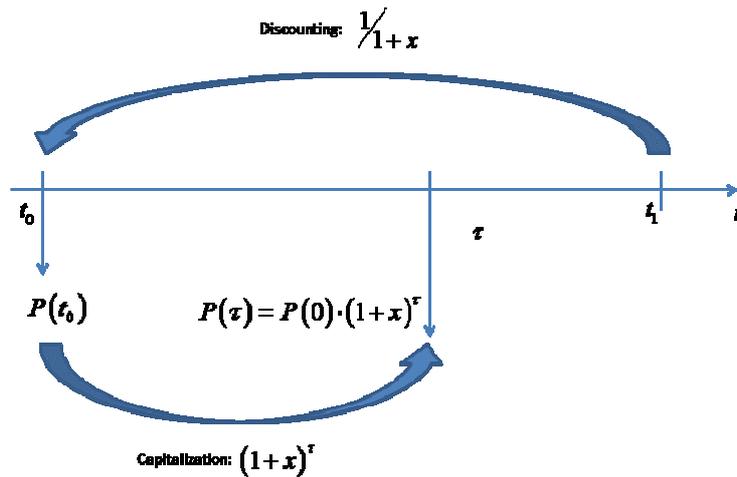


Fig. 7 – Bond price between two coupon detachments.

3.3.3. Alternative method

An alternative method could be to split the observation period in K sub periods $I_1 = [t_{1,1}, \dots, t_{1,m_1}]$, \dots , $I_i = [t_{i,1}, \dots, t_{i,m_i}]$, \dots , $I_K = [t_{K,1}, \dots, t_{K,m_K}]$, potentially of equal length. We would apply on each of them the calibration algorithm described in the section 3.3.1 to compute model parameters. The borderline case would be $K = N$ where each sub period $I_i = \{t_i\}$ would be reduced to a singleton. For calibrating on a sub period I_{i+1} , we would use as initial quadruplet the quadruplet found during the calibration of the previous sub period I_i : $(\alpha_{i+1}^{(0)}, \beta_{i+1}^{(0)}, \sigma_{i+1}^{(0)}, \eta_{i+1}^{(0)}) = (\alpha_i^*, \beta_i^*, \sigma_i^*, \eta_i^*)$. The figure Fig. 8 describes this procedure.

This approach can turn out to be interesting, if the parameters are stable in time. The advantage to calibrate on the whole period is to capture integrally parameters dynamics, which improves model predictability. Nevertheless, it would affect fitting quality. By contrast, operating a different calibration on each sub period would provide a better fitting, but less relevant information would be captured. Furthermore, parameters can vary from one sub period to another one.

As for each sub period's size, two approaches can be considered:

- The sub periods are of equivalent duration; they ideally have the same number of observation dates.
- The sub periods are not of equivalent duration. Their respective size depends on the hypothesis on phenomena currently occurring in market.

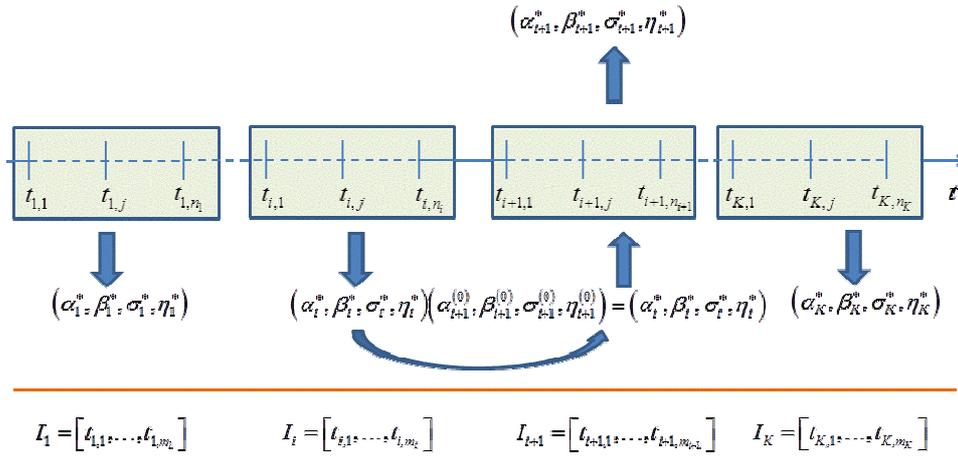


Fig. 8 – Splitting of the observed period into several sub periods potentially of equivalent duration.

To conclude, we will not choose this method, because it is more suitable to focus on model predictability for implementing an ESG.

4. ANALYSING THE RESULTS

In this section, we will comment the fitting quality obtained for the securities used in the calibration process, we will analyse the different components of the spread of a corporate bond and finally we will examine the results obtained from the out of sample test.

4.1. FITTING QUALITY

Fitting is nearly perfect for the CDS. As for the bonds, fitting is satisfactory, the error being less than 6.20%, as the figure Fig. 12 in the appendix shows. The table Tab. 4 displays the parameter values determined in this way.

α	β	σ	η
0.006585229	0.005000000	0.007124079	0.040527902

Tab. 4 – Parameters optimal values.

As shown in the figure Fig. 9, the intensity process tends to be higher (with a 100% ratio at several quotation dates) and “jerkier” than the liquidity process. Those observations will be corroborated at the section 4.2. Other tables and graphs are available at the section 7.1 in the appendix.

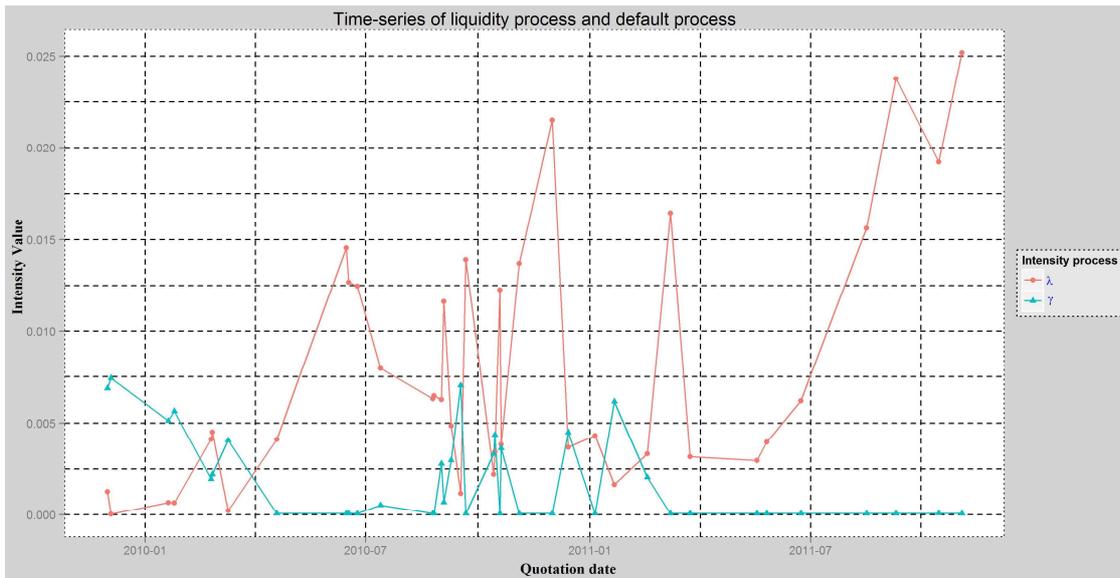


Fig. 9 – Evolution of intensity and liquidity process during the observed period.

4.2. DECOMPOSING THE YIELD TO MATURITY OF A BOND

As can be observed in the figure Fig. 10, the riskless rate curve generated from swap rates vs. 6-month Euribor embeds an implied spread, entailing a negative liquidity rate. The theoretical default component actually embeds a part of the actual liquidity component.

We note:

- $r^{(mod)}$ the theoretical riskless rate.
- $r^{(réel)}$ the actual riskless rate.
- $s^{(impl)}$ the implied spread embedded in the theoretical riskless rate.
- $y^{(mod)}$ the theoretical total actuarial yield.
- s the theoretical spread (sum of theoretical and default costs).
- $s_{liq}^{(mod)}$ the theoretical liquidity component.
- $s_{def}^{(mod)}$ the theoretical default component.
- s_{CDS} the theoretical CDS premium.
- b the bias between the default component of the bond actuarial yield and the CDS premium.

The theoretical total yield is the sum of the theoretical riskless rate and the theoretical default and liquidity components:

$$\begin{aligned}
y^{(mod)} &= \underbrace{r^{(réel)} + s^{(impl)}}_{=r^{(mod)}} + s \\
&= r^{(réel)} + s_{def}^{(réel)} + s_{liq}^{(réel)} \\
&= r^{(mod)} + s_{def}^{(mod)} + \underbrace{s_{liq}^{(mod)}}_{\leq 0}
\end{aligned} \tag{22}$$

The actual default component can be expressed as:

$$s_{def}^{(réel)} = s^{(impl)} + \theta \left(s_{def}^{(mod)} - |s_{liq}^{(mod)}| \right), \theta \in [0,1] \tag{23}$$

The actual liquidity component can be expressed as:

$$s_{liq}^{(réel)} = s^{(impl)} + (1-\theta) \left(s_{def}^{(mod)} - |s_{liq}^{(mod)}| \right), \theta \in [0,1] \tag{24}$$

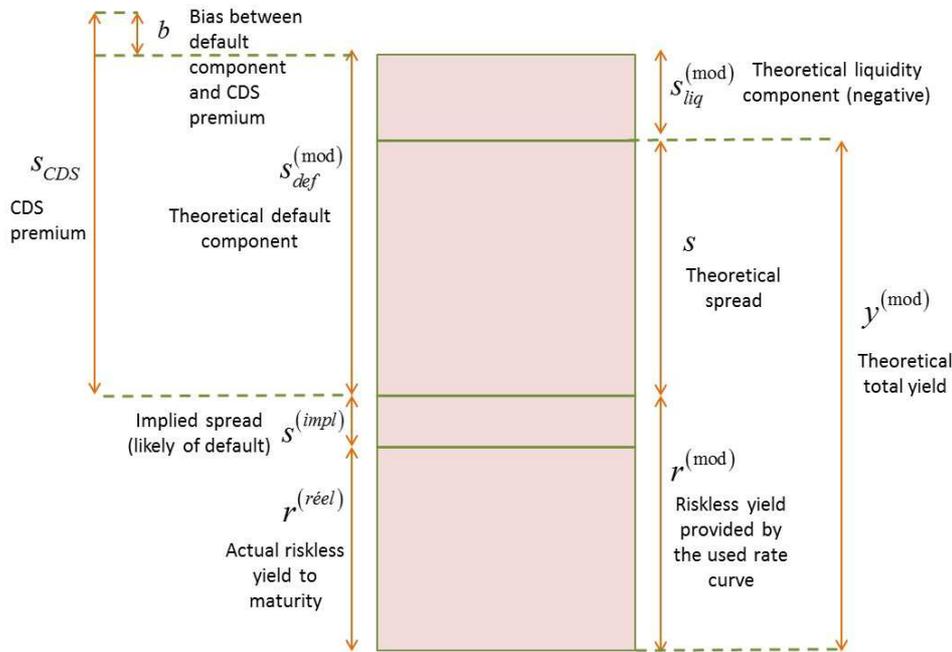


Fig. 10 – Decomposition of a coupon bond yield such as provided by the model from observed data.

Other tables and graphs can be seen in the appendix (see section 7.2).

4.3. OUT OF SAMPLE TEST

As explained in the section 3.3.3, we split the observed period into two sub periods ($K = 2$):

- The first one includes the quotations from 1 December 2009 to 21 September 2010. We note this sub period $I_1 = [t_{1,1}, \dots, t_{1,n_1}]$.
- The second one includes the quotations from 14 October 2010 to 2 December 2011. We note this sub period $I_2 = [t_{2,1}, \dots, t_{2,n_2}]$.

We calibrate the model from data included in the sub period $I_1 = [t_{1,1}, \dots, t_{1,n_1}]$ and we price the different instruments at the dates $I_2 = [t_{2,1}, \dots, t_{2,n_2}]$ with the previously computed parameters values $(\alpha_1^*, \beta_1^*, \sigma_1^*, \eta_1^*)$. We then get the following values:

- $s^{(\text{mod})}(t_{2,i}, \lambda_{2,i}^*)$ for the CDS premiums at the different dates $t_{2,i}$ ($i \in \llbracket 1, m_2 \rrbracket$).
- $CB^{(\text{mod})}(t_{2,i}, j, \lambda_{2,i}^*, \gamma_{2,i}^*)$ for the bond prices at the dates $t_{2,i}$ ($i \in \llbracket 1, m_2 \rrbracket$ and $j \in \llbracket 1, n_{2,i} \rrbracket$).

We then linearly regress the observed values $s^{(\text{mkt})}(t_{2,i})$ and $CB^{(\text{mkt})}(t_{2,i}, j)$ against the theoretical values $s^{(\text{mod})}(t_{2,i}, \lambda_{2,i}^*)$ and $CB^{(\text{mod})}(t_{2,i}, j, \lambda_{2,i}^*, \gamma_{2,i}^*)$ for $i \in \llbracket 1, m_2 \rrbracket$ and $j \in \llbracket 1, n_{2,i} \rrbracket$. The figure Fig. 11 describes this procedure.

We finally operate several statistical tests:

- Newey-West test,
- Test on Newey-West coefficients,
- Jarque-Bera test,
- Annova test.

We find that the test on Newey-West coefficients is conclusive for the CDS. Moreover, predictability is satisfactory for the bonds DE000DB5S501 and DE000DB5S5U8. By contrast, this is hardly the case for the bond DE000DB7URS2. Those results are anyway relevant given that LMN algorithm is not predictable. The tables Tab. 5 and Tab. 6 present the parameter values respectively in the 1st sub period and in the 2nd sub period. Parameters cannot be regarded as stable. Fitting quality is obviously better than in the case of a calibration on the whole period. In addition, for a given instrument, if the curves generated on the sub periods $I_1 = [t_{1,1}, \dots, t_{1,n_1}]$ and $I_2 = [t_{2,1}, \dots, t_{2,n_2}]$ are concatenated, the shape of such a curve looks like the shape of the curve generated by calibrating on the whole period.

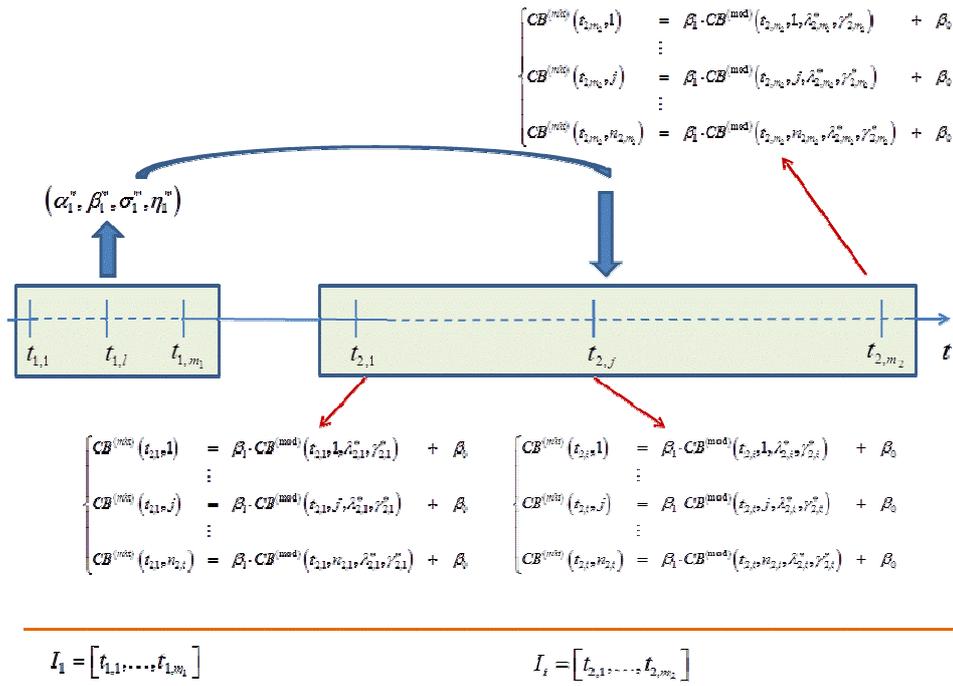


Fig. 11 – Testing parameters on a sample not used in the calibration process.

α	β	σ	η
0.007680684	0.034969190	0.051498691	0.042351664

Tab. 5 – Parameter values in the 1st sub period.

α	β	σ	η
0.006303657	0.012212810	0.013242982	0.037600166

Tab. 6 – Parameter values in the 2nd sub period.

5. CONCLUSION

LMN model is convenient to implement and with an important dataset one can set up the parameters in a “market consistent” way that preserves also a reasonable stability when adding new data. The results show that LMN model is predictable to a certain extent.

However, they show the existence of implied spreads in swap rates vs. 6-month Euribor curve. Selecting data to build a rate curve happens to be crucial in the current economic context. It would be relevant to explore several solutions.

- The existence of a correlation between default and liquidity risks could be considered. As a matter of fact, intuitively, a bond will be all the less liquid (all the more difficult to purchase, because all the more coveted) as the issuing firm will have a higher default probability. Likewise, a bond buyer bears a liquidity risk (*i.e.* difficulty to sell this bond on the market), if the issuing firm sees its default probability increasing (fewer potential buyers). Subsequently, this buyer, if he must get rid of this bond, will have “to sell it off”, hence an obvious shortfall.
- For determining 5-year maturity bond properties, it could be appropriate to consider a non-linear regression method.

➤ It would be consistent to use bonds issued by different entities and the associated CDS. For this purpose, let's consider the entities E_1, \dots, E_m . Suppose that the respective premiums of the CDS on those entities are s_1, \dots, s_m . Let t_1, \dots, t_n be the observation dates. We note $O(t_i, E_k, j)$ the $T_{k,j}$ -maturity bond j available at the date t_i and issued by the entity E_k . $n_{i,k}$ is the number of bonds available at the date t_i and issued by the entity E_k . The goal is to build a unique entity E representing the existing entities E_1, \dots, E_m . At a date t_i , the collection of the maturities of the bonds issued by E is made up by the collection of the maturities of the bonds issued by all the considered entities: $\mathfrak{A} = \bigcup_{k=1}^m \bigcup_{j=1}^{n_{i,k}} \{T_{i,k,j}\}$. The issue then consists of computing the premium of a CDS on the entity E as well as the prices of the bonds issued by this entity. The baseline approach would be taking:

- As CDS premium the average of the premiums of the entities E_1, \dots, E_m : $\hat{s} = \frac{1}{m} \sum_{k=1}^m s_k$.
- As M -maturity bond price the average prices of the available M -maturity bonds:

$$CB(t_i, M_{i,l}) = \frac{1}{m} \sum_{k=1}^m \frac{1}{a_{i,k}} \sum_{j=1}^{n_{i,k}} 1_{T_{i,k,j}=M_{i,l}} \cdot CB(t_i, E_k, T_{i,k,j}) \quad (25)$$

with:

$$a_{i,k} = \sum_{j=1}^{n_{i,k}} 1_{T_{i,k,j}=M_{i,l}} \quad (26)$$

and:

$$1_{T=M} = \begin{cases} 1 & \text{if } T = M \\ 0 & \text{if } T \neq M \end{cases} \quad (27)$$

Nevertheless, as mentioned in BCE [2009], due to market participants concentration and risk circularity, the correlations between the CDS premiums and the bond prices must be taken into account.

6. REFERENCES

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7. APPENDIX

In this appendix, the reader will find all the other graphics and tables relative to results obtained in this paper.

7.1. FITTING QUALITY

	DE000DB5S5U8	DE000DB5S501	DE000DB7URS2	5-year maturity bond	CDS
Average	-2,31%	2,92%	3,10%	1,05%	0,00%
Standard deviation	1,53%	1,34%	1,42%	1,13%	0,00%
Minimum	0,14%	0,26%	0,15%	0,02%	0,00%
Maximum	6,20%	5,61%	5,01%	2,58%	0,00%

Tab. 7 – Fitting quality (error in %) for the used instruments.

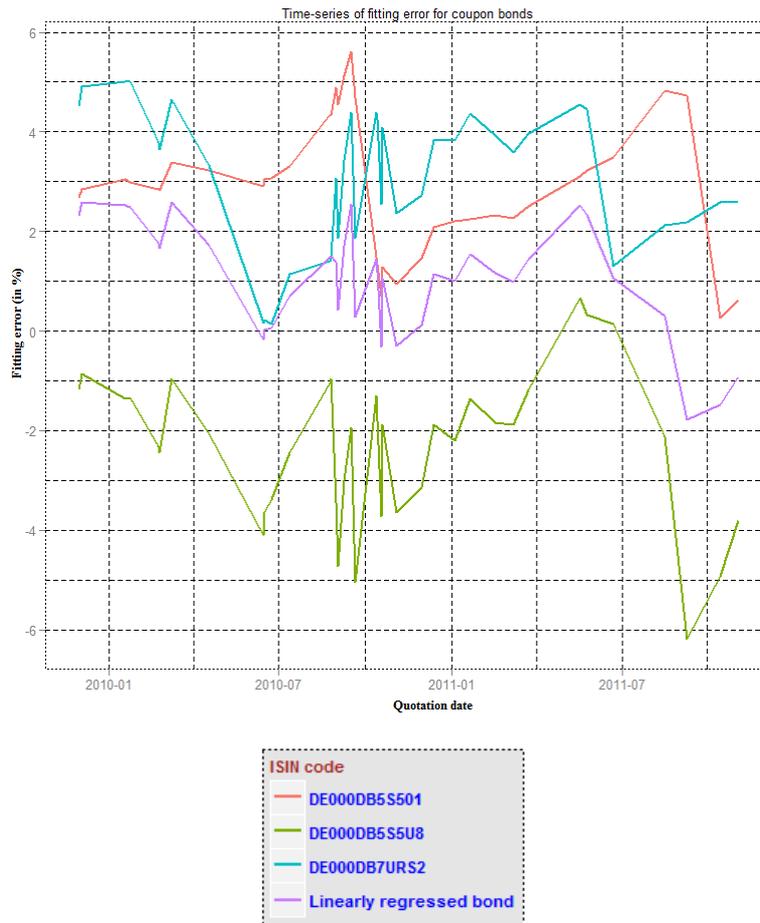


Fig. 12 – Fitting quality of coupon bonds.

7.2. DECOMPOSING THE YIELD TO MATURITY OF A BOND

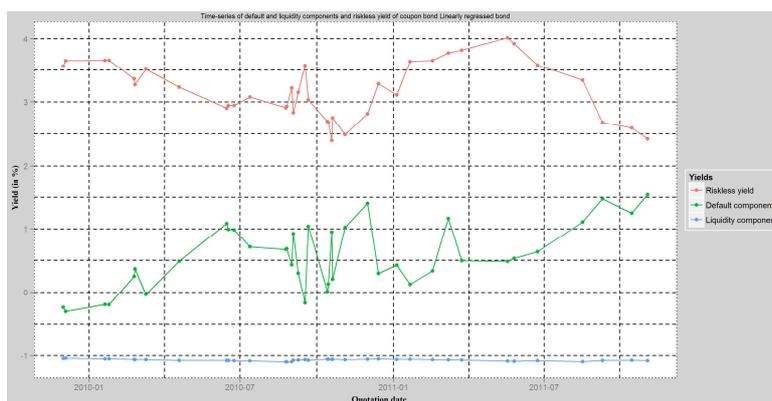


Fig. 13 – Time series of the different components of the 5-year maturity bond yield.

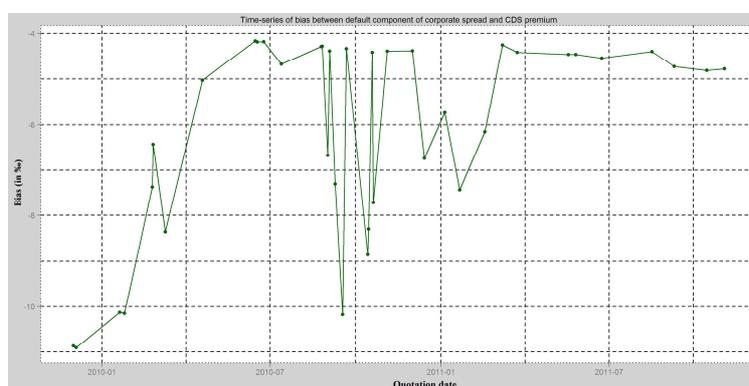


Fig. 14 – Bias is always negative during the observed period.

7.3. OUT OF SAMPLE TEST

	DE000DB5S5U8	DE000DB5S501	DE000DB7URS2	5-year maturity bond	CDS	
Moy.	2,36%	3,82%	2,96%	1,50%	0,00%	1st period
Ecart-type	1,26%	1,01%	1,83%	0,95%	0,00%	
Min.	0,85%	2,76%	0,15%	0,02%	0,00%	
Max.	4,80%	5,95%	5,27%	2,65%	0,00%	
Moy	2,24%	2,23%	3,48%	1,17%	0,00%	2nd period
Ecart-type	1,48%	1,35%	1,05%	0,65%	0,00%	
Min.	0,14%	0,27%	1,33%	0,12%	0,00%	
Max.	5,83%	5,08%	4,77%	2,58%	0,00%	

Tab. 8 – Fitting quality (error in %) for the instruments used in the two studied sub periods.

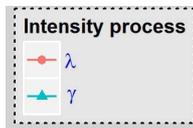
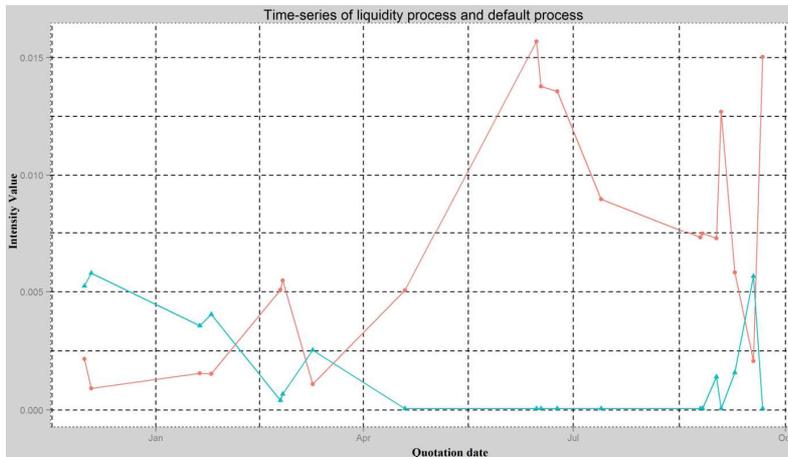
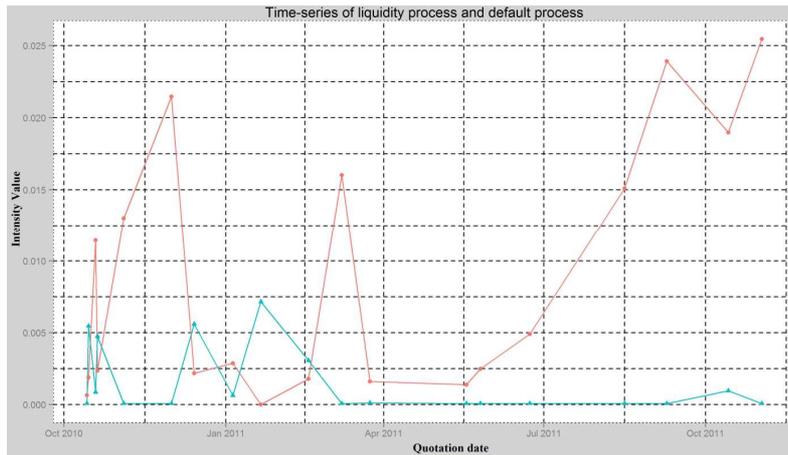


Fig. 15 – Times series of intensity and liquidity process.