LONG-TERM CARE INSURANCE: A MULTI-STATE SEMI-MARKOV MODEL TO DESCRIBE THE DEPENDENCY PROCESS IN ELDERLY PEOPLE

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Abstract:

The pricing of today's long-term care insurance products relies on simple models where dependency is considered as a single homogeneous state. Because of aging population and rapid evolution in the field of medicine, it becomes paramount to get a clearer picture of the underlying risk. We believe it may only be achieved by taking into account several levels of dependency. A semi-Markov process is a multi-state process whose transition probabilities not only depend on the current state but also on the time spent in this state. This process has proven more flexible than the simple Markov process, and is core to numerous publications in the field of epidemiology. However its use in relation with long-term care insurance has remained mostly theoretical, mainly because of the lack of data available to insurers.

The present article aims at introducing the construction process of a semi-Markov model with 4 levels of dependency. This work is based on data from the French long-term care public aid: the "Allocation Personnalisée d'Autonomie" (APA). Firstly, we introduce the parameters used to model transitions between states. We then proceed to the calibration of those parameters, using a likelihood maximization method, while taking into account the peculiarities of the APA data set. Finally, we apply this model to the pricing of a fictive long-term care insurance product, using a Monte Carlo method.

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BULLETIN FRANÇAIS D'ACTUARIAT, Vol. 15, n° 29, janvier - juin 2015, pp. 41 - 73

Résumé :

La tarification des produits d'assurance dépendance se base aujourd'hui sur des modèles simples, où la dépendance est considérée comme un état unique et homogène. En raison du vieillissement de la population et des progrès rapides de la médecine, il est primordial d'acquérir une vision plus claire de ce risque, aujourd'hui très peu maîtrisé. Nous pensons que cet objectif ne peut être atteint qu'en prenant en compte plusieurs niveaux de dépendance. Un processus multi-états est dit semi-markovien lorsque les probabilités de transition du processus dépendent à la fois de l'état actuel et du temps passé dans cet état. De tels processus s'avèrent plus flexibles que les processus markoviens simples, et ont fait l'objet de nombreuses publications dans le domaine de l'épidémiologie. Cependant, leur application à l'assurance dépendance est restée principalement théorique, en raison notamment du manque de données accessibles aux assureurs.

Cet article a pour but de présenter la démarche de construction d'un modèle semi-markovien considérant 4 niveaux de dépendance. Ce travail s'appuie sur des données recueillies dans le cadre de l'Allocation Personnalisée d'Autonomie (APA). Tout d'abord, nous introduisons les paramètres intervenant dans la modélisation des transitions entre les états. Nous procédons ensuite à l'estimation de ces paramètres par la méthode du maximum de vraisemblance, en tenant compte des spécificités liées aux données APA. Enfin, nous proposons une application du modèle à la tarification d'un produit d'assurance dépendance fictif, à l'aide d'une méthode de type Monte Carlo.

Keywords: Long-Term Care Insurance, semi-Markov process, APA, right censoring, Weibull law, semi-proportional hazard, static frailty, maximum likelihood, Monte Carlo.

1. INTRODUCTION

In developed countries, since the beginning of the 20th century, there has been a steady increase in life expectancy at birth of around one quarter every year, as a result of the rapid evolution in medical techniques. This, along with the aging of the baby-boomer generations has resulted in the expectation that the number of French people aged 65 or more will double by 2060 (Blanplain and Chardon, 2010). Among other consequences, it will cause a spike in the number of elderly dependent people (Léocrart, 2011).

Paradoxically enough, the dependency risk can be called a young risk, because, while mortality and longevity have been studied for more than a century, the first long-term care insurance products only appeared in the mid 1980s, products covering partial dependency being even more recent. As all products include a maximum age of subscription, the number of people who reached higher ages where the incidence of dependency becomes significant is quite low, and so is the number of claims. Therefore, the amount of data available to insurers is still very limited.

To price long-term care insurance products, most insurers use discrete time models with 3 states: autonomy, dependency and death (see figure 1). Most products today also cover partial dependency, providing a percentage of the benefit granted in total dependency. The pricing of such products is achieved by considering that they offer two distinct guarantees whose cost can be calculated using two separate 3-state models. This approach yields very robust results and allows the use of experience data. However, the underlying assumption is that partial and total dependency are two completely independent phenomena, with probabilities of becoming totally dependent being the same for both autonomous and partially dependent insured lives.

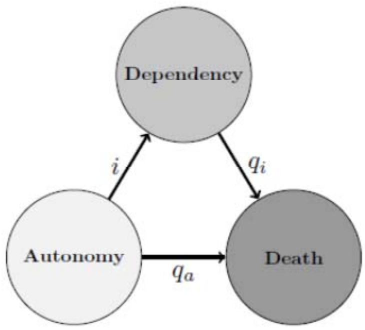


Figure 1: Simple model with 3 states: autonomy, dependency and death; i is the incidence rate and q_a (resp. q_i) the mortality rate for autonomous (resp. dependent) people.

To get a better understanding of the underlying dependency process, we believe it needs to be studied as a single multi-state process. The Markov process, for which transition probabilities only depend on the current state of the process, has already been used to this extent [?, ?]. However, survival times in dependency depend heavily on the age of the individual at entry, but also on the time already spent in dependency, as very high death rates are observed during the first few months after entry. To take both phenomena into account, a classic Markov process is not enough and therefore a more flexible process is needed to get a model which matches insurers experience.

For a semi-Markov process, transition probabilities depend not only on the current state but also on the time spent in the current state. Such process has been extensively used in the field of epidemiology (Commenges, 2002) and yielded better results than Markov process when modeling complex phenomena like for example the evolution of HIV (Mathieu, 2006) or follow-up of kidney transplant (Foucher et al., 2007). For long, the semi-Markov process has been identified as a powerful tool for long-term care insurance as well, in (Haberman and Pitacco, 1998; Denuit and Robert, 2007) and more recently in (Christiansen, 2012) while (Janssen and Manca, 2007) discussed numerical and computational issues. However, to the best of our knowledge, few papers focused on applications based on real data, one can however refer to (Lepez, 2006) due to the unavailability of such data. As a consequence, issues that arise when working on censored data coming from longitudinal studies are rarely addressed, although developing methods to handle such data proves necessary to the application of semi-Markov models for insurance purposes.

This paper provides an application based on data from the French public aid for dependent elderly people: the APA: "Allocation Personnalisée d'Autonomie". We consider a model with 4 different states of dependency. To define those states we rely on the "Autonomie Gérontologue Groupes Iso-Ressources" (AGGIR) grid. The AGGIR grid aims to categorize people by groups of similar needs based on their level of dependency. It is used in France for the attribution of the APA. This grid describes 6 levels of dependency, from the more severe level Gir 1 to the less severe Gir 6. However, only people in states Gir 1 to Gir 4 may actually benefit from the public aid, hence we only consider these 4 levels of dependency in our model. A description for each level of dependency can be found in table 1. To determine to which group one belongs, the ability to perform 8 activities of daily living is assessed, in a similar way to definition used by most insurers around the world. However, in the case of the AGGIR grid, the degree of incapacity is also taken into account, and some activities have more weight than others. This translates into a complex algorithm [?, see]]Vetel1998.

Gir level	Associated definition
Gir 1	People confined to bed or to a wheelchair, and whose mental abilities are greatly impaired, who need constant care. Also people at the end of their lives.
Gir 2	People confined to bed or to a wheelchair, whose mental abilities are not impaired, and who require care for most activities of daily living <u>or</u> people with impaired mental abilities but able to move by themselves, who need permanent oversight.
Gir 3	People with mental autonomy and partial physical autonomy, who need help for cleaning and bathing several times a day.
Gir 4	People who can walk inside their home but who require help for cleaning, clothing and possibly transfers.
Gir 5	People who need occasional help for cleaning, cooking and houseclean.
Gir 6	People still autonomous for the main activities of daily living.

Table 1: Levels of the AGGIR grid from most severe (Gir 1) to least severe (Gir 6).

Most insurers use their own definition of dependency, based on activities of daily living (ADL), in order to make it easier for insured lives to understand and not to be impacted by future changes in the public definition. Those definitions and the AGGIR definition can nevertheless be compared to some extent. In our paper, the choice of an AGGIR-based definition is driven by the use of data from the French public aid. This data is gathered over the whole population, and the number of dependent people observed this way outweighs most insurers portfolio's, especially at higher ages. As this data is gathered using a very specific observation process, we develop a specific methodology to limit the associated observation bias. Besides, as the data only includes information about dependent people, incidence rates and mortality rates for autonomous people cannot be inferred from it and therefore need to be obtained from another source.

In this work, we focus on APA data which gathers the assessments of dependency states on individuals. Among other features, this data is right-censored and contains missing values. Our main assumptions on this data are that the stock effect observed can be removed by left truncating of the data, and that transitions times and evaluations times can be assimilated (see section 2 for more details). We fit a homogeneous semi-Markov model with four dependency states (plus death) relying on Weibull distribution laws that integrate Cox proportional hazard rates.

In the next section, we introduce the APA data and its peculiarities in terms of censoring and truncating. We discuss several assumptions that are necessary to process the data and use it in the following sections of the paper.

The third section then provides a definition for the semi-Markov process, and introduces the elements of our parametric model. For every transition, the duration is assumed to follow a Weibull law. The impact of sex and gender is then taken into account through a semi-proportional hazard model. The impact of pathologies, which are not observed in the data, but, we believe, explain the heterogeneity between trajectories, is modeled through a static frailty which takes only two value. The value of frailty is determined at entry in dependency with a probability which depends on both the gender and the age of entry of the individual, through a generalized linear model with a logistic link function. The impact of frailty on the duration law is also modeled through a semi-proportional hazard rate. At last, this section also provides an expression for the likelihood function associated with the model, which is used for the calibration of parameters.

The penultimate section presents the parameters estimated through the maximization of the likelihood function. An algorithm to generate trajectories from these estimations is developed, and used to get descriptive statistics about the modeled dependency process.

The last section introduces a specific methodology for the pricing of insurance products using the calibrated model. This method relies on Monte Carlo simulations as a closed formula for the premium is not available due to the complexity of the process. Using the Central Limit Theorem and the delta method, we then compute an upper-bound for the uncertainty on the estimated premium. This methodology is finally applied to the pricing of a fictive long-term care insurance product, with a quick analysis about reserves and sensitivity to different risk factors.

2. THE APA DATA

2.1 Introducing the data

The APA: "Allocation Personnalisée d'Autonomie" is the French public aid for elderly dependent people. It has been introduced in 2002, and is only available to people aged 60 and more. People who want to benefit from the aid need to have their level of dependency evaluated by a public service team, and be assigned to group Gir 4 or more severe. Then, they agree with the team on a solution to cope with their dependency, and part of the cost is supported by the public aid, up to a maximum amount and depending on the people own resources.

The aid is managed locally by the French administrative area. Therefore each area gathers its own data. The data we were able to get are the same as in (Lepez, 2006). They have been gathered by 4 French administrative areas over the years 2002 to 2005. Only the individuals who have been granted the aid appear in the data.

Content of the data includes the following information

- The date of birth of the individual,
- The gender of the individual,
- The date of death of the individual, if death occurred during the observation period,
- The first date of evaluation which allowed the individual to benefit from the aid, with the result of this evaluation,
- Up to three subsequent evaluations with the result of those evaluations.

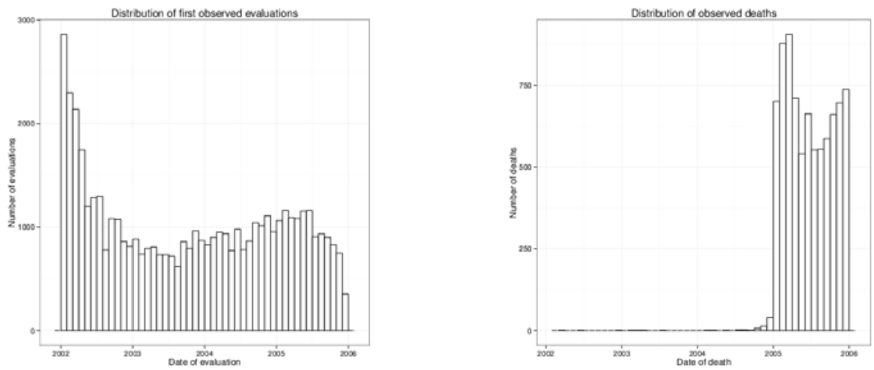


Figure 2: Left: distribution of first evaluations of Gir observed; right: distribution of observed deaths.

We observe on figure 2.1 (left) that the number of first evaluations in 2002 is way higher than during the subsequent years. This is due to the fact that the APA was created in 2002, hence many people had not taken any evaluation of their dependency level beforehand because they had no incentive to. We note that those people do not enter the APA all at once, but progressively, over the course of year 2002. This phenomenon is known as the "stock effect". On the other hand, figure 2.1 (right) shows that there is hardly any death observed before 2005. It appears that the information about deaths has only been collected from the year 2005. Consequently, deaths which occurred in 2002, 2003 or 2004 will be missing.

2.2 Discussion about the observation process

Before using the data in a model, we need to make and discuss several assumptions on the associated observation process. First of all, we note that it only contains evaluation dates whereas we are looking for the exact times at which transitions between states occurred. When we have two consecutive evaluations giving different results, we know that the transition occurred between the two evaluation times. Such phenomenon is called interval censoring. Methods to cope with interval-censored data have been developed in cases where the censoring process is non-informative (Foucher et al, 2007), for example when we have pre-scheduled evaluations, which is not the case for our process. In the case of an informative observation process, we need to be able to specify a model for the dependency between transitions and evaluations and misspecification of the model can alter the results (Chen et al., 2010).

For the APA data, however, evaluations can be requested by individuals or their usual doctor as they please, are free and can generally be obtained on short notice. We can therefore assume that, as soon as a transition occurs, the individual will request an evaluation, and consequently, it can be assumed that transitions can only occur at the evaluation times. Nevertheless, it should be noted that evaluations can still take place when no transitions has occurred. Besides, should the transition time and the evaluation time be very different, the results of the model still holds in an insurance context. Indeed, the relevant information in this context would be the time at which the claim is filled, and therefore the same delay we observed between the transition time and the evaluation time would still be present in this case. Hence, for the remaining of the paper, we work under the assumption that transition only occur at evaluation times.

Another issue that needs to be discussed is that, at the start of the observation period, some individuals are already dependent. We have seen previously that, because of the "stock effect", any individual who entered the APA in 2002 may have already been dependent for some time already. This phenomenon, known as left-truncating, cannot be handled easily, except in the case of simple memoryless time-homogeneous Markov process. Therefore, we decide to remove individuals who became dependent before year 2003 from the data. This way, we obtain left truncated data which should not be affected by the "stock effect". This approach should not generate any bias, as the people who became dependent in 2002 should not be any different than their 2003 counterparts. However it reduces the number of data available, and the observation period is shortened from 4 to 3 years.

The same issue arises when we look at the age of the individuals at first evaluation. The APA is only granted to people aged 60 and more. Hence, for individuals entering the APA at 60, we don't know exactly for how long they have been dependent. Hence, we decide to remove individuals who were less than 61 at first evaluation from the data. As dependency before 60 is quite rare, the impact is very limited in this case.

In the case where there are more than 4 evaluations for an individual, the subsequent evaluations are not recorded. This is actually a form of censoring because the only event that can be observed past the fourth evaluation is death. Information about death (resp. survival) of the individual after this point could still be taken into account, but we would not know the state of dependency of the individual at the moment of death (resp. at the end of the observation period). We consider instead that the observation period ends with the fourth evaluation. This could induce bias in the observation, but there are only few individuals for which 4 evaluations were observed, which may explain why this number was picked in the first place.

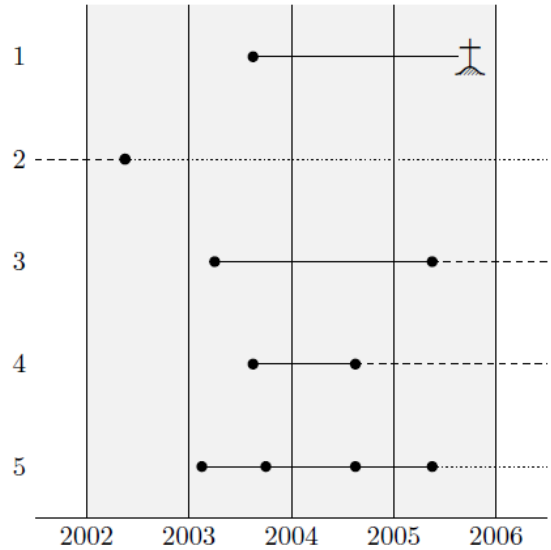
At the end of the observation period, due to the fact that death is not observed until 2005, we have several possible configurations:

- If death of the individual has been observed, then we have a full trajectory.
- If death of the individual has not been observed, and the last observed evaluation occurred in 2005, then we know that the individual is alive at the end of the observation period, because otherwise his death would have been recorded. The trajectory is then right censored, and the information about survival of the individual between his last observation and the end of the observation period should be taken into account through a term in the

likelihood function.

- If death of the individual has not been observed, and the last observed evaluation occurred before year 2005, then the individual has either died before 2005, or he is still alive at the end of the observation period, because otherwise death during year 2005 would have been recorded. In both cases, as no new evaluation has been made, we know that no other transition may have happened. We call this phenomenon partial censoring, as only the death of the individual is actually censored. This last configuration may seem quite complex, but its probability can actually be expressed quite easily in terms of likelihood, as we will see in next section.

Finally, let's note that in the data, there are cases where the dependency level of the individual improves between two consecutive evaluations. According to the definition used by French insurers, dependency is considered to be a consolidated and irreversible state. A temporary disability is not considered as dependency. Those improvements should thus be considered as errors in the evaluation diagnosis. Indeed, elderly people can have good days or bad days, which may cause the result of the evaluations to vary from one visit to another. Considering dependency as an irreversible process makes the estimation of the model much easier. Besides, from an insurance point of view, it is safer to consider there is no improvement, as an insured life will not be eager to declare any improvement in his health status that means no more benefit from the insurance. Unfortunately, the insurer has no way to detect such improvements as the cost of periodic controls would be way over their potential benefits. Consequently, in case of improvements, we consider that the state of dependency of the individual remains the same. Figure 3 provides an example of different kind of trajectories encountered and previously described.



1: Fully observed trajectory, from the entry into dependency to the moment of death.

2: Discarded trajectory, since the entry into dependency occurs during year 2002.

3: Right-censored trajectory, with no missing information.

4: Partially censored trajectory, as death may have occurred but not been observed.

5: Frequency-censored trajectory, the observation stops after the fourth evaluation.

Figure 3: Evaluation process for the APA data: examples of trajectories. Plain (resp. dashed/dotted) lines correspond to observed (resp. censored/removed) parts of the trajectories in dependency.

Features of the observation process can be summarized as follows:

- It is left-truncated, according to both the calendar year and the age. People who are already dependent or dead on the 1st of January 2003, or are already dependent or dead at the age of 61 do not appear in the data.
- It is right-truncated. Only people who become dependent before the 31st of December 2005 appear in the data.
- It is partially censored. Death is not observed until the 1st of January 2005.
- It is frequency censored. The observation period ends after the 4th evaluation.
- It is right-censored. The observation period ends at the 31st of December 2005.

After processing the data, we have information about 31,731 dependent individuals, but only 9,270 observed transitions.

3. DESCRIPTION OF THE MODEL

3.1 Introduction of the model

The model we present (see Figure 3.1) has 6 states: autonomy, death, and 4 different levels of dependency, Gir 1 being the most severe and Gir 4 the least severe (refer to table 1 for the description of those levels). Numbers will be associated with states: 5 for autonomy, 4 to 1 for Gir 4 to Gir 1 respectively and 0 for death. The model is unidirectional: transitions can only occur toward a more severe state of dependency or death. The dependency incidence rate $i(s)$ as well as the autonomous mortality rate $q^a(s)$, where s is the age of the individual, cannot be estimated from the APA data and we use exogenous information for those laws. In addition to the incidence rates, we also need to determine the distribution of the initial state of dependency, which will be estimated later in this section. This leaves us with 10 transitions for which we provide a semi-Markov model.

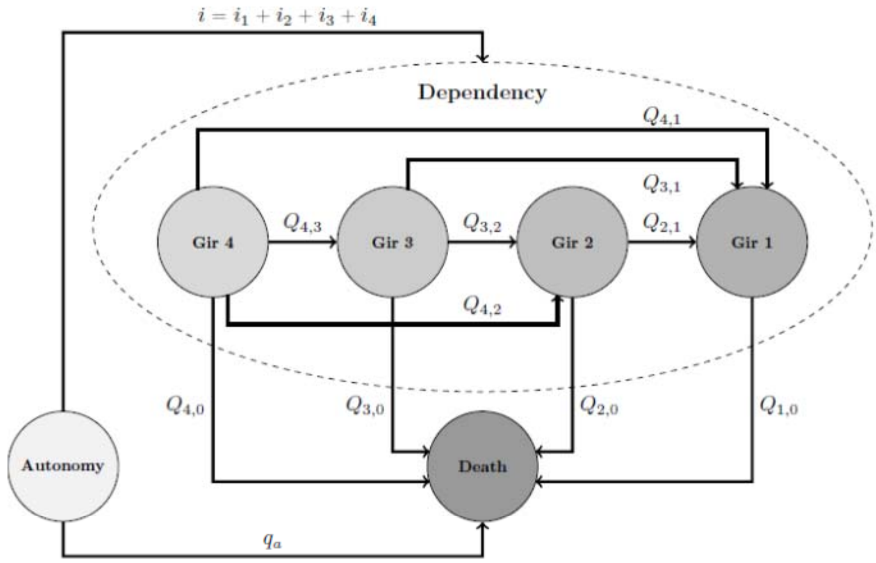


Figure 4: Semi-Markov model with 4 states of dependency. Transitions probabilities originating from dependency states are defined using their semi-Markov kernel $Q_{i,j}$.

3.2 Elements of semi-Markov theory

Definition: *Let $Y = (Y_t)_{t\geq 0}$ be a right-continuous process which takes its values in a finite set of states $E \subset \mathbb{N}$. Let $X = (X_n)_{n\in\mathbb{N}}$ be the sequence of consecutive states visited by the process and $T = (T_n)_{n\in\mathbb{N}}$ the sequence of consecutive times at which changes in the value of Y occur. Y is called a semi-Markov process if (X, T) is a multidimensional Markov process, or more formally, for all $n \in \mathbb{N}$, $x > 0$ and $j \in E$*

$$P(X_{n+1} = j, \quad T_{n+1} - T_n \leq x \mid X_0, \quad T_0, \quad \dots, \quad X_n, \quad T_n) = P(X_{n+1} = j, \quad T_{n+1} - T_n \leq x \mid X_n, T_n).$$

Furthermore, if those probabilities do not depend on T_n, then Y is called a time-homogeneous semi-Markov process.

The semi-Markov process keeps a memory of how long it has been in the current state. Nevertheless, similarly to the classical Markov process, it does not keep any memory about the previously visited states or transition times.

Definition: *Let Y be a time-homogeneous semi-Markov process, X (resp. T) the sequence of visited states (resp. transition times) associated with Y. We define*

- *The semi-Markov kernel*

$$\forall i, j \in E, \quad \forall\ 0 \leq x, \quad Q_{i,j}(x) = P(X_{n+1} = j, \quad T_{n+1} - T_n \leq x \mid X_n = i).$$

- *The jump probabilities*

$$\forall i, j \in E, \quad , \quad p_{i,j} = P(X_{n+1} = j | X_n = i) = \lim_{x\to+\infty} Q_{i,j}(x).$$

- *The duration laws*

$$\forall i, j \in E, \forall\ x \geq 0, \quad F_{i,j}(x) = P(T_{n+1} - T_n \leq x \mid X_n = i, X_{n+1} = j) = \begin{cases} \dfrac{Q_{i,j}(x)}{p_{i,j}} & \text{if} p_{i,j} > 0, \\ 0 & \text{otherwise.} \end{cases}$$

A semi-Markov process is entirely determined by its semi-Markov kernel and the initial states distribution. Besides, it can be noted that X is a discrete time Markov chain with values in E, whose transition probabilities are precisely the jump probabilities $p_{i,j}$.

We have the fundamental relation: $Q_{i,j}(x) = p_{i,j} \times F_{i,j}(x)$. To fully describe a semi-Markov process, we therefore need to model both the jump probabilities and the duration laws.

3.3 Model

3.3.1 Jump process

According to the previous definition, jump probabilities are indeed probabilities constrained by the following relations

$$\begin{cases} \forall i \neq j \in E, & 0 \leq p_{i,j} \leq 1, \\ \forall i \in E, \sum_{j \neq i} & p_{i,j} = 1. \end{cases}$$

Those probabilities will be estimated later alongside other parameters of the model.

For sake of clarity, we omit indexes of parameters corresponding to transitions $i \to j$ in the remain of this section.

3.3.2 Base duration laws

The base element of our model for duration law is the Weibull distribution. The hazard rate associated with this distribution is a single factor polynom, which degree depends on the shape parameters, which makes it very flexible. Besides, the distribution only consists of two parameters, and the survival function can easily be inverted, making it an excellent choice for optimization purposes. This distribution is commonly used in reliability theory, one can refer to Jiang and Murphy (1997) or Bucar et al. (2004), and was applied to model the dependency process by Lepez (2006).

The Weibull distribution can be described using either of those functions

- Survival function $S_0(x) = e^{-\sigma x^\nu}$,
- Density probability $f_0(x) = -\frac{\mathrm{d}S_0(x)}{\mathrm{d}x} = \sigma \nu x^{\nu-1} e^{-\sigma x^\nu}$,
- Hazard rate $h_0(x) = \frac{f_0(x)}{S_0(x)} = \sigma \nu x^{\nu-1}$,

with $\sigma, \nu > 0$.

The case $\nu = 1$ corresponds to an exponential distribution with constant hazard rate which brings us back to a time-homogeneous Markov model.

3.3.3 Integration of covariates

To take the covariates into account in the model, we make the assumption of proportional hazard rates, which was introduced by Cox, see for example (Cox and Oakes, 1984), and has since been used in a lot of publications. Our model will only consider two covariates: gender and age of entry in dependency. Gender is a binary covariate, so we only need to introduce a single parameter α for each transition in the model to account for its effect on hazard rates. On the other hand, age at entry in dependency can vary on a continuous scale. Nevertheless, we still decide to use a single parameter β so that if s is the age of entry in dependency, the hazard rate is multiplied by a factor $e^{\beta s}$. In this case, the choice of the exponential function is no longer neutral, the underlying assumption is that being one year older will have the same multiplicative effect on the hazard rate, regardless of the age s.

Finally, we have, for $x > 0$, $g \in \{1; 2\}, s > 0$

$$h_1(x|g,s) = h_0(x)e^{\alpha g+\beta s}$$
$$S_1(x|g,s) = S_0(x)^{\exp(\alpha g+\beta s)}$$

where $\alpha, \beta \in \mathbb{R}$.

The proportional hazard model is widespread in survival data analysis, it is very simple to implement and requires few additional parameters. As we have proportional hazard for every transition in the process, our model is said to have semi-proportional hazard. It means that the propotional hazard assumption is less restrictive in our case that it would be otherwise.

3.3.4 Introduction of a transverse static frailty

While the previously introduced covariates should explain part of the heterogeneity in the trajectories, we believe that the pathologies, which are unobserved in the data, remain the main source of heterogeneity. A rough categorization of pathologies causing dependency would give us two groups, with on one hand, cancer, strokes and some other cardiovascular diseases, on the other hand, dementia, which is mainly caused by Alzheimer's disease, neurological diseases and arthrosis. The first group of diseases is associated with quick trajectories, while the second group goes in pair with a slower degenerative process which results in longer trajectories.

To take this heterogeneity into account, we introduce a transverse static frailty in the model. Frailty can be seen as an additional covariate whose value has an impact on the trajectory but the variable itself cannot be observed. We consider that each individual has its own frailty, which does not vary over time and impacts every transition the individual will undergo. Hence, it is both static and transverse. One of the main limitations of the semi-Markov model is that no information can be carried over to the next transition, and therefore the duration of transitions are uncorrelated. The introduction of frailty allows us to bypass this limitation. Instead of a propotional hazard through frailty, we could consider a hidden mixture model which is a more general case, but it would require additional parameters.

We assume that frailty u is distributed according to a Bernouilli law with parameter $\eta(g,s) \in]0;1[$ where g is the gender and s the age of entry in dependency. Furthermore, we use a generalized linear model with a logit link function to express the impact of g and s on $\eta(g,s)$

$$\log\left(\frac{\eta}{1-\eta}\right) = \eta_0 + \eta_1 \times g + \eta_2 \times s.$$

For each transition, the impact of frailty will be modeled by a single parameter γ through a proportional hazard rate e^γ, so this impact may be different for each transition. The conditional laws associated with this frailty u are

$$h(x|g,s,u) = e^{\gamma u} h_1(x|g,s) = \begin{cases} h_1(x|g,s) & \text{if} u = 0, \\ e^\gamma h_1(x|g,s) & \text{if} u = 1, \end{cases}$$

$$S(x|g,s,u) = S_1(x|g,s)^{\exp(\gamma u)}$$

$$f(x|g,s,u) = h(x|g,s,u) \times S(x|g,s,u) = e^{\gamma u} h_1(x|g,s) S_1(x|g,s)^{\exp(\gamma u)},$$

with $\gamma > 0$ to guarantee the identifiability of the model.

3.3.5 Summary of the model

Finally, our duration model is characterized by the following laws

$$u \sim \mathcal{B}\left(\frac{e^{\eta_0+\eta_1\times g+\eta_2\times s}}{1+e^{\eta_0+\eta_1\times g+\eta_2\times s}}\right),$$

$$\lambda(g,s,u) = \sigma e^{\alpha g+\beta s+\gamma u},$$

$$h(x|g,s,u) = \nu\lambda(g,s,u)x^{\nu-1},$$

$$S(x|g,s,u) = \exp(-\lambda(g,s,u)x^\nu),$$

$$f(x|g,s,u) = \nu\lambda(g,s,u)x^{\nu-1}\exp(-\lambda(g,s,u)x^\nu),$$

where

- g the gender and s the age of entry in dependency,
- $\sigma, \nu, \alpha, \beta, \gamma, \eta$ are parameters defined for each transition whose domains of definition are summarized in Table 2,
- η_0, η_1, η_2 are the parameters of the frailty, defined globally.

With the jump probabilities, we have a total of 59 parameters that need to be estimated. Figure 3.3 illustrates the roles the different parameters play in the determination of the duration law, and Table 2 their domain of definition and short descriptions as a reminder.

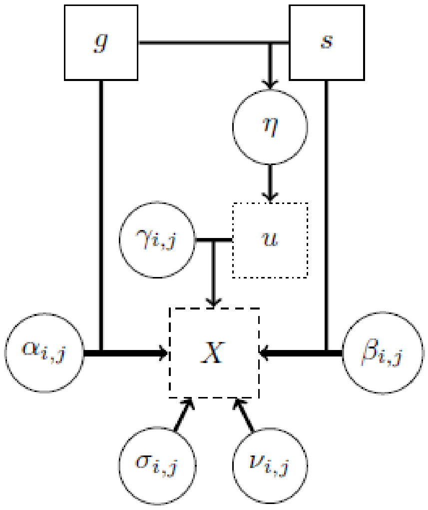


Figure 5: Role of parameters in the determination of the duration law X.

Parameter	Count	Domain of definition	Description
$p_{i,j}$	6	$0 \leq p_{i,j} \leq 1, \sum_{k \neq i} p_{i,k} = 1$	jump probabilities
$\sigma_{i,j}$	10	$\sigma_{i,j} > 0$	scale parameters of Weibull laws
$\nu_{i,j}$	10	$\nu_{i,j} > 0$	shape parameters of Weibull laws
$\alpha_{i,j}$	10	$\alpha_{i,j} \in \mathbb{R}$	impact of gender
$\beta_{i,j}$	10	$\beta_{i,j} \in \mathbb{R}$	impact of age of entry in dependency
$\gamma_{i,j}$	10	$\gamma_{i,j} > 0$	impact of frailty
$\eta 0, \eta 1, \eta 2$	3	$(\eta 0, \eta 1, \eta 2) \in \mathbb{R}^3$	distribution of frailty

Table 2: Summary of the different parameters used in the model.

3.4 Likelihood function

We denote by T_1 (resp. T_2) the beginning of the death observation period (resp. the end of the observation period). We recall that in our data, T_1 is the 1 of January 2005 and T_2 the 31 of December 2005. In addition, for each individual p, we introduce

- n_p the number of observed transitions,
- $X^p = (X_k^p)_{1 \leq k \leq n_p}$ the sequence of visited states,

- $t^p = (t_k^p)_{1\le k\le n_p}$ the sequence of transition times,
- indexes (δ_1^p, δ_2^p) indicating if the trajectory is right censored or partially censored, where
$$(\delta_1^p, \delta_2^p) = \left(\mathrm{I}[X_{n_p}^p \neq 0, T_1 \le t_{n_p} < T_2], \mathrm{I}[X_{n_p}^p \neq 0, t_{n_p} < T_1]\right),$$
- a vector of covariates $Z_p = (g_p, s_p)$ where g_p is the gender and s_p the age of entry in dependency.

The log-likelihood function has the following expression

$$l = \sum_{p=1}^{N} \log\left(\eta(g_p, s_p) \times l_p^1 + (1 - \eta(g_p, s_p)) \times l_p^0\right),$$

$$! \; l_p^u = \left(\prod_{k=1}^{n_p-1} \underbrace{C_{X_k^p, X_{k+1}^p}(t_{k+1}^p - t_k^p | g_p, s_p, u)}_{\text{observed transition}}\right) \times \underbrace{C_{X_{n_p}^p}^1 (T_2 - t_{n_p}^p | g_p, s_p, u)^{\delta_1^p}}_{\text{right censoring}}$$
$$\times \underbrace{C_{X_{n_p}^p}^2 (T_1 - t_{n_p}^p, T_2 - t_{n_p}^p | g_p, s_p, u)^{\delta_2^p}}_{\text{partial censoring}}$$

where, for $i, j \in E$, $x > 0, x_1 \ge x_2 > 0$, $g \in \{1,2\}, s > 0, u \in \{0,1\}$, N is the number of observed individuals and

- $C_{i,j}(x|g,s,u) = p_{i,j} \times f_{i,j}(x|g,s,u)$ is a term associated with an observed transition. First term $p_{i,j}$ gives the probability of observing the transition $i \to j$ and $f_{i,j}(x|g,s,u)$ the conditional probability of this transition happening precisely after a duration x has passed,
- $C_i^1(x|g,s,u) = \sum_{j<i} p_{i,j} \times S_{i,j}(x|g,s,u)$ is a right-censoring term, whose value is the probability of remaining in state i for a duration x,
- $C_i^2(x_1, x_2|g,s,u) = p_{i,0} \times \left(1 - S_{i,0}(x_1|g,s,u)\right) + \sum_{j<i} p_{i,j} \times S_{i,j}(x_2|g,s,u)$ is a composed censoring term. First term $p_{i,0} \times \left(1 - S_{i,0}(x_1|g,s,u)\right)$ is the probability of dying before a time x_1 has passed, and second term $\sum_{j<i} p_{i,j} \times S_{i,j}(x_2|g,s,u)$ the probability of remaining in state i for a duration x_2. As $x_1 < x_2$, both events are exclusive and their sum gives the likelihood associated with individuals for which death may have happened before T_1 and has not been observed.

4. RESULTS AND TRAJECTORIES

In this section, we present results of the model described in the previous section.

4.1.1 Estimation of parameters

We need to estimate 59 parameters by maximizing a likelihood function gathering information about the 31,731 trajectories we extracted from the APA data. We use the Nelder-Mead algorithm (Nelder and Mead, 1965) to maximize the likelihood function. This algorithm is based on successive evaluations of the optimization function on the vertices of a simplex which evolves in accordance with the results of those evaluations. Geometrical transformations like reduction, extension, or reflection are applied to the simplex in order to explore the most promising parts of the solution space. This algorithm offers a very powerful alternative to the Newton-Raphson algorithm when the computation of derivatives is not possible, and the solution space too large to use evolutionary algorithms. An implementation of the Nedler-Mead algorithm is provided in the programming language R (R Core Team, 2014) trough the function constrOptim(), which allows for linearly constrained optimization.

Transitions	$p_{i,j}$	$\sigma_{i,j}$	$\nu_{i,j}$	$\alpha_{i,j}$	$\beta_{i,j}$	$\gamma_{i,j}$	$\eta 0$	$\eta 1$	$\eta 2$
4 →3	0.27	0.0107	1.43	-0.23	0.044	0.13	0.93	-0.06	-0.04
4 →2	0.34	0.0043	1.43	-0.15	0.046	0.62			
4 → 1	0.03	0.0005	1.65	-0.11	0.070	1.17			
4 →0	0.37	0.0413	1.39	-0.90	0.039	3.09			
3 →2	0.43	0.0375	1.43	-0.12	0.029	0.57			
3 → 1	0.05	0.0136	1.59	-0.22	0.044	0.22			
3 →0	0.52	0.0439	1.23	-0.73	0.037	2.95			
2 →1	0.13	0.1279	1.49	0.06	0.008	0.21			
2 →0	0.87	0.0515	1.23	-0.82	0.037	3.38			
1 →0	1.00	0.0711	1.14	-0.61	0.036	3.64			

Table 3: Estimated values of parameters in the final model.

As the optimization algorithm can converge toward a local optimum of the likelihood function, we perform 100 iterations of the algorithm with randomly generated values for the initial parameters, and keep the best solution found at the end. The likelihood of the solution can vary a lot between two iterations, which prevents the use of tests based on likelihood. However, it seems to always converge toward one of a few local optima. With 100 iterations, relative stability of the result is achieved, with the best five solutions yielding similar parameters results. The estimated parameters can be found in Table 3. We note that gender has a significant impact on the duration laws. Women ($g = 2$) have lower hazard rates than men ($g = 1$) for every transition, especially the transitions that lead to death. For example the hazard rate for the transition $4 \to 0$ is 2.5 times higher for men than for women. Consequently women survive longer than men in dependency. Besides, hazard rates also increase with age, for every transition. The hazard rate for transition $4 \to 2$ is 4 times higher for someone aged 95 than for someone aged 65.

Estimated probability of frailty can be found in Figure 6. The probability of having a positive frailty decreases with age, from 25 % at 50 to 5 % at 100. This probability is very close for men and women. The impact of frailty is directly related to the severity of the transition. Indeed, frailty has very high impact on transitions toward death or a non-consecutive state, increasing hazard rate by up to 3700 % in the case of transition $1 \to 0$ but a lower impact on transitions between consecutive states, with only a 14 % increase for transition $4 \to 3$.

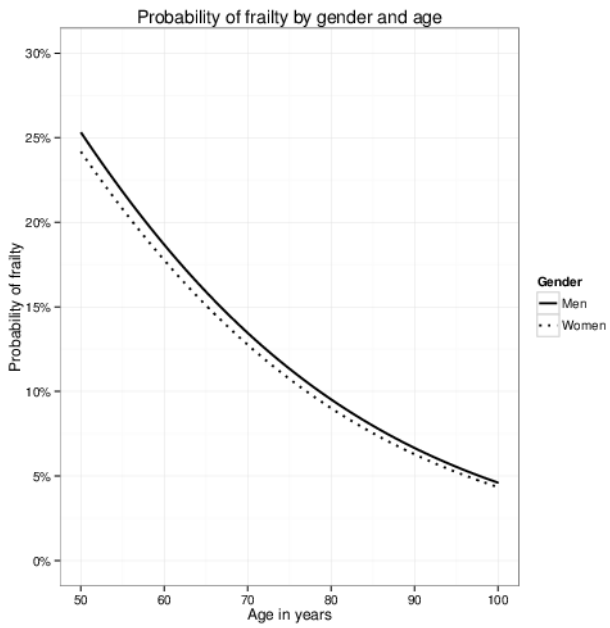


Figure 6: Estimated probability of frailty with respect to gender and age of entry.

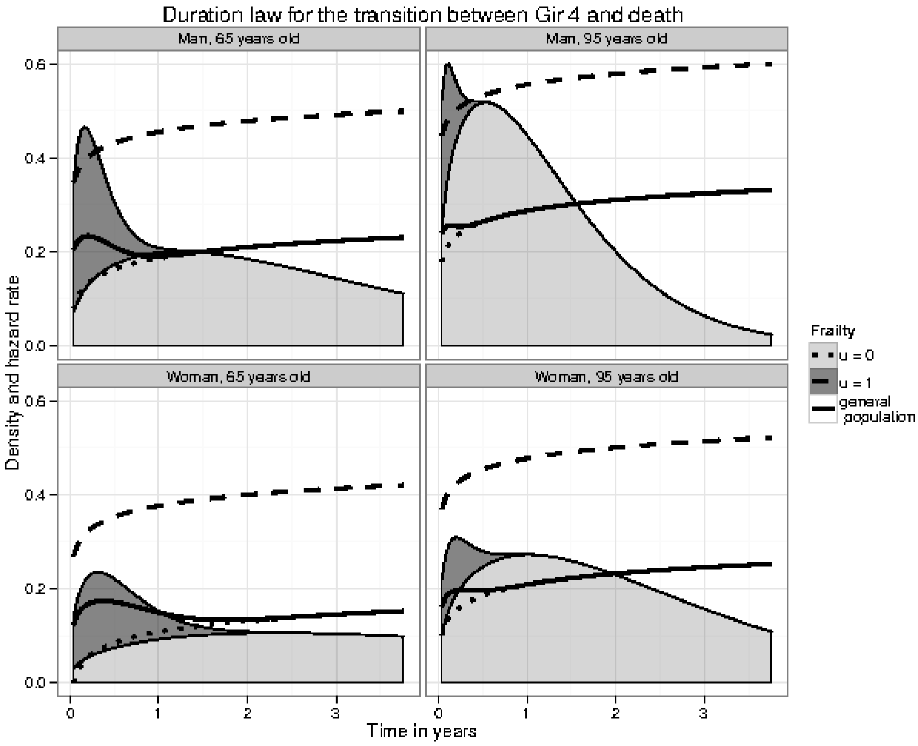


Figure 7: Duration law for the transition from Gir 4 to death; Grayed areas represent the associated density for people with and without frailty. Plain (resp. dashed/dotted) lines represent the hazard rate for the general population (resp. for people with/without frailty).

Figure 7 shows the duration law we obtain for the transition from Gir 4 to death. The individuals with frailty have very high hazard rates, and therefore they also die very quickly. This results in high hazard rates for the general population over the first year of dependency, with a decrease as the individuals with frailty die.

In addition, we also need to determine the distribution of the initial state of dependency. For an individual of gender g who becomes dependent at age s, we want to estimate the probability for the initial state of dependency to be Gir i for $i \in \{1; 2; 3; 4\}$.

For each gender and each age of entry in dependency between 65 and 95, we look at the state of entry in dependency for the people who became dependent at that age. It can be seen as a sample of a multinomial distribution of parameters the probabilities of entries in dependency at that age. We use empirical estimators to determine those probabilities. Furthermore, as we have at least 5 entries in dependency for each age and each state of dependency, the Fischer condition is met and it makes sense to compute normal confidence intervals for those rates.

For each state of dependency, we then perform smoothing of the empirical probabilities by age of entry using the unidimensional Whittaker-Henderson method, as described in Planchet and Thérond (2006), with parameters $h = 3$ and $z = 2$, and the weight of each probability being equal to the number of entries in dependency at that age. This choice of weights ensures that for each age, the sum of the smoothed probabilities for the different states of entry is still equal to 1, while this result would not hold for other types of interpolation as for example a generalized linear model. For an individual of gender g and age of entry in dependency s, the state of entry is determined using the estimated probabilities that we note $(e_i(g,s))_{i\in\{1,4\}}$ and which will be used latter for the simulation of trajectories.

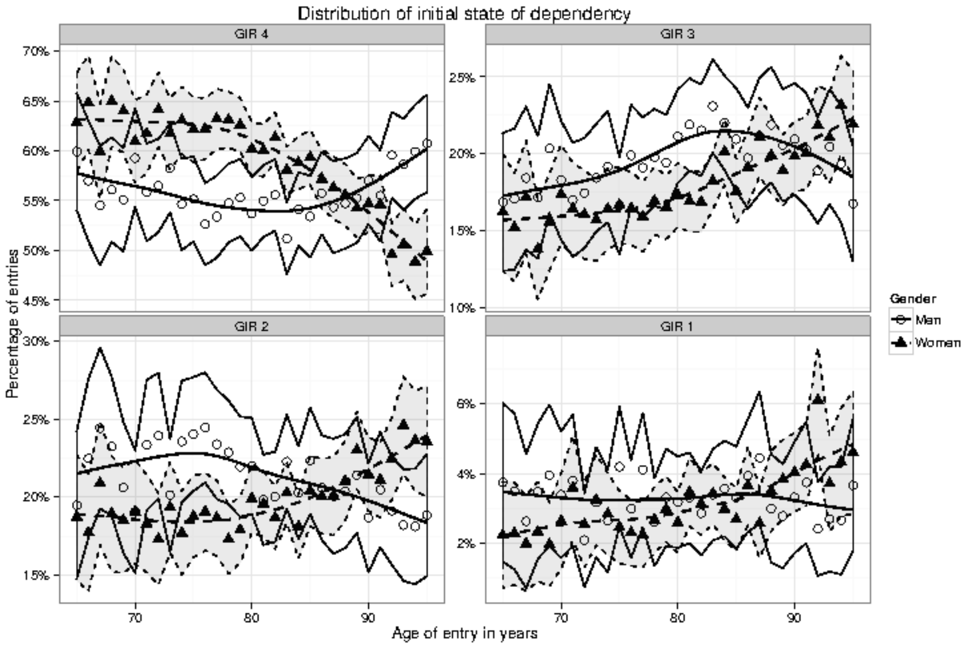


Figure 8: Distribution of initial states of dependency. Empirical probabilities (circles for men, triangles for women) with associated normal 95 % confidence intervals , and results of Whittaker-Henderson smoothing (plain line for men, dashed line for women).

Figure 8 highlights a significant difference between men and women. At age 65, men are more likely than women to directly enter a severe state of dependency. However, this trend shifts over time and the situation is reversed for ages over 90. This phenomenon can be interpreted by considering the underlying pathologies. On one hand, we know that men are more subject to cancers, which result in very severe dependency very quickly, and the incidence rate for dependency caused by cancer becomes lower with age. On the other hand, women are more affected by dementia, which become more frequent at higher ages. Early states of dementia are not recognized by the AGGIR grid as dependency as long as they are not associated with functional limitations. When those limitations occur, the dependency state may already be very severe.

4.2 Simulation of trajectories

In order to generate trajectories, in an algorithmically efficient way, we need to define several quantities, for individuals of gender $g \in \{1; 2\}$. Note that incidence rate and autonomous mortality rate are defined so that every year i is applied first and q_a is applied on the remaining autonomous people.

- $p_i^g(s,x)$ the probability at age s to become dependent at age $s+x$,
- $p_a^g(s,x)$ the probability at age s to die at age $s+x$ without ever becoming dependent,
- $p_i^g(s)$ the probability at age s to become dependent one day,
- $p_a^g(s)$ the probability at age s to die without ever becoming dependent,
- $p_{|i}(s,x)$ the probability at age s, knowing one will become dependent before dying, that the entry in dependency occurs at age $s+x$,
- $p_{|a}(s,x)$ the probability at age s, knowing one will die without ever becoming dependent, that the death occurs at age $s+x$,

where $s, x \in \mathbb{N}$. We note that $p_i^g(s) + p_a^g(s) = 1$ for every $s \in \mathbb{N}$.

Those quantities can be linked to the incidence and mortality rates

$$p_i^g(s,x) = \left(\prod_{k=0}^{x-1} (1 - i^g(s+k))(1 - q_a^g(s+k))\right) i^g(s+x),$$

$$p_a^g(s,x) = \left(\prod_{k=0}^{x-1} (1 - i^g(s+k))(1 - q_a^g(s+k))\right)(1 - i^g(s+x)) \times q_a^g(s+x),$$

$$p_i^g(s) = \sum_{x=0}^{\infty} p_i^g(s,x),$$

$$p_a^g(s) = \sum_{x=0}^{\infty} p_a^g(s,x),$$
$$p_{|i}^g(s,x) = \frac{p_i^g(s,x)}{p_i^g(s)},$$
$$p_{|a}^g(s,x) = \frac{p_a^g(s,x)}{p_a^g(s)},$$

where $s, x \in \mathbb{N}$.

4.2.1 Simulation algorithm

The trajectory of an individual p characterized by his gender g_p and his age s_p^0 at the start of the simulation consists of

- the number of visited states: n_p. As our dependency process is assumed to be unidirectional, and we have 4 states of dependency and death as a terminal state, we have $2 \le n_p \le 6$,
- a set of visited states: $X^p = (X_k^p)_{1\le k\le n_p}$,
- a set of transition times: $t^p = (t_k^p)_{1\le k\le n_p}$,

where t_0^p is the time at which the simulation starts, and $X_1^p = 5$, as we only consider individuals who are autonomous at the start of the simulation.

To simulate the trajectory of an individual p, we use the following algorithm

1. We set $X_1^p = 5$, and $t_1^p = s_p^0$.
2. With probability $p_i(s_p^0)$, the individual dies without becoming dependent. In this case, we set $X_2^p = 0$ and go to step 3. Otherwise the individual becomes dependent and we go to step 4.
3. The age of death is $t_2^p = s_p^0 + x + r$ where x is distributed according to the probabilities $p_{|a}(s_p^0, l)$ for $l \in \mathbb{N}$ and r is the fractional part of the year, uniformly distributed on $[0;1]$. The trajectory ends with the death of the individual and the algorithm stops, with $n_p = 2$.
4. The age of entry in dependency is $t_2^p = s_p^0 + x + r$ where x is distributed according to the probabilities $p_{|i}(s_p^0, l)$ for $l \in \mathbb{N}$ and r is the fractional part of the year, uniformly distributed on $[0;1]$. We note $s_p = t_2^p$ the age of entry in dependency and we set $k = 2$.
5. The probabilities $(e_i(g_p, s_p))_{i\in\{1,4\}}$ are used to determine the state of entry in dependency X_2^p.
6. The frailty u_p is set to 1 with probability $\eta(g_p, s_p)$ and to 0 otherwise.

7. - The next state X_{k+1}^p is determined with respect to probabilities $p_{X_k^p,j}$ for all $j \in \{0, \dots, X_k^p - 1\}$.

- To determine the time spent in state X_k^p, we first draw a random variable x distributed uniformly on $[0; 1]$. We then set $t_{k+1}^p = t_k^p + y$ where y is defined below

$$y = \left[\frac{e^{-(\alpha g_p + \beta s_p + \gamma u_p)}}{\sigma} \ln\left(\frac{1}{1-x}\right)\right]^{\frac{1}{\nu}}.$$

We deliberately omitted indexes in the previous formula for the sake of clarity. They should be X_k^p and X_{k+1}^p.

- We increment k. If the individual is still alive, i.e. $X_k^p \neq 0$, we repeat the steps of 7. Otherwise, the trajectory ends with the death of the individual and $n_p = k$.

4.3 Statistics on simulated trajectories

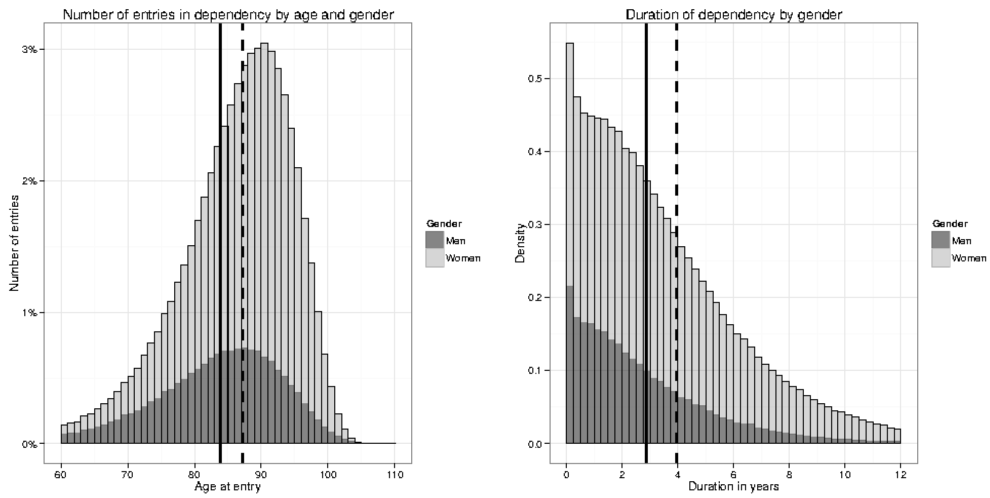


Figure 9: Left: distribution of entries in dependency by age as a percentage of total entries for a population of 1,000,000 at 60. Right: density for the distribution of survival time in dependency, computed on the same population. For both graphs, plain lines (resp. dashed lines) represent the mean value for men (resp. women).

Figure 9 (left) gives the distribution of the age of entry in dependency, which only depends on exogenous data used for incidence and autonomous mortality rates. According to those laws, for a population of individuals aged 60, men become dependent at 84 on average and women at 87. Figure 9 (right) gives the distribution of survival time in dependency, regardless of age of entry. The density is very high during the first year, due to frailty. Besides, women survive in average 4 years in dependency while men survive a little less than 3. Those results could be compared to those presented in (Debout, 2010), a study of the APA data performed over 300,000 trajectories over a 6 years period. Figure 9 gives the life expectancy in dependency and the average time spent in each state, as a function of the age of entry. Despite the probability of frailty being lower at higher ages of entry, the life expectancy decreases with age, for both men and women.

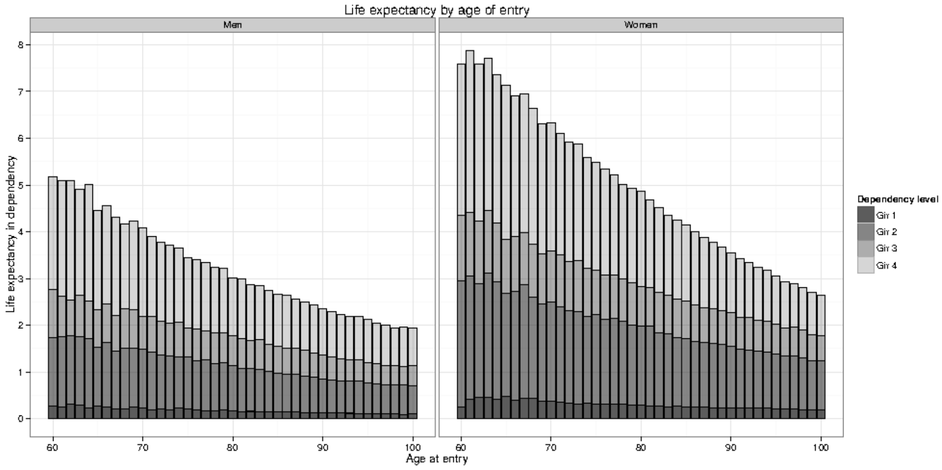


Figure 10: Life expectancy, and its breakdown by state of dependency, with respect to the age of entry in dependency, for men (left) and women (right), based on 1,000,000 simulations.

5. APPLICATION TO PRICING

5.1 Pricing methodology

5.1.1 An estimator for the premium

We consider a long-term care insurance product characterized, on one hand by a sequence of periodic benefit cash flows B, and on the other hand by a sequence of periodic premium cash flows P such that B and P have the same periodicity. We assume that conditions for the payment of the benefit (resp. the premium) has been set in the product description. For a fixed amount of benefit, we define the pricing of the product as finding the corresponding amount of premium so that the expectancy of the discounted cash flow of premium matches the expectancy of the discounted cash flows of benefit.

For a sequence of periodic cash flows $F = (F_i)_{i\in\mathbb{N}}$, and a fixed actuarial rate $\tau \geq 0$, we define the associated Net Present Value (NPV)

$$\mathrm{NPV}(F) = \sum_{i=0}^{\infty} \frac{F_i}{(1+\tau)^i}.$$

If we further assume that the amount of every premium cash flow is either null or equal to a fixed amount p^*, which covers the case of single premium and level premiums product, the problem becomes finding p^* such that

$$p^* = \frac{\mathrm{NPV}(B)}{\mathrm{NPV}(\mathbb{I}_P)},$$

where $\mathbb{I}_P = \frac{P}{p^*}$ is the sequence of premium unit cash flows.

Most insurance models rely on a discrete time scale model, for which a closed formula for the premium p^* can be calculated. In a multi-states continuous scale model however, multiple integrals appear in the equivalent formula, for which an analytical solution does not exist. Therefore we have to rely on another method for the pricing.

We decide to use a Monte Carlo method, which relies on the simulation of trajectories in order to find an estimate which converges toward the right amount of premium. We use the following methodology:

- We generate n trajectories using the algorithm provided in the previous section.
- For each trajectory $k \in \{1; \dots; n\}$, we determine the NPV of both the benefit cash flows NPV_k^B and the premium unit cash flows NPV_k^P.
- We use the following estimator for the amount of premium

$$\hat{p}_n = \frac{\frac{1}{n}\sum_{k=1}^{n} \mathrm{NPV}_k^B}{\frac{1}{n}\sum_{k=1}^{n} \mathrm{NPV}_k^P}.$$

According to the law of large numbers, this is a consistent estimator of p^*, i.e. $\hat{p}_n \underset{n\to+\infty}{\to} p^*$ almost surely.

5.1.2 Uncertainty on premium estimation

For a sample of n trajectories, let us denote by $\widehat{\mu_B^n}$ and $\widehat{\sigma_B^n}$ (resp. $\widehat{\mu_P^n}$ and $\widehat{\sigma_P^n}$) the empirical estimators of mean and variance of $\mathrm{NPV}(B)$ (resp. $\mathrm{NPV}(P)$) and $\widehat{\rho^n}$ the empirical estimator of the correlation between $\mathrm{NPV}(B)$ and $\mathrm{NPV}(P)$.

According to the central limit theorem, we have

$$\sqrt{n}\left[\begin{pmatrix}\widehat{\mu_B^n}\\ \widehat{\mu_P^n}\end{pmatrix}-\begin{pmatrix}\mu_B\\ \mu_P\end{pmatrix}\right]\overset{n\to\infty}{\to}\mathcal{N}(0,\Sigma)\quad \text{where}\Sigma=\begin{pmatrix}\sigma_B^2 & \rho\sigma_B\sigma_P\\ \rho\sigma_B\sigma_P & \sigma_P^2\end{pmatrix}.$$

Let us denote by g the function

$$\begin{array}{rccc} g: & \mathbb{R}\times\mathbb{R}^* & \to & \mathbb{R}\\ & (x,y) & \mapsto & \dfrac{x}{y}.\end{array}$$

We have $p_n = g(\widehat{\mu_B^n},\widehat{\mu_P^n})$ and, for $(x,y)\in\mathbb{R}^*\times\mathbb{R}$, $\vec{\nabla}g(x,y)=\left(\frac{1}{y},-\frac{x}{y^2}\right)$.

We have according to the Delta method

$$\sqrt{n}[\hat{p}_n-p^*]\overset{n\to\infty}{\sim}\mathcal{N}\left(0,\vec{\nabla}g(\mu_B,\mu_P)^t\Sigma\vec{\nabla}g(\mu_B,\mu_P)\right).$$

with

$$\begin{aligned}\vec{\nabla}g(\mu_B,\mu_P)^t\Sigma\vec{\nabla}g(\mu_B,\mu_P) &= \begin{pmatrix}\dfrac{1}{\mu_P} & -\dfrac{\mu_B}{\mu_P^2}\end{pmatrix}\begin{pmatrix}\sigma_B^2 & \rho\sigma_B\sigma_P\\ \rho\sigma_B\sigma_P & \sigma_P^2\end{pmatrix}\begin{pmatrix}\dfrac{1}{\mu_P}\\ -\dfrac{\mu_B}{\mu_P^2}.\end{pmatrix}\\ &= \frac{1}{\mu_P^2}\left(\sigma_B^2-2\rho\frac{\mu_B}{\mu_P}\sigma_B\sigma_P+\frac{\mu_B^2}{\mu_P^2}\sigma_P^2\right).\end{aligned}$$

Slutsky's theorem ensures that the former convergence still holds when we replace the different quantities by their empirical estimators and therefore, for $\alpha\in]0;1[$, we have the following asymptotic upper-bound for the distance between the premium and its estimator

$$|\hat{p}_n-p^*|\leq\sqrt{\widehat{\sigma_B^n}^2-2\widehat{\rho^n}\frac{\widehat{\mu_B^n}}{\widehat{\mu_P^n}}\widehat{\sigma_B^n}\widehat{\sigma_P^n}+\frac{\widehat{\mu_B^n}^2}{\widehat{\mu_P^n}^2}\widehat{\sigma_P^n}^2}\times\frac{\Phi^{-1}(1-\frac{\alpha}{2})}{\widehat{\mu_P^n}\sqrt{n}},$$

with an asymptotic level of confidence of $1-\alpha$, where Φ is the cumulative distribution function of the standard normal law.

5.2 Practical case

5.2.1 Product description

For this product, the claims are assessed using the AGGIR grid. Only states Gir 1 to Gir 4 are considered as dependency. A constant level premium is paid by the insured life, at the beginning of every month, as long as he is alive and autonomous. Should the insured life become dependent, the premium is no longer due, and benefit will be granted instead at the end of every month, while he is still alive. The amount of benefit depends on the state of dependency

- Gir 1: 1,300 ,

- Gir 2: 1,100 ,
- Gir 3: 800 ,
- Gir 4: no benefit.

An additional cash amount of 1650 € is also granted at the end of the first month of dependency, regardless of the dependency state. Besides, several additional features are added to the product. First of all, a deferral period of 3 months is fixed, so that no payment is made during the first three months spent in dependency, except for the 1650 €. Furthermore, an elimination period of 2 years is added to the product, with counter-insurance on the premium. It means that, should the insured life become dependent during the two years period after subscribing, the contract would be canceled and all premium paid would be refunded to the insured life. Finally, a technical interest rate of 2 % will be set.

5.2.2 *Results of pricing*

We determine the price of the previous product for several ages of subscription, based on 1,000,000 simulations, which gives us a relative uncertainty on the premium lower than 0.4 %. We compute a single price for both men and women, based on exogenous assumptions on the gender distribution in the initial portfolio. A summary of the results is provided in table 4.

Age of subscription	40 years	50 years	60 years	70 years	80 years
Monthly premium	17.42	25.14	38.36	63.18	119.12
% Confidence interval	± 0.07	± 0.09	± 0.14	± 0.22	± 0.40

Table 4: Premium for several ages of subscription.

5.2.3 *Reserves*

In long-term care insurance, we are mostly concerned about two categories of technical reserves, the reserve for premium and the reserve for claim.

On one hand, the incidence rate of dependency increases with age, and so does the associated risk. On the other hand, with level premiums, the amount of premium remains the same over the years. Therefore a reserve for premium should be constituted with incoming premiums to cope for this increase of the risk. The amount of reserve is defined as the difference between the net present value of the future benefit and the future payments, for insured lives who are not dependent yet.

Whenever a claim occurs, a reserve for claim should be constituted to account for the future payments of benefit associated with this claim. The amount of reserve corresponds to the best estimate of the net present value of the benefits. In classic long-term care insurance models, the amount of reserve only depends on the gender, age of entry in dependency and time spent in dependency. With our model, the current state of dependency and the time spent in this state also give additional information and therefore should be used to get a more accurate estimation of the required amount of reserve.

Figure 11 provides the projected average amount of reserve required for one insured live aged 60 at subscription, computed at the time of subscription.

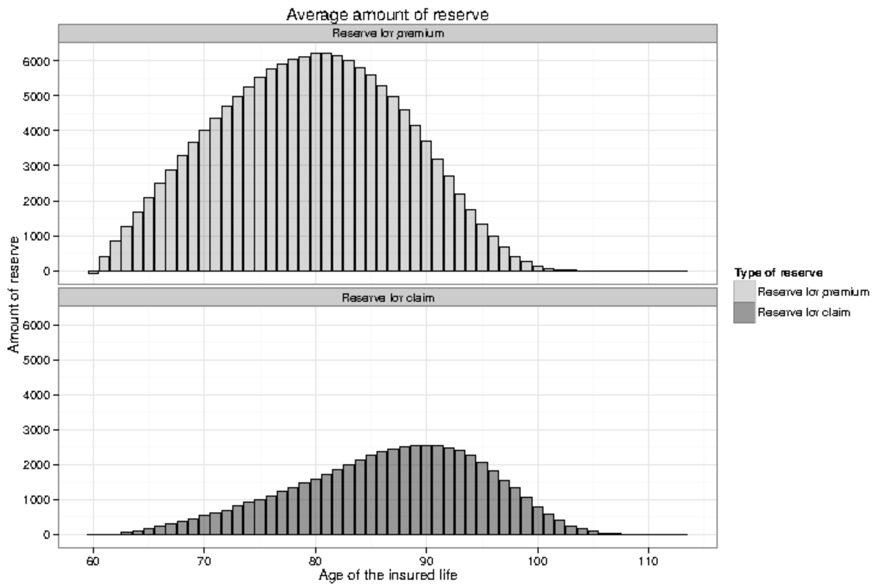


Figure 11: Projected amount of reserve for one insured live aged 60 at subscription, computed on a portfolio of 1,000,000 insured lives.

If the model was to be used for actual pricing, computation of actual reserves would need to be performed every year. A simulation method for already dependent people, similar to the one we introduced in the model section, should therefore be used. The only difference is that in this case, based on the history of the trajectory, we would first have to compute the probability of frailty, draw the frailty accordingly, and finally complete the trajectory.

5.2.4 Sensitivity to different risk factors

We study the impact of several factors of risk on the premium, such as incidence rate, mortality rate for autonomous people, hazard rates in dependency and interest rates. The impact of factors of risk such as incidence rate, mortality rate for autonomous people, hazard rates in dependency and interest rates are given in table 5, for a population of 1,000,000 insured lives aged 60 at subscription. It can be noted that the dependency risk and the longevity risks are positively correlated, as people surviving to higher ages means more premiums but also more people likely to become dependent, the second effect outweighing the first. Besides, long-term care insurance products have a very high sensitivity to the interest rate.

Shocks	Incidence rate	Autonomous mortality rate	Hazard rates in dependency	Interest rate
Value of shock	+ 10 %	- 10 %	- 10 %	- 50 bps
Resulting premium (38.36 with no shock)	40.70	39.36	41.33	41.20
Relative variation	+ 6.1 %	+ 2.6 %	+ 7.9 %	+ 7.4 %

Table 5: Sensitivity to different risk factors on the premium at 60.

6. DISCUSSION

In this paper, we presented the construction steps of a 4-state semi-Markov model for the dependency process, based on data from the French public aid, the APA: "Allocation Personnalisée d'Autonomie". Semi-Markov models have been widely described in the actuarial literature, but there are only few applications based on real long-term care insurance data, because the available data is very scarce. As a consequence, methods to deal with censored data, which is encountered in longitudinal studies, have rarely been described.

The model we developed accounts for the effect of covariates like gender and age of entry in dependency, through semi-proportional hazard rates. Heterogeneity caused by underlying pathologies, which are not observed in the data, is also taken into account through a static frailty. Estimation of parameters was performed using the maximum likelihood method, with the introduction of specific terms to deal with right censoring and missing dates of death in the data. We then provided an algorithm to generate trajectories as well as a Monte Carlo method for the pricing of long-term care insurance products. At last, we presented an application to a fictive product with a quick look at reserve and sensitivity to risk factors.

The data at our disposal provides information about 31,731 individuals, over an observation period of 3 years, with only 1 year for which death was observed. Nevertheless, the results we obtained from the model proved quite close to those presented in Debout (2010), a report based on a much larger sample of the APA data, gathering trajectories about 300,000 individuals over a 6 years period.

The underlying pathology is one of the main causes of heterogeneity in the trajectories. A study about incapacitating pathologies and their relative importance can be found in Monod-Zorzi et al. (2007). In our paper, those pathologies were not observed and we introduced a static Bernoulli frailty to account for their effect. In the future we plan on working on a portfolio which contains pathologies. This will allow us to get a better interpretation of our results, and see if it is necessary to use a more complex model for frailty with three or more levels.

Besides, we mentioned that, with a multi-state semi-Markov model, the amount of reserve for claim should be calculated while taking into account the current state of dependency and the time spent in this state. This could lead to a more accurate estimation of reserves? However, it requires us to be able to generate trajectories for people who are already dependent, and specific methods need to be developed.

At last, if we had access to more recent data from the APA, we would be able to test the adequacy of our model, especially for trajectories longer than 3 years which can only be partially observed in the current data set.

7. ACKNOWLEDGEMENTS

First of all, I would like to acknowledge the help of Catherine Matias, CNRS research director [1] and Vincent Lepez, chief pricing actuary at SCOR Global Life. Catherine and Vincent both oversee my research as a PhD candidate and they provided invaluable help through the whole writing process. I am also grateful to an anonymous

[1] CNRS: Centre National de la Recherche Scientifique, France's largest public organism for scientific research.

referee for a very thorough reading of an earlier version of the paper that led to many improvements. Finally, I would like to thank my colleagues from the long-term care insurance research center at SCOR, for sharing their knowledge of the risk with me.

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