Pricing and Risk Management of guarantees in unit-linked life insurance

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SÉPIA, PARIS, DECEMBER 12, 2007
1. Introduction

- Life insurance products
- Definition
- Advantages
- Guarantees
- Pricing and Risk management
- Outline

2. Pricing: the classical approach

3. Risk management strategies

4. Pricing and risk management

Conclusions

References
Life insurance products

2 broad types of life insurance products in Western Europe:

- **Policies with a minimum guaranteed return**
  - guaranteed return (at least 0%)
  - variable bonuses depending on the performance of insurer’s assets
  - investment decisions left to the insurer
  - only a limited part of the assets may be invested in equities

- **Unit-linked policies**
  - investment risk is fully borne by the policyholder
  - investment decisions are (partly) left to the policyholder
Unit-linked policies - definition

Life insurance contract, in which premiums are used to purchase units in a particular fund or combination of funds, and the value of the units is directly linked to the performance of the assets held in the fund(s).

The investment risk is borne by the policyholder.
Unit-linked policies - advantages

- For the **insurance company**:
  - Allows to transfer the investment risk to the policyholder
  - Much less capital consuming than traditional life insurance

- For the **policyholder**:
  - Allows to participate into stock market results
  - Usually offers tax advantages, as compared to mutual funds
  - High degree of transparency
  - (Partial) control over asset allocation
  - Introduction of protection elements (guarantees)
Unit-linked policies - usual guarantees

- **GMMB** - Guaranteed Minimum Maturity Benefit
  - guarantees the policyholder a specific monetary amount at the maturity of the contract
  - may be fixed or subject to regular or equity-dependent increases

- **GMDB** - Guaranteed Minimum Maturity Benefit
  - guarantees the policyholder a specific monetary amount upon death, during the term of the contract
  - may be fixed or subject to regular or equity-dependent increases
Unit-linked policies - usual guarantees

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  - may be fixed or subject to regular or equity-dependent increases

- **Other guarantees** do exist, like surrender options, annuity conversion options, ...

In this presentation, we will focus on GMDB.
Pricing and Risk management

- Guarantees reintroduce financial risk for the insurer

- Issues:
  - What price should be asked to the policyholder?
  - How to manage the risk associated with these guarantees?
  - How much capital should be allocated for such guarantees?
  - Should the price depend on the risk management strategy?

- Important in the context of Solvency II
Outline

1. Introduction to unit-linked policies
2. Pricing the guarantees: the classical approach
3. Various approaches to risk management
4. Link between pricing and risk management
5. Conclusions
2. Pricing: the classical approach

- Optional nature
- Black & Scholes
- Actuarial approach
- Financial vs. Actuarial
The optional nature of the guarantee

Insurer’s liability for a death at time \( t \):

\[
\max(K, S_t) = S_t + \max(0, K - S_t)
\]

- The first term is simply the value of the underlying assets. The associated risk is borne by the policyholder.
- The second term is the payout of a \textit{european put option} contract with maturity \( t \) and strike price \( K \). The associated risk is borne by the insurer.
The optional nature of the guarantee

The GMDB is equal to a weighted sum of european put options with varying maturities.

- The maturities correspond to the possible death times.
- The weights correspond to the probability that the insured will die at those times.

$$\sum_{t=1}^{T} t p_x q_x + t \max(0, K - S_t)$$
The optional nature of the guarantee

The GMDB is equal to a weighted sum of european put options with varying maturities.

- The maturities correspond to the possible death times.
- The weights correspond to the probability that the insured will die at those times.

The price (single premium) of the GMDB, for an individual aged $x$, is then simply equal to the same weighted sum of the put option prices:

$$SP = \sum_{t=1}^{T} t p_x q_x + t P(K, t)$$

We still have to evaluate the price of the european put options.
The Black & Scholes model

- The simplest model for equity options is the Black & Scholes model.

- It is based on strong hypotheses:
  - Complete, frictionless and arbitrage free financial market
  - Constant risk-free interest rate
  - The mortality risk is completely diversified
  - The underlying asset follows a Geometric Brownian Motion:

\[ dS_t = \mu S_t \, dt + \sigma S_t \, dW_t \]

- Under these assumptions, analytical expressions may be obtained for the put option prices, and hence for the GMDB.
Black & Scholes : single premium

\[ SP = \sum_{t=1}^{T} \left( K e^{-r t} \Phi(-d_2(0, t)) - S_0 \Phi(-d_1(0, t)) \right) p_x q_{x+t} \]

where

\[ d_2(s, t) = \frac{\log(S_0/K) + (r - \sigma^2/2)(t - s)}{\sigma \sqrt{t - s}} \]

\[ d_1(s, t) = d_2(s, t) + \sigma \sqrt{t - s} \]

where \( \Phi() \) is the standard normal distribution function.
B-S model: Is it the right approach?

The assumptions underlying the B-S assumptions are usually not fulfilled:

- The market is not complete; the mortality risk cannot be replicated.
- The guarantees are not actively traded. It is difficult to assume a no arbitrage principle on risks linked to the human life.
- If a hedging strategy is applied, the hedging portfolio should be continuously rebalanced. This would imply huge transaction costs.
- If it is not continuously rebalanced, a hedging error is introduced.
- Equities usually do not follow a GBM.

Question: does it make sense to use the Black & Scholes approach to price the guarantees?
Financial vs. Actuarial approach

- The financial engineer considers the GMDB as a contingent claim and uses a hedging argument to price it.

- The actuary considers the GMDB as an insurance contract and uses the equivalence principle to price it.
The actuarial approach

- In the actuarial approach, the future losses are modeled, and the single (pure) premium is equal to the expected value of these discounted future losses.

- If we again assume that the underlying asset follows a GBM, an analytical expression of the pure premium may also be derived:

\[
SP^A = \sum_{t=1}^{T} (Ke^{-rt} \Phi(-d_2^A(0, t)) - S_0 \Phi(-d_1^A(0, t)))t p_x q_{x+t}
\]

where

\[
d_2^A(s, t) = \frac{\log(S_0/K) + (\mu - \sigma^2/2)(t - s)}{\sigma \sqrt{t - s}}
\]

\[
d_1^A(s, t) = d_2^A(s, t) + \sigma \sqrt{t - s}
\]

- Expression quite similar to the financial price.
Financial and Actuarial single premiums

- The financial single (pure) premium:

\[
SP^F = \sum_{t=1}^{T} (Ke^{-rt} \Phi(-d^F_2(0,t)) - S_0 \Phi(-d^F_1(0,t))) tp_x q_{x+t}
\]

where

\[
d^F_2(s,t) = \frac{\log(S_0/K) + (r - \sigma^2/2)(t-s)}{\sigma \sqrt{t-s}}
\]

\[
d^F_1(s,t) = d^F_2(s,t) + \sigma \sqrt{t-s}
\]

- The actuarial single (pure) premium:

\[
SP^A = \sum_{t=1}^{T} (Ke^{-rt} \Phi(-d^A_2(0,t)) - S_0 \Phi(-d^A_1(0,t))) tp_x q_{x+t}
\]

where

\[
d^A_2(s,t) = \frac{\log(S_0/K) + (\mu - \sigma^2/2)(t-s)}{\sigma \sqrt{t-s}}
\]

\[
d^A_1(s,t) = d^A_2(s,t) + \sigma \sqrt{t-s}
\]
Financial and Actuarial single premiums

- The difference lies in the probability measure used:
  - Financial approach: risk-neutral probability measure
  - Actuarial approach: physical (or real) probability measure

- As the expected return on shares is (usually) larger than the risk-free rate, the "actuarial" (pure) premium is smaller than the "financial" (pure) premium.

- But this does not take risk into account ...

- What is the right approach?
3. Risk management strategies

- Risk landscape
- Risk management strategies
- Insurance approach
- Static hedging
- Dynamic hedging
- Risk premium
- Reinsurance
Risk landscape

- **Mortality risk**: this is a typical insurance risk, that can reasonably be considered as *diversifiable*. I.e. it can be effectively reduced by increasing the size of the portfolio covered.

- **Financial risk**: this is typically a risk that is *undiversifiable*. I.e. increasing the size of the portfolio does not help. When the stock market goes down, it goes down for all contracts. We call it a *systematic* risk.

- Note that both risks are inter-related. It is not possible to completely isolate the one from the other.
Possible risk management strategies

- Traditional insurance approach: provisions and capital
- Static hedging
- Dynamic hedging
- Adjustable mortality risk premium, based on capital at risk
- Reinsurance
Traditional insurance approach

- A probability distribution of the discounted future costs is determined. For the GMDB, the following quantity has to be modeled:

\[ DFC = \sum_{t=1}^{T} e^{-rt} tp_{x} q_{x} + t \max(0, K - S_{t}) \]

- Based on this distribution, a **Total Solvency Level** (TSL) is calculated, using any acceptable risk measure (VaR, Tail-VaR, ...). Let us assume that we use the VaR at 99% confidence level. The TSL is then equal to:

\[ TSL = \text{VaR}_{99\%}(DFC) \]

- This amount of money is invested in risk-free bonds.

- This method has been used in insurance for many years, especially in non-life insurance.
distribution of the future costs

- It is usually necessary to resort to stochastic simulations to determine an approximate distribution of the expected future costs.

- 2 independent processes need to be simulated:
  - the financial index
  - the death process

- For the financial index, any model of stocks return may be used. The simplest model, but still widely used, is the log-normal model, which naturally results from the assumption that the stock returns follow a geometric Brownian motion.

- This model is however not quite appropriate. In particular, it fails to capture more extreme price fluctuations. Other types of models may be thought of:
  - Autoregressive models (AR, ARCH, GARCH, ...)
  - Regime-switching models
  - ...
Some statistics on EuroStoxx 50

- Data from 1987 to 2002
- Q-Q plot of the monthly log-returns:

![Q-Q plot of the monthly log-returns](image)

- Moments of monthly log-returns:

<table>
<thead>
<tr>
<th>Data</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Skewness</th>
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<td>0.0032847</td>
<td>-1.062209</td>
<td>5.4192254</td>
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</tbody>
</table>
Some statistics on EuroStoxx 50

- Monthly volatilities:

![Volatility Chart](image)
Distribution of the future costs - example

Base case assumptions:

- $S_0 = 1$, $K = 1$
- The stock follows a log-normal model, with parameters $\mu = 8.5\%$ and $\sigma = 25\%$
- constant risk-free interest rate $r = 5\%$
- 1,000 insured persons aged 45
- 10,000 simulations
Distribution of the future costs

Distribution DFC (insurance approach)
Insurance approach - results

<table>
<thead>
<tr>
<th>Mean</th>
<th>Std Dev</th>
<th>VaR95</th>
<th>VaR99</th>
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<td>26,49</td>
<td>40,80</td>
<td>36,53</td>
<td>47,51</td>
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</table>

The Total Solvency Level may be further divided in:

- Provision: best estimate of the discounted future payments.
- Capital: TSL - provision

<table>
<thead>
<tr>
<th>Provision</th>
<th>VaR95</th>
<th>VaR99</th>
<th>TVaR95</th>
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### Sensitivity to $\mu$

<table>
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<th>$\mu$</th>
<th>mean</th>
<th>Std Dev</th>
<th>VaR$_{95}$</th>
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<td>2.01</td>
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<td>15%</td>
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<td>19.98</td>
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<tr>
<td>10%</td>
<td>3.51</td>
<td>7.28</td>
<td>19.68</td>
<td>35.71</td>
<td>29.39</td>
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<tr>
<td>8,5%</td>
<td>5.28</td>
<td>9.38</td>
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</tr>
<tr>
<td>5%</td>
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<td>12.93</td>
<td>38.58</td>
<td>48.88</td>
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<td>65.48</td>
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## TSL - sensitivity analysis

- Sensitivity to $\sigma$

<table>
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<th>$\mu$</th>
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<th>VaR_{95}</th>
<th>VaR_{99}</th>
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<tbody>
<tr>
<td>5%</td>
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<td>0.03</td>
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<td>0.08</td>
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<td>0.11</td>
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<td>0.55</td>
<td>1.67</td>
<td>1.33</td>
<td>2.99</td>
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<td>15%</td>
<td>0.71</td>
<td>2.21</td>
<td>3.48</td>
<td>11.53</td>
<td>8.28</td>
<td>17.37</td>
</tr>
<tr>
<td>20%</td>
<td>2.46</td>
<td>5.64</td>
<td>13.99</td>
<td>29.58</td>
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<td>35.50</td>
</tr>
<tr>
<td>25%</td>
<td>5.28</td>
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<td>40.80</td>
<td>36.53</td>
<td>47.51</td>
</tr>
<tr>
<td>30%</td>
<td>8.94</td>
<td>12.77</td>
<td>38.47</td>
<td>50.25</td>
<td>45.61</td>
<td>54.61</td>
</tr>
<tr>
<td>35%</td>
<td>13.17</td>
<td>15.88</td>
<td>46.65</td>
<td>56.89</td>
<td>52.86</td>
<td>60.43</td>
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<tr>
<td>40%</td>
<td>17.64</td>
<td>18.28</td>
<td>52.92</td>
<td>61.53</td>
<td>58.15</td>
<td>64.79</td>
</tr>
</tbody>
</table>
• Advantages:
  - Natural way of working in insurance. Easy to interpret and to understand.
  - Simple investment portfolio: everything is invested in risk-free assets. No rebalancing needed.

• Drawbacks:
  - Results very sensitive to the assumptions underlying the financial model.
  - Large amounts of capital needed, and this capital has to be remunerated.
Static hedging

- Consists in effectively purchasing the put options from a financial institution.

- Advantages:
  - The financial risk is almost entirely eliminated
  - Easy to fix the price of the guarantee. It is given by the price of the options bought.

- Drawbacks:
  - Difficult to find options with long-term maturities
  - If available, their price could be prohibitive
  - What does happen in case of lapses?
Dynamic hedging

- Instead of purchasing the options externally, it is also possible to replicate them.

- The replicating portfolio is built by taking positions on the underlying asset(s) and on the risk-free asset.

- The positions must be adjusted regularly (in theory, continuously) in order to have an efficient replication.

- The Black & Scholes model may be used to determine the replicating portfolio.
Replicating portfolio for a put option

- A hedging strategy for an option on a single stock, with maturity $T$, is defined as a bivariate process $\theta = (\eta_t, \xi_t)$, $t = 0, ..., T$ where

  $\eta_t$ is the quantity of risk-free asset to be held at time $t$

  $\xi_t$ is the quantity of stocks to be held at time $t$

- It can be shown that, under the Black & Scholes assumptions, the following hedging strategy duplicates the final pay-out of a European put option:

\[
\eta_t = Ke^{-r(T-t)}\Phi(-d_2^F(t,T))
\]
\[
\xi_t = -\Phi(-d_1^F(t,T))
\]

- Under the same assumptions, this strategy is self-financing. i.e. no additional money must be reinjected through time.
Replicating portfolio for the GMDB

- As mortality is not "replicable", it is impossible to design a self-financing hedging strategy for the GMDB.

- However, if we assume that the mortality risk has been sufficiently mutualised, a logical hedging strategy would be the following:

\[
\eta_t^* = (n - N_t) \sum_{s=t+1}^{T} sp_{x+t}q_{x+t + s}\eta_t
\]

\[
\xi_t^* = (n - N_t) \sum_{s=t+1}^{T} sp_{x+t}q_{x+t + s}\xi_t
\]

where

- \( n \) is the initial number of insured, considered all aged \( x \) at time \( t = 0 \)
- \( N_t \) is the number of deaths up to time \( t \)

- This means that, at each time step, we adapt our hedge to the observed number of survivors.
- This strategy is also risk-minimizing in the sense of Möller (1998)
Simulations

- If the replication strategy is perfect, all risk disappears.

- This is not our case, at least because of
  - the combination with the mortality risk
  - the impossibility to rebalance our hedging portfolio continuously
  - the behaviour of the stock, which does not follow exactly a log-normal distribution.

- This implies **hedging errors**, that have to be evaluated through simulations.

- Moreover, **transaction costs** are implied when rebalancing the hedging portfolio.

- In this case, in addition to the financial index and death process, the **hedging process** has to be simulated to obtain the distribution of the future costs.
Distribution of the future costs - example

Same example as for the insurance approach:

- \( S_0 = 1, K = 1 \)
- The stock follows a log-normal model, with parameters \( \mu = 8.5\% \) and \( \sigma = 25\% \)
- constant risk-free interest rate \( r = 5\% \)
- 1,000 insured persons aged 45
- 10,000 simulations

Only the hedging errors due to mortality and discrete hedging frequency are modeled. No frictional costs are included.
Distribution of the future costs

Distribution DFC (dynamic hedging)

F(x)

x

0 10 20 30 40 50

0 0.2 0.4 0.6 0.8 1
Comparison with insurance approach

Comparison distributions DFC

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Dynamic hedging - results

<table>
<thead>
<tr>
<th>Mean</th>
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<tr>
<td>10,11</td>
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<td>16.22</td>
</tr>
</tbody>
</table>

Again, the Total Solvency Level may be further divided in:

- **Provision**: best estimate of the discounted future costs.
- **Capital**: TSL -provision

<table>
<thead>
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<th></th>
<th>VaR95</th>
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<td>2,69</td>
<td>4.83</td>
<td>3.91</td>
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## Comparison with insurance approach

### TSL

<table>
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<th>VaR(_{95})</th>
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<tr>
<td><strong>Insurance approach</strong></td>
<td>5.28</td>
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### Capital

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### TSL - sensitivity analysis

- **Sensitivity to $\mu$**

<table>
<thead>
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<td>22,70</td>
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</table>
1. Introduction

2. Pricing: the classical approach

3. Risk management strategies
   - Risk landscape
   - Risk management strategies
   - Insurance approach
   - Static hedging
   - Dynamic hedging

4. Pricing and risk management

Conclusions

References

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**TSL - sensitivity analysis**

- Sensitivity to $\sigma$

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<thead>
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<th>$\mu$</th>
<th>mean</th>
<th>Std Dev</th>
<th>VaR$_{95}$</th>
<th>VaR$_{99}$</th>
<th>TVaR$_{95}$</th>
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<td>2.56</td>
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<tr>
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<td>3.14</td>
<td>28.69</td>
<td>39.94</td>
<td>30.69</td>
<td>33.68</td>
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</tbody>
</table>
Sensitivity analysis - comparison

Mean and standard deviation

Mu

- Mean (Insurance approach)
- StDev (Insurance approach)
- Mean (Dynamic hedging)
- StDev (Dynamic hedging)
Sensitivity analysis - comparison

Mean and standard deviation

Sigma

Mean (Insurance approach)  StDev (Insurance approach)
Mean (Dynamic hedging)  StDev (Dynamic hedging)
Sensitivity analysis - comparison

Total Solvency Levels

- VaR 99% (Insurance approach)
- TVaR 99% (Insurance approach)
- VaR 99% (Dynamic hedging)
- TVaR 99% (Dynamic hedging)
Sensitivity analysis - comparison

![Graph showing Total Solvency Levels with different risk management strategies and insurance approaches.

- VaR 99% (Insurance approach)
- TVaR 99% (insurance approach)
- VaR 99% (Dynamic hedging)
- TVaR 99% (Dynamic hedging)
Dynamic hedging - Pro’s and Con’s

- Advantages:
  - Financial risk is strongly reduced
  - Requires less capital than the insurance approach
  - More flexible than the static hedging approach

- Drawbacks:
  - Management of the hedging portfolio
  - Transaction costs
  - Hedging errors
Variable risk premium

- For each individual, the capital at risk is calculated periodically (weekly, monthly, ...) as the positive difference between the guarantee and the value of the fund.

- A premium rate is applied to this capital at risk, and the amount is charged to the policyholder.

- Advantages:
  - Depending on the frequency of calculation, the financial risk can be importantly reduced. Almost only the mortality risk remains.
  - Pricing and reserving is much easier.

- Drawbacks:
  - The administration is more complicated, and more costly.
  - The policyholder does not know in advance the cost of the guarantee.
Reinsurance

- As the guarantee may be isolated from the main product, it can be easily ceded to a reinsurer on a quota-share basis, often at 100%.

- In order to better control its financial risk, the reinsurer may prefer to offer a cover that works on an index-linked basis, the actual fund value being replaced by a reference financial index.

- In this case, the insurance company is subject to a basis risk.

- The reinsurance may also work on a variable risk premium basis.

- Advantages :
  - The risk can be completely ceded to the reinsurer.
  - Support for direct pricing.

- Drawbacks :
  - Reinsurance has a price ...
  - A basis risk may remain if the cover works on an index-linked basis
  - Counterparty risk.
4. Link pricing - risk management

1. Introduction
2. Pricing: the classical approach
3. Risk management strategies
4. Pricing and risk management
   • Pricing methodology
   • Actuarial and financial price
   • Types of risk
   • Consequences for pricing
   • Optimal strategy

Conclusions

References
Pricing methodology

- Premium = expected discounted future costs + loading for risk

- The loading for risk can be determined by applying a cost of capital to the capital allocated.

- Multi-periodic capital allocation.

- Questions:
  - How much capital to allocate?
  - Which cost of capital to use?
Actuarial and financial price

- Depending on the risk management strategy, both elements of the premium will be different.

- The pricing method may be applied to each case.

- Results will depend on the capital allocation and cost of capital hypotheses:
  - The "actuarial" price (based on the insurance approach) depends greatly upon the capital requirements and the cost of capital. The "financial" price is only impacted in a limited way.
  - If capital requirements are low, the insurance approach leads to a lower price than the dynamic hedging, and conversely.
  - If the cost of capital is low, the actuarial price is lower than the financial one, and conversely.
  - The financial price is close to the classical (B-S) price.
Actuarial or financial price?

- Which price is the "correct" one?
- Should it depend on the risk management strategy applied?
- If the dynamic (or static) hedging strategy is applied, it seems logical to use the financial price.
- And if the insurance risk management approach is applied? Does it still make sense?
- To answer this question, a distinction must be made between 2 types of risk involved: systematic and diversifiable risks.
Systematic and diversifiable risk

- **Specific** (or diversifiable) risk: risk that can potentially be eliminated by diversification. In perfect markets, it should not be rewarded.

- **Systematic** (or market) risk: risk that cannot be diversified away. It is correlated with the "market" returns.

- Systematic risk has one and only one price. It is a market price. It should not be influenced by the risk management strategy or the capital allocated.

- Diversifiable risk (or pure insurance risk) has no cost in perfect markets. However, when capital is allocated, it incurs costs, called frictional costs.

- Examples of frictional capital costs:
  - double taxation costs
  - agency costs
  - financial distress costs

Systematic and diversifiable risk

- **Specific** (or diversifiable) risk: risk that can potentially be eliminated by diversification. In perfect markets, it should not be rewarded.

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- Examples of frictional capital costs:
  - double taxation costs
  - agency costs
  - financial distress costs
Consequences for pricing

- The pure premium is to be based on the financial price, even in the insurance risk management approach.

- Depending on the risk management strategy, the allocated capital will defer.

- The pure premium is then loaded by the frictional costs on the capital allocated.

   → Pricing depends on risk management through the costs incurred on the capital allocated.
Optimal risk management strategy

- Risk management creates value by reducing the capital to be allocated.

- Particularly important in the context of Solvency II.

- It can be shown that it is always optimal for the shareholders to fully hedge the exposure to any tradeable (market) risk ...

- ... as long as the terms of trade are set on an efficient market.

- Why ?
  - return is reduced, but risk is reduced accordingly: systematic risk has only 1 price !
  - less capital to hold and less capital costs.

- For the dynamic hedging, transaction costs will temper this argument.
Conclusions

- The evaluation and pricing of guarantees in unit-linked contracts should be based on the financial approach.

- Different risk management strategies are possible. They have an impact on the capital required.

- Simulation methods must be used to evaluate this impact.

- The premium must be charged with the frictional costs on this capital. Premium depends on the risk management strategy.

- Risk management creates value through the amount of capital needed.

- Leaving transaction costs aside, it is optimal to fully hedge the market risk involved (static or dynamic hedging).
On the classical pricing approach:


References (cont.)

- On the comparison between the financial and the actuarial approaches:
  


- On the multi-periodic capital allocation in unit-linked contracts:


References (cont.)

• On pricing and risk management in incomplete markets:


• On risk management for (re)insurance companies:
