TAKING INTO ACCOUNT THE PANDEMIC RISK IN A PARTIAL INTERNAL MODEL

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RESUME :

Depuis la médiatisation à outrance du virus de la grippe A (H1N1) en 2009, une prise de conscience de l’importance du risque de pandémie s’est opérée, notamment chez les assureurs. En effet, la survenance d’une pandémie sévère engendrerait une augmentation significative du nombre de décès, pouvant aller jusqu’à mettre en péril la solvabilité des compagnies d’assurance. De ce fait, il est fondamental pour un assureur de mieux appréhender ce risque. Ceci est tout à fait conforme aux exigences de la nouvelle Directive européenne Solvabilité 2, qui incite les assureurs à maîtriser l’ensemble des risques auxquels ils font face, et à détenir un capital suffisant pour ces risques. Ce capital peut être calculé par une formule standard, un modèle interne ou un modèle interne partiel. L’objet de cet article est de proposer une manière d’intégrer le risque de pandémie de grippe dans un modèle interne partiel, pour le risque décès en Prévoyance. Pour ce faire, le risque de mortalité est décomposé en deux parties : une tendance et un choc pandémique. Le modèle permet de prendre en compte la nature des données disponibles et du niveau de segmentation choisi par la compagnie d’assurance (données individuelles ou données agrégées).

Mots-clés: Risque de pandémie, Modèles épidémologiques, Modèle Interne, Solvabilité II, Modèles de mortalité, Tables de mortalité prospectives, Prévoyance

ABSTRACT:

Since the influenza A (H1N1) virus has made headlines of newspapers in 2009, awareness of the importance of pandemic risk has risen, especially among insurers. In fact, the occurrence of a severe pandemic could cause a significant increase in mortality, and consequently might cause the bankruptcy of insurance companies. Thus, it is essential for insurers to better understand pandemic risk. This is entirely consistent with the

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requirements of the new European Directive Solvency 2, which encourages insurers to control all the risks they face and to hold sufficient capital for these risks. This capital can be calculated by a standard formula, an internal model or a partial internal model. The purpose of this paper is to propose a way of integrating the risk of pandemic influenza in a partial internal model for the death guarantee and protection business. To do so, mortality risk is split into a trend and a pandemic shock event. The model takes into account the nature of available data and of the level of segmentation chosen by the insurance company (individual or aggregate data).

**Keywords:** Pandemic Risk, Epidemiological Models, Internal Model, Solvency II, Mortality models, Prospective mortality tables, Protection Business

1. **INTRODUCTION**

In 2009, pandemic risk has been high on the agenda because of influenza A (H1N1) virus. The spread of this virus has been strongly popularized through the media, and this has highlighted the importance of this risk, not only for human kind but also for insurance companies. In fact, according to the World Health Organisation, a disease epidemic occurs when there are more cases of that disease than normal, and a pandemic is a worldwide epidemic of a disease. Pandemics belong to extreme risk events: hopefully, it rarely happens, but when it happens, consequences are enormous.

Many viruses can cause a pandemic (viruses already known as SARS, Ebola, AIDS, influenza, or viruses that are currently unknown), and they can be transmitted by various modes of spreading (for example by air, by blood or by mosquitoes). Among all those viruses, the influenza virus is considered by the World Health Organisation as the most important health threat, as stated in Société Française de Statistique (2008). In fact, pandemic influenzas have been periodically observed in the past, for example three times in the 20th century, and it is expected to happen again, whether it be the influenza A (H1N1) virus or not. Such events remain very hard to foresee, because observed data from the past is probably not reliable and exhaustive, because influenza virus has the property to easily mutate, which leads to a high volatility of the parameters of spreading, and because of healthcare changes. Thus, building a robust model is difficult.

From an insurer’s point of view, if a pandemic occurs, claims of his portfolio will on the one hand significantly increase (for death, temporary or permanent disability, or even

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involuntary unemployment), except for profits on annuities. On the other hand, assets will be negatively impacted. Thus, in the worst scenarios, this can lead to a possible bankruptcy. Therefore, it is of paramount importance for an insurance company to better quantify this risk. This is totally consistent with the new Solvency II Directive of the European Parliament (2009), which incites insurance companies to take into account all the risks they face, and to hold sufficient capital with respect to these risks. To do so, insurance companies have three possibilities: they can use a standard formula, which is prescribed by the regulator, build a full internal model, which completely corresponds to the business of the company, or build a partial internal model, which is a mix of those two first possibilities. Actually, insurance companies are strongly encouraged to build an internal model.

The aim of this article is to focus on mortality risk, and to describe how this risk can be modelled in order to integrate pandemic risk in a partial internal model. To do this, it is possible to describe mortality risk as a trend, on which catastrophic shocks events are added. As the level of information on insured portfolio can be full (a portfolio with individual data) or partial (a portfolio with aggregated data), it is important to take into account the nature of available data in such a forecast of mortality. In the framework of this study, we focus on Protection business (Creditor Insurance and Individual Protection), and obviously on death risk. The methodology described in this article can be applied to any protection portfolio in aggregate data, to model death risk.

After a few reminders about the different methods to calculate capital requirements in Solvency II in section 2, we will explain the split of mortality into a trend and pandemic shocks in section 3, and for each part of this split, we will explore different models in function of available data, respectively in sections 4 and 5. Lastly, an application of this partial internal model will be done in section 6.

2. THE CALCULATION OF CAPITAL REQUIREMENTS IN SOLVENCY II

First of all, Solvency II Directive (2009) introduces new solvency requirements, which are more risk-sensitive and sophisticated than in the past. These new solvency capital calculations should be aligned to the specific risk profile of the undertaking. In a word, in Solvency II, the emphasis is on risk management.

More precisely, the capital requirement is determined in a two levels approach. The first level is the Minimum Capital Requirement (MCR), which is the capital representing
the threshold that triggers ultimate supervisory measures in the event that it is breached. The second level is the Solvency Capital Requirement (SCR), which corresponds to the value-at-risk of the net assets of the insurance company subject to a confidence level of 99.5% over a one-year period.

The SCR can be calculated using either the standard formula, an internal model or a partial internal model. Those three possibilities are described in the following paragraphs.

2.1 Calculation of SCR with the Standard Formula

The standard formula is based on a risk cartography, which includes risks modules (non life underwriting, market, health underwriting, default, life underwriting, and operational risks), and risks sub-modules (such as mortality, longevity, catastrophe, revision, lapse, expense, and disability risks for life underwriting risk). Capital charges are determined using a bottom-up approach. First of all, the capital requirements are evaluated for each risk sub-module (via stress tests or factor-based capital charges). Secondly, the capital requirements of sub-modules are aggregated using a prescribed correlation matrix to obtain the capital requirement of risk modules. Thirdly, another aggregation using a prescribed correlation matrix is done between the capital requirements of risk modules to give the Basic SCR (BSCR). Lastly, a separate loading for operational risk is added to obtain the company SCR.

Solvency II Risk Cartography is represented in the following figure.

![Figure 1 - QIS 4 Risk Cartography, source: QIS 4 Technical Specifications](image-url)
The standard formula is easier to implement and treats risks consistently across companies. However it does not reflect any characteristics specific to an insurance company.

2.2 Calculation of SCR with an internal model

The goal of an internal model is to create a stochastic model with structures and relationships that best depict the company’s business. The risks incorporated into an internal model are likely to be the same or very similar to those included in the standard model. The calculation of the SCR using an internal model is based on the distribution of equities in a year, as described in Devineau, Loisel (2009). To calculate the SCR, two steps are needed:

- To project assets and liabilities under the historical probability in order to evaluate the quantile 99.5% of the Net Asset Value at time t=1;
- To discount at time t=0 the value of the quantile 99.5% in order to calculate the amount of capital, which invested at risk-free rate will enable the insurance company to avoid bankruptcy at time t=1 in 99.5% of cases.

Internal models enable the insurance company to quantify risk and to determine the capital requirement on the basis of the company’s specific risk profile. Nevertheless, they require expertise and resources for model building, calibration, validation, interpretation and communication. Last, they require supervisory approval.

2.3 Calculation of SCR with a partial internal model

The partial model option allows companies to replace some components of the standard formula SCR with results from an internal model. In other words, a partial internal model is used to substitute parts of the standard formula for the computation of the SCR. The SCR derived from the partial model and the SCR computed by the standard formula are correlated using either the standard model or company aggregation methodology. More precisely, according to CEIOPS (2007), the aggregation method defined by the standard formula is applied to any aggregation step that contains non-modelled parts. Aggregation steps that do not include standard formula parts have the same flexibility as full internal models.

Using a partial internal model has some advantages and drawbacks.

- On the one hand, implementing a partial internal model enables the company to better reflect the undertaking’s risk profile than the SCR
standard formula. It also encourages innovation and specialization to certain business areas. Last, but not least, it eases transition from the standard formula to full internal models.

- On the other hand, a partial internal model has some drawbacks. In order to avoid cherry picking, a company that develops a partial internal model must explain the reasons why some components of the SCR standard formula are replaced by internal SCR, and why the other risks are not included in the partial model. Furthermore, a partial internal model does not take part of the overall power of full internal models.

Currently, partial internal models are widely used by companies. For example, ACP has stated in the reporting of the QIS 4¹ results that over 40% of insurance companies surveyed have developed a partial internal model (knowing that 92 insurance companies were interviewed). Moreover, the three modules most modelled are non life underwriting risk (90%), market risk (80%) and life underwriting risk (75%).

In the future, partial internal models are intended to gradually disappear. In fact, insurance companies are strongly encouraged to develop a full internal model. Therefore, a partial internal model is considered as a transitional phase between standard formula and a full internal model. If the company wants to continue using its partial internal model, it will have to provide additional justification (justification for an improvement over the standard formula, justification that the non-modeled part is inferior to 20% of the total SCR). To put it in a nutshell, a partial internal model is a compulsory step between standard formula and full internal model.

2.4 The scope of our study: The sub-modules Life_{mort} and Life_{cat}

Among the risks described in the Solvency II cartography, we will focus on Life underwriting risk module. This corresponds to the value of the uncertainty of the results to come, relating to the sinistrality of in force contracts. More precisely, the aim of this study is to model mortality risk as a trend, on which catastrophe shocks events are added. Thus, a partial internal model will be described in this paper: the submodules Life_{mort} and Life_{Cat} will be replaced by an internal model. The following figure defines the perimeter of internal model.

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¹ Source: Blog Actu d’Actuaires, “Modèles internes partiels : résultats du QIS4”, http://actudactuaires.typepad.com/laboratoire/analyse_et mesure_des_risques/page/2/, website accessed on September, the 17th of 2010
3. SPLIT OF MORTALITY RISK INTO A TREND AND PANDEMIC SHOCK EVENT

As explained in Juillard et al. (2008), an internal model should integrate two components in order to consider all the volatility of mortality:

- The uncertainty of the trend of mortality (also called systematic risk of mortality): it encompasses risk tendency and the risk of oscillations around these tendencies;
- Extreme shocks of mortality, such as pandemics.

The following figures represent death rates of 40 year old men in Italy. The first figure covers the period 1885 – 2006, the second one focuses on the period 1965 – 2006, and the third one focuses on the period 1912 – 1923. The first figure illustrates the death rates tendency. The two peaks correspond to the First World War combined with the Spanish flu, and the Second World War. The second figure illustrates oscillations around these tendencies. The third figure focuses on the mortality shock caused by the Spanish flu in 1918, combined with the First World War.
Modelling the components “trend of mortality” and “pandemic shock” at the same time is difficult, because existing models that take into account systematic death risk cannot
include extreme shocks. Therefore, splitting both components of mortality risk makes sense.

On the one hand, the “trend of mortality” component, noted $Mortality(t)_{\text{Trend}}$, can be modelled differently depending on available data of the insurance company, and of the segmentation level chosen. In fact, the finest models are based on individual data: the insurance company has large reliable archives of data at the insured people level, and one “model point” of the model corresponds to one person. Nevertheless, most of insurance companies do not have this level of information on whole or part of their portfolios. Available data can be incomplete or aggregate. For example, instead of having individual’s age at the beginning of the contract for each insured person in the portfolio, the insurance company only has the mean, minimum and maximum ages at the beginning of the contracts for the entire portfolio. Moreover, if the insurance company has individual data, computation time corresponding to such a detailed segmentation level could be too long. Further, Planchet and Leroy (2009) consider that a detailed segmentation level could cause a model risk, a risk in the calibration of the model, and operational risk.

Therefore, two situations can be taken into account:

- If the company has individual data, a prospective stochastic mortality table can be implemented (mortality risk is monitored by the probability to die $q_x$);
- If the company has aggregated or partial data, a stochastic loss ratio model can be implemented (mortality risk is monitored by actuarial loss ratio $S/P$, which represents the claims charge divided by the risk premium of the portfolio).

On the other hand, the “pandemic shock” component, noted $Mortality(t)_{\text{Pandemic Shock}}$, can be included by a frequency x cost approach. Frequency is usually modelled by a Bernouilli distribution with a parameter $p$. This random variable represents an indicator function, which takes value 1 when a pandemic occurs (it corresponds to a “success” with the probability $p$), and value 0 when there is no pandemic (it corresponds to a “failure” with probability $1 - p$). Cost can be modelled by an Exponential distribution with a mean $\varepsilon$, as explained in Juillard et al. (2008). In fact, $\varepsilon$ represents the average cost of a pandemic event (in term of mortality rate) and can be calculated by an epidemiological model. The calibration of this cost is of paramount importance, and we will explore it in detail thereafter. To sum up, the “pandemic shock”
component is given by: \( \text{Mortality}(t)_{\text{Pandémie shock}} = B_t(p) \cdot \exp_t(\varepsilon) \).

The combination of the trend of mortality and the pandemic shock event can be additive or multiplicative. If it is additive, mortality table is uniformly translated upward. If it is multiplicative, as the probability of dying increases with age, adding a pandemic shock event would have more impact for older than for the young. The analysis of the three pandemics of the 20th century (and particularly the Spanish Flu in 1918) has shown that young adults (25 to 35 years old) could be more affected than the elderly. Therefore, the impact on mortality can be considered as additive.

As the “trend of mortality” and “pandemic shock” are supposed to be independent, the full model is simply the sum of those two components: 
\( \text{Mortality}(t) = \text{Mortality}(t)_{\text{Trend}} + \text{Mortality}(t)_{\text{Pandemic Shock}} \).

The following figure sums up the process of the model:

![Figure 6 - Process of the model](image-url)

On the one hand, the trend of mortality is modelled by a stochastic mortality table or by a stochastic loss ratio projection, depending on the nature of data. On the other hand, the pandemic shock event is modelled by a Frequency x Cost approach.

4. MODELLING OF THE “TREND OF MORTALITY” COMPONENT

In this section, we will describe the first step of the process, which is the assessment of the trend of mortality. Two methodologies can be implemented, depending on the nature of available data. Concerning individual data, prospective mortality tables are usually used to reflect the trend of future mortality and oscillations around the trend. This type of models has been the subject of many papers and the reader can refer to Planchet et Thérond (2006), Cairns et al. (2007), or Soutiras (2009) for more details. In this paper, we will focus on the case of aggregate data, and we will build a stochastic loss ratio model for Protection business (Creditor Insurance and Individual Protection).

This stochastic loss ratio model has been the subject of several research papers,
whether it is Corlosquet (2004), Aguir (2006), Batiu (2007), Felix (2009), Felix et Planchet (2009). It includes three stages: the deterministic evolution of the loss ratio, stochastic simulations of the loss ratio at \( t=1 \) and the dynamic evolution of loss ratios given the stochastic simulations at \( t=1 \). More precisely:

- The deterministic evolution of the loss ratio is modelled on the projection horizon, in function of the nature of the product (Individual Protection or Creditor Insurance);
- Stochastic simulations of the loss ratio at time \( t=1 \) are done by using a Log Normal distribution;
- The loss ratio is then modelled in a dynamic way: given the stochastic simulation at time \( t=1 \), the loss ratio progressively returns towards the deterministic evolution described at stage one.

Those three stages are described in the following paragraphs. The projection horizon has been chosen in function of the maximum duration of contracts, and it is here set at 30 years.

4.1 Deterministic evolution of the loss ratio to the projection horizon

The first stage consists of modelling the deterministic evolution of the loss ratio to the projection horizon in function of the nature of the product. The business covered in the framework of this study includes Individual Protection (IP) and Creditor Insurance (CI).

On the one hand, Individual Protection products aim at protecting family against unpleasant events while handling the financial consequences of a death, a disability or an accident (in this study, only death guarantee is included). Individual Protection (IP) products are not linked to a credit. The capital under risk is the contractual capital given to the beneficiary in the event of a loss.

Aguir (2006) has shown that an increasing linear model can correctly represent the projection of the loss ratio for IP products. Intuitively, the older the insured portfolio is, the higher its loss ratio becomes. This study has lead to the following deterministic loss ratio projection:

\[
\forall t \in [1; 10], \quad E \left[ \frac{S}{P}(t) \right] = (1 + \mu_\lambda) \cdot E \left[ \frac{S}{P}(t-1) \right]
\]

\[
\forall t \in [11; 30], \quad E \left[ \frac{S}{P}(t) \right] = E \left[ \frac{S}{P}(t-1) \right]
\]
Where $\mu$ represents the linear growth rate of the loss ratio over time. This rate depends on the average age of the insured, noted $\lambda$, and also on medical selection.

In fact, loss ratio increases linearly during the first ten years, and then it stabilizes over the remainder of the projection. The underlying assumption is that, on a time horizon so distant, if the loss ratio deteriorated significantly, actuaries would have taken actions to restore a constant loss ratio (for example by reviewing the technical bases).

The following graph illustrates the deterministic projection of the loss ratio for an IP portfolio:

![Figure 7 - Deterministic projection of the loss ratio for an IP portfolio](image)

On the other hand, in a Creditor Insurance (CI) product, the three following parts are involved: the insurer, the credit institution and the insured people. This kind of product enables the credit institution to guard against a failure of the insured persons who have a loan, and it enables the insured people to guard against their own failure. If an insured person dies, the insurer is obligated to pay the balance of the loan.

Batiu (2007) has shown that the projection of the loss ratio of CI products has the shape of a bell. In fact, the loss ratio is affected over time by two opposite effects:

- As for IP products, mortality increases over time because the age of insured people increases;
- Unlike IP products where the capital at risk is constant, the capital at risk of CI products decreases (it is the outstanding capital of the underlying loan, and it is amortized over time).

Batiu (2007) represents the projection bell shaped through four line segments: the first line segment is constant, the second one increases, the third one is constant, and the last one decreases so that the loss ratio equals to zero at the maximum duration of contracts.

Therefore, the deterministic projection of loss ratio is as following:
\[
E \left[ \frac{S}{P}(t) \right] = \left\{ \begin{array}{l}
\forall t \in [1; t_1] \left[ \frac{S}{P}(0) \right] \\
\forall t \in [t_1 ; t_2] \left[ (1 + \alpha) \cdot E \left[ \frac{S}{P}(t-1) \right] \right] \\
\forall t \in [t_2 ; t_3] \left[ E \left[ \frac{S}{P}(t-1) \right] \right] \\
\forall t \in [t_3 ; D_{\text{max}}] \left[ \frac{D_{\text{max}} - t + 1}{D_{\text{max}} - t + 2} \cdot E \left[ \frac{S}{P}(t-1) \right] \right]
\end{array} \right.
\]

Where \( D_{\text{max}} \) is the maximum duration of contracts, \( t_1, t_2, t_3 \) are years when intersecting line segments, and \( \alpha \) is the slope of the first line segment. These parameters are calibrated according to the growth rate of the portfolio (the number of contracts is increasing, decreasing or flat), the average duration of contracts and the average age of the insured people.

Therefore, the deterministic projection of loss ratio for CI products is the following:

\[ S/P \]

\[ S/P(0) \]

\[ \text{Slope } \alpha \]

\[ 0 \]

\[ t_1 \]

\[ t_2 \]

\[ t_3 \]

\[ D_{\text{max}} \]

\[ t \]

\[ \text{Figure 8 - Deterministic projection of the loss ratio for a CI portfolio} \]

Thus, the deterministic projection of the loss ratio is now modelled in function of the nature of the product (IP or CI).

4.2 Stochastic simulations of the loss ratio at time t=1

Underwriting risk for Protection products is caused by the underlying volatility of the mortality of contracts. It is therefore necessary to build a stochastic mortality model to take into account this volatility. Felix (2009), and Felix and Planchet (2009) describe such a model in detail. At time \( t=1 \), \( N \) values of loss ratio are drawn according to a given distribution, which is calibrated according to available information of the portfolio. Sütterlin (2004) has shown that loss ratio for Protection business can be represented by a LogNormal distribution, with parameters \( \mu \) and \( \sigma \).
Those parameters $\mu$ and $\sigma$ are given by the following formulas:

$$\mu = \ln \left[ E \left( \frac{S}{P} \right) \right] - \frac{\sigma^2}{2}$$

$$\sigma = \sqrt{\ln \left( \frac{1 + \left( E \left( \frac{S}{P} \right)^2 \text{Vol} \left( \frac{S}{P} \right)^2 \right)}{\left( \frac{S}{P} \right)^2} \right)}$$

The calibration of Log Normal distribution is therefore based on expectancy and volatility of the portfolio loss ratio. The expected loss ratio of the portfolio equals to the economic loss ratio. The volatility of loss ratio is calculated according to some qualitative criteria (such as the quality of technical bases, moral risk, etc.), which can for example be calibrated by a Principal Component Analysis (Felix 2009).

Given the expectancy and volatility of the loss ratio, the loss ratio matrix is therefore given by:

$$\forall i \in [1..N], \quad \frac{S}{P_i}(1) = \exp \left[ \Phi^{-1}(u_i) \cdot \sigma_i + \mu_i \right]$$

Where $\Phi^{-1}$ is the quantile function of the standard normal distribution, and $u_i$ are uniform random variables defined on $[0,1]$ for $i \in [1..N]$.

4.3 Dynamic evolution of the loss ratio to the projection horizon

After having done stochastic simulations of loss ratio at time $t=1$, it is necessary to model the projection of loss ratio from $t=2$ to the projection horizon. Felix and Planchet (2009) have described a dynamic projection (or conditional expectation), which enables us to calculate the loss ratio at time $t$ given the loss ratio at time $t=1$. This modelisation consists of a gradual return to the expectancy with an adjustable speed to the average.

Therefore, for each loss ratio $i$ drawn at time $t=1$, the loss ratio at time $t$ is given by adding to the deterministic projection a trend factor, which enables us to have a gradual return to the average. The corresponding formula is the following:

$$\forall i \in [1..N], \quad \frac{S}{P_i}(t) = E \left[ \frac{S}{P}(t) \right] + \left( \frac{S}{P_i}(1) - E \left[ \frac{S}{P}(1) \right] \right) \cdot \nu^{t-1}$$

Where $\nu$ is the speed to the average.
4.4 Conclusion

To conclude, the trend of mortality can be modelled depending on the nature of data. For individual data, different stochastic mortality models exist, such as Lee-Carter and its derivatives, CBD or P-Splines. No model emerges as obviously better, and results between models are reasonably consistent. For aggregate data, a stochastic loss ratio projection can be done in three steps: the trend in function of the nature of the product, stochastic simulations at \( t=1 \), and dynamic evolution from \( t=2 \) to the projection horizon.

5. MODELLING OF THE “PA NDEMIC SHOCK” COMPONENT

Once the trend of mortality has been determined, it is necessary to model a shock due to a pandemic event. This can be done by a Frequency x Cost approach.

5.1 Frequency

To determine the probability of occurrence of a pandemic, it is possible to rely on the history of past pandemics. The problem is that calibration is difficult:

- According to Linfoot (2007), there have been 31 pandemic influenzas since 1580, making an annual frequency of 7%.
- According to Swiss Re (2007), there have been between 10 and 13 pandemic influenzas for the past 300 years, making an annual frequency between 3% and 4%.
- According to the World Health Organisation (WHO), there have been 3 pandemic influenzas in the 20\(^{th}\) century (in 1918, 1957 and 1968), making an annual frequency of 3%.

In fact, all past pandemics did not have the same impact, and it is necessary to link frequency and severity of a pandemic. In the framework of Solvency II, the aim is to find the mortality shock that corresponds to a pandemic that occurs on average once every 200 years. The frequency is therefore \(1/200\), and the calibration process of the epidemiological model will give the corresponding mortality shock.

Mathematically, the occurrence of a pandemic is represented by a random experiment that has two possible ways: “success” if a pandemic influenza happens (with a probability equals to \( p \)), and “failure” otherwise (with a probability equals to \(1−p\)). This is a Bernouilli distribution with a parameter \( p = 1/200 \) at each time step on the projection horizon.
5.2 Cost: Implementation and calibration of a compartmental epidemiological model

5.2.1 State of art

The spread of a pandemic can be studied by non-compartmental or by compartmental models.

Non-compartmental models are for example branching processes or binomial chain models. In a branching process, like Galton-Watson (1874), people who die at each step time create a random number of descendants, regardless of the size of this population and other individuals. This model can be represented as a tree. In a binomial chain model, like Greenwood (1931) or Reed- Frost (1928), the number of newly infected individuals follows a binomial distribution. The disadvantage of those models is that they are focused on small populations.

In compartmental models, the population is divided into compartments, which correspond to a few key characteristics of the disease. For example:

- The dynamic of a cold can be represented by two compartments: individuals who are susceptible to the disease (noted S), and individuals who are infected (noted I). This model is called SI.
- The dynamic of rubella can be represented by three compartments: S, I and the compartment for individuals who have recovered and are immune, or who are dead (noted R). This model is called SIR.
- The dynamic of influenza can be represented by four compartments: S, I, R and the compartment for individuals who are in latent phase, which is the period of time during which the individual has been infected but is not yet infectious himself. This compartment is noted E for Exposed. The corresponding model is called SEIR.

Our paper focused on influenza pandemic based on an epidemiological SEIR model.

From the initial conditions (namely the number of individuals in each class), the rate of occurrence of new cases and deaths caused by disease, compartmental models determine the number of individuals in each class at each moment. Furthermore, it has been proven, in particular in Tatem et al. (2006), that transportation is a key vector in the spread of the disease because of the interconnectivity of cities. Therefore, including geographic movements of population by transportation enables us to add a spatial component to a classical compartmental model.
The SEIR original model, which includes transportation, was created by Baroyan et al. (1969). This model was not based on actual data transportation, but assumed that interactions between cities were proportional to their population size. The model was improved by Rvachev et Baroyan (1977), and tested on actual data by the National Institute of Influenza Research. Then, Rvachev and Longini (1985) have improved and implemented this model. This deterministic model was based on the 1968 influenza pandemic, and took into account air traffic between 52 cities. Grais et al. (2003, 2004) have extended that model to update population levels, incorporate more recent air travel data, and adjust seasonality parameters. Cooper et al. (2006) have developed a stochastic model of the spread of influenza based on those models. Recently, Epstein et al. (2007) have extended the model by adding travel restrictions and vaccination.

This class of models has been shown to be capable of accurately forecasting the spread of pandemic influenza. In the framework of our study, a description of this model will be given (for more details, the reader can refer to Habart and Lenca (2008) and Habart et al. (2009)), and the topic of calibration of this model, to meet Solvency II expectations, will be addressed.

5.2.2 Description of a SEIR stochastic model including transportation

The model consists of a system of difference equations in discrete time domain, describing the disease dynamics within each city and air travel by individuals from one city to another. The population in each city \( i \) on day \( t \) is partitioned into five disjoint disease states: Nonsusceptible \( NS_i(t) \), Susceptible \( S_i(t) \), Exposed \( E_i(\tau,t) \), Infectious \( I_i(\tau,t) \) and Removed states. A Removed individual is either Recovered \( R_i(t) \) or Dead \( D_i(t) \). \( E_i(\tau,t) \) and \( I_i(\tau,t) \) were infected \( \tau \) days earlier, on day \( t-\tau \). A Nonsusceptible individual has immunity to the virus through either previous exposure or vaccination. Then, the total population \( T_i(t) \) can be expressed as:

\[
T_i(t) = NS_i(t) + S_i(t) + \sum_{\tau=0}^{\tau_1} E_i(\tau,t) + \sum_{\tau=0}^{\tau_2} I_i(\tau,t) + R_i(t) + D_i(t)
\]

Where \( \tau_1 \) is the maximum length of the exposed period, and \( \tau_2 \) is the maximum length of the infected period, which equals to the exposed plus the infectious periods.

The following assumptions are made:
- Mortality due to the disease is modeled as a fraction \( d \) of the individuals
who are removed from the Infectious state.
- Within each city, individuals are assumed to be well-mixed, therefore contacts between Susceptible and Infectious persons are uniformly distributed. Let $\lambda$ be the daily infectious contact rate. Then the average number of new infections caused by one Infectious person is proportional to the number of Susceptibles and is equal to $\lambda \cdot \frac{S_i(t)}{T_i(t)}$.
- The seasonality of influenza is taken into account, because cities within the tropics have peak viral transmission year round, and for cities outside the tropics, transmissibility varies sinusoidally, with peak transmission occurring on January in the northern hemisphere and on July in the southern hemisphere. Let $sf$ be the seasonality factor.
- Concerning transportation, Infectious individuals are supposed not to travel. For the others, the number of individuals traveling is proportional to the fraction of the population in that state. Let $A_i(t)$ represents the number of individuals in city $i$ on day $t$, and let $\sigma_{ij}$ be the average daily number of travelers from city $i$ to city $j$. The probability of travel is: $pT_{ij}(t) = \frac{\sigma_{ij}}{T_i(t)}$
The change in $A_i(t)$ due to travel can be included in a transportation operator $\Omega$: $\Omega[A_i(t)] = \sum_{j=1}^{n} \left( A_j(t) \cdot pT_{ji}(t) - A_i(t) \cdot pT_{ij}(t) \right)$
- The number of random contacts between individuals follows a Poisson distribution, because contacts between pairs of individuals are independent and new contacts between $t$ and $t + \Delta t$ does not depend either on the number of previous contacts or on the time $t$.
- Random travel between cities is implemented as a series of draws from binomial distributions.
- Two strategies of intervention are implemented in order to limit the spread of the virus: travel restriction and vaccination. Concerning travel restriction, global travel is reduced by a fraction $f_r$ to and from every city, if the total number of infectious cases in the initially exposed city reaches a certain threshold. Vaccination is implemented as a transfer of a percentage $V$ of the susceptible population to the nonsusceptible population.
The following figure illustrates how the model works at time $t$, between two cities $i$ and $j$.

![Diagram of the model](image)

**Figure 9 - Operating diagram of the model at time $t$**

From a mathematical point of view, the model integrates 8 equations:

\[
\tilde{N}S_i(t+1) = NS_i(t) + \Omega[NS_i(t)] + v \cdot (S_i(t) + \Omega[S_i(t)])
\]

\[
E_i(0,t+1) = \frac{sf(I_i,t) \cdot \lambda \cdot (S_i(t) + \Omega[S_i(t)])}{T_i(t) - D_i(t) + \omega[T_i(t) - D_i(t)]} \sum_{\tau=0}^{\tau_2} I_i(\tau,t)
\]

\[
\tilde{S}_i(t+1) = S_i(t) + \Omega[S_i(t)] - E_i(0,t+1) - v \cdot (S_i(t) + \Omega[S_i(t)])
\]

\[
E_i(\tau+1,t+1) = (1 - \gamma(t)) \cdot (E_i(\tau,t) + \Omega[E_i(\tau,t)]) \quad \text{for } \tau = 0,1,\ldots,\tau_1 - 1
\]

\[
I_i(\tau+1,t+1) = \begin{cases} 
\gamma(t) \cdot (E_i(\tau,t) + \Omega[E_i(\tau,t)]) + (1 - \delta(t)) \cdot I_i(\tau,t) & \text{for } \tau = 0,1,\ldots,\tau_1 \\
(1 - \delta(t)) \cdot I_i(\tau,t) & \text{for } \tau = \tau_1 + 1, \tau_1 + 2, \ldots, \tau_2 - 1
\end{cases}
\]

\[
R_i(t+1) = R_i(t) + \Omega[R_i(t)] + (1 - d) \sum_{\tau=0}^{\tau_z} \delta(t) \cdot I_i(\tau,t)
\]
\[
D_i(t + 1) = D_i(t) + d \sum_{\tau=0}^{\tau_1} \delta(\tau) \cdot I_i(\tau, t)
\]

\[
T_i(t + 1) = NS_i(t + 1) + S_i(+1) + \sum_{\tau=0}^{\tau_1} E_i(\tau, t + 1) + \sum_{\tau=0}^{\tau_2} I_i(\tau, t + 1) + R_i(t + 1) + D_i(t + 1)
\]

Where \( \gamma(t) \) is the probability that an individual becomes Infectious on day \( \tau + 1 \), given that that person was still in the Exposed state on day \( \tau \), and \( \delta(\tau) \) is the probability that an individual is Recovered or Dead on day \( \tau + 1 \), given that that person was still in the Infectious state on day \( \tau \).

Therefore, the SEIR model gives the number of dead people in each city \( i \) at the end of the projection period of one year (\( t_{end} \)). The mortality shock is consequently the following:

\[
\text{Mortality Shock}_{i}(t_{end}) = \frac{D_{i}(t_{end})}{T_{i}(t_{end})}
\]

We assume that the mortality shock at the county level equals to the mortality shock at the level of the city that represents the country.

The following table sums up the parameters of the model and their calibration, when possible.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Default value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>Daily time step</td>
<td>Varies from 0 to 365 days</td>
</tr>
<tr>
<td>( i )</td>
<td>City</td>
<td>Varies from 1 to 155</td>
</tr>
<tr>
<td>( i^* )</td>
<td>Initial city exposed</td>
<td></td>
</tr>
<tr>
<td>( NS_i(0), S_i(0), E_i(0), I_i(0), R_i(0) )</td>
<td>Population input file (number of people in each city at t=0)</td>
<td></td>
</tr>
<tr>
<td>( \sigma_{ij} )</td>
<td>Average number of travellers per day between city ( i ) to city ( j )</td>
<td>Transport matrix</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Daily infectious contact rate</td>
<td></td>
</tr>
</tbody>
</table>
TAKING INTO ACCOUNT THE PANDEMIC RISK IN A PARTIAL INTERNAL MODEL

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Default value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$sf$</td>
<td>Saisonnality factor</td>
<td></td>
</tr>
<tr>
<td>$\tau_1$</td>
<td>Maximum day of the Exposed period</td>
<td>1</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>Maximum day of the Infectious period</td>
<td>7</td>
</tr>
<tr>
<td>$d$</td>
<td>Fraction of infected persons who die</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma(t)$</td>
<td>Probability that an individual becomes Infectious on day $\tau + 1$ given that that person was Exposed on day $\tau$</td>
<td></td>
</tr>
<tr>
<td>$\delta(t)$</td>
<td>Probability that an individual becomes Recovered or Dead on day $\tau + 1$ given that that person was Infectious on day $\tau$</td>
<td></td>
</tr>
<tr>
<td>$f_r$</td>
<td>Fraction by which to restrict travel</td>
<td>0.9</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Fraction of susceptibles to be vaccinated</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 1: Table of parameters of the model

The following figure illustrates the model by giving the evolution of the number of people in each class, in one city, on one year.
5.2.3 Calibration of the model

In order to be Solvency II consistent, the model must give a mortality shock that corresponds to a pandemic with a return period of 200 years. Nevertheless, parameters of spread of such an event are not known. Therefore, given historical parameters observed in the past, the model has to be calibrated in order to determine the percentage of overmortality (i.e. mortality over the normal mortality) due to a pandemic with a return period of 200 years. The chosen methodology includes three stages:

- Calculating mortality shocks of the three pandemics of the 20th century using the SEIR model;
- Matching a quantile to each of the three pandemics;
- Calculating the shock corresponding to a pandemic with a return period of 200 years.

5.2.3.1 Calculating mortality shocks of the three pandemics of the 20th century

The SEIR epidemiological model described above enables us to calculate overmortality due to the spread of a virus around the world during one year.

The first step is to calculate mortality shocks of the pandemics of 1918, 1957 and 1968. Assessing observed historical parameters that represent the spread of the virus is difficult, and assumptions can be volatile. For example, attack rate for the 1968 influenza pandemic has been estimated to 25% according to INVS (2005) and between 11% and 49% according to Office Fédéral de la Santé Publique (2007). Lethality rate for the 1918 influenza pandemic has been estimated between 1% to 3% according to Flahaut (2008), and to 3.6% according to Office Fédéral de la Santé Publique (2007). This variability is, on the one hand, due to the nature of influenza: virus moves among different communities with different impacts. On the other hand, the ability to measure impact in the last century was
even more limited than today (for example in 1918). The following table gives some parameters assumptions:

Given observed historical data, we consider for each parameter a minimum, mean and maximum value. Those parameters are described in the following table:

<table>
<thead>
<tr>
<th></th>
<th>1918 Influenza</th>
<th>1957 Influenza</th>
<th>1968 Influenza</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attack rate</td>
<td>[25%; 50%; 60%]</td>
<td>[25%; 50%; 60%]</td>
<td>[25%; 50%; 60%]</td>
</tr>
<tr>
<td>Lethality rate</td>
<td>[1%; 3%; 4%]</td>
<td>[0, 3%; 0, 4%, 0, 5%]</td>
<td>[0, 15%; 0, 25%; 0, 40%]</td>
</tr>
</tbody>
</table>

Table 2: Assumptions retained in this paper

Therefore, 5 scenarios are tested, as described in the following table: a central scenario, and 4 sensitivity tests that correspond to minimum and maximum values of parameters.

<table>
<thead>
<tr>
<th></th>
<th>1918 Influenza</th>
<th>1957 Influenza</th>
<th>1968 Influenza</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Central scenario</td>
<td>Attack rate</td>
<td>50,00%</td>
<td>50,00%</td>
</tr>
<tr>
<td>1. Central scenario</td>
<td>Lethality rate</td>
<td>0,4%</td>
<td>0,4%</td>
</tr>
<tr>
<td>2. Sensitivity test on attack rate (minimum)</td>
<td>Attack rate</td>
<td>25,00%</td>
<td>25,00%</td>
</tr>
<tr>
<td>2. Sensitivity test on attack rate (minimum)</td>
<td>Lethality rate</td>
<td>0,4%</td>
<td>0,4%</td>
</tr>
<tr>
<td>3. Sensitivity test on attack rate (maximum)</td>
<td>Attack rate</td>
<td>60,00%</td>
<td>60,00%</td>
</tr>
<tr>
<td>3. Sensitivity test on attack rate (maximum)</td>
<td>Lethality rate</td>
<td>0,4%</td>
<td>0,4%</td>
</tr>
<tr>
<td>4. Sensitivity test on lethality rate (minimum)</td>
<td>Attack rate</td>
<td>50,00%</td>
<td>50,00%</td>
</tr>
<tr>
<td>4. Sensitivity test on lethality rate (minimum)</td>
<td>Lethality rate</td>
<td>0,3%</td>
<td>0,3%</td>
</tr>
<tr>
<td>5. Sensitivity test on lethality rate (maximum)</td>
<td>Attack rate</td>
<td>50,00%</td>
<td>50,00%</td>
</tr>
<tr>
<td>5. Sensitivity test on lethality rate (maximum)</td>
<td>Lethality rate</td>
<td>0,3%</td>
<td>0,5%</td>
</tr>
</tbody>
</table>

Table 3: Description of the scenarios tested

The following graphs represent the mortality shocks calculated for each pandemic for the sensitivity tests on attack rate and on lethality rate. Vertical segments link minimum and maximum values, and horizontal segments represent the central scenarios. Observed data has been added to this graph: according to Patterson (1986), mortality shock for the US population was 0,53% in 1918, 0,041% in 1957 and 0,017% in 1968. According to Simonsen et al. (1998), mortality shock for the US population was also 0,53% in 1918. And according to INVS (2005), mortality shock for the French population was 0,06% in 1968.
For the Spanish influenza in 1918, the central scenario of assumptions gives a mortality shock of 0.31%. By stressing the attack rate, this mortality shock varies between 0.15% and 1%. For the pandemic of 1957, the central scenario gives a mortality shock of 0.10%, and sensitivity test on attack rate gives a range of [0.04%; 0.18%]. For the pandemic of 1968, the central scenario gives a mortality shock of 0.06%, and sensitivity test on attack rate gives a range of [0.02%; 0.11%].

By stressing the lethality rate, the shock varies between 0.09% and 0.45% for the Spanish influenza in 1918, between [0.07%; 0.12%] for the pandemic of 1957 and between [0.04%; 0.10%] for the pandemic of 1968. Those sensitivity tests show that mortality shocks given by the model are consistent with observed data. It also confirms that mortality shocks are extremely sensitive to the parameters of spread. Because those parameters are difficult to assess, mortality shocks can be volatile.
5.2.3.2 Matching a quantile to each the three pandemics of the 20th century

To match quantiles to mortality shocks of the 20th century, Guette (2010) takes into account quantiles of probability, which are defined as the number of pandemics worse or equal to the considered pandemic, divided by the total number of pandemics. For example, Linfoot (2007) assesses the annual frequency of occurrence of a pandemic to: \( \frac{31}{420} = 7.4\% \).

Besides, 1918 pandemic influenza is known as the worst case in 420 years. Therefore, the corresponding quantile to 1918 pandemic influenza is: \( \frac{1}{420} \cdot \frac{1}{7.4\%} = 3.2\% \). That corresponds to a return period of 31 pandemic events. Using the same method for the other pandemics gives the following results:

<table>
<thead>
<tr>
<th>Pandemic</th>
<th>Quantile</th>
<th>Return period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1918</td>
<td>3.20%</td>
<td>31</td>
</tr>
<tr>
<td>1957</td>
<td>27.40%</td>
<td>4</td>
</tr>
<tr>
<td>1968</td>
<td>75.80%</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4: Quantiles matching to the pandemics of the 20th century

5.2.3.3 Calculating the shock corresponding to a pandemic with a return period of 200 years

Considering the points (quantiles; mortality shocks) found above, a distribution can be adjusted and calculating the shock that corresponds to a return period of 200 years is easy (it is the quantile 0.5%).

Graphically, the following adjustment is obtained:

Figure 13 - Adjustment of a distribution to the points (quantile; mortality shocks)

Exponential adjustments give satisfactory R² coefficients, and mortality shocks

...
The results are given in the following graph:

![Graph of mortality shocks for a return period of 200 years]

**Figure 14 - Mortality shocks of each scenario for a return period of 200 years**

For the central scenario, the mortality shock is 0.26%. For comparison, the study of Doyle et al. (2005) for INVS, based on the three pandemics of the 20th century, leads to a shock of 0.35%, with no public health policy. The study of Mutualité Française (2008), with the same assumptions as INVS, gets a shock between 0.22% and 0.50%. Therefore, the shock obtained by our study, with quite similar hypothesis, seems to be consistent with other studies. Furthermore, this shock is also consistent with Consultation Paper n°49, which advocates a shock of 0.25%.

5.2.4 Discussion

Some points of the model can be improved. First, the virus only spreads by air travel, whereas it could spread by other modes of transportation. Nevertheless, it is very difficult to have exhaustive and reliable data on the other modes of transportation.

Moreover, parameter values are unique, whereas transmission from an infectious to a susceptible individual actually varies by geography, age, economic, social and cultural factors, and access to health care. For example, we use a flat mortality curve, whereas we should have used a distribution of excess deaths by age. Usually, for example in Taubenberger and Morens (2006), a “U” curve is applied to the moderate scenarios, because excess deaths are at very young ages and at ages 65-plus, and a “W” curve is applied to severe scenarios because excess deaths are at the very young ages and there is a spike at ages 20-40. Therefore, we should have included specific population characteristics in the assessment of the parameters of spread.
Nevertheless, as explained above, observed data for the past pandemics are not exhaustive (it is difficult to have this level of information), and are extremely volatile, and this can obviously lead to a model risk. This is the reason why we tested a triangular distribution (including minimum, maximum and average parameters), but our model can be improved by including a statistic distribution for each parameter of spreading.

Last, the adjustment of the distribution is made on only three points, which is too small for robust results. This is due to the fact that, hopefully, pandemics rarely occur and there have only been three pandemics in the last century. For oldest pandemics, there is again a lack of available data.

6. APPLICATION: IMPACT OF TAKING INTO ACCOUNT PANDEMIC RISK IN A PARTIAL INTERNAL MODEL

This part is dedicated to an application of the model previously developed. After a description of the portfolio, the process will be explained, and a calculation of SCR Lifemort and SCR Lifecat will be done by the standard formula and by the partial internal model in order to be compared.

6.1 Description of the portfolio

The application relates to a portfolio in aggregate data. The covered business is Creditor Insurance, in periodic premiums, for death guarantee. Contract duration ranges between 5 and 28 years, with an average of 16 years, and there are 853 contracts. The average actuarial age of the portfolio is 43. At t=0, written premiums equal to 457 K€, claims load equals to 210 K€.

6.2 Step by step process

The following diagram sums up the step by step process of the application:
6.3 SCR calculated with the standard formula

6.3.1 SCR Life mort

According to the QIS 4 Technical specifications, SCR Life mortality risk is defined by the following formula:

$$Life_{Mort} = \sum_i (\Delta \text{NAV}| \text{mortshock})$$

Where:

- $i$ denotes each policy where the payment of benefits is contingent on mortality risk;
- $\Delta \text{NAV}$ is the change in the net value of assets minus liabilities;
- $\text{mortshock}$ is a permanent 10% increase in mortality rates for each age.

In our example, calculations give the following results:

$$Life_{Mort} = \sum_i (\Delta \text{NAV}| \text{mortshock})$$

$$Life_{Mort} = 612,310 \text{ €}$$

6.3.2 SCR Life Cat

According to the QIS 4 Technical specifications, SCR Life catastrophe risk is defined by the following formula:

$$Life_{Cat} = \sum_i 0.0015 \cdot \text{Capital at Risk}$$

Where $i$ denotes each policy where the payment of benefits is contingent on either mortality or disability risk.

$\text{Capital at Risk}$ is determined as:

$$\text{Capital at Risk} = \sum_i \left( \text{SA}_i + \text{AB}_i \cdot \text{Annuity Factor} - TP_i \right)$$

Where:

- $TP_i$ : the technical provision (net of reinsurance) for each policy $i$;
- $\text{SA}_i$ : For each policy $i$ : where benefits are payable as a single lump sum, the Sum Assured (net of reinsurance) on death or disability. Otherwise,
TAKING INTO ACCOUNT THE PANDEMIC RISK IN A PARTIAL INTERNAL MODEL

zero;
- \( AB_i \): For each policy i: where benefits are not payable as a single lump sum, the Annualised amount of Benefit (net of reinsurance) payable on death or disability. Otherwise, zero;
- \textit{Annuity \_ Factor}: Average annuity factor for the expected duration over which benefits may be payable in the event of a claim.

Calculations give the following result:

\[
\text{Capital \_ at \_ Risk} = 145\ 324\ 279 \text{ \€}
\]

\[
\text{Life}_{\text{Cat}} = 217\ 986 \text{ \€}
\]

6.3.3 Aggregation by correlation matrix

According to the QIS 4 Technical specifications, the capital charge for life underwriting risk is derived by combining the capital charges for the life sub-risks using a correlation matrix as follows:

\[
SCR_{\text{Life}} = \sqrt{\sum_{r\times x} Corr_{\text{Life}}^{r\times x} \cdot Life_r \cdot Life_x}
\]

Where:
- \( SCR_{\text{Life}} \): Capital charge for life underwriting risk;
- \( Corr_{\text{Life}}^{r\times x} \): the cells of the correlation matrix \( Corr_{\text{Life}} \);
- \( Life_r, Life_x \): Capital charges for individual life underwriting sub-risks according to the rows and columns of correlation matrix \( Corr_{\text{Life}} \).

And where the correlation matrix \( Corr_{\text{Life}} \) is defined as:

<table>
<thead>
<tr>
<th>CorrLife</th>
<th>Life_{mor}</th>
<th>Life_{lon}</th>
<th>Life_{dis}</th>
<th>Life_{lapse}</th>
<th>Life_{exp}</th>
<th>Life_{rev}</th>
<th>Life_{CA}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life_{mor}</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Life_{lon}</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Life_{dis}</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Life_{lapse}</td>
<td>0</td>
<td>0.25</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Life_{exp}</td>
<td>0.25</td>
<td>0.25</td>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Life_{rev}</td>
<td>0</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
<td>0.25</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Life_{CAT}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Calculations give the following result:

\[ SCR_{Life}^{\text{Standard Formula}} = 1344178 \, \text{€} \]

Knowing that:

\[ Life_{\text{Lapse}} = 714716 \, \text{€} \]
\[ Life_{\text{Exp}} = 554505 \, \text{€} \]

And that other risks have no impact on our portfolio.

### 6.4 SCR calculated with the partial internal model

By using the internal model described in part 4.2., the stochastic projection of loss ratios is the following:

As this portfolio contains Credit insurance products, the deterministic projection of loss ratio has the shape of a bell. Loss ratio at \( t=0 \) is 60%, and then, stochastic simulations are done at \( t=1 \) using a Log Normal distribution. For \( t > 1 \), loss ratios returned to the deterministic trend.

Concerning the pandemic shock event, simulations of Bernouilli and Exponential distributions give a pandemic that occurs year 10 with a mortality shock of 2.23%. We suppose that, after this shock, the loss ratio immediately returns to its normal level. By adding the pandemic shock component, the stochastic projection of loss ratios is the following:
Given those projections of loss ratios, calculations of the capital for Life mortality risk and catastrophe risk, by using the internal model, gives the following result:

\[ \text{Life}_{\text{Mort-Cat}} = 197,686 \, \text{€} \]

Lapse and expense shocks are also applied, and the results are the following:

\[ \text{Life}_{\text{Lapse}} = 715,000 \, \text{€} \]
\[ \text{Life}_{\text{Exp}} = 555,043 \, \text{€} \]

By using correlation matrix, the SCR Life for the internal model is the following:

\[ \text{SCR}_{\text{Life}}^{\text{Internal Model}} = 1,144,594 \, \text{€} \]

### 6.5 Comparison

By comparing the two capitals obtained, we find that the SCR calculated with the internal model, \( \text{SCR}_{\text{Life}}^{\text{Internal Model}} \), is inferior by 15% to the SCR calculated with the Standard Formula, \( \text{SCR}_{\text{Life}}^{\text{Standard Formula}} \). By definition, the SCR calculated by the internal model better matches the need of capital of the company to face its real risks.

### 6.6 Sensitivity tests

In order to test the robustness of the model, two sensitivity tests have been made, respectively on the initial loss ratio, and on regulation. For each of this test, \( \text{SCR}_{\text{Life}}^{\text{Internal Model}} \) and \( \text{SCR}_{\text{Life}}^{\text{Standard Formula}} \) of central scenario and sensitivity tests are compared.
6.6.1 Sensitivity test on Loss Ratio at t=0

In the central scenario, loss ratio at t=0 has been set to 60%. The sensitivity tests consist of setting an initial loss ratio at 30% and at 120%. The same methodology as the methodology described 6.3 and 6.4 in has been applied. The results are given in the following figure.

![Sensitivity tests on S/P(0)](image)

*Figure 19 - Sensitivity tests on Loss Ratio at t=0*

Therefore, SCR given by the internal model is always inferior to SCR given by the standard formula. The gap between SCR internal model and standard formula equals to 17% if S/P(0)=60%, it equals to 20% if S/P(0)=30% and it equals to 15% if S/P(0)=120%.

6.6.2 Sensitivity test on regulation

Building an internal model is subject to potential change in the regulation. The application presented in part 6 is QIS 4 compliant, but in 2010, risk cartography and correlation matrix have been changed in the QIS 5 Draft and in the QIS 5 (final version). The following figure compares the results obtained with the methodology compliant with QIS 4, QIS 5 draft and QIS 5.
Therefore, SCR standard formula QIS 4 and QIS 5 (final version) are relatively close when the simplified approach is chosen. A difference of 7% exists and is due to the new calibration of the mortality shock (15% instead of 10%). Concerning QIS 5 results, the simplified approach is less prudent than approach with shock. SCR internal model remains below the SCR standard formula, whatever the regulation.

7. CONCLUSION

To put it in a nutshell, calibration of the mortality shock due to a pandemic event raises a controversy. Existing models give volatile results, especially because of the lack of reliable existing data. Nevertheless, this risk will actually occur, as shown by the influenza A (H1N1) virus, and it is necessary to take it into account in the calculation of SCR, as described in Solvency II Directive. Therefore, even if pandemic risk remains difficult to assess, insurance companies have to model it at best as they can.

To do so, the insurance company can use the standard formula or an internal model (partial or full). This paper gives a way to integrate pandemic risk in a partial internal model, in function of the nature of available data. Mortality risk is split into a trend and a pandemic shock, which is given by an epidemiological model. An application of the model has been made in aggregate data, and the SCR found by the partial internal model was inferior by 9% to the SCR of the standard formula, mainly because the internal model better matches the business of the company.

To go further in the study of pandemic models, it would be interesting to try to improve the robustness of such models. Therefore, model risk would be minimized.
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TAKING INTO ACCOUNT THE PANDEMIC RISK IN A PARTIAL INTERNAL MODEL


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