

# Economic Scenarios Generator in the Solvency II Framework

### With R

Version 1.0

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Economic Scenarios Generator in the Solvency II Framework



In the last ten years, we have observed a generalization of the use of "economic valuations" in various frameworks used by insurers: regulation (Solvency 2), accounting (IFRS) and financial reporting (MCEV).

This led to the use by insurers of methods originally developed for pricing financial instruments to calculate their liabilities. On this occasion, many challenges have emerged, particularly in life insurance:

- long duration of life insurance liabilities;
- no market;
- partially endogenous risk factors;
- volatility of the value which does not reflect the risks carried.

The aim of this presentation is to present the specific elements induced by the use of "economic valuations" in the insurance business and the consequences for construction of a Economic Scenarios generator (ESG).



In order to implement the "standard model" of Solvency II, we need to compute the "economic balance sheet":

- get the prices of different assets at the date of calculation;
- calculate the value ("price") of liabilities;

To compute the Solvency Capital Requirement (SCR) with the standard formula, we have to recalculate the change in price of assets and liabilities at date 0 when a shock is applied.

For basic assets (equities, sovereign and corporate bonds mainly), prices comes directly form the market. For derivative assets (convertible bonds, options, etc.), we should strictly speaking have closed formulas to recalculate prices depending on the level of underlying risk factors (equity, interest rate, credit, liquidity).

Economic Scenario Generator (ESG) provides the answer to the second item.



 $\Lambda_{x} = \sum_{t=1}^{\infty} \frac{1}{\left(1+i\right)^{t}} I_{]t;\infty[}\left(T_{x}\right)$ 

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## Background and Goals of the Workshop



Computing the economic balance sheet et time t=0 fulfills the requirements of Pilar 1 of Solvency II. To meet the requirements of Pilar 2, one must also be able to project this balance sheet in the future.

An ALM model in life insurance should then be able :

- to compute prices (assets and liabilities);

- to compute quantiles of the distribution of the net asset value, that is prices distributions.

The first item uses risk neutral measure, the second one uses historical measure.

As part of a comprehensive modeling which aims to provide distributions of economic value, we use a two-tiered approach:

- The construction of a functional *g* providing the vector of prices based on state variables Y to the calculation date,  $\pi_0 = g(Y_0)$ 

- Building a dynamic model for risk factors, $Y_t$ 

We can then determine prices at any time using:

The construction of the functional *g* is based on the classical assumptions of financial markets including the "no arbitrage assumption", which leads to construct "risk neutral" probability that make the discounted prices processes martingales.

 $\pi_{t} = g\left(Y_{t}\right)$ 

The construction of the dynamics of Y is a problem of econometrics.



For example, with the classical Vasicek model, the following dynamics are used for the unique risk factor (the short rate

 $dr_t = a(b_1 - r_t)dt + \sigma dW_t^Q$  $dY_{t} = dr_{t} = a(b - r_{t})dt + \sigma dW_{t}$  $b_{\lambda} = b - \frac{\lambda \sigma}{\sigma} \qquad W_t^Q = W_t + \lambda \times t$ 

Quantiles

and the pricing function is:

with  $r_{\infty} = b_{\lambda} - \frac{\sigma^2}{2}$ 

We observe here that the link between the two representations is made via the barameter 
$$\lambda$$
.

 $g(r_{t}) = P(r_{t}, T-t) = \exp\left(\frac{1-e^{-a(T-t)}}{a}(r_{\infty}-r_{t})-(T-t)r_{\infty}-\frac{\sigma^{2}}{4a^{3}}(1-e^{-a(T-t)})^{2}\right)$ 

Note: The parameter  $\sigma$  is theoretically invariant.



Pricing





What is a Economic Scenarios Generator (ESG)?

An ESG is a tool to project economic and financial risk factors.

It should be noted that the link between risk factors and asset prices may be more or less direct:

- For equities and real estate, the risk factor is the price of the asset ;

- For bonds we use usually a limited number of explanatory factors, typically the short rate. So we need a model to calculate the price of the asset from a description of the risk factor. The asset is a derivative.

How to project risk factors depends on the use that will be made of the ESG and lead to define the concepts historical probability (for distribution purposes) and risk neutral probability (for pricing purposes).





The modeled risk factors typically include:

- Stock price;
- Short-term rate and riskless zero-coupon bonds;
- Default process bond counterparties;
- Process of liquidity;
- Real estate prices;
- Inflation.

Other economic factors may be taken into account, such as unemployment level. Modeling choices are tailored to the objective: price calculation or analysis of the price distribution.



The main objective of this presentation is to show that the R software is well suited to the construction of a ESG for insurance business.

It can cover all needs, price calculations, projections of risk factors and calibration of the models (particularly complex task).

R is much more relevant than existing commercial solutions on the market because, by construction, everything that is built is documented and open.

We will in particular present two packages

- ESG (<u>http://cran.r-project.org/web/packages/ESG/index.html</u>)
- ESGtoolkit (<u>http://cran.r-project.org/web/packages/ESGtoolkit/index.html</u>)

We will now focus on stock and riskless bonds prices.

R codes used for the presentation are available from

http://www.ressources-actuarielles.net/C1256F13006585B2/0/A5E99E9ABF5D3674C125772F00600F6C







#### **Illustration #1 : convergence**

We consider an unit linked contract with an underlying asset modeled by a lognormal process and the risk free rate modeled by a Vasicek model, so that:

$$S(t) = S_0 \exp\left(\int_0^t \left(r(u) - \frac{\sigma^2}{2}\right) du + \sigma B(t)\right) \qquad dr(t) = k\left(\theta - r(t)\right) dt + \sigma_r dB_r(t)$$

The contract duration is 10 years, fully redeemed at the end. In the meantime, the structural ratchet rate is 2% and a cyclical ratchet rate equals 5% is added when the value of the share at time *t* is lower than the initial value.

The best estimate value of the contract is simply the initial value of the share.

### **1. Computing best estimates**





#### Illustration #1 : convergence

The convergence of the empirical best estimate to its theoretical value is slow and, after 1000 runs, a difference of about 1.5% remains. This difference leads to a difference of about 15% on own funds...

Dividing this difference by 10 multiply the number of simulations by 100.

Therefore it may be useful (essential) to optimize this scheme (see e.g. Nteukam and Planchet [2012]).

4. 1.3 Ņ Ecart :-1.0 0.0 0.8 0 200 600 1000 400 800

Nombre de simulations

Note: cf. Bauer et al. [2010] on the issue of budgets simulation.

#### Ecart de convergence en fonction du nombre de simulations





#### **Illustration #1 : convergence**

We can also remember that calculating the price of a single ZC by simulation is not simple.

Convergence for this type of asset is slower than for more complex cash-flows such as those of a savings contract.







#### **Illustration #2 : bias**

Euler discretization process of the short rate for, for example, H&W model leads to:  $\sigma^2 (c_1 + c_2)^2 = \sqrt{1 - (c_1 + c_2)^2} = \sqrt{1 - (c_1 + c_2)^2}$ 

$$r_{\delta}(n\delta) = f(0,n\delta) + \frac{\sigma^{2}}{2k^{2}}(1 - e^{-kn\delta})^{2} + \sigma\sqrt{\delta}e^{-kn\delta}\sum_{i=1}^{n}e^{+k(i-1)\delta}\varepsilon$$

This discretization does not introduce bias for the mean, but:

$$V\left(r_{\delta}\left(n\delta\right)\right) = \frac{\sigma^{2}}{2k}\left(1 - e^{-2kn\delta}\right) \times \frac{2 \times k \times \delta}{e^{2k\delta} - 1} = V\left(r\left(n\delta\right)\right) \times \frac{2 \times k \times \delta}{e^{2k\delta} - 1}$$

Constant  $c_{\delta} = \frac{2 \times k \times \delta}{e^{2k\delta} - 1}$  tends to 1 when the discretization step tends to 0, but can lead to poor results with a monthly or annual discretization.

It is then necessary to reduce the bias (directly, with a Milstein's discretization, or an exact one when available, *etc.*).





#### **Illustration #2 : bias**

For example, we consider the approximation of the price of a ZC bond by simulation. With a sample size of 10,000 we have:



The bias is material, even with a huge sample size.

### 1. Computing best estimates





#### **Illustration #2 : bias**

One can see the same phenomenon with more complex processes, e.g. the calculation of a "real" best estimate.

We must distinguish the cashflows projection step (usually annual) and the discretization used to approximate the discount factors (see Ifergan [2013]).



In practice in the projection models used to calculate a best estimate, we can consider that the sampling error is of the order of 0.50%.



- 2. R Package: ESG
- 3. From Pilar 1 to Pilar 2
- 4. R Packages: ESG and ESGtoolkit







#### http://cran.r-project.org/web/packages/ESG/index.html

ESG package was designed to provide a model as simple as possible that meets the basic constraints imposed by Solvency 2 for best estimates calculations.

Given its importance in asset allocation, it is a priori risks associated with fixed income products that require the most attention. So, the package provides basically, under risk neutral measure:

- B&S model for equity and real estate (cf. Planchet et al. [2009]);
- H&W model for risk free ZC bonds (*cf.* Hull and White [1990]);
- LMN model for corporate bonds (cf. Longstaff et al. [2005]).

The content of this package is detailed there :

Economic Scenarios Generation for Insurance: ESG package and other tools (Pt. 1 : ESG) Rmetrics Workshop on B in Finance and Insurance 2014, Paris

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June 27, 2014



- 2. R Package: ESG
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- 4. R Package: ESGtoolkit







The SCR calculation is based on the projection of one-year balance sheet (see Guibert et *al.* [2010].)



The projection of risk factors is performed with the historical measure and the SCR is the solution of the (implicit) equation  $P(E_1 < 0 | E_0 = x) \le 0.5\%$ 

often approached in practice by

$$SCR \approx E_0 - VaR_{0,5\%} \left( \delta(1) \times E_1 \right) \approx E_0 - P(0,1) \times VaR_{0,5\%} \left( E_1 \right)$$





We show the associated calculation structure with the very simple example of a GMDB guarantee on an unit-linked saving contracts (this is the most simple example of a "variable annuity" contract).

In case of death at time *t*, the insurer guarantees the repayment of:

$$G_t = S_t + \left[K - S_t\right]^+$$

The present value of the cash flow associated with this guarantee is written

$$\Lambda_x = e^{-rT_x} \left[ K - S_{T_x} \right]^+ \mathbb{1}_{T_x \le T}$$

Thus, the value of the cash-flow is

$$V = \mathbf{E}^{P^{a} \otimes Q^{F}} \left[ e^{-rT_{x}} \left[ K - S_{T_{x}} \right]^{+} \mathbb{1}_{T_{x} \leq T} \right] = \sum_{t=1}^{T} {}_{t-1} p_{x} \times q_{x+t-1} \times \mathbf{E}^{Q^{F}} \left[ e^{-rt} \left[ K - S_{t} \right]^{+} \right]$$
$$= \sum_{t=1}^{T} {}_{t-1} p_{x} \times q_{x+t-1} \times \left[ Ke^{-rt} N \left( -d_{2} \left( t \right) \right) - S_{0} N \left( -d_{1} \left( t \right) \right) \right]$$





Wen building the hedging portfolio, two factors mechanically generate imperfections:

- imperfect pooling of death;
- the discrete time readjustment of the position.

We will focus on discretization error. By introducing transaction costs proportional to the volumes traded, it is possible to assess the cost of these imperfections:

- on one hand the cost of coverage error due to the discrete nature of the reallocations;

- other costs associated transactions.

We start from the following decomposition of the value of the guarantee, for an insured aged x:

$$V = \sum_{t=1}^{T} p_{x} \times q_{x+t-1} \times E^{Q^{F}} \left[ e^{-rt} \left[ K - S_{t} \right]^{+} \right]$$





The value of the arbitrage portfolio at the origin is written with obvious notations (for one put option):

$$W = -P(S_0, T, K, r, \sigma) + \alpha_0 S_0 + \beta_0$$

The immunization strategy against changes in the underlying asset (delta hedging) added to the fact that the initial value of the portfolio is zero leads to:

$$\alpha_{i}(T) = -N\left(-d_{1}(S_{i}, T - i\delta_{N})\right) \qquad \beta_{i}(T) = P\left(S_{i\delta_{N}}, T - i\delta_{N}, K, r, \sigma\right) - \alpha_{i}(T)S_{i\delta_{N}}$$

The associated costs are :

- transaction costs : 
$$c_i = \gamma \times \left( \left| \alpha_i - \alpha_{i-1} \right| S_{i\delta_N} + \left| \beta_i - \beta_{i-1} e^{r\delta_N} \right| \right)$$
  
- hedging error :  $w_i^- = -P\left( S_{i\delta_N}, T\left(1 - \frac{i}{N}\right), K, r, \sigma \right) + \alpha_{i-1}(T) S_{i\delta_N} + \beta_{i-1}(T) \exp(r\delta_N)$ 





The contract involves several interrelated options. The amount of risky asset held at time t to cover the options until the maturity T is equal to (under the assumption that the mortality risk is perfectly pooled):

$$a_{t} = \sum_{j \in J} \sum_{k=t\delta_{N}^{-1}+1}^{N-1} \alpha_{t} (k\delta_{N})_{(k-1)\delta_{N}} p_{x(j)} q_{x(j)+(k-1)\delta_{N}}$$

The cash-flows are projected using monte-carlo simulation under the physical measure and exact calculation under risk neutral measure:

$$S_t^i = S_{t-\delta_N}^i \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)\delta_N + \sigma\sqrt{\delta_N}\varepsilon\right\}$$

 $\mathcal{E} =$ 

$$\sum_{n=0}^{N-1} e^{-rn\delta_N} \left( k \times \left( \left| a_{n\delta_N} - a_{n\delta_N - \delta_N} \right| S_{n\delta_N \delta_N} + \left| b_{n\delta_N} - b_{n\delta_N - \delta_N} e^{r\delta_N} \right| \right) + S_{n\delta_N \delta_N} \left( a_{n\delta_N} - a_{n\delta_N - \delta_N} \right) + \left( b_{n\delta_N} - b_{n\delta_N - \delta_N} e^{r\delta_N} \right) \right)$$





The present value of costs involved by these imperfections is

$$\varepsilon = \sum_{i=1}^{N} \left( c_i + w_i^{-} \right) \exp\left( -ri\delta_N \right)$$

We need information about the distribution if this global additional cost.

The illustration is built based on the following assumptions:

- A risk free rate of 3%;
- A contract period of 10 years;
- Volatility of the assets of 16%;
- Transaction fee of 0.1% of transaction volume.

It assumes that the reallocations are performed on a weekly basis, which leads to the empirical distribution of the costs.





We find, for weekly reallocations, a value of the 99.5% quantile equals 2.1%.

The average cost of the strategy of 0.40%.

Passing daily adjustments, the quantile equals 0.4% and the average cost 0.1%.

The shape of the distribution is as follows:





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#### http://cran.r-project.org/web/packages/ESGtoolkit/index.html

As we saw in the first part of this presentation, the ESG is an essential component of the calculations to be done to meet the different demands of the regulator in the context of Solvency 2 and an insurer needs not only one ESG but (at least) two:

- one built under the "risk neutral framework";
- the other built with historical probabilities.

The "risk neutral" package ESG addresses the basic step of best estimate calculations for saving contracts. The use of second involves more complex models. ESGtoolkit package has been design for this "advanced use" of ESGs.

The content of this package is detailed there :

Economic Scenarios Generation for Insurance: ESG package and other tools (Pt. II : ESGtoolkit) Rmetrics Workshop on R in Finance and Insurance 2014, Paris

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R software provides flexible tools for building ESG adapted to Solvency II calculations :

- under a risk neutral measure (reserves);
- under the physical measure (economic capital and ORSA).

Further work will be dedicated to a deeper integration between the 2 packages ESG and ESGtoolkit.





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