MATURITY GUARANTEES EMBEDDED IN UNIT-LINKED CONTRACTS VALUATION & RISK MANAGEMENT*

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ABSTRACT

A key feature of maturity guarantees attached to unit-linked life insurance contracts is the uncertainty surrounding the benefit. With these products an insurance company is directly exposed to the financial market and to the need to explain to regulators, rating agencies and shareholders how they measure and plan to control this risk. One approach by direct insurance companies to these problems was to retrocede the risk to a reinsurer. Hence the risk is transferred and with it many questions regarding valuation and risk management. But the sharp and prolonged fall of global capital markets have occasioned many reinsurers to exit the market and moreover pushed insurance companies to reassess how they measure and manage the risk associated with maturity guarantees attached to unit-linked life insurance contracts. Here we present a method to evaluate and control the risk of such products under a risk neutral world and examine some practical issues concerning the hedging of such products.

KEYWORDS

Garantie Plancher, GMAB, GMDB, Fair Value, Mark-to-Market, Asset Liability Management, Dynamic Hedging, Capital & Reserves requirements.

1. INTRODUCTION

Most traditional actuarial Life products have deterministic benefits, whereas some contracts in addition include profits or bonuses allocated to the policy from the surplus earned by the insurance company. A new type of life insurance products links the payoff of the insurance benefits to various assets and not the general account. These assets could be a certain stock, a basket of stocks or simply shares of a mutual fund.

The latter type of products is the topic of this article, and we refer to these contracts as Unit-Linked (UL) products though other names are used depending on the country: Variable Annuities in the US, Segregated Funds in Canada, Unités de Compte in France.

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Such products seem to offer the insurance companies as well as the insurance customers advantages compared to traditional products. Customers may benefit from higher yields in financial markets and then again, the insurance industry may benefit from offering more competitive and flexible savings products.

However insurers are facing new and unfamiliar risks today as they race to design and distribute innovative UL products that include minimum benefits features. Not only competition but also certain regulators have imposed new standards in the UL industry. A Guaranteed Minimum Death Benefit (Garantie Plancher en cas de Décès) is now embedded in all recent UL contracts sold in Europe or North America. More recently living benefits (Garantie Plancher en cas de Vie) have appeared for the purpose of protecting policyholders’ wealth against poor capital markets performance if the insured survives a certain period. What is foreseen as the common life insurance product in the near future, is the UL endowment insurance, which is a combination of a pure endowment policy (minimum maturity benefit) and a term insurance policy (minimum death benefit) linked to a mutual fund.

Insurers now confront substantial exposure to capital market risks. Capital market risk in a UL arises from two main sources. First, the revenue of the product is achieved by charging a guarantee fee, assessed as a percentage charge against the market value of the contract (which is linked by definition to the underlying asset performance). Therefore, if the asset moves down, the insurer collects a fee applied to a reduced value. The nominal amount collected therefore fluctuates in relation to the capital markets levels. Incidentally, this source of risk concerns the whole savings insurance industry regardless of any minimum benefit embedded in the contracts.

The second main source of market risk obviously originates from policy minimum benefits contingent to either death or survival. At policyholder death or at the maturity date if he survived, if the market value of his investment is less than the guaranteed value, the insurer commits to making up the difference. Obviously the higher the guaranteed amount, the larger the risk for the insurance company. Together, as the market declines an insurer has exposure to increasing claims in addition to reduced revenues.

Applied to the worldwide UL market size and taking into account the recent crash of international equity markets, valuation and risk management of such embedded guarantees is a fundamental challenge to the actuarial profession. Unfortunately, the actuarial world and local regulators seem to have retreated into a strange silence. For the time being the most detailed and documented official actuarial guideline comes from the Canadian
Institute of Actuaries (CIA) which has devoted an industry-wide task force to guarantees associated to Canadian Segregated Funds. One point of this excellent work is that it has raised the awareness of the unique and significant risks associated to such guarantees. On a methodological point of view the CIA’s recommendation for valuation and reserving basically consists in: 1) estimating the probability distribution function of the present value of revenues and claims under a historical market return model 2) setting reserves as a percentile of this distribution. From our prospective this methodology has two major drawbacks. First, the results rely heavily on the market return model and its parameters and it thus gives rise to non robust and heterogeneous risk measures among the insurance industry. The second one is linked to Risk Management: indeed this method assumes a poor Asset Liability Management strategy that consists in running the risk naked and investing revenues and reserves at the risk free rate. To that extent the CIA mentions that, if the insurer has another ALM strategy, it could be included in the projections of futures cash flows: we are back to square one!

Another point is also mitigating the proposal of the CIA: the International Accounting Standards (IAS) and the Financial Accounting Standards from US GAAP states the concept of Fair Valuation of risks.

According to these accounting rules the value a financial market risk has to be the same whoever is carrying it. Such benefits could be easily analyzed as derivative products contingent to actuarial factors. Therefore we claim that the valuation of such financial risk embedded in an insurance contract has to be consistent with no-arbitrage conditions in the financial markets.

In this article we treat the case of a pure endowment benefit i.e Guaranteed Minimum Accumulation Benefit (GMAB) i.e a maturity guarantee on a single premium UL contract. We restrict our attention to this product because it embeds a high degree of financial risk compared to term insurance i.e death benefit this product and also because it is a truly innovative product for the savings insurance industry.

The major goal of this article is to provide insurance practitioners with a comprehensive valuation methodology and a risk management strategy to manage maturity guarantees embedded in a simple unit-linked contract.

The article is structured into two parts. The first one focuses on the valuation model using stochastic interest rates and provides with numerical examples. The second part describes the outcomes of dynamic hedging and proposes a consistent approach for reserves and capital concerns.
2. GMAB VALUATION

The following study focuses on “maturity guarantees” or “guaranteed minimum accumulation benefit” (US market denomination), that are added to classical Unit-linked policies. Boyle and Hardy (1997) examine different types of techniques to evaluate those guarantees. For us though a Unit-linked insurance contract with guarantee has a payoff that can effectively be represented by simple financial options, therefore option pricing theory will be applied for valuation purposes.

2.1 Contract description

We consider a single premium UL contract with:

- a single underlying financial vehicle
- a maturity $T$
- a guaranteed minimum surrender value at maturity.

2.1.1 Structure of the benefit

At inception a single premium, $S_0$, is invested in a mutual fund. During the lifetime of the contract the value of the investment evolves according to the market value of the mutual fund. If the policyholder withdraws his money before maturity or in case of death, he or his beneficiaries will get the market value of his contract from which a surrender charge is deducted. However in case of survival at the maturity date he benefits from the guaranteed surrender value i.e he will get the higher between the market value of the contract and the guaranteed amount. Let us define $S_t$, the market value of the investment in the mutual fund at time $t$. In general the minimum guaranteed amount, denoted $K_T$, is a deterministic function of the initial investment, but it can also be a path-dependent value of market value (for instance $x\%\text{Max}\{S_i\}_{i=1...N}$).

The benefit of the contract can be written as follow:

$$
\begin{cases}
S_T + (K_T - S_T) I_{[T_L < T]} I_{[T_A > T]} \\
(1-\varsigma)S_{T_L} I_{[T_L < T]} \\
S_{T_A} I_{[T_A < T]}
\end{cases}
$$

with $T_L$ representing the date when the policyholder leaves the contract, $\varsigma$ the surrender charge and $T_A$ representing the date of death of the policyholder.

As the insurance company holds in its balance sheet shares of the mutual fund valued $S_t$ i.e. the market value of the contract at each date, the valuation problem can be summarized by:

$$
(K_T - S_T) I_{[T_L < T]} I_{[T_A > T]}.
$$
2.1.2 Revenue structure

Instead of charging a single up-front premium as for traditional life insurance, these kind of guarantees are financed through installments i.e. regular fees proportional to the market value of the contract (as asset management fees).

Another part of the revenue comes from the surrender charges associated with early withdrawals. The level of those penalties essentially depends on the legislation and the product design. For the sake of simplicity and conservatism the model described in this article will not take into account this source of revenue for the insurance company.

2.2. The Model

The framework for modeling those guarantees is a contingent claim framework.

2.2.1 Capital Markets factors

Financial markets are assumed to be frictionless and complete. Under these conditions Harrison and Kreps (1979) have shown there is a unique probability measure $Q$, called risk neutral probability, under which the continuously discounted price of securities is a martingale.

- Mutual fund process

Let $S_t$ be the value of the market value of a unit of mutual fund at time $t$. Under $Q$, we assume that $S_t$ is determined by the following stochastic differential equation:

$$dS_t = (r_t - \varphi_t)S_t dt + \rho \sigma_S S_t dZ_t + \sqrt{1 - \rho^2} \sigma_S S_t dY_t$$  \hspace{1cm} Eq 1

$\sigma_S$ is the instantaneous standard deviation of the return on mutual fund and $\rho$ is the correlation coefficient between the mutual fund and the risk-free interest rate. $Z_t$ and $Y_t$ are independent standard Wiener processes.

- Interest rate process

We will use the classical one factor HJM model with a constant volatility explained by Heath, Jarrow and Morton (1992) that allows us to integrate the curve as the one that is observed in the market. Under the risk neutral probability we assume:

$$r_t = f(0,t) + \sigma_r^2 \frac{t^2}{2} - \sigma_r Y^1$$  \hspace{1cm} Eq 2

Where $f(0,t)$ is the instantaneous rate given by the market, and $\sigma_r$ is the instantaneous standard deviation of the nominal rates.

2.2.2 Actuarial factors

- Mortality
For convenience and simplicity we assume that the age-at-death random variable $T_A$ can be expressed in a continuous-time manner. Thus, for an individual aged $x$, the probability of death after time $t \geq 0$ i.e. after age $x + t$, can be written as:

$$P(T_A \geq t) = e^{-\int_0^t \lambda_s(x) ds},$$

where $\lambda_s(x)$ is called the hazard rate function. Without loss of generality the hazard rate function can be fitted to a mortality table.

Obviously we assume that $T_A$ is independent of $(S_t, r_t)$.

- Lapsation

Modeling withdrawing behavior is a tricky exercise. It is very tempting to try to model lapse rates jointly depending with interest rates and mutual fund performance. However such models are more complex to implement and understand and our focus is on models that have shown themselves able to manage GMAB risk in practice.

For conservatism sake we assume that the time-at-lapse random variable $T_L$ can be expressed the same way as mortality:

$$P(T_L \geq t) = e^{-\int_0^t \delta_s(x) ds}$$

and that $T_L$ is independent of $(S_t, r_t, T_A)$.

2.3 Valuation of the Asset-Liability system

We present in this part the valuation at the policy holder level. At a portfolio level, the valuation consists in summing the head by head values.

Let’s define $P(t,T)$ the price at time $t$ of a zero coupon default free bond with a nominal 1 maturing at time $T$ (with $T \geq t$).

$$P(t,T) = E_Q \left[ e^{-\int_t^T \eta(u) du} e^{-\int_t^T f(t,u) du} \right] = e^{-\int_t^T f(t,u) du}.$$ 

Let’s define the 3 following variables:

$R$: Present Value of the revenue side (Asset)

$L$: Present Value of the claim side (Liability)

$MTM_{Naked} = R - L$ is the net position, also called mark-to-market value of the GMAB (Naked refers to the fact that it is not related to the ALM strategy).
2.3.1 Revenue valuation

We assume that all the fees are calculated along the year (continuously) based on the market value of the investment, but we assume that this amount is cashed only at the end of each calendar year.

We will suppose here that we receive our revenue ($\alpha$ annual premium rate) based on the account value of each policyholder in a yearly basis. Let be $T$ the maturity of the guarantee and $N$ the number of corresponding years.

$$R(0) = E_Q \left[ \sum_{i=1}^{N} e^{-\int_{0}^{T_i} \delta_t dt} S_i \times 1_{[T_L>\tau]} \times 1_{[T_A>\tau]} \right] = \alpha S_0 \sum_{i=1}^{N} e^{-\int_{0}^{T_i} \delta_t dt} - \int_{0}^{T} (\delta(s) + \lambda(s))ds$$

\[ Eq \ 3 \]

2.3.2 Liability valuation

We assume here that the maturity of the guarantee is $T$ and the level of the guarantee is a fixed amount $K$ (typically $K = S_0$)

$$L(0) = E_Q \left[ \left( K - S_T \right)^+ \times 1_{[T_L>\tau]} \times 1_{[T_A>\tau]} \right]$$

\[ Eq \ 4 \]

Let us define a new probability measure, called $T$-neutral forward probability define as follow:

$$\frac{dQ_T}{dQ} = e^{-\int_{0}^{T} \delta_t ds}$$

\[ Eq \ 5 \]

So we can rewrite under $Q_T$ both equations (1) and (2):

$$r_t = f(0,t) + \sigma_{\tau}^2 t^2 - \sigma_{\tau}^2 (T \times t - \frac{t^2}{2}) - \sigma_{\tau} Z_t^T$$

\[ Eq \ 6 \]
\[ S_T = S_0 e^{\int_0^T \left( r_u - \frac{\sigma_x^2}{2} \right) du - \rho \sigma_x \sigma_r \left( \frac{1}{2} T \mu^r + \frac{1}{2} \sigma_r^2 \right)} + \sigma_r \left( \rho Z_t + \sqrt{1 - \rho^2} Y_t \right) \]  
\text{Eq 7}

At this point we have all the elements to evaluate the Liability component:

\[ L(0) = P(0,T)e^{-\int_0^T (\lambda(s) + \dot{\lambda}(s))ds} \]

\[ E_Q \left[ (K - S_T)^+ \right] \]

We know from the last relationship that \( \ln \left( \frac{S(T)}{S(0)} \right) \) is Normally distributed with the following parameters:

\[ m(T) = \int_0^T (\phi(0,u) - \varphi_u) du - \sigma_r^2 \frac{T^3}{6} - \frac{1}{2} \sigma_r^2 T^2 - \rho \sigma_r \sigma_r \frac{T^2}{2} \quad \text{(Mean)} \]

\[ \text{Var}(T) = \sigma_r^2 T + \rho \sigma_r \sigma_r T^2 + \sigma_r^2 \frac{T^3}{3} \quad \text{(Variance)} \]

So we can explicitly write the value of the payoff function:

\[ L(0) = e^{-\int_0^T (\lambda(s) + \dot{\lambda}(s))ds} K N(d_2) P(0,T) - S_0 \ N(d_1) e^{0} \]

\[ \text{Eq 8} \]

\[ d_2 = \frac{\ln \left( \frac{K}{S(0)} \right) - m(T)}{ \text{Var}(T)^{1/2} } \quad , \quad d_1 = d_2 - \text{Var}(T)^{1/2} \]

2.3.3 Global valuation

If we assume that \( MTM_{\text{Naked}}(0) \) represents the mark-to-market value of the asset and liability attached to a given policyholder at inception we can conclude that:

\[ MTM_{\text{Naked}}(0) = aS_0 \sum_{i=1}^{N} \left[ e^{-\int_0^T (\lambda(s) + \dot{\lambda}(s))ds} - \int_0^T e^{-\phi_x ds} \right] \]

\[ KN(d_2) P(0,T) - S_0 N(d_1) e^{0} \]

\[ \text{Eq 9} \]
For any given time $0 \leq t \leq T$ it is straightforward to expand this formula.

### Special case:

For an illustrative and practical sake, if we assume that:
- Fees are constant: $\varphi_t = \varphi$
- Lapse rates are constant: $\delta_t = \delta$
- Mortality follows a table $\{l_x\}$

then Eq 9 becomes:

$$MTM_{Naked}(0) = \alpha S_0 \sum_{i=1}^{N} \left[ \frac{l_{x+i} e^{-(\varphi+\delta)i}}{l_x} \right] - \frac{l_{x+T}}{l_x} e^{-\delta T} \left( K \{d_2\} P(0,T) - S_0 N(d_1) e^{-\varphi T} \right).$$

#### 2.3.4 Numerical example

**Context**

The mutual fund is supposed to be an indexed fund that tracks the CAC 40, assuming that all dividends of the stock are instantaneously reinvested, and all the fees are taken continuously from the fund. We assume that we can represent all the policyholders by a single synthetic person with the following characteristics:

**Actuarial assumptions**

- Age: 50 years old
- Sex: 50% male-50% female
- Mortality table: French certified table
- Lapse assumption: annual exponential rate 3%
- Net single Premium: 10 000€

**Insurance Policy terms**

- Maturity: 8 years
- Guarantee: 100% of the net single invested premium
- Fees: asset management + insurance loading + cost of guarantee = 3%/year

**Capital Market assumptions:**

- Interest rates structure: flat $r = 5\%$
- Mutual fund volatility: $\sigma_S = 23\%$

Solving $MTM_{Naked}(0)=0$ comes up with a fair price for $\alpha$ of 166bps i.e. charging 1.66% per annum for the guarantee allows the risk-taker to balance at inception the value of future premiums and the value of the guarantee.

Now, let us imagine that the equity market falls immediately by 2% and the entire curve is shifted down by 20bps. Then the market value of the transaction (asset – liability)
becomes: -102.63€ i.e. 1.03% of the invested premium. At this point the fair price is now 183bps.

3. RISK MANAGEMENT

Section 2 provides with a comprehensive valuation tool that allows the risk-taker not only to set the price of a UL guarantee but also to value the product at any point in time. To this extent it is an ALM tool. As shown in the numerical example, setting a fair price at inception doesn’t remove the risk. The whole A/L system has a high sensitivity to capital market environment and has to be managed through time. The purpose of this section is to provide with a clear quantification of the different source of risk and their contribution to the entire system. Then we will show the efficiency of a classical Δ-strategy and its contribution to the whole risk reduction.

3.1 Sources of Risk

Before presenting our risk management strategy, we want to underline clearly where the risk is coming from.

- **Financial Risk**
  - Mutual fund performance
    \[
    S_t = S_0 e^{\int_0^t \left( r_u - \varphi_u \sigma_S^2 \right) du - \rho \sigma_S \sigma_\rho \left( t T - t^2 \right) + \sigma_\rho \rho Z_t^T + \sqrt{1 - \rho^2} \psi_t^T}.
    \]
  - Interest rate performance
    \[
    r_t = f(0,t) + \frac{\sigma_r^2 t^2}{2} - \frac{\sigma_r^2 (T,t)}{2} - \sigma_r Z_t^T.
    \]

- **Actuarial Risk**
  - Death uncertainty
    \[
    i Q_X(t, T - t) = i Q_X(t - dt) + \sigma Q_X^H dt.
    \]
  - Lapsation factor uncertainty
    \[
    i L(t, T - t) = i L(t - dt) + \sigma p Z_t^T dt.
    \]

Of course with our simple models for Actuarial Risk it is theoretically possible to obtain negative lapse and mortality rates. But we have found this to be the case in practice when working with some databases.

3.2 Nature of the Risks

GMAB is a portfolio of simple options whose multipliers are determined by actuarial best estimates of future lapse and mortality. As such a GMAB position will possess all the “greeks” or instantaneous risk measures of a simple option will. These measures like “rho” and “delta” are valid for only an instant in time and for infinitesimally small moves. Useful as these figures are they can not provide much insight into some basic and simple questions.
What is the relative sensitivity of the GMAB position to interest rates and equities moves in the marketplace? To answer this question one can construct a two-way table of one-month standard deviations moves in the underlying asset and the 8-year Swap rate and then re-evaluate our GMAB under these new conditions holding everything else constant.

To explore the nature of the risk of our simple GMAB we will examine the discounted Profit & Loss distribution, its sensitivity in a two way table and its Greeks.

- **P&L analysis**

  We can try to capture the risk by Monte Carlo simulated all the risky parameters, and look at the distribution of the discounted P&L.

  At this step we only introduce, the Underlying asset source of risk, assuming an initial deposit of 100€.

  ![P&L distribution](image)

  It appears that the risk profile is asymmetric and that large potential losses can occur.

- **Marginal contributions to risk**

  What is the contribution of each factor to the overall risk profile?

  We define the marginal contribution to risk by its contribution to the global variance:

  \[ c_{source(i)} = \frac{\text{Var}(P&L_i)}{\text{Var}(P&L_{global})} \]

  with \( i \) being a source of risk among the four previously described sources of risk.

  To get the overall picture, we use accurate stochastic parameters coming from the capital market place and actuarial studies.
This simple analysis underscores the financial risks associated with a GMAB. Unlike traditional Life Insurance products GMAB guarantees are mainly driven by systematic risks.

- Two way table

This table presents the value of $\text{MTM}_{\text{Naked}}(0)$ as a percentage of notional with a 8-year swap rate moving from 4.8% to 5.2% and an account value moving from 95 to 105.

<table>
<thead>
<tr>
<th>AV\IR</th>
<th>4.8%</th>
<th>5.0%</th>
<th>5.2%</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>-2.0%</td>
<td>-1.5%</td>
<td>-1.0%</td>
</tr>
<tr>
<td>100</td>
<td>-0.5%</td>
<td>0.0%</td>
<td>0.4%</td>
</tr>
<tr>
<td>105</td>
<td>0.8%</td>
<td>1.2%</td>
<td>1.6%</td>
</tr>
</tbody>
</table>

This table shows how volatile the value of a GMAB is and how complex the risk profile is.

- Greeks

Greeks are the standard quantitative measure for such complex risk profiles. We use the assumption of the numerical application to come up with quantitative values of the Greeks.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta</td>
<td>26.93%</td>
</tr>
<tr>
<td>Rho</td>
<td>215.51</td>
</tr>
<tr>
<td>Vega</td>
<td>-57</td>
</tr>
<tr>
<td>Gamma</td>
<td>0</td>
</tr>
</tbody>
</table>

The Delta and the Rho quantify the systematic risks of a standard GMAB at inception.

### 3.3 Hedging a GMAB

One way an insurance company can hedge the risk of a GMAB is to buy put options with the same maturity as the GMAB. One problem with this approach is figuring out exactly how many puts to buy in practice because the correct amount depends on policyholder behavior and on terminal fund performance. A second problem with this approach is realizing it fails immunize the revenue stream of a GMAB—the very thing that
will finance the upfront cost of buying puts in the first place. Another practical problem with this risk management technique is the volatility it introduces in the financial results under mark-to-market accounting.

We advocate, based on our experience, using a dynamic hedging program to minimize the volatility of a GMAB’s profit. This approach is consistent with best outcome for hedging as best hedge can be put in place when new information arrives regarding policyholders, fund performance and capital market changes. For us a practical hedging strategy would consist of removing systematic equity risk and controlling interest rate risk using futures. It is our view a real time risk and trading system should be in place in order to properly manage these risks. We also suggest that a hedged GMAB be a GMAB with an appropriate surrender charge schedule as this can mitigate the hedging problem created by “rational” lapse behavior.

In order to assess how this dynamic hedging strategy contributes to minimize the volatility of the P&L, we simulated the P&L including a “naive” Delta neutral strategy with a weekly rebalancing.

The following graph shows the distributions of both naked and hedged P & Ls under Black & Scholes universe.

![Graph showing the distribution of both naked and hedged P&L under Black & Scholes universe.](image-url)
In this simple test it is clear that the volatility and shape of the terminal distribution is dramatically altered. Therefore it becomes possible for an Insurance company to set a price such as the probability of ruin is minimized.

3.4 Reserves & Capital methodology

3.4.1 Reserves

Most of the standard methods for reserves valuation are moving to the banking standard or the mark-to-market standard. In case of hedging we advocate the reserve being equal to the overall net position which includes the mark-to-market value of the naked GMAB and the hedge portfolio

\[ \text{Reserve}(t) = \text{Min}(0, \text{MTM}_{\text{naked}}(t) + \text{HedgingPortfolio}(t)) \]

Note how with our simple example this reserve figure can be quickly arrived at in practice on a large number of policyholders as no simulation work is required.

3.4.2 Capital

How should capital be set for Hedged GMAB business? Whatever the methodology it should reward companies that are hedging effectively. We feel a simple measure and a simple rule are the best. Our simple measure of hedging effectiveness is the r-squared based on a regression of the cumulative weekly changes in the hedge portfolio on changes in the GMAB or naked portfolio. We then considered hedging to be efficient if our hedging effectiveness measure was greater than .65. In other words the hedge portfolio could explain at least 65% of the variability in the weekly changes in the GMAB’s value. We suggest on a quarterly basis hedging effectiveness be measured and capital be reallocated.

How do we assess Capital requirement?

At this point all we have said is that capital for GMAB should be related to hedging and only if hedging is actually efficient in practice. We still need to discuss what we feel is an appropriate technique to assess capital on unhedged GMAB business. All the most common techniques such as Value at Risk are suffering when the normality disappears, and in our case if you put uncertainty on mortality or lapsation, we loose this property. To capture all the extreme events, we think that the Conditional Tail Expectation (CTE) will be appropriate. The concept of CTE to determine capital is quite intuitive, the capital required is precisely the expected shortfall when a shortfall occurs (Artzner et al. 1999). However using a tail measure is always a challenge as far as it requires MC simulation techniques. To capture the accuracy of a CTE measure a confidence interval is necessary.
Application to our problematic

The Office of the Superintendent of Financial Institutions in Canada (OFSI) recently imposed strict rules concerning capital allocation for product like ours. The insurer must allocate capital defined by the CTE at 95%, if no hedging is put in place. In case of hedging strategies, the insurer can reduce the OFSI capital requirement up to a maximum of 50% of this amount.

Why 50%? Why would people hedge knowing they will only get 50% credit for?
We advocate a conservative and less capricious approach to capital requirement.

OSFI: \( C = CTE_{95\%} (\text{Naked}) - \frac{1}{2}(CTE_{95\%} (\text{Naked}) - CTE_{95\%} (\text{Hedged})) - \text{Re serves} \)

Proposed Capital Requirement: \( C = (1 - R^2) \times CTE_{95\%} (\text{naked}) \)

with being the realized \( R^2 \) defining Hedging Efficiency.

Remarks

If the insurance or reinsurance company is not hedging the product the \( R^2 \) will be equal to zero, so the Capital requirement will be the same as the OSFI one.

At inception the capital deployment will be the same, if you’re hedging or not. Why is it a good thing? The model can lead to a perfect result but the reality can be totally different, especially for people that try to beta hedge actively managed funds. Our conservative method is designed to protect the policyholder.

The capital requirement is directly linked to the Hedging effectiveness in practice and not to theoretical projections of hedging.

4. CONCLUSION

• The main goal of this document was to provide an uniform valuation technique for the GMAB product, that is consistent with capital markets risk management. Most of the time people only price the Equity risk but forget the impact of interest rate deviation; using HJM framework allow people to incorporate this element in their pricing target.

• Our recommendation for risk management concern is simple : Life insurers should structure GMAB in such ways that the lapsation risk is paid by the policyholder using surrender charges and that the financial risks are transferable to the capital market place through dynamic hedging strategy.

• The accounting rules for those products should reward risk management strategies, that are different from the classical techniques of Life insurers
promoting the fact that new sales offset the effect of a bad block of business. This is key to encourage Life insurer to adopt the right attitude. We describe in this paper valuation techniques for reserve and capital that can be used by any Life insurers using or not risk management strategies.

- GMABs are a major innovation in the field of guaranteed products (GPs). Like all guaranteed products, they offer a protection or a guarantee to investors on their capital and an upside to the performance of a risky assets. GMABs differ from traditional structured GPs that are available in Europe from the investor’s point of view as they offer:
  - Transparency: the underlying security for a GMAB is an index fund compared to a portfolio of zero coupon bonds and options in structured GP. With a GMAB the guarantee and its associated volatility is effectively stripped out from the investor’s security.
  - Flexibility: with a GMAB an investor can withdraw his money at any time with a visible price for a known surrender charge versus a structured GP product where a “special” price must be made for the client.

- We can extend our methodology and techniques to the Guaranteed Minimum Death Benefit (GMDB) sold over the world aggressively the last 10 years. The only difference between those two products is the trigger of payment. That is why you can extend the « mark-to-market » methodology and its associated risk management strategies to the GMDB products.

- After the 2002 bearish Equity market, most of the Life insurer have to find risk management strategies to avoid future volatility in their balance sheet as well as to eliminate all the downside equity risk.
REFERENCES


